Solution

Math 1401 **Weekly Assignment 6** **Due Date:** 3/11/10

Show all work and box all answers. No work shown or messy work = no credit. Make sure your final answers are in proper form. 40 pts possible divided by 2 to give your total out of 20 pts for handout and 5 points for book work.

Book Work

Sec 3.4 # 11, 13, 17, 25

Sec 3.5 # 7, 11, 43

Sec 3.6 # 7, 11, 15, 21, 25, 35, 37, 39

$\frac{38}{2} + 5 = 19 + 5 = 24$

1. (3 pts) A stone dropped into a still pond sends out a circular ripple whose radius increases at a constant rate of 3 ft/s. How rapidly is the area enclosed by the ripple increasing at the end of 10 s?

$A = \pi r^2$

$\frac{dr}{dt} = 3 \text{ ft/s}$

$t = 10 \text{ s}$

$r = \frac{dr}{dt} \cdot t$

$r = 3 \text{ ft/s} \cdot 10 \text{ s}$

$r = 30 \text{ ft}$

$\left.\frac{dA}{dt}\right|_{t=10} = 2\pi r \frac{dr}{dt}$

$\left.\frac{dA}{dt}\right|_{r=30 \text{ ft}} 2\pi(30 \text{ ft})(3 \tfrac{\text{ft}}{\text{s}})$

$\boxed{\frac{dA}{dt} = 180\pi \text{ ft}^2/\text{s}}$

Ans: $180\pi \frac{\text{ft}^2}{\text{s}}$

2. (3 pts) A spherical balloon is inflated so that its volume is increasing at the rate of 3 feet^3/min. How fast is the diameter of the balloon increasing when the radius is 1 ft?

$r = 1 \text{ ft}$

$\frac{dV}{dt} = 3 \text{ ft}^3/\text{min}$

$t = ?$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = \frac{4}{\cancel{3}} \cdot \cancel{3} \cdot \pi \cdot r^2 \cdot \frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{1}{4\pi r^2} \cdot \frac{dV}{dt}$

$\left.\frac{dr}{dt}\right|_{r=1 \text{ ft}} = \frac{1}{4\pi(1)^2} \cdot 3 \text{ ft}^3/\text{min}$

$\frac{dr}{dt} = \frac{3 \text{ ft/min}}{4\pi} \cdot 2 = \frac{3}{2\pi}$

$D = 2r$

Ans: $\frac{3}{2\pi}$ ft/min

3. (3 pts) A 17 ft ladder is leaning against a wall. If the bottom of the ladder is pulled along the ground away from the wall at a constant rate of 5 ft/s, how fast will the top of the ladder be moving down the wall when it is 8 ft above the ground?

$D = 17\text{ ft}$

$\frac{dx}{dt} = 5\text{ ft/s}$

$y = 8\text{ ft}$

$\frac{dy}{dt} = ?$

$\sqrt{17^2 - 8^2} = \sqrt{x^2}$

$15 = x$

$17^2 = x^2 + y^2$

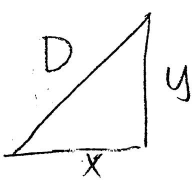
$0 = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$

$\frac{-\cancel{2}x\frac{dx}{dt}}{\cancel{2}y} = \frac{\cancel{2y}\frac{dy}{dt}}{\cancel{2y}}$

$-\frac{x}{y}\frac{dx}{dt} = \frac{dy}{dt}$

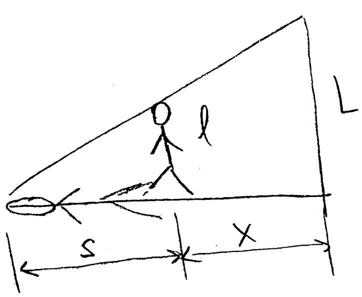
$-\frac{15}{8} \cdot 5 = \frac{dy}{dy}$

$-\frac{75}{8}\text{ ft/s}$



Ans: $-\frac{75}{8}$ ft/s ✓

4. (3 pts) A man 6 ft tall is walking at what rate of 3 ft/s toward a streetlight 18 ft high. At what rate is his shadow length changing?



$l = 6\text{ ft}$

$L = 18\text{ ft}$

$\frac{dx}{dt} = 3\text{ ft/s}$

$\frac{ds}{dt} = ?$

Similarity of Δ's:

$\frac{l}{s} = \frac{L}{s+x}$

$l(s+x) = Ls$

$s = \frac{lx}{L-l}$

$\frac{ds}{dt} = \frac{l}{L-l}\frac{dx}{dt}$

$= \frac{6}{18-6} \cdot 3 = \frac{6}{12} \cdot 3 = \frac{1}{2} \cdot 3 = \frac{3}{2}\text{ ft/s}$

Note: if $\frac{dx}{dt} = -3\text{ ft/s} \Rightarrow \frac{ds}{dt} = -\frac{3}{2}\text{ ft/s}$

but here we rather care about the abs. value.

5. (2 pts) Find the local linear approximation of $f(x) = \frac{1}{x}$ at $x_0 = 1$. Use this linear approximation to estimate $\frac{1}{1.1}$.

$\frac{1}{x} \approx \frac{1}{x_0} - \frac{1}{x_0^2}(x - x_0)$

$\frac{1}{x} \approx \frac{1}{1} - \frac{1}{1}(x-1) \Rightarrow 1-(x-1)$

$\frac{1}{1.1} \approx 1-(1.1-1) = \boxed{.9}$

Ans: 0.9

$\frac{1}{1.1} \approx$ 0.91

6. (22 pts) Find the following limits.

(a) $\lim_{x\to 0} \frac{\sin(2x)}{sin(5x)}$ $= \frac{0}{0}$

$\lim_{x\to 0} \frac{\frac{d}{dx}(\sin 2x)}{\frac{d}{dx}(\sin 5x)}$

$\lim_{x\to 0} \frac{2\cos 2x}{5\cos 5x}$

$\lim_{x\to 0} \frac{2\cos 2(0)}{5\cos 5(0)} = \frac{2\cdot 1}{5\cdot 1}$

$= \boxed{\frac{2}{5}}$

Ans: $\frac{2}{5}$

(b) $\lim_{t\to 0} \frac{te^t}{1-e^t}$ $\frac{0\cdot e^0}{1-e^0} = \frac{0\cdot 1}{1-1} = \frac{0}{0}$

$\lim_{t\to 0} \frac{\frac{d}{dx} te^t}{\frac{d}{dx} 1-e^t} = \lim_{t\to 0} \frac{(t\cdot e^t + e^t\cdot 1)}{-e^t} = \lim_{t\to 0} \frac{te^t+e^t}{-e^t}$

$\lim_{t\to 0} \frac{0\cdot e^0 + e^0}{-e^0} = \frac{0\cdot 1+1}{-1} = \frac{1}{-1} = \boxed{-1}$

Ans: -1

(c) $\lim_{x\to 0^+} \frac{1-\ln(x)}{e^{\frac{1}{x}}} = \frac{1-\ln(0)}{e^{\frac{1}{0}}} = \frac{1+(+\infty)}{\infty} = \frac{\infty}{\infty}$

$\lim_{x\to 0^+} \frac{\frac{d}{dx}(1-\ln x)}{\frac{d}{dx} e^{1/x}} = \lim_{x\to 0^+} \frac{-\frac{1}{x}}{e^{1/x}} =$

$\lim_{x\to 0^+} \frac{-\frac{1}{0}}{e^{1/0}} = \frac{-\infty}{+\infty} = -\infty$

$\lim_{x\to 0^+} \frac{\frac{d}{dx} -\frac{1}{x}}{\frac{d}{dx} e^{1/x}} = \frac{-\frac{1}{x^2}}{e^{1/x}} = \frac{0}{+\infty} = \boxed{0}$

Ans: 0

(d) $\lim_{x\to 0^+} \tan(x)\ln(x) = \tan(0)\cdot\ln(0) = 0\cdot(-\infty)$

$\lim_{x\to 0^+} \frac{\ln x}{\cot x} = \lim_{x\to 0^+} \frac{\frac{d}{dx}\ln x}{\frac{d}{dx}\cot x} = \frac{-\infty}{\infty}$

$\lim_{x\to 0^+} \frac{\frac{1}{x}}{-\csc^2 x} = -\frac{\sin^2 x}{x} = \frac{0}{0}$

$\lim_{x\to 0^+} \frac{-2\sin x\cos x}{1} = \frac{-2(0)(1)}{1} = \frac{0}{1} = 0$

Ans: 0

(e) $\lim_{x\to 0} \left(\frac{1}{x^2} - \frac{\cos(3x)}{x^2}\right) = \infty - \infty$

$\lim_{x\to 0} \frac{1-\cos 3x}{x^2}$

$\lim_{x\to 0} \frac{\frac{d}{dx}(1-\cos 3x)}{\frac{d}{dx} x^2}$

$\lim_{x\to 0} \frac{(3\sin 3x)}{2x}$

$\lim_{x\to 0} \frac{3\sin 3(0)}{2(0)} = \frac{3\cdot 0}{0} = \frac{0}{0}$

$\lim_{x\to 0} \frac{\frac{d}{dx}(3\sin 3x)}{\frac{d}{dx} 2x} = \frac{9\cos 3x}{2}$

$\lim_{x\to 0} \frac{9\cos(3\cdot 0)}{2} = \boxed{\frac{9}{2}}$

Ans: $\frac{9}{2}$

(f) $\lim_{x\to 0^+} (e^{2x}-1)^x = (e^{2\cdot 0}-1)^0 = (1-1)^0 = 0^0$

-1

$y = (e^{2x}-1)^x$

$\ln y = \ln(e^{2x}-1)^x = \cancel{x}\ln(e^{2x}-1)\cdot\frac{1}{\cancel{x}}$

$\ln y = \frac{x\ln(e^{2x}-1)}{\frac{1}{x}} = \lim_{x\to 0^+}\ln y = \lim_{x\to 0^+} \frac{\frac{2e^{2x}}{e^{2x}-1}}{-\frac{1}{x^2}}$

$\lim_{x\to 0^+} \frac{x^2 2e^{2x}}{e^{2x}-1} = \frac{0}{0}$

$\lim_{x\to 0^+} \frac{-2x\cancel{2e^{2x}} - x^2(\cancel{4e^{2x}})}{\cancel{2e^{2x}}} = \lim_{x\to 0^+} \frac{-2x-2x^2}{2}$

$\lim_{x\to 0^+} \frac{-2(0)-2(0)^2}{2} = \frac{0}{2} = 0$

$e^{\ln y} = e^0 = \boxed{1}$

Ans: 1

$\lim \ln y = 0$

$\ln \lim y = 0$

$\lim y = e^0 = 1$