01

Date 1-Oct 2021

Muhammad NAIMATULLAH Khan
21K - 4540 Math
Exercises = 2:3 , 2:4 , 2:5
Miss Javeria Iftikhar
Assignment # 01

Exercise NO (2:3)

3) $y = 3x^8 + 2x + 1$

differentiate w.r.t x on b/s

$$\frac{dy}{dx} = \frac{d}{dx} 3x^8 + \frac{d}{dx} 2x + \frac{d}{dx}(1)$$

$\therefore \frac{d}{dx} \boxed{x}^n = nx^{n-1}$ (Power Rule).

$$\frac{dy}{dx} = 24x^7 + 2$$

Ans

) $y = \sqrt{2}\,x + \frac{1}{\sqrt{2}}$

differentiate w.r.t x on b/s

$$\frac{dy}{dx} = \frac{d}{dx} (2)^{1/2} x + \frac{d}{dx} (2)^{-1/2}$$

$$\frac{dy}{dx} = (2)^{1/2} \text{ OR } \sqrt{2}$$

Ans

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7) $y = \frac{-1}{3}(x^7 + 2x - 9)$

differentiate w.r.t x on b/s

$\frac{dy}{dx} = \frac{-1}{3} \frac{d}{dx}(x^7 + 2x - 9)$

$\frac{dy}{dx} = \frac{-1}{3} \cdot \frac{d}{dx} \cdot x^7 + \frac{d}{dx} 2x - \frac{d}{dx} 9$

$\frac{dy}{dx} = \frac{-1}{3}(7x^6 + 2)$ Ans

10) $\sqrt{x} + \frac{1}{x} = f(x)$

diff w.r.t x

$\frac{d(f(x))}{dx} = \frac{d}{dx} (x)^{1/2} + x^{-1}$

Apply Power Rule

$f'(x) = \frac{1}{2}(x)^{-1/2} + (-1)(x)^{-2}$

$f'(x) = \frac{1}{2\sqrt{x}} - \frac{1}{x^2}$

Ans

13) $f(x) = x^e + \frac{1}{x^{\sqrt{10}}}$

differentiate w.r.t x

$f'(x) = \frac{d}{dx} x^e + \frac{d}{dx} \frac{1}{x^{\sqrt{10}}}$

$= ex^{e-1} + \frac{d}{dx} (x^{\sqrt{10}})^{-1}$

$= ex^{e-1} - \sqrt{10}\, x^{-1-\sqrt{10}}$

Ans

) $f(x) = (3x^2+1)^2$

Applying $(a+b)^2 = a^2+2ab+b^2$

$f(x) = 9x^4 + 6x^2 + 1$

diff w.r.t x

$$f'(x) = \frac{d}{dx}(9x^4) + \frac{d}{dx}(6x^2) + \frac{d}{dx}(1)$$

$$f'(x) = 36x^3 + 12x$$

Ans

) Find $y'(1)$

$$y = \frac{x^{3/2}+2}{x}$$

$$y = \frac{x^{3/2}}{x} + \frac{2}{x}$$

$$y = (x^{3/2-1}) + 2^{x^{-1}}$$

$$y = (x)^{1/2} + 2x^{-1}$$

diff w.r.t x on b/s

$$\frac{dy}{dx} = \frac{d}{dx}(x)^{1/2} + \frac{d}{dx} 2^{x^{-1}}$$

$$\frac{dy}{dx} = \frac{1}{2}(x)^{-1/2} - 2^{(x^{-2})}$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}} - \frac{2}{x^2}$$

$\frac{1}{2} - 2$

$-\frac{3}{2}$

At $y = 1$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{1}} - \frac{2}{(1)^2}$$

$$\frac{dy}{dx} = \frac{-3}{2}$$

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20). $x = \frac{t^2+1}{3t}$

$x = \frac{t^{\cancel{2}}}{3\cancel{t}} + \frac{1}{3t}$

$x = \frac{t}{3} + \frac{1}{3t}$

$x = \frac{1}{3}\left(t + \frac{1}{t}\right)$

diff w.r.t t on b/s

$\frac{dx}{dt} = \frac{1}{3} \cdot \frac{d}{dt}(t + t^{-1})$

$\frac{dx}{dt} = \frac{1}{3}(1 - t^{-2})$

$\frac{dx}{dt} = \frac{1}{3} - \frac{1}{3t^2}$

Ans

22) $y = \frac{1 + x + x^2 + x^3 + x^4 + x^5 + x^6}{x^3}$

$y = \frac{1}{x^3} + \frac{x}{x^{3}} + \frac{x^2}{x^3} + \frac{x^3}{x^3} + \frac{x^4}{x^3} + \frac{x^5}{x^3} + \frac{x^6}{x^3}$

$y = x^{-3} + x^{-2} + x^{-1} + 1 + x + x^2 + x^3$

diff w.r.t x on b/s

$\frac{dy}{dx} = -3(x)^{-4} - 2(x)^{-3} - (x)^{-2} + 0 + 1 + 2x + 3$

$\frac{dy}{dx} = \frac{-3}{x^4} - \frac{2}{x^3} - \frac{1}{x^2} + 1 + 2x + 3x^2$

at $x = 1$

$\frac{dy}{dx} = -6 + 6 = 0$

Ans

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Exercise 2:3

3) $y = (1-x)(1+x)(1+x^2)(1+x^4)$

Apply formula, $(a-b)(a+b) = a^2 - b^2$

$y = (1-x^2)(1+x^2)(1+x^4)$

Again Apply formula.

$y = (1-x^4)(1+x^4)$

Again Apply formula.

$y = 1 - x^8$

diff w.r.t x on b/s

$\frac{dy}{dx} = \frac{d}{dx}1 - \frac{d}{dx}x^8$

$\frac{dy}{dx} = -8x^7$

at $x = 1$

$\frac{dy}{dx} = -8$

Ans

d^2y/dx^2 (d) $y = (5x^2-3)(7x^3+x)$

$y = 35x^5 + 5x^3 - 21x^3 - 3x$

$y = 35x^5 - 16x^3 - 3x$

Diff. w.r.t x

$\frac{dy}{dx} = 175x^4 - 48x^2 - 3$

Again diff w.r.t x

$\frac{d^2y}{dx^2} = 700x^3 - 96x$

Ans

42) (c) $y = \frac{3x - 2}{5x}$

$y = \frac{3x}{5x} - \frac{2}{5x}$

$y = \frac{3}{5} - \frac{2x^{-1}}{5}$

Diff w.r.t x on b/s

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{3}{5}\right) - \frac{d}{dx} \cdot \frac{(2x)^{-1}}{5}$

$f'(x) = 0 + \frac{2}{5x^2}$

Again diff w.r.t x b/s

$\frac{d^2y}{dx^2} = \frac{2x^{-2}}{5}$

$= \frac{4x^{-4}}{5}$

$f''(x) = \frac{4}{5x^4}$ Ans

45) (c) $\frac{d^4}{dx^4}[x^{-3}] \Big|_{x=1}$

Diff w.r.t x

$f'(x) = \frac{d}{dx}(x^{-3})$

$f'(x) = -3x^{-4}$

Again Diff w.r.t x

$f''(x) = \frac{d}{dx} -3x^{-4}$

$f''(x) = 12x^{-5}$

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Again Differientiate

$f'''(x) = \frac{d}{dx} 12x^{-5}$

$f'''(x) = -60x^{-6}$

Again diff w.r.t x

$f''''(x) = \frac{d}{dx}(-60x^{-6})$

$f''''(x) = \frac{d^4}{dx^4} = 360x^{-7}$

put $x = 1$

$= 360(1)^{-7}$

$= 360$

Ans

7) Show that $y = x^3 + 3x + 1$ satisfies $y''' + xy'' - 2y' = 0$

$y = x^3 + 3x + 1$

Diff w.r.t x

$\frac{dy}{dx} = \frac{d}{dx} x^3 + \frac{d}{dx} 3x + \frac{d}{dx}(1)$

$y' = 3x^2 + 3$ — eq (i)

Again Diff w.r.t x

$y' = \frac{d}{dx} 3x^2 + \frac{d}{dx}(3)$

$y'' = 6x$ — eq (ii)

Again Diffirien tiat w.r.t x

$y'' = \frac{d}{dx} 6x$

$y''' = 6$ — eq (iii)

Requaried

$y''' + xy'' - 2(y') = 0$ —— (A)

put eq i, ii, iii in A

$6 + x(6x) - 2(3x^2+3) = 0$

$\cancel{6} + 6x^2 - 6x^2 - \cancel{6} = 0$

$0 = 0$

Satisfied.

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Exercise: (2:4)

$$\frac{d}{dx} U\cdot V = U\cdot\frac{d}{dx}V + V\cdot\frac{d}{dx}U$$

$$\frac{d}{dx}\frac{U}{V} = \frac{V\cdot\frac{d}{dx}U - U\cdot\frac{d}{dx}V}{V^2}$$

1) $\underset{(U)}{(x+1)}\underset{(V)}{(2x-1)} = f(x)$

diff w.r.t x on b/s

$$f(x) = \frac{d}{dx}(x+1)(2x-1)$$

By Using product rule:

$$\frac{d}{dx} U\cdot V = U\cdot\frac{d}{dx}V + V\cdot\frac{d}{dx}U$$

$$f(x) = (x+1)\cdot\frac{d}{dx}(2x-1) + (2x-1)\cdot\frac{d}{dx}(x+1)$$

$$(x) = (x+1)\cdot(2) + (2x-1)\cdot(1)$$

$$= 2x+2 + 2x-1$$

$$(x) = 4x+1$$

4) $f(x) = (x+1)(x^2-x+1)$

diff w.r.t x

$$f(x) = \frac{d}{dx}(x+1)(x^2-x+1)$$

Using product formula:

$f(x) = (x+1) \cdot \frac{d}{dx}(x^2 - x + 1) + (x^2 - x + 1) \cdot \frac{d}{dx}(x+1)$

$= (x+1) \cdot \frac{d}{dx} x^2 - \frac{d}{dx}(-x) + \frac{d}{dx}(1) + (x^2 - x + 1) \cdot \frac{dx}{dx}$

$= (x+1) \cdot (2x \mp 1) + (x^2 - x + 1)(1)$

$= 2x^2 \cancel{\mp x} + 2x \cancel{\mp 1} + x^2 \cancel{- x} \cancel{+ 1}$

$= 3x^2 + \cancel{2x} + \cancel{x} - \cancel{2x} - \cancel{x}$

$f(x) = 3x^2$

Q.5) $(3x^2 + 6)(2x - 1/4)$

Diff w.r.t x

$f'(x) = \frac{d}{dx}(3x^2 + 6)(2x - 1/4)$

Using Product Rule,

$f'(x) = (3x^2 + 6) \cdot \frac{d}{dx}(2x - 1/4) + (2x - 1/4) \cdot \frac{d}{dx}(3x$

$= (3x^2 + 6)(2) + (2x - 1/4)(\overset{}{\underset{3}{6}}x)$

$= 6x^2 + 12 + 12x^2 - \frac{\cancel{6}x}{\cancel{4}2}$

$f'(x) = 18x^2 - \frac{3x}{2} + 12$

Ans

1) $\left(\frac{1}{x}+\frac{1}{x^2}\right)(3x^3+27)$

diff w.r.t x

$f'(x) = \frac{d}{dx}(x^{-1}+x^{-2})(3x^3+27)$

By Using Product Rule

$f'(x) = (x^{-1}+x^{-2})\cdot\frac{d}{dx}(3x^3+27)+(3x^3+27)\frac{d}{dx}(x^{-1}+x^{-2})$

$= (x^{-1}+x^{-2})\cdot\frac{d}{dx}(3x^3)+\frac{d}{dx}(27)+(3x^3+27)\cdot\frac{d(x^{-1})}{dx}+\frac{d(x^{-2})}{dx}$

$=(x^{-1}+x^{-2})(9x^2)+(3x^3+27)(-x^{-2}-2x^{-3})$

$=(9x+9)+(-3x^{+1}-27x^{-2}-6-54x^{-3}$

$= 9x+9-3x-27x^{-2}-54x^{-3}$

$f'(x) = 3+6x-\frac{27}{x^2}-\frac{54}{x^3}$

Ans

2. $f(x) = \frac{x-2}{x^4+x+1}$

diff w.r.t x

$f'(x) = \frac{d}{dx}\frac{(x-2)}{x^4-x+1}$

= By Using Quotient Rule:

$= \frac{(x^4-x+1)\cdot\frac{d}{dx}(x-2) - (x-2)\cdot\frac{d}{dx}(x^4-x+1)}{(x^4-x+1)^2}$

$= \frac{(x^4-x+1)(1)-(x-2)(4x^3+1)}{(x^4-x+1)^2}$

$= \frac{(x^4-x+1)-(4x^4+x-8x^3+2)}{(x^4-x+1)^2}$

$= \frac{-3x^4+8x^3+3}{(x^4-x+1)^2}$ Ans

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16) $f(x) = (2\sqrt{x}+1)\left(\frac{2-x}{x^2+3x}\right)$

= Simplifying

$(2(x)^{1/2}+1)\left(\frac{2-x}{x^2+3x}\right)$

$= \frac{4(x)^{1/2} - 2(x)^{3/2} + 2 - x}{x^2+3x}$

Now diff w.r.t x

$f(x) = \frac{d}{dx} \frac{4(x)^{1/2} - 2(x)^{3/2} + 2 - x}{x^2+3x}$

Using Quatient Rule:

$f'(x) = \frac{(x^2+3x)\cdot \frac{d}{dx}(4(x)^{1/2} - \frac{d}{dx} 2(x)^{3/2} + \frac{d}{dx}(2) - \frac{d}{dx}(+x - (4x^{1/2} - 2(x)^{3/2} + 2 - x)\cdot \frac{d}{dx}(x^2+3x}{(x^2+3x)^2}$

$= \frac{(x^2+3x)\cdot(\frac{1}{\cancel{2}}\cdot \cancel{4}^{2}(x)^{1/2-1} - \frac{3}{\cancel{2}}\cdot\cancel{2}(x)^{3/2-1} + 0 - \cancel{x} - (4x^{1/2} - 2(x)^{3/2} + 2 - x)\cdot(2x+3}{(x^2+3x)^2}$

$= \frac{(x^2+3x)(2(x)^{-1/2} - 3(x)^{1/2} - 1) - (4x^{1/2} - 2(x)^{3/2} +}{(x^2+3x)^2}$

$= \frac{(2(x)^{3/2} - 3(x)^{5/2} - x^2 + 6x^{1/2} - 9(x)^{3/2} - 3x - (8(x)^{3/2} - 4(x)^{5/2} + 4x - 2x^2 + 12x^{1/2} - 6(x)^{3/2}}{(x^2+3x)^2}$

$= (2(x)^{3/2} - 3((x)^{5/2}) \cancel{-x^2} \cancel{+6x^{1/2}} - 9(x)^{3/2} - \cancel{3x} -$
$+ 4x^{5/2} - 4x + \cancel{2}x^2 - 12x^{1/2} + 6(x)^{3/2} -$

$= \frac{x^{5/2} + x^2 - 9x^{3/2} - 4x - 6x^{1/2} - 6}{(x^2+3x)^2}$

Ans

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). $f(x) = (2x+1)\left(1+\frac{1}{x}\right)(x^{-3}+7)$

simplify

$= (2x+1)(1+x^{-1})(x^{-3}+7)$

$= (2x+2+1+x^{-1})(x^{-3}+7)$

$= (2x+3+x^{-1})(x^{-3}+7)$

It can be find by both method but it simple to expand:

$= 2x^{-2}+14x+3x^{-3}+21+x^{-4}+7x^{-1}$

Diff w.r.t x

$f'(x) = -4(x)^{-3}+14-9(x)^{-4}+0-4(x)^{-5}-7(x)^{-2}$

Ans

) $f(x) = (x^2+1)^4$

Formula:

$(x+1)^n = n(x+1) \cdot \frac{d}{dx}(x+1)$

Diff w.r.t x

$f'(x) = \frac{d}{dx}(x^2+1)^4$

$= 4(x^2+1)^3 \cdot \frac{d}{dx}(x^2+1)$

$= 4(x^2+1)^3 \cdot (2x)+0$

$f'(x) = 8x(x^2+1)^3.$

Ans

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Q23) $Y = \left(\frac{3x+2}{x}\right)(x^{-5}+1)$

Simplify: $(3x+2+x^{-1})(x^{-5}+1)$

$= 3x^{-4} + 3x + 2x^{-5} + 2 + x^{-6} + x^{-1}$

OR

By Using Quatient Rule A

Product Rule Both:

diff w.r.t x

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{3x+2}{x}\right)(x^{-5}+1)$

Apply Product Rule:

$f'(x) = \left(\frac{3x+2}{x}\right)\cdot\frac{d}{dx}(x^{-5}+1) + (x^{-5}+1)\cdot\frac{d}{dx}\left(\frac{3x}{}\right.$

$f'(x) = \left[\left(\frac{3x+2}{x}\right)\cdot(-5x^{-6}) + (x^{-5}+1)\left\{\frac{(x)\cdot\frac{d}{dx}(3x+2) - (3x)(}{(x)^2}\right.\right.$

$= \left[\left(\frac{3x+2}{x}\right)(-5x^{-6}) + (x^{-5}+1)\left(\frac{x\cdot(3) - (3x+2)(}{x^2}\right.\right.$

$= -15x^{-5} - 10x^{-6} + (x^{-5}+1)\left\{\frac{3x - 3x - 2}{x^2}\right.$

$= -15x^{-5} - 10x^{-6} + \cancel{3x^{-4}} + \cancel{3x} - \cancel{3x^{-4}} - \cancel{3x} - 2(x$

$= -15x^{-5} - 10x^{-6} - 2(x^{-5}) - 2$

$= -17x^{-5} - 10x^{-6} - 2$

$x = 1$

$= -17(1)^{-5} - 10(1)^{-6} - 2$

$= -29$

A

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Exercise No 2:5

Find $f'(x)$.

$f(x) = \frac{5}{x^2} + \sin x$

Differentiate w.r.t x on b/s

$f'(x) = \frac{d}{dx} 5(x)^{-2} + \frac{d}{dx}\sin x$

$= \frac{-10}{x^3} + \cos x$

$f(x) = -4x^2 \cos x$

Diff w.r.t x

$f'(x) = \frac{d}{dx}(-4x^2)(\cos x)$

Apply Product Rule:

$f'(x) = (-4x^2)\cdot\frac{d}{dx}\cos x + \cos x\cdot\frac{d}{dx}(-4x^2)$

$= -4x^2\cdot -\sin x + \cos x \; -8x$

$= 4x^2\cdot\sin x - 8x\cos x$

$f(x) = \frac{\sin x}{x^2+\sin x}$

Diff w.r.t x

$f'(x) = \frac{d}{dx}\frac{\sin x}{x^2+\sin x}$

$f'(x) = \frac{(x^2+\sin x)\cdot\frac{d}{dx}(\sin x) - (\sin x)\cdot\frac{d}{dx}(x^2+\sin x)}{(x^2+\sin x)^2}$

$= \frac{(x^2+\sin x)(\cos x) - (\sin x)(2x+\cos x)}{(x^2+\sin x)^2}$

$f'(x) = \dfrac{x^2\cos x + \cancel{\sin x\cos x} - 2x\sin x - \cancel{\sin x\cos x}}{(x^2+\sin x)^2}$

$f'(x) = \dfrac{x^2\cos x - 2x\sin x}{(x^2+\sin x)^2}$

Ans

8) $f(x) = (x^2+1)(\sec x)$

Diff w.r.t x

$f'(x) = \dfrac{d}{dx}(x^2+1)(\sec x)$

$= (x^2+1)\cdot\dfrac{d}{dx}\sec x + \sec x\cdot\dfrac{d}{dx}(x^2+1)$

$= (x^2+1)\cdot\sec x\tan x + \sec x\cdot 2x$

10) $\cos x - x\csc x$

Diff w.r.t x

$f'(x) = \dfrac{d}{dx}(\cos x) - \dfrac{d}{dx}(x)(\csc x)$

$= -\sin x - \left[x\cdot\dfrac{d}{dx}\csc x + \csc x\cdot\dfrac{d}{dx}(x)\right]$

$= -\sin x - [-x\csc x\cot x + \csc x]$

$= -\sin x + x\operatorname{cosec} x\cot x - \operatorname{cosec} x$

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Q14) $f(x) = \dfrac{\sec x}{1+\tan x}$

Diff w.r.t x

$f'(x) = \dfrac{d}{dx} \dfrac{\sec x}{1+\tan x}$

$= \dfrac{(1+\tan x)\cdot \frac{d}{dx}(\sec x) - (\sec x)\cdot \frac{d}{dx}(1+\tan x)}{(1+\tan x)^2}$

$= \dfrac{(1+\tan x)\cdot \sec x \tan x - \sec x \cdot (\sec^2 x)}{(1+\tan x)^2}$

$= \dfrac{\sec x \tan x + \sec x \tan^2 x - \sec^3 x}{(1+\tan x)^2}$

$= \dfrac{\sec x(\tan x + \tan^2 x - \sec^2 x)}{(1+\tan x)^2}$

$\because \tan^2 x - \sec^2 x = -1$

$= \dfrac{\sec x\,(\tan x - 1)}{(1+\tan x)^2}$

Ans

$= f(x) = \sec^2 x - \tan^2 x$

Diff w.r.t x

$f(x) = \dfrac{d}{dx}\sec^2 x - \dfrac{d}{dx}\tan^2 x$

$= 2\sec x \tan x \sec x - 2\tan x \cdot \sec^2 x.$

$= 2\dfrac{1}{\cos x}\cdot\dfrac{\sin x}{\cos x}\cdot\dfrac{1}{\cos x} - 2\dfrac{\sin x}{\cos x}\cdot\dfrac{1}{\cos^2 x}$

$= \cancel{\dfrac{2\sin x}{\cos^3 x}} - 2\cancel{\dfrac{\sin x}{\cos^3 x}}$

$f'(x) = 0$

Ans

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18) $f(x) = \dfrac{(x^2+1)\cot x}{3 - \cos x \csc x}$

Diff w.r.t x

$f'(x) = \dfrac{d}{dx} \dfrac{(x^2+1)\cot x}{3 - \cos x \cdot \csc x}$

$\therefore \csc x = \dfrac{1}{\sin x}$ so $\dfrac{\cos x}{\sin x} = \cot x$

$= \dfrac{d}{dx} \dfrac{(x^2+1)(\cot x)}{(3 - \cot x)}$

= Applying Quatient Rule

$= \dfrac{(3-\cot x) \cdot \frac{d}{dx}\left[\cot x \cdot \frac{d}{dx}(x^2+1) + (x^2+1) \cdot \frac{d}{dx}\right. - (x^2+1)(\cot x) \cdot (-\operatorname{cosec}^2 x)}{(3 - \cot x)^2}$

$\Rightarrow \dfrac{(3-\cot x) \cdot [2x\cot x + (x^2+1) \cdot (-c - (x^2+1)\cot x (\operatorname{cosec}^2 x)}{(3 - \cot x)^2}$

$\Rightarrow \dfrac{6x\cot x - 2x\cot^2 x - 3(x^2+1)(\cdot \operatorname{cosec}^2 x) + \cancel{\cot x}(-\cancel{\operatorname{cosec}^2 x})(\cancel{x^2+1}) - (\cancel{x^2+1})\cot x}{(3 - \cot x)^2}$

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$$(x) = \frac{6x\cot x - 2x\cot^2 x - 3(x^2+1)\csc^2 x}{(3-\cot)^2}$$

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$y = x^2\cos x + 4\sin x$

Diff w.r.t x

$$\frac{dy}{dx} = \frac{d}{dx}(x^2)(\cos x) + \frac{d}{dx}4\sin x$$

Applying product Rule:

$$\frac{dy}{dx} = x^2 \cdot \frac{d}{dx}(\cos x) + (\cos x)\cdot\frac{d}{dx}(x^2) + 4\cos x$$

$$\frac{dy}{dx} = x^2 \cdot -\sin x + \cos x \cdot 2x + 4\cos x$$

$$\frac{dy}{dx} = -x^2\sin x + 2x\cos x + 4\cos x.$$

~~Here~~ given $x'' = 1$

~~Again~~ Derivative.

$$\frac{d^2y}{dx^2} = \left\{\left[(-x^2)\cdot\frac{d}{dx}(\sin x) + (\sin x)\cdot\frac{d}{dx}(-x^2)\right] + 2\right.$$

$$\left[x\cdot\frac{d}{dx}\cos x + \cos x \cdot\frac{d}{dx}(x)\right] + \frac{d}{dx}4\cos x$$

$$\frac{d^2y}{dx^2} = -\left[x^2 \cdot \cos x + \sin x\,(2x)\right] + 2\left[-x\sin x + c\right.$$
$$- 4\sin x$$

$$\frac{d^2y}{dx^2} = -x^2\cos x - 2x\sin x - 2x\sin x + 2\cos$$
$$- 4\sin x$$

$$\frac{d^2y}{dx^2} = (2-x^2)\cos x - 4(x+1)\sin x$$

Ans

24) $Y = \tan x$

diff w.r.t x

$$\frac{dy}{dx} = \frac{d}{dx}\tan x$$

$$\frac{dy}{dx} = \sec^2 x$$

Again diff w.r.t x.

$$\frac{d^2y}{dx^2} = 2\sec x \cdot \frac{d}{dx}\sec x$$

$$\frac{d^2y}{dx^2} = 2\sec x \cdot \sec x\tan x$$

$$\frac{d^2y}{dx^2} = 2\sec^2 x\tan x$$

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Exercise NO: 2:6

$f(x) = \left(x^3 - \frac{7}{x}\right)^{-2}$

$= (x^3 - 7x^{-1})^{-2}$

$=$ Diff w.r.t x

$= \frac{d}{dx}(x^3 - 7x^{-1})^{-2}$

$= -2(x^3 - 7x^{-1}) \cdot \frac{d}{dx}(x^3 - 7x^{-1})$

$= -2\left(x^3 - \frac{7}{x}\right)^{-3} \cdot \left(3x^2 + \frac{7}{x^2}\right)$

A

$f(x) = \sqrt[3]{12 + \sqrt{x}}$

$= (12 + \sqrt{x})^{1/3}$

$=$ Diff w.r.t x

$\frac{1}{3}(12 + \sqrt{x})^{1/3 - 1} \cdot \frac{d}{dx}(12 + (x)^{1/2})$

$= \frac{1}{3}(12 + \sqrt{x})^{-3/2} \cdot 0 + \frac{1}{2}(x)^{-1/2}$

$= \frac{1}{6(12 + \sqrt{x})^{3/2} \cdot \sqrt{x}}$

Ans

$fx = \tan\sqrt{x}$

Diff w.r.t x

$f'(x) = \frac{d}{dx}\tan\sqrt{x}$

$= \sec^2\sqrt{x} \cdot \frac{d}{dx}\sqrt{x}$

$= \sec^2\sqrt{x} \cdot \frac{1}{2}(x)^{-1/2}$

$= \sec^2\sqrt{x} \cdot \frac{1}{2\sqrt{x}}$

Ans

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$f(x) = 4x + 5\sin^4 x$

Diff w.r.t x

$= \frac{d}{dx}(4x) + \frac{d}{dx}(5\sin^4 x)$

$= 4 + 20\sin^3 x \cdot \frac{d}{dx}\sin x$

$= 4 + 20\sin^3 x\cos x$

Ans

19) $f(x) = \cos^2(3\sqrt{x})$

Diff w.r.t x

$\frac{d}{dx}\left((\cos(3\sqrt{x}))^2\right)$

$= 2\cos(3\sqrt{x}) \cdot \frac{d}{dx}(\cos(3\sqrt{x})) \cdot \frac{d}{dx}(3\sqrt{x})$

$= \cancel{2}\cos(3\sqrt{x}) \cdot -\sin(3\sqrt{x}) \cdot \frac{3}{\cancel{2}\sqrt{x}}$

$= \frac{-3\cos(3\sqrt{x}) \cdot \sin(3\sqrt{x})}{\sqrt{x}}$

Ans

22). $f(x) = \cos^3\left(\frac{x}{x+1}\right)$

Diff w.r.t x

$f'(x) = \frac{d}{dx}\left(\cos\left(\frac{x}{x+1}\right)\right)^3$

$= 3\cos\left(\frac{x}{x+1}\right) \cdot \frac{d}{dx}\left(\cos\left(\frac{x}{x+1}\right)\right)$

$= 3\cos\left(\frac{x}{x+1}\right) \cdot \left\{-\sin\left(\frac{x}{x+1}\right)\right\} \cdot \frac{d}{dx}\left(\frac{x}{x+1}\right)$

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$3\cos^2\left(\frac{x}{x+1}\right)\left[-\sin\left(\frac{x}{x+1}\right)\right]\cdot\left[\frac{(x+1)\cdot\frac{d}{dx}(x) - x\cdot\frac{d}{dx}(x+1)}{(x+1)^2}\right]$

$= 3\cos^2\left(\frac{x}{x+1}\right)\left[-\sin\left(\frac{x}{x+1}\right)\right]\cdot\left(\frac{(x+1)(1)-(x)(1)}{(x+1)^2}\right)$

$= -\frac{3}{(x+1)^2}\cos^2\left(\frac{x}{x+1}\right)\sin\left(\frac{x}{x+1}\right)\cdot 1$

Ans

4) $f(x) = \sqrt{3x - \sin^2(4x)}$

diff w.r.t x

$= \frac{d}{dx}\left(3x - (\sin(4x))^2\right)^{1/2}$

$f'(x) = \frac{1}{2}\left(3x - (\sin(4x))^2\right)^{-1/2}\cdot\frac{d}{dx}(3x - (\sin(4x))^2)$

$f'(x) = \frac{1}{2\sqrt{3x - \sin^2 4x}}\cdot 3 - 2\sin 4x\cdot\cos 4x\cdot\frac{d}{dx}(4x)$

$= \frac{1}{2\sqrt{3x - \sin^2 4x}}\cdot 3 - 8\sin(4x)\cdot\cos(4x)$

Ans

5) $f(x) = [x^4 - \sec(4x^2-2)]^{-4}$

Diff w.r.t x on b/s

$f'(x) = \frac{d}{dx}[x^4 - \sec(4x^2-2)]^{-4}$

$= -4[x^4 - \sec(4x^2-2)]^{-5}\cdot$

$= \frac{d}{dx}(x^4 - \sec(4x^2-2))$

$= -4[x^4 - \sec(4x^2-2)]^{-5} \cdot 4x^3 - \sec(4x^2$
$\tan(4x^2-2) \cdot \frac{d}{dx}(4x^2-2)$.

$= -4[x^4 - \sec(4x^2-2)]^{-5} \cdot [4x^3 - \sec(4x^2$
$\tan(4x^2-2) \cdot (8x)$

$= -16x[x^4 - \sec(4x^2-2)]^{-5} \cdot (x^2 - 2\sec(4x$
$\tan(4x^2-2)$

Ans

28) $y = \sqrt{x}\tan^3(\sqrt{x})$

Diff w.r.t x on b/s

$$\frac{dy}{dx} = \frac{d}{dx}(x)^{1/2} \cdot (\tan\sqrt{x})]^3$$

$$\frac{dy}{dx} = (x)^{1/2} \cdot \frac{d}{dx}(\tan(\sqrt{x}))^3 + \tan^3\sqrt{x} \cdot \frac{d}{dx}(x)$$

$$\frac{dy}{dx} = \cancel{(x)^{1/2}} \cdot 3(\tan\sqrt{x})^2 \cdot \sec^2(\sqrt{x}) \cdot \frac{1}{2\cancel{\sqrt{x}}} + \tan^3\sqrt{x} \cdot \frac{1}{2\sqrt{x}}$$

$$\frac{dy}{dx} = \frac{3}{2}(\tan^2\sqrt{x}) \cdot \sec^2(\sqrt{x}) + \frac{1}{2\sqrt{x}} \cdot \tan$$

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) $Y = \cos(\cos x)$

Diff w.r.t x on b/s

$$\frac{dy}{dx} = \frac{d}{dx}\cos(\cos x)$$

$$= -\sin(\cos x)\cdot\frac{d}{dx}\cos x$$

$$= -\sin(\cos x)\cdot -\sin x$$

$$= \sin(\cos x)\cdot\sin x$$

Ans

). $y = \dfrac{1+\csc(x^2)}{1-\cot(x^2)}$

diff w.r.t x on b/s

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1+\csc(x^2)}{1-\cot(x^2)}\right)$$

$$\frac{dy}{dx} = \frac{(1-\cot(x^2))\cdot\frac{d}{dx}(1+\csc(x)^2) - (1+\csc x^2)\cdot\frac{d}{dx}(1-\cot x^2)}{(1-\cot x^2)^2}$$

$$\frac{dy}{dx} = \frac{(1-\cot x^2)\cdot(0-\cot x^2\csc^2\cdot 2x) - (1+\csc x^2)(0+\operatorname{cosec}^2 x^2\cdot 2x)}{(1-\cot x^2)^2}$$

$$= \frac{(1-\cot x^2)(\cancel{2x}\cot x^2\csc x^2) - (1+\csc x^2)(2x\operatorname{cosec}^2 x^2)}{(1\mp\cot x)^2}$$

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$= \dfrac{(2\cot x^2 \operatorname{cosec} x^2) + (\cot^2 x^2 \operatorname{cosec} x^2) - (2x \operatorname{cosec} + (2x \operatorname{cosec}^3 x^2)}{(1-\cot x)^2}$

$= \dfrac{2x(\operatorname{cosec}^2 x^2)(1+\cot x^2 + \csc x^2 +}{(1-\cot x)^2}$

Ans.

36). $y = (x^2+x)^5 (\sin^8 x)$

Diff w.r.t x on b/s

$\dfrac{dy}{dx} = \dfrac{d}{dx} (x^2+x)^5 (\sin x)^8$

$= (x^2+x)^5 \cdot \dfrac{d}{dx} (\sin x)^8 + (\sin x)^8 \cdot \dfrac{d}{dx} (x^2$

$= (x^2+x)^5 \cdot 8\sin^7 x \cdot \cos x + \sin^8 x \cdot 5$

$= 8(x^2+x)^5 (\sin^7 x \cdot \cos x) + 5(\sin^8 x \cdot ($

37) $y = \left(\dfrac{1+x^2}{1-x^2}\right)^{17}$

diff w.r.t x

$\dfrac{dy}{dx} = \dfrac{d}{dx} \left(\dfrac{1+x^2}{1-x^2}\right)^{17}$

$= 17\left(\dfrac{1+x^2}{1-x^2}\right) \cdot \dfrac{d}{dx} \dfrac{1+x^2}{1-x^2}$

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$$17\left(\frac{1+x^2}{1-x^2}\right)^{16}\cdot\left[\frac{(1-x^2)\cdot\frac{d}{dx}(1+x^2)-(1+x^2)\cdot\frac{d}{dx}(1-x^2)}{(1-x^2)^2}\right]$$

$$17\left(\frac{1+x^2}{1-x^2}\right)^{16}\left[\frac{(1-x^2)\cdot(2x)-(1+x^2)(-2x)}{(1-x^2)^2}\right]$$

$$17\left(\frac{1+x^2}{1-x^2}\right)^{16}\left[\frac{2x-2x^3-(-2x-2x^3)}{(1-x^2)^2}\right]$$

$$17\left(\frac{1+x^2}{1-x^2}\right)^{16}\left[\frac{2x-\cancel{2x^3}+2x+\cancel{2x^3}}{(1-x^2)^2}\right]$$

$$=17\left(\frac{(1+x^2)^{16}}{(1-x^2)^{16}}\cdot\frac{4x}{(1-x^2)^2}\right.$$

$$=\frac{68x\,(1+x^2)^{16}}{(1-x^2)^{18}}$$

Ans

$y=[1+\sin^3(x^5)]^{12}$

diff w.r.t x

$$\frac{dy}{dx}=\frac{d}{dx}\left[1+(\sin(x^5))^3\right]^{12}$$

$$=12\left[1+\sin^3(x^5)\right]^{11}\cdot\frac{d}{dx}\left[1+(\sin(x^5))^3\right].$$

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$$\frac{dy}{dx} = 12[1+\sin^3(x^5)]^{11} \cdot (0 + 3\sin^2(x^5)) \cdot \frac{d}{dx} \sin$$

$$\frac{dy}{dx} = 12[1+(\sin^3 x^5)]^{11} \cdot (3\sin^2(x^5)) \cdot \cos x^5$$

$$\therefore \frac{d\, x^5}{dx} = 5x^4$$

$$\frac{dy}{dx} = 12[1+(\sin^3(x^5))^{11} \cdot (3\sin^2(x^5)) \cdot \cos x^5 \cdot 5x$$

$$\frac{dy}{dx} = \{180\, x^4 [1+(\sin^3(x^5))^{11} \cdot (\sin^2(x^5))] \cdot \cos x^5 \cdot$$

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Exercise NO 3:1

Find dy/dx by implicit differentation:

1) $x^3 + y^3 = 3xy^2$

Diff w.r.t x on both side

$$\frac{d}{dx}x^3 + \frac{d}{dx}y^3 = \frac{d}{dx}3xy^2$$

$$3x^2 + 3y^2 \cdot \frac{dy}{dx} = 3\left(x \cdot 2y + y^2\frac{dy}{dx}\right)$$

$$3x^2 + 3y^2 \cdot \frac{dy}{dx} = 6xy\frac{dy}{dx} + 3y^2 \cdot \frac{dy}{dx}$$

$$3y^2 \cdot \frac{dy}{dx} - 6xy \cdot \frac{dy}{dx} = 3y^2 - 3x^2$$

$$\frac{dy}{dx}(3y^2 - 6xy) = 3y^2 - 3x^2$$

$$\frac{dy}{dx} = \frac{3y^2 - 3x^2}{3y^2 - 6xy}$$

$$\frac{dy}{dx} = \frac{\cancel{3}(y^2 - x^2)}{\cancel{3}(y^2 - 2xy)}$$

$$\frac{dy}{dx} = \frac{(y^2 - x^2)}{y^2 - 2xy}$$

Ans

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7) $\frac{1}{\sqrt{x}} + \frac{1}{\sqrt{y}} = 1$

$(x)^{-\frac{1}{2}} + (y)^{-\frac{1}{2}} = 1$

diff w.r.t x on b/s

$\frac{d}{dx}(x)^{-\frac{1}{2}} + \frac{d}{dx}(y)^{-\frac{1}{2}} = \frac{d}{dx}(1)$

$-\frac{1}{2}(x)^{-\frac{1}{2}-1} - \frac{1}{2}(y)^{-\frac{1}{2}-1} \cdot \frac{dy}{dx} = 0$

$-\frac{1}{2}(x)^{-\frac{3}{2}} - \frac{1}{2}(y)^{-\frac{3}{2}} \cdot \frac{dy}{dx} = 0$

$-\frac{1}{\cancel{2}}(y)^{-\frac{3}{2}} \cdot \frac{dy}{dx} = \frac{1}{\cancel{2}}(x)^{-\frac{3}{2}}$

$\frac{-1}{y^{3/2}} \cdot \frac{dy}{dx} = \frac{1}{x^{3/2}}$

$\frac{dy}{dx} = -\frac{y^{3/2}}{x^{3/2}}$

Ans

9) $\sin(x^2y^2) = x$

diff w.r.t x b/s

$\frac{d}{dx}\sin(x^2y^2) = \frac{d}{dx}(x)$

$\cos(x^2 \cdot y^2) \cdot \frac{d}{dx}(x^2 \cdot y^2) = 1$

$\cos(x^2y^2) \cdot \left[x^2 \cdot 2y \cdot \frac{dy}{dx} + y^2 \cdot 2x\right] = 1$

$\cos(x^2y^2) \cdot \left[2x^2y \cdot \frac{dy}{dx} + 2xy^2\right] = 1$

Multiplying

$\cos(x^2y^2) \cdot (2x^2y) \cdot \frac{dy}{dx} + 2xy^2\cos(x^2y^2) =$

d

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$$= \frac{1 - 2xy^2\cos(x^2y^2)}{2x^2y\cos(x^2y^2)}$$

Ans

$$\frac{xy^3}{1+\sec y} = 1+y^4$$

Diff w.r.t x on b/s

$$\frac{d}{dx}\left(\frac{xy^3}{1+\sec y}\right) = \frac{d(1+y^4)}{dx}$$

$$\frac{(1+\sec y)\left[\cancel{x}\cdot 3xy^2\frac{dy}{dx} + \cancel{x}y^3\cdot(\sec y\cdot\tan y)\frac{dy}{dx}\right] - xy^3\cdot(\sec y\tan y)\frac{dy}{dx}}{(1+\sec y)^2} = 4y^3\cdot\frac{dy}{dx}$$

$$(1+\sec y)\left(3xy^2\cdot\frac{dy}{dx} + \cancel{x}y^3\ \text{[illegible]}\ y\right) - xy^3\sec y\tan y\cdot\frac{dy}{dx} =$$

$$y^3(1+\sec y) + (1+\sec y)(3xy^2)\cdot\frac{dy}{dx} - xy^3\sec y\tan\cdot\frac{dy}{dx}$$

$$= 4y^3(1+\sec y)^2$$

$$y^3(1+\sec y) = 4y^3(1+\sec y)^2 - (1+\sec y)(3xy^2) + (x\cancel{3}y^3\sec y\tan y\cdot\frac{dy}{dx})$$

$$\frac{dy}{dx} = \frac{y^{\cancel{3}\,\cancel{2}\,1}(1+\sec y)}{4y^{\cancel{3}\,1}(1+\sec y)^2 - (3xy^2(1+\sec y)) + xy^3\sec y\tan}$$

$$\frac{dy}{dx} = \frac{y\,(1+\sec y)}{4y(1+\sec y)^2 - 3x\cancel{y}(1+\sec y) + xy\sec y\,t}$$

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18). $x \cos y = y$

diff w.r.t x b/s

$$\frac{d}{dx} x \cos y = y \cdot \frac{dy}{dx}$$

$$-\left[x \cdot \sin y \cdot \frac{dy}{dx} + \cos y \cdot (1)\right] = \frac{dy}{dx}$$

$$\cos y = \frac{dy}{dx} + x \sin y \cdot \frac{dy}{dx}$$

$$\cos y = \frac{dy}{dx}(x \sin y + 1)$$

$$\frac{\cos y}{1 + x \sin y} = \frac{dy}{dx}$$

Now Again diff

$$\frac{dy'}{dx} = \frac{d}{dx} \frac{\cos y}{1 + x \sin y}$$

$$\frac{dy}{dx} =$$

$$\frac{dy''}{dx} = \frac{(1+x\sin y)(-\sin y) \cdot \frac{dy}{dx} - (\cos y)\left[x \cos y \frac{dy}{dx} + \right.}{(1 + x \sin y)^2}$$

$$= = \frac{-(1 + x \sin y)(\sin y) \cdot \frac{dy}{dx} - \left[x \cos y - \cos y\right.}{}$$

$$\frac{(x \cos y)\ (2 \sin^2 y + \cos^2 y)}{(1 + x \sin y)^3}$$

$$\frac{-2 \sin y \cos y + (x \cos y)(2 \sin^2 y + \cos^2 y)}{(1 + x \sin y)^3}$$

$$\frac{dy''}{dx} = -\sin 2y + \frac{y(\sin^2 y + 1)}{(1 + \cancel{x\sin}\cancel{\cos} y)^3}$$

Ans.

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Exercise 3:1 Q26

Q26. $y^3 + yx^2 + x^2 - 3y^2 = 0$; $(0, 3)$

slope of 1st derivative =

$m_{Tan} = \frac{dy}{dx}$

Diff w.r.t x

$$\frac{d}{dx} y^3 + \frac{d}{dx} yx^2 + \frac{d}{dx} x^2 - \frac{d}{dx} - 3y^2 = 0$$

$$3y^2 \cdot \frac{dy}{dx} + \left\{ 2xy \cdot \frac{dy}{dx} + x^2 \cdot \frac{dy}{dx} \right\} + 2x - \left\{ 6y \cdot \frac{dy}{dx} \right.$$

$$\frac{dy}{dx}(3y^2 + x^2 - 6y) = -2x - 2xy$$

$$\frac{dy}{dx} = \frac{-2x(1+y)}{3y^2 + x^2 - 6y} = 0$$

here $x = 0$ or $y = 3$

$$\frac{dy}{dx} = \frac{-2(0) \cdot (1+3)}{3((3)^2) + (0)^2 - 6(3)}$$

$$= \frac{4 \cdot (0)}{27 - 18}$$

$$\frac{dy}{dx} = 0 \longrightarrow \text{slope}$$

Ans

$\frac{2}{3}-1$

$-\frac{1}{3}$

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$(-1, 3\sqrt{3})$

Q28). $x^{2/3} + y^{2/3} = 4;$

Diff w.r.t x

$\frac{d}{dx}(x)^{2/3} + \frac{d}{dx}(y)^{2/3} = \frac{d}{dx}(4)$

$\frac{2}{3}(x)^{2/3-1} + \frac{2}{3}(y)^{2/3-1} \cdot \frac{dy}{dx} = 0$

$\frac{2}{3}(x)^{-1/3} + \frac{2}{3}(y)^{-1/3} \cdot \frac{dy}{dx} = 0$

$-\frac{\cancel{2}}{\cancel{3}}(x)^{-1/3} = +\frac{\cancel{2}}{\cancel{3}}(y)^{-1/3} \cdot \frac{dy}{dx}$

$\frac{-(x)^{-1/3}}{(y)^{-1/3}}$

power change by reciprocal

$-\frac{(y)^{1/3}}{(x)^{1/3}} = 0$

here $x = -1$, $y = 3\sqrt{3}$

$-\frac{(3\sqrt{3})^{1/3}}{(-1)^{1/3}} = 0$

$\sqrt{3} = \frac{dy}{dx}$

Point slope formul: $m = \sqrt{3}$ (

$y - 3\sqrt{3} = \sqrt{3}(x+1)$

$y = (x+1)\sqrt{3} + 3\sqrt{3}$

Ans