1) $V = \int_a^b (A(x))\, dx \longrightarrow$ Slicing Method

2) $V = \int_a^b (\pi [f(x)]^2)\, dx \longrightarrow$ Disk Method

3) $V = \int_a^b [\pi (f(x)^2 - g(x)^2]\, dx \longrightarrow$ Washers Method.

Date:

Exercise No 6.2

Q_1

Applying Disk Method

$$= \int_a^b \pi [f(x)^2]\, dx$$

$$= \int_{-1}^{3} \pi [(\sqrt{3-x})^2]\, dx$$

$$= \int_{-1}^{3} \pi (3-x)\, dx$$

$$= \int_{-1}^{3} \pi \left(3x - \frac{x^2}{2}\right)$$

Now applying limit

= Limit = Upper limit - lower limit

$$= \pi \left(3(3) - \frac{(3)^2}{2}\right) - \pi \left(3(-1) - \frac{(-1)^2}{2}\right)$$

$$= \pi \left(\frac{9 - 9}{2}\right) - \pi \left(\frac{-3 - 1}{2}\right)$$

$$= \pi \left(\frac{18 - 9}{2}\right) - \pi \left(\frac{-6 - 1}{2}\right)$$

$$= \pi \left(\frac{9}{2}\right) - \pi \left(\frac{-7}{2}\right)$$

$$= \pi \left(\frac{9 + 7}{2}\right) \Rightarrow 8\pi$$

Q2)

Applying Washers Method

$$V = \int_a^b [f(x)^2 - g(x)^2]\,dx$$

so

$f(x) = 2-x^2$ & $g(x) = x$

$$V = \int_0^1 \pi\left[(2-x^2)^2 - (x)^2\right]dx$$

$$= \int_0^1 \pi\left[4 - 4x^2 + x^4 - x^2\right]dx$$

$$= \int_0^1 \pi\{x^4 - 5x^2 + 4\}\,dx$$

$=$ so

$$\int_0^1 \pi\left[\frac{x^5}{5} - \frac{5x^3}{3} + 4x\right]$$

Now applying limit.

$=$ Upper $-$ lower.

$$= \pi\left[\frac{(1)^5}{5} - \frac{5(1)^3}{3} + 4(1)\right] - \left[\frac{0}{5} - \frac{5(0)}{3} + 4(0)\right]$$

$$= \pi\left[\frac{1}{5} - \frac{5}{3} + 4\right]$$

$$= \pi\left[\frac{3 - 25 + 60}{15}\right]$$

$$V = \frac{38\pi}{15}$$

A

Shajeel

Q_3

Applying Disk Method with respect to y axis:

$y = 3-2x$

so $x = \frac{1}{2}(3-y)$

$$V = \int_c^d \pi [f(x)]^2 dy$$

$$V = \int_0^2 \pi \left[\frac{1}{2}(3-y)\right]^2 dy$$

$$V = \int_0^2 \pi . \frac{1}{4}[(3-y)]^2 dx$$

$$V = \pi \int_0^2 \frac{1}{4}[(9-6y+y^2)] dx$$

$$V = \pi \int_0^2 \frac{1}{4}\left(\frac{y^3}{3} - \frac{6y^2}{2} + 9y\right)$$

$$V = \frac{1}{4}\pi \int_0^2 \left(\frac{y^3}{3} - 3y^2 + 9y\right)$$

= Applying limit.

$$V = \frac{1}{4}\pi \left(\frac{(2)^3}{3} - 3(2)^2 - 9(2)\right) - \left(\frac{0}{3} - 3(0) - 9(0)\right)$$

$$= \frac{1}{4}\pi\left(\frac{8}{3} - 12 + 18\right) \Rightarrow \frac{1}{4}\pi\left(\frac{8}{3} + \cancel{6}\right)$$

$$= \frac{1}{\cancel{4}2}\pi\left(\frac{\cancel{26}\,13}{3}\right) \Rightarrow \frac{13\pi}{6} \text{ Ans}$$

y^{-2+1}

y^{-1}

Date:

Q4

Applying Washer Method: with respect to y axis:

$y = \frac{1}{x}$ so $x = \frac{1}{y}$

$g(x) = \frac{1}{y}$ and $f(x) = 2$

By ~~Applying~~ ~~limit~~ formula.

$$V = \int_{1/2}^{2} \left((2)^2 - \left(\frac{1}{y}\right)^2 \right) dx$$

$$V = \int_{1/2}^{2} \left[4 - \frac{1}{y^2} \right] dx$$

$$V = \int_{1/2}^{2} \left[4x - y^{-1} \right]$$

$$V = \int_{1/2}^{2} \left[4x + \frac{1}{y} \right]$$

$$V = \int_{1/2}^{2} \left[4(2) + \frac{1}{2} \right] - \left[4\left(\frac{1}{2}\right) + \frac{1}{1/2} \right]$$

$$= \left(8 + \frac{1}{2} \right) - \left[2 + 2 \right]$$

$$= \pi \left(\frac{17}{2} \right) - 4$$

$$= \pi \left(\frac{17 - 8}{2} \right)$$

$$V = \frac{9\pi}{2}$$

Ans

Date:

Q7

~~Applying~~ Disk Method

$[a,b] = [-1, 3]$

$$V = \pi \int_{-1}^{3} [(\sqrt{1+y})^2]\, d\cancel{x}y$$

$$V = \pi \int_{-1}^{3} (1+y)\, d\cancel{x}y$$

$$V = \pi \int_{-1}^{3} \left(\cancel{1}y + \frac{y^2}{2}\right)$$

V = ~~App~~ lying limit

$$V = \pi\left(3 + \frac{(3)^2}{2}\right) - \left(-1 + \frac{(-1)^2}{2}\right)$$

$$= \left(3 + \frac{9}{2}\right) - \left(\frac{-\cancel{2} + 1}{2}\right)$$

$$= \pi\left(\frac{6+9}{2}\right) - \left(\frac{-1}{2}\right)$$

$$= \frac{15}{2} + \frac{1}{2}$$

$$= \pi \frac{16}{2}$$

$$= 8\pi$$

~~Ans.~~

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Date:

Q8)

$f(x) = 2$ & $g(x) = x^2 - 1$

It is interms of x so applying washers method with respect to y.

So = $y = x^2 - 1 \Rightarrow y + 1 = x^2 \Rightarrow x = \sqrt{y+1}$

Now put $f(x)$ and $g(x)$ in formula:

$$= \pi \int_0^3 [(2)^2 - (\sqrt{y+1})^2] \, dy$$

$$= \pi \int_0^3 [4 - (y+1)] \, dy$$

$$= \pi \int_0^3 (3 - y) \, dy$$

= Integrating

$$= \pi \Big|_0^3 \left(3y - \frac{y^2}{2}\right)$$

= Applying limit

$$= \pi \left(3(3) - \frac{(3)^2}{2}\right) - \left(0(3) - \frac{(0)^2}{2}\right)$$

$$= \pi \left(9 - \frac{9}{2}\right)$$

$$= \left(\frac{18 - 9}{2}\right)\pi$$

$$= \frac{9\pi}{2}$$

Ans

Date:

9)

Data

$[a, b] = [0, 2]$

$A(x) = \text{Square} \quad (A(x))^2 \quad \Rightarrow$ given condition

Applying Slicing Method.

$$V = \int_a^b (A(x)) \, dx.$$

so

$$V = \int_0^2 [(x^2)^2] \, dx$$

$$= \int_0^2 (x^4) \, dx$$

$$= \int_0^2 \frac{(x^5)}{5} \, \cancel{d}$$

= Applying limit

$$= \frac{(2)^5}{5} - 0$$

$$= \frac{32}{5}$$

Ans.

Date:

10)

Data:

$[a, b] = [\pi/4, \pi/3]$

$y = \sec x$ — condition is square

$A(x) = \sec^2 x$

Applying Volume by Slicing Method.

$$= \int_a^b A(x)\, dx.$$

$$= \int_{\pi/4}^{\pi/3} \sec^2 x \, dx.$$

= Integration

$$\int_{\pi/4}^{\pi/3} (\tan x)$$

Applying limit

$$V = \tan\left(\frac{\pi}{3}\right) - \tan\left(\frac{\pi}{4}\right)$$

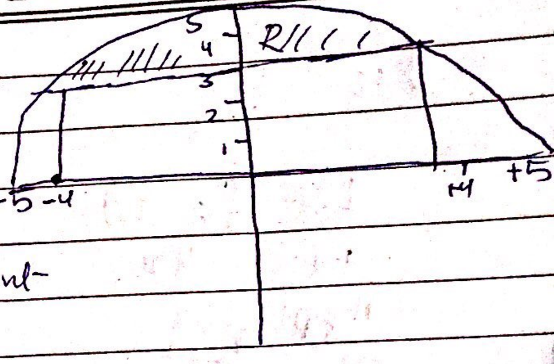
$$V = \sqrt{3} - 1$$

Ans

Same as Q12.

Date:

Q11)



$y = \sqrt{25 - x^2}$, $y = 3$

For finding intersecting point

$y = \sqrt{25 - (3)^2}$

$y = \pm 4$

so $[a, b] = [-4, 4]$

$f(x) = \sqrt{25 - x^2}$ & $g(x) = 3$

Applying Washers method:

$$V = \int_{-4}^{+4} [(\sqrt{25 - x^2})^2 - (3)^2]\, dx$$

$$= \pi \int_{-4}^{+4} ((25 - x^2) - 9)\, dx$$

~~Integ~~

$$= \pi \int_{-4}^{+4} (16 - x^2)\, dx$$

Integrating.

$$= \pi \left[\left(16x - \frac{x^3}{3}\right) \right]_{-4}^{+4}$$

Applying limit

$$= \pi \left(16(4) - \frac{(4)^3}{3}\right) - \left(16(-4) - \frac{(-4)^3}{3}\right)$$

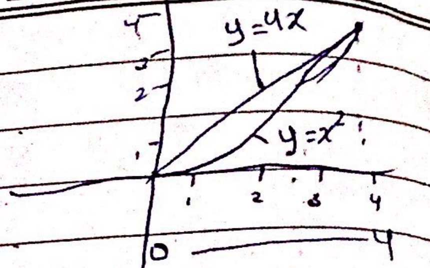
$$= \pi \left(64 - \frac{64}{3}\right) - \left(-64 + \frac{64}{3}\right)$$

$$\left(\frac{192 - 64}{3}\right) - \left(\frac{-192 + 64}{3}\right)$$

$$= \pi \left(\frac{128}{3}\right) - \left(\frac{-128}{3}\right) \Rightarrow \frac{128}{3} + \frac{128}{3}$$

$$= \frac{256\pi}{3}$$ Ans.

Date:



Q15

Let $[a, b] = [0, 4]$

$f(x) = 4x$ & $g(x) = x^2$

Applying Washers Method. with respect to x.

$$= \pi \int_0^4 [(4x)^2 - (x^2)^2]\, dx$$

$$= \pi \int_0^4 [16x^2 - x^4]\, dx.$$

Integrating

$$= \pi \int_0^4 \left[\frac{16(x^3)}{3} - \frac{x^5}{5} \right] \cancel{dx}.$$

$$= \pi \int_0^4 \left| \frac{16x^3}{3} - \frac{x^5}{5} \right|.$$

Applying limit.

$$= \left(\frac{16(4)^3}{3} - \frac{(4)^5}{5} \right) - (0)$$

$$= \left(\frac{16(64)}{3} - \frac{1024}{5} \right)$$

$$= \frac{1024}{3} - \frac{1024}{5}$$

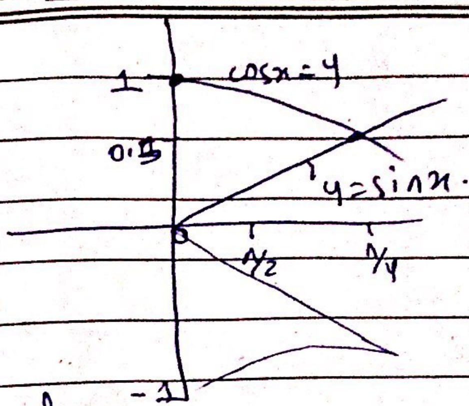
$$= \frac{5120 - 3072}{15}$$

$$= \frac{2048\pi}{15}$$

A

Date:

(4)



Let $[a,b] = [0, \pi/4]$

$f(x) = \cos x$, $g(x) = \sin x$

Applying Washers method :

$$\int_0^{\pi/4} \pi[(\cos x)^2 - (\sin x)^2]\, dx$$

$$\int_0^{\pi/4} [\cos^2 x - \sin^2 x]\, dx$$

$\because \cos^2 x - \sin^2 x = \cos 2x$

$$\int_0^{\pi/4} [\cos 2x]\, dx .$$

Integrating.

$$\int_0^{\pi/4} \frac{\sin 2x}{2}$$

Applying limits -

$$= \pi \frac{\sin 2(\pi/4)}{2} - 0$$

$$= \pi \frac{(1)}{2}$$

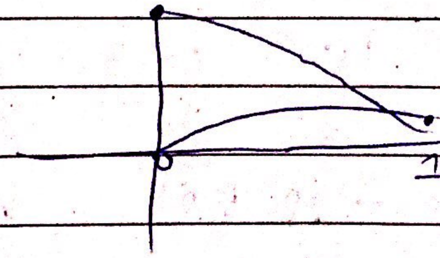
$$= \frac{\pi}{2}$$

Ans

16)

$y = e^{-2x}$, $y = 0$, $x = 0$, $x = 1$

let $[a, b] = [0, 1]$



so $f(x) = e^{-2x}$

$$= \int_0^1 [e^{-2x}]^2 \, dx$$

$$= \int_0^1 [e^{-4x}] \, dx$$

Integrating

$$= \pi \int_0^1 \frac{-1}{4} \cdot e^{-4x}$$

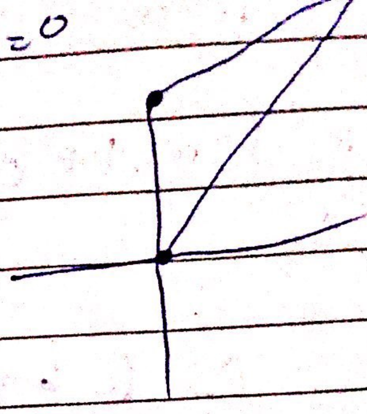
$$= \frac{1}{4} \pi \int_0^1 -e^{-4x}$$

Applying limits:

so $\frac{1}{4} \pi (1 - e^{-4})$

Ans.

$Y = \frac{e^{3x}}{\sqrt{1+e^{6x}}}$, $x=0$, $x=1$, $y=0$



let $[a,b] = [0,1]$

$f(x)$

Now

Applying Washers method:

$= V_x \int_0^1 \pi \left[\frac{e^{3x}}{\sqrt{1+e^{6x}}} \right]^2 dx$

$= \pi \int_0^1 \left[\frac{e^{6x}}{1+e^{6x}} \right] dx$

= Integrating.

$= \pi \int_0^1 \left[\frac{(e^x)^6}{1+(e^x)^6} \right] dx$

let $u = e^x$

$du = e^x . dx$

$\frac{du}{e^x} = dx$

$= \pi \int_0^1 \left[\frac{(u)^{\cancel{6}5}}{1+(u)^6} . \frac{du}{\cancel{u}} \right]$

$= \pi \int_0^1 \left[\frac{u^5}{1+(u)^6} . du \right]$

$= \pi \int_0^1 \frac{1}{6} (\ln(1+e^{6x})$

$\frac{2}{3}+1$

$\frac{2+3}{3}$ $\frac{5}{3}$

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Now Applying limit:

$\frac{\pi}{6}\left(\ln(1+e^{6(1)}) - \ln(1+e^{6(0)})\right)$

$\frac{\pi}{6}(6.002 - 0.693)$

$V = \frac{5.309\pi}{6}$

Ans.

19)

Data

$[a,b] = [0,1]$

$y = x^3$, $y = 0$, $y = 1$

so taking cuberoot on b/s

$y^{1/3} = x^{3\times\frac{1}{3}}$

$y^{1/3} = x$

so

Applying Slicing Method:

$A(x) = (y^{1/3})^2$

$V = \int_0^1 (y^{1/3})^2 \, dx$

$V = \int_0^1 (y^{2/3}) \, dx$

$V = \int_0^1 \left(\frac{y^{5/3}}{5/3}\right)$

Date:

Applying limit

$$= \frac{(1)^{5/3}}{5/3} - (0)$$

$$= \frac{3}{5}$$

Ans

Q20

Data,

$x = 1-y^2$ & $A(x) = (1-y^2)^2$

$[a,b] = [-1, 1] \longrightarrow$ They are making equation continuous.

So Applying Volume By slicing Method.

$$= \int_{-1}^{1} (1-y^2)^2 \, dy$$

$$= \int_{-1}^{1} (1-2y^2+y^4)\, dy$$

Integrating.

$$= \int_{-1}^{1} \left(y - \frac{2y^3}{3} + \frac{y^5}{5}\right) d$$

$$\left(1 - \frac{2(1)^3}{3} + \frac{1^5}{5}\right) - \left(-1 - \frac{2(-1)^3}{3} + \frac{(-1)^5}{5}\right)$$

$$= \left(1 - \frac{2}{3} + \frac{1}{5}\right) - \left(-1 - \frac{-2}{3} - \frac{1}{5}\right) \Rightarrow \left(-2 - \frac{1}{3}\right)$$

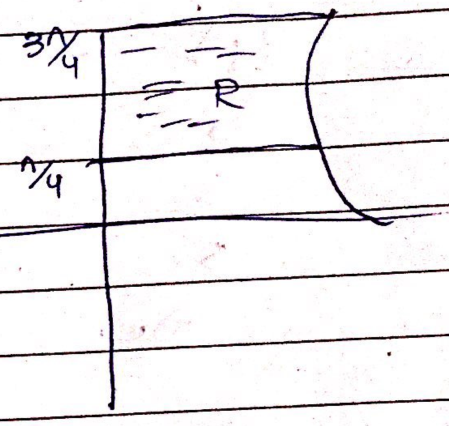
$$\left(\frac{1}{3}\right) - \left(\frac{-7}{3}\right) \Rightarrow \frac{16}{15}$$

Ans.

Date:

2) $x = \operatorname{cosec} y$, $x = 0$, $y = \pi/4$, $y = 3\pi/4$

so $[c, d] = [\pi/4, 3\pi/4]$



Using Disk Method for that.

so

$$V = \pi \int_c^d [(f(y))^2] \, dy$$

$$V = \pi \int_{\pi/4}^{3\pi/4} (\csc y)^2 \, dy$$

Integrating

$$= \pi \int_{\pi/4}^{3\pi/4} -\cot x$$

Applying limit

$$= \pi(-\cot(3\pi/4) - (-\cot(\pi/4)))$$

$$= \pi(+1 + 1)$$

$$= 2\pi$$

Ans.

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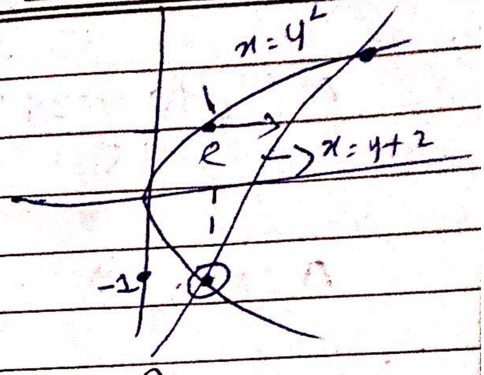

Q_{23}

$x = y^2 \quad , \quad x = y+2$

so $y^2 - y + 2 = 0$

$(y+1)(y-2) = 0$

$y = -1 \qquad y = 2$ (end points)

so $f(x) = y+2$ & $g(x) = y^2$

Now Applying Washer's Method.

$= \pi \int_{-1}^{2} [(y+2)^2 - (y^2)^2] \, dy$

$= \pi \int_{-1}^{2} [y^2 + 4y + 4 - y^4] \, dy$

= Integrating

$= \pi \int_{-1}^{2} \left[\frac{y^3}{3} + \frac{4y^2}{2} + 4x - \frac{y^5}{5} \right]$

Applying limit

$= \pi \left(\frac{(2)^3}{3} + \frac{\cancel{4}(2)^2}{\cancel{2}_1} + 4(2) - \frac{(2)^5}{5} \right) - \left(\frac{(-1)^3}{3} + \frac{4(-1)^2}{2} + 4(-1) - \frac{(-1)}{5} \right)$

$= \pi \left(\frac{8}{3} + 8 + 8 - \frac{32}{5} \right) - \left(\frac{-1}{3} + 2 - 4 + \frac{1}{5} \right)$

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Now Applying limit.

$$= \frac{\pi}{2}\left(e^{2(1)}\right) - \left(e^{2(0)}\right)$$

$$= \frac{\pi}{2}\left(e^{2} - 1\right)$$

Ans.

26) $y = \sqrt{\frac{1-x^2}{x^2}}$ $(x>0)$, $x=0$, $y=0$, $y=2$

$[a, b] = [0, 2]$

$f(x) = \sqrt{\frac{1-x^2}{x^2}}$

Applying changing to $f(y)$

$y^2 = \frac{1-x^2}{x^2}$

$x^2 = \frac{1+y^2}{x^2}$

$x^4 = 1+y^2$

$x^2 = \sqrt{1+y^2}$

taking root

$x = 2\sqrt{1+y^2}$

So

Date:

$$\pi\left(\frac{40+240-96}{15}\right)-\left(\frac{-5-30+3}{15}\right)$$

$$\pi\left(\frac{184}{15}\right)-\left(\frac{-32}{15}\right)$$

$$V=\frac{72\pi}{5}$$

Ans.

Q25

$y = \ln x \quad , \quad x = 0 \quad , \quad y = 0 \quad , \quad y = 1$

$[a,b] = [0,1]$

Now

$$\int_0^1 \pi\left[(\ln x)^2 - (0)^2\right]dx$$

$\therefore \ln x = e^y$

$$\int_0^1 \pi\,(e^y)^2 \cdot dy$$

$$\int_0^1 \pi\,(e^{2y}) \cdot dy$$

$$= \int_0^1 \pi\,\frac{1}{2} \cdot e^{2y}$$

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$$V = \pi \int_0^2 \left(\frac{1}{1+y^2} \right) dy$$

$$\therefore \int \tan^{-1} x = \frac{1}{1+x^2}$$

so

$$V = \pi \int_0^2 \tan^{-1} y$$

Now Applying limit

$$V = \pi \left(\tan^{-1}(2) \right) - \left(\tan^{-1}(0) \right]$$

$$V = 1.01\,\pi \quad \text{OR} \quad \pi \tan^{-1}(2)$$

Ans.