National University of Computer & Emerging Sciences, Karachi

SPRING - 2020 CS –Department

Final Examination

June 8 , 2020, 9:00 am – 12:00 noon

Course Code: MT-104	**Course Name: Linear Algebra**
Instructor Name: Mr. Muhammad Amjad	
Student Roll No:	**Section No:**

Instructions:

- Read each question completely before answering it. There are **11 questions (short & long) and 3 page**.
- In case of any ambiguity, you may make assumption. But your assumption should not contradict any statement in the question paper.
- All the answers must be solved according to the sequence given in the question paper.
- **Do not write anything on question paper.** Return the question paper.
- **This is an open book paper, If Plagiarism or copied from other candidate is found strict action will be taken. Do use your own stuff.**

Time: 180 minutes **Max Marks**: 70 points

Q1. Determine whether the set of all pairs of real number (x, y) with the operations

$$x + y = xy, \& kx = x^k$$

If it is not a vector space , then list all the axioms that fail to hold. [05]

Q2. Are the vectors v_1 = $(1,0,1,2)$, v_2 = $(0,1,1,2)$, and v_3 = $(1,1,1,3)$ in R^4 linearly dependent or linearly independent. (**note: you may find the values from calculator directly there is no need to apply long method**). [03]

Q3 . Complete the following statements/ theorem as follows. [04]

(i) A nonempty set $S = \{v_1, v_2 \ldots\ldots\ldots v_r\}$ in a vector space V is linearly -----------

(ii) if k is a positive integer, λ is an eigen value of matrix A and X is a ------------

(iii)If W is a finite -dimensional inner product space, then , --------------------------------

(iv) If S is an orthogonal basis for an n - dimensional inner product space V , and if $(U)_S = (u_1, u_2, \ldots\ldots\ldots u_n) and (V)_S = (v_1, v_2, \ldots\ldots\ldots v_n)$ then: --------------------------

Q4.(a) Find a basis for the row space of A consisting entirely row vectors from A. [05]

$$A = \begin{pmatrix} 1 & 3 & 3 & 2 \\ 2 & 6 & 9 & 7 \\ -1 & -3 & 3 & 4 \end{pmatrix}$$

(a) Find Rank and Nullity of A defined in part (a). [02]

(b) Also , show /verify the theorem: rank A + nullity A = n. [01]

Q5. Let $S = \{v_1, v_2, v_3, v_4\}$ [05]

$$v_1 = (1,-2,0,3,-4), v_2 = (3,2,8,1,4), v_3 = (2,3,7,2,3), \& v_4 = (-1,2,0,4,-3)$$

and let V be the subspace of R^5 given by V = Span S . Find the basis for V.

Q6. For the following matrix

$$A = \begin{pmatrix} 5 & 6 & 2 \\ 0 & -1 & -8 \\ 1 & 0 & -2 \end{pmatrix}$$

(a) Find Eigen values [03]
(b) Find Eigen vectors [04]
(c) Find Algebraic and Geometric multiplicity of each Eigen value [02]
(d) Inspect if A is Diagonalizable by AM and GM and why? [01]
(e) Do the same for A^5 ? [02]

Q7. For the following Matrix B, Eigenvalues and Eigen basis are given , (i)Find the Diagonal matrix D which is the result of Diagonalization (show all the working) [05]

$$B = \begin{pmatrix} 2 & 0 & -2 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

For $\lambda_1 = 2,\ v_1 = (1, 0, 0)$ **;** $\lambda_2 = 3,\ v_2 = (-2, 0, 1)$ and $v_3 = (0, 1, 0)$.

(ii) Also, find A^{15} ? [02]

Q8. Find the Standard matrix for the orthogonal projection of R^2 onto the stated line,and then use that matrix to find the orthogonal projection of (3,4) onto the line makes an angle of $\frac{\pi}{3}$ (= 60^0)along positive x - axis. [04]

Q9. For the following matrix [10]

$$A = \begin{bmatrix} -3 & 1 & 2 \\ 1 & -3 & 2 \\ 2 & 2 & 0 \end{bmatrix}$$

(i) Find a matrix *P* that orthogonally diagonalizes the matrix *A*.
(ii) Find the spectral decomposition of the matrix

Q10. Find the QR - decomposition of [10]

$$A = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$$

{HINT: Take u_1, u_2, u_3 as column vectors of matrix A }
