RELATIONS

Let $A = \{1,2\}$, $B = \{1,2,3\}$

$A \times B = \{(1,1),(1,2),(1,3),(2,1),(2,2),(2,3)\}$

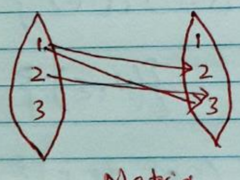
All subsets of $A \times B$ are relations from A to B.

Domain of a Relation $= \{a \in A \mid (a,b) \in R\}$
Range of a Relation $= \{b \in B \mid (a,b) \in R\}$

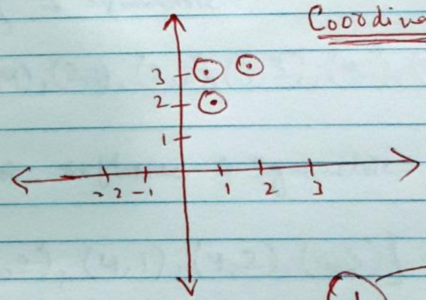
Let $R = \{(a,b) \in A \times B \mid a < b\}$

(a) Find the ordered Pair in R: $R = \{(1,2),(1,3),(2,3)\}$
(b) Find the Domain & range in R: $D = \{1,2\}$, $R = \{2,3\}$
(c) IS 1R3 , 2R2 , 3R2 ?? I
Yes ~~No~~ No

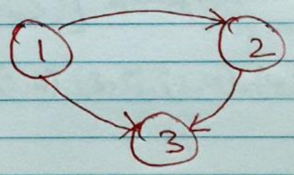
Arrow Diagram



Coordinate Graph



Directed Graph.



Matrix

$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0 & 1 & 1 \\ 2 & 0 & 0 & 1 \\ 3 & 0 & 0 & 0 \end{array}$$

$R = \{(1,1), (1,2), (2,1), (1,-1), (2,2)\}$

$R_1 = \{(a,b) \mid a \leq b\} = \{(1,1), (1,2), (2,2)\}$

$R_2 = \{(a,b) \mid a > b\} = \{(2,1), (1,-1)\}$

$R_3 = \{(a,b) \mid a = b \vee a = -b\} = \{(1,1), (1,-1), (2,2)\}$

$R_4 = \{(a,b) \mid a = b\} = \{(1,1), (2,2)\}$

$R_5 = \{(a,b) \mid a = b+1\} = \{(2,1)\}$

$R_6 = \{(a,b)\ a+b \leq 3\} = \{(1,1), (1,2), (2,1), (1,-1)\}$

Date.

$A \times A$ is known as the universal Relation.

TYPES OF RELATION

1- Reflexive $\forall a \in A, \forall (a,a) \in R$

2- Symmetric $\forall (a,b) \in R$, if $(a,b) \in R$ than $(b,a) \in R$

3- Antisymmetric $\forall (a,b) \in R$, if $(a,b) \wedge (b,a) \in R$ than $a = b$

4- Transitive $\forall (a,b,c) \in R$, if $(a,b) \wedge (b,c) \in R$ than $(a,c) \in R$.

5- Irreflexive $\forall a \in A, \forall (a,a) \notin R$

6- Asymmetric Asymmetric = Irreflexive & Antisymmetric

Examples: $A = \{1, 2, 3, 4\}$

$R_1 = \{(1,1), (1,2), (2,1), (2,2), (3,4), (4,1), (4,4)$

Holds no Property

$R_2 = \{(1,1), (1,2), (2,1)\}$ Symmetric

$R_3 = \{(1,1), (1,2), (1,4), (2,1), (2,2), (3,3), (4,1), (4,4)\}$ Reflexive & Symmetric

$R_6 = \{(3,4)\}$ not reflexive & Symmetric

$R_4 = \{(2,1), (3,1), (3,2), (4,1), (4,2), (4,3)\}$ Antisymmetric & irreflexive Asymmetric

$R_5 = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,3), (3,4), (4,4)\}$

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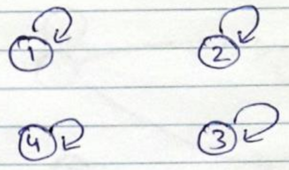
Reflexive

$\rightarrow \forall a \in A, \forall (a,a) \in R$

$R_1 = \{(1,1), (2,2), (3,3), (4,4)\}$

$$\begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & & 0 & 1 \end{bmatrix} \end{array}$$



Diagonal has all 1's

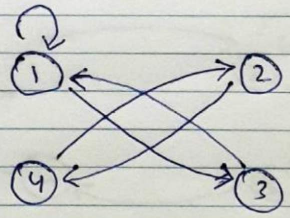
Every node has loop.

Symmetric

$\rightarrow \forall (a,b) \in R$, If $(a,b) \in R$ than (b,a)

$R_1 = \{(1,1), (1,3), (2,4), (3,1), (4,2)\}$

$$\begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$



$M = M^t$

Pair of incoming & outgoing node

bidirectional links

Date.

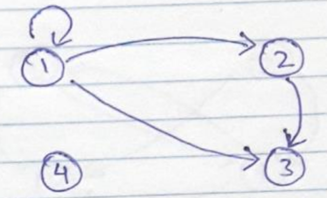
3- Transitive

$\forall a,b,c \in A$, If $((a,b) \in R \wedge (b,c) \in R)$ than $(a,c) \in R$

$R_1 = \{(1,1),(1,2),(1,3),(2,3)\}$

$$\begin{array}{c c} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

No identification



~~If~~ three nodes have indirect Path than 1^{st} & 3^{rd} have direct Path.

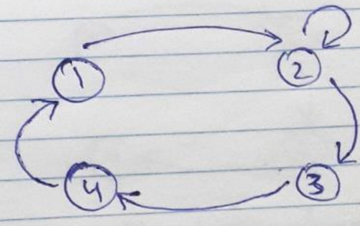
or Forms a triangle

4- Antisymmetric

$\forall a,b \in A$, If $((a,b) \wedge (b,a)) \in R$ than $a = b \in R$

$R_1 = \{(1,2),(2,2),(2,3),(3,4),(4,1)\}$

$$\begin{array}{c c} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$



$M_{ij} = M_{ji} \quad i \neq j$

$M_{12} = M_{21}$

$M_{23} = M_{32} \quad 1 = 0$

$1 = 0$

no Pair of arrows between distinct node

No bidirectional link

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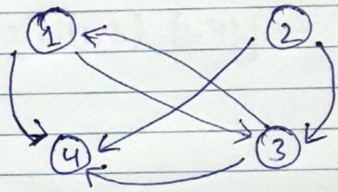
5- Irreflexive:

$\forall a \in A, \forall (a,a) \notin R$

$R_1 = \{(1,3), (1,4), (2,3), (2,4), (3,1), (3,4)\}$

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 0 & 1 & 1 \\ 2 & 0 & 0 & 1 & 1 \\ 3 & 1 & 0 & 0 & 1 \\ 4 & 0 & 0 & 0 & 0 \end{array}$$



All diagonal have 0's

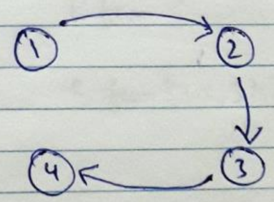
→ no loops

6- Asymmetric:

Asymmetric = Antisymmetric + Irreflexive

$R = \{(1,2), (2,3), (3,4)\}$

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 1 & 0 \\ 3 & 0 & 0 & 0 & 1 \\ 4 & 0 & 0 & 0 & 0 \end{array}$$



No bidirectional link

No loop

All 0's in diagnol

$M_{ij} \neq M_{ji}$ $\quad i \neq j$

$0 \neq 1$

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Combining Relations:

let $A = \{1,2,3\}$, $B = \{1,2,3,4\}$

$R_1 = \{(1,1),(2,2),(3,3)\}$ $R_2 = \{(1,1),(1,2),(1,3)(1,4)\}$

$R_1 \cup R_2 = \{(1,1),(2,2),(3,3),(1,2),(1,3),(1,4)\}$

$R_1 \cap R_2 = \{(1,1)\}$

$R_1 - R_2 = \{(2,2),(3,3)\}$

$R_2 - R_1 = \{(1,2),(1,3),(1,4)\}$

$R_1 \oplus R_2 = \{(1,2),(1,3),(1,4),(2,2),(3,3)\}$

$\underline{R \circ S} = \{(1,0),(1,1),(2,1),(2,2),(3,0),(3,1)\}$

$R = \{(1,1),(1,4),(2,3),(3,1),(3,4)\}$ R from $\{1,2,3\}$ to $\{1,2,3,4\}$

$S = \{(1,0),(2,0),(3,1),(3,2),(4,1)\}$ S from $\{1,2,3,4\}$ to $\{0,1,2\}$

Inverse of Relation

$R^{-1} = \{(1,1),(4,1),(3,2),(1,3),(4,3)\}$

Equivalence relation: Reflexive, Symmetric & transitive.
Partial order relation: Reflexive, Antisymmetric & transitive.