hw1

April 11, 2024

1 1.

For this problem, there are 7 spots for (u u u r r r r). When we are ploting the path, we dont care if we use the first u or second u which means that **each u is same and same thing for r**. All we need is to make sure that there are 3 u and 4 r in the 7 spots. So, we can think this question as a **combination**. Also, once we find the all combination of u, we also find the all ombination of r because after we decided the spots for u, the rest spots have to be r. So, we can think this question as a **combination of u in 7 spots or otherwise for r which give the same results.** We will use the equation of combination that order does not matter.

Solution:

for u in 7 spots:

$$Path = \binom{7}{3} = \frac{7!}{(7-3)!3!} = 35$$

and for r in 7 spots:

$$Path = \binom{7}{4} = \frac{7!}{(7-4)!4!} = 35$$

as we can see the answer is same

2 2.

For this problem, as the question state that "any buffer is just as likely to ba available (or occupied) as any other", we can think that all buffers are same. So, we can think this as a combintation problem where order does not matter.

1. **A** = "A least two but no more than five buffers are occupied".
   We can think this **A** as the compliment of the possible that none of the buffers are occupied or only one buffer is occupied or all buffers are occupied. In this way, it is easier to do because each event is independent and we can simply just add them up.

Solution:

$$S = 2^6 = 64$$

$$P(noneoccpuied) = \frac{1}{S} = \frac{1}{64}$$

$$P(oneoccupied) = \frac{6}{S} = \frac{6}{64}$$

$$P(alloccupied) = \frac{1}{S} = \frac{1}{64}$$

$$P(\overline{A}) = P(noneoccpuied) + P(oneoccupied) + P(alloccupied) = \frac{8}{64}$$

$$P(A) = 1 - P(\overline{A}) = 1 - \frac{8}{64} = \frac{56}{64} = \frac{7}{8}$$

2. B = "At least one buffer is occupied".
   Same as the question 1 which we can think B as the complement of the event that none buffer is occupied.

Solution

$$P(\overline{B}) = P(noneoccpuied) = \frac{1}{S} = \frac{1}{64} P(B) = 1 - P(\overline{B}) = 1 - \frac{1}{64} = \frac{63}{64}$$

$$P(B) = 1 - P(\overline{B}) = 1 - \frac{1}{64} = \frac{63}{64}$$

3 3.

For this problem, we can use two events to solve the problem.
1. **A** is the first four songs are not Cake song.
2. **B** is the 5th song is a Cake song.

Now, because we first have to have the first 4 songs are not Cake song first and then the 5th song is a Cake song. This is turns out a probability that event **B** given event **A**

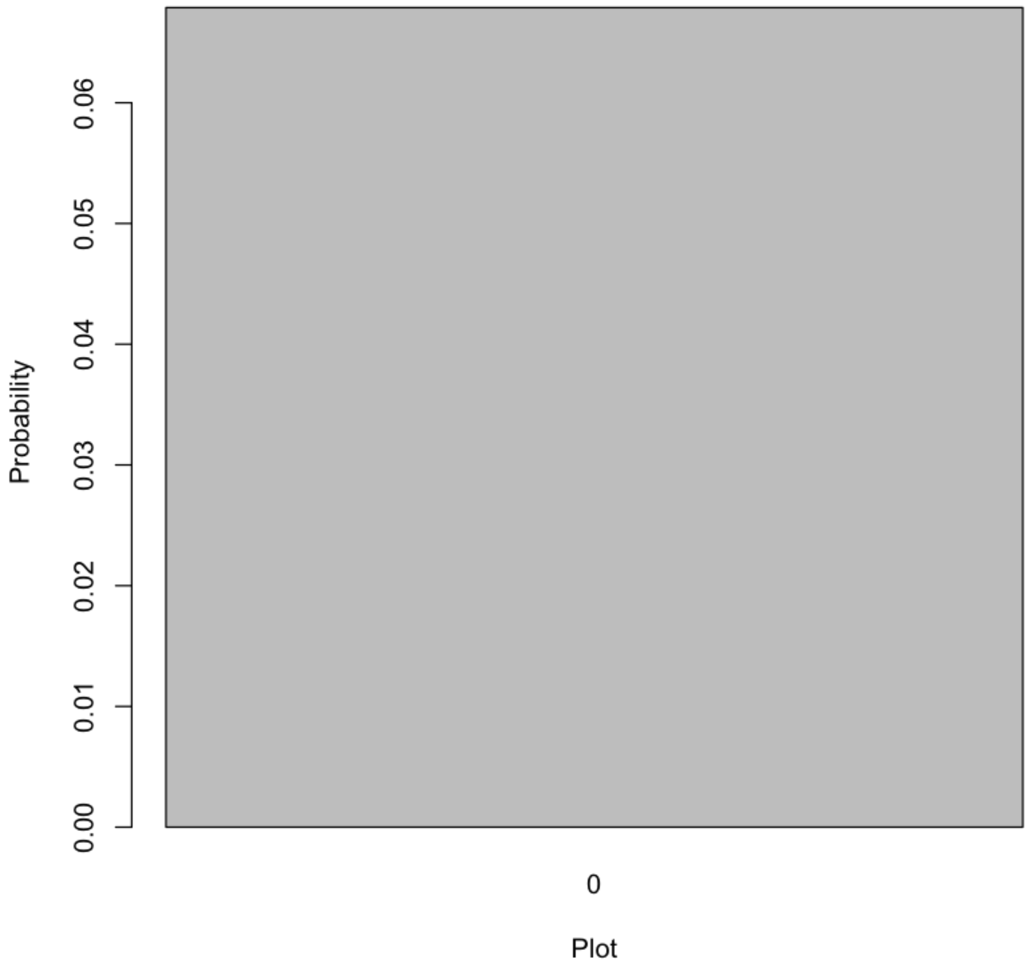
3.1
Solution

$$P(B|A) = P(A) * P(B) = \frac{90 * 89 * 88 * 87 * 86}{100 * 99 * 98 * 97} * \frac{10}{96} = \frac{2581}{38024} = 0.067878$$

3.2
Solution

[2]:
```
set.seed(1237)
n = 50000 # number of samples
numSong = 100 # number of song
y = numeric(n) # vector of sample that win
for (i in 1:n) {
    x = sample(1:numSong, numSong, repl = F)
    if (x[1] > 10 && x[2] > 10 && x[3] > 10 && x[4] > 10 &&
        x[5] >= 1 && x[5] <=10 ) {
            y[i] = 1
        }
}
# print(x)
# print(y)
p = mean(y == 1)
sprintf("P = %f", p)
barplot(p, names.arg=c(0), ylab="Probability", xlab="Plot", horiz=FALSE)
```

'P = 0.067880'



4 4.

Solution

$$Number = \binom{100}{20} * \binom{80}{30} * \binom{50}{40} * \binom{10}{10} \approx 4.88 * 10^{52}$$

5 5.

Solution
Set event A = the program will access the first location.
event B = the program will access the second location.

event $(A \cap B)$ = the program will access both locations.
event $(\overline{A \cup B})$ = the program will access neither location.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.5 + 0.4 - 0.3 = 0.6$$

$$P(\overline{A \cup B}) = 1 - P(A \cup B) = 1 - 0.6 = 0.4$$

6 6.

For this problem, A is more probable. Event A is a subset of the large space S. Event B is a subset of event A because in order be a computer programmer and a activist in the environment movement at the same time, he have to be given by he is a comouter programmer first. So, the the probability of event B will smaller than A.