CS211 DISCRETE STRUCTURES

MID-II SOLUTION

Instructions:

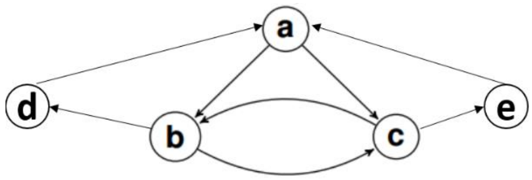
- Return the question paper together with the answer script. Read each question completely before answering it. There are **4 questions** written on **2 pages.**
- In case of any ambiguity, you may make assumptions. However, your assumptions should not contradict any statement in the question paper.
- Attempt all the questions in given sequence of the question paper to get bonus point.

Total Time: 60 Minutes **Maximum Points**: 26

Question # 1: [CLO-1] **[4x2= 08 points]**

(a) Find the smallest relation on {cup, glass, soccer}, that is Asymmetric and Transitive, but not Symmetric.
Solution:
R= {(cup, glass)}

(b) Represent the following digraph as shown in figure # 1 in matrix form.



Solution:

$$\begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Figure # 1

(c) Find the value of the sum: $\sum_{k=1}^{4}(k^2 - 1)$ ·
Solution:
$\sum_{k=1}^{4}(k^2 - 1) = (1^2 - 1) + (2^2 - 1) + (3^2 - 1) + (4^2 - 1) = 26.$

(d) Find the sum of number between 200 and 950 which are divisible by 11.
Solution:
a= 209, d= 11, T_n = 946.
$T_n = a + (n-1)d$; $946 = 209 + (n-1)(11)$ n =68.

Now for Sum; $S_n = \frac{n}{2}[2a + (n-1)d]$; $S_{68} = \frac{68}{2}[2(209) + (68-1)(11)]$ = 39,270

Question # 2: [CLO-2] **[3x2=06 points]**

(a) Prove using mathematical induction that $1^3 + 2^3 + 3^3 + \dots + n^3 = (\frac{n(n+1)}{2})^2$, whenever n is a nonnegative integer.

Solution:

$1^3 + 2^3 + 3^3 + \dots + n^3 = (n(n+1)/2)^2$

STEP 1: We first show that p (1) is true.

Left Side = $1^3 = 1$

Right Side = $1^2 (1 + 1)^2 / 4 = 1$

hence p (1) is true.

STEP 2: We now assume that p (k) is true

$1^3 + 2^3 + 3^3 + \dots + k^3 = (k(k+1)/2)^2$ (1)

add $(k + 1)^3$ to both sides

$1^3 + 2^3 + 3^3 + \dots + k^3 + (k+1)^3 = [(k+1)(k+1+1)/2]^2$

$1^3 + 2^3 + 3^3 + \dots + k^3 + (k+1)^3 = [(k+1)(k+2)/2]^2$ (2)

put eq(1) in eq(2)

=> $(k(k+1)/2)^2 + (k+1)^3 = [(k+1)(k+2)/2]^2$

=> $k^2 (k+1)^2 /4 + (k+1)^3$

=> $(k+1)^2 [k^2 + 4k + 4] / 4$

=> $(k+1)^2 [(k+2)^2] / 4$

=> $[(k+1)(k+2)/2]^2 = [(k+1)(k+2)/2]^2$

LHS = RHS, Hence proved!

(b) Let x be an integer and P is the following statement. P: "If $x^2 - (x - 2)^2$ is not divisible by 8, then x is even."
Prove by contraposition.

Solution: Contraposition: If x is odd then $x^2 - (x - 2)^2$ is divisible by 8.

Let x =2k + 1 be an odd number.

$x^2 - (x - 2)^2 = x^2 - (x^2 - 2x + 4) = x^2 - x^2 + 4x - 4) = 4(x - 1) = 4(2k + 1 - 1) = 8k$

which is an integer multiple of 8. Therefore $x^2 - (x - 2)^2$ is divisible by 8.

(c) Express in sigma notation the sum of the first 50 terms of the series 3 + 6 + 9 + 12 + 15 +. . ..

Solution:

In sigma notation we have $\sum_{i=1}^{50} 3i$. Note that we could also write this in other forms, for example $\sum_{j=1}^{50} 3j$ or $\sum_{k=1}^{50} 3k$ (we can use any variable as the index of summation). We can also change the limits of summation, obtaining forms such as the sum $\sum_{i=0}^{49} 3(i+1)$. Note: It is not correct to write $\sum_{i=1}^{50}(3+i)$; this represents the sum $4 + 5 + 6 + \cdots + 53$.

<u>Question # 3:</u> [CLO-3] **[4x2=08 points]**

(a) A message has been encrypted using the function f(x) = (x + 5) mod 26. If the message in coded form is ***VZJXYNTS UFUJW***, decode the message.

Solution:

QUESTION PAPER is the encrypted message.

(b) Find the greatest common divisor, d, of 250 and 29 and determine integers x and y such that d = 250x + 29y.

Solution:

$$\begin{aligned} 250 &= 8.29 + 18 \\ 29 &= 1.18 + 11 \\ 18 &= 1.11 + 7 \\ 11 &= 1.7 + 4 \\ 7 &= 1.4 + 3 \\ 4 &= 1.3 + 1 \\ 3 &= 3.1. \end{aligned}$$

$$\begin{aligned} 1 &= 4 - 3 \\ &= 4 - (7 - 4) = 2.4 - 7 \\ &= 2(11 - 7) - 7 = 2.11 - 3.7 \\ &= 2.11 - 3(18 - 11) = 5.11 - 3.18 \\ &= 5(29 - 18) - 3.18 = 5.29 - 8.18 \\ &= 5.29 - 8(250 - 8.29) = \underline{69}.29 - \underline{8}.250 \end{aligned}$$

(c) List all integers between −100 and 100 that are congruent to −1 modulo 25.
Solution:
-76, -51, -26, -1, 24, 49, 74, 99 are the integers.

(d) Suppose that a computer has only the memory locations 0, 1, 2. . . 64. Use the hashing function h(x) = (x + 9) mod 65 to determine the memory locations at which the following values are stored:
63, 509, 197, 832, and 652.
Solution:
63 will be stored on memory location 7,
509 will be stored on memory location 63,
197 will be stored on memory location 11,
832 will be stored on memory location 61, and
652 will be stored on memory location 11+1=12.

<u>Question # 4:</u> [CLO-4] **[2x2=04 points]**

(a) Determine whether the relation in Question # 1 part (a) is a partial-order relation? Show all of your steps.
Solution:
It holds antisymmetric and transitive property but it does not hold reflexive property hence not a partial order relation.

(b) Determine whether the relation in Question # 1 part (b) is an equivalence relation? Show all of your steps.
Solution:
It is not an equivalence relation. Since it does not hold reflective, symmetric and Transitive properties.

ALL THE BEST