National University of Computer & Emerging Sciences, Karachi

Fall-2019 CS-Department

CS211-Discrete Structures

Practice Assignment-I

Note:

1- This is hand written assignment.
2- Just write the question number instead of writing the whole question.
3- You can only use A4 size paper for solving the assignment.

Submission date: Tuesday,24th September, 2019 by 12 noon at my office

1. Which of these sentences are propositions? What are the truth values of those that are propositions?

a) Boston is the capital of Massachusetts.
b) Miami is the capital of Florida.
c) 2 + 3 = 5.
d) 5 + 7 = 10.
e) x + 2 = 11.
f) Answer this question.

2. What is the negation of each of these propositions?

a) Jennifer and Teja are friends.
b) There are 13 items in a baker's dozen.
c) Abby sent more than 100 text messages every day.
d) 121 is a perfect square.
e) Steve has more than 100 GB free disk space on his laptop.
f) Zach blocks e-mails and texts from Jennifer.
g) $7 \cdot 11 \cdot 13 = 999$.
h) Diane rode her bicycle 100 miles on Sunday.

3. Suppose that Smartphone A has 256MBRAMand 32GB ROM, and the resolution of its camera is 8 MP; Smartphone B has 288 MB RAM and 64 GB ROM, and the resolution of its camera is 4 MP; and Smartphone C has 128 MB RAM and 32 GB ROM, and the resolution of its camera is 5 MP. Determine the truth value of each of these propositions.

a) Smartphone B has the most RAM of these three smartphones.
b) Smartphone C has more ROM or a higher resolution camera than Smartphone B.
c) Smartphone B has more RAM, more ROM, and a higher resolution camera than Smartphone A.
d) If Smartphone B has more RAM and more ROM than Smartphone C, then it also has a higher resolution camera.
e) Smartphone A has more RAM than Smartphone B if and only if Smartphone B has more RAM than Smartphone A.

4. Suppose that during the most recent fiscal year, the annual revenue of Acme Computer was 138 billion dollars and its net profit was 8 billion dollars, the annual revenue of Nadir Software was 87 billion dollars and its net profit was 5 billion dollars, and the annual revenue of Quixote Media was 111 billion dollars and its net profit was 13 billion dollars. Determine the truth value of each of these propositions for the most recent fiscal year.

a) Quixote Media had the largest annual revenue.
b) Nadir Software had the lowest net profit and Acme Computer had the largest annual revenue.
c) Acme Computer had the largest net profit or Quixote Media had the largest net profit.
d) If Quixote Media had the smallest net profit, then Acme Computer had the largest annual revenue.
e) Nadir Software had the smallest net profit if and only if Acme Computer had the largest annual revenue.

5. Let p, q, and r be the propositions

p: You have the flu. q: You miss the final examination. r: You pass the course.

Express each of these propositions as an English sentence.

a) $\neg p$ b) $p \lor q$ c) $p \rightarrow q$ d) $p \land q$ e) $p \leftrightarrow q$
f) $\neg p \rightarrow \neg q$ g) $\neg p \land \neg q$ h) $\neg p \lor (p \land q)$ i) $p \rightarrow q$ j) $\neg q \leftrightarrow r$
k) $q \rightarrow \neg r$ l) $p \lor q \lor r$ m) $(p \rightarrow \neg r) \lor (q \rightarrow \neg r)$ n) $(p \land q) \lor (\neg q \land r)$

6. Let p, q, and r be the propositions

p: You get an A on the final exam. q: You do every exercise in this book. r: You get an A in this class.

Write these propositions using p, q, and r and logical connectives.

a) You get an A in this class, but you do not do every exercise in this book.
b) You get an A on the final, you do every exercise in this book, and you get an A in this class.
c) To get an A in this class, it is necessary for you to get an A on the final.
d) You get an A on the final, but you don't do every exercise in this book; nevertheless, you get an A in this class.
e) Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.
f) You will get an A in this class if and only if you either do every exercise in this book or you get an A on the final.

7. Write each of these statements in the form "if p, then q" in English. [Hint: Refer to the list of common ways to express conditional statements provided in this section.]

a) I will remember to send you the address only if you send me an e-mail message.
b) To be a citizen of this country, it is sufficient that you were born in the United States.
c) If you keep your textbook, it will be a useful reference in your future courses.
d) The Red Wings will win the Stanley Cup if their goalie plays well.
e) That you get the job implies that you had the best credentials.
f) The beach erodes whenever there is a storm.
g) It is necessary to have a valid password to log on to the server.
h) You will reach the summit unless you begin your climb too late.

8. What are the disjunction, conjunction, exclusive or, conditional, and biconditional of the propositions "I'll go to the movies tonight" and "I'll finish my discrete mathematics homework"?

9. "If it is sunny tomorrow, then I will go for a walk in the woods."

a) Describe at least five different ways to write the conditional statement $p \rightarrow q$ in English.
b) State the converse, inverse and contrapositive of a conditional statement.
c) Given a conditional statement $p \rightarrow q$, find the inverse of its inverse, the inverse of its converse, and the inverse of its contrapositive.

10. Show in at least two different ways that the compound propositions $\neg p \lor (r \rightarrow \neg q)$ and $\neg p \lor \neg q \lor \neg r$ are equivalent.

11. Let P(x) be the statement "Student x knows calculus" and let Q(y) be the statement "Class y contains a student who knows calculus." Express each of these as quantifications of P(x) and Q(y).

a) Some students know calculus.
b) Not every student knows calculus.
c) Every class has a student in it who knows calculus.
d) Every student in every class knows calculus.
e) There is at least one class with no students who know calculus.

12. Let P(m, n) be the statement "m divides n," where the domain for both variables consists of all positive integers. (By "m divides n" we mean that n = km for some integer k.) Determine the truth values of each of these statements.

a) $P(4, 5)$ b) $P(2, 4)$ c) $\forall m \forall n P(m, n)$
d) $\exists m \forall n P(m, n)$ e) $\exists n \forall m P(m, n)$ f) $\forall n P(1, n)$

13. Show that these compound propositions are tautologies.
a) $(p \to q) \wedge (q \to r) \to (p \to r)$ b) $(p \vee q) \wedge (\neg p \vee r) \to (q \vee r)$
c) $(\neg q \wedge (p \to q)) \to \neg p$ d) $((p \vee q) \wedge \neg p) \to q$

14. Show that these compound propositions are not logically equivalent.
a) $(p \to q) \to r$ and $p \to (q \to r)$
b) $(p \wedge q) \to r$ and $(p \to r) \wedge (q \to r)$
c) $p \to q) \to (r \to s)$ and $(p \to r) \to (q \to s)$

15. Express these system specifications using the propositions p "The message is scanned for viruses" and q "The message was sent from an unknown system" together with logical connectives (including negations).

a) "The message is scanned for viruses whenever the message was sent from an unknown system."
b) "The message was sent unknown system but it was not scanned for viruses."

16. Let F(x, y) be the statement "x can fool y," where the domain consists of all people in the world. Use quantifiers to express each of these statements.

a) Everybody can fool Fred.
b) Evelyn can fool everybody.
c) Everybody can fool somebody.

17. Let Q(x, y) be the statement "x has sent an e-mail message to y," where the domain for both x and y consists of all students in your class. Express each of these quantifications in English.

a) $\exists x \exists y Q(x, y)$ b) $\exists x \forall y Q(x, y)$
c) $\forall x \exists y Q(x, y)$ d) $\exists y \forall x Q(x, y)$

18. Use De Morgan's laws to find the negation of each of the following statements.

a) Jan is rich and happy. b) Carlos will bicycle or run tomorrow

19. Show that $p \leftrightarrow q$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$ are logically equivalent.

20. Determine the truth value of each of these statements if the domain of each variable consists of all real numbers.
a) $\exists x(x^2= 2)$ b) $\exists x(x^2 = -1)$
c) $\forall x(x^2 + 2 \geq 1)$ d) $\forall x(x^2= x)$

21. Let P(x) be the statement "x can speak Russian" and let Q(x) be the statement "x knows the computer language C++." Express each of these sentences in terms of P(x), Q(x), quantifiers, and logical connectives. The domain for quantifiers consists of all students at your school.

a) There is a student at your school who can speak Russian and who knows C++.
b) There is a student at your school who can speak Russian but who doesn't know C++.
c) Every student at your school either can speak Russian or knows C++.
d) No student at your school can speak Russian or knows C++.

22. Let P(x, y) be the statement "Student x has taken class y," where the domain for x consists of all students in your class and for y consists of all computer science courses at your school. Express each of these quantifications in English.

a) $\exists x \exists y P(x, y)$ b) $\exists x \forall y P(x, y)$ c) $\forall x \exists y P(x, y)$
d) $\exists y \forall x P(x, y)$ e) $\forall y \exists x P(x, y)$ f) $\forall x \forall y P(x,y)$

23. Let Q(x, y) be the statement $x + y = x - y$ where the universe for x and y is the set of all real numbers. Determine the truth value of:

(a) $Q(5,-2)$. (b) $Q(4.7, 0)$.
(c) Determine the set of all pairs of numbers, x and y, such that Q(x, y) is true.

24. Find a universe for x such that $\forall x$ (x2 < x) is true.

25. The following proposition uses the English connective "or". Determine from the context whether "or" is intended to be used in the inclusive or exclusive sense.
"If you do not wear a shirt or do not wear shoes, then you will be denied service in the restaurant."

26. Show that by using laws of logic: $(p \wedge (\sim(\sim p \vee q))) \vee (p \wedge q) \cong p$

27. Show by the Venn diagram when

If $A \subseteq B$ $A \cap C = \varnothing$ and B and C are overlapping

(i) $A \cup (B \cap C)$

If A and B are overlapping, B and Care overlapping but A and C are different

(ii)$(A \cap B) \cup (A \cap C)$ (iii)$A \cup (B \cap C)$

28. Simplify the following expression by using the set identities $(A-(A \cap B)) \cap (B-(A \cap B))$

29. Let P(x), Q(x), R(x), and S(x) be the statements "x is a baby," "x is logical," "x is able to manage a crocodile," and "x is despised," respectively. Suppose that the domain consists of all people. Express each of these statements using quantifiers; logical connectives; and P(x), Q(x), R(x), and S(x).

a) Babies are illogical.
b) Nobody is despised who can manage a crocodile.
c) Illogical persons are despised.
d) Babies cannot manage crocodiles.
∗e) does (d) follow from (a), (b), and (c)? If not, is there a correct conclusion?

30. Let F(x, y) be the statement "x can fool y," where the domain consists of all people in the world. Use quantifiers to express each of these statements.

a) Everybody can fool Fred.
b) Evelyn can fool everybody.
c) Everybody can fool somebody.
d) There is no one who can fool everybody.
e) Everyone can be fooled by somebody.
f) No one can fool both Fred and Jerry.
g) Nancy can fool exactly two people.

31. Show that the following statement is valid:

a) If today is Tuesday, I have a test in maths or economics. If my economics professor is sick, I will not have a test in economics. Today is Tuesday and my economics professor is sick. Therefore I have a test of maths.

b) If Ali is a lawyer, then he is ambitious. If Ali is an early riser then he does not like chocolates. If Ali is ambitious then he is an early riser. Then if Ali is a lawyer then he does not like chocolates.

32. What rule of inference is used in each of these arguments?

a) Alice is a mathematics major. Therefore, Alice is either a mathematics major or a computer science major.
b) Jerry is a mathematics major and a computer science major. Therefore, Jerry is a mathematics major.
c) If it is rainy, then the pool will be closed. It is rainy. Therefore, the pool is closed.
d) If it snows today, the university will close. The university is not closed today. Therefore, it did not snow today.
e) If I go swimming, then I will stay in the sun too long. If I stay in the sun too long, then I will sunburn. Therefore, if I go swimming, then I will sunburn.

33. What rule of inference is used in each of these arguments?

a) Kangaroos live in Australia and are marsupials. Therefore, kangaroos are marsupials.
b) It is either hotter than 100 degrees today or the pollution is dangerous. It is less than 100 degrees outside today. Therefore, the pollution is dangerous.
c) Linda is an excellent swimmer. If Linda is an excellent swimmer, then she can work as a lifeguard. Therefore, Linda can work as a lifeguard.
d) Steve will work at a computer company this summer. Therefore, this summer Steve will work at a computer company or he will be a beach bum.
e) If I work all night on this homework, then I can answer all the exercises. If I answer all the exercises, I will understand the material. Therefore, if I work all night on this homework, then I will understand the material.

34. Use rules of inference to show that the hypotheses "Randy works hard," "If Randy works hard, then he is a dull boy," and "If Randy is a dull boy, then he will not get the job" imply the conclusion "Randy will not get the job."

35. Use rules of inference to show that the hypotheses "If it does not rain or if it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on," "If the sailing race is held, then the trophy will be awarded, "and "The trophy was not awarded" imply the conclusion "It rained.

36. Let A = {0, 2, 4, 6, 8, 10}, B = {0, 1, 2, 3, 4, 5, 6}, and C = {4, 5, 6, 7, 8, 9, 10}. Find:

a) $A \cap B \cap C$.   b) $A \cup B \cup C$.   c) $(A \cup B) \cap C$.   d) $(A \cap B) \cup C$.

37. Draw the Venn diagrams for each of these combinations of the sets *A*, *B*, and *C*.
a) $A \cap (B - C)$ b) $(A \cap B) \cup (A \cap C)$ c) $(A \cap B) \cup (A \cap C)$

38. Apply principle of inclusion-exclusion. Suppose that there are 520 people on an airplane. All of them eat meat, but 200 only like beef and 400 like chicken only. How many of them like both chicken and beef and neither of them. Also draw Venn diagram.

39. In a survey on the gelato preferences of college students, the following data was obtained: 78 like mixed berry, 32 like Irish cream, 57 like tiramisu, 13 like both mixed berry and Irish cream, 21 like both Irish cream and tiramisu, 16 like both tiramisu and mixed berry, 5 like all three flavors, and 14 like none of these three flavors. How many students were surveyed? Apply principle of inclusion-exclusion.

40.

Let $f(x) = \lfloor x^2/3 \rfloor$. Find $f(S)$ if

a) $S = \{-2, -1, 0, 1, 2, 3\}$.
b) $S = \{0, 1, 2, 3, 4, 5\}$.
c) $S = \{1, 5, 7, 11\}$.
d) $S = \{2, 6, 10, 14\}$.

41.

Why is f not a function from $\mathbf{R}$ to $\mathbf{R}$ if

a) $f(x) = 1/x$?
b) $f(x) = \sqrt{x}$?
c) $f(x) = \pm\sqrt{(x^2+1)}$?

42.

Determine whether f is a function from $\mathbf{Z}$ to $\mathbf{R}$ if

a) $f(n) = \pm n$.
b) $f(n) = \sqrt{n^2+1}$.
c) $f(n) = 1/(n^2-4)$.

43.

Find these values.

a) $\lceil \frac{3}{4} \rceil$
b) $\lfloor \frac{7}{8} \rfloor$
c) $\lceil -\frac{3}{4} \rceil$
d) $\lfloor -\frac{7}{8} \rfloor$
e) $\lceil 3 \rceil$
f) $\lfloor -1 \rfloor$
g) $\lfloor \frac{1}{2} + \lceil \frac{3}{2} \rceil \rfloor$
h) $\lfloor \frac{1}{2} \cdot \lfloor \frac{5}{2} \rfloor \rfloor$

44.

Determine whether each of these functions from $\{a, b, c, d\}$ to itself is one-to-one.

a) $f(a) = b$, $f(b) = a$, $f(c) = c$, $f(d) = d$
b) $f(a) = b$, $f(b) = b$, $f(c) = d$, $f(d) = c$
c) $f(a) = d$, $f(b) = b$, $f(c) = c$, $f(d) = d$

45.

Determine whether each of these functions is a bijection from $\mathbf{R}$ to $\mathbf{R}$.

a) $f(x) = 2x + 1$
b) $f(x) = x^2 + 1$
c) $f(x) = x^3$
d) $f(x) = (x^2+1)/(x^2+2)$

46.

Let $f: \mathbf{R} \to \mathbf{R}$ and let $f(x) > 0$ for all $x \in \mathbf{R}$. Show that $f(x)$ is strictly decreasing if and only if the function $g(x) = 1/f(x)$ is strictly increasing.

47.

Find $f \circ g$ and $g \circ f$, where $f(x) = x^2 + 1$ and $g(x) = x + 2$, are functions from $\mathbf{R}$ to $\mathbf{R}$.

48.

Prove that if x is a real number, then $\lfloor -x \rfloor = -\lceil x \rceil$ and $\lceil -x \rceil = -\lfloor x \rfloor$.

49. Show that by using laws of logic:

$\neg(p \leftrightarrow q) \equiv (p \leftrightarrow \neg q)$

50. Show that by using laws of logic:

$\neg p \leftrightarrow q$ **and** $p \leftrightarrow \neg q$ **are logically equivalent.**