hw3

April 26, 2024

1

Let:
Event A = the probability of spam email
Event B = the probability of email that contain the phrase “Large inheritance”

$$\begin{aligned} P(A|B) &= \frac{P(A) \times P(B|A)}{P(B)} \\ &= \frac{80\% \times 10\%}{80\% \times 10\% + 20\% \times 1\%} = 97.6\% \end{aligned}$$

2

A = transmit 1 B = transmit 0 C = receive 1 D = receive 0

2.1

$$P(C) = P(C|A) + P(C|B) = 55\% \times 91\% + 45\% \times 6\% = 52.75\%$$

2.2

$$P(D) = P(D|A) + P(D|B) = 55\% \times 9\% + 45\% \times 94\% = 47.25\%$$

2.3

$$\begin{aligned} P(A|C) &= \frac{P(A) \times P(C|A)}{P(C)} \\ &= \frac{55\% \times 91\%}{52.75\%} = 94.882\% \end{aligned}$$

2.4

$$\begin{aligned} P(B|D) &= \frac{P(B) \times P(D|B)}{P(D)} \\ &= \frac{45\% \times 94\%}{47.25\%} = 89.52\% \end{aligned}$$

2.5

$$P(error) = P(D|A) + P(C|B) = 1 - P(C|A) - P(D|B) = 7.65\%$$

3

3.1

$$P(X = 1) = (\frac{1}{2})^6 = \frac{1}{64}$$

3.2

$$X \sim BernoulliDistribution(\frac{1}{64})$$

3.3

$$E(X) = \sum_i p_i x_i = 1 \cdot p + 0 \cdot (1 - p) = 1 \cdot \frac{1}{64} + 0 \cdot \frac{63}{64} = \frac{1}{64}$$

3.4

$$Var(X) = p(1 - p) = \frac{1}{64} \times \frac{63}{64} = \frac{63}{4096}$$

4

4.1

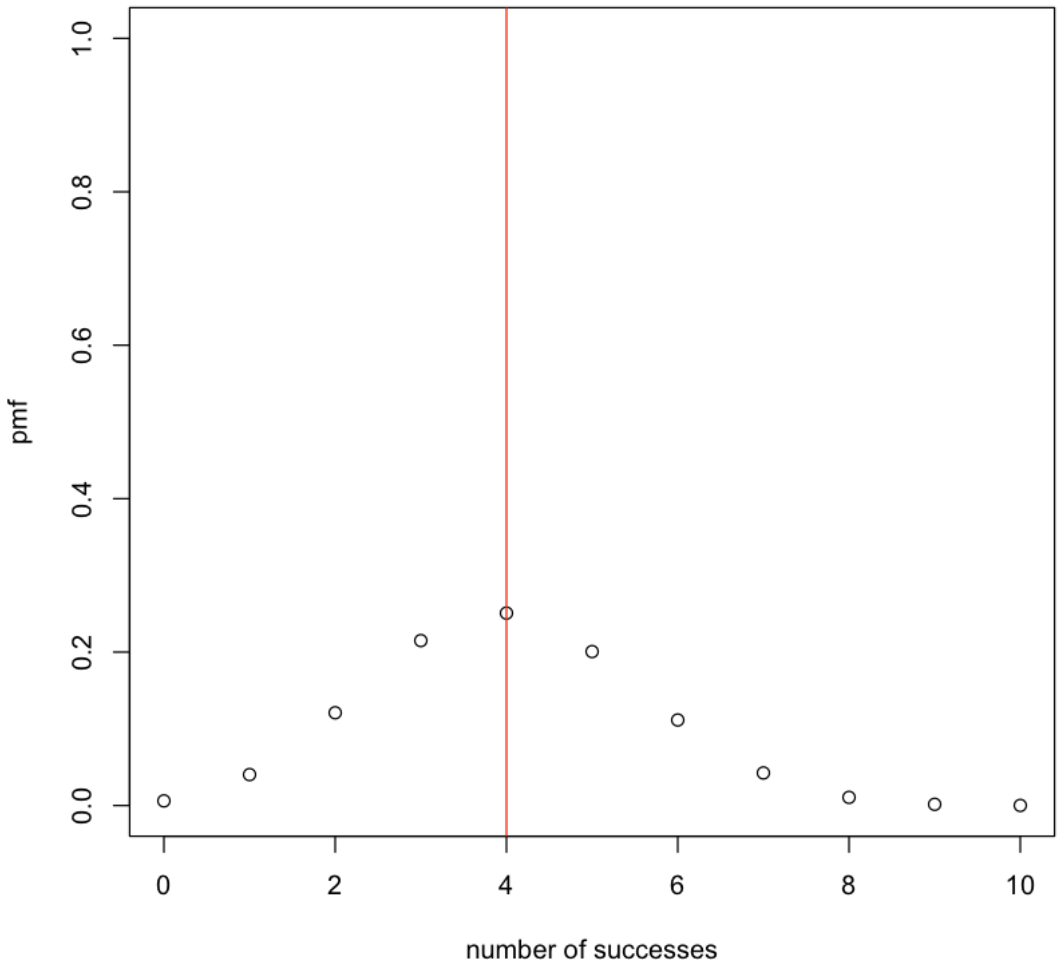
$$P(X = k) = \binom{10}{k} 0.4^k (1 - 0.4)^{10-k} \qquad 0 \leq k \leq 10 \tag{1}$$

4.2

$$X \sim Binom(10, 0.4) \tag{2}$$

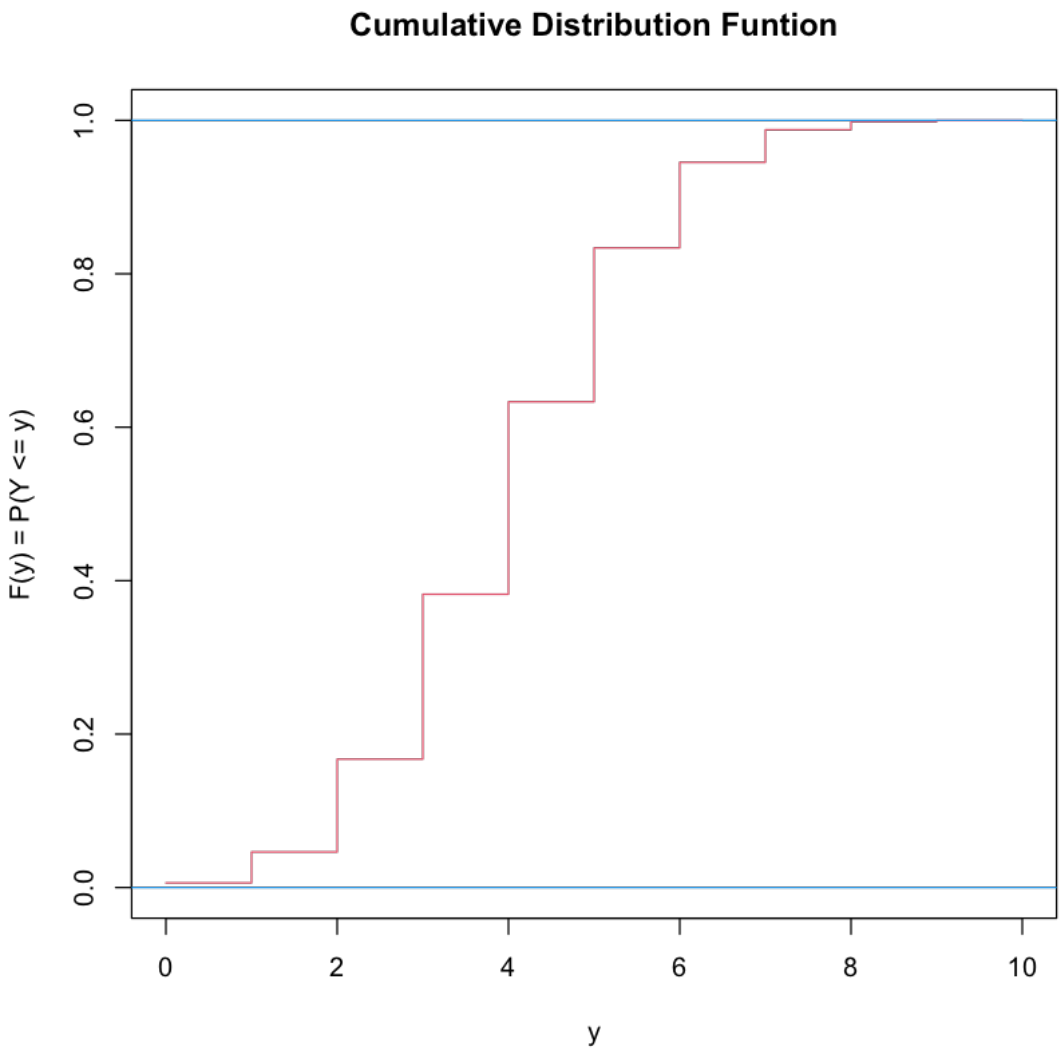
4.3

[44]:
```
n = 10
p = 0.4
plot(x=0:n, dbinom(0:n, n, p), ylim=c(0,1.0),ylab="pmf", xlab="number of␣
  ↪successes") # note dbinom R function
mean = n*p
abline(v=mean, col="red")
```



4.4

[45]:
```
##
pmf = dbinom(0:10, 10,0.4)
yvalues = c(0,1,2,3,4,5,6,7,8,9,10)
cdf = cumsum(pmf)    # note the use of the R function "cumsum"
#### Writing a general PMF ####
plot(yvalues, cdf, type = 's', ylim = c(0,1), ylab="F(y) = P(Y <=␣
↪y)",col=2,xlab="y", main="Cumulative Distribution Funtion");abline(h=0:
↪1,col=4)
```



4.5

[46]:
```
n = 10
p = 0.4
x = 7
prob = 1 - pbinom(x, n, p)
cat("P (X   8) = ",prob)
```

```
P (X   8) =  0.01229455
```

4.6

[47]:
```
n = 10
p = 0.4
prob = pbinom(8, n, p)- pbinom(2, n, p)
cat("P (3   X   8) = ",prob)
```

```
P (3   X   8) =  0.8310325
```

5

5.1

$$P(X = k + 1) = (1 - p)^k p$$

5.2

$$E(Y) = \frac{q}{p}$$

5.3

$$E(X) = \frac{1}{p}$$

[ ]:
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