Title: An Overview of Morse Theory: Slicing Technique in Manifolds

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Date: 06/03/2024

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本科生毕业论文

论文题目：Morse理论综述：流形中的切片技术

学　　院：暨南大学伯明翰大学联合学院

学　　系：

专　　业：数学与应用数学

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2024 年 3 月 12 日

Statement of Originality

I hereby declare that the thesis presented is the result of research performed by me personally, under guidance from my supervisor. This thesis does not contain any content (other than those cited with references) that has been previously published or written by others, nor does it contain any material previously presented to other educational institutions for degree or certificate purpose to the best of my knowledge. I promise that all facts presented in this thesis are true and creditable.

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An Overview of Morse Theory: Slicing Technique in Manifolds

Abstract A concise abstract should briefly state the purpose of the research and the main results. An abstract is often presented separate from the article, so it must be able to stand alone. The abstract should be about 150 words with 3-8 key words.

Keywords One, Two, Three, Four

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0. Notation and Conventions

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- abc

1. Introduction

Morse theory is the study of flow in differential topology using critical point analysis of differentiable functions (i.e. Morse functions) on differential manifolds A powerful tool for geometric topology and overall geometric structure. Harold Marston · Morse found it in the 1920s. Currently, both domestically and internationally The basic research on this theory has become more comprehensive, and there are a large number of research results in various mathematical fields worth studying and researching for undergraduate students.

This paper aims to provide a clear and systematic exposition of the basic theory of Morse Theory, as well as introduce the relevant concepts of power systems. And conduct in-depth research on the definition of Morse Small flow and related examples. This article will be based on the notebooks and lectures of a large number of mathematical predecessors Systematically review and summarize notes, published articles, and books.

2. Morse Functions and The Topology of Morse Functions

2.1 Fundations About Morse Functions

Let $f : M \to \mathbb{R}$. A **critical point** of f is the point $p \in M$ such that $d_p f = 0$. If there is a local coordinate system $(x^1, \cdots, x^n)$, that means

$$\frac{\partial f}{\partial x_1}(p) = \cdots = \frac{\partial f}{\partial x_n} = 0.$$

And the real value $f(p)$ is called a **critical value** of f. Let $\mathbf{Cr}_f$ be the set of all critical points of f. For $p \in \mathbf{Cr}_f$, a **Hessian** of f at p is a $n \times n$-matrix

$$H_{f,p} := \left(\frac{\partial^2 f}{\partial x_i \partial x_j}(p) \right).$$

Moreover, $H_{f,p} : T_pM \times T_pM \to \mathbb{R}$ is symmetric. This describes the local properties near the critical point p. A critical point p is non-degenerate(nondegenerate) if and only if Hessian is non-singular i.e.the determinant $|H_{f,p}| \neq 0$. We talk about the negative definiteness of matrix Hessian. Index $\lambda(f, p)$ of a critical point p means the integer number of negative eigenvalue of $H_{f,p}$.

We call a function $f : M \to \mathbb{R}$ **Morse function** if it is proper and $H_{f,p}$ is nondegenerate for any $p \in \mathbf{Cr}_f$.

2.2 Existence of Morse Function

After we define the Morse functions on a manifold, it is necessary for us to show Morse function can be found on any manifold.

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2.3 Morse Lemma and Morse Inequalities

3. Morse-Smale Dynamics

4. Symplectic Manifolds and Hamiltonian Flows

5. Conclusion

Acknowledgements

感谢我的导师XXX老师，谢谢他对我的悉心指导。他无私的关爱和严谨的治学态度，将激励我不断的进取，走好以后的道路。其次，还要感谢在这四年的学习中教过我的所有老师们，谢谢他们传授给了我知识。我的同学XXX，在写作的过程中给我提供了一些宝贵的资料和建议，在此一并感谢!

Appendix

/*是正文主体的补充项目，并不是必需的。下列内容可以作为附录：（1）为了整篇材料的完整，插入正文又有损于编排条理性和逻辑性的材料；（2）由于篇幅过大，或取材于复制件不便编入正文的材料；（3）对一般读者并非必须阅读，但对本专业人员有参考价值的资料；（如外文文献复印件及中文译文、公式的推导、程序流程图、图纸、数据表格等）附录按“附录A，附录B，附录A1“等编号。请单击样式“附录1”为第1级的附录编号，样式“附录2”为第二级的附录编号，样式“附录3”控制第三级别的样式。*/

References

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