FAST- National University of Computer & Emerging Sciences, Karachi.

Department of Computer Science

Assignment # 4 -- Solution, Fall 2020.

CS211-Discrete Structures

<u>Instructions:</u> Max. Points: 100

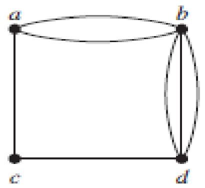
1- This is hand written assignment.
2- Just write the question number instead of writing the whole question.
3- You can only use A4 size paper for solving the assignment.

---

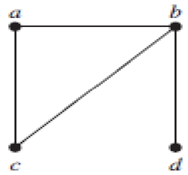
1. Determine whether the graph shown in figure i to iv has directed or undirected edges, whether it has multiple edges, and whether it has one or more loops. Use your answers to determine the type of graph.

Solution:

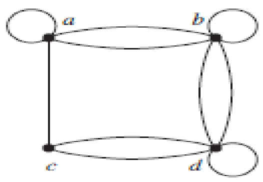
i) It has undirected edges.
It has multiple edges.
It has no loops.
It is undirected Multigraph.



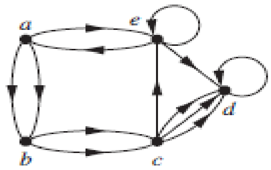
ii) It has undirected edges.
It has no multiple edges.
It has no loops.
It is undirected simple graph.



iii) It has undirected edges.
It has multiple edges.
It has three loops.
It is undirected Pseudo graph.



iv)It has directed edges.
It has multiple edges.
It has two loops.
It is directed Multi graph.



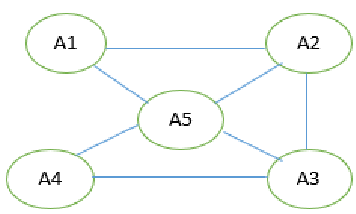
2. The intersection graph of a collection of sets A1, A2,…, An is the graph that has a vertex for each of these sets and has an edge connecting the vertices representing two sets if these sets have a nonempty intersection. Construct the intersection graph of these collections of sets.

i) A1 = {0, 2, 4, 6, 8}, A2 = {0, 1, 2, 3, 4}, A3 = {1, 3, 5, 7, 9}, A4 = {5, 6, 7, 8, 9}, A5 = {0, 1, 8, 9}

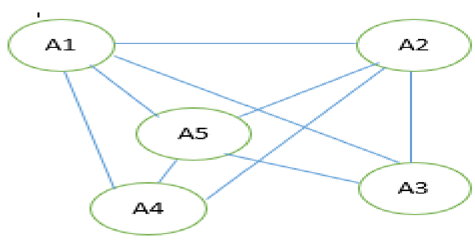
ii) A1 = {…,−4,−3,−2,−1, 0}, A2 = {…,−2,−1, 0, 1, 2,…}, A3 = {…,−6,−4,−2, 0, 2, 4, 6,…}, A4 = {…,−5,−3,−1, 1, 3, 5,…}, A5 = {…,−6,−3, 0, 3, 6,…}

Solution:

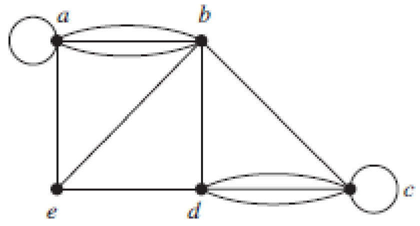
i)



ii)



3. (a) Find the number of vertices, the number of edges, and the degree of each vertex in the given undirected graph. Also find the neighborhood vertices of each vertex in given graphs.



i) Number of Vertices: 5 Number of edges:13

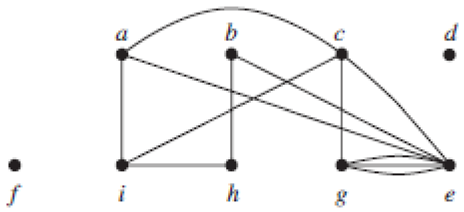
Degree of vertices:

deg(a) = deg(b) = deg(c) = 6, deg(d) = 5, deg(e) = 3.

Neighborhood Vertices:

$N(a)$ = {a, b, e}, $N(b)$ = {a, c, d, e}, $N(c)$ = {b, c, d}, $N(d)$ = {b, c, e},

$N(e)$ = a, b, d}



ii) Number of Vertices: 9 Number of edges:12

Degree of vertices:

deg(a) =3, deg(b) =2, deg(c) = 4, deg(d) = 0, deg(e) = 6.

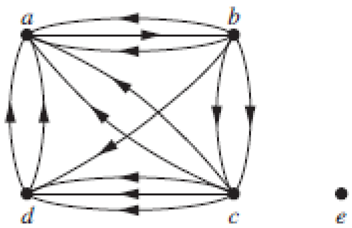
deg(f) =0, deg(g) =4, deg(h) = 2, deg(i) = 3.

Neighborhood Vertices:

$N(a)$ = {c, e, i}, $N(b)$ = {e, h}, $N(c)$ = {a, e, g, i}, $N(d)$ = $\varnothing$,

N(e) = {a, b, c, g}, $N(f) = \varnothing$, $N(g)$ = {c, e}, $N(h)$ = {b, i}, $N(i)$ = {a, c, h}.

(b) Determine the number of vertices and edges and find the in-degree and out-degree of each vertex for the given directed multigraph.

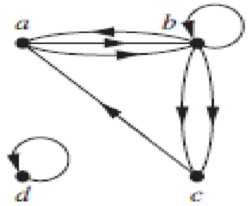


i) In-degree of a vertices

$\deg^-(a) = 6$, $\deg^-(b) = 1$, $\deg^-(c) = 2$, $\deg^-(d) = 4$, $\deg^-(e) = 0$.

Out-degree of a vertices

$\deg^+(a) = 1$, $\deg^+(b) = 5$, $\deg^+(c) = 5$, $\deg^+(d) = 2$, $\deg^+(e) = 0$.



ii) In-degree of a vertices

$\deg^-(a) = 2$, $\deg^-(b) = 3$, $\deg^-(c) = 2$, $\deg^-(d) = 1$.

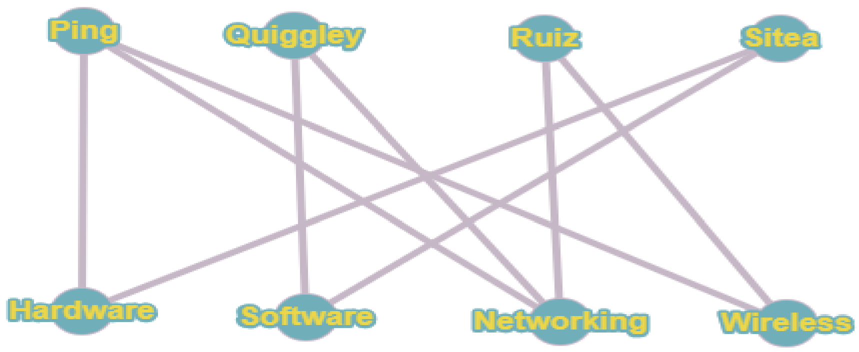
Out-degree of a vertices

$\deg^+(a) = 2$, $\deg^+(b) = 4$, $\deg^+(c) = 1$, $\deg^+(d) = 1$.

4. (a) Suppose that a new company has five employees: Zamora, Agraharam, Smith, Chou, and Macintyre. Each employee will assume one of six responsibilities: planning, publicity, sales, marketing, development, and industry relations. Each employee is capable of doing one or more of these jobs: Zamora could do planning, sales, marketing, or industry relations; Agraharam could do planning or development; Smith could do publicity, sales, or industry relations; Chou could do planning, sales, or industry relations; and Macintyre could do planning, publicity, sales, or industry relations. Model the capabilities of these employees using appropriate graph.

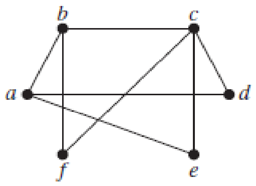
Solution: Bipartite Graph

(b) Suppose that there are four employees in the computer support group of the School of Engineering of a large university. Each employee will be assigned to support one of four different areas: hardware, software, networking, and wireless. Suppose that Ping is qualified to support hardware, networking, and wireless; Quiggley is qualified to support software and networking; Ruiz is qualified to support networking and wireless, and Sitea is qualified to support hardware and software. Use appropriate graph to model the four employees and their qualifications.
Solution: Bipartite Graph

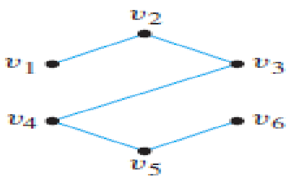


5. Find which of the following graphs are bipartite. Redraw the bipartite graphs so that their bipartite nature is evident. Also write the disjoint set of vertices.

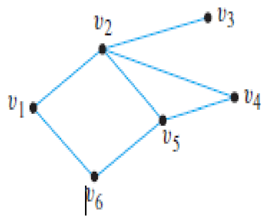
i)



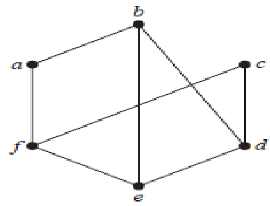
ii)



iii)

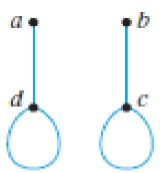


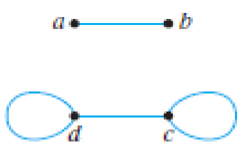
iv)

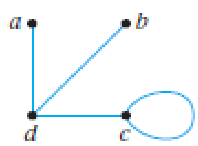


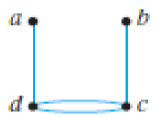
Solution:
(a) Not bipartite (since a is adjacent to b & f vertices)
(b) Bipartite (A (V_1, V_3, V_5) & B (V_2, V_4, V_6))
(c) Not bipartite (since V_4 & V_5 are adjacent vertices)
(d) Not Bipartite (since b is adjacent to d & e vertices)

6. Draw a graph with the specified properties or show that no such graph exists.
a) A graph with four vertices of degrees 1, 1, 2, and 3
b) A graph with four vertices of degrees 1, 1, 3, and 3
c) A simple graph with four vertices of degrees 1, 1, 3, and 3
Solution:
a) No such graph is possible. By Handshaking theorem, the total degree of a graph is even.
But a graph with four vertices of degrees 1, 1, 2, and 3 would have a total degree of 1 + 1 + 2 + 3 = 7, which is odd.
b) Let G be any of the graphs shown below.









In each case, no matter how the edges are labeled, deg(a) = 1, deg(b) = 1, deg(c) =3, and deg(d) = 3.

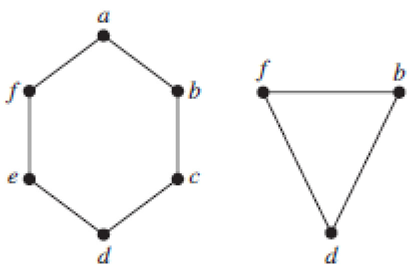
c) There is no simple graph with four vertices of degrees 1, 1, 3, and 3.

7. a) In a group of 15 people, is it possible for each person to have exactly 3 friends? Explain. (Assume that friendship is a symmetric relationship: If x is a friend of y, then y is a friend of x.)
Solution:
By using Handshaking theorem.
No! there is no graph possible, such that 15 vertices have degree 3. Since (15 * 3) ≠ 2e.

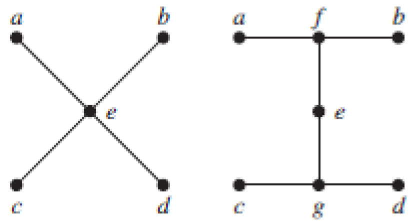
b) In a group of 4 people, is it possible for each person to have exactly 3 friends? Why?
Solution:
By using Handshaking theorem.
Yes! there is graph possible, such that 4 vertices have degree 3. Since (4 * 3) = 2e.

8. (a) Find the union of the given pair of simple graphs. (Assume edges with the same endpoints are the same.)

i)

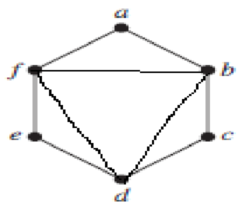


ii)

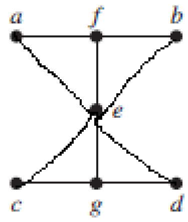


Solution:

i)



ii)



b) How many vertices does a regular graph of degree four with 10 edges have?
Solution:
We want to determine a regular graph of degree four with $m = 10$ edges.

Let the graph contain n vertices $v_1, v_2, ..., v_n$, then each of these n vertices have degree 4.

$$\deg(v_i) = 4$$
$$i = 1, 2, ..., n$$

By the Handshaking theorem, the sum of degrees of all vertices is equal to twice the number of edges:

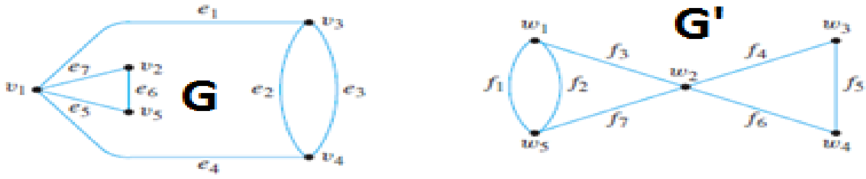
$$20 = 2(10) = 2m = \sum_{v=1}^{n} \deg(v_i) = \sum_{v=1}^{n} 4 = 4n$$

We then obtained the equation $20 = 4n$. Divide each side of the equation by 4:

$$n = \frac{20}{4} = 5$$

9. For given pair (G, G') of graphs. Determine whether they are isomorphic. If they are, give function g: V (G) →V (G') that define the isomorphism. If they are not, give an invariant for graph isomorphism that they do not share.

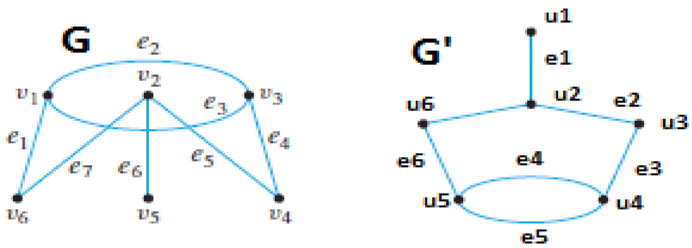
i)



Solution: Both graph G and G' are satisfying all the invariant. Hence, they are isomorphic.

Function: $g(V_1) = W_2$, $g(V_2) = W_3$ $g(V_3) = W_1$ $g(V_4) = W_5$ $g(V_5) = W_4$

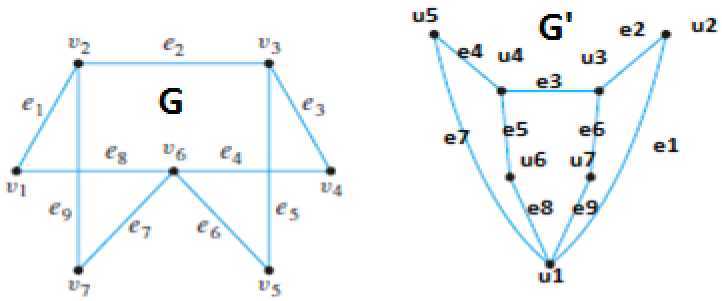
ii)



Solution: Both graph G and G' are satisfying all the invariant. Hence, they are isomorphic.

Function: $g(V_1) = U_5$, $g(V_2) = U_2$ $g(V_3) = U_4$ $g(V_4) = U_3$ $g(V_5) = U_1$ $g(V_6) = U_6$

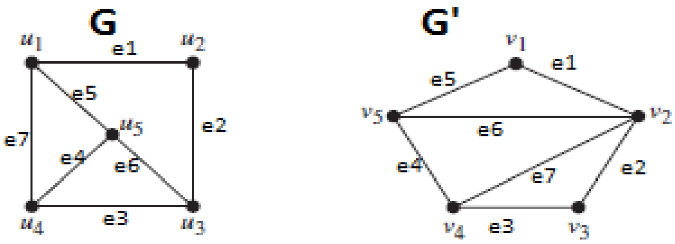
iii)



Solution: Both graph G and G' are satisfying all the invariant. Hence, they are isomorphic.

Function: $g(V_1) = U_5$, $g(V_2) = U_4$ $g(V_3) = U_3$ $g(V_4) = U_2$
$g(V_5) = U_7$ $g(V_6) = U_1$ $g(V_7) = U_6$

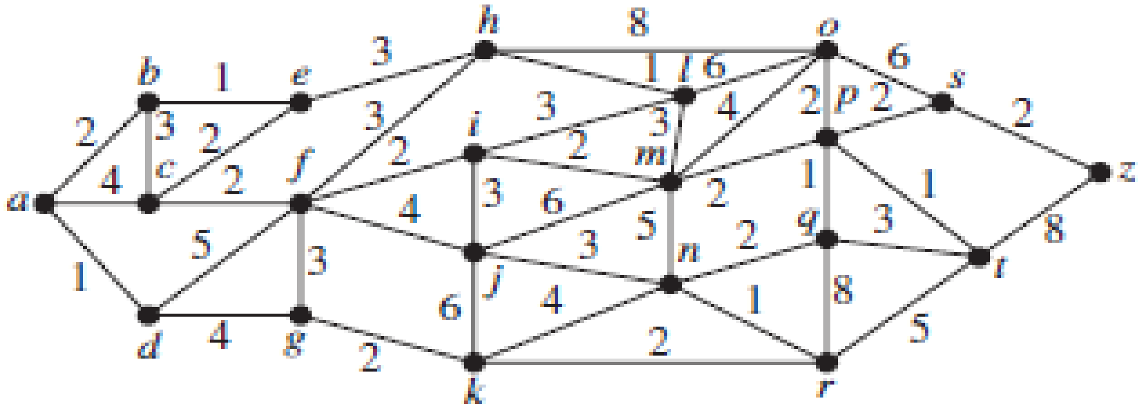
iv)



Solution: Graph G has no vertex of degree 4 where G' has vertex V_2 with degree 4. Hence, they are not isomorphic.

10. **Find the length of a shortest path between *a* and *z* in the given weighted graph by using Dijkstra's algorithm.**

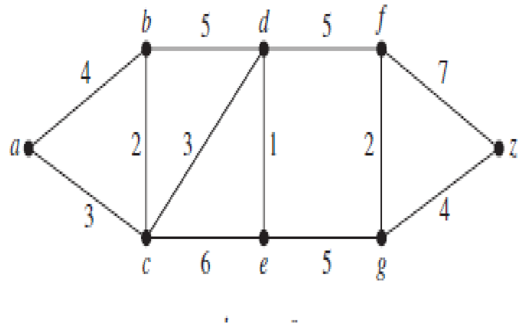
i)



N	D(b)	D (c)	D(d)	D(e)	D(f)	D(g)	D(h)	D(i)	D(j)	D(k)	D(l)	D(m)	D(n)	D(o)	D(p)	D(q)	D(r)	D(s)	D(t)	D(z)
a	2,a	4,a	1,a																	
ad	2,a	4,a			6,d	5,d														
adb		4,a		3,b	6,d	5,d														
adbe		4,a			6,d	5,d	6,e													
adbec					6,c	5,d	6,e													
adbeg					6,c		6,e			7,g										
adbegf							6,e	8,f	10,f	7,g										
adbegfh								8,f	10,f	7,g	7,h			14,h						
adbegfhk								8,f	10,f		7,h		11,k	14,h			9,k			
adbegfhkl								8,f	10,f			10,l	11,k	13,l			9,k			
adbegfhkli									10,f			10,l	11,k	13,l			9,k			
adbegfhklir									10,f			10,l	10,r	13,l		17,r			14,r	
adbegfhklirj												10,l	10,r	13,l		17,r			14,r	
adbegfhklirjm													10,r	13,l	12,m	17,r			14,r	
adbegfhklirjmn														13,l	12,m	12,n			14,r	
adbegfhklirjmnp														13,l		12,n		14,p	13,p	
adbegfhklirjmnpq														13,l				14,p	13,p	
adbegfhklirjmnpqo																		14,p	13,p	
adbegfhklirjmnpqot																		14,p		21,t
adbegfhklirjmnpqots																				16,s
adbegfhklirjmnpqotsz																				

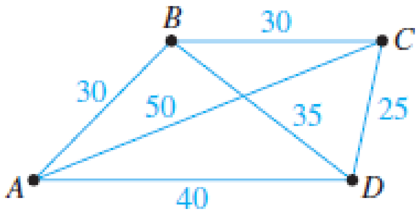
ii)

N	*D(b)*	*D(c)*	*D(d)*	*D(e)*	*D(f)*	*D(g)*	*D(z)*
a	*4,a*	*3,a*	∞	∞	∞	∞	∞
ac			*6,c*	*9,c*	∞	∞	∞
acb			*6,c*	*9,c*	∞	∞	∞
acbd				*7,d*	*11,d*	∞	∞
acbde					*11,d*	*12,e*	∞
acbdef						*12,e*	*18,f*
acbdefg							*16,g*
acbdefgz	*4,a*	*3,a*	*6,c*	*7,d*	*11,d*	*12,e*	*16,g*

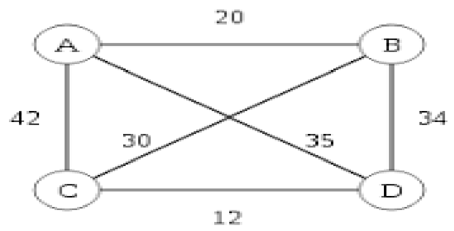


11. Imagine that the drawing below is a map showing four cities and the distances in kilometers between them. Suppose that a salesman must travel to each city exactly once, starting and ending in city A. Which route from city to city will minimize the total distance that must be traveled?

i)



ii)



i) Solution:

Hamiltonian Circuit are: ABCDA = 125; ABDCA = 140; ACBDA = 155.

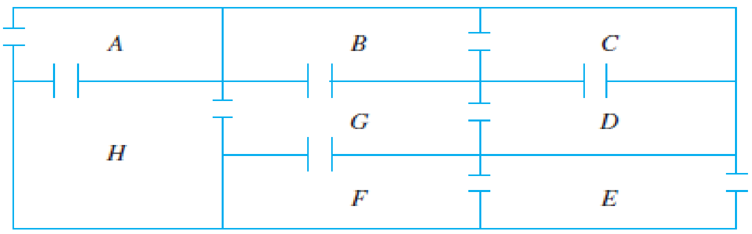
Hence ABCDA = 125 is the minimum distance travelled.

ii) Solution:

Hamiltonian Circuit are: ABCDA = 97; ABDCA = 108; ACBDA = 141.

Hence ABCDA = 97 is the minimum distance travelled.

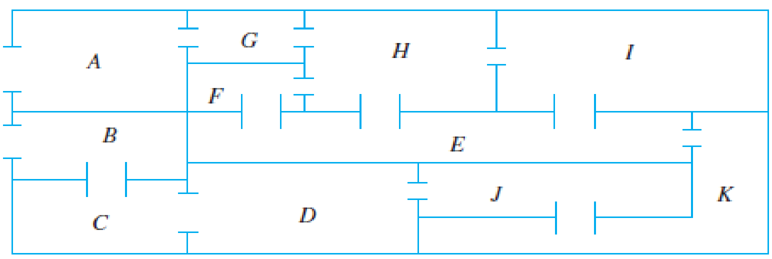
12. (a) The following is a floor plan of a house. Is it possible to enter the house in room A, travel through every interior doorway of the house exactly once, and exit out of room E? If so, how can this be done?



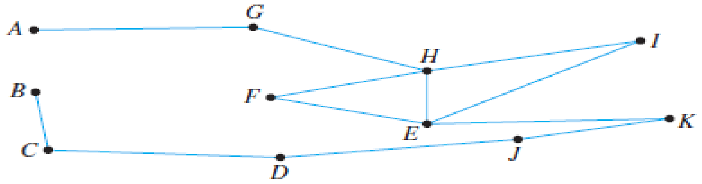
Solution:

Yes! Path: A →H→G→B→C→D→G→F→E

(b) The floor plan shown below is for a house that is open for public viewing. Is it possible to find a trail that starts in room A, ends in room B, and passes through every interior doorway of the house exactly once? If so, find such a trail.



Solution Let the floor plan of the house be represented by the graph below.

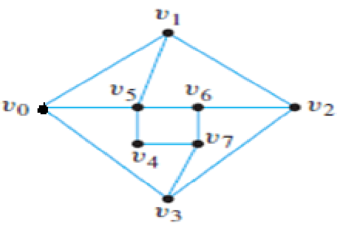


Each vertex of this graph has even degree except for A and B, each of which has degree 1. Hence by Corollary 10.2.5, there is an Euler path from A to B. One such trail is

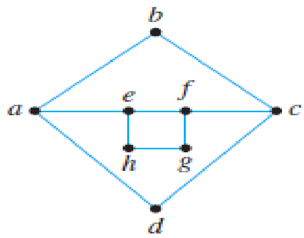
$$AGHFEIHEKJDCB.$$

13. Find Hamiltonian circuits AND Path for those graphs that have them. Explain why the other graphs do not.

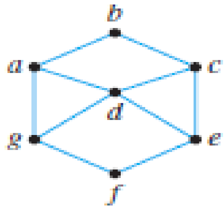
i)



ii)



iii)



i) Solution:

Hamiltonian Circuit: $V_0, V_1, V_2, V_6, V_5, V_4, V_7, V_3, V_0$

Hamiltonian Path: $V_0, V_1, V_2, V_6, V_5, V_4, V_7, V_3$

ii) Solution:

Hamiltonian Circuit: doesn't exist

Hamiltonian Path: b, c, f, g, h, e, a, d

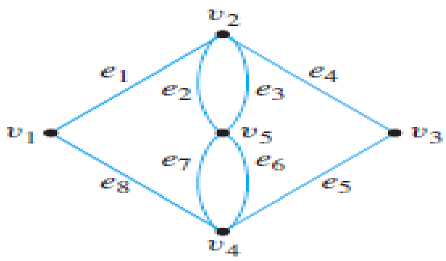
iii) Solution:

Hamiltonian Circuit: d, c, b, a, g, f, e, d

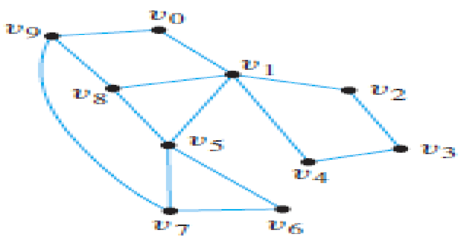
Hamiltonian Path: d, c, b, a, g, f, e

14. a) Determine which of the graphs have Euler circuits. If the graph does not have an Euler circuit, explain why not. If it does have an Euler circuit, describe one.

i)



ii)



i) Solution: All vertices have even degree so circuit exists.

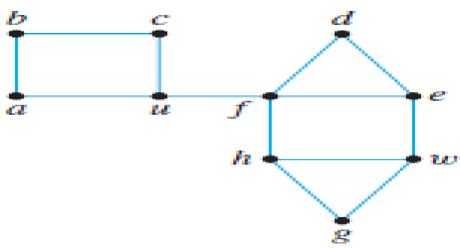
Euler Circuit: $V_1, V_2, V_5, V_4, V_5, V_2, V_3, V_4, V_1$

ii) Solution:

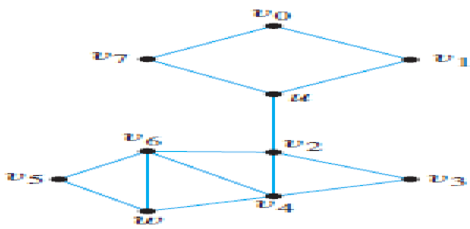
Euler Circuit do not exist because all vertices don't have even degree.

b) Determine whether there is an Euler path from u to w. If the graph does not have an Euler path, explain why not. If it does have an Euler path, describe one.

i)



ii)



i) Solution: Euler Path exists because exact two vertices have odd degree.

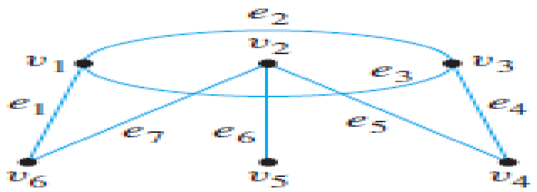
Euler path: U, V_1, V_0, V_7, U, V_2, V_3, V_4, V_2, V_6, V_5, W, V_6, V_4, W

ii)Solution:

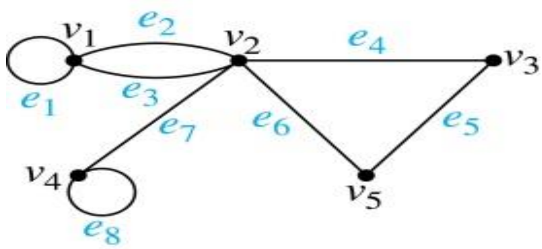
Euler Path doesn't exist because four vertices have odd degree.

15. (a) Use an incidence matrix to represent the graph shown below.

i)



ii)



Solution:

i)

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

ii)

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

(b) Draw a graph using below given incidence matrix.

i)

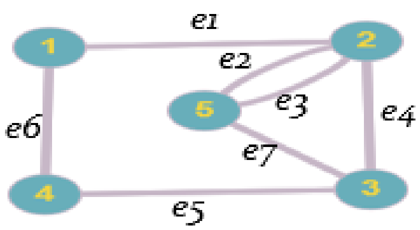
$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

ii)

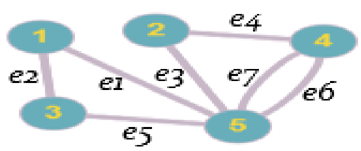
$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix}$$

Solution:

i)

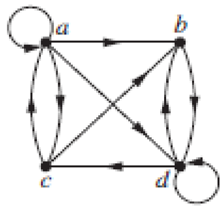


ii)



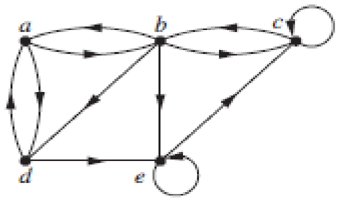
16. Use an adjacency list and adjacency matrix to represent the given graph.

i)



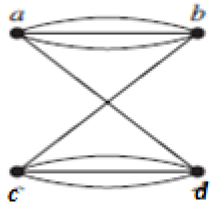
Initial Vertex	Terminal Vertices
a	a, b, c, d
b	d
c	a, b
d	b, c, d

(ii)



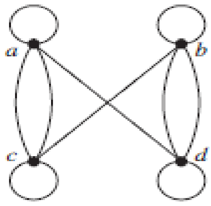
Initial Vertex	Terminal Vertices
a	b, d
b	a, c, d, e
c	b, c,
d	a, e
e	c, e

iii)



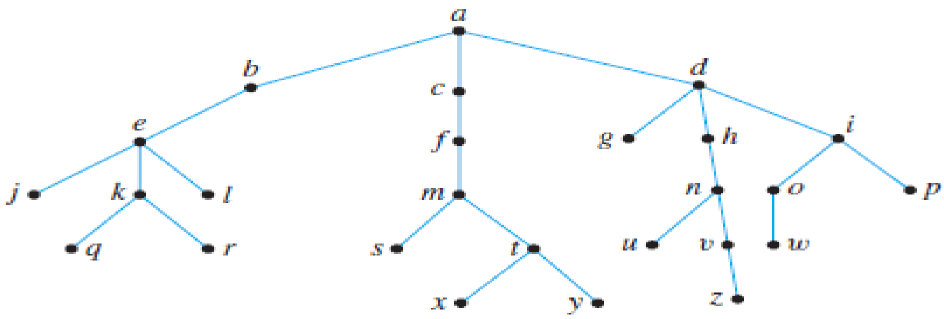
Vertex	Adjacent Vertices
a	b, d
b	a, c
c	b, d
d	a, c

iv)



Vertex	Adjacent Vertices
a	a, c, d
b	b, c, d
c	a, b, c
d	a, c, d

17. Consider the tree shown at right with root a.



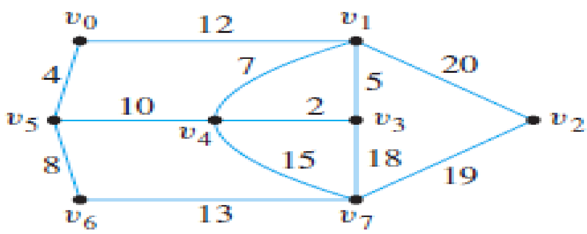
Solution:

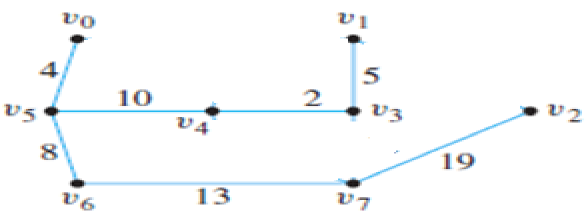
i) What is the level of n?	Level of n is 3.
ii) What is the level of a?	Level of a is 0.
iii) What is the height of this rooted tree?	Height of this rooted tree is 5
iv) What are the children of n?	u & v are the children of n.
v) What is the parent of g?	d is the parent of g.
vi) What are the siblings of j?	k & l are the siblings of j.
vii) What are the descendants of f?	m, s, t, x & y are the descendants of f.
viii) What are the internal nodes?	a, b, e, k, c, f, m, t, d, h, i, n, o & v are the internal nodes.
ix) What are the ancestors of z?	v, n, h, d & a are the ancestors of z.
x) What are the leaves?	j, l, q, r, s, x, y, g, p, u, w & z are the leaves.

18. Use Prim's algorithm to find a minimum spanning tree starting from V_0 for given graphs. Indicate the order in which edges are added to form each tree.

i) Solution: MST Cost = 61

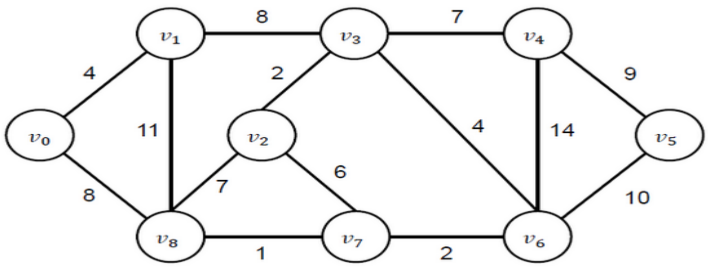
$(V_0,V_5) = 4$, $(V_5,V_6) = 8$, $(V_4,V_5) = 10$,
$(V_3,V_4) = 2$, $(V_1,V_3) = 5$, $(V_6,V_7) = 13$,
$(V_2,V_7) = 19$.

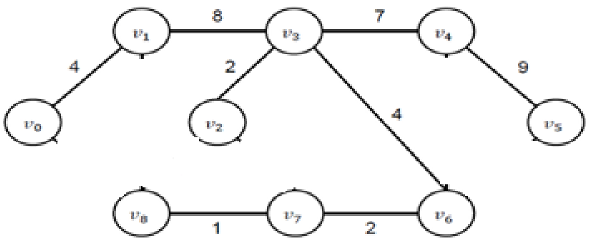


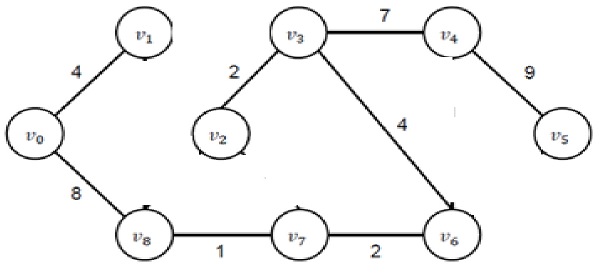


ii) **Solution: MST Cost = 37**

(V_0,V_1) = 4, (V_0,V_8) = 8, (V_7,V_8) = 1,
(V_6,V_7) = 2, (V_3,V_6) = 4, (V_2,V_3) = 2,
(V_3,V_4) = 7, (V_4,V_5) = 9,

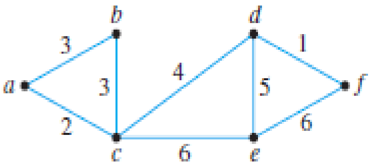




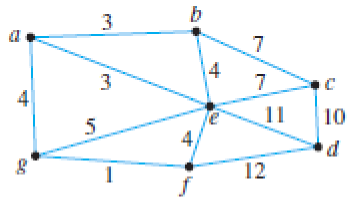


19. Use Kruskal's algorithm to find a minimum spanning tree for given graphs. Indicate the order in which edges are added to form each tree.

i)



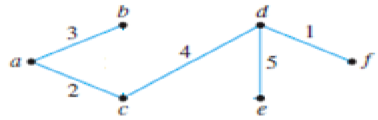
ii)



i) Solution: MST cost = 15

Order of edges added is:

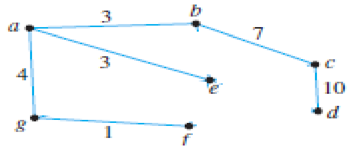
(d, f) =1, (a, c) =2, (a, b) =3, ~~(b, c) =3,~~
(c, d) =4, (d, e) =5, ~~(c, e) =6,~~ ~~(e, f) =6~~



ii) Solution: MST cost = 28

Order of edges added is:

(g, f) =1, (a, b) =3, (a, e) =3, (a, g) =4,
~~(b, e) =4,~~ ~~(e, f) =4,~~ ~~(g, e) =5,~~ (b, c) =7
~~(c, e) =7,~~ (c, d) =10, ~~(d, e) =11,~~ ~~(d, f) =12~~

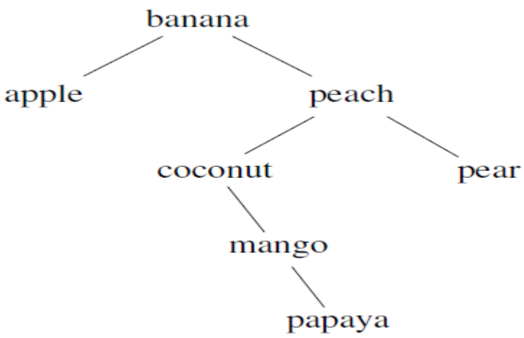


20. (a) i) Build a binary search tree for the word's banana, peach, apple, pear, coconut, mango, and papaya using alphabetical order.

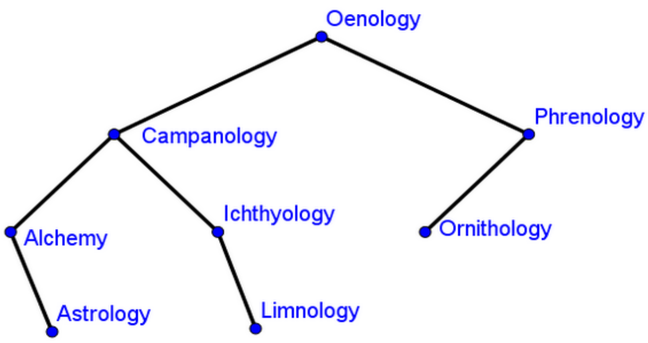
ii) Build a binary search tree for the word's oenology, phrenology, campanology, ornithology, ichthyology, limnology, alchemy, and astrology using alphabetical order.

Solution:

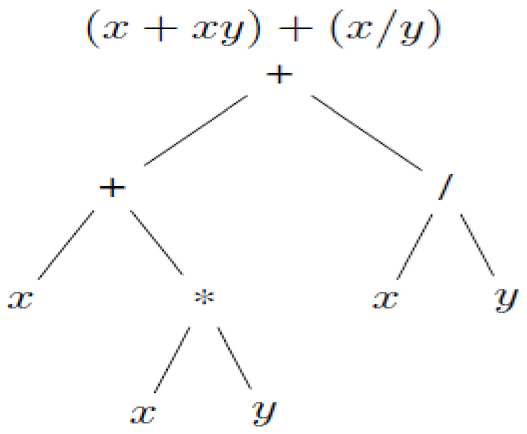
i)



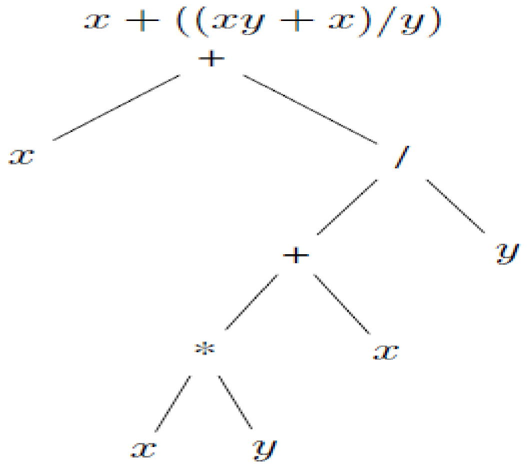
ii)



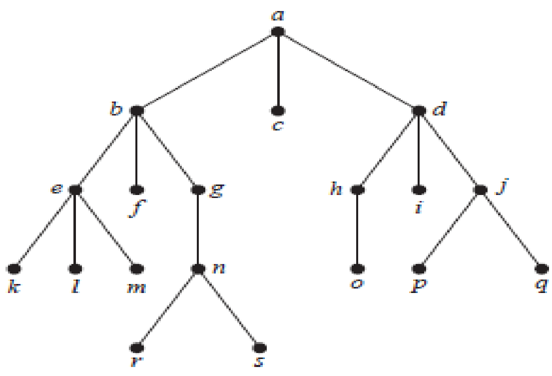
(b) Represent these expressions using binary trees.
(i) (x + xy) + (x / y)
(ii) x + ((xy + x) / y)
Solution:
i)



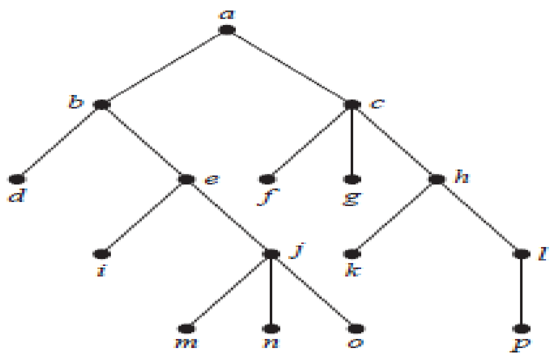
ii)



21. Determine the order in which preorder, Inorder and Postorder traversal visits the vertices of the given ordered rooted tree.
i)



ii)



Solution:
i)
Preorder: a b e k l m f g n r s c d h o I j p q
Inorder: k e l m b f r n s g a c o h d i p j q
Postorder: k l m e f r s n g b c o h I p q j d a
ii)
Preorder: a b d e i j m n o c f g h k l p
Inorder: d b i e m j n o a f c g k h p l
Postorder: d i m n o j e b f g k p l h c a

22. (a) How many edges does a tree with 10000 vertices have?
Solution:
A tree with n vertices has n – 1 edge. Hence 10000 – 1 = 9999 edges.
(b) How many edges does a full binary tree with 1000 internal vertices have?
Solution:
A full binary tree has two edges for each internal vertex. So, we'll just multiply the number of internal vertices by the number of edges. Hence 1000 * 2 = 2000 edges.

(c) How many vertices does a full 5-ary tree with 100 internal vertices have?
Solution:

A full $m-ary$ tree with I internal vertices has $n = mi + 1$ vertices.
From the given information, we have $m = 5$, $i = 100$
So $n = 5 \times 100 + 1 = 501$
Therefore a full 5-ary tree with 100 internal vertices has 501 vertices.

23. a) Write these expressions in Prefix and Postfix notation:

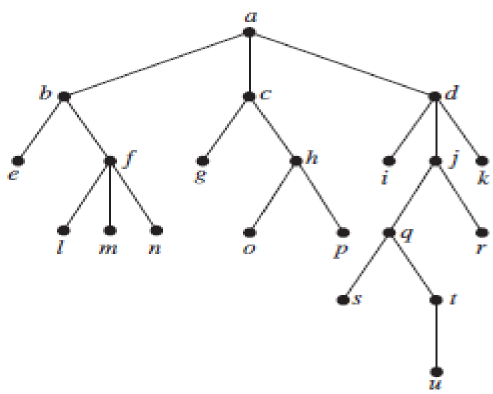
i) (x + xy) + (x / y)
Solution:
Prefix: + + x * x y / x y
Postfix: x x y * + x y / +

ii) x + ((xy + x) / y)
Solution:
Prefix: + x / + * x y x y
Postfix: x x y * x + y / +

b) i) What is the value of this prefix expression + – ↑ 3 2 ↑ 2 3 / 6 – 4 2
Solution: 4

ii) What is the value of this postfix expression 4 8 + 6 5 – * 3 2 – 2 2 + * /
Solution: 3

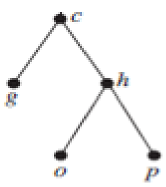
24. Answer these questions about the rooted tree illustrated.



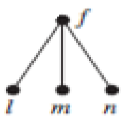
i) Is the rooted tree a full m-ary tree?
Solution: It is not a full m-ary tree for any m because some of its internal vertices have two children and others have three children.
ii) Is the rooted tree a balanced m-ary tree?
Solution: It is not balanced m-ary tree because it has leaves at levels 2, 3, 4 and 5.

iii) Draw the subtree of the tree that is rooted at
a) c. b) f. c) q.
Solution:

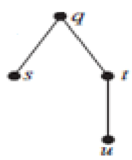
a)



b)

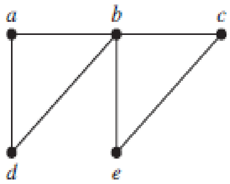


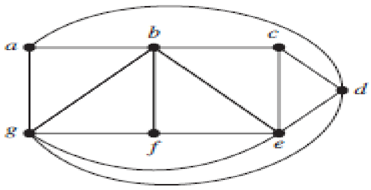
c)

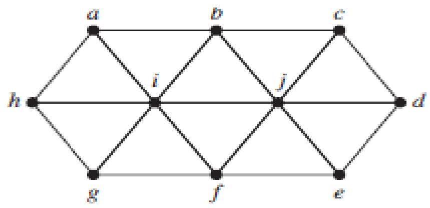


25. Find a spanning tree for the graph shown by removing edges in simple circuits. Write down the removed edges.

(i) ii) iii)
Solution:







i) ii) iii)