Date: 03/16/2023

Prob & Stats.

Binomial exp is majorly for the case when populati[on] size is too large >50

Problem 2 $p = 0.18$ $q = 0.82$

x could be 0–12

b) $P(x=3)$ when $n=12$

$^{12}C_3 \times (0.18)^3 (0.82)^9$

Problem 3) $p = 0.5$ $n = 13$ $P(X \leq 5) = ?$ $P(X=6)+P(X=7)+P(X=8)+P(X=9)$

$P(X \geq 7) = ?$ $(P(X=7)+P(X=8)+\ldots+P(X=13)) = ?$

$P(X=0)+P(X=1)+P(X=2)+P(X=3)+P(X=4)+P(X=5)$

Problem 4) $n = 10$ $p = 0.05$ $q = 0.95$

x	0	1	2	3	4	5	6	7	8	9	10
$P(x=X)$											

$^{10}C_0 \times (0.05)^0 (0.95)^{10}$

$^{10}C_1 \times (0.05)^1 (0.95)^9$

If success rate is 50% shape of histogra[m] is symmetric

If $p < 0.5$ then graph is right skew[ed]

If $p > 0.5$ then graph is left ske[wed]

no. of success in population

$$P(x) = \frac{{}^{r}C_x \times {}^{N-r}C_{n-x}}{{}^{N}C_n}$$

N population size

n sample size

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Date: March 21,

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Prob. & Stats.

Poisson Probability Distribution

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$ where λ is parameter of this distribution

x → no. of successes in fixed time interval

λ = mean no. of occurrences in an interval

x = no. of occurrences in an interval

mean, $\mu = \lambda$

Variance, $\sigma^2 = \lambda$

S.D, $\sigma = \sqrt{\lambda}$

Q/ On average, a household receives 9.5 (avg.) telemarking phone calls per week (interval). Find the probability that a randomly selected household receives exactly 6 calls (x) during next week.

$$P(X=6) = \frac{(9.5)^6 e^{-9.5}}{6!} = 0.076$$

Q/ An avg. of 2 of every 10 products (avg. with respect to volume) sold by a company are returned for a refund, within 7 days (interval). Find the probability that exactly 6 of 40 products sold will be returned.

$n=40$ $x=6$ $p=\frac{2}{10}$ $q=8/10$ { binomial way }

$\lambda = 2$ out of 10

$\lambda = 2\times4 = 8$ out of 40 (Poisson distribution)

$x = 6$

$$P(X=6) \Rightarrow \frac{8^6 \times e^{-8}}{6!}$$

Prob. & Stats

Q/ An auto salesperson sells an average of $\underbrace{0.9}_{\mu=\lambda}$ cars per day (interval). Write probability distribution of x.

$$P(x) = \frac{e^{-\lambda} \cdot \lambda^{x}}{x!}$$

$\boxed{\mu = \lambda = \text{variance}}$ $\boxed{\sigma = \sqrt{\mu}}$

x	0	1	2	3	4	5	6	7
$P(x)$	0.4066	0.3659	0.1647	0.0494	0.0111	0.0020	0.0003	0.0000

No. of trials are NOT fixed like n

Take it to x until $P(x)$ is almost zero

$P(x)$ can be written as $f(x)$ is known as prob. distribution function.

x	$P(x)/f(x)$	$F(X)$
0	0.4066	0.4066
1	0.35659	0.7 (0.4066+0.35659)
⋮	⋮	
7	0.000	
	$\Sigma = 1$	

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Q/ 2% of the fuses manufactured by a firm are defective. Find the prob. that a box containing 200 fuses has
(a) at least 1 defected fuse (b) Three or more.

Poisson : (~~n>25 10 sign~~)

(a) $P(X \geq 1) \Rightarrow 1 - P(X = 1)$

$\lambda = \sqrt{}$

(b) $P(X \geq 3)$:

$P(x = 3, 4, 5, 6\}$ or $1 - P\{x_0, x_1, x_2\}$

General Rule

For approximation of binomial

* A binomial prob. distribution with $n > 25$, $\mu \leq 25$. Use poisson prob. for approximation.
* with $n > 25$, $\mu > 25$. use Normal prob. distribution.

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Discrete Probability Distribution

The set of ordered pairs $(x, f(x))$ is a probability function, Prob mass functions or prob. distribution of discrete random variable.
x if, for each possible outcome x

1- $f(x) \geq 0$

2 - $\sum_x f(x) = 01$

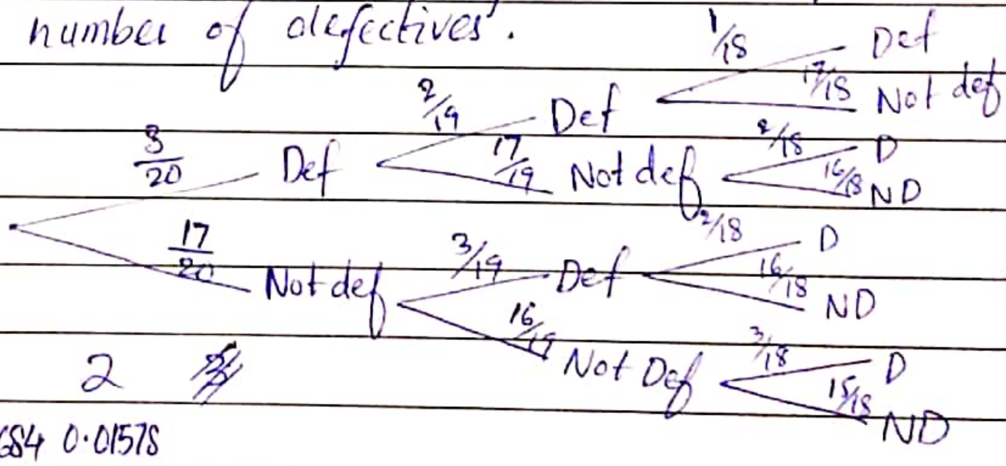
3 - $P(X = x) = f(x)$

Cummulative Distribution Function

$F(x)$ of a discrete random variable x with prob. $f(x)$ is
$F(x) = P(X \leq x) = \sum_{t \leq x} f(t) \quad - \infty < x < \infty$

Q/ A shipment of 20 similar laptops of a retail outlet contains 3 defectives. In a random purchase of 2, find the prob. distribution for number of defectives.

~~x~~



x	0	1	2	~~3~~
$P(x)$	$\frac{17}{20} \times \frac{16}{19}$ 0.7157	0.2684	0.01578	

$$P(x) = \frac{{}_rC_x \times {}_{N-r}C_{n-x}}{{}_NC_n}$$

$N = 20 \quad n = 2$

$r = 3 \quad x = 0, 1, 2$

Cummulative distribution function	0.7157	0.7157+0.2684	0.7157+0.2684+0.01578

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$F(X=0) = 0.7157$

$F(X=1) = P(x \leq 1) = 0.2684 + 0.7157$

$$F(X) = \begin{cases} 0 & X<0 \\ 0.7157 & 0 \leq X < 1 \\ 0.2684+0.7157 & 1 \leq X < 2 \\ 1 & 2 \leq X \end{cases}$$

$\to 0.7157 + 0.2684 + 0.01578$

Cummulative frequency graph

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Date: 03-28-2023

Prob. & Stats.

Joint Prob. Distribution

1) $f(x,y) \geq 0 \ \forall (x,y)$

2) $\sum_x \sum_y f(x,y) = 1$

3) $P(X=x, Y=y) = f(x,y)$

Marginal Distribution

1) $g(x) = \sum_y f(x,y)$

2) $h(y) = \sum_x f(x,y)$

Q/ Two ballpoints are selected at random from a box, that contain 3 blue, 2 red, 3 green pens. X is the number that blue pen selected.
Y is the number that red pen selected.
Find (a) The Joint Prob. (b) The marginal prob. (c) $P[(x,y) \in A]$ A is the region $\{(x,y) \mid x+y \leq 1\}$ (d) $E(X,Y)$ (e) The covariance of X and Y (f) The correlation coefficient

$x = 0,1,2,3 \quad y = 0,1,2$

a)

$f(0,0)$ all 2 pens are green	$f(1,0)$	$f(2,0)$
$f(0,1)$ 1 red & 1 green	$f(1,1)$	
$f(0,2)$		

$f(x,y)$	x = 0	1	2	$h(y)$
y = 0	$f(0,0)$	$f(1,0)$	$f(2,0)$	← prob of $y=0$
1	$f(0,1)$	$f(1,1)$	$\boxed{f(2,1)}$	← prob. of $y=1$
2	$f(0,2)$	$\boxed{f(1,2)}$	$\boxed{f(2,2)}$	← Prob of $y=2$
$g(x)$	Sum_1	Sum_2	Sum_3	1

marginal prob.

sum of all prob. is equal to 1

$$f(x,y) = \frac{3C_x \, 2C_y \, 3C_{2-x-y}}{8C_2}$$

$$f(0,0) = \frac{3C_0 \times 2C_0 \times 3C_2}{8C_2} = \frac{3}{28}$$

$$f(1,0) = \frac{3C_1 \times 2C_0 \times 3C_1}{8C_2} = \frac{9}{28}$$

$$f(2,0) = \frac{3C_2 \times 2C_0 \times 3C_0}{8C_2} = \frac{3}{28}$$

$$f(0,1) = \frac{3C_0 \times 2C_1 \times 3C_1}{8C_2} = \frac{3}{14} = f(1,1)$$

$$f(2,1) = f(1,2) = f(2,2) = 0$$

$$f(0,2) = \frac{1}{28}$$

$Sum_2 = \frac{15}{28}$ $Sum_3 = \frac{3}{28}$, $Sum_1 = \frac{5}{14}$

$P(y=0) = \frac{15}{28}$ $P(y=1) = \frac{3}{7}$, $P(y=2) = \frac{1}{28}$

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b) Marginal prob of x

x	0	1	2
$g(x)$	$5/14$	$15/28$	$3/28$

Marginal prob. of y

y	0	1	2
$h(y)$	$15/28$	$3/7$	$1/28$

c) $x+y \leq 1$

$$P(x+y \leq 1) = f(0,0) + f(1,0) + f(0,1)$$

$$\frac{3}{28} + \frac{9}{28} + \frac{3}{14} =$$

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Prob. & Stats.

d) $E(\overset{\text{multiplied}}{X \cdot Y}) = \sum_{x=0}^{2} \sum_{y=0}^{2} \overset{\text{multiplied}}{xy} f(x,y) = \mu_{xy}$

$x_0 y_0 f(0,0) + x_0 y_1 f(0,1) + x_1 y_0 f(1,0) + x_1 y_1 f(1,1) + x_0 y_2 f(0,2)$

$+ x_2 y_0 f(2,0)$

$(0)(0)\left[\frac{3}{28}\right] + (0)(1)\left[\frac{3}{14}\right] + \dots$

$= \frac{3}{14}$

e) Find the covariance of X and Y

$\sigma_{xy} = E(XY) - \mu_x \mu_y$

$= \frac{3}{14} - \left(0\left(\frac{5}{14}\right) + 1\left(\frac{15}{28}\right) + 2\left(\frac{3}{28}\right)\right)$

$= \frac{3}{14} - \left(\frac{3}{4} \cdot \frac{1}{2}\right) = -\frac{9}{56}$

use (marginal prob)

$\mu_x = E(X)$

$= \sum_{x=0}^{2} x \cdot g(x)$

$\mu_y = E(Y)$

$= \sum_{y=0}^{2} y \cdot h(y)$

$0\left(\frac{15}{28}\right) + 1\left(\frac{3}{7}\right) + 2\left(\frac{1}{28}\right)$

$= \frac{1}{2}$

) Correlation coefficient

$\rho_{xy} = \frac{\sigma_{xy}}{\sigma_x \sigma_y} = \frac{-9/56}{45/112 \cdot 9/28} =$

$\sigma_x^2 = E(X^2) - (\mu_x)^2$

where $E(X^2) = \sum_{x=0}^{2} x^2 f(x)$

$0\left(\frac{5}{4}\right) + 1\left(\frac{15}{28}\right) + 2^2\left(\frac{3}{28}\right)$

$\sigma_y^2 = E(y^2) - (\mu_y)^2$

$E(Y^2) = \sum \dots$

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Q/

x	4	5	6	7	8	9
$f(x)$	1/12	1/12	1/4	1/4	1/6	1/6

let $g(x) = 2x - 1$ (Salary for workers)

Find $E(g(x))$

Sol

$$E\{g(x)\} = E(2x-1) = \sum_{x=4}^{9} (2x-1) \cdot f(x)$$

$$(2(4)-1)(1/12) + (2(5)-1)(1/12) + (2(6)-1)(1/4) + \ldots$$

$= \$12.67$

$\mu_x = x \cdot f(x) = 6.8333$

Q/ Calculate the variance $g(X) = 2X + 3$

x	$f(x)$	$E[g(x)^2]$ $g(x)^2 \times f(x)$	$E[g(x)]$ $g(x) \cdot f(x)$
0	1/4		
1	1/8		
2	1/2		
3	1/8		

$\sigma_x^2 = E(X^2) - \mu_x^2$

where μ_x is also $[E(X)]^2$

$$\sigma^2_{g(x)} = \underbrace{E[g(x)^2]}_{\substack{\downarrow \\ g(x)^2 \times f(x) \\ (2x+3)^2 \cdot f(x)}} - [E g(x)]^2$$

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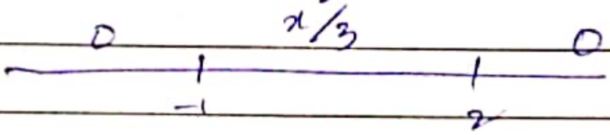
Q/ $f(x) = \begin{cases} x^2/3 & -1<x<2 \\ 0 & \text{elsewhere} \end{cases}$

a) Verify that $f(x)$ is a density function
b) Find $P(0 < X \leq 1)$
c) Find $F(x)$ Cumm. distrib. function → ?

a. Prove 1) $f(x) \geq 0 \;\forall x \in R$ & 2) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

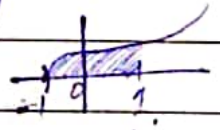


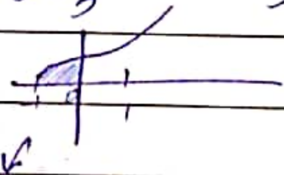
$\frac{x^2}{3}$ is always ≥ 0 due to squaring ~~between~~ at -1 & 2 (interval)
so 1st is proven

$\frac{1}{3}\int_{-\infty}^{\infty} x^2 dx = \frac{1}{3}\int_{-1}^{2} x^2 = \frac{1}{3}\left[\frac{x^3}{3}\right]_{-1}^{2} = \left[\frac{x^3}{9}\right]_{-1}^{2} = 1$ hence proven

b) $P(\overset{a}{0} < X \leq \overset{b}{1}) = ?\; \int_0^1 \frac{x^2}{3} = \left[\frac{x^3}{9}\right]_0^1 = 1/9$

c) $F(x) = ?\; \int_{-\infty}^{x} f(t)\,dt \rightarrow \int_{-1}^{x} \frac{t^2}{3} \longrightarrow \frac{1}{3}\left[\frac{t^3}{3}\right]_{-1}^{x} = \left[\frac{t^3}{9}\right]_{-1}^{x} = \frac{x^3 \ldots}{9}$

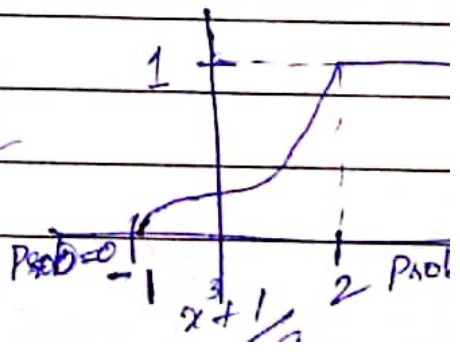




~~Instead~~ $F(b) - F(a)$ for (b) as an alternative

$F(x) = \begin{cases} 0 & x<-1 \\ \frac{x^3+1}{9} & -1 \leq x < 2 \\ 1 & x \geq 2 \end{cases}$

Cummulative probabilities since $\frac{x^3+1}{9}$ is 1 so next is $n+1=1$



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Date:
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$$2(x-1) \quad 1<x<2 \leftarrow \text{integrate from 1 to 2}$$

Q/ Pdf is $f(x) = \begin{cases} 2(x-1) & 1<x<2 \\ 0 & \text{elsewhere} \end{cases}$

Find mean and variance of X

representation of min & max values which are 1 & 2 here

$$\text{Mean} = \mu = E(X) = \int_{-\infty}^{\infty} x f(x)\, dx$$

$\boxed{\text{Probability of a point is zero}}$

$$\text{Variance } \sigma^2 = E(X^2) - \mu^2 \quad \text{OR} \quad E(X^2) - [E(X)]^2$$

$$\text{where } E(X^2) = \int_{-\infty}^{\infty} x^2 \cdot f(x)\, dx$$

$$E(X) = \int_1^2 x[2(x-1)]\, dx$$

$$2\int_1^2 x^2 - x\, dx \longrightarrow 2\left[\frac{x^3}{3} - \frac{x^2}{2}\right]_1^2 = \frac{5}{3}$$

$$E(X^2) \; \cancel{\text{Variance}} \; E(X^2) = \int_1^2 x^2 \cdot 2(x-1)\, dx$$

$$2\int_1^2 x^3 - x^2 \longrightarrow 2\left[\frac{x^4}{4} - \frac{x^3}{3}\right]_1^2 = 17/6$$

$$\text{Variance} = 17/6 - (5/3)^2 = 1/18$$

Date:
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Joint Density Function

1) $f(x,y) \geq 0 \ \forall (x,y)$

2) $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y)\,dx\,dy = 1$

3) $P[(X,Y) \in A] = \iint_A f(x,y)\,dx\,dy$

Marginal Distribution

$g(x) = \int_{-\infty}^{\infty} f(x,y)\,dy$ and $h(y) = \int_{-\infty}^{\infty} f(x,y)\,dx$

Q/ The Joint Density Function is $f(x,y) = \begin{cases} \frac{2}{5}(2x+3y) & 0 \leq x \leq 1 \\ & 0 \leq y \leq 1 \\ 0 & \text{elsewhere} \end{cases}$

a) Verify condition 2 of definition

b) Find $P[(X,Y) \in A]$ where $A = \{(x,y) / 0 < x < \frac{1}{2} ; \frac{1}{4} < y < \frac{1}{2}\}$

c) $g(x)$ and $h(y)$

Sol $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y)\,dx\,dy$. $\int_{y=0}^{y=1} \int_{x=0}^{x=1} \frac{2}{5}(2x+3y)\,dx\,dy$

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integrated 3y in terms of x

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$$\int_{y=0}^{y=1} \frac{2}{5}\left[x^2+3xy\right]_{x=0}^{x=1} dy$$

$$\int_{y=0}^{y=1} \frac{2}{5}(1+3y)\,dy$$

$$\frac{2}{5}\left[y+\frac{3y^2}{2}\right]_0^1 = \frac{2}{5}\left[1+\frac{3}{2}\right] = 1$$ hence verified (2)

b) $P[(X,Y)\in A] = \int_{y=1/4}^{y=1/2}\int_{x=0}^{x=1/2} \frac{2}{5}(2x+3y)\,dx\,dy$ (Limits are diff of stated in Q)

$$\int_{y=1/4}^{y=1/2} \frac{2}{5}\left[x^2+3xy\right]_{x=0}^{x=1/2} dy = \frac{2}{5}\int_{1/4}^{1/2} \frac{1}{4}+\frac{3}{2}y\,dy = \frac{2}{5}\left[\frac{y}{4}+\frac{3y^2}{4}\right]_{1/4}^{1/2} = \frac{13}{160}$$

c) marginal prob. = ?

$$\int_{-\infty}^{\infty} f(x)\,dy = ?$$

$$\int_0^1 \frac{2}{5}(2x+3y)\,dy$$ Integrating for x with respect to y & vice versa

$$\left[\frac{4xy+6}{5}\right]_0^1 \quad 0<x\leq 1$$

$$g(x) = \begin{cases} \frac{4x+6}{5} & 0\leq x\leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

$$h(y) = \begin{cases} \frac{2+6y}{5} & 0\leq y\leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

$$h(y) = \int_0^1 \frac{2}{5}(2x+3y)\,dx = \frac{2}{5}\left[x^2+3xy\right]_0^1 = \frac{2}{5}+\frac{6y}{5}$$

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Discrete

Mean, $\mu = \sum x P(x)$

S.D, $\sigma = \sqrt{\sum x^2 P(x) - \mu^2}$

σ^2 = variance = $(S.D)^2$

Continuous

Mean, $\mu = \int_{-\infty}^{\infty} x f(x)\, dx = E(X)$

S.D, $\sigma = \sqrt{E(X^2) - \mu^2}$

where $E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx$

Variance, $\sigma^2 = (S.D)^2$

$f(x) = \begin{cases} x^2 \div 3 & -1 < x < 2 \\ 0 & \text{elsewhere} \end{cases}$

$F(x) = ?$ $\quad \frac{1}{3}\int_{-1}^{x} x^2\, dt = \left[\frac{1}{9}t^3\right]_{-1}^{x} = \frac{x^3 - 1}{9}$

$F(x) = \begin{cases} 0 & x < -1 \\ (x^3-1)/9 & -1 \le x < 2 \\ 1 & x \geq 2 \end{cases}$

JOINT PROBABILITY $(f(x,y))$

mean = $\mu_x = \sum x f(x)$

$\mu_y = \sum y f(y)$

marginal prob:-

$g(x) = \sum_x f(x,y)$ keep x same and for diff. values of y, take sum and build prob. dist. for X

$h(y) = \sum_y f(x,y)$

~~mean = E(XY)~~ $= \int_{y=-\infty}^{y=\infty}\int_{x=-\infty}^{x=\infty} f(x,y)\, dx\, dy$ 2

~~where outer integral must have limits in whole no's~~

marginal prob :-

$g(x) = \int_{-\infty}^{\infty} f(x,y)\, dy$ when ∞ & $-\infty$ are extreme values of y

$h(y) = \int_{-\infty}^{\infty} f(x,y)\, dx$

Covariance, $\sigma_{xy} = E(X \cdot Y) - \mu_x \mu_y$

mean $= E(X \cdot Y) = \sum_{x=-\infty}^{\infty} \sum_{y=-\infty}^{\infty} xy f(x,y)$

$\sigma_x^2 = \sum x^2 P(x) - (E(X))^2$

$\sigma_y^2 = \sum y^2 P(y) - (E(Y))^2$

where $\mu_x = \int_{-\infty}^{\infty} \cancel{f(x,y)}\, g(x)\, dx$

and $\mu_y = \int_{-\infty}^{\infty} h(y)\, dy$, $E(X \cdot Y) = \int_{y=-\infty}^{y=\infty}\int_{x=-\infty}^{x=\infty} xy f(x,y)\, dx\, dy$

where outer interval must have whole no intervals

Correlation coefficient $= \dfrac{\sigma_{xy}}{\sigma_x \sigma_y}$

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Date: 03/15/2023

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Prob. & Stats.

Binomial Formula

$$P(x) = {}_nC_x \, p^x \, q^{(n-x)}$$

$P(x)$ — prob. of success for some x

n — total no. of trials

x — No. of successes in n trials

p — prob. of success

q — prob. of failure

$(n-x)$ — no. of failure in n trials

n, p are binomial parameters

$p + q = 1$

Mean and Standard Deviation

$$\mu = np \qquad \sigma = \sqrt{npq}$$

- * n identical trials (tossing coin 10 times, etc.)
- * each trial has only 2 possible outcomes, success or failure (head/tail)
- * probability of two outcomes remains constant.
- * trials are independent

→ CONDITIONS OF BINOMIAL EXPERIMENT

n	x	$p = 0.05$	$p = 0.10$	$p = 0.20$
6	0	failed all times		
	1	succeeded once		
	2			
	3			
	4			
	5			
	6			

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