

# hw5

May 31, 2024

## 1

### 1.1

$$P(I) = (1 - 0.4)^6 = 0.046656 \quad (1)$$

$$P(S) = \binom{6}{1} \times 0.4 \times 0.6^5 = 0.187 \quad (2)$$

$$P(C) = 1 - P(I) - P(S) = 0.766344 \quad (3)$$

$$P(C|E) = 1 - P(I|E) - P(S|E) = 1 - 0.2 - 0.3 = 0.5 \quad (4)$$

$$P(E|C) = \frac{P(E) \times P(E|C)}{P(E)} \quad (5)$$

$$= \frac{0.5 \times 0.5}{0.766344} = 0.326 \quad (6)$$

## 2

### 2.1

$$f_x = \int_x^{10} \frac{10}{x^2} dx \quad (7)$$

$$= -10x^{-1} \Big|_{10}^x \quad (8)$$

$$= -10x^{-1} + 10 \times 10^{-1} \quad (9)$$

$$= -10x^{-1} + 1 \quad (10)$$

### 2.2

$$P(X \leq 15) = -10 \times 15^{-1} + 1 = 0.3333 \quad (11)$$

### 2.3

$$P = \frac{1^3}{3} \times \frac{2}{3} = \frac{2}{81} \quad (12)$$

### 3

$$P(T > 8) = = P(\min(X_1, X_2) > 8) \quad (13)$$

$$= P(X_1 > 8 \cap X_2 > 8) \quad (14)$$

$$= \underbrace{P(X_1 > 8) \times P(X_2 > 8)}_{\text{due to independence}} \quad (15)$$

$$= e^{-\frac{1}{10} \times 8} e^{-\frac{1}{5} \times 8} \quad (16)$$

$$= e^{-\frac{12}{5}} \quad (17)$$

### 4

#### 4.1

$$Z = \frac{72 - \mu}{\sigma} = \frac{72 - 71}{2.5} = 0.4 \quad (18)$$

$$\Phi(0.4) = 0.6554 \quad (19)$$

$$P(x > 72) = 1 - 0.6554 = 0.3446 \quad (20)$$

#### 4.2

$$\Phi\left(\frac{m - \mu}{\sigma}\right) = 0.883 \quad (21)$$

$$\frac{m - \mu}{\sigma} = 1.19 \quad (22)$$

$$m = 1.19 \times \sigma + \mu \quad (23)$$

$$m = 73.975 \quad (24)$$

### 5

#### 5.1

$$\eta = P(F = 1|L = 1) = 1 - P(F = 0|L = 1) = 1 - 0.1 = 0.9 \quad (25)$$

#### 5.2

$$\theta = P(F = 0|L = 0) = 1 - P(F = 1|L = 0) = 1 - 0.2 = 0.8 \quad (26)$$