Entrega Final 1er Corte

Punto 1

> ### A. Sea $Z = 3 - 4i$, $W = 6i$, $P = 3 + 2i$

a. Calcule $(2z + w) * p$
   a. $(6 - 8i + 6i)(3 + 2i)$
   b. $(6 - 2i)(3 + 2i)$
   c. $(18 + 12i - 6i + 4)$
   d. RTA: $(22 - 6i)$

b. Calcule $\frac{P}{Z} - W$

$$\frac{3+2i}{3-4i} - 6i \; = \; \left(\frac{3+2i}{3-4i} \cdot \frac{3+4i}{3+4i}\right) - 6i$$

$$\frac{9 + 12i + 6i - 8}{9 + 16} = \frac{1 + 18i}{25}$$

$$\left(\frac{1}{25} + \frac{18}{25}i\right) - 6i = \frac{1}{25} + \frac{132}{25}i$$

> ### B. Escriba en forma polar y en forma Euler los siguientes complejos:

1. $Z = -3 + 5i$
   - Polar:

     $|Z| = \sqrt{34}$

     $\theta = tan^{-1}(-\frac{5}{3}) + \pi \quad = 2.1112$

     RTA: $\sqrt{34} \cdot CIS(2.1112)$
   - Euler

     $Z = \sqrt{34} \cdot e^{2.112 \ i}$
2. $Z = -2 + -i$
   - Polar:

     $|Z| = \sqrt{5}$

     $\theta = tan^{-1}\left(\frac{1}{2}\right) + \pi = 3.605$

     RTA: $\sqrt{5} \ \cdot \ CIS(3.605)$
   - Euler

$Z = \sqrt{5} \cdot e^{3.605\ i}$

3. $Z = 1 - \frac{\sqrt{3}}{3}$
   - Polar:

     $|Z| = \sqrt{1 + \frac{1}{3}}$

     $|Z| = \sqrt{\frac{4}{3}}$

     $\theta = tan^{-1}\left(\frac{\sqrt{3}}{3}\right) + 2\pi = 5.76$

     RTA: $\sqrt{\frac{4}{3}} \cdot CIS\,(5.76)$
   - Euler

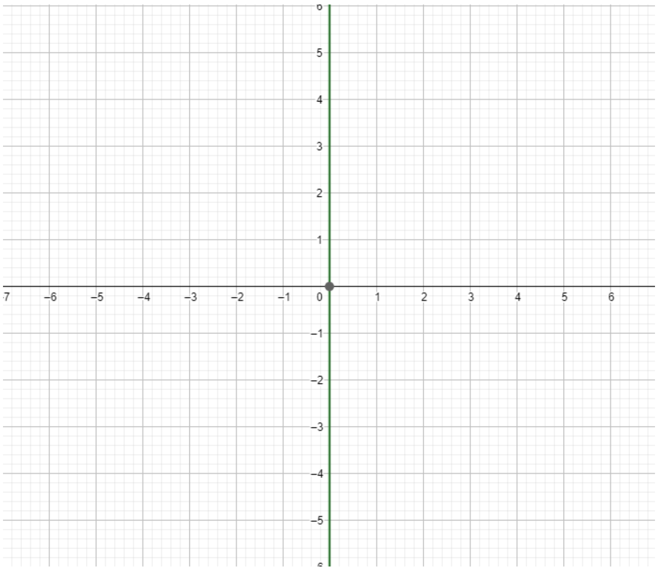
     $\sqrt{\frac{4}{3}} \cdot e^{5.76\ i}$

> C. Encuentre la imagen de la lineal indicada bajo la transformación $f(z) = z2$ (realice un bosquejo de la grafica), donde

💡 En este caso, para todos los ejercicios, la transformación es: $Z^2 = U : (x^2 - y^2)\ V : (2xyi)$

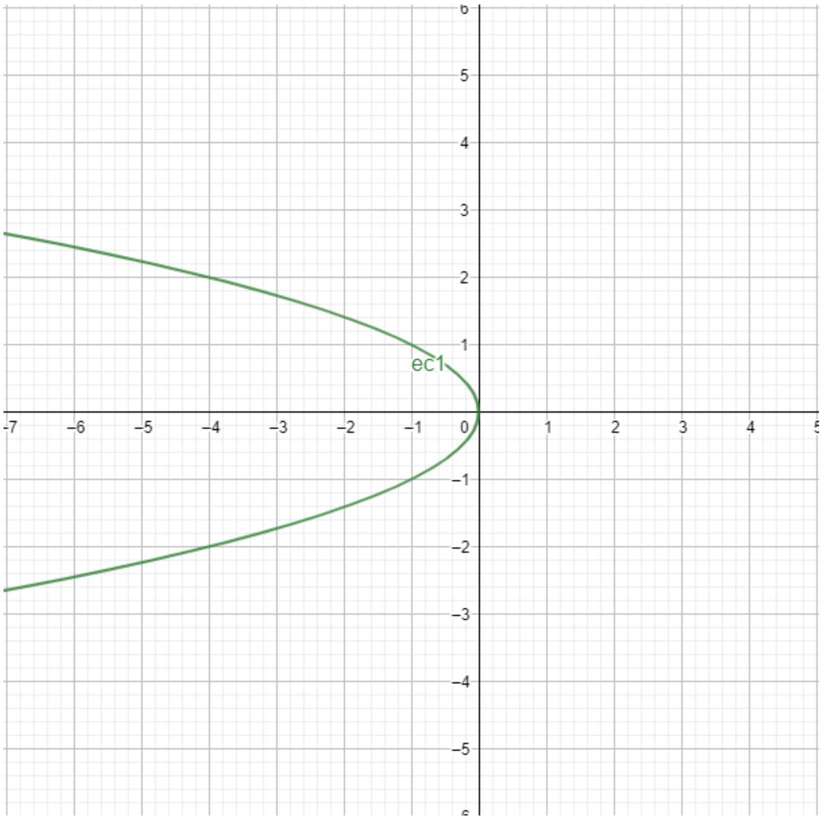
1. $X = 0$

   Antes de la transformación



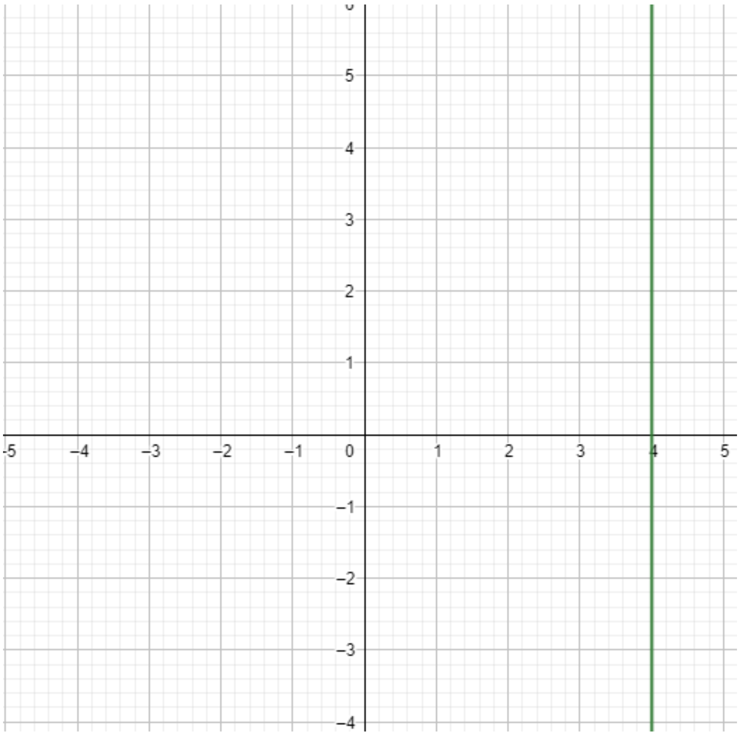
   Después de la transformación:

   $U = -V^2$



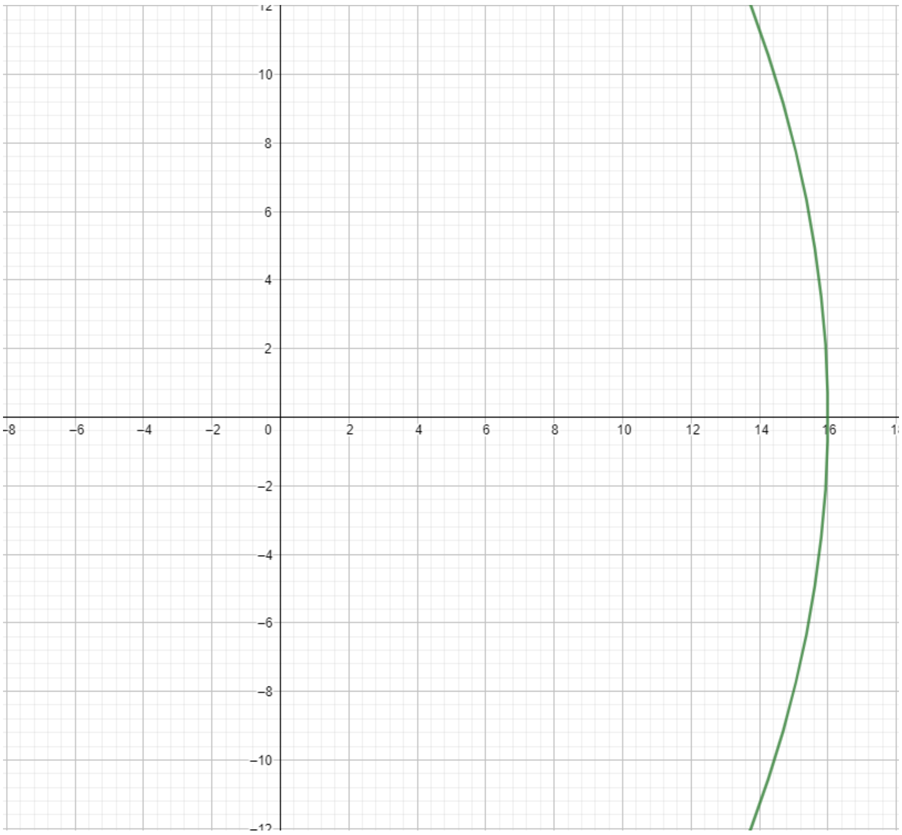
2. $X = 4$

Antes de la transformación:



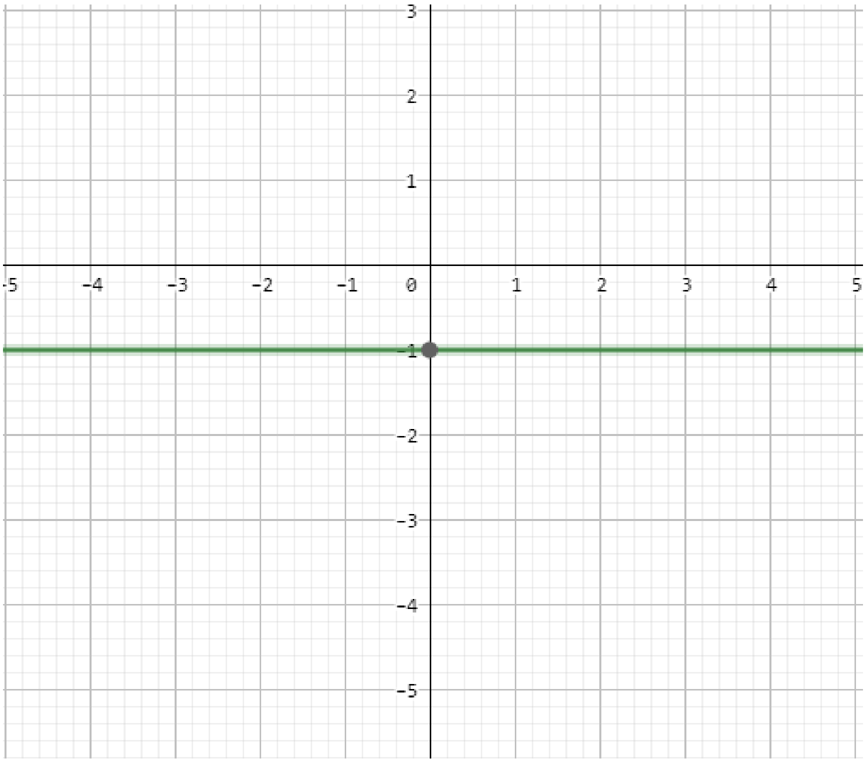
Después de la transformación:

$$U = 16 - Y^2$$
$$V = -8Yi$$
$$Y = \frac{V}{-8}$$
$$U = 16 - \frac{V^2}{64}$$



3. $y = -1$

   Antes de la transformacion:

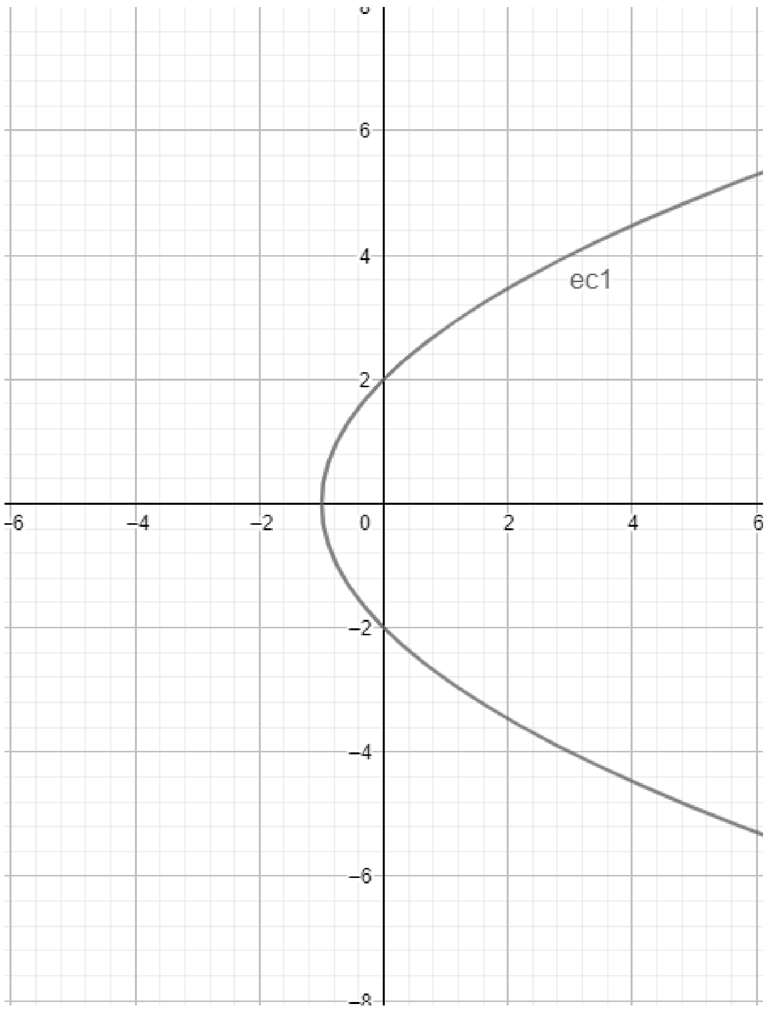


$U = x^2 - 1$

$V = 2x$

$\frac{V}{2} = X$

$U = \left(\frac{V^2}{4}\right) - 1$



Punto 2

> Para las siguientes funciones, escriba cada $f(z) = u(x,y) + iv(x,y)$ y Determine si $f(z)$ es o no, analítica

1. $f(z) = ez + z$
   - Forma $U + iV$

$$
\begin{aligned}
e^z + z &= e^{x+iy} + (x+iy) \\
&= e^x \cdot Cos(y) + iSen(y) + (x+iy) \\
&= (e^x \cdot Cos(y) + x) + i(e^x \cdot Sen(y) + y) \\
U &= e^x \cdot Cos(y) + x \\
V &= i(e^x \cdot Sen(y) + y)
\end{aligned}
$$

   - Analiticidad

$$\frac{\partial U}{\partial x} = (e^x cos\,(y) + 1) = \frac{\partial V}{\partial x} = (e^x cos\,(y) + 1) \checkmark$$

$$\frac{\partial U}{\partial y} = -e^x Sen(y) = -\frac{\partial V}{\partial x} = e^x Sen(y) ✔️$$

La función es analítica

2. $f(z) = z + \overline{z}$

> 💡 Uso de la propiedad $Z + \overline{Z} = 2Re(z)$

- Forma $U + iV$

  $U = 2X$

- Analiticidad

  No es analítica, debido a que la ecuación no tiene una parte imaginaria (V)

3. $f(z) = z^2 + i$

- Forma $U + iV$

$$\begin{aligned}(x+iy)^2 + i &= x^2 + 2xyi - y^2 + i \\ U &= X^2 - Y^2 \\ V &= 2xyi + i\end{aligned}$$

- Analiticidad

$$\frac{\partial U}{\partial x} 2x = \frac{\partial V}{\partial y} = 2x ✔️$$

$$\frac{\partial U}{\partial x} - 2y = -\frac{\partial V}{\partial y} = 2y ✔️$$

La función es analítica

Punto 3

> ### Evalúe el lımite que se indica

1. $\lim_{Z \to (6-8i)} \frac{Z^3 - Z^2(6+i) + Z(30+30i) + (252-136i)}{Z^3 - Z^2(16-16i) + Z(8-164i) + 216 + 312i}$

$$\begin{cases}(6-8i)^3 & = 216 - 864i - 1125 - 512 = -936 + 352i \\ (6-8i)^2 & = -28 - 96i\end{cases}$$

$$\lim_{Z \to (6-8i)} \frac{(-936+352i) - (-28-96i)(6+i) + (6-8i)(30+30i) + (252-136i)}{(-936+352i) - (-28-96i)(16-16i) + (6-8i)(8-164i) + 216 + 312i}$$

$$\frac{(-936+352i)-(-264-548i)+(150-150i)+252-136i}{(-936+352i)-(-1088-1088i)+(-1264-1012i)+216+312i} \tag{1}$$

$$\frac{-270+614i}{-896+740i} \tag{2}$$

$$\frac{-270+614i}{-896+740i} = \frac{-270+614i}{-896+740i} \cdot \frac{-896-740i}{-896-740i}$$

$$= \frac{696280-350344i}{1360416} = \frac{696280}{1360416} + \frac{350344i}{1360416} \tag{3}$$

$$R = 0.512 + 0.258i$$

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2. $\lim_{Z \to (1-i)} \frac{5Z^2-2Z+2}{z+1}$

$$\lim_{Z \to (1-i)} \frac{5Z^2-2Z+2}{z+1}$$

$$\frac{5Z^2-2Z+2}{z+1} = \frac{-10i-(2-2i)+2}{2-i} = \frac{8i}{2-i}$$

$$\frac{8i}{2-i} \cdot \frac{2+i}{2+i} = \frac{16i-8}{5} = R = \frac{-8}{5} + \frac{16}{5}i$$

Punto 4

> Para los siguientes valores de z, exprese a Ln z en la forma $a+bi$

1. $\sqrt{2}+\sqrt{16}i$

   $|Z| = \sqrt{8}$

   $\theta^o = tan^{-1}(\sqrt{\frac{6}{2}})$ = 60^o

   Rta = $Ln(\sqrt{8}) + (60^o + 2n \cdot 180^o)$

2. $-\sqrt{3}+i$

   $|Z| = 2$

   $\theta^o = tan^{-1}(-\frac{1}{\sqrt{3}}) + 180^o = 150^o$

   Rta = $Ln(2) + (150^o + 2n \cdot 180^o)$

3. $-2+2i$

   $|Z| = \sqrt{8}$

   $\theta^o = tan^{-1}(-1) + 180^o = 135^o$

   Rta = $Ln(\sqrt{8}) + (135^o + 2n \cdot 180^o)$

Exprese la cantidad indicada de la forma $a + bi$

1. $Sec = (\frac{1}{cos(\pi+i)})$

$$Sec = (\frac{1}{cos(\pi + i)}) = \frac{1}{e^{\pi i-1} + e^{-\pi i+1}} \cdot \frac{1}{2}$$
$$= \frac{1}{2e^{\pi i-1} + 2e^{-\pi i+1}}$$

$$= \frac{1}{2} \cdot \frac{1}{e^{-1}Cos(\pi) + iSen(\pi) + e^{1}Cos(\pi) - iSen(\pi)}$$
$$= \frac{1}{3} \cdot \frac{1}{e^{-1}Cos(\pi) + e^{1}Cos(\pi)} = \frac{1}{-e^{-1} - e^{1}}$$
$$Rta = \frac{1}{-3.08616127 + 0i}$$

2. $Senh(1 + \frac{\pi}{3}i)$

$$\frac{e^{z} - e^{-z}}{2} \rightarrow \frac{e^{1+\frac{\pi}{3}i} - e^{-1-\frac{\pi}{3}i}}{2}$$

$$\frac{e^{1}Cos(\frac{\pi}{3}) + iSen(\frac{\pi}{3}) - e^{-1}Cos(\frac{\pi}{3}) - iSen(\frac{\pi}{3})}{2} = ...$$
$$... = \frac{cos(\frac{\pi}{3})(e^{1} - e^{-1}) + iSen(\frac{\pi}{3})(e^{1} - e^{-1})}{2} = ...$$
$$... = \frac{cos(\frac{\pi}{3})(e^{1} - e^{-1})}{2} + \frac{iSen(\frac{\pi}{3})(e^{1} - e^{-1})}{2}$$
$$...Rta = 0.587601 + 1.01776i$$

3. $Cos(2 - 4i)$

$$\frac{e^{i(2-4i)} + e^{-i(2-4i)}}{2} = \frac{e^{4+2i} + e^{-4-2i}}{2} = ...$$

$$... = \frac{e^{4}Cos(2) + iSen(2) + e^{4}Cos(-2) - iSen(-2)}{2} = ...$$

$$... = \frac{Cos(2)(e^{4} - e^{-4})}{2} + \frac{iSen(2)(e^{4} - e^{-4})}{2} = Rta = -11.3642 + 24.8146i$$

Punto 5

Determine una serie de Furier para la función

1. Primer caso

$$f(x)\begin{cases} x-1, & -\pi < x < 0 \\ x+1, & 0 < x < \pi \end{cases}$$

Desarrollo:

a. a_0

$$a_0 = \frac{1}{\pi}\left[\int_{-\pi}^{0}(x-1)dx \;+\; \int_{0}^{\pi}(x+1)dx\right] = ...$$

$$... = \frac{1}{\pi}\left[\left(\frac{x^2}{2}-x\right)\Bigg|_{-\pi}^{0} + \left(\frac{x^2}{2}-x\right)\Bigg|_{0}^{\pi}\right] = \frac{1}{\pi}\left[-2\pi^2\right] = Rta = -2\pi$$

b. a_n

$$a_n = \frac{1}{\pi}\left[\int_{-\pi}^{0}(x-1)\cdot Cos(nx)\,dx \;+\; \int_{0}^{\pi}(x+1)\cdot Cos(nx)\,dx\right] = ...$$

$$... = \left[\frac{1}{n}\sin(nx)(x-1) - \int\frac{1}{n}\sin(nx)\,dx\right]_{-\pi}^{0} = ...$$

$$... = \left[\frac{1}{n}\sin(nx)(x-1) + \frac{1}{n^2}\cos(nx)\right]_{-\pi}^{0} = ...$$

$$... = \left[\frac{1}{n^2} - (-1)^n\frac{1}{n^2} + \int_{0}^{\pi}(x+1)\cdot Cos(nx)\,dx\right] = ...$$

$$... = \left[\frac{1}{n}\left(\sin(nx)(x+1) - n\cdot\int\frac{1}{n}\sin(nx)\,dx\right)\right]_{0}^{\pi} = ...$$

$$... = \left[\frac{1}{n}\left(\sin(nx)(x+1) - n\left(-\frac{1}{n^2}\cos(nx)\right)\right)\right]_{0}^{\pi} = ...$$

$$... = \left[\frac{1}{n}\left(\sin(nx)(x+1) + \frac{1}{n}\cos(nx)\right)\right]_{0}^{\pi} = ...$$

$$a_n = \left[\frac{1}{n^2} - (-1)^n\frac{1}{n^2} + \frac{(-1)^n - 1}{n^2}\right] = \frac{0}{n^2\pi} = 0$$

c. b_n

$$b_n = \frac{1}{\pi}\left[\int_{-\pi}^{0}(x-1)\cdot Sen(nx)\,dx \;+\; \int_{0}^{\pi}(x+1)\cdot Sen(nx)\,dx\right] = ...$$

$$... = \left[-\frac{(x-1)\cdot Cos(nx)}{n} + \frac{1}{n}\int Cos(nx)\cdot dx \right] = ...$$

$$... = \left[-\frac{(x-1)\cdot Cos(nx)}{n} + \frac{Sen(nx)}{n^2}\Big|_{-\pi}^{0} \right] = ...$$

$$... = \frac{1}{n} - \left[-\frac{(-\pi-1)(-1)^n}{n} \right] = \frac{\pi(-1)^n}{n}$$

$$... = \left[-\frac{(x+1)\cdot Cos(nx)}{n} + \frac{1}{n}\int Cos(nx)\cdot dx \right] = ...$$

$$... = \left[-\frac{(x+1)\cdot Cos(nx)}{n} + \frac{Sen(nx)}{n^2}\Big|_{0}^{\pi} \right] = ...$$

$$... = \left[-\frac{\pi+1(-1)^n}{n} - \frac{(-1)}{n} \right] = \frac{-\pi(-1)^n}{n}$$

$$... \, b_n = \frac{1}{\pi}\left[-\frac{\pi(-1)^n}{n} + \frac{\pi(-1)^n}{n} \right] = 0$$

💡 Formula general del primer caso

Para pares e impares

$$f(x) = -2\pi \sum_{n=1}^{+\infty} [0+0] = -2\pi$$

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2. Seguzazando caso

$$f(x) \begin{cases} 1 & 0 < x < 1 \\ 2-x & 1 < x < 2 \end{cases}$$

a. a_o

$$\frac{1}{2}\left[\int_0^1 dx + \int_1^2 2-x \; dx \right] = \frac{1}{2}\left[x\Big|_0^1 + 2x - \frac{x^2}{2}\Big|_1^2 \right] = \frac{1}{4}$$

b. a_n

$$\frac{1}{2}\left[\int_0^1 Cos\left(\frac{n\pi x}{2}\right)\,dx + \int_1^2 (2-x)Cos\left(\frac{n\pi x}{2}\right)\,dx\right] = ...$$

$$\frac{1}{2}\left[\frac{2}{n\pi}Sen(\frac{n\pi x}{2})\Big|_0^1 + \left[\frac{2(2-x)}{n\pi}Sen\left(\frac{n\pi x}{2}\right) + \frac{2}{n\pi}\int Sen\left(\frac{n\pi x}{2}\right)\,dx\right]\right]$$

$$... + \left[\frac{2(2-x)}{n\pi}Sen\left(\frac{n\pi x}{2}\right) - \frac{4}{n^2\pi^2}Cos\left(\frac{n\pi x}{2}\right)\right] = ...$$

$$= \frac{1}{2}\left[\frac{2}{n\pi}Sen(\frac{n\pi x}{2})\Big|_0^1 - \frac{2}{n\pi}\left[-(2-x)\cdot Sen\left(\frac{n\pi x}{2}\right) + \frac{2Cos\left(\frac{n\pi x}{2}\right)}{n\pi}\right]\Big|_1^2\right]$$

$$= \frac{1}{2}\left[\frac{2}{n\pi}Sen(\frac{n\pi x}{2})\Big|_0^1 - \frac{4}{n^2\pi^2}\left[(-1)^n - Sen(\frac{n\pi}{2}) - \frac{2Cos(\frac{n\pi}{2})}{n\pi}\right]\right]$$

$$= \frac{1}{2}\left[\frac{2Sen(\frac{n\pi}{2})}{n\pi} - \frac{4(-1)^n}{n^2\pi^2} + \frac{4Sen(\frac{n\pi}{2})}{n^2\pi^2} + \frac{8Cos(\frac{n\pi}{2})}{n^3\pi^3}\right] = ...$$

$$a_n = \begin{cases} Pares, & \frac{1}{2}\left[-\frac{4}{n^2\pi^2} + \frac{8(-1)^n}{n^3\pi^3}\right] \rightarrow -\frac{4}{n^2\pi^2}\left[1 - \frac{2(-1)^n}{n\pi}\right] \\ Impares, & \frac{1}{2}\left[\frac{2(-1)^n}{n\pi} + \frac{4}{n^2\pi^2} + \frac{4(-1)^n}{n^2\pi^2}\right] \rightarrow \frac{1}{n\pi}\left[(-1)^n + \frac{4+4(-1)^n}{n\pi}\right] \end{cases}$$

c. b_n

$$b_n = \frac{1}{2}\left[\int_0^1 Sen(\frac{nx\pi}{2})\,dx + \int_1^2 (2-x)Sen(\frac{nx\pi}{2})\,dx\right]$$

$$b_n = \frac{1}{2}\left[-\frac{2}{n\pi}Cos(\frac{nx\pi}{2})\Big|_0^1 + \int_1^2 (2-x)Sen(\frac{nx\pi}{2})\,dx\right] = ...$$

$$b_n = \frac{1}{2}\left[-\frac{2}{n\pi}\left(Cos(\frac{nx\pi}{2}) + 1\right) + \int_1^2 (2-x)Sen(\frac{nx\pi}{2})\,dx\right] = ...$$

$$= ... + \left[-(2-x)\frac{2}{n\pi}Cos(\frac{nx\pi}{2}) + \frac{4}{n^2\pi^2}Sen(\frac{nx\pi}{2})\Big|_1^2\right] = ...$$

$$= \frac{1}{2}\left[-\frac{2}{n\pi}\left(Cos(\frac{nx\pi}{2}) + 1\right) + \frac{2Cos(\frac{n\pi}{2})}{n\pi} - \frac{4Sen(\frac{n\pi}{2})}{n^2\pi^2}\right] = ...$$

$$= \frac{1}{2}\left[-\frac{2}{n\pi}\left(Cos(\frac{nx\pi}{2}) + 1\right) + \frac{2Cos(\frac{n\pi}{2})}{n\pi} - \frac{4Sen(\frac{n\pi}{2})}{n^2\pi^2}\right] =$$

$$... = -\frac{1}{n\pi}\left[1 + \frac{2Sen(\frac{n\pi}{2})}{n\pi}\right]$$

$$b_n = \begin{cases} Pares, & -\frac{1}{n\pi} \\ Impares, & -\frac{1}{n\pi}\left[1 + \frac{2(-1)^n}{n\pi}\right] \end{cases}$$

💡 Formula general de Fourier para el segundo caso

$$\begin{cases} Pares, & \frac{1}{8} + \sum_{n=1}^{+\infty}\left[-\frac{4Cos(\frac{nx\pi}{2})}{n^2\pi^2}\left[1 - \frac{2(-1)^n}{n\pi}\right] - \frac{Sen(\frac{nx\pi}{2})}{n\pi}\right] \\ Imprar, & \frac{1}{8} + \sum_{n=1}^{+\infty}\left[\frac{Cos(\frac{nx\pi}{2})}{n\pi}\left[(-1)^n + \frac{4+4(-1)^n}{n\pi}\right] - \frac{Sen(\frac{nx\pi}{2})}{n\pi}\left[1 + \frac{2(-1)^n}{n\pi}\right]\right] \end{cases}$$