Invitation to TDA – Theoretical Exercises

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University of Warsaw, Summer 2022

Problem 1 Show that a unit square and unit circle are homeomorphic.

Problem 2 Show that an interval $[0, 1]$ is *not* homeomorphic to $[0, \frac{1}{3}] \cup [\frac{2}{3}, 1]$.

Problem 3 For a given matrix, check if it represent a distance matrix of some discrete metric space. Search for algorithmic criteria that makes a given matrix a distance matrix of some metric space.

Problem 4 Show that a map between metric spaces $f\colon (X, d) \to (X', d')$ is continuous in the sense of epsilon-delta if and only if it is continuous in the sense that preimages of open sets are open.

Problem 5 Prove that a norm $\|\cdot\|$ on a real vector spaces induces a metric via $d(x, y) = \|x - y\|$.

Problem 6 Let X, Y be i.i.d. random variables sampled from the uniform distribution on $[0, 1]$. Show that $\mathbb{E}(|X - Y|) = 1/3$. (In the lecture, it was incorrectly stated that it would be $1/2$).

Problem 7 Search the literature for the proof that Peano curve indeed visits each point in a square.