Date 2

Normalized decimal floating form.

$\pm 0.d_1 d_2 \ldots d_n \times 10^n$ $\qquad 1 < d_1 \leq 9$

$0 < d_2 \leq 9$ for $n = 2, 3 \ldots$

e.g 0.3102×10^{-2} is normalized

Error

$AE = |\text{True value} - \text{Approx value}|$

$ARE = \left| \frac{AE}{\text{True value}} \right|$

Taylor polynomial

$P_n(x) = f(x_0) + \frac{f'(x_0)}{1!}(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 + \ldots$

$R_n(x) = \frac{f^{n+1}((\xi))}{(n+1)!}(x - x_0)^{n+1}$

$f(x) = P_n(x) + R_n(x)$

Root finding methods:

$f(a) \cdot f(b) < 0$ Immediate theorem. (to check if root exists)

1) Bisection Method

$f(a) \cdot f(b) < 0$

$c = \frac{a+b}{2}$

$f(a) \cdot f(c) > 0 ; a = c$

$f(b) \cdot f(c) < 0 ; b = c$

ii) Fixed base method

Make x subject (highest power on one side)

e.g $x^4 - 3x^2 - 3 = 0$

$x^4 = 3x^2 + 3$

$x_n = (3x_{n-1}^2 + 3)^{1/4}$

iii) Newton Method

$$P_n = P_{n-1} - \frac{f(P_{n-1})}{f'(P_{n-1})}$$

iv) Secant Method

$$x_{n+1} = x_n - \frac{f(x_n)(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}$$

$$P_n = P_{n-1} - \frac{f(P_{n-1})(P_{n-1} - P_{n-2})}{f(P_{n-1}) - f(n-2)} \quad n \geq 2$$

v) False position $f(a)f(b) < 0$

$$c = \frac{a f(b) - b f(a)}{f(b) - f(a}$$

$f(a) . f(c) > 0 ; a = c$

$f(a) . f(c) < 0 ; b = c$

No. of iterations

$2^{-N}(b-a) < 10^{\text{some power}}$

Date

Lagrange Interpolation Polynomial

One degree $P(x) = L_0(x) f(x_0) + L_1(x) f(x_1)$

$$L_0(x) = \frac{(x-x_1)}{(x_0-x_1)} \qquad L_1(x) = \frac{(x-x_0)}{(x_1-x_0)}$$

2nd Degree $P(x) = L_0(x) f(x_0) + L_1(x) f(x_1) + L_2(x) f(x_2)$

$$L_0(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} \qquad L_1(x) = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} \qquad L_2(x) = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}$$

Bond error

$$f(x) = P(x) + \frac{f^{n+1}(\xi(x))}{(n+1)!}(x-x_0)(x-x_1)+\cdots(x-x_n)$$

Divided Difference table.

x	y	1st Diff	2nd Diff	3rd Diff
x_0	y_0	$D_0 = \frac{y_1-y_0}{x_1-x_0}$		
x_1	y_1		$D_0 = \frac{D_1-D_0}{x_2-x_0}$	
x_2	y_2	$D_1 = \frac{y_2-y_1}{x_2-x_1}$		$D_0 = \frac{D_1-D_0}{x_3-x_0}$
x_3	y_3		$D_1 = \frac{D_2-D_1}{x_3-x_1}$	
		$D_2 = \frac{y_3-y_2}{x_3-x_2}$		

One Degree

$P_1(x) = f(x_0) + (x-x_0) f(x_0, x_1)$

$f(x_0, x_1) =$ 1st Diff

$f(x_0, x_1, x_2) =$ 2nd Diff

$f(x_0, x_1, x_2, x_3) =$ 3rd Diff

2nd Degree

$P_2(x) = f(x_0) + (x-x_0) f(x_0, x_1) + (x-x_0)(x-x_1) f(x_0, x_1, x_2)$