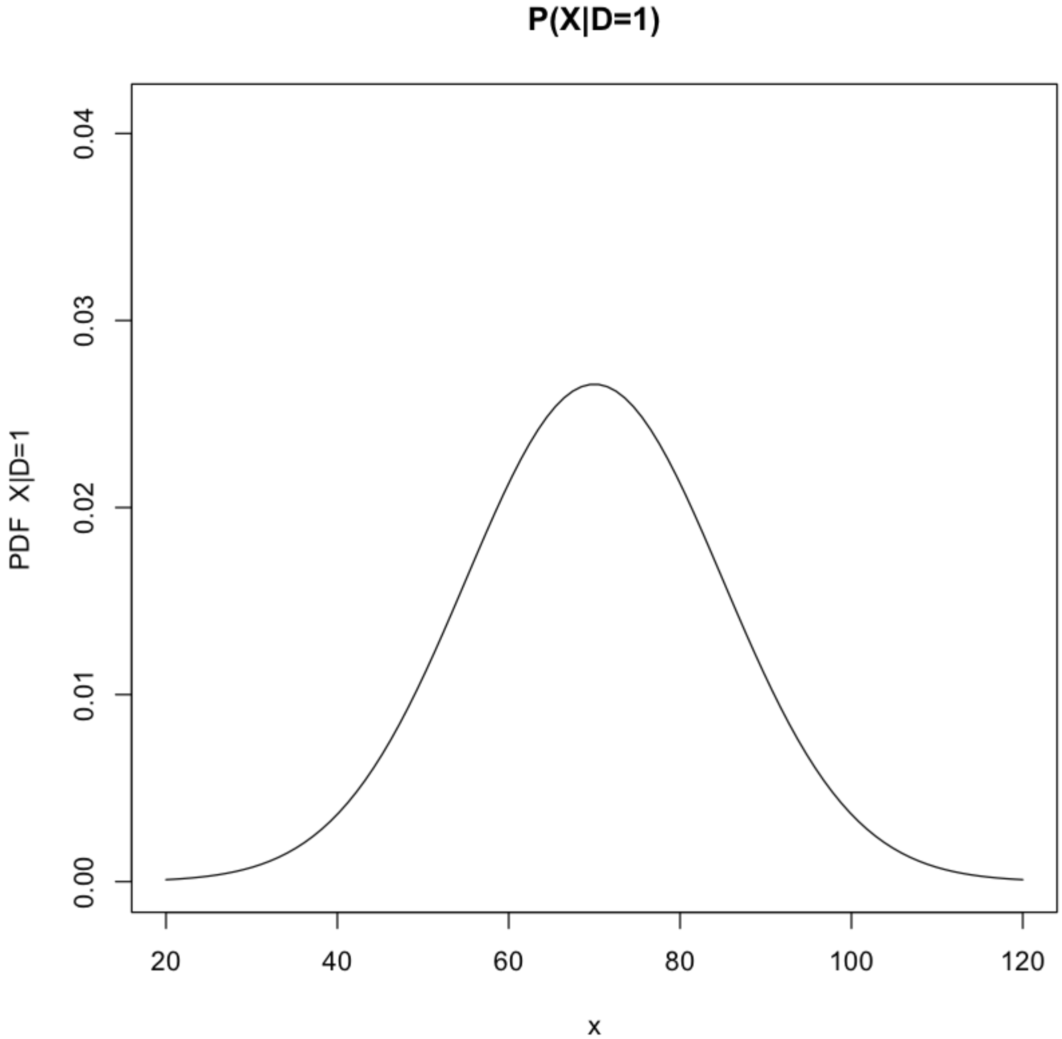
hw6

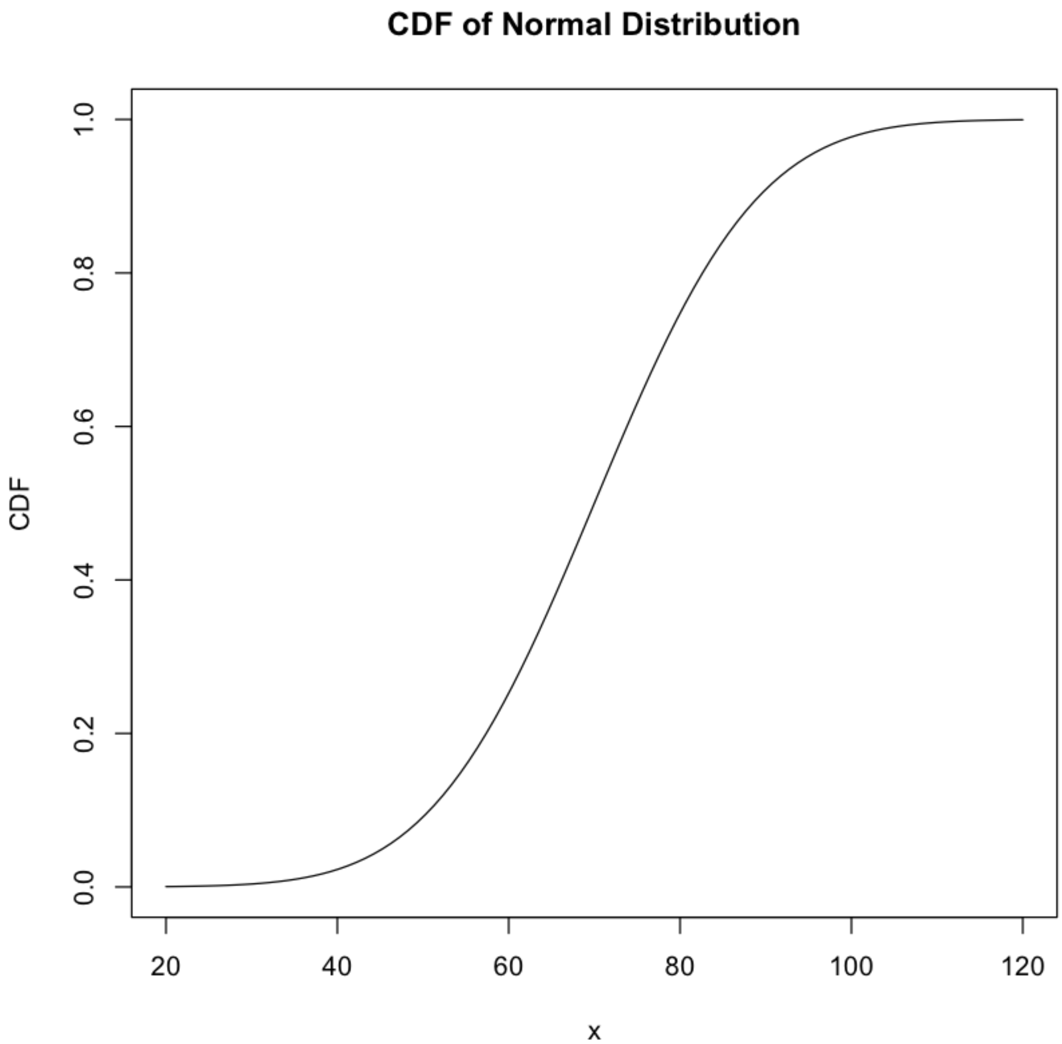
May 31, 2024

1

1.1

[39]:
```
x = seq(20, 120, length = 100)
hx = dnorm(x, 70, 15)
plot(x, hx, xlim = c(20,120), ylim = c(0, 0.041), type='l', ylab="PDF  X|D=1",␣
 ↪xlab="x", main="P(X|D=1)")
mean=70
sd =15
y = pnorm(x,70,15)  # Calculate the CDF of x
plot(x,y, type="l", main = "CDF of Normal Distribution", ylab = "CDF")
```


P(X|D=1)
PDF X|D=1
0.00
0.01
0.02
0.03
0.04
20
40
60
80
100
120
x

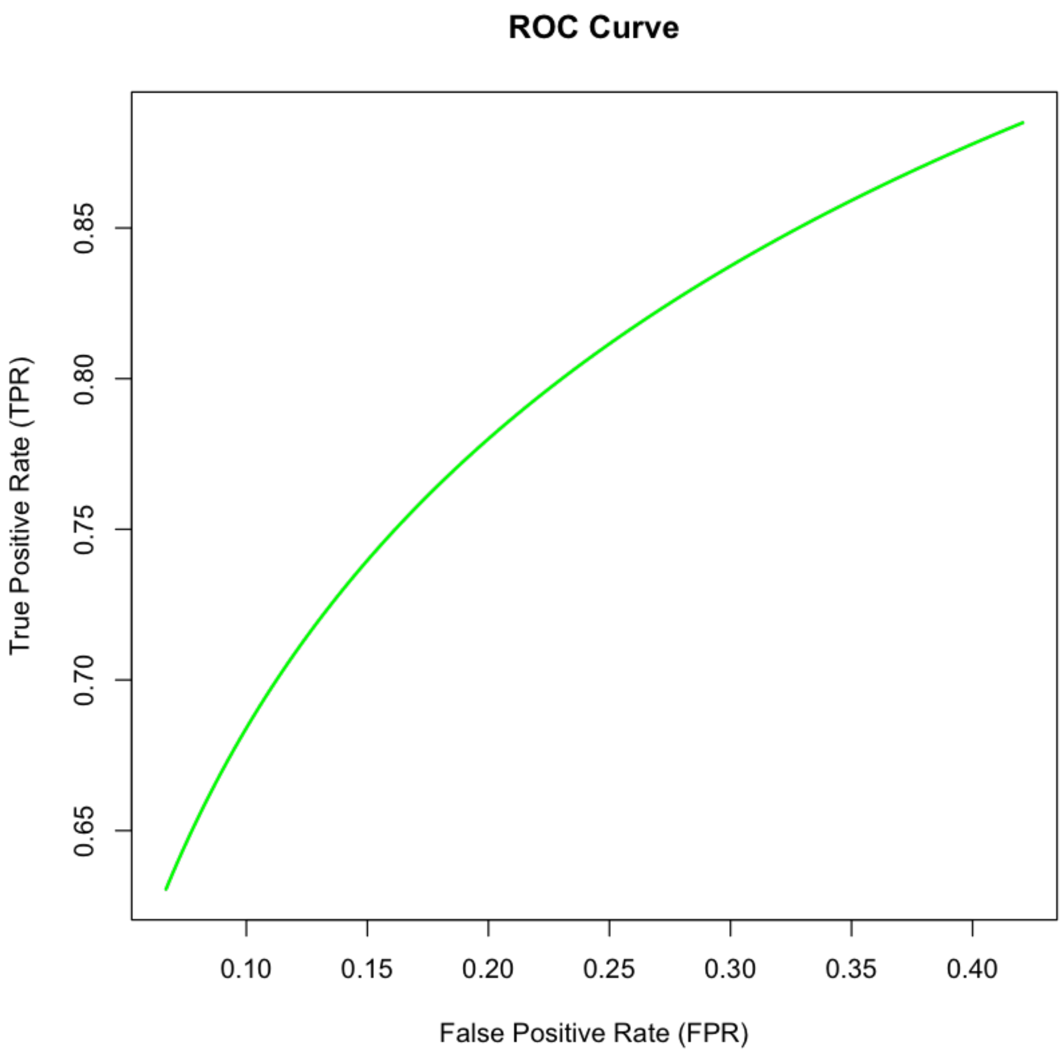


1.2

[40]:
```
mean2 <- 50
sd2 <- 10
calculate_tpr_fpr <- function(x_star) {
  tpr <- 1 - pnorm(x_star, mean = mean, sd = sd)
  fpr <- 1 - pnorm(x_star, mean = mean2, sd = sd2)
  return(c(tpr, fpr))
}

x_star_vals <- seq(52, 65, by = 0.1)
roc_data <- t(sapply(x_star_vals, calculate_tpr_fpr))
```

```
plot(roc_data[,2], roc_data[,1], type = "l", col = "green", lwd = 2,
     main = "ROC Curve", xlab = "False Positive Rate (FPR)", ylab = "True␣
 ↪Positive Rate (TPR)")
```



1.3

We want to let fpr p fnr. So, pnorm(x_star, mean = 70, sd = 15) = 1 - pnorm(x_star, mean = 50, sd = 10)

$$\frac{x^* - 70}{15} = 1 - \frac{x^* - 50}{10} \quad (1)$$

$$x^* = 58 \quad (2)$$

2

2.1

$$specificity = \frac{TN}{TN + FP} = \frac{100}{100 + 10} = 0.9090 \tag{3}$$

2.2

$$sensitivity = \frac{TP}{TP + FN} = \frac{15}{15 + 10} = 0.6 \tag{4}$$

2.3

$$prevalenceofthedisease = \frac{TP + NP}{TN + FN + TP + NP} = \frac{15 + 10}{15 + 10 + 100 + 10} = 0.185 \tag{5}$$

3

3.1

$$P = \begin{bmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{bmatrix} = \begin{bmatrix} 1-\alpha & \alpha \\ \beta & 1-\beta \end{bmatrix}. \tag{6}$$

3.2

$$\begin{aligned} \pi P &= (0.8, 0.2) \begin{bmatrix} 1-\alpha & \alpha \\ \beta & 1-\beta \end{bmatrix} \\ &= [(1-\alpha)0.8 + \beta 0.2, \alpha 0.8 + (1-\beta)0.2] \\ &= (0.8, 0.2) \end{aligned}$$

3.3

$$\pi P^5 = (0.8, 0.2)(\frac{1}{\alpha + \beta} \begin{bmatrix} \beta & \alpha \\ \beta & \alpha \end{bmatrix} + \frac{(1-\alpha-\beta)^5}{\alpha+\beta} \begin{bmatrix} \alpha & -\alpha \\ -\beta & \beta \end{bmatrix})$$

3.4

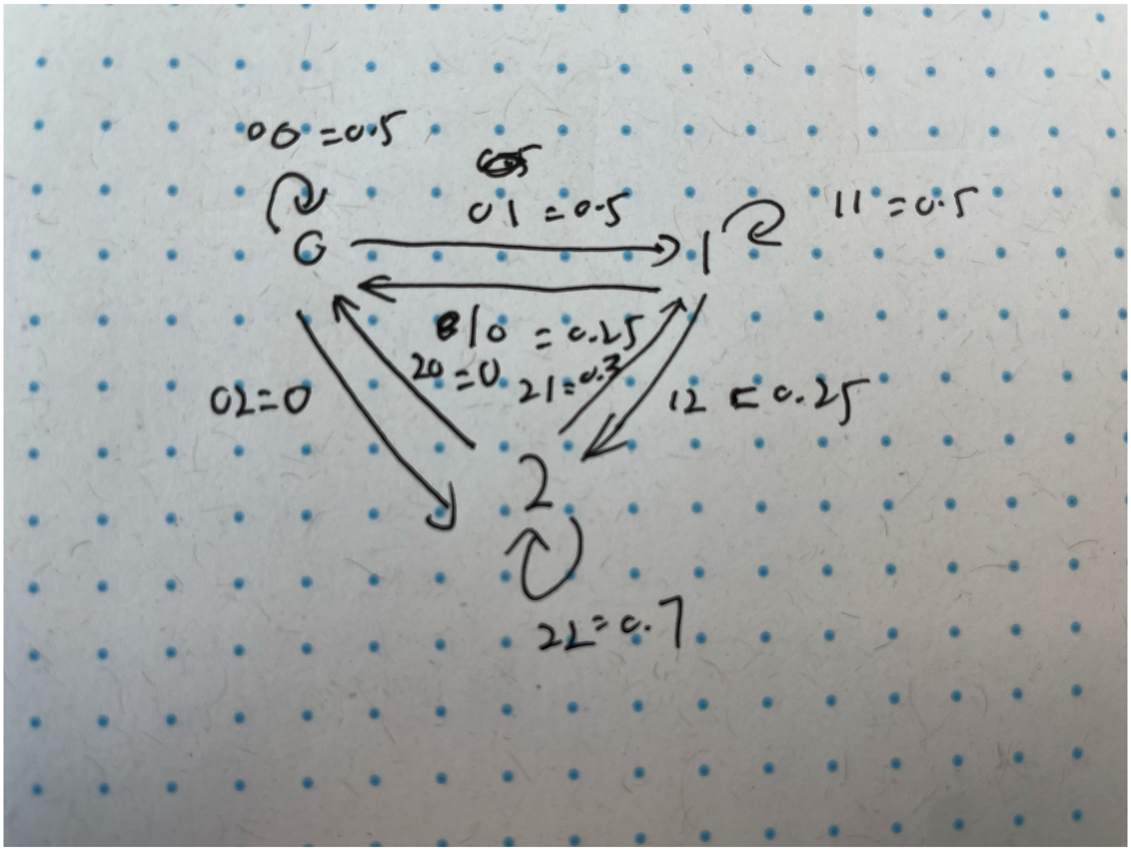
$$\lambda_0 P^\infty = [0.8, 0.2] \left(\frac{1}{\alpha + \beta} \begin{bmatrix} \beta & \alpha \\ \beta & \alpha \end{bmatrix} \right) = \left[\frac{\beta}{\alpha + \beta}, \frac{\alpha}{\alpha + \beta} \right]$$

4

4.1

$$p = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.25 & 0.5 & 0.25 \\ 0 & 0.3 & 0.7 \end{bmatrix}$$

4.2



4.3

```
[41]: m = matrix(0, nrow=3,ncol=3)  #define a vector
m[1,] = c(0.5,0.5,0)        #specify the row entries
m[2,] = c(0.25,0.5,0.25)
m[3,] = c(0,0.3,0.7)
print(m)

e = eigen(t(m))            #solve for the eigenvalues and eigenvectors of the transpose matrix
print(e)                   #Note, the leading eigenvalue is 1, and all the rest are smaller

pi = e$vectors[,1]/sum(e$vectors[,1])  #Extract the corresponding eigenvector and normalize it
print(pi)
```

```
     [,1] [,2] [,3]
[1,] 0.50  0.5 0.00
[2,] 0.25  0.5 0.25
[3,] 0.00  0.3 0.70
eigen() decomposition
```

```
$values
[1] 1.00000000 0.61925824 0.08074176

$vectors
          [,1]       [,2]       [,3]
[1,] 0.3585686  0.5415949  0.4838878
[2,] 0.7171372  0.2583586 -0.8114959
[3,] 0.5976143 -0.7999536  0.3276080

[1] 0.2142857 0.4285714 0.3571429
```

$$P_1 = 0.4285 \tag{7}$$