FAST- National University of Computer & Emerging Sciences, Karachi.

FAST School of Computing

Class Participation Written-I, Spring 2021 -- Solution

12th March 2021

Course Code: CS 211	Course Name: Discrete Structures
Instructors: Mr. Shoaib Raza	
Student Roll No:	Section:

Time Allowed: 50 minutes. Maximum Points: 30 points

NOTE: Each question carries equal points. In order to get maximum marks, step-by-step solutions are required.

Question #1:

Let p, q, r and s be the propositions.

p: I play cricket. q: I play hockey. r: I am tired. s: I do exercise.

Write these propositions using *p, q, r and s* and logical connectives (including negations):

a) I play cricket and hockey. Solution: $p \wedge q$

b) I am tired next day if I play hockey. Solution: $q \rightarrow r$

c) I am tired only if I will not do exercise. Solution: $r \rightarrow \neg s$

Question #2:

Using the premises(statements) from Question #1, apply rules of inference to obtain conclusion from those premises.

Solution:

Now we can write the premises as, $(p \wedge q) \wedge (q \rightarrow r) \wedge (r \rightarrow \neg s)$

$\equiv \underline{(p \wedge q)} \wedge (q \rightarrow r) \wedge (r \rightarrow \neg s)$ Simplification

$\equiv \underline{q \wedge (q \rightarrow r)} \wedge (r \rightarrow \neg s)$ Modus Ponen

$\equiv \underline{r \wedge (r \rightarrow \neg s)}$ Modus Ponen

$\equiv \neg s$ Hence, the conclusion is "I will not do exercise."

Question #3:

Prove or disprove the following logical equivalence using the laws of logic: $p \rightarrow (\neg q \wedge r) \cong \neg p \vee \neg (r \rightarrow q)$

Solution:

$= \neg p \vee \neg (r \rightarrow q)$

$= \neg p \vee \neg (\neg r \vee q)$ Implication Law

$= \neg p \vee (\neg (\neg r) \wedge \neg q)$ De-Morgan Law

$= \neg p \vee (r \wedge \neg q)$ Double Negation Law

$= \neg p \vee (\neg q \wedge r)$ Commutative Law

$= p \rightarrow (\neg q \wedge r)$ hence proved

Question #4:

Use truth table to prove that the given statement $((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$ is a Tautology OR Contradiction.

Solution:

p	q	r	$\neg p$	$(p \vee q)$	$(\neg p \vee r)$	$(p \vee q) \wedge (\neg p \vee r)$	$(q \vee r)$	$((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$
T	T	T	F	T	T	T	T	T
T	T	F	F	T	F	F	T	T
T	F	T	F	T	T	T	T	T
T	F	F	F	T	F	F	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T	T
F	F	T	T	F	T	F	T	T
F	F	F	T	F	T	F	F	T

<u>Question #5:</u>

Suppose the variable x represents all adults in your neighborhood.

P(x): x knows kung Fu. **Q (x): x knows karate.**
R(y): y Knows karate. **S(y): y knows Kung Fu.**

(a) Translate the statement into English: $\exists x\ (p(x) \wedge \neg\ Q(x))$

Solution: There is an adult in your neighborhood who knows Kung Fu but not Karate.

(b) Use quantifiers to express the statements. “No adult in your neighborhood knows kung Fu and karate.”

Solution: $\neg\ \exists x\ (p(x) \wedge\ \ Q(x)) = \forall x\ \neg\ (p(x) \wedge\ \ Q(x)) = \forall x\ (\neg\ p(x) \wedge \neg\ Q(x))$

<u>Question #6:</u>

Let P(a, b) means “$a + b = 0$”, where a and b are integers. Determine the truth value of the statement.

(a) $\forall a\ \exists\ b\ \neg\ P\ (a, b)$

Solution: True **$\neg\ P\ (a, b) = a + b \neq 0$** **$a + b \neq 0 + 1 \neq 0$ and $a + b \neq 1 + 1 \neq 0$**

(b) $\exists\ b\ \exists\ a\ P(a, b)$

Solution: True **$P\ (a, b) = x + y = 0$** **$a + b = 0 + 0 = 0$ and $a + b = 1 + (-1) = 0$**

<u>Question #7:</u>

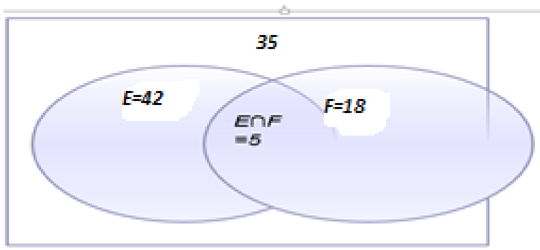
Use set-builder notation and logical equivalences to establish the given expression. **$A - (A - B) = (A \cap B)$**

$\equiv \{x\ /\ x \in A \wedge (x \notin (A - B)\}$
$\equiv \{x\ /\ x \in A \wedge x \notin (x \in A \wedge x \notin B)\}$
$\equiv \{x\ /\ x \in A \wedge \neg\ x \in (x \in A \wedge \neg x \in B)\}$
$\equiv \{x\ /\ x \in A \wedge x \in (x \notin\ A\ \ v\ x \in B)\}$ **De-Morgan Law**
$\equiv \{x\ /\ (x \in A\ x \notin\ A)\ v\ (x \in A \wedge x \in B)\}$ **Distributive Law**
$\equiv \{x\ /\ (\Phi)\ v\ (x \in A \wedge x \in B)\}$ **Complement or Negation Law**
$\equiv \{x\ /\ (x \in A \wedge x \in B)\} \equiv$ R.H.S

<u>Question #8:</u>

There are 100 students in a class, 47 are learning English and 23 are learning French Language whereas 5 learning both languages. How many learning either and how many learning neither? Draw Venn diagram.

Solution:



Total number of students, n(μ) = 100
Number of English language students, n(E) = 47
Number of French language students, n(F) = 23
Number of students who learning both, $n(E \cap F) = 5$
Number of students who learning either of them,
$n(E \cup F) = n(E) + n(F) - n(E \cap F) = 47+23-5 = 65$
Number of students who learning neither = $n(\mu) - n(E \cup F) = 100 - 65 = 35$

<u>Question #9:</u>

Determine whether the function from Z to Z is Injective OR Surjective. **$f(n) = n^2 + 1$**

Solution:

It is neither Injective nor Surjective.

<u>Question #10:</u>

Let f be the function from {p, q, r, s} to {7,8,9,10} such that f(p) = 7, f(q) = 8, f(r) = 10 and f(s) = 9. Is f invertible and if so, what is its inverse?

Solution:

The function *f* is invertible because it is a one-to-one correspondence. The inverse function f^{-1} reverses the correspondence given by *f*, so $f^{1}(7) = p$, $f^{1}(8) = q$, $f^{1}(10) = r$ and $f^{1}(9) = s$.

BEST OF LUCK 😊