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Date ______ 20 ____

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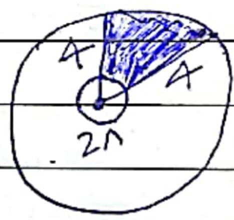
Q11)

We have to find $\frac{dA}{dt} = ?$

The π is equal to $r\theta$.

The minute had taken 60 min for one revolution then

$$\frac{d\theta}{dt} = \frac{\pi}{30} \text{ rad/min}$$



$$A = \frac{1}{2}\pi^2 = \frac{1}{2}r^2\theta$$

so

$$\frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt} = \cancel{8}$$ (r is constant)

By solving we get

$$\frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt} = \frac{1}{2}(4)^2 \cdot \left(\frac{\pi}{30}\right) =$$

so

$$\frac{dA}{dt} = \boxed{\frac{4\pi}{15} \text{ in}^2/\text{min}}$$ Ans

(2)

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Q12)

Solution:

Area of a circle:

$A = \pi r^2$

radius increases at a constant rate of 3 ft/s.

so

$\frac{dr}{dt} = 3 \text{ ft/sec}$

The time given is

$t = 10 \text{ sec}$.

so r will be

$r = \frac{dr}{dt} \cdot t$

$r = 3 \text{ ft/s} \times 10 \cancel{s}$

$r = 30 \text{ ft}$.

We have to find $\frac{dA}{dt}$ so

$\frac{dA}{dt} = \pi r^2$

Diff w.r.t t

$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$

$\therefore r = 30$ & $\frac{dr}{dt} = 3 \text{ ft}$

$\frac{dA}{dt} = 2\pi(30 \text{ ft})(3 \text{ ft/s}) = \boxed{180\pi \text{ ft}^2/\text{s}}$

dr

Q13)

Solution:

Area of circles

$A = \pi r^2$

Diff w.r.t t b/s

So

$$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt} \quad \text{— eq (1)}$$

We have given that Area.

$A = \pi r^2$

$9 = \pi r^2$

$\frac{9}{\pi} = r^2$

root on b/s

$\frac{3}{\sqrt{\pi}} = r$

We haven given that $\frac{dA}{dt} = 6 mi^2/h$.

So put r & $\frac{dA}{dt}$ in eq 1

eq 1 Employes:

$$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$$

$$6 = 2\pi \frac{(3)}{(\sqrt{\pi})} \cdot \frac{dr}{dt}$$

$$6 = \frac{6\cancel{\sqrt{\pi}} \cdot \sqrt{\pi}}{\cancel{\sqrt{\pi}}} \cdot \frac{dr}{dt}$$

$$6 = 6\sqrt{\pi} \cdot \frac{dr}{dt}$$

So $\frac{dr}{dt} = \frac{\cancel{6}}{\cancel{6}\sqrt{\pi}}$

$\frac{dr}{dt} = \frac{1}{\sqrt{\pi}}$ mi/h

Ans

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Q14)

Solution:

Volume of circle is

$V = \frac{4}{3} \pi r^3$ — eq A

here given $\frac{dt}{dx} = 3$

given $r = 1$

$r = \frac{d}{2}$ — eq i

$d = 2r$

$d = 2(1)$

$d = 2$

put eq i to eq A

$V = \frac{4}{3} \pi r^3$

so $V = \frac{dt}{dx}$

$\frac{dt}{dx} = \frac{4}{3} \pi \left(\frac{d}{2}\right)^3$

$\frac{dt}{dx} = \frac{\cancel{4}^1}{3} \pi \frac{d^3}{\cancel{8}\,2}$

$\frac{dt}{dx} = \frac{\pi d^3}{6}$

Q4

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diff w.r.t x

$$\frac{dt}{dx} = \frac{1}{\cancel{6}_2} \cancel{3}\pi d^2 \cdot \frac{dD}{dt}$$

$$\frac{dt}{dx} = \frac{1}{2} \pi d^2 \cdot \frac{dD}{dt}$$

$$\frac{2}{\pi d^2} \frac{dt}{dx} = \frac{dD}{dt}$$

this is because we have to find diameter.

here $d = 2$

$$\frac{2}{\pi (2)^2} \cdot \frac{dt}{dx} = \frac{dD}{dt}$$

here $\frac{dt}{dx} = 3$ (given)

$$\frac{\cancel{2} \cdot 3}{\cancel{4}_2 \pi} = \frac{dD}{dt}$$

$$\frac{3}{2\pi} = \frac{dD}{dt}$$

Q15)

$r = 9$

$\frac{dr}{dt} = -15$ (given)

Volume of ~~circle~~ (sphere) is

$V = \frac{4}{3}\pi r^3$

diff w.r.t t b/s

$\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$

$r = 9$ $\frac{dr}{dt} = -15$ (we take +ve) Because air is removing from Ballon.

$= 4\pi(9)^2(+15)$

$\frac{dV}{dt} = 4\pi(81)(+15)$

$\frac{dV}{dt} = 4860\pi \ cm^3/min$

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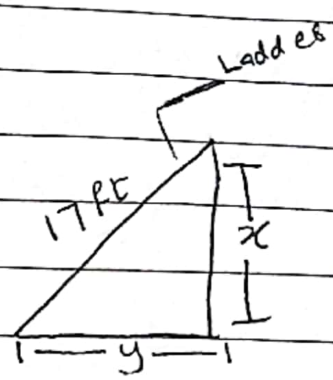
Q16)

Solution:



Given

$\frac{dy}{dt} = 5$ ft/s

$x = 8$ ft

We have to find $\frac{dx}{dt}$

Applying hypotenios formula.

$(B)^2 + (P)^2 = H^2$ OR $x^2 + y^2 = (17)^2$ — A

$(8)^2 + (y)^2 = (17)^2$

$64 + y^2 = 289$

$y^2 = 225$

$y = 15$

Diff eq A

eq A employes

$x^2 + y^2 = (17)^2$

diff w.r.t t on b/s

$\frac{d}{dt} x^2 + \frac{d}{dt} y^2 = \frac{d}{dt} (17)^2$

$2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} = 0$ — i

$\therefore$ $x = 8$ $y = 15$ & $\frac{dy}{dt} = 5$

put in eq (i)

$2(8) \frac{dx}{dt} = -2(15)(5)$

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Q7

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$$16 \frac{dx}{dt} = -150$$

$$\frac{dx}{dt} = \frac{-150}{16}$$

$$\frac{dx}{dt} = -9.375 \text{ ft/s}$$

Ans

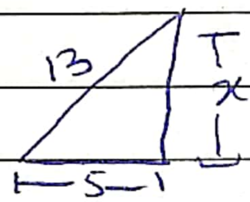
Q17)

Solution:

Let y be the distance from the top Ladder to the ground.
Let x be the distance from the foot of the ladder to the wall.

Given that $\frac{dy}{dt} = -2$ ft/sec

$y = 5$ feet



Find : $\frac{dx}{dt}$

$x^2 + y^2 = (13)^2$

diff w.r.t t b/s

$2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} = 0$

$\frac{dx}{dt} = \frac{-\cancel{2}y}{\cancel{2}x} \cdot \frac{dy}{dt} \Rightarrow \frac{-y}{x} \cdot \frac{dy}{dt}$ — (A)

$x^2 + y^2 = (13)^2$ here $y = 5$

$x^2 + (5)^2 = (13)^2$

$x = 12$ feet so

$\frac{dx}{dt} = \frac{-5}{\cancel{12}\,6} \cdot (-\cancel{2}\,1)$

$\frac{dx}{dt} = \frac{5}{6}$ ft/sec.

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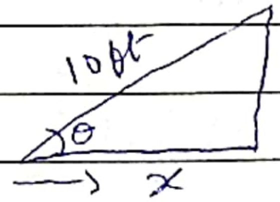
Q18)

Solution:



$$\cos\theta = \frac{B}{H} = \frac{x}{10}$$

$$x = 10\cos\theta$$

Now Diff w.r.t t on b/s

$$\frac{dx}{dt} = -10\sin\theta \cdot \frac{d\theta}{dt}$$

$$\therefore \quad \sin\theta = \frac{\sqrt{96}}{10} \quad \& \quad \frac{dx}{dt} = -6 \text{ in/sec} \Rightarrow \frac{-1}{2} \text{ ft/sec}$$

$$\frac{-1}{2} = -\cancel{10}\left(\frac{\sqrt{96}}{\cancel{10}}\right) \cdot \frac{d\theta}{dt}$$

$$\frac{-1}{2} \cdot \frac{-1}{\sqrt{96}} = \frac{d\theta}{dt}$$

$$\frac{1}{2\sqrt{96}} = \frac{d\theta}{dt}$$

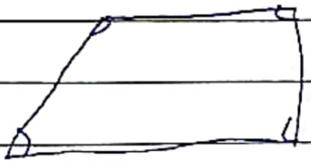
When the bottom of the plank is 2 ft from the wall the angle θ increase at $\frac{1}{2\sqrt{96}}$ OR 0.051 rad/sec

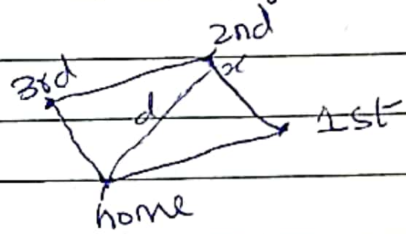
Q19)

Solution:

According to pythagoras theorem

$(B)^2 + (\cancel{H})^2 = H^2$ OR $x^2 + y^2 = z^2$





$\therefore z^2 = x^2 + (60)^2$ — eq 1

Diff eq i w.r.t t on b/s

$$\cancel{2}z \cdot \frac{dz}{dt} = \cancel{2}x \cdot \frac{dx}{dt} + 0$$

$$\cancel{z} \cdot \frac{dz}{dt} = \frac{x}{z} \cdot \frac{dx}{dt}$$

The $x = 50$ (given in question)

so put x in eq i

$z^2 = (50)^2 + (60)^2 = 6100$

$z = 10\sqrt{61}$ ft

Also the speed is given from 1st Base to 2nd Base 25 ft/s

so

$$\frac{dz}{dt} = \frac{x}{z} \cdot \frac{dx}{dt}$$

$$\frac{dz}{dt} = \frac{50}{10\sqrt{61}}(25)$$

$$= \frac{125}{}$$

Q20)

Solution:

Let z be the distance between the radar station and the rocket.
Let h be the height of rocket.

We Know $\frac{dz}{dt} = 2000$

Find $\frac{dh}{dt} = ?$

By pythagoras theorem

$x^2+y^2=z^2$ OR $5^2+h^2=z^2$ — eq i

Diff eq i w.r.t t b/s

$2z \cdot \frac{dz}{dt} = 0 + \frac{dh}{dt} \cdot 2h$

$\frac{\cancel{2}z}{\cancel{2}h} \cdot \frac{dz}{dt} = \frac{dh}{dt}$ — eq ii

$\therefore$ $h = 4$ so

$5^2 + (4)^2 = z^2$

$\sqrt{41} = z$, $h = 4$, & $\frac{dz}{dt} = 2000$

put all in eq ii

$\frac{\sqrt{41}}{4} \times 2000 = \frac{dh}{dt}$

$\frac{dh}{dt} = 3201.56$ miles per hour

$(500\sqrt{41})$