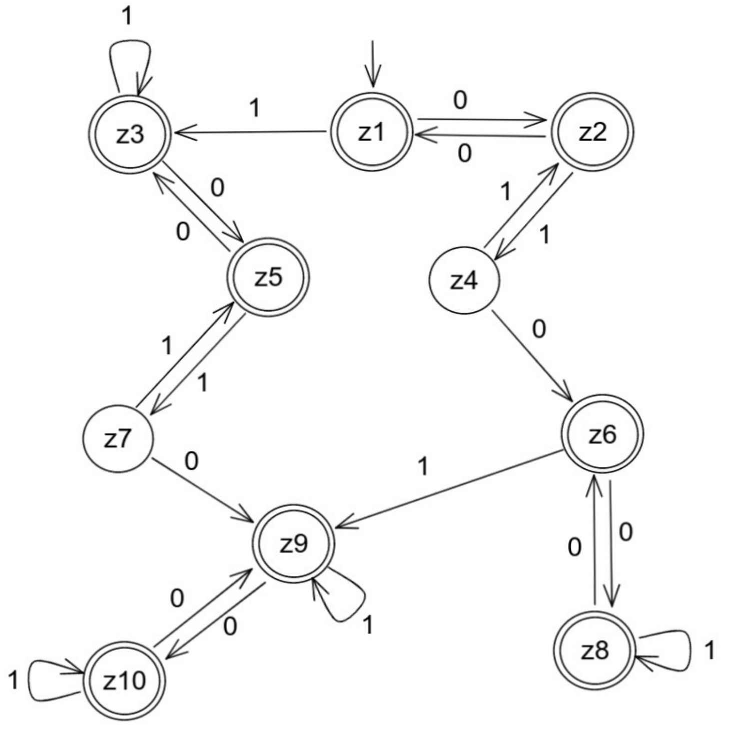
Theory Of Automata

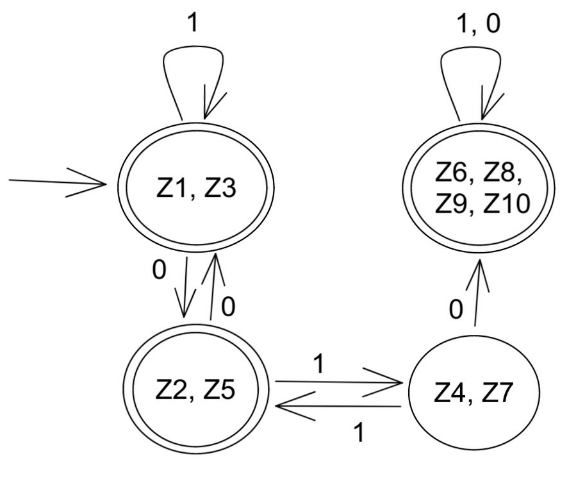
Assignment # 3

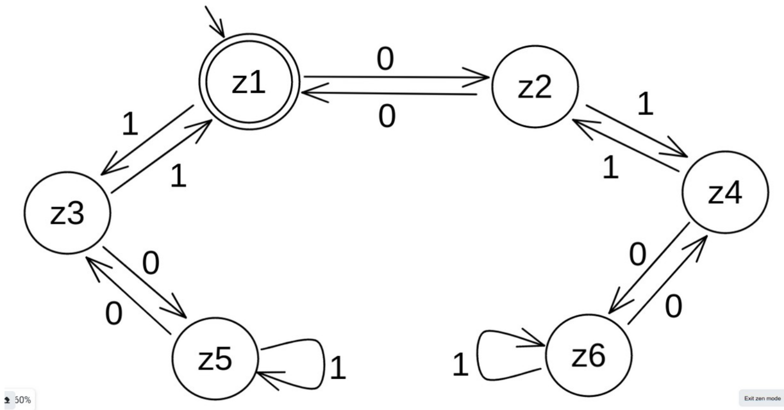
Saad Bin Khalid 20K-0161 Section F

Question 1 – DFA minimization

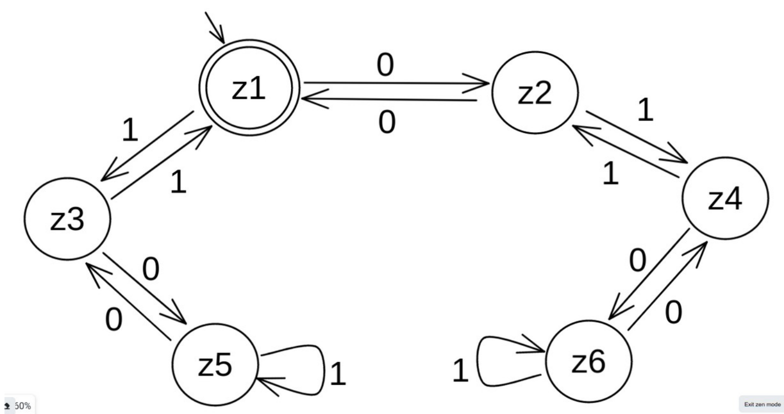


	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6	Z_7	Z_8	Z_9
Z_2									
Z_3	2,5								
Z_4									
Z_5		1,3 4,7							
Z_6		1,8			3,8				
Z_7				2,5					
Z_8					3,6				
Z_9		1,10	1,6		3,10	4,7 8,10		6,8 6,10	
Z_{10}		1,9			3,9	8,9		6,9	6,10





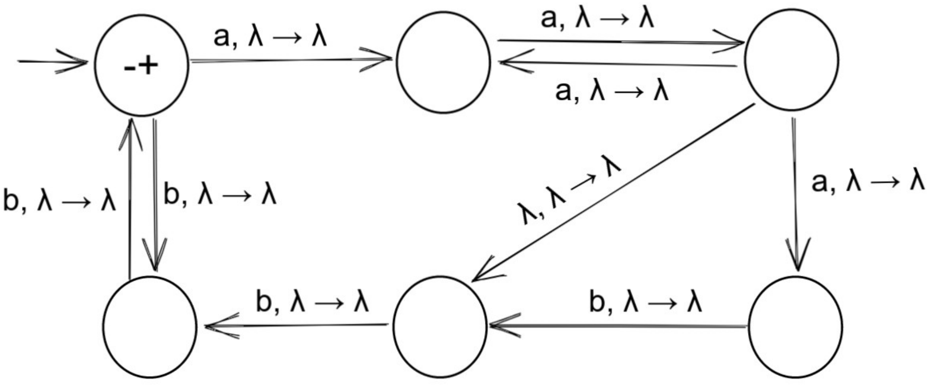
	Z_1	Z_2	Z_3	Z_4	Z_5
Z_2					
Z_3					
Z_4					
Z_5					
Z_6					



Question 2 – RG and PDA

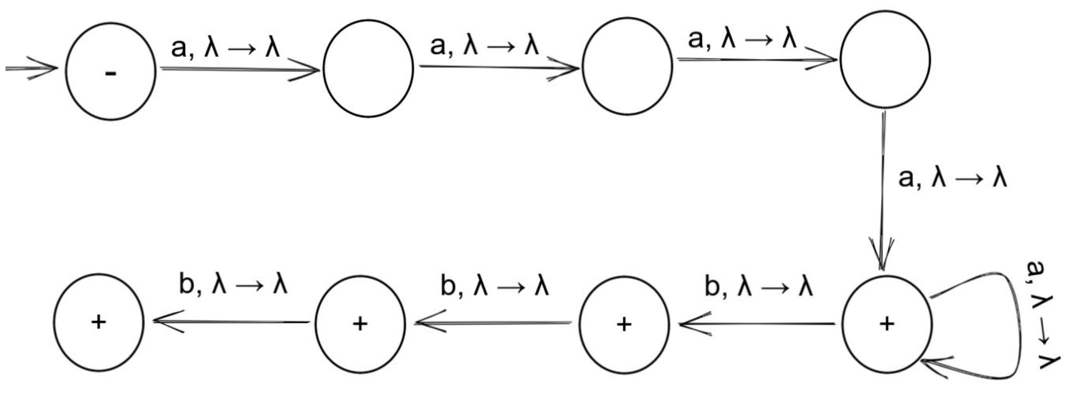
$\{ a^n b^m : (n+m) \text{ is even} \}$

$(aa)^*(ab+\lambda)(bb)^*$



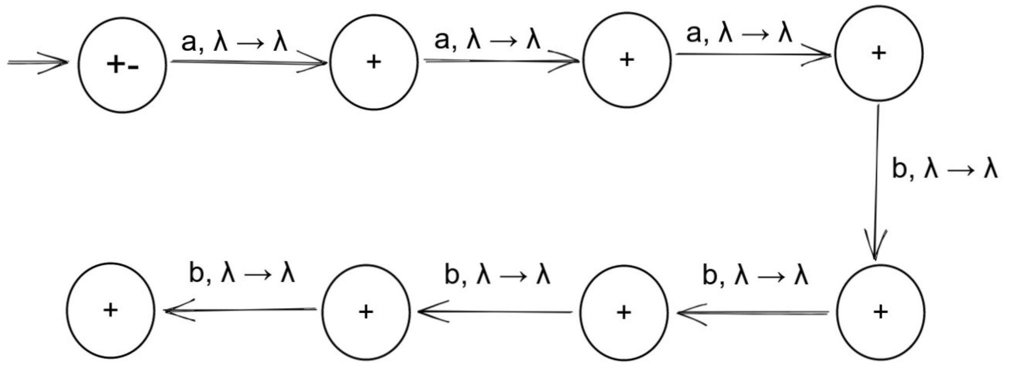
$\{a^n b^m, n \geq 4, m \leq 3\}$

$a^4 a^*(b^3+b^2+b+\lambda)$



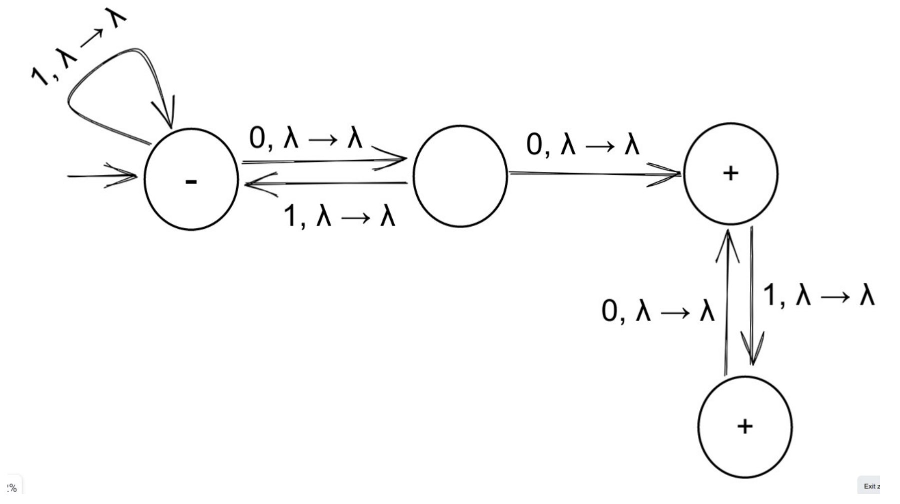
$\{a^n b^m, n < 4, m \leq 4\}$

$(a^3+a^2+a+\lambda)(b^4+b^3+b^2+b+\lambda)$



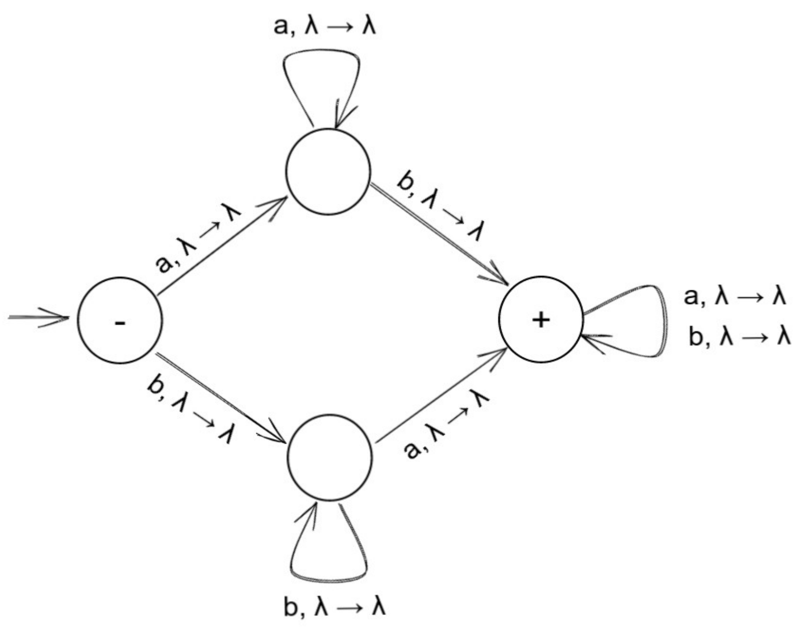
having exactly one pair of consecutive zeros.

$(1+01)^*00(1+10)^*$



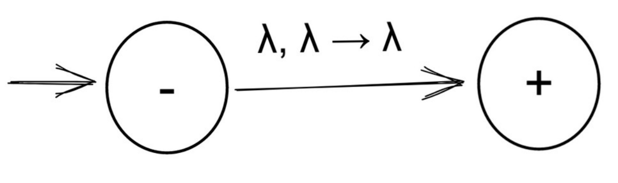
all strings that contain at least one occurrence of each symbol in alphabet

$(a+b)^*a(a+b)^*b(a+b)^* + (a+b)^*b(a+b)^*a(a+b)^*$



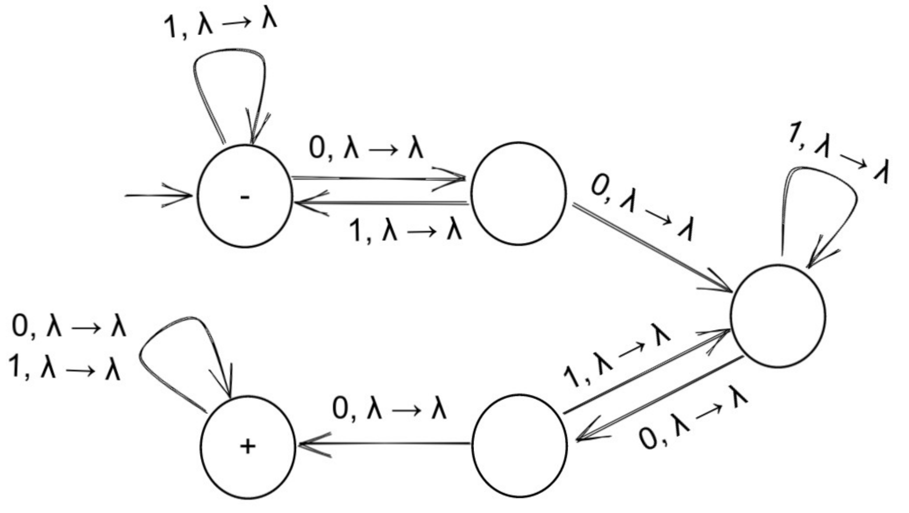
all string not ending in 0, 1

λ



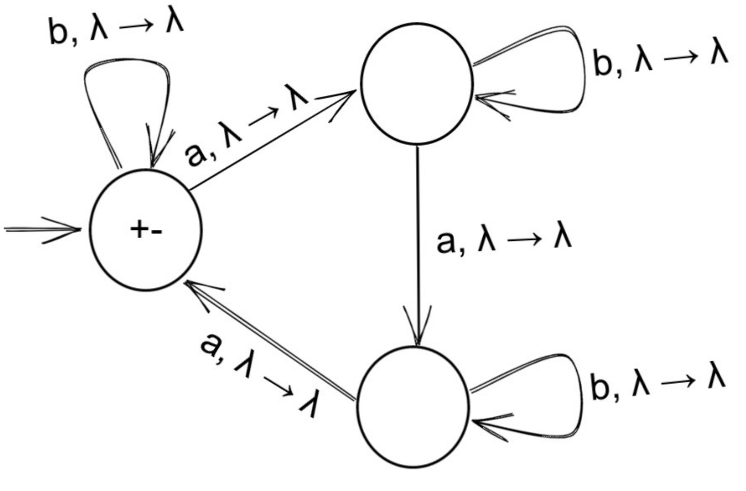
all string having at least two occurrences of substring 00

$(0+1)^*00(0+1)^*00(0+1)^*$



$\{w : n_a(w) \bmod 3 = 0\}$

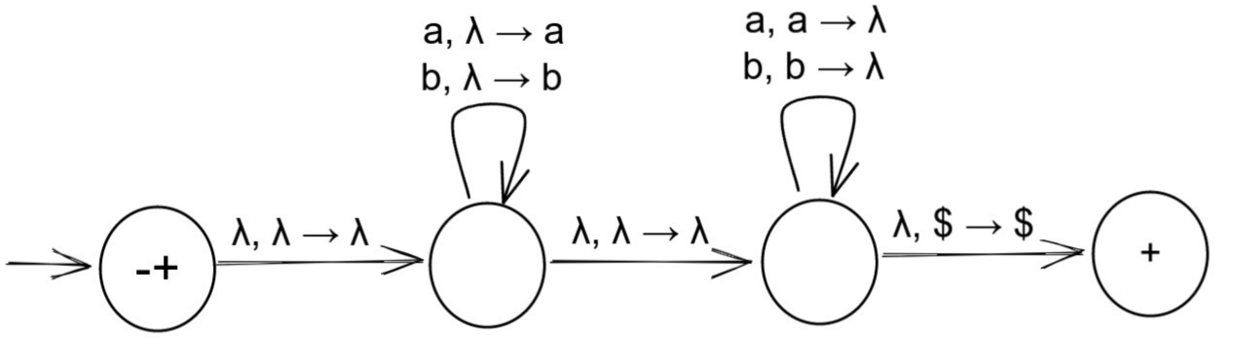
$(b^*ab^*ab^*ab^*)^*$



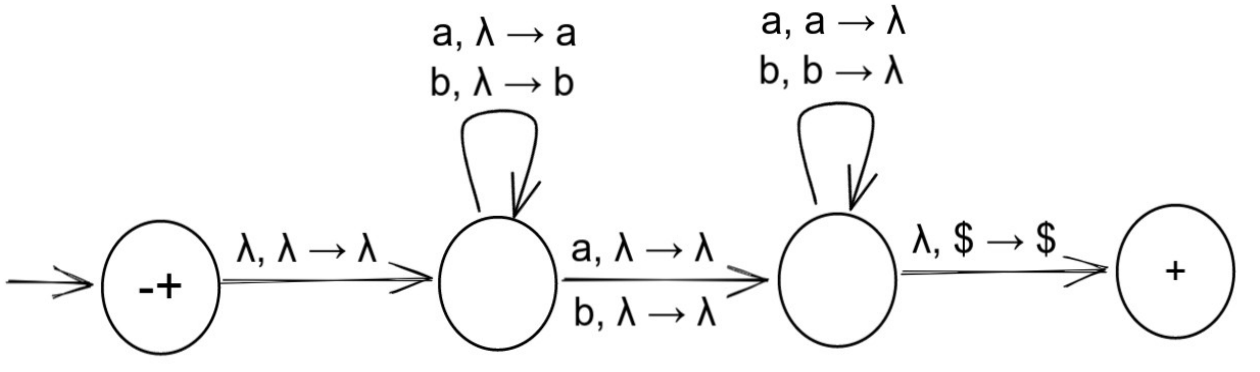
Question 3

3.1 $\rightarrow$ CFG & PDA

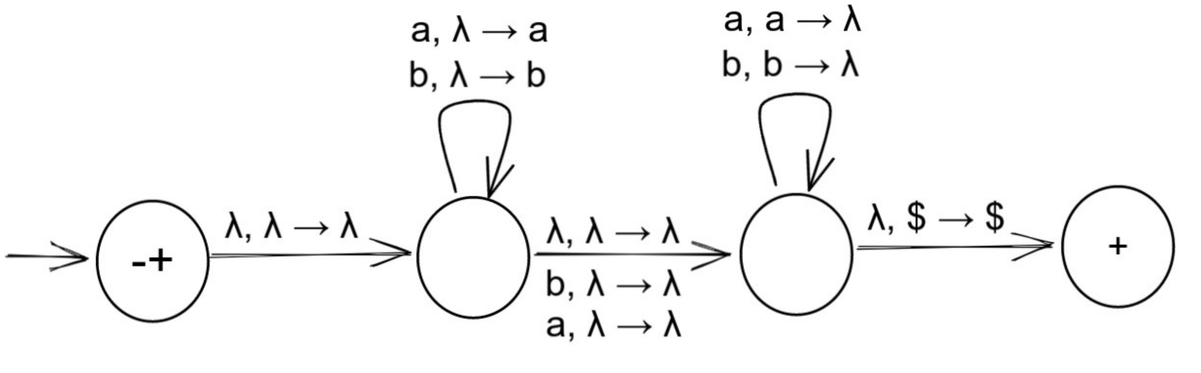
The language of even length palindromes
$S \rightarrow aSa \mid bSb \mid \lambda$



The language of odd length palindromes
$S \rightarrow aSa \mid bSb \mid a \mid b$

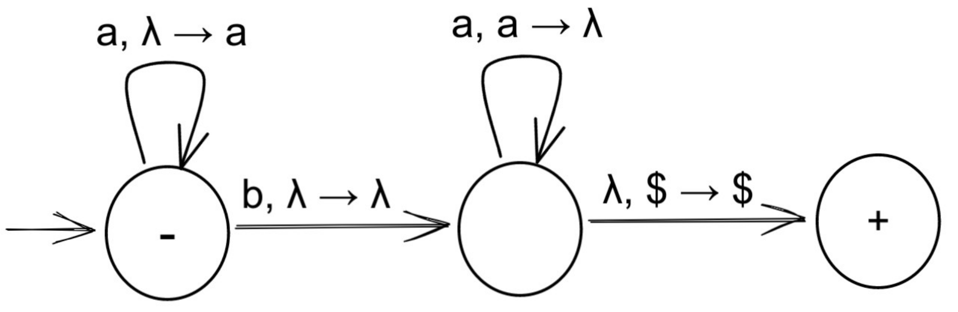


The language of all length palindromes
$S \rightarrow aSa \mid bSb \mid a \mid b \mid \lambda$



a^nba^n

$S \rightarrow aSa \mid b$



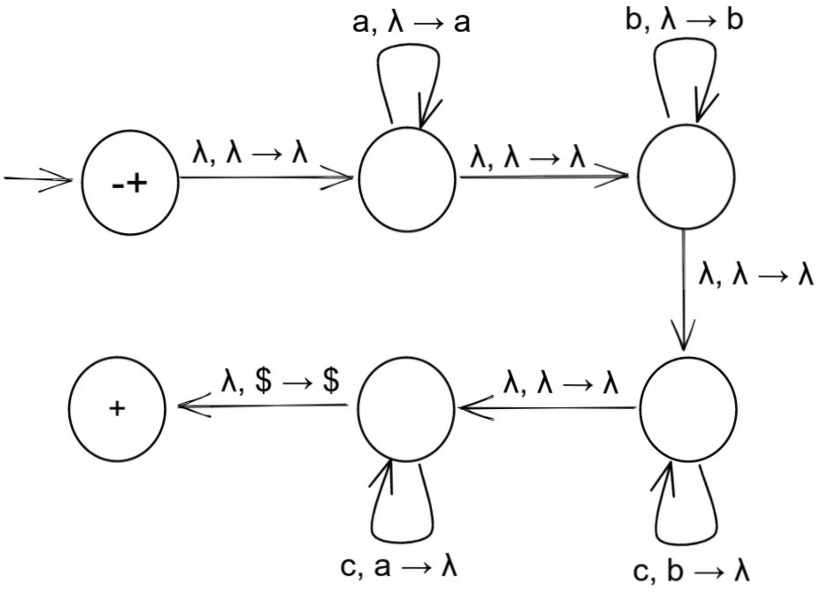
ww: w € {a, b}∗

not possible in PDA

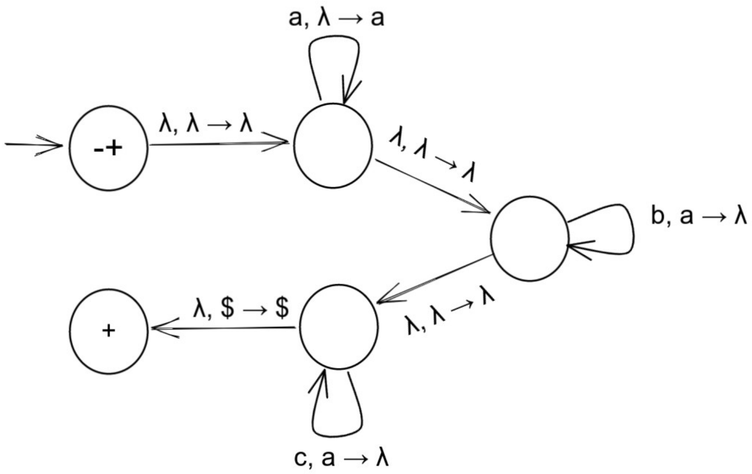
$\{a^nb^mc^{n+m}\}$

$S \rightarrow aSc \mid B$

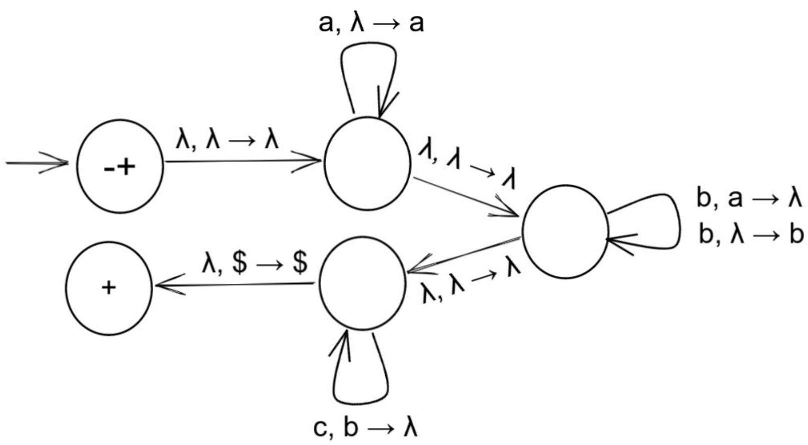
$B \rightarrow bBc \mid \lambda$



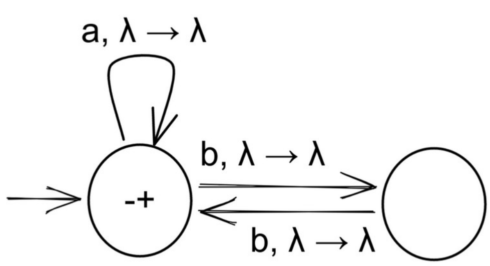
$\{a^{n+m}b^{n}c^{m}\}$
$S \rightarrow aSb \mid C$
$C \rightarrow aCc \mid \lambda$



$\{a^{n}b^{n+m}c^{m}\}$
$S \rightarrow AC$
$A \rightarrow aAb \mid \lambda$
$C \rightarrow bCc \mid \lambda$

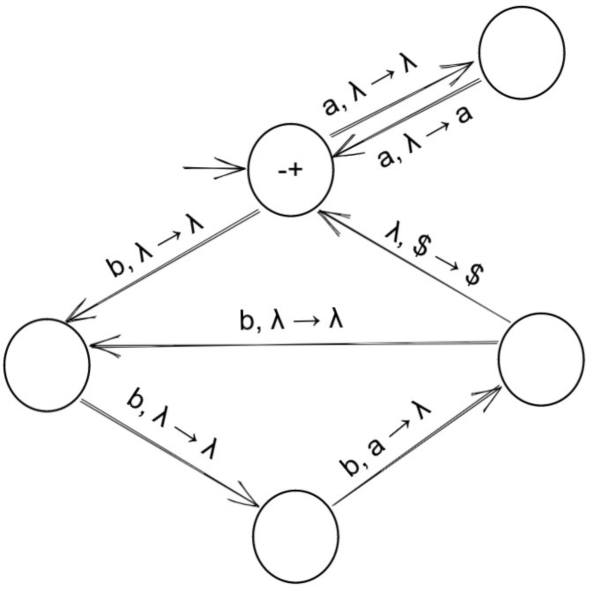


$\{a^{n}b^{2m}\}$
$S \rightarrow AB$
$A \rightarrow aA \mid \lambda$
$B \rightarrow bbB \mid \lambda$



$\{a^{2n}b^{3n}\}$

$S \rightarrow aaSbbb \mid \lambda$



$3.2 \rightarrow$ Pumping lemma

$L = \{a^{2n}b^{3n}\}$

If $N = 3$, then let $w = aabbb \rightarrow x{=}a,\ y{=}a,\ z{=}bbb.$

<u>i=0:</u> $xy^iz = abbb$ → *does not belongs to L*

<u>i=1:</u> $xy^iz = aabbb$ → *belongs to L*

<u>i=2:</u> $xy^iz = aaabbb$ → *does not belongs to L*

<u>i=3:</u> $xy^iz = aaaabbb$ → *does not belongs to L*

<u>i=4:</u> $xy^iz = aaaaabbb$ → *does not belongs to L*

<u>i=5:</u> $xy^iz = aaaaaabbb$ → *does not belongs to L*

Hence, the language is not regular

$L = \{a^nb^{n+m}c^m\}$

If $N = 3$, then let $w = abbc \rightarrow x{=}a,\ y{=}b,\ z{=}bc.$

<u>i=0:</u> $xy^iz = abc$ → *does not belongs to L*

<u>i=1:</u> $xy^iz = abbc$ → *belongs to L*

<u>i=2:</u> $xy^iz = abbbc$ → *does not belongs to L*

<u>i=3:</u> $xy^iz = abbbbc$ → *does not belongs to L*

<u>i=4:</u> $xy^iz = abbbbbc$ → *does not belongs to L*

<u>i=5:</u> $xy^iz = abbbbbbc$ → *does not belongs to L*

Hence, the language is not regular

$L = \{a^n b a^n\}$

If N = 3, then let w = aabaa → $x=a$, $y=a$, $z=baa$.

<u>*i=0:*</u> xy^iz = *abaa* → *does not belongs to L*
<u>*i=1:*</u> xy^iz = *aabaa* → *belongs to L*
<u>*i=2:*</u> xy^iz = *aaabaa* → *does not belongs to L*
<u>*i=3:*</u> xy^iz = *aaaabaa* → *does not belongs to L*
<u>*i=4:*</u> xy^iz = *aaaaabaa* → *does not belongs to L*
<u>*i=5:*</u> xy^iz = *aaaaaabaa* → *does not belongs to L*

Hence, the language is not regular

3.3 → Stack operations

$\{a^{2n}b^{3n}\}$

S → aaSbbb | λ

a, a → λ
b, b → λ
S, S → λ
S, S → aaSbbb

String to derive: aabbb

	a	λ		
	a	λ		
	S	S	λ	
	b	b	b	λ
	b	b	b	λ
S	b	b	b	λ
λ	λ	aa	aa	aabbb

$\{a^n b^{n+m} c^m\}$
$S \rightarrow AC$
$A \rightarrow aAb \mid \lambda$
$C \rightarrow bCc \mid \lambda$
$a, a \rightarrow \lambda$
$b, b \rightarrow \lambda$
$c, c \rightarrow \lambda$
$A, A \rightarrow \lambda$
$C, C \rightarrow \lambda$
$S, S \rightarrow AC$
$A, A \rightarrow aAb$
$C, C \rightarrow bCc$

String to derive: abbc

		a	λ		
		A	λ		
		b	b	λ	
		b	b	λ	
	A	C	C	C	λ
S	C	c	c	c	λ
λ	λ	λ	a	abb	abbc

$a^n b a^n$
$S \rightarrow aSa \mid b$
$a, a \rightarrow \lambda$
$b, b \rightarrow \lambda$
$S, S \rightarrow aSa$
$S, S \rightarrow b$

String to derive: aabaa

			a	λ		
	a	λ	S	S	b	
	S	S	a	a	a	λ
S	a	a	a	a	a	λ
λ	λ	a	a	aa	aab	aabaa

Question 4 – Simplification

S → abS | abA | abB
A→ cd
B → aB
C → dc

S → abS | abA
A → cd

S → abS | abcd

S → ABC | a
A→ b
B → c
C → d
E → e
F → f
G → g

S → ABC | a
A → b
B → c
C → d

S → bcd | a

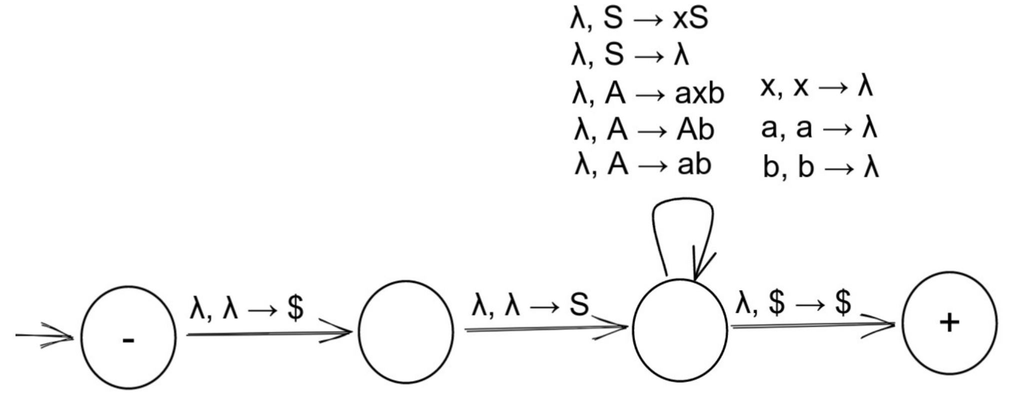
S → aB | bX
A → Bad | bSX | a
B → aSB | bBX
X → SBD | aBX | ad

S → bX
X → ad

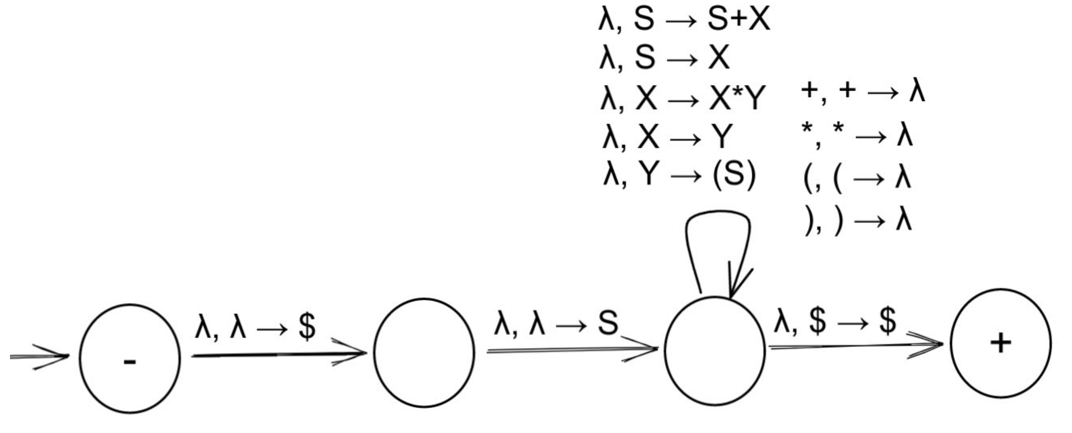
S → bad

Question 5 – PDA

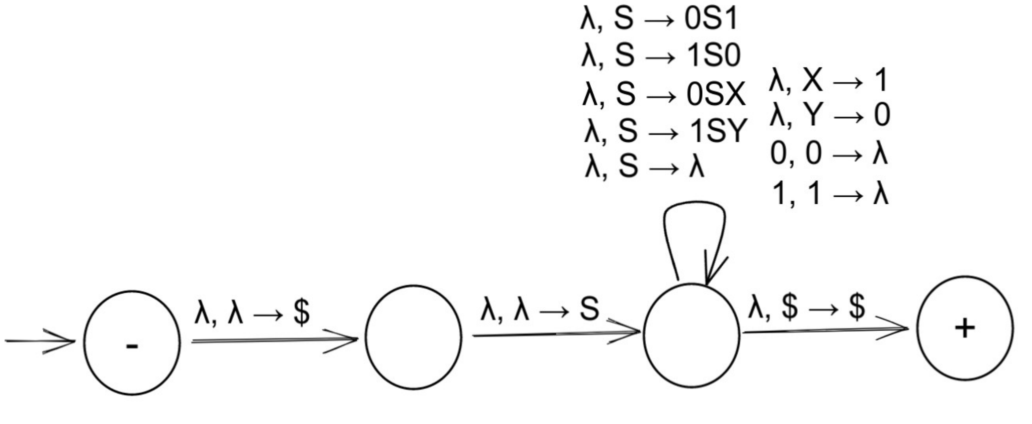
S → xS | ε
A → axb | Ab | ab



S → S+X | X
X → X*Y | Y
Y → (S)

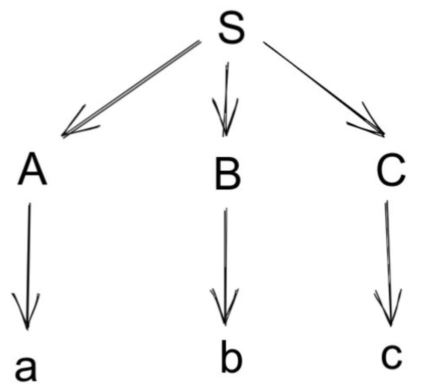


S → 0S1 | 1S0 | ε
S → 0SX | 1SY | ε
X → 1, Y → 0



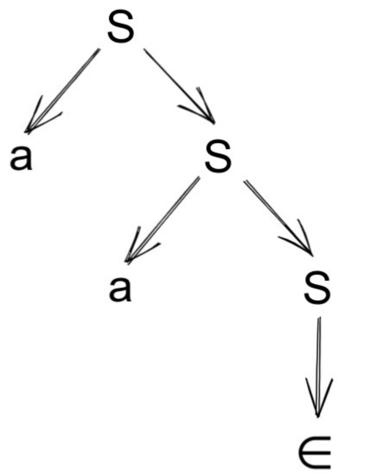
Question 6 – Ambiguity

S → ABC
A → a, B → b, C → c



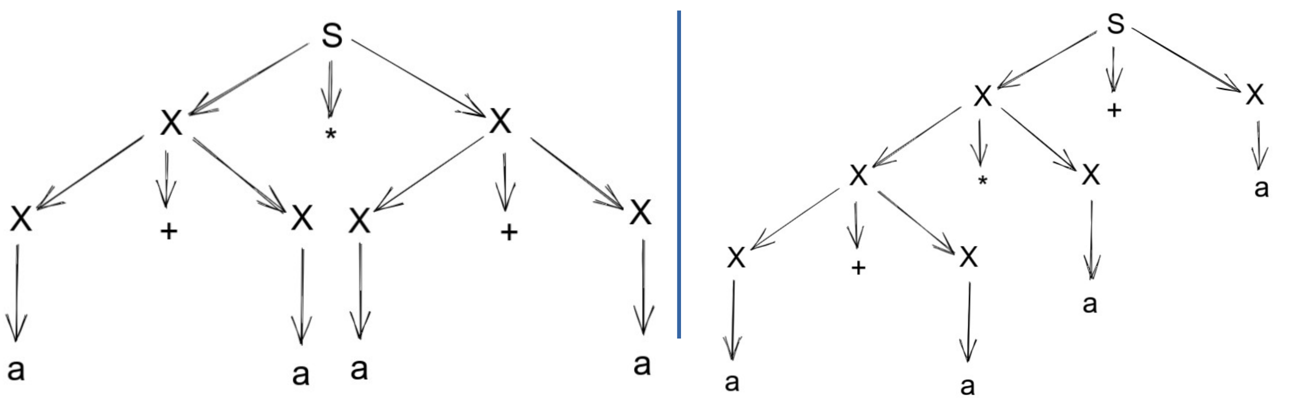
unambigious

S → aS | λ



unambigious

X → X+X | X*X |X| a



ambigious

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