Chapter 6
Continuous Random Variables and the Normal Distribution

Section 6.1

6.1 The probability distribution of a discrete random variable assigns probabilities to points while that of a continuous random variable assigns probabilities to intervals.

6.3 Since $P(a) = 0$ and $P(b) = 0$ for a continuous random variable,
$P(a < x < b) = P(a < x \le b) = P(a \le x < b) = P(a \le x \le b)$.
This means that the probability that x assumes a value in the interval a to b is the same whether or not the values a and b are included in the interval.

6.5 The **standard normal distribution** is a special case of the normal distribution. For the standard normal distribution, the value of the mean is equal to zero and the value of the standard deviation is 1. The units of the standard normal distribution curve are denoted by z and are called the z-values or z-scores. The z-values on the right side of the mean (which is zero) are positive and those on the left side are negative. A specific value of z gives the distance between the mean and the point represented by z in terms of the standard deviation.

6.7 As its standard deviation decreases, the spread of a normal distribution curve decreases and its height increases.

6.9 For a standard normal distribution, the z value gives the distance between the mean and the point represented by z in terms of the standard deviation. For example, z = 1 is a distance of 1 standard deviation to the right of the mean. The z values on the right side of the mean are positive and those on the left side are negative.

6.11 Area within 2.5 standard deviations of the mean is:
$P(-2.50 < z < 2.50) = P(z < 2.50) - P(z < -2.50) = 0.9938 - 0.0062 = 0.9876$

6.13 **a**. $P(0 < z < 1.95) = P(z < 1.95) - P(z < 0) = 0.9744 - 0.5000 = 0.4744$

b. $P(-2.05 < z < 0) = P(z < 0) - P(z < -2.05) = 0.5000 - 0.0202 = 0.4798$

c. $P(1.15 < z < 2.37) = P(z < 2.37) - P(z < 1.15) = 0.9911 - 0.8749 = 0.1162$

d. $P(-2.88 \le z \le -1.53) = P(z \le -1.53) - P(z \le -2.88) = 0.0630 - 0.0020 = 0.0610$

e. $P(-1.67 \le z \le 2.24) = P(z \le 2.24) - P(z \le -1.67) = 0.9875 - 0.0475 = 0.9400$

6.15 **a.** $P(z > 1.43) = 1 - P(z \le 1.43) = 1 - 0.9236 = 0.0764$

b. $P(z < -1.65) = 0.0495$

c. $P(z > -0.65) = 1 - P(z \le -0.65) = 1 - 0.2578 = 0.7422$

d. $P(z < 0.89) = 0.8133$

6.17 **a.** $P(z < -2.34) = 0.0096$

b. $P(0.67 \le z \le 2.59) = P(z \le 2.59) - P(z \le 0.67) = 0.9952 - 0.7486 = 0.2466$

c. $P(-2.07 \le z \le -0.93) = P(z \le -0.93) - P(z \le -2.07) = 0.1762 - 0.0192 = 0.1570$

d. $P(z < 1.78) = 0.9625$

Section 6.2

6.19 $\mu = 30$ and $\sigma = 5$

a. $z = (x - \mu)/\sigma = (39 - 30)/5 = 1.80$

b. $z = (x - \mu)/\sigma = (19 - 30)/5 = -2.20$

c. $z = (x - \mu)/\sigma = (24 - 30)/5 = -1.20$

d. $z = (x - \mu)/\sigma = (44 - 30)/5 = 2.80$

6.21 $\mu = 55$ and $\sigma = 7$

a. For $x = 58$: $z = (x - \mu)/\sigma = (58 - 55)/7 = 0.43$
$P(x > 58) = P(z > 0.43) = 1 - P(z \le 0.43) = 1 - 0.6664 = 0.3336$

b. For $x = 43$: $z = (x - \mu)/\sigma = (43 - 55)/7 = -1.71$
$P(x > 43) = P(z > -1.71) = 1 - P(z \le -1.71) = 1 - 0.0436 = 0.9564$

c. For $x = 68$: $z = (x - \mu)/\sigma = (68 - 55)/7 = 1.86$
$P(x < 68) = P(z < 1.86) = 0.9686$

d. For $x = 22$: $z = (x - \mu)/\sigma = (22 - 55)/7 = -4.71$
$P(x < 22) = P(z < -4.71) = 0$ approximately

6.23 $\mu = 117.6$ and $\sigma = 14.6$

a. For $x = 77.9$: $z = (x - \mu)/\sigma = (77.9 - 117.6)/14.6 = -2.72$
For $x = 98.3$: $z = (x - \mu)/\sigma = (98.3 - 117.6)/14.6 = -1.32$
$P(77.9 < x < 98.3) = P(-2.72 < z < -1.32) = P(z < -1.32) - P(z < -2.72) = 0.0934 - 0.0033 = 0.0901$

b. For $x = 85.3$: $z = (x - \mu)/\sigma = (85.3 - 117.6)/14.6 = -2.21$
For $x = 142.6$: $z = (x - \mu)/\sigma = (142.6 - 117.6)/14.6 = 1.71$
$P(85.3 < x < 142.6) = P(-2.21 < z < 1.71) = P(z < 1.71) - P(z < -2.21) = 0.9564 - 0.0136 = 0.9428$

Section 6.3

6.25 $\mu = 190$ minutes and $\sigma = 21$ minutes

a. For $x = 160$: $z = (x - \mu)/\sigma = (160 - 190)/21 = -1.43$
$P(x < 160) = P(z < -1.43) = 0.0764$

b. For $x = 215$: $z = (x - \mu)/\sigma = (215 - 190)/21 = 1.19$
For $x = 245$: $z = (x - \mu)/\sigma = (245 - 190)/21 = 2.62$
$P(215 \le x \le 245) = P(1.19 \le z \le 2.62) = P(z \le 2.62) - P(z \le 1.19) = 0.9956 - 0.8830 = 0.1126$

6.27 $\mu = 46$ miles per hour and $\sigma = 4$ miles per hour

a. For $x = 40$: $z = (x - \mu)/\sigma = (40 - 46)/4 = -1.50$
$P(x > 40) = P(z > -1.50) = 1 - P(z \le -1.50) = 1 - 0.0668 = 0.9332$ or 93.32%

b. For $x = 50$: $z = (x - \mu)/\sigma = (50 - 46)/4 = 1.00$
For $x = 57$: $z = (x - \mu)/\sigma = (57 - 46)/4 = 2.75$
$P(50 < x < 57) = P(1.00 < z < 2.75) = P(z < 2.75) - P(z < 1.00) = 0.9970 - 0.8413 = 0.1557$ or 15.47%

6.29 $\mu = 1650$ kwh and $\sigma = 320$ kwh

a. For $x = 1950$: $z = (x - \mu)/\sigma = (1950 - 1650)/320 = 0.94$
$P(x < 1950) = P(z < 0.94) = 0.8264$

b. For $x = 900$: $z = (x - \mu)/\sigma = (900 - 1650)/320 = -2.34$
For $x = 1300$: $z = (x - \mu)/\sigma = (1300 - 1650)/320 = -1.09$
$P(900 \le z \le 1300) = P(-2.34 \le z \le -1.09) = P(z \le -1.09) - P(z \le -2.34) = 0.1379 - 0.0096 = 0.1283$ or 12.83%

6.31 $\mu = \$19{,}800$ and $\sigma = \$350$

a. For $x = 19{,}445$: $z = (x - \mu)/\sigma = (19{,}445 - 19{,}800)/350 = -1.01$
$P(x < 19{,}445) = P(z < -1.01) = 0.1562$ or 15.62%

b. For $x = 20{,}300$: $z = (x - \mu)/\sigma = (20{,}300 - 19{,}800)/350 = 1.43$
$P(x > 20{,}300) = P(z > 1.43) = 1 - P(z \le 1.43) = 1 - 0.9236 = 0.0764$ or 7.64%

6.33 $\mu = 15$ minutes and $\sigma = 2.4$ minutes

a. For $x = 20$: $z = (x - \mu)/\sigma = (20 - 15)/2.4 = 2.08$
$P(x > 20) = P(z > 2.08) = 1 - P(z \le 2.08) = 1 - 0.9812 = 0.0188$ or 1.88%

b. For $x = 25$: $z = (x - \mu)/\sigma = (25 - 15)/2.4 = 4.17$
$P(x > 25) = P(z > 4.17) = 1 - P(z \le 4.17) = 1 - 1.0 = 0.0$ approximately
Although it is possible that a given car may take more than 25 minutes for oil and lube service, the probability is close to zero.

6.35 $\mu = 5.75$ ounces and $\sigma = 0.11$ ounce
For $x = 5.5$: $z = (x - \mu)/\sigma = (5.5 - 5.75)/0.11 = -2.27$
For $x = 6.0$: $z = (x - \mu)/\sigma = (6.0 - 5.75)/0.11 = 2.27$
$P(x < 5.5) + P(x > 6.0) = P(z < -2.27) + P(z > 2.27) = 0.0116 + 0.0116 = 0.0232$ or 2.32%

Section 6.4

6.37 **a.** $z = 1.65$ approximately

b. $z = -1.96$

c. $z = -2.33$ approximately

d. $z = 2.58$ approximately

6.39 $\mu = 550$ and $\sigma = 75$

a. To find the z value that corresponds to the required x value, we look for 0.0250 in the body of the normal distribution table. The corresponding z value is -1.96.
$z = -1.96$
$x = \mu + z\sigma = 550 + (-1.96)(75) = 403$

b. To find the z value that corresponds to the required x value, we look for $1 - 0.9345 = 0.0655$ in the body of the normal distribution table. The corresponding z value is -1.51.
$z = -1.51$
$x = \mu + z\sigma = 550 + (-1.51)(75) = 436.75$

c. To find the z value that corresponds to the required x value, we look for 1 – 0.0275 = 0.9725 in the body of the normal distribution table. The corresponding z value is 1.92.
$z = 1.92$
$x = \mu + z\sigma = 550 + (1.92)(75) = 694$

d. To find the z value that corresponds to the required x value, we look for 0.9600 in the body of the normal distribution table. The corresponding z value is 1.75.
$z = 1.75$
$x = \mu + z\sigma = 550 + (1.75)(75) = 681.25$

e. To find the z value that corresponds to the required x value, we look for 0.5000 – 0.4700 = 0.0300 in the body of the normal distribution table. The corresponding z value is –1.88.
$z = -1.88$
$x = \mu + z\sigma = 550 + (-1.88)(75) = 409$

f. To find the z value that corresponds to the required x value, we look for 1 – 0.0900 = 0.9100 in the body of the normal distribution table. The corresponding z value is 1.34.
$z = 1.34$
$x = \mu + z\sigma = 550 + (1.34)(75) = 650.5$

6.41 $\mu = \$95$ and $\sigma = \$20$
Let x denote the amount spent by a randomly chosen customer on a visit to this store. We are to find x such that the area in the right tail of the normal distribution curve is 0.10. Thus, $z = 1.28$ and
$x = \mu + z\sigma = 95 + (1.28)(20) = \$120.60$
So, a required minimum purchase of \$121 would meet the condition.

6.43 $\sigma = \$50$ and $P(x \geq 250) = 0.20$
The area to the left of $x = 250$ is $1 - 0.20 = 0.80$ and $z = 0.84$ approximately.
Then, we have $x = \mu + z\sigma \Rightarrow \mu = x - z\sigma = 250 - (0.84)(50) = \$208$
The mean price of all college textbooks is approximately \$208.

<u>Section 6.5</u>

6.45 The normal distribution may be used as an approximation to a binomial distribution when both $np > 5$ and $nq > 5$.

6.47 **a.** $\mu = np = 120(0.60) = 72$ and $\sigma = \sqrt{npq} = \sqrt{120(0.60)(0.40)} = 5.36656315$

b. In this example, $np = 120(0.60)=72 > 5$ and $nq = 120(0.40)=48 > 5$, so we can use the normal probability distribution to approximate the binomial probability distribution.

For $x = 69.5$: $z = (69.5 - 72)/5.36656315 = -0.47$
$P(x \le 69.5) = P(z \le -0.47) = 0.3192$

c. For $x = 66.5$: $z = (66.5 - 72)/5.36656315 = -1.02$
For $x = 73.5$: $z = (73.5 - 72)/5.36656315 = 0.28$
$P(66.5 \le x \le 73.5) = P(-1.02 \le z \le 0.28) = P(z \le 0.28) - P(z \le -1.02) = 0.6103 - 0.1539 = 0.4564$

6.49 In this example, $np = 400(0.151)=60.4 > 5$ and $nq = 400(0.849)=339.6 > 5$, so we can use the normal probability distribution to approximate the binomial probability distribution.
$\mu = np = 400(0.151) = 60.4$ and $\sigma = \sqrt{npq} = \sqrt{400(0.151)(0.849)} = 7.160977587$
For $x = 54.5$: $z = (54.5 - 60.4)/7.160977587 = -0.82$
For $x = 70.5$: $z = (70.5 - 60.4)/7.160977587 = 1.41$
$P(54.5 \le x \le 70.5) = P(-0.82 \le z \le 1.41) = P(z \le 1.41) - P(z \le -0.82) = 0.9207 - 0.2061 = 0.7146$

6.51 In this example, $np = 100(0.80)=80 > 5$ and $nq = 100(0.20)=20 > 5$, so we can use the normal probability distribution to approximate the binomial probability distribution.
$\mu = np = 100(0.80) = 80$ and $\sigma = \sqrt{npq} = \sqrt{100(0.80)(0.20)} = 4$

a. For $x = 74.5$: $z = (74.5 - 80)/4 = -1.38$
For $x = 75.5$: $z = (75.5 - 80)/4 = -1.13$
$P(74.5 \le x \le 75.5) = P(-1.38 \le z \le -1.13) = P(z \le -1.13) - P(z \le -1.38) = 0.1292 - 0.0838 = 0.0454$

b. For $x = 73.5$: $z = (73.5 - 80)/4 = -1.63$
$P(x \le 73.5) = P(z \le -1.63) = 0.0516$

c. For $x = 73.5$: $z = (73.5 - 80)/4 = -1.63$
For $x = 85.5$: $z = (85.5 - 80)/4 = 1.38$
$P(73.5 \le x \le 85.5) = P(-1.63 \le z \le 1.38) = P(z \le 1.38) - P(z \le -1.63) = 0.9162 - 0.0516 = 0.8646$

6.53 In this example, $np = 100(0.06)=6 > 5$ and $nq = 100(0.94)=94 > 5$, so we can use the normal probability distribution to approximate the binomial probability distribution.
$\mu = np = 100(0.06) = 6$ and $\sigma = \sqrt{npq} = \sqrt{100(0.06)(0.94)} = 2.37486842$

a. For $x = 8.5$: $z = (8.5 - 6)/2.37486842 = 1.05$
$P(\text{shipment is returned}) = P(x \ge 8.5) = P(z \ge 1.05) = 1 - P(z < 1.05) = 1 - 0.8531 = 0.1469$

b. $P(\text{shipment is not returned}) = 1 - P(\text{shipment is returned}) = 1 - 0.1469 = 0.8531$

Supplementary Exercises

6.55 $\mu = 16$ ounces and $\sigma = 0.18$ ounce

a. For $x = 16.20$: $z = (16.20 - 16)/0.18 = 1.11$
For $x = 16.50$: $z = (16.50 - 16)/0.18 = 2.78$
$P(16.20 \le x \le 16.50) = P(1.11 \le z \le 2.78) = P(z \le 2.78) - P(z \le 1.11) = 0.9973 - 0.8665 = 0.1308$

b. For $x = 15.70$: $z = (15.70 - 16)/0.18 = -1.67$
$P(x < 15.70) = P(z < -1.67) = 0.0475$ or 4.75%

c. For $x = 15.20$: $z = (15.20 - 16)/.18 = -4.44$
$P(x < 15.20) = P(z < -4.44) = 0.0$ approximately
Although it is possible for a carton to contain less than 15.20 ounces, the probability of this is very close to zero.

6.57 $\mu = 28$ minutes and $\sigma = 5$ minutes
Let x denote the morning commute time. We are to find x so that the area in the right tail of the normal distribution curve is 0.01. Thus, $z = 2.33$ and $x = \mu + z\sigma = 28 + (2.33)(5) = 39.65$ minutes. Thus, Jenn must leave by approximately 7:50 am, 40 minutes before she is due to arrive at work.

6.59 In this example, $np = 100(0.80) = 80 > 5$ and $nq = 100(0.20) = 20 > 5$, so we can use the normal probability distribution to approximate the binomial probability distribution.
$\mu = np = 100(0.80) = 80$ and $\sigma = \sqrt{npq} = \sqrt{100(0.80)(0.20)} = 4$

a. For $x = 84.5$: $z = (84.5 - 80)/4 = 1.13$
For $x = 85.5$: $z = (85.5 - 80)/4 = 1.38$
$P(84.5 \le x \le 85.5) = P(1.13 \le z \le 1.38) = P(z \le 1.38) - P(z \le 1.13) = 0.9162 - 0.8708 = 0.0454$

b. For $x = 74.5$: $z = (74.5 - 80)/4 = -1.38$
$P(x \le 74.5) = P(z \le -1.38) = 0.0838$

c. For $x = 74.5$: $z = (74.5 - 80)/4 = -1.38$
For $x = 87.5$: $z = (87.5 - 80)/4 = 1.88$
$P(74.5 \le x \le 87.5) = P(-1.38 \le z \le 1.88) = P(z \le 1.88) - P(z \le -1.38) = 0.9699 - 0.0838 = 0.8861$

d. For $x = 71.5$: $z = (71.5 - 80)/4 = -2.13$
For $x = 77.5$: $z = (77.5 - 80) / 4 = -0.63$
$P(71.5 \le x \le 77.5) = P(-2.13 \le z \le -0.63) = P(z \le -0.63) - P(z \le -2.13) = 0.2643 - 0.0166 = 0.2477$

Supplementary Exercises

6.61 $\sigma = 0.18$ ounce and $P(x > 16) = 0.90$
The area to the left of $x = 16$ is $1 - 0.90 = 0.10$ and $z = -1.28$ approximately.
Then, $\mu = x - z\sigma = 16 - (-1.28)(0.18) = 16.23$ ounces. Thus, the mean amount of ice cream put into these cartons by this machine should be approximately 16.23 ounces.

6.63 $\mu = 45{,}000$ and $\sigma = 2000$
First, we find the probability that one tire lasts at least 46,000 miles.
For $x = 46{,}000$: $z = (46{,}000 - 45{,}000)/2000 = 0.50$
$P(x \geq 46{,}000) = P(z \geq 0.50) = 1 - P(z \leq 0.50) = 1 - 0.6915 = 0.3085$
So, the probability of one tire lasting at least 46,000 miles is 0.3085.
Then, P(all four tires last more than 46,000 miles) $= (0.3085)^4 = 0.0091$.

6.65 $\mu = 45$ minutes and $\sigma = 3$ minutes
Let x be the amount of time Ashley spends commuting to work. We are given that the area to the left of x is 0.95, which gives $z = 1.65$ approximately. Then, $x = \mu + z\sigma = 45 + (1.65)(3) = 49.95 \approx 50$ minutes. Ashley should leave home at about 8:10 am in order to arrive at work by 9 am 95% of the time.

6.67 **a.** The gambler will win more money if he hits on a single-number bet than if he hits on a color bet.

b. Single-number bet: $np = (25)\left(\frac{1}{38}\right) = 0.658 < 5$, so we cannot use the standard normal distribution to find the probability. The gambler comes out ahead if his number comes up at least once:

P (at least one success) = 1 – P(all losses) = 1 – $\left(1 - \frac{1}{38}\right)^{25} = 0.4866$

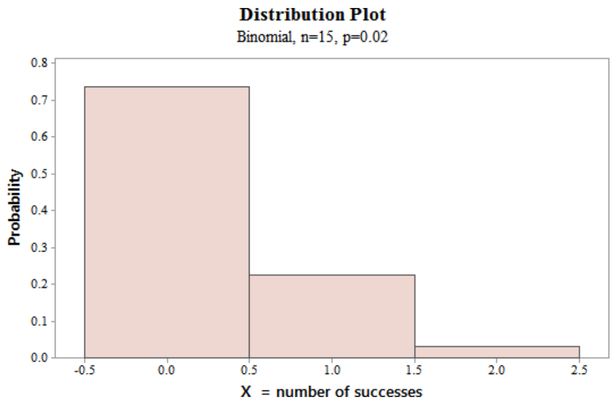
Color bet: $np = (25)\left(\frac{18}{38}\right) = 11.84 > 5$ and $nq = (25)\left(\frac{20}{38}\right) = 13.16 > 5$, so we can use the normal distribution to approximate the probability.

$\mu = np = 25\left(\frac{18}{38}\right) = 11.84$ and $\sigma = \sqrt{npq} = \sqrt{25\left(\frac{18}{38}\right)\left(\frac{20}{38}\right)} = 2.49653500$

The gambler will come out ahead if he wins 13 bets or more since he would need to win over half of his bets.
For $x = 12.5$: $z = (12.5 - 11.84)/2.49653500 = 0.26$
$P(x \geq 12.5) = P(z \geq 0.26) = 1 - P(z \leq 0.26) = 1 - 0.6026 = 0.3974$
Since $0.4866 > 0.3974$, the gambler has a better chance of coming out ahead with the single-number bet.

6.69 $\mu = np = 15(0.02) = 0.30$ and $\sigma = \sqrt{npq} = \sqrt{15(0.02)(0.98)} = 0.54221767$

Since $np < 5$, the normal approximation to the binomial is not appropriate. The Empirical Rule requires a bell-shaped distribution, and this distribution is strongly skewed right.



By the Empirical Rule, approximately 68% of the observations fall in the interval $\mu \pm \sigma$, approximately 95% fall in the interval $\mu \pm 2\sigma$, and about 99.7% fall in the interval $\mu \pm 3\sigma$. These intervals are –0.24 to 0.84, –0.78 to 1.38, and –1.33 to 1.93, respectively. Using the normal approximation with continuity correction,

For $x = -0.74$: $z = (-0.74 - 0.3)/0.54221767 = -1.92$

For $x = 1.34$: $z = (1.34 - 0.3)/0.54221767 = 1.92$

$P(-0.74 \le x \le 1.34) = P(-1.92 \le z \le 1.92) = P(z \le 1.92) - P(z \le -1.92) = 0.9726 - 0.0274 = 0.9452 > 0.68$

For $x = -1.28$: $z = (-1.28 - 0.3)/0.54221767 = -2.91$

For $x = 1.88$ $z = (1.88 - 0.3)/0.54221767 = 2.91$

$P(-1.28 \le x \le 1.88) = P(-2.91 \le z \le 2.91) = P(z \le 2.91) - P(z \le -2.91) = 0.9982 - 0.0018 = 0.9964 > 0.95$

For $x = -1.83$: $z = (-1.83 - 0.3)0/.54221767 = -3.93$

For $x = 2.43$: $z = (2.43 - 0.3)/0.54221767 = 3.93$

$P(-1.83 \le x \le 2.43) = P(-3.93 \le z \le 3.93) = P(z \le 3.93) - P(z \le -3.93) = 1.0 - 0.0 =$ 1.0 approximately

Self-Review Test

1. a **3.** d **5.** a **7.** b

9. **a.** $P(0.85 \le z \le 2.33) = P(z \le 2.33) - P(z \le 0.85) = 0.9901 - 0.8023 = 0.1878$

b. $P(-2.97 \le z \le 1.49) = P(z \le 1.49) - P(z \le -2.97) = 0.9319 - 0.0015 = 0.9304$

c. $P(z \le -1.29) = 0.0985$

d. $P(z > -0.74) = 1 - P(z \le -0.74) = 1 - 0.2296 = 0.7704$

11. $\mu = 152.5$ minutes and $\sigma = 8.48$ minutes

a. For $x = 142$: $z = (142 - 152.5)/8.48 = -1.24$

For $x = 156$: $z = (156 - 152.5)/8.48 = 0.41$

$P(142 \le x \le 156) = P(-1.24 \le z \le 0.41) = P(z \le 0.41) - P(z \le -1.24) = 0.6591 - 0.1075 = 0.5516$

b. For $x = 168$: $z = (168 - 152.5)/8.48 = 1.83$

$P(x \ge 168) = P(z \ge 1.83) = 1 - 0.9664 = 0.0336$

c. For $x = 163$: $z = (163 - 152.5)/8.48 = 1.24$

$P(x \le 163) = P(z \le 1.24) = 0.8925$

d. For $x = 159$: $z = (159 - 152.5)/8.48 = 0.77$
For $x = 169$: $z = (169 - 152.5)/8.48 = 1.95$
$P(158 \le x \le 169) = P(0.77 \le z \le 1.95) = P(z \le 1.95) - P(z \le 0.77) = 0.9744 - 0.7794 = 0.1950$

13. In this example, $np = 800(0.15) = 120 > 5$ and $nq = 800(0.85) = 680 > 5$, so we can use the normal probability distribution to approximate the binomial probability distribution.
$\mu = np = 800(0.15) = 120$ and $\sigma = \sqrt{npq} = \sqrt{800(0.15)(0.85)} = 10.09950494$

a. **i.** For $x = 114.5$: $z = (114.5 - 120)/10.09950494 = -0.54$
For $x = 115.5$: $z = (115.5 - 120)/10.09950494 = -0.45$
$P(114.5 \le x \le 115.5) = P(-0.54 \le z \le -0.45) = P(z \le -0.45) - P(z \le -0.54) = 0.3264 - 0.2946 = 0.0318$

ii. For $x = 102.5$: $z = (102.5 - 120)/10.09950494 = -1.73$
For $x = 142.5$: $z = (142.5 - 120)/10.09950494 = 2.23$
$P(102.5 \le x \le 142.5) = P(-1.73 \le z \le 2.23) = P(z \le 2.23) - P(z \le -1.73) = 0.9871 - 0.0418 = 0.9453$

iii. For $x = 106.5$: $z = (106.5 - 120)/10.09950494 = -1.34$
$P(x \ge 106.5) = P(z \ge -1.34) = 1 - P(z \le -1.34) = 1 - .0901 = .9099$

iv. For $x = 100.5$: $z = (100.5 - 120)/10.09950494 = -1.93$
$P(x \le 100.5) = P(z \le -1.93) = 0.0268$

v. For $x = 110.5$: $z = (110.5 - 120)/10.09950494 = -.94$
For $x = 123.5$: $z = (123.5 - 120)/10.09950494 = 0.35$
$P(110.5 \le x \le 123.5) = P(-0.94 \le z \le 0.35) = P(z \le 0.35) - P(z \le -0.94) = 0.6368 - 0.1736 = 0.4632$

b. P(at least 675 do not have wheat intolerance) = P(less than 125 have wheat intolerance) = $P(x < 125) = P(x \underline{<} 124) = P(x \underline{<} 124.5)$
For $x = 124.5$: $z = (124.5 - 120)/10.09950494 = 0.45$
$P(x < 124.5) = P(z < 0.45) = 0.6736$

c. P(do not have wheat tolerance) = 1 – 0.15 = 0.85
$\mu = 800(0.85) = 680,\ \sigma = \sqrt{800(0.85)(0.15)} = 10.09950494$
For x = 681.5: z = (681.5 – 680)/10.09950494 = 0.15
For x = 697.5: z = (697.5 – 680)/10.09950494 = 1.73
P(between 682 and 697 do not have wheat intolerance) = P(682 to 697) = P(681.5 to 697.5) = $P(0.15 < z < 1.73) = P(z < 1.73) - P(z < 0.15) = 0.9582 - 0.5596 = 0.3986$