hw2

April 26, 2024

1

sicnce they are equally like to born with green or brown eys. So, the probability that one kid was born with green eye is

$$P(green) = \frac{1}{2}$$

and same for the brown eye

$$P(brown) = \frac{1}{2}$$

1.1

let's define two events.
Event A: the first kid has green eyes. Event B: the second kid hsa green eyes. Since these two events are independent, so the probability that both of them have green eyes is the mutilply of the probability that one kid has green eyes.

$$P(A \cap B) = \frac{1}{2} * \frac{1}{2} = \frac{1}{4}$$

1.2

using the same two events as in the part a). Now, given by the condition that the first kid is green eyes, find the probability that both two kids have green eyes. This is P(B|A).

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$$

2

2.1) So, since we discarding the keys that dont work, we dont use the Geometric Distribution for this question. First, we find the space (X)for the k^{th} to open the door. That is $X = (n-1)!$. The sapce of S is n!

$$P(k^{th}) = \frac{x}{S} = \frac{(n-1)!}{n!} = \frac{(n-1)!}{n(n-1)!} = \frac{1}{n}$$

2.2) For this question, we use Geomertic Distribution.

$$P(k^{th}) = (1-p)^{k-1}p$$

$$p = \frac{1}{n}$$

$$P(k^{th}) = (1-\frac{1}{n})^{k-1}\frac{1}{n}$$

3

For this problem we can use Binomial Distribution to solve. $p = \frac{1}{3}$, $k = 10$, $n = 20$

$$P(x = 10) = \binom{20}{10}\frac{1}{3}^{10}(1-\frac{1}{3})^{20-10} = 0.0543$$

4

For this problem, there is only one possible that the player get all 13 hearts. And the space is combination of choose 13 cards out of 52 crads

$$P(allHearts) = \frac{1}{\binom{52}{13}} = 1.5748 \times 10^{11}$$

5

1. Since all the sensors and all the time frames are independent from others. We can use $P(A \cap B) = P(A) \times P(B)$

$$P(A_i) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

2. $$P(A_i\ capA_j) = P(A_i) \times P(A_j) \times P(A_k{}^c) = \frac{1}{4} \times \frac{1}{4} \times \frac{3}{4} = \frac{3}{16}$$

3. $$P(A_1 \cap A_2 \cap A_3) = \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{64}$$

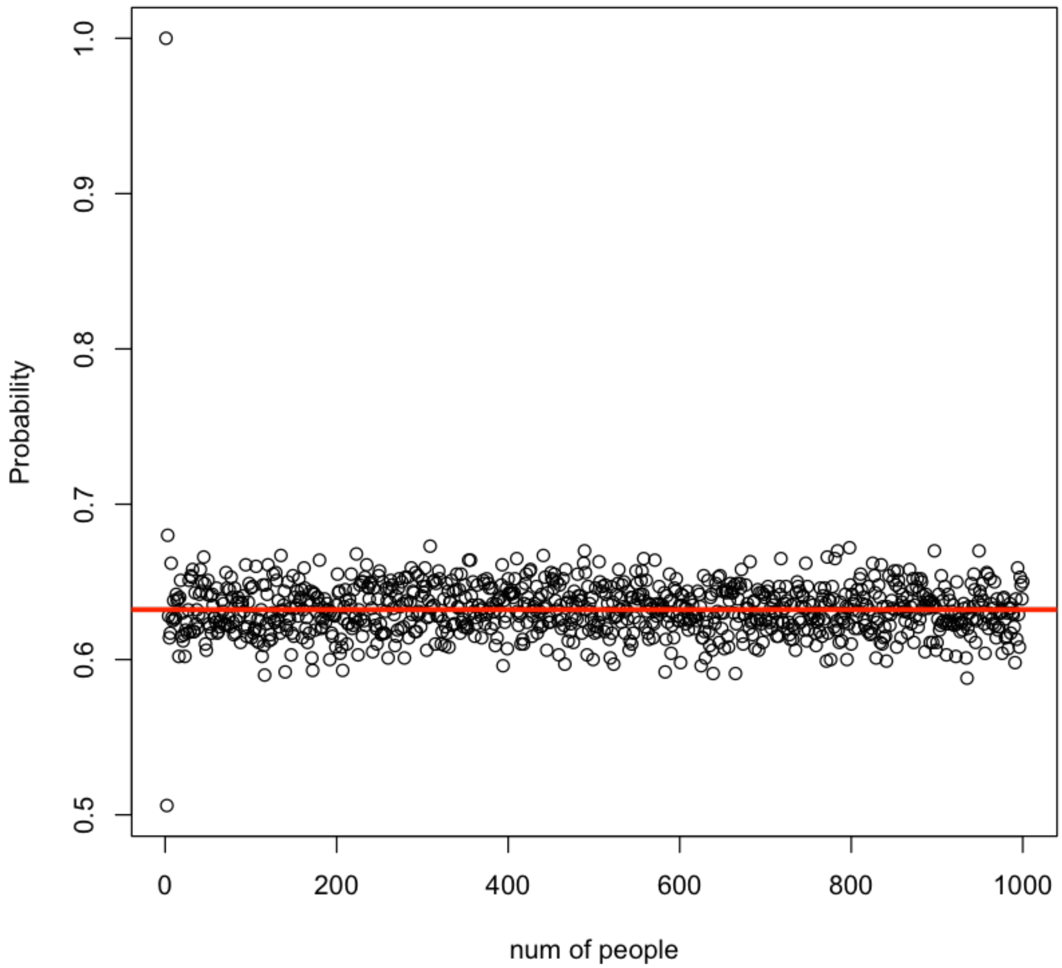
4. $$P(A_1 \cup A_2 \cup A_3) = 1 - P(A_1{}^c \cap A_2{}^c \cap A_3{}^c) = 1 - \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{37}{64}$$

6

[1]:
```
set.seed(1002)
numExp = 1000
numPeople = 1000
result = numeric(numPeople)
for(i in 1:numPeople){
    match = numeric(numExp)
    for(j in 1:numExp){
        x = sample(1:i, i, repl = F)
        for(k in 1:i){
            if(x[k] == k){
                match[j] = 1
                break;
            }
        }
    }

    p = mean(match == 1)
    result[i] = p
}

plot(result, ylab = "Probability", xlab="num of people")
abline(h = 1 - 1/exp(1), col = "red",lwd = 3)
```



Probability
1.0
0.9
0.8
0.7
0.6
0.5
0
200
400
600
800
1000
num of people