National University of Computer and Emerging Sciences, Lahore Campus				
	Course:	**Probability & Stats**	**Course Code:**	**MT205**
	Program:	**BS CS**	**Semester:**	**Spring 20**
	Duration:	**3 hours**	**Total Marks:**	**130**
	Paper Date:	**July 06; 2020**	**Weight**	**50%**
	Section:	**All**	**Page(s):**	**03**
	Exam:	**Final Term**	**Paper Time**	**9:00am - 12:00noon**
Instruction/Notes:	(i) Attempt All Questions. (ii) Solve the following problems and upload your handwritten solutions to Google class room as a single PDF within given time. (iii) Handwritten solutions should be scanned using camscanner app. Ensure the scans are taken on level surface and with proper lighting. The solution PDF file should be named as FirstName-LastName_18L-1234. (iv) Upload your PDF solution file till 12:15pm (15 minutes for scanning and uploading)			

Q1. **(a)** In the manufacture of a certain scientific instrument great importance is attached to the life of particular critical components. This component is obtained in bulk from source ***A*** and in the course of inspection, the lives of 1150 of the components from source ***A*** are determined. The following frequency table is obtained: **Points (10)**

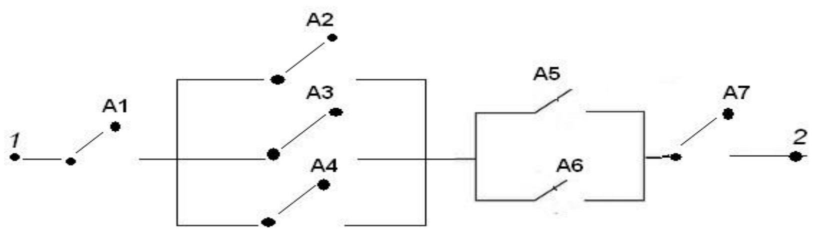
Life in hours	No. of components
1000 - 1020	40
1020 - 1040	96
1040 - 1060	364
1060 - 1080	372
1080 - 1100	85
1100 - 1120	76
1120 - 1140	65
1140 - 1160	52

Find Arithmetic Mean, Median and Mode.

(b) Use method of least square to fit an equation of the form $y = ab^x$ to the following data in which y represents the number of bacteria per unit volume existing in a culture at the end of x hours. **Points (10)**

x	0	1	2	3	4	5
y	73	91	112	131	162	250

Q2. **(a)** In the circuit, given below, the switches open and close independently and randomly. The probability that the particular switch is closed is "0.65" which is same for all the switches. Find the probability of finding closed path from 1 to 2. **Points (10)**



Hint: Use addition law for two and three events. Also use the definition of independent events.

(b) In an experiment, *A*, *B*, *C* and *D* are events with probabilities **Points (10)**
$P(A \cup B) = \frac{5}{8}$, $P(A) = \frac{3}{8}$, $P(C \cap D) = \frac{1}{3}$ and $P(C) = \frac{1}{2}$. Furthermore, *A* and *B* are disjoint while *C* and *D* are independent. Answer the following questions with proper mathematical justification:

(i) What is $P(B)$? (ii) What is $P(A \cap B^c)$? (iii) What is $P(A \cup B^c)$?
(iv) Are A and B independent? (v) What is $P(D)$? (vi) What is $P(C \cup D)$?
(vii) What is $P(C|D)$? (viii) What is $P(C \cap D^c)$? (ix) What is $P(C \cup D^c)$?
(x) What is $P(C^c \cap D^c)$?

(c) A machine produces photo detectors in pairs. Test show that the first photo detector is acceptable with probability $\frac{2}{5}$. When the first photo detector is acceptable, the second photo detector is acceptable with probability $\frac{4}{5}$. Otherwise, if the first photo detector is defective, the second photo detector is acceptable with probability $\frac{3}{5}$. **Points (05)**

(i) Draw a tree diagram for the experiment.
(ii) What is the probability that exactly one photo detector of a pair is acceptable?
(iii) What is the probability that both photo detectors in a pair are defective?

(d) Dr. James has been teaching basic statistics for many years. He knows that 90% of the students will completed the assigned problems. He has also determined that among those who do their assignments, 95% will pass the course. Among those students who do not do their assignments, 60% will pass. Peter took statistics in last semester with Dr. James and received a passing grade. What is the probability that he completed the assignments? **Points (05)**

Q**3.** **(a)** Each time a modem transmits one bit, the receiving modem analyzes the signal that arrives and decides whether the transmitted bit is 0 or 1. It makes an error with probability *p*, independent of whether any other bit is received correctly. Answer the following questions with justification (reason). **Points (10)**

(i) If the transmission continuous until the receiving modem makes its first error, what is the Probability Mass Function (*PMF*) of *X*, the number of bits transmitted?
(ii) If the probability of error is $p = 0.1$, what is the probability that $X \geq 6$?
(iii) If the modem transmits 100 bits, what is the *PMF* of *Y*, the number of errors?
(iv) If the probability of error is $p = 0.01$ and the modem transmits 200 bits, what is the probability that $Y \leq 3$?

(b) The painted light bulbs produced by a company are 60% white, 25% blue and 15% green. In a sample of 10 bulbs, find the probability that 5 are white, 2 are green and 3 are blue. **Points (5)**

(c) Telephone calls are being placed through a certain exchange at random times on the average of ten per minute. Assuming a Poisson random variable, determine the probability that in a 12 second interval, there are 3 or more calls. **Point (5)**

Q4. **(a)** The density function of a random variable *X* is given by **Points (10)**

$$f(x) = \begin{cases} a + bx^2, & 0 \leq x \leq 1 \\ 0, & otherwise \end{cases}$$

If $E[X] = \frac{3}{5}$, find *a* and *b*?

(b) Find moment generating function $M_X(t)$ of the random variables having following probability functions: **Points (10)**

$$P_X(x) = \binom{n}{x} p^x (1-p)^{n-x}$$

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

(c) The random vector (X, Y) has a joint PDF **Points (10)**

$$f(x,y) = \begin{pmatrix} 2e^{-x}e^{-2y}, & x > 0,\ y > 0 \\ 0, & otherwise \end{pmatrix}$$

Find the following:

(i) Marginal Probability Density Function (PDF) of X.

(ii) Find Expected value of X and XY.

(d) Let X have zero mean and unit variance, and put $Y = 3X$. Find the following: **Points (10)**

(i) Correlation between X and Y. i.e. $E(XY)$

(ii) Covariance $Cov(X, Y) = \sigma_{XY}$

(iii) Coefficient of correlation ρ_{XY}

(iv) Make conclusions on the basis of obtained results.

Q5. (a) Let $Y = 5X - 10$, and X be normally distributed with mean 10 and variance 25. Find $P(Y \geq 68)$. **Points (10)**

Hint: Use basic properties of mean and variance to proceed further.

(b) A process is in control when the average amount of instant coffee that is packed in a jar, is 6 oz. The standard deviation is 0.2 oz. A sample of 100 jars is selected at random and the sample average is found to be 6.1 oz. Is the process out of control? **Points (05)**

(c) A random sample of size 20, from a normal population, has mean 182 and standard deviation is 2.3. Test the following hypothesis at 5% significance level. **Points (05)**

H_0: $\mu \leq 181$

H_1: $\mu > 181$