Stirling's Interpolation formula (Assumed equal step size)
centered different formula.

x	$f(x)$	1DD	2DD	3DD
x_{-2}	$f(x_{-2})$	$f[x_{-2}, x_{-1}]$	$f[x_{-2}, x_{-1}, x_0]$	$f[x_{-2}, x_{-1}, x_0, x_1]$
x_{-1}	$f(x_{-1})$	$f[x_{-1}, x_0]$	$f[x_{-1}, x_0, x_1]$	$f[x_{-1}, x_0, x_1, x_2]$
x_0	$f(x_0)$	$f[x_0, x_1]$		
x_1	$f(x_1)$	$f[x_1, x_2]$	$f[x_0, x_1, x_2]$	
x_2	$f(x_2)$			

4DD

$f[x_{-2}, x_{-1}, x_0, x_1, x_2]$.

formula:

$$P(x) = f(x_0) + Sh\left(\frac{f[x_{-1}, x_0] + f[x_0, x_1]}{2}\right) + S^2h^2 f[x_{-1}, x_0, x_1]$$

$$+ S(S^2-1^2)h^3\left[\frac{f[x_{-2}, x_{-1}, x_0, x_1] + f[x_{-1}, x_0, x_1, x_2]}{2}\right]$$

$$+ S^2(S^2-1^2)h^4 f[x_{-2}, x_{-1}, x_0, x_1, x_2]$$

where

$$S = \frac{x - x_0}{h}$$

odd DD	even DD.
$S(S^2-1^2)$	$S^2(S^2-1^2)$
$S(S^2-1^2)(S^2-2^2)$	$S^2(S^2-1^2)(S^2-2^2)$

Striling (Sirka method).

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x	$f(x)$	1FD	2FD	3FD	4FD
x_{-2}	$f(x_{-2})$	Δf_{-2}	$\Delta^2 f_{-2}$		
x_{-1}	$f(x_{-1})$	Δf_{-1}		$\Delta^3 f_{-2}$	
x_0	$f(x_0)$		$\Delta^2 f_{-1}$		$\Delta^4 f_{-2}$
x_1	$f(x_1)$	Δf_0		$\Delta^3 f_{-1}$	
x_2	$f(x_2)$	Δf_1	$\Delta^2 f_0$		

formula:

odd   even.

$$p(x) = f(x_0) + \left(\frac{\Delta f_{-1} + \Delta f_0}{2}\right)\frac{s}{1!} + \Delta^2 f_{-1}\frac{s^2}{2!}$$

$$+ \left(\Delta^3 f_{-2} + \Delta^3 f_{-1}\right)\frac{s(s^2-1^2)}{3!} + \Delta^4 f_{-2}\frac{s^2(s^2-1^2)}{4!}$$

odd   even.

Stirling's Interpolation:

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Ex 3.3 (Past Paper)

0.43

x ke closest wala se decide hoga.

	x_{-2}	x_{-1}	x_0	x_1	x_2
x	0	0.2	0.4	0.6	0.8
$f(x)$	1	1.22140	1.49182	1.82212	2.22554

Use Stirling's formula to approximate $f(0.43)$

x	$f(x)$	ΔFD	Δ^2FD	Δ^3FD	Δ^4FD
x_{-2} 0	1	0.22140			
x_{-1} 0.2	1.22140	0.27042	0.04902		
x_0 0.4	1.49182		0.05988	0.01086	0.00238
x_1 0.6	1.82212	0.33030	0.07312	0.01324	
x_2 0.8	2.22554	0.40342			

$$P(x) = f(x_0) + \frac{s}{1!}\frac{(\Delta f_{-1} + \Delta f_0)}{2} + \frac{s^2}{2!}(\Delta^2 f_{-1}) + \frac{s(s^2-1^2)}{3!}\frac{(\Delta^3 f_{-2} + \Delta^3 f_{-1})}{2} + \frac{s^2(s^2-1^2)}{4!}\Delta^4 f_{-2}$$

$\therefore \boxed{s = \frac{x - x_0}{h}}$

$$P(0.43) = 1.49182 + s\frac{(0.27042 + 0.33030)}{2} + \frac{s^2}{2!}(0.05988) + \frac{s(s^2-1^2)}{3!}\frac{(0.01086 + 0.01324)}{2} + \frac{s^2(s^2-1^2)}{4!}(0.00238)$$

$P(0.43) = 1.53725$

Question:

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x	0	0.5	1.8	1.5
$f(x)$	1	1.6	3.8	4.1

Use stirling's formula to approximate $f(0.6)$

$x_0 = 0.5$

	x	$f(x)$	$\Delta^1 f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
x_{-1}	0	1			
x_0	0.5	$\boxed{1.6}$	$\boxed{0.6}$	1.6	-3.5
x_1	1	3.8	$\boxed{2.2}$	-1.9	
x_2	1.5	4.1	0.3		

$\because$ we cannot make diamond so we neglect $\Delta^3 f(x)$ to complete diamond.

$$p(x) = f(x_0) + \frac{s}{1!}\left(\frac{\Delta f_{-1} + \Delta f_0}{2}\right) + \frac{s^2}{2!}\left(\Delta^2 f_{-1}\right)$$

$\because s = \frac{x - x_0}{h}$

$$p(x) = 1.6 + \frac{s}{1!}\left(\frac{0.6 + 2.2}{2}\right) + \frac{s^2}{2!}(1.6)$$

$s = 0.2.$

$$p(0.6) = 1.91$$

Chap-4:

Numerical Differentiation:

Analytical Derivative:

$$f'(x_0) = \lim_{x \to 0} \frac{f(x_0+h) - f(x_0)}{h}.$$

↳ gives intantaneous rate of change of f at x_0

Functions $f(x)$

closed forms
$y = f(x)$.

Error computation is possible.

Discretize form/Tabular form

x	$f(x)$
x_0	f_0
x_1	f_1
⋮	⋮

(Analytical Derivati is not possible)

x	$f(x)$	$f'(x)$
0.5	0.4794	0.852 → FD
0.6	0.5646	0.796, 0.852
0.7	0.6442	0.796 → BD

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Problem 1) a).

1) $f'(0.5) = ?$ $x_0 = 0.5$, $x_0 + h = 0.6$

$$f'(0.5) = \frac{f(x_0+h) - f(x_0)}{h}.$$: error always taken as zero.

$$= \frac{0.5646 - 0.4794}{0.1} = 0.852$$ → forward Difference.

→ middle ke points mei forward aur backward bhi nikal sakte hain.

2) $f'(0.6) = ?$ $x_0 = 0.6$, $x_0 + h = 0.7$

$$= \frac{f(x_0+h) - f(x_0)}{h} = \frac{0.6442 - 0.5646}{0.1} = 0.796$$ forward

$f'(0.6) = ?$ $x_0 = 0.6$, $x_0 - h = 0.5$

$$= \frac{f(x_0-h) - f(x_0)}{h} = \frac{\cancel{0.5646}\ 0.4794 - 0.5646}{0.1}$$

$= 0.852$

~~backward~~

3) $f'(0.7) = ?$ $x_0 = 0.7$, $x_0 - h = 0.6$

$$= \cancel{\frac{f(x_0-h) - f(x_0)}{h}} = \frac{0.5646 - 0.6442}{0.1} = 0.796$$ backward

$$\frac{f(x_0) - f(x_0-h)}{h}.$$

5) a) Most accurate formula.

x	$f(x)$	$f'(x)$
1.1	9.025013	17.76971
1.2	11.02318	22.1936.
1.3	13.46374	27.10735
1.4	16.44465	32.51085

1) $x_0 = 1.1$, $x_0 + h = 1.2$, $x_0 + 2h = 1.3$.

$$= \frac{1}{2\times 0.1}\left[-3\times(9.025013) + 4(11.02318) - 13.46376\right]$$

$= 17.76971 \rightarrow$ Three point End formula (left End).

2) $x_0 = 1.2$. $x_0 - h = 1.1$, $x_0 + h = 1.2$.

$$f'(x_0) = \frac{1}{2(0.1)}(13.46374) - (9.025013)$$

$= 22.1936 \rightarrow$ Threepoint midpoint formula.

3). $x_0 = 1.3$, $x_0 - h = 1.2$, $x_0 + h = 1.4$.

$$f'(x_0) = \frac{1}{2(0.1)}(16.44465) - (11.02318)$$

$= 27.10735$

$\hookrightarrow$ ~~Three point Midpoint~~.

4). $x_0 = 1.4$, $x_0 - h = 1.3$, $x_0 - 2h = 1.2$.

$$= \frac{1}{-2\times 0.1}\left[-3\times f(x_0) + 4(f(x_0 - h)) - f(x_0 - 2h)\right]$$

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x	0.2	0.4	0.6	0.8	1.0
$f(x)$	0.97986	0.91777	0.80803	0.63860	0.38437

a. Use all for $f'(0.4)$ and $f''(0.4)$.

↳ we only do best (Three point Midpoint).

$$f'(x_0) = \frac{1}{2(0.2)}\left[(0.80803) - (0.97986)\right] \cancel{+ \frac{(0.2)^2}{6}}$$

$= -0.42958$

Second Derivative Midpoint formula.

$$f''(x_0) = \frac{1}{(0.2)^2}\left[(0.97986) - 2(0.91777) + f(0.63860)\right].$$

$= -5.42700$.

→ do for all possible answers.

25).

$\frac{d}{dt}$ (distance/displacement) = speed / velocity.

$\frac{d^2}{dt^2}$ (distance / displacement) = acceleration.

0	5	10
0	38.3	~~623~~ 742

3	8	13
225	623	993

$x_0 = 0$ Three point endpoint left :

$$f'(0) = \frac{1}{2(5)} \left(-3(0) + 4(38.3) - (742) \right).$$

$f'(5)$ = Three point Midpoint .

$f'(10)$ = ~~Backward~~. Three point end right.

$f'(3)$ = Three point endpoint left .

$f'(8)$ = Three point Mid point

$f'(13)$ = Three point end right.

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26) . $\varepsilon(t) = L\frac{di}{dt} + Ri$ $\because$ $L = 0.98$.
$R = 0.142$.

t	1.00	1.01	1.02	1.03	1.04
i	3.10	3.12	3.14	3.18	3.24
di/dt	5 Left end (a)	4 Threepoint midpoint (b)	fivepoint Midpoint (c)	Threepoint Midpoint (d)	fivepoint end point (right) (e)
$L\frac{di}{dt} + Ri$	$0.98(a) + (0.142)(3.10)$	"	"	"	"

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Ex 4.1

Q#7a → continue from (Q 5) a).

closed form of above function $f(x)$ is $\boxed{f(x) = e^{2x}}$

compute actual error and error bound

at $x = 1.1, 1.2, 1.3, 1.4$

closed form error computable.

$f(x) = e^{2x}$

$f'(x) = 2e^{2x}$

$f'(1.1) = 2e^{2\times 1.1} = 18.05002$

x	$f(x)$	~~Absolute~~ Approximate value $f'(x)$	True value $f'(x)$	Actual Error
1.1	9.028013	17.76971	18.05002	$\lvert 17.76971 - 18.05002\rvert = 0.28031$
1.2	11.02318	22.19364	22.04635	$\lvert 22.19364 - 22.04635\rvert = 0.14729$
1.3	13.46374	27.10735	26.92748	$= 0.17987$
1.4	16.44465	32.31085	32.88929	$= 0.37844$

maximum possible error ← Error Bound

→ Error bound sirf closed form mei niklega.

→ Not necessary that every function is bonded.

$-\frac{h}{2} f''(\xi)$ → Error function of 1st formula.

~~for~~ $x_0 = 1.1$	Error function	Error bound
$f(x) = \frac{h^2}{3} f'''(\xi_0)$	$h^2/3\, f'''(\xi_0)$	0.35908
$\to x_0 \le \xi_0 \le x_0+2h$	$h^2/6\, f'''(\xi_0)$	0.17955
now check if function is bound	$h^2/6\, f'''(\xi_0)$	~~0.43858~~ 0.21926
$\frac{(0.1)^2}{3}(8e^{2\xi_0})$		0.43852

~~|0.2803|~~

$= \lvert 0.2667 e^{2\xi_0}\rvert = \lvert 0.2667\rvert \lvert e^{2\xi_0}\rvert$

maximum value $(x_0+2h) = \lvert 0.26671\rvert\, e^{2(1.3)}\rvert = 0.35908$

EX # 4.3 & 4.4.

Numerical Integration.

↳ only possible when integral is definite.

$$\int_a^b f(x)dx = \sum_{i=0}^{n} a_i f(x_i)$$ → Numerical Quadratic formula.

$f(x)$ is replaced by n^{th} degree Lagrange's Interpolation formula, i.e.

$$p(x) = f(x_0)L_0 + f(x_1)L_1 + \dots + f(x_n)L_n + \frac{(x-x_0)(x-x_1)\dots(x-x_n)}{(n+1)!} f^{(n+1)}(\xi)$$

$x_0 \le \xi \le x_n$

Closed Newton-cotes formula. Error function

→ Trapezium

(Trapezoidal Rule) ← n=1, ∵ $h = \frac{b-a}{1} \to n$.

$$\int_{a=x_0}^{b=x_n/x_1} f(x)dx = \frac{h}{2}\left(f(x_0)+f(x_1)\right) - \frac{h^3}{12} f''(\xi)$$ → Error function,

$x_0 \le \xi \le x_1$

→ Parabola. give exact ans with 3 degree

(Simpson's 1/3rd Rule) ← n=2. ∵ $h = b-a/2 \to n$

$$\int_{a=x_0}^{b=x_n/x_2} f(x)dx = \frac{h}{3}\left(f(x_0)+4f(x_1)+f(x_2)\right) - \frac{h^5}{90} f^{(4)}(\xi)$$ → Error function

$x_0 \le \xi \le x_2$.

(Simpson's 3/8th Rule) ← n=3. ∵ $h = b-a/3 \to n$.

$$\int_{a=x_0}^{b=x_n/x_4} f(x)dx = \frac{3h}{8}\left(f(x_0)+3f(x_1)+3f(x_2)+f(x_3)\right) - \frac{3}{80}h^5 f^{(4)}(\xi)$$

$x_0 \le \xi \le x_3$ Error function

Ex4.3:

Q#1 (b) Evaluate $\int_0^{0.5} \frac{2}{x-4}\,dx$ by trapezoidal Rule.

Q#3 (Actual error, Error bound).
Q#5 (Simpson's 1/3rd Rule).

$$\int_0^{0.5} \frac{2}{x-4}\,dx = -0.26706 = \text{True value}$$

Trapezoid Rule $= \int_{a=x_0}^{b=x_1} f(x)\,dx = \frac{h}{2}\left(f(x_0)+f(x_1)\right).$

$a = 0 = x_0$ $\quad$ $b = 0.5 = x_1$

$f(x_0) = f(0) = -0.5$ $\quad$ $f(x_1) = f(0.5) = -0.57143.$

$h = \frac{b-a}{n} = \frac{0.5-0}{1} = 0.5.$

$= \frac{0.5}{2}\left(-0.5 - 0.571418\right)$

$\left|T.V - \text{Trapezoid value}\right| = 0.26706 - 0.2678$

$= 0.2678$

Simpson's 1/3rd Rule: $\int_0^{0.5} 2/x-4 \,dx.$

$h = \frac{b-a}{2} = \frac{0.5-0}{2} = 0.25.$

$x_0 = 0$, $x_1 = 0.25$, $x_2 = 0.5$

$f(x_0) = -0.5.$ $\quad$ $f(x_2) = -0.57143$

$f(x_1) = 0.53333$

$= \frac{h}{3}\left(f(x_0) + 4f(x_1) + f(x_2)\right)$

$= 0.26547$

$\left|TV - \text{Simpson's 1/3rd Rule}\right| = \left|0.26706 - 0.26547\right|$

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Numerical Integration:

closed Newton-cotes formula:

$\because h = \frac{b-a}{n}$.

① $n=1$

Trapezoidal Rule: $\frac{h}{2}\left(f(x_0)+f(x_1)\right) - \frac{h^3}{12} f''(\xi)$ $\quad x_0 < \xi < x_1$

② $n=2$.

Simpson's 1/3rd Rule. $\frac{h}{3}\left(f(x_0)+4(f(x_1))+f(x_2)\right) - \frac{h^5}{90} f^{(4)}(\xi)$

③ $n=3$

Simpson's 3/8th Rule $\frac{3h}{8}\left(f(x_0)+3f(x_1)+3f(x_2)+f(x_3)\right) - \frac{3h^5}{80} f^{(4)}(\xi)$.

Ex 4.3:

Q#1 (b) Evaluate $\int_0^{0.5} \frac{2}{x-4}\, dx$ by using Trapezoidal Rule.

Q#3 (b) compute actual error & error bound in Q# 1b.

Q#5 (b) Repeat Q# 1b for Simpson's rule (1/3rd).

Calculator :

x_0	x_1	x_2	x_3
0	0.16667	0.33334	0.5
$f(x_0)$	$f(x_1)$	$f(x_2)$	$f(x_3)$
-0.5	-0.52173	-0.54547	-0.57142

$n=3$

Simpson's 3/8th Rule. = -0.26706.

Q#3(b): Error bound.

Trapezoid Rule → Error Function $= \frac{h^3}{12} f''(\xi)$ $\quad \underset{0}{x_0} < \xi < \underset{0.5}{x_1}$

$|f(x)| \le K \rightarrow$ max value.

$\left|\frac{h^3}{12} f''(\xi)\right| = \left|\frac{0.5^3}{12} \times 4(\xi-4)^{-3}\right|$ $\quad \because f = \frac{2}{\xi-4}$

$= \frac{0.5^3 \times 4}{12} \left|(\xi-4)^{-3}\right|$ $\quad f' = -2(\xi-4)^{-2}(1)$

$= 0.04167 \left|(\xi-4)^{-3}\right|$ $\quad f'' = 4(\xi-4)^{-3}$

vary: 0 – 0.5

~~put this in~~ calculator

found: decreasing function!

put: 0.0214 in place of $|(\xi-4)^{-3}|$

$0.04167 * 0.0214$

~~0.00089~~

≤ 0.00097

Error bound.

0	0.1	0.2	0.3	0.4
-0.01562	-0.01685	-0.01822	-0.01974	-0.0214

max: $|-0.01562|$ → 0.01562 → min

min: $|-0.0214|$ ~~|-0.01974|~~ → 0.0214 ~~0.01974~~ → max always $\rightarrow \int_{a=x_{-1}}^{b=x_{n+1}}$ $\because h = \frac{b-a}{n+2}$

Date:

open Newton-cotes formulas with their error terms:

$n=0$; Midpoint Rule. $\int_{a=x_{-1}}^{b=x_1} f(x)dx = 2hf(x_0) + \frac{h^3}{3} f''(\xi)$

$x_{-1} < \xi < x_1$

$n=1$;

$$\int_{a=x_{-1}}^{b=x_2} f(x)dx = \frac{3h}{2}\left[f(x_0)+f(x_1)\right] + \frac{3h^3}{4} f''(\xi)$$

$a = x_{-1} \rightarrow$ always x_{-1} b/c we have to exclude this

$x_{-1} < \xi < x_2$

$n=2$;

$$\int_{a=x_{-1}}^{b=x_3 \rightarrow x_{n+1}} f(x)dx = \frac{4h}{3}\left[2f(x_0) - f(x_1) + 2f(x_0)\right] + \frac{14h^5}{45} f^{(4)}(\xi).$$

$x_{-1} < \xi < x_3$

~~Numerical Integration~~

~~(2) open Newton cotes formulae:~~

Q# 1d $\int_0^1 x^2 e^{-x}$ by using Trapezoid Rule.

Q# 3d compute actual error & error bound.

Q# 5d. Repeat 1 by Simpson's 1/3rd Rule.

Q# 7 Repeat 5d by Simpson's 3/8th Rule.

Q# 9. Repeat 1D by Midpoint Rule.

↳ open Newton.

Q#9. $\int_{a=0}^{1} x^2 e^{-x}\, dx$; Midpoint Rule, $n=0$.

$h = \frac{b-a}{n+2} = \frac{1-0}{0+2} = \frac{1}{2} = 0.5$

a	x_0	b
x_{-1}	0.5	$x_{n+1} = x_1$
0		1

$\int_a^b f(x) = 2(0.5) f(0.5)$

$= 0.15163$ ✓ sawal mei itna hi tha.

for $n=1$: $\int_{a=x_{-1}=0}^{b=x_2=1} = \frac{3h}{2}\left[f(x_0) + f(x_1) \right]$. $\because h = \frac{b-a}{n+2}$

$h = \frac{1-0}{3}$

$h = 1/3$

	x_{-1}	x_0	x_1	x_2
	a			b
	0	1/3	2/3	1
$f(x)$ →		0.07961	0.22818	

$\int_a^b f(x) = \frac{3(1/3)}{2}\left(0.07961\right) + \left(0.22818\right)$

$= 0.15390$

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Composite Methods of Numerical Integration (closed).

1- Composite trapezoid Rule :

first value, last value

$$\int_{a=x_0}^{b=x_n} f(x)dx = \frac{h}{2}\left(f(a)+f(b)+2\left(f(x_1)+f(x_2)+\dots f(x_{n-1})\right)\right).$$

n = # of subintervals. , no. of points = $n+1$

$$h = \left(\frac{b-a}{n}\right)$$

$n\uparrow$ $h\downarrow$ $\rightarrow$ Error $\downarrow$

$n\downarrow$ $h\uparrow$ $\rightarrow$ Error $\uparrow$.

Simpson's 1/3rd Rule

$$\int_{a=x_0}^{b=x_2} f(x)\,dx = \frac{h}{3}\left(f(x_0)+4f(x_1)+f(x_2)\right).$$

↳ n should be even.
↳ b/c parabola has 3 points

$$\int_a^b f(x)dx = \frac{h}{3}\left(f(a)+f(b)+4\sum_{j=1}^{n-1} f(x_{2j-1}) + 2\sum^{n-2} f(x_{2j})\right)$$

odd

even.

Ex 4.4: (Q# 1e, Q# 3e).

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Evaluate

$\int_0^2 e^{2x} \sin(3x)\,dx$, $n=8$. by composite trapezoidal Rule &

composite simpson's 1/3rd Rule.

$= \frac{h}{2}\left(f(0) + f(2) + 2(f(x_1) + f(x_2 \cdots f(x_7))\right)$.

$x_0 \quad x_8$

$h = \frac{2-0}{8} = 2/8 = 0.25$.

$f(x) = 0 \quad f(x) = 1.1238 \quad f(x) = 2.7114 \quad f(x) = 3.487 \quad f(x) = 1.0427 \quad f(x) = -6.963$

$x_0 = 0, \quad x_1 = 0.25 \quad x_2 = 0.5, \quad x_3 = 0.75, \quad x_4 = 1, \quad x_5 = 1.25$.

~~$f(x_0) = 1 \quad f(x_1) = 1.406 \quad f(x_2) = 2.7114 \quad f(x_3) = 3.21269, f(x_4))$~~

$x_6 = 1.5 \quad x_7 = 1.75, \quad x_8 = 2$.

$f(x) = -19.63 \quad f(x) = -28.44 \quad f(x) = -15.25$.

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Chap # 5 Numerical solution to Initial value problems of ordinary differential Equations (Particular Solution in Discrete form Only).

(i) Euler's Method.

$y_{i+1} = y_i + h y_i' + \underbrace{O(h^2)}_{\text{Error bound}}$

order of convergence (h^2) pe series neglect hogi.

$y_i' = f(x_i, y_i)$

$$\boxed{y_{i+1} = y_i + h f(x_i, y_i)}$$

Given:

$h = 0.2$, $\frac{dy}{dx} = \underbrace{x^2 + y^2}_{f(x,y)}$.

$y(0) = 1$

$x_0 \quad y_0$.

$i=0$, $y_1 = ?$ at $x_1 = 0.2$.

$i=1$, $y_2 = ?$ at $x_2 = 0.4$

Taylor Series

$y(x) = y_0 + (x - x_0) y_0' + \frac{(x - x_0)}{2!} y_0'' + \cdots$

$y(x_1) = y_0 + \underbrace{(x_1 - x_0)}_{h} y_0' + \frac{(x_1 - x_0)}{2!} y_0''$

$y_1 = y_0 + h y_0' + \frac{h^2}{2!} y_0'' + O(h^3)$

$y_{i+1} = y_i + h y_i' + \frac{h^2}{2!} y_i'' + O(h^3)$

order of convergence. (h^3) pe series neglect hogi

EX 5.2 Q# 1 & Q# 2 .

① Euler Method → Non iterative .

dependent ← $\frac{dy}{dt} = f(t, y)$, $y(t_0) = y_0$, $a \le t \le b$.

Q# 1. Use Euler Method to approximate the solution for the following initial value problem .

$y' = 1 + y/t$, $1 \le t \le 2$, $y(1) = 2$ with $h = 0.25$

$y(t_{i+1}) = y(t_i) + h y'(t_i) + \frac{h^2}{2!} y''(\xi_i)$

↳ Error function .

$f(t, y) = 1 + y/t$

$t_0 = 1$, $y_0 = 2$

$t_1 = 1.25$, $y_1 = ?$

$t_2 = 1.50$ $y_2 = ?$

$t_3 = 1.75$ $y_3 = ?$

$t_4 = 2$ $y_4 = ?$

$y_{i+1} = y_i + 0.25 (1 + y_i / t_i)$

i=0, ~~$y_i + 0.25($~~

$y_1 = y_0 + 0.25 (1 + 2/1) = y_1 = 2.75$.

i	t_i (A)	y_i (B)	y_{i+1}
0	1	2	2.75
1	1.25	2.75	3.85
2	1.50	3.55	4.39107
3	1.75	4.39107	5.26905

E F

Date: M T W T F S S

① $y' = \cos(2t) + \sin(3t)$, $0 \le t \le 1$
$y(0) = 1$ with $h = 0.25$

compute actual error at each t.

$y' = \cos(2t) + \sin(3t)$.

$\frac{dy}{dt} = \cos(2t) + \sin(3t)$.

integrate it.

$y = \frac{\sin(2t)}{2} - \frac{\cos(3t)}{3} + c$.

$y(0) = 1$.

$1 = -\frac{1}{3} + c \Rightarrow c = 4/3$.

$y(0.25)$ = True value. , $|A - \text{True value}|$

$t_0 = 0$, $y_0 = 1$ Error.

$t_1 = 0.25$, $y_1 = A$ $|A - TV|$

$t_2 = 0.5$, $y_2 = B$ $|B - TV|$

$t_3 = 0.75$, $y_3 = C$ $|C - TV|$

$t_4 = 1$, $y_4 = D$. $|D - TV|$