National University of Computer and Emerging Sciences, Karachi

FAST School of Computing

Midterm-II Examination -- Solution, Fall-2022

November 02, 2022, 08:30 AM – 09:30AM

Course Code: CS1005	Course Name: Discrete Structures
Instructor Names: Mr. Shoaib Raza, Ms. Bakhtawer, Ms. Safia, Ms. Fizza Aqeel, Mr. Fahad Hussain and Mr. Sudais	
Student Roll No:	Section No:

Instructions:

- Return the question paper together with the answer script. Read each question completely before answering it. There are **3 questions** written on **2 pages.**
- In case of any ambiguity, you may make assumptions. However, your assumptions should not contradict any statement in the question paper.
- Attempt all the questions in the given sequence of the question paper. Show all steps properly in order to get full points.

Total Time: 01 Hour **Maximum Points**: 24

Question # 1: **[CLO -1, C2]** **[4 x 2 = 08 points]**

(a) Let $X_n = 2^n + 5.\ 3^n$ for n = 0, 1, 2, 3… Show that $X_4 = 5X_3 – 6X_2$

Solution:

$x_0= 2^0+ 5 \cdot 3^0= 6$,

$x_1= 2^1+ 5 \cdot 3^1= 17$,

$x_2= 2^2+ 5 \cdot 3^2= 49$,

$x_3= 2^3+ 5 \cdot 3^3= 143$,

$x_4= 2^4+ 5 \cdot 3^4= 421$

Since we have $X_0 = 6$, $X_1 = 17$, $X_2 =49$, $X_3 = 143$, $X_4 = 421$.

Now, $X_4 = 5(143) – 6(49)$

$X_4 = 715 – 294$

$X_4 = 421$ proved.

(b) Find the sum of numbers between 250 and 1000 which are divisible by 17.

Solution:

a= 255, d= 17, $T_n = 986$.

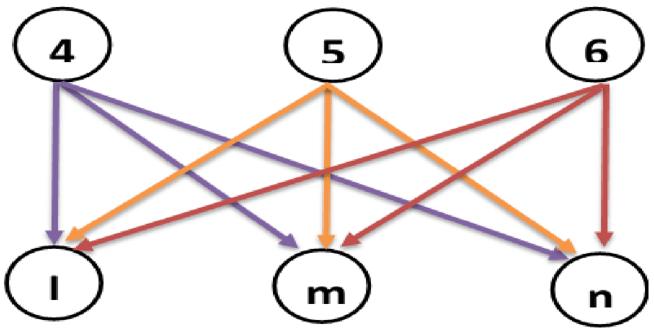
$T_n = a + (n-1)d;\quad 986 = 255 + (n-1)(17)$ n = 44.

Now for Sum; $S_n = \frac{n}{2}[2a + (n-1)d];\quad S_{54} = \frac{44}{2}[2(255) + (44-1)(17)]$ = 27,302.

(c) Draw the directed graph for R_1 o R_2. Consider the relation R_1 = {(4, a), (4, b), (5, c), (6, a), (6, c)} from X to Y and R_2 = {(a, l), (a, n), (b, l), (b, m), (c, l), (c, m), (c, n)} from Y to Z where X = {4, 5, 6}, Y = {a, b, c} and Z = {l, m, n}.

Solution:

R_1 o R_2 = {(4, l), (4, n), (4, m), (5, l), (5, m), (5, n), (6, l), (6, m), (6, n)}



(d) Prove or Disprove that the less than (a<b) relation on Set A = {1, 2, 3, 4} is a partial order relation. Discuss all its properties.
Solution:
For partial Order relation, relation should be reflexive, Antisymmetric and Transitive.
Reflexive: a<a = 1<1 – No
AntiSymmetric: a<b and b<a; a=b - Yes
Transitive: a<b, b<c then a<c. - 1<2, 2<3 then 1<3 – Yes
Relation is not Reflexive so not a partial order relation.

Question # 2: **[CLO -2, C3]** **[4 x 2 = 08 points]**

(a) For a given pair of graphs G and G1 in Figure # 1. Determine whether G and G1 are isomorphic. If they are, give function F: V(G) →V(G1) that define the isomorphism. If they are not, give an invariant for graph isomorphism that they do not share.
Solution:
G is isomorphic to G1.
Hence,
F(a) = t, F(b) = x, F(c) = u, F(d) = y, F(e) = v, F(f) = z, F(g) = w
OR
F(a) = v, F(b) = y, F(c) = u, F(d) = x, F(e) = t, F(f) = w, F(g) = z.

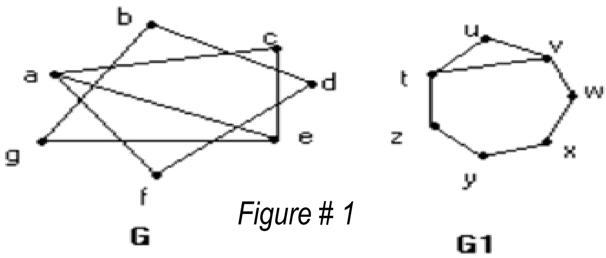


Figure # 1

(b) Consider the graph given in Figure # 2. With the indicated link costs, use Dijkstra's shortest-path algorithm to compute the shortest path from a to all other nodes. Use the table given below for computations.
Solution:

Path	D(b)	D(c)	D(d)	D(e)	D(f)
a	**1,a**	∞	6,a	∞	∞
ab		∞	3,b	**2,b**	∞
abe		7,e	**3,b**		15,e
abed		**7,e**			14,e
abedc					**14,e**

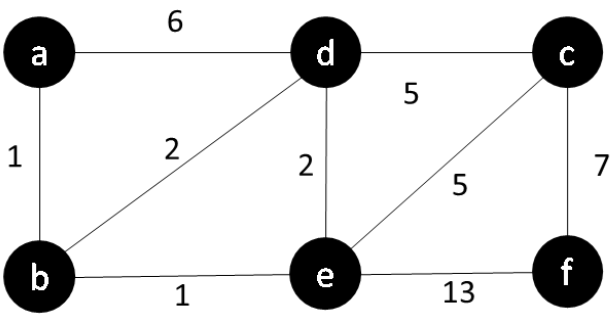


Figure # 2

(c) Determine whether the graph G in Figure # 3 has an Euler circuit OR an Euler path. Construct such a circuit or path when one exists. If no Euler circuit or path exists, justify with a valid reason.
Solution:
Graph G doesn't have an Euler path because all vertices have even degree but Euler circuit a, b, c, d, f, a, e, d, a exists or vice versa.

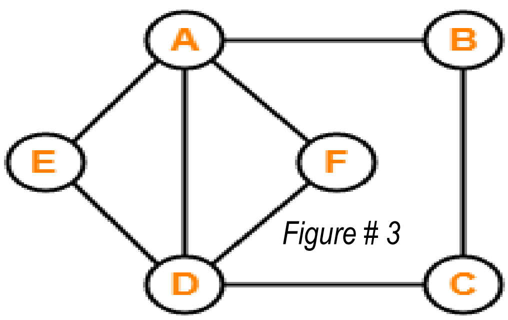


Figure # 3

(d) Determine whether the graph G in Figure # 3 has a Hamilton circuit OR a Hamilton path. Construct such a circuit or path when one exists. If no Hamilton circuit or path exists, justify with a valid reason.
Solution:
Graph G doesn't have the Hamilton Circuit but Hamilton path e, a, f, d, c, b exists or vice versa.

Question # 3: **[CLO -2, C3]** **[4 x 2 = 08 points]**

(a) Use Kruskal's algorithm to find a minimal spanning tree for the graph shown in Figure # 4. Indicate the order in which edges are added to form the tree.

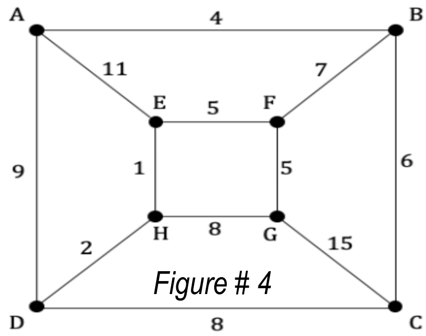
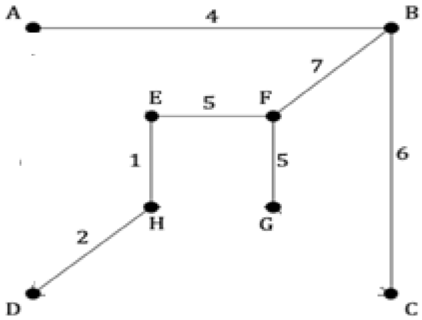


Figure # 4

Solution:

Order of edges:

E,H = 1
D,H = 2
A,B = 4
E,F = 5
F,G = 5
B,C = 6
B,F = 7
MST = 30



(b) Calculate the regions for a graph given in Figure # 4 using Euler Formula.

Solution:

# of edges, e = 12; # of vertices, v = 8; # of regions, r = e - v + 2 = 12 – 8 + 2 = 6.

(c) Show step by step inorder traversal of the tree given in Figure # 5.

Solution:

H, D, I, B, E, A, J, F, K, C, G

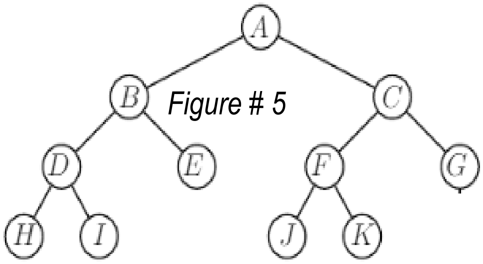


Figure # 5

(d)

(i) Determine whether the tree given in Figure # 5 is a Full m-ary tree or not. Give reason.

Solution:

It's a full m-ary tree. All internal vertices have exact two children**.**

(ii) Determine whether the tree given in Figure # 5 is a Balanced m-ary tree or not. Give reason.

Solution:

It's a balanced m-ary tree since child exists at only h or h-1.

ALL THE BEST