National University of Computer & Emerging Sciences, Karachi

Fall-2019 - Department of Computer Science

Bachelor of Science (Computer Science)

Midterm-2 Examination -- Solution

November 05, 2019, 01:00 pm – 02:00 pm

Course Code: CS211	**Course Name: Discrete Structures**
Instructor Names: Mr. Saif-ul-Islam, Mr. Shoaib Raza, and Dr. Fahad Samad	
Student Roll No:	**Section No:**

Instructions:

- Return the question paper together with the answer script.
- Read each question completely before answering it.
- **Answer all the questions in given sequence of the question paper.**
- There are **3** questions on **2** pages and each question carries 10 marks and each part carries 5 marks.

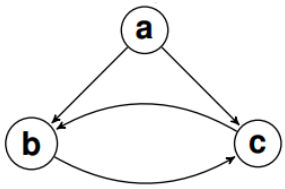
Total Time: 60 minutes **Maximum Marks**: 30

Question # 1 (Relations) **[2 x 5 = 10 Marks]**

Consider a scenario from World War II where two German soldiers are communicating with each other through separate channels. Head of Platoon wants to send two different messages to each of them through separate one-way communication channels. Both of them can receive the message but cannot reply back.

(a) Transform the given scenario into a digraph. Is this relation irreflexive? Show the in-degree and out-degree of each vertex. **(2 + 2 + 1 = 5)**

Solution:



It is irreflexive because there are no loops.

In Degree: $\deg^-(a) = 0$, $\deg^-(b) = 2$, $\deg^-(c) = 2$

Out Degree: $\deg^+(a) = 2$, $\deg^+(b) = 1$, $\deg^+(c) = 1$

(b) Determine whether the relation is an equivalence relation? Is this a partial-order relation? Show all of your steps. **(5)**

Solution:

For equivalence relation, relation should be reflexive, Symmetric and Transitive.

For partial Order relation, relation should be reflexive, Antisymmetric and Transitive.

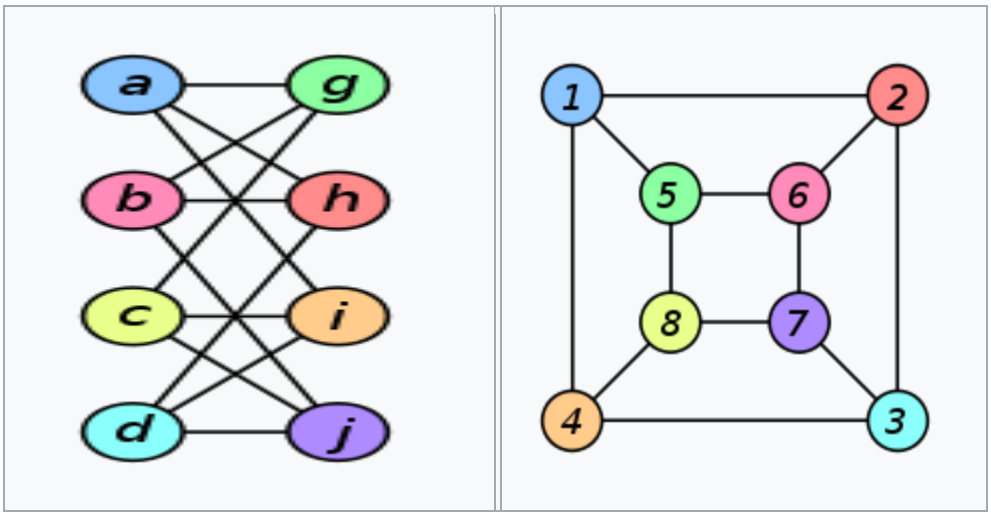
Relation	**Reason**
Not reflexive	Because every vertex does not have a self-loop.
Not symmetric	Because we do not have (b, a) for (a, b).
Not antisymmetric	Because we have (b, c) and (c, b).
Not transitive	Because the path for (b, c), (c, b) from b to b is missing the edge (b, b).

Given relation is neither Equivalence nor Partial order relation.

Question # 2 (Graph Theory) **[2 x 5 = 10 Marks]**

(a) Determine if the following two graphs ***G and H*** are isomorphic. If they are, give function ***g: V (G) → V (H)*** that define the isomorphism. If they are not, give the reason why? **(5)**

Graph G **Graph H**



(b) Consider the graphs ***G and H*** in question 2(a). **(2 + 3 = 5)**

(i) Is ***Graph G*** a complete Bipartite Graph? Explain.

Solution:

Both Graph G and H are isomorphic.

f(a) = 1 f(b) = 6 f(c) = 8 f(d) = 3

f(g) = 5 f(h) = 2 f(i) = 4 f(j) = 7

(ii) Determine if Euler and Hamilton circuits exist in ***Graph H.*** If yes, show the circuit, if not explain why?

Solution:

Graph H doesn't contain Euler Circuit as all the vertices have odd degree.

Graph H Contains Hamilton's Circuits:

H1: 1,2,3,7,6,5,8,4,1 **H2:** 1,2,6,7,3,4,8,5,1 **H3:** 1,2,6,5,8,7,3,4,1

Question # 3 (Number Theory) **[2 x 5 = 10 Marks]**

An Army general after a battle orders his sub ordinate general to counts the number of surviving soldiers of a battle by aligning them successively in rows of certain sizes. Each time, he counts the number of remaining soldiers who failed to fill a row. The Army general initially had 2000 soldiers before the battle. After the battle:

aligning them in rows of 5 soldiers leaves 1 remaining soldier;
aligning them in rows of 7 soldiers leaves 2 remaining soldiers;
aligning them in rows of 9 soldiers leaves 3 remaining soldiers;

(a) How many soldiers survived the battle? Use Chinese Remainder Theorem and include all steps. **(5)**

Solution:

We will follow the notation used in the proof of the Chinese remainder theorem.
We have m=m1 * m2 * m3 * m4 = 5 * 7 * 9 = 315.
M1 = 315/5 = 63, M2 = 315/7 = 45, and M3 = 315/9 = 35
Also, by simple inspection we see that:
y1 = 2 is an inverse for M1 = 63 modulo 5,
y2 = 5 is an inverse for M2 = 45 modulo 7 and
y3 = 8 is an inverse for M3 = 35 modulo 9.
The solutions to the system are then all numbers x such that
x = a1M1y1 + a2M2y2 + a3M3y3 mod m
= ((1 * 63 * 2) + (2*45*5) + (3*35*8)) mod 315
= 1416 (mod 315) = 156.
156 soldiers survived the battle.

(b) After counting the number of soldiers survived, the army general communicated this to head office using Shift / Caesar Cipher (with a shift of 3). What will be the Encrypted Word of the count? **(5)**

Hint: you have to convert the number of soldiers in words. (e.g. 21 will be converted to twenty one)

Solution:

Total Number of Soldiers Survived: One hundred and fifty six
Encrypted Number word: RQH KXQGUHG DQG ILIWB VLA

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ALL THE BEST