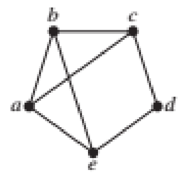
Graph Theory

In Exercises 1–8 determine whether the given graph has an Euler circuit. Construct such a circuit when one exists. If no Euler circuit exists, determine whether the graph has an Euler path and construct such a path if one exists.

1.



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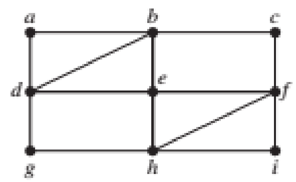

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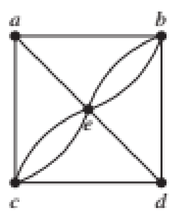

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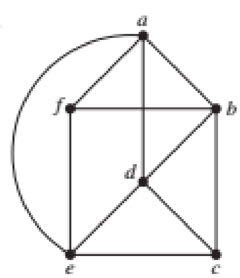

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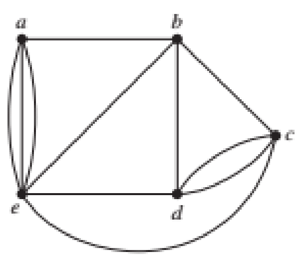

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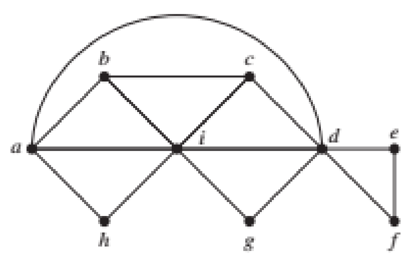

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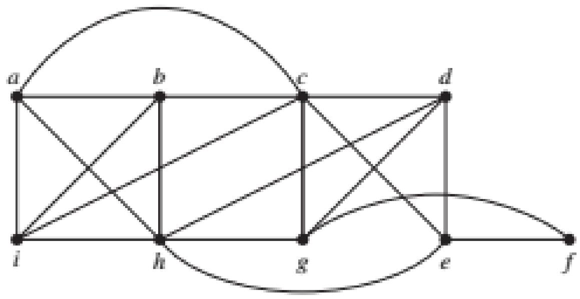

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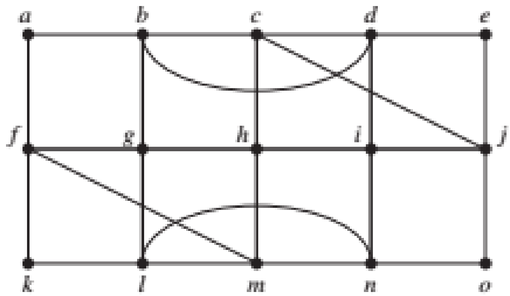
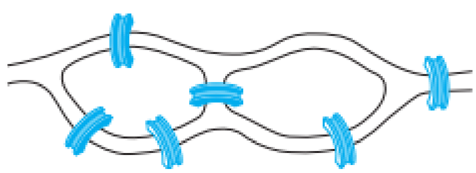
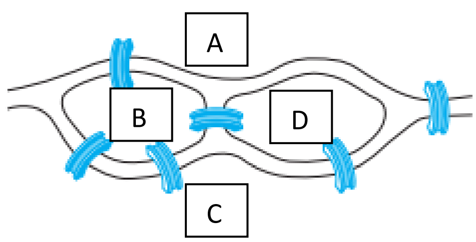


Can someone cross all the bridges shown in this map exactly once and return to the starting point?

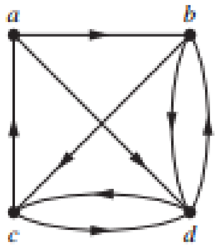


Solution: [Convert it into a graph, we will have Degree of A=2, B=4, C=4, D=2]



In Exercises 18–23 determine whether the directed graph shown has an Euler circuit. Construct an Euler circuit if one exists. If no Euler circuit exists, determine whether the directed graph has an Euler path. Construct an Euler path if one exists.

18.



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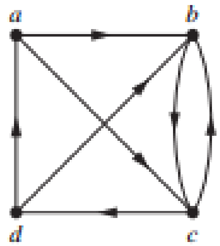

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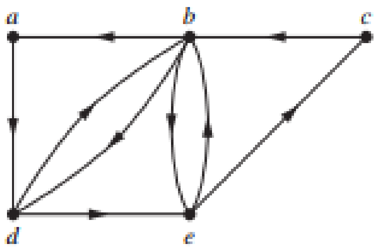

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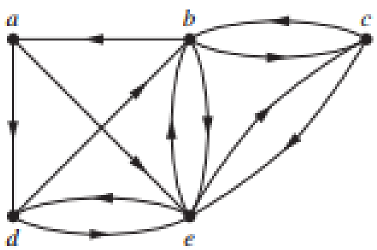

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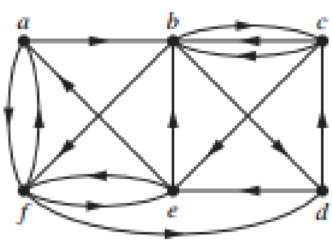
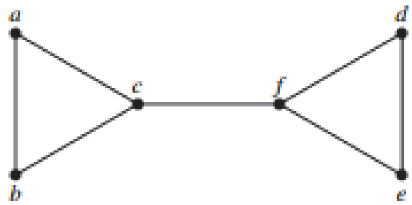


In Exercises 30–36 determine whether the given graph has a Hamilton circuit. If it does, find such a circuit. If it does not, give an argument to show why no such circuit exists.

30.



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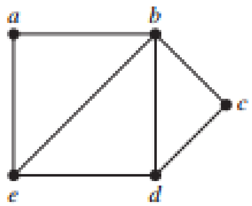

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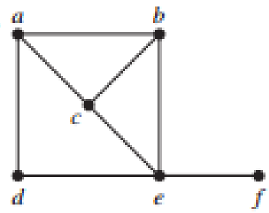

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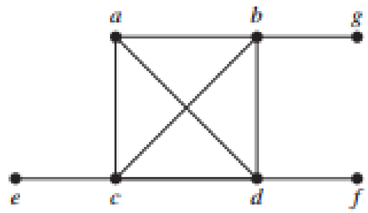

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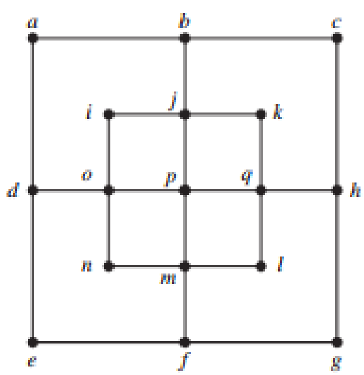

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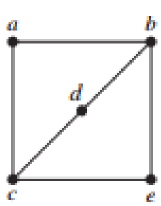

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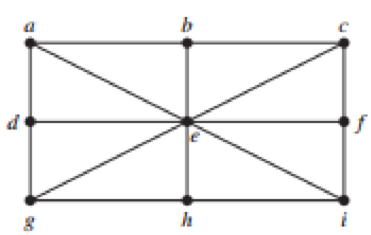


SOLUTION 30&31:

SOLUTION

Let us first determine the degree of every vertex in the given graph:

$$\begin{aligned}\deg(a) &= 2\\ \deg(b) &= 2\\ \deg(c) &= 3\\ \deg(d) &= 3\\ \deg(e) &= 2\\ \deg(f) &= 2\end{aligned}$$

We then note that Dirac's theorem is not satisfied, but this does not necessarily mean that no Hamilton circuit exists.

SOLUTION

Let us first determine the degree of every vertex in the given graph:

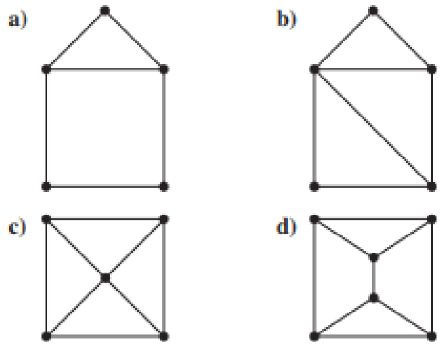
$$\begin{aligned}\deg(a) &= 2\\ \deg(b) &= 4\\ \deg(c) &= 2\\ \deg(d) &= 3\\ \deg(e) &= 3\end{aligned}$$

We then note that Dirac's theorem is not satisfied (since some degrees are less than $n/2 = 5/2 = 2.5$), but this does not necessarily mean that no Hamilton circuit exists.

However, we do note that the given graph graph contains the cycle C_5 and the cycle C_5 within the given graph forms a Hamilton circuit (as the circuit will pass through all vertices exactly once).

A possible Hamilton circuit is thus the path of C_5: a, b, c, d, e, a.

For each of these graphs, determine (*i*) whether Dirac's theorem can be used to show that the graph has a Hamilton circuit, (*ii*) whether Ore's theorem can be used to show that the graph has a Hamilton circuit, and (*iii*) whether the graph has a Hamilton circuit

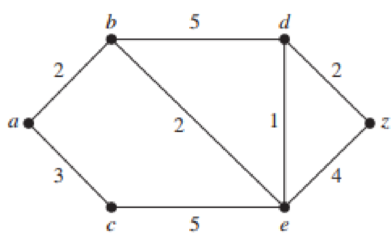


Shortest path Problems

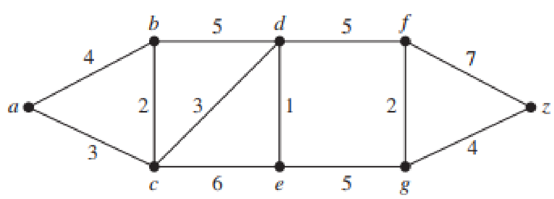
1. For each of these problems about a subway system, describe a weighted graph model that can be used to solve the problem.
a) What is the least amount of time required to travel between two stops?
b) What is the minimum distance that can be traveled to reach a stop from another stop?
c) What is the least fare required to travel between two stops if fares between stops are added to give the total fare?

In Exercises 2–4 find the shortest path between *a* and *z and its length* in the given weighted graph.

2.



3.



4.