Question 2.from textbook

(a)

Now the weight of the pig becomes a function of t. And we want to find the velocity of this function, we need to find the differential of it. Then we get

$$\begin{aligned}\omega'(t) &= (\frac{800}{1+3e^{-\frac{t}{30}}})' \\ &= \frac{80e^{-\frac{t}{30}}}{1+3e^{-\frac{t}{30}}{}^{2}}\end{aligned}$$

Then we set $t = 0$, we can get

$$\omega'(0) = \frac{80}{16} = 5(lbs)$$

To ensure it is right, we can calculate that $w(0) = \frac{800}{4} = 200$, and $w(1) = \frac{800}{1+3e^{-\frac{1}{30}}} = 205.0415 \approx 205$. So that $\Delta w \approx 5$. So the pig is gaining about 5 lbs/day at $t = 0$.

After that, we want to find how is the weight ω would happen when t increases. I've written a project in MATLAB, in order to draw 2 picture of $t - \omega$ and $t - \omega'$ relationship.

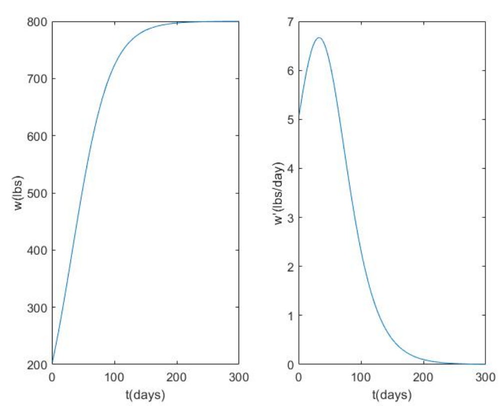
```
t=1:0.01:300;
w=800./(1+3*exp(-t/30));
y=80*exp(-t/30).*(1+3*exp(-t/30)).^(-2);

subplot(1,2,1)
plot(t,w);
xlabel('t(days)');ylabel('w(lbs)');

subplot(1,2,2)
plot(t,y);
xlabel('t(days)');ylabel('w''(lbs/day)');

```

(a) Codes



(b) 2 picture

Figure 1: Relationship between time t and weight ω

We can easily find that when $t \to \infty$ then $\omega \to 800$ and $\omega' \to 0$. And ω' is going upward firstly and then falling to 0 when t becomes larger. Analying this, we can know that there is a fast grown of the weight of the pig (ω) and after that, we can sell the pig to gain the greatest profit.

(b)

Step 1. Ask the question

There three stages in step 1.

Variables:	t = time (days)
	w = weight of pig (lbs)
	p = price for pigs (\$/lb)
	C = cost of keeping pig t days (\$)
	R = revenue obtained by selling pig (\$)
	P = profit from sale of pig (\$)
Assumptions:	$w = \frac{800}{1+3e^{-\frac{t}{30}}}$
	$p = 0.65 - 0.01t$
	$C = 0.45t$
	$R = p \cdot w$
	$P = R - C$
	$t \geq 0$
Objective:	Maximize P

Step 2. Select the modeling approach

We will model it as one-variable optimization problem. And we will use Newton method of one-variable to solve it.

Step 3. Formulate the model Let $y = f(x) = P$, $x = t$. We can get

$$\begin{aligned} y &= f(x) \\ &= (0.65 - 0.01x)(\frac{800}{1+3e^{-\frac{x}{30}}}) - 0.45x \end{aligned}$$

And we can get the differential of y.

$$f'(x) = -\frac{8}{1+3e^{-\frac{x}{30}}} + \frac{80(0.65-0.01x)e^{-\frac{x}{30}}}{(1+3e^{-\frac{x}{30}})^2} - 0.45$$

Step 4. Solve the model

After that we can apply the Newton method of one-variable to $f'(x)$. We can get when $x^* = t^* = 12.3349$, profit goes to maximum which is $P^* = 135.4236$.

Step 5. Answer the question

We should hold the pigs for 12.3349 days, and then sell them to get the max profit which is 12.3349\$.

```
clc,clear
syms x;

f=-8/(3*exp(-x/30)+1)+(80*(13/20-x/100)*exp(-x/30))/(3*exp(-x/30)+1)^2-9/20;
x0=8;
eps=0.0001;
res=Newton_1(f,x,x0,eps,100);

```

(a) Main codes

```
function res=Newton_1(f,x,x0,eps,N)
df=diff(f);
for i=1:N
    x1=x0-subs(f,x,x0)/subs(df,x,x0);    %solution after iteration
    if abs(x1-x0)<eps
        break;                           %solution good enough
    end
    x0=x1;
end
res=double(x0);
end
```

(b) Function codes

Two codes solving question (b) by Newton method of one-variable

(c)

Now we treat 800 as a new parameter c. The objective function becomes to

$$f(x) = (0.65 - 0.01x)(\frac{c}{1+3e^{-\frac{x}{30}}}) - 0.45x$$

And the differential of $f(x)$ becomes to

$$F = f'(x) = -\frac{c}{100\left(3\mathrm{e}^{-\frac{x}{30}}+1\right)} + \frac{c\cdot\left(\frac{13}{20}-\frac{x}{100}\right)\mathrm{e}^{-\frac{x}{30}}}{10(3\mathrm{e}^{-\frac{x}{30}}+1)^2} - 0.45$$

Since the sensitivity is about how effect the objective variables when the parameters change under 1%, so we can let c become 808, and calculate the time t and the profit P. objective function $f(x)$ becomes

$$f(x) = (0.65 - 0.01x)(\frac{808}{1+3e^{-\frac{x}{30}}}) - 0.45x$$

And $x^* = 12.3877, P^* = 136.8334$, from the definition of sensitivity, the sensitivity is

$$S(c,x) = \frac{12.3877 - 12.3349}{12.3349}\cdot 100 = 0.428$$
$$S(c,P) = \frac{136.8334 - 135.4236}{135.4236}\cdot 100 = 1.041$$

We can also use Newton method to F and find all optimal time t and profit P over $c \in [750:10:850]$.

c	t	P
750	11.97648279119	140.357603321875
760	12.0523387777815	140.393774763666
770	12.1260168949276	140.428448403757
780	12.1976105236101	140.46170627831
790	12.2672077293703	140.493624850193
800	12.3348916388565	140.524275459953
810	12.4007407845062	140.553724734524
820	12.464829420497	140.582034958174
830	12.5272278127437	140.609264409627
840	12.5880025054188	140.635467668889
850	12.6472165662017	140.660695896839

Sensitivity of final weight c, time t and profit P

```matlab
clc,clear
syms x;
res=zeros(11,1);
fw=(750:10:850)';
p=zeros(11,1);
i=1;
% N=1;
for c=750:10:850
    f=-((10*c+450)*exp(x/15)+(c*x-35*c+2700)*exp(x/30)+4050)/(1000*(exp(x/30)+3)^2);      %the objective function
    x0=8;
    eps=0.001;
    res(i,1)=Newton_1(f,x,x0,eps,100);
    p(i,1)=(0.65-0.01*res(i,1))*(800/(1+3*exp(-res(i,1)/30)))-0.45;
    i=i+1;
%     N=N+1;
end
table(fw,res,p,'VariableNames',{'c' 't' 'P'})                                              %draw a table of c and x

```

Codes in MATLAB

Question 6.from textbook

(a)

In the question 6, the response time relationship becomes to a linear relationship. Then let's apply the five-step method.

Step 1. Ask the question

There three stages in step 1. (Notice that n and t are matrices)

Variables:	x = x-index of position
	y = y-index of potition
	n = average call times (times)
	t = each average response time (min)
	T = total average response time (min)
Assumptions:	$t = \|x - x_0\| + \|y - y_0\|$
	$T = \frac{n \cdot t}{84}$
	$0 \le x \le 6, 0 \le y \le 6$
Objective:	Minimize T

Step 2. Select the modeling approach

In this question, we measure it as a multivariable unstrained optimization problem. If we solve it directly it would be very difficult since its objective function is very complicate. So we use random search model to solve it. Then it will become more easy.

Step 3. Formulate the model

The opitmal function $z = T$ is

$$\begin{aligned} z &= T \\ &= (6(|x-1| + |y-5|) + 8(|x-3| + |y-5|) + 8(|x-5| + |y-5|) \\ &\quad + 21(|x-1| + |y-3|) + 6(|x-3| + |y-3|) + v3(|x-5| + |y-3|) \\ &\quad + 18(|x-1| + |y-1|) + 8(|x-3| + |y-1|) + 6(|x-5| + |y-1|))/84 \end{aligned}$$

Step 4. Solve the model

Then we use random search method to solve this objective function. We set the boundary $x \in [0, 6]$ and $y \in [0.6]$, and the iteration times becomes to 10000000. We can get the solution approaching to

$$x_{\min} = 1, \qquad y_{\min} = 3, \qquad z_{\min} = 2.6194$$

Step 5. Answer the question

The optimal position to build the new facility is at position (1,3). Then the least average response time is 2.6194 minutes

```
function r=RandomS(a,b,c,d,N)
xm=a+(b-a)*rand(1);                             %random number of x, y, z
ym=c+(d-c)*rand(1);

zm=(6*(abs(xm-1)+abs(ym-5))+8*(abs(xm-3)+abs(ym-5))+8*(abs(xm-5)+abs(ym-5))+...
    21*(abs(xm-1)+abs(ym-3))+6*(abs(xm-3)+abs(ym-3))+3*(abs(xm-5)+abs(ym-3))+...
    18*(abs(xm-1)+abs(ym-1))+8*(abs(xm-3)+abs(ym-1))+6*(abs(xm-5)+abs(ym-1)))/84;

for n=1:N                                       %zm is the objective funtion
    x=a+(b-a)*rand(1);
    y=c+(d-c)*rand(1);

    z=(6*(abs(x-1)+abs(y-5))+8*(abs(x-3)+abs(y-5))+8*(abs(x-5)+abs(y-5))+...
        21*(abs(x-1)+abs(y-3))+6*(abs(x-3)+abs(y-3))+3*(abs(x-5)+abs(y-3))+...
        18*(abs(x-1)+abs(y-1))+8*(abs(x-3)+abs(y-1))+6*(abs(x-5)+abs(y-1)))/84;

    if z<zm                                     %whether better or not
        xm=x;
        ym=y;
        zm=z;
    end
end
r=[xm,ym,zm];                                   %output the solution
end
```

Codes of question 6 in MATLAB