# National University of Computer and Emerging Sciences, Karachi

## FAST School of Computing

## Midterm 1 Examination, Fall 2022

## September 28, 2022, 08:30 am – 09:30 pm

| Course Code: CS1005 | Course Name: Discrete Structures |
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| Instructor Names: Mr. Shoaib Raza, Ms. Bakhtawer, Ms. Safia, Ms. Fizza Aqeel, Mr. Fahad Hussain and Mr. Sudais | |
| Student Roll No: | Section No: |

**Instructions:**

- Return the question paper along with the answer script. Read each question completely before answering it. There are **3 questions and 2 pages.**
- In case of any ambiguity, you may make assumption. But your assumption should not contradict any statement in the question paper.
- Answering all the questions in given sequence of the question paper.

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**Total Time**: 60 minutes **Maximum Points**: 24

**Question # 1 (Propositional Logic and Rules of Inference)** **[CLO-3 C3]**

(a) Let P, Q, and R be the propositions. [2 Points]

P: Niagara Falls is in New York. Q: New York City is the capital state of United State.

R: New York City will have more snow in 2050.

Write these propositions using P, Q, and R and logical connectives (including negations):

(i) If Niagara Falls is in New York, New York City will not have more snow in 2050. Solution: $(P \rightarrow \neg R)$

(ii) Neither Niagara Falls is in New York nor will New York City have more snow in 2050. Solution: $(\neg P \wedge \neg R)$

(iii) It is not the case that New York City is not the state capital of the United States. Solution: $\neg(\neg Q)$

(iv) New York City will not have more snow in 2050 only if New York City is not the state capital of the United States.
Solution: $(\neg R \rightarrow \neg Q)$

(b) Using the truth table, prove or disprove that the contrapositive of statement (i) in part (a) is equivalent to the converse of its inverse. [2 Points]

Solution:

| P | Q | R | ¬ P | ¬ R | R → ¬ P Contrapositive | ¬ P → R Inverse | R → ¬ P Converse of Inverse |
|---|---|---|---|---|---|---|---|
| T | T | T | F | F | F | T | F |
| T | T | F | F | T | T | T | T |
| T | F | T | F | F | F | T | F |
| T | F | F | F | T | T | T | T |
| F | T | T | T | F | T | T | T |
| F | T | F | T | T | T | F | T |
| F | F | T | T | F | T | T | T |
| F | F | F | T | T | T | F | T |

(c) Using the premises (statements) from part (a), apply rules of inference to obtain conclusion(s). [2 Points]

Solution:

| | |
|---|---|
| $= (P \rightarrow \neg R) \wedge (\neg P \wedge \neg Q) \wedge \underline{\neg(\neg Q)} \wedge (\neg R \rightarrow \neg Q)$ | Double negation |
| $= \underline{(P \rightarrow \neg R) \wedge (\neg R \rightarrow \neg Q)} \wedge \underline{(Q)} \wedge (\neg P \wedge \neg Q)$ | Re-arrange the premises and Hypothetical Syllogism |
| $= \underline{(P \rightarrow \neg Q) \wedge (Q)} \wedge (\neg P \wedge \neg Q)$ | Modus Tollen |
| $\underline{= (\neg P) \wedge (\neg P \wedge \neg Q)}$ | Commutative |
| $\underline{= (\neg P \wedge \neg P) \wedge \neg Q}$ | Associative |
| $= \underline{(\neg P \wedge \neg Q)}$ | Simplification |
| $= \neg P \wedge \neg Q$ | |

Conclusion: ¬P: "Niagara Falls is not in New York." **OR** ¬Q "New York city is not the capital state of United State."

(d) Using laws of Logic, determine if the following statement is a tautology, contradiction or a contingency. [2 Points]

$$((P \vee Q) \wedge (P \rightarrow R)) \rightarrow (Q \vee R)$$

Solution:

The statement is a tautology.

| | | |
|---|---|---|
| | $((a \vee b) \wedge (a \rightarrow c)) \rightarrow (b \vee c)$ | |
| $\equiv$ | $\neg((a \vee b) \wedge (\neg a \vee c)) \vee (b \vee c)$ | Implication equivalence(x2). |
| $\equiv$ | $(\neg(a \vee b) \vee \neg(\neg a \vee c)) \vee (b \vee c)$ | De Morgans. |
| $\equiv$ | $((\neg a \wedge \neg b) \vee (\neg\neg a \wedge \neg c)) \vee (b \vee c)$ | De Morgans. |
| $\equiv$ | $((\neg a \wedge \neg b) \vee (a \wedge \neg c)) \vee (b \vee c)$ | Double negation. |
| $\equiv$ | $(\neg a \wedge \neg b) \vee b \vee (a \wedge \neg c) \vee c$ | Assocative and commutative. |
| $\equiv$ | $((\neg a \vee b) \wedge (\neg b \vee b)) \vee ((a \vee c)) \wedge (\neg c \vee c))$ | Distributive. |
| $\equiv$ | $((\neg a \vee b) \wedge T) \vee ((a \vee c)) \wedge T)$ | Negation. |
| $\equiv$ | $(\neg a \vee b) \vee (a \vee c)$ | Identity laws (x2). |
| $\equiv$ | $a \vee \neg a \vee b \vee c)$ | Associative and commutative. |
| $\equiv$ | $T \vee b \vee c$ | Negation |
| $\equiv$ | $T$ | Domination |

**Question # 2 (Predicates and Quantifiers)** **[CLO-2 C2]**

(a) Let F(x, y) means "x + y = 1", where 'x' and 'y' are integers. Determine the truth value of the following statements. [2 Points]

(i) ∀x ∃y F(x, y) Solution: TRUE

(ii) ∃x ∀y F(x, y) Solution: FALSE

(b) Translate each of the following statements into logical expressions using predicates, quantifiers, and logical connectives where C(x) is "x is a comedian" and F(x) is "x is funny" and the domain consists of all people. [2 Points]

(i) All comedians are funny. Solution: ∀x(C(x) → F(x))

(ii) Some comedians are funny. Solution: ∃x(C(x) ∧ F(x))

(c) Translate each of the following statements into English where P(x) is "x is a professor,", Q(x) is "x is ignorant,", and R(x) is "x is vain," and the domain consists of all people. [2 Points]

(i) ¬ ∃x (P(x) ∧ Q(x)) Solution: No professors are ignorant.

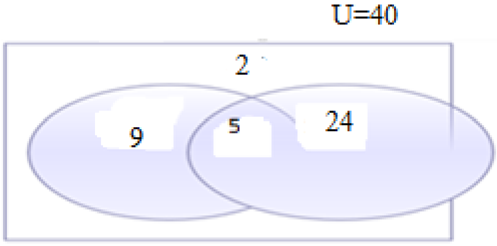
(ii) ∀x (Q(x) → R(x)) Solution: All ignorant people are vain.

**Question # 3 (Set Theory and Functions)** **[CLO-2 C2]**

(a) Out of 40 students, 14 are taking English Composition and 29 are taking Chemistry. If five students are in both classes. Using a Venn diagram, determine how many students are in either class and how many are in neither of the classes? [2 Points]

Solution:

Total number of students, n(μ) = 40
Number of English Composition students, n(E) = 14
Number of Chemistry students, n(C) = 29
Number of students who learning both, n(E∩C) = 5
Number of students who learning either of them,
n(E∪C) = n(E) + n(C) – n(E∩C) = 14 + 29 - 5 = 38
Number of students who learning neither = n(μ) – n(E∪C) = 40– 38 = 2.




(b) Using Set identities, prove or disprove that $\overline{A \cap \overline{B}} \cup B = \bar{A} \cup B$ [2 Points]

Solution:

= $(A^c \cup (B^c)^c) \cup B$ de Morgan's
= $(A^c \cup B) \cup B$ double complement
= $A^c \cup (B \cup B)$ associative
= $A^c \cup B$ idempotent

(c) Suppose $f$: Z→Z where $f$ (m, n) = $x^3 + 1$. Determine whether the function is an onto (surjective) and/or a one-to-one (injective) or both (bijective). [2 Points]

Solution: one-to-one (injective)

(d) Given f(x) = $x^3$ + 18 and g(x) = 4x + 1, find (f o g) (x) [2 Points]

Solution:

(f o g) (x) = $64x^3$ + 60x +19

(e) Prove or disprove the statement $\lceil -x \rceil = -\lfloor x \rfloor$ for real number $x$.

Solution:

$\lceil -2.5 \rceil = -\lfloor 2.5 \rfloor$

-2 = -2 Proved.

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***ALL THE BEST***