Chapter 5
Discrete Random Variables and Their Probability Distributions

Section 5.1

5.1 A variable whose value is determined by the outcome of a random experiment is called a **random variable**. A random variable that assumes countable values is called a **discrete random variable**. The number of cars owned by a randomly selected individual is an example of a discrete random variable. A random variable that can assume any value contained in one or more intervals is called a **continuous random variable**. An example of a continuous random variable is the amount of time taken by a randomly selected student to complete a statistics exam.

5.3 The number of adults with no ideological affiliation is a discrete random variable because the values of x are countable: 0, 1, 2, 3, 4 and 5.

Section 5.2

5.5
1. The probability assigned to each value of a random variable x lies in the range 0 to 1; that is, $0 \le P(x) \le 1$ for each x.
2. The sum of the probabilities assigned to all possible values of x is equal to 1; that is, $\sum P(x) = 1$.

5.7
a. $P(x = 3) = 0.15$
b. $P(x \le 2) = P(x = 0) + P(x = 1) + P(x = 2) = 0.11 + 0.19 + 0.28 = 0.58$
c. $P(x \ge 4) = P(x = 4) + P(x = 5) + P(x = 6) = 0.12 + 0.09 + 0.06 = 0.27$
d. $P(1 \le x \le 4) = P(x = 1) + P(x = 2) + P(x = 3) + P(x = 4) = 0.19 + 0.28 + 0.15 + 0.12 = 0.74$
e. $P(x < 4) = P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3) = 0.11 + 0.19 + 0.28 + 0.15 = 0.73$
f. $P(x > 2) = P(x = 3) + P(x = 4) + P(x = 5) + P(x = 6) = 0.15 + 0.12 + 0.09 + 0.06 = 0.42$
g. $P(2 \le x \le 5) = P(x = 2) + P(x = 3) + P(x = 4) + P(x = 5) = 0.28 + 0.15 + 0.12 + 0.09 = 0.64$

5.9 **a.**

x	$P(x)$
1	8/80 = 0.10
2	20/80 = 0.25
3	24/80 = 0.30
4	16/80 = 0.20
5	12/80 = 0.15

b. The probabilities listed in the table of part a are approximate because they are obtained from a sample of 80 days.

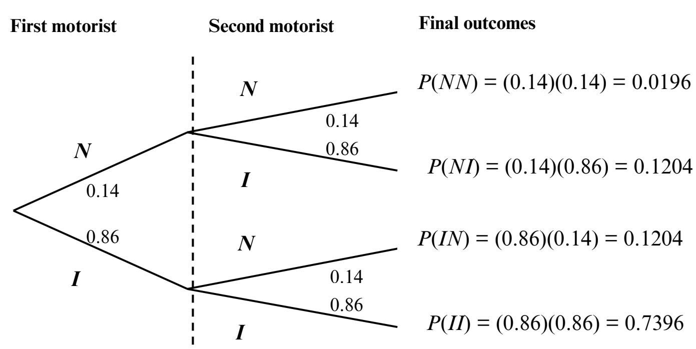
c. i. $P(x = 3) = 0.30$

ii. $P(x \geq 3) = \text{P}(3 \text{ or } 4 \text{ or } 5) = P(3) + P(4) + P(5) = 0.30 + 0.20 + 0.15 = 0.65$

iii. $P(2 \leq x \leq 4) = \text{P}(2 \text{ or } 3 \text{ or } 4) = P(2) + P(3) + P(4) = 0.25 + 0.30 + 0.20 = 0.75$

iv. $P(x < 4) = \text{P}(1 \text{ or } 2 \text{ or } 3) = P(1) + P(2) + P(3) = 0.10 + 0.25 + 0.30 = 0.65$

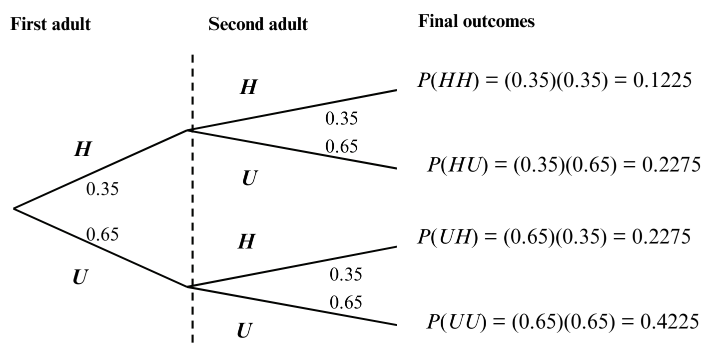
5.11 Let N = the motorist is uninsured and I = the motorist is insured.
Then $P(N) = 0.14$ and $P(I) = 1 - 0.14 = 0.86$. Note that the first and second events are independent.



$P(x = 0) = P(II) = 0.7396$, $P(x = 1) = P(NI) + P(IN) = 0.1204 + 0.1204 = 0.2408$, $P(x = 2) = P(NN) = 0.0196$

x	$P(x)$
0	0.7396
1	0.2408
2	0.0196

5.13 Let H = person happy with job and U = person unhappy with job.
Then $P(H) = 0.35$ and $P(U) = 1 - 0.35 = 0.65$. Note that the first and second events are independent.



$P(x = 0) = P(UU) = 0.4225$,
$P(x = 1) = P(HU) + P(UH) = 0.4550$,
$P(x = 2) = P(HH) = 0.1225$

x	$P(x)$
0	0.4225
1	0.4450
2	0.1225

Section 5.3

5.15 The **mean** of discrete random variable x is the value that is expected to occur per repetition, on average, if an experiment is repeated a large number of times. It is denoted by μ and calculated as $\mu = \sum xP(x)$. The **standard deviation** of a discrete random variable x measures the spread of its probability distribution. It is denoted by σ and is calculated as $\sigma = \sqrt{\sum x^2 P(x) - \mu^2}$.

5.17

x	$P(x)$	$xP(x)$	x^2	$x^2P(x)$
0	0.73	0.00	0	0.00
1	0.16	0.16	1	0.16
2	0.06	0.12	4	0.24
3	0.04	0.12	9	0.36
4	0.01	0.04	16	0.16
		$\sum xP(x) = 0.44$		$\sum x^2P(x) = 0.92$

$\mu = \sum xP(x) = 0.44$ error

$\sigma = \sqrt{\sum x^2 P(x) - \mu^2} = \sqrt{0.92 - (0.44)^2} = 0.852$ error

5.19 Let x be the number of patients entering the emergency room during a one-hour period at Millard Fellmore Memorial Hospital.

x	$P(x)$	$xP(x)$	x^2	$x^2P(x)$
0	0.2725	0.0000	0	0.0000
1	0.3543	0.3543	1	0.3543
2	0.2303	0.4606	4	0.9212
3	0.0998	0.2994	9	0.8982
4	0.0324	0.1296	16	0.5184
5	0.0084	0.0420	25	0.2100
6	0.0023	0.0138	36	0.0828
		$\sum xP(x) = 1.2997$		$\sum x^2P(x) = 2.9849$

$\mu = \sum xP(x) = 1.2997\, patients$

$\sigma = \sqrt{\sum x^2 P(x) - \mu^2} = \sqrt{2.9849 - (1.2997)^2} = 1.1383$ patients

5.21 Let x be the number of customers that arrive at a convenience store.

x	$P(x)$	$xP(x)$	x^2	$x^2P(x)$
0	0.03	0.00	0	0.00
1	0.09	0.09	1	0.09
2	0.16	0.32	4	0.64
3	0.23	0.69	9	2.07
4	0.27	1.08	16	4.32
5	0.11	0.55	25	2.75
6	0.07	0.42	36	2.52
7	0.04	0.28	49	1.96
		$\sum xP(x) = 3.43$		$\sum x^2P(x) = 14.35$

$\mu = \sum xP(x) = 3.43$ customers

$\sigma = \sqrt{\sum x^2 P(x) - \mu^2} = \sqrt{14.35 - (3.43)^2} = 1.61$ customers

There is an average of 3.43 customers arriving per half-hour, with a standard deviation of 1.61 customers.

5.23

x	$P(x)$	$xP(x)$	x^2	$x^2P(x)$
0	0.10	0.00	0	0.00
2	0.45	0.90	4	1.80
5	0.30	1.50	25	7.50
10	0.15	1.50	100	15.00
		$\sum xP(x) = 3.9$		$\sum x^2P(x) = 24.30$

$\mu = \sum xP(x) = \$3.9$ million

$\sigma = \sqrt{\sum x^2 P(x) - \mu^2} = \sqrt{24.30 - (3.9)^2} = \$3.015$ million

The contractor is expected to make an average of \$3.9 million profit with a standard deviation of \$3.015 million.

5.25

x	$P(x)$	$xP(x)$	x^2	$x^2P(x)$
0	.5455	.0000	0	.0000
1	.4090	.4090	1	.4090
2	.0455	.0910	4	.1820
		$\sum xP(x) = .500$		$\sum x^2P(x) = .591$

$$\mu = \sum xP(x) = 0.500 \text{ person}$$
$$\sigma = \sqrt{\sum x^2P(x) - \mu^2} = \sqrt{0.591 - (0.500)^2} = 0.584 \text{ person}$$

<u>Section 5.4</u>

5.27 The parameters of the binomial distribution are n and p, which stand for the total number of trials and the probability of success, respectively.

5.29 **a.** This is an example of a binomial experiment because it satisfies all four conditions of a binomial experiment.

i. There are three identical trials (selections).
ii. Each trial has two outcomes: a red ball is drawn and a blue ball is drawn.
iii. The probability of drawing a red ball is 6/10 and that of a blue ball is 4/10. These probabilities add up to 1. The two probabilities remain constant for all draws because the draws are made with replacement.
iv. All draws are independent.

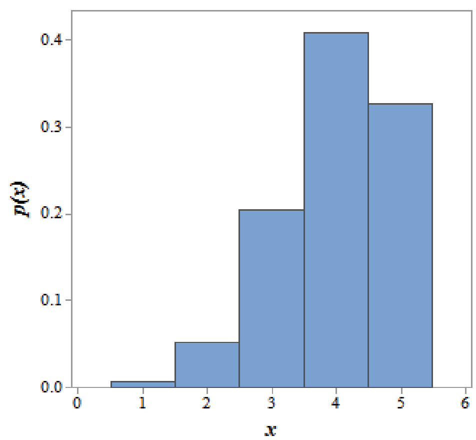
b. This is not a binomial experiment because the draws are not independent since the selections are made without replacement and, hence, the probabilities of drawing a red and a blue ball change with every selection.

c. This is an example of a binomial experiment because it satisfies all four conditions of a binomial experiment:

i There are a few identical trials (selection of households).
ii. Each trial has two outcomes: a household holds stocks and a household does not hold stocks.
iii. The probabilities of these two outcomes are .28 and .72, respectively. These probabilities add up to 1. The two probabilities remain the same for all selections because the number of households in NYC is very large.
iv. All households are independent.

5.31. **a.**

x	$P(x)$
0	0.0003
1	0.0064
2	0.0512
3	0.2048
4	0.4096
5	0.3277



b. $\mu = np = (5)(0.80) = 4.000$

$\sigma = \sqrt{npq} = \sqrt{(5)(0.80)(0.20)} = 0.894$

5.33 **a.** The random variable x can assume any of the values 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

b. $n = 10$ and $p = 0.70$

$P(6) = {}_nC_x p^x q^{n-x} = {}_{10}C_6(.70)^6(.30)^4 = 210(0.117649)(0.0081) = 0.2001$

The probability that exactly six say they never purchased organic fruit or vegetables in a random sample of 10 who answered the survey is about 0.2001.

5.35 $n = 15$ and $p = 0.30$

Let x denote the number men in a random sample of adult men aged 25–35 who say they will definitely consider a hair transplant if they can afford it.

a. $P(\text{at least } 4) = P(x \geq 4) = 1 - P(\text{less than } 4) = 1 - (0.0047 + 0.0305 + 0.0916 + 0.1700) = 0.7032$

b. $P(1 \text{ to } 3) = P(1 \leq x \leq 3) = P(1) + P(2) + P(3) = 0.0305 + 0.0906 + 0.1700 = 0.2921$

c. $P(\text{at most } 5) = P(x \leq 5) = P(0) + P(1) + P(2) + P(3) + P(4) + P(5)$

$= 0.0047 + 0.0305 + 0.0916 + 0.1700 + 0.2186 + 0.2061 = 0.7215$

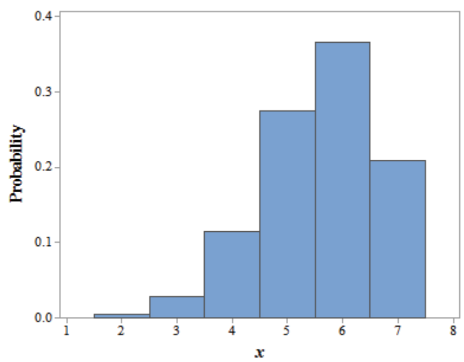
5.37 $n = 4$, $p = 0.63$, $q = 1 - p = 1 - 0.63 = 0.37$

a. $P(4) = {}_nC_x p^x q^{n-x} = {}_4C_4(0.63)^4(0.37)^0 = (1)(0.15752961)(1) = 0.1575$

b. $P(0) = {}_nC_x p^x q^{n-x} = {}_4C_4(0.63)^0(0.37)^4 = (1)(1)(0.01874161) = 0.0187$

5.39 **a.** $n = 7$ and $p = 0.80$

x	$P(x)$
0	0.0000
1	0.0004
2	0.0043
3	0.0287
4	0.1147
5	0.2753
6	0.3670
7	0.2097



$\mu = np = (7)(0.80) = 5.6$ customers

$\sigma = \sqrt{npq} = \sqrt{(7)(0.80)(0.20)} = 1.058$ customers

b. $P(\text{exactly } 4) = P(4) = 0.1147$

Section 5.5

5.41 The **hypergeometric probability distribution** gives probabilities for the number of successes in a fixed number of trials. It is used for sampling without replacement from a finite (small) population since the trials are not independent. Example 5-16 in the text is an example of the application of a hypergeometric probability distribution.

5.43 $N = 11$, $r = 4$, $N - r = 7$, and $n = 4$

a. $P(2) = \dfrac{{}_rC_x \; {}_{N-r}C_{n-x}}{{}_NC_n} = \dfrac{{}_4C_2 \; {}_7C_2}{{}_{11}C_4} = \dfrac{(6)(21)}{330} = 0.3818$

b. $P(4) = \dfrac{{}_rC_x \; {}_{N-r}C_{n-x}}{{}_NC_n} = \dfrac{{}_4C_4 \; {}_7C_0}{{}_{11}C_4} = \dfrac{(1)(1)}{330} = 0.0030$

c. $P(x \le 1) = P(0) + P(1) = \dfrac{{}_4C_0 \; {}_7C_4}{{}_{11}C_4} + \dfrac{{}_4C_1 \; {}_7C_3}{{}_{11}C_4} = \dfrac{(1)(35)}{330} + \dfrac{(4)(35)}{330} = 0.1061 + 0.4242$

$= 0.5303$

5.45 $N = 18$, $r = 11$, and $N - r = 7$, and $n = 4$

Let x be the number of unspoiled eggs in a random sample of 4, and r be the number of unspoiled eggs in 18.

a. $P(\text{exactly } 4) = P(4) = \dfrac{{}_{r}C_{x}\ {}_{N-r}C_{n-x}}{{}_{N}C_{n}} = \dfrac{{}_{11}C_{4}\ {}_{7}C_{0}}{{}_{18}C_{4}} = \dfrac{(330)(1)}{3060} = 0.1078$

b. $P(2 \text{ or fewer}) = P(x \le 2) = P(0) + P(1) + P(2)$

$= \dfrac{{}_{11}C_{0}\ {}_{7}C_{4}}{{}_{18}C_{4}} + \dfrac{{}_{11}C_{1}\ {}_{7}C_{3}}{{}_{18}C_{4}} + \dfrac{{}_{11}C_{2}\ {}_{7}C_{2}}{{}_{18}C_{4}} = \dfrac{(1)(35)}{3060} + \dfrac{(11)(35)}{3060} + \dfrac{(55)(21)}{3060}$

$= 0.0114 + 0.1258 + 0.3775 = 0.5147$

c. $P(\text{more than } 1) = P(x > 1) = P(2) + P(3) + P(4) = \dfrac{{}_{11}C_{2}\ {}_{7}C_{2}}{{}_{18}C_{4}} + \dfrac{{}_{11}C_{3}\ {}_{7}C_{1}}{{}_{18}C_{4}} + \dfrac{{}_{11}C_{4}\ {}_{7}C_{0}}{{}_{18}C_{4}}$

$= \dfrac{(55)(21)}{3060} + \dfrac{(165)(7)}{3060} + \dfrac{(330)(1)}{3060} = 0.3775 + 0.3775 + 0.1078 = 0.8628$

Section 5.6

5.47 The following three conditions must be satisfied to apply the Poisson probability distribution:

1) x is a discrete random variable.
2) The occurrences are random.
3) The occurrences are independent.

5.49 a. $P(x \le 1) = P(x = 0) + P(x = 1) =$

$\dfrac{(5)^0 e^{-5}}{0!} + \dfrac{(5)^1 e^{-5}}{1!} = \dfrac{(1)(0.00673795)}{1} + \dfrac{(5)(0.00673795)}{1} = 0.0067 + 0.0337 = 0.0404$

Note that the value of e^{-5} is obtained from Table II of Appendix B of the text.

b. $P(x = 2) = \dfrac{\lambda^x e^{-\lambda}}{x!} = \dfrac{(2.5)^2 e^{-2.5}}{2!} = \dfrac{(6.25)(0.08208500)}{2} = 0.2565$

5.51 $\lambda = 5.4$ shoplifting incidents per week

$P(\text{exactly three}) = P(3) = \dfrac{\lambda^x e^{-\lambda}}{x!} = \dfrac{(5.4)^3 e^{-5.4}}{3!} = \dfrac{(157.464)(0.00451658)}{6} = 0.1185$

5.53 $\lambda = 3.7$ reports of lost student ID cards per week

a. $P(\text{at most } 1) = P(x \le 1) = P(0) + P(1) = \dfrac{(3.7)^0 e^{-3.7}}{0!} + \dfrac{(3.7)^1 e^{-3.7}}{1!}$

$= \dfrac{(1)(0.02472353)}{1} + \dfrac{(3.7)(0.02472353)}{1} = 0.0247 + 0.0915 = 0.1162$

b. i. $P(1 \text{ to } 4) = P(1 \le x \le 4) = P(1) + P(2) + P(3) + P(4)$
$= 0.0915 + 0.1692 + 0.2087 + 0.1931 = 0.6625$

ii. $P(\text{at least } 6) = P(x \ge 6)$
$= P(6) + P(7) + P(8) + P(9) + P(10) + P(11) + P(12) + P(13)$
$= 0.0881 + 0.0466 + 0.0215 + 0.0089 + 0.0033 + 0.0011 + 0.0003 + 0.0001$
$= 0.1699$

iii. $P(\text{at most } 3) = P(x \le 3) = P(0) + P(1) + P(2) + P(3)$
$= 0.0247 + 0.0915 + 0.1692 + 0.2087 = 0.4941$

5.55 Let x be the number of telemarketing calls received by Fred Johnson during a given week. $\lambda = 20$ calls per week

a. $P(\text{exactly } 13) = P(13)$

$$= \frac{\lambda^x e^{-\lambda}}{x!} = \frac{(20)^{13} e^{-20}}{13!} = \frac{(81{,}920{,}000{,}000{,}000{,}000)(0.00000000206115)}{6{,}227{,}020{,}800} = 0.0271$$

b. i. $P(12 \text{ to } 15) = P(12 \le x \le 15) = P(12) + P(13) + P(14) + P(15)$
$= 0.0176 + 0.0271 + 0.0387 + 0.0516 = 0.1350$

ii. $P(\text{fewer than } 9) = P(x < 9) = P(0) + P(1) + P(2) + P(3) + P(4) + P(5) + P(6) + P(7) + P(8)$
$= 0.0000 + 0.0000 + 0.0000 + 0.0000 + 0.0000 + 0.0001 + 0.0002 + 0.0005 + 0.0013 = 0.0021$

5.57 Let x denote the number of accidents on a given day. $\lambda = 0.8$ accident per day

a. $P(\text{none}) = P(0) = \dfrac{\lambda^x e^{-\lambda}}{x!} = \dfrac{(.8)^0 e^{-0.8}}{0!} = \dfrac{(1)(0.44932896)}{1} = 0.4493$

b.

x	$P(x)$
0	0.4493
1	0.3595
2	0.1438
3	0.0383
4	0.0077
5	0.0012
6	0.0002

c. $\mu = \lambda = 0.8, \sigma^2 = \lambda = 0.8, \text{ and } \sigma = \sqrt{\lambda} = \sqrt{0.8} = 0.894$

5.59 Let x denote the number of cars passing through a school zone exceeding the speed limit. The average number of cars speeding by at least ten miles per hour is 20 percent. Thus, $\lambda = (0.20)(100) = 20$ cars per 100.

a. $P(\text{exactly } 25) = P(25) = \dfrac{\lambda^x e^{-\lambda}}{x!} = \dfrac{(20)^{25} e^{-20}}{25!} = 0.0446$

b. i. $P(\text{at most } 8) = P(x \le 8)$
$= P(0) + P(1) + P(2) + P(3) + P(4) + P(5) + P(6) + P(7) + P(8)$
$= 0.0000 + 0.0000 + 0.0000 + 0.0000 + 0.0000 + 0.0001 + 0.0002 + 0.0005 + 0.0013 = 0.0021$

ii. $P(15 \text{ to } 20) = P(15 \le x \le 20)$
$= P(15) + P(16) + P(17) + P(18) + P(19) + P(20)$
$= 0.0516 + 0.0646 + 0.0760 + 0.0844 + 0.0888 + 0.0888 = 0.4542$

iii. $P(\text{at least } 30) = P(x \ge 30)$
$= P(30) + P(31) + P(32) + P(33) + P(34) + P(35) + P(36) + P(37) + P(38) + P(39) + P(x \ge 40)$
$= 0.0083 + 0.0054 + 0.0034 + 0.0020 + 0.0012 + 0.0007 + 0.0004 + 0.0002 + 0.0001 + 0.0001 + 0.0000 = 0.0218$

Supplementary Exercises

5.61

x	$P(x)$	$xP(x)$	x^2	$x^2P(x)$
0	0.13	0.00	0	0.00
1	0.28	0.28	1	0.28
2	0.30	0.60	4	1.20
3	0.17	0.51	9	1.53
4	0.08	0.32	16	1.28
5	0.04	0.20	25	1.00
		$\sum xP(x) = 1.91$		$\sum x^2P(x) = 5.29$

$\mu = \sum xP(x) = 1.91$ root canals

$\sigma = \sqrt{\sum x^2P(x) - \mu^2} = \sqrt{5.29 - (1.91)^2} = 1.281$ root canals

Dr. Sharp performs an average of 1.91 root canals on Monday.

5.63 Let x denote the number of machines that are broken down at a given time. Assuming machines are independent, x is a binomial random variable with $n = 8$ and $p = 0.04$.

a. $P(\text{exactly } 8) = P(8) = {}_nC_x p^x q^{n-x} = {}_8C_8 (0.04)^8(0.96)^0 = (1)(0.000000000007)(1) \approx 0.0000$

b. $P(\text{exactly } 2) = P(2) = {}_nC_x p^x q^{n-x} = {}_8C_2 (0.04)^2(0.96)^6 = (28)(0.0016)(0.78275779) = 0.0351$

c. $P(\text{none}) = P(0) = {}_nC_x p^x q^{n-x} = {}_8C_0 (0.04)^0(0.96)^8 = (1)(1)(0.72138958) = 0.7214$

5.65 Let x denote the number of defective motors in a random sample of 20. Then x is a binomial random variable with $n = 20$ and $p = 0.05$.

a. $P(\text{shipment accepted}) = P(x \le 2) = P(0) + P(1) + P(2) = 0.3585 + 0.3774 + 0.1887 = 0.9246$

b. $P(\text{shipment rejected}) = 1 - P(\text{shipment accepted}) = 1 - 0.9246 = 0.0754$

5.67 Let x denote the number of tax returns in a random sample of 3 that contain errors. Then x is a hypergeometric random variable with $N = 12$, $r = 2$, $N - r = 10$, and $n = 3$.

a. $P(\text{exactly }1) = P(1) = \dfrac{{}_rC_x \; {}_{N-r}C_{n-x}}{{}_NC_n} = \dfrac{{}_2C_1 \; {}_{10}C_2}{{}_{12}C_3} = \dfrac{(2)(45)}{220} = 0.4091$

b. $P(\text{none}) = P(0) = \dfrac{{}_rC_x \; {}_{N-r}C_{n-x}}{{}_NC_n} = \dfrac{{}_2C_0 \; {}_{10}C_3}{{}_{12}C_3} = \dfrac{(1)(120)}{220} = 0.5455$

c. $P(\text{exactly }2) = P(2) = \dfrac{{}_rC_x \; {}_{N-r}C_{n-x}}{{}_NC_n} = \dfrac{{}_2C_2 \; {}_{10}C_1}{{}_{12}C_3} = \dfrac{(1)(10)}{220} = 0.0455$

5.69 $\lambda = 1.4$ airplanes per hour

a. $P(\text{none}) = P(0) = \dfrac{\lambda^x e^{-\lambda}}{x!} = \dfrac{(1.4)^0 e^{-1.4}}{0!} = \dfrac{(1)(0.24659696)}{1} = 0.2466$

b.

x	$P(x)$
0	0.2466
1	0.3452
2	0.2417
3	0.1128
4	0.0395
5	0.0111
6	0.0026
7	0.0005
8	0.0001

Advanced Exercises

5.71 Let x be a random variable that denotes the gain you have from this game. There are 36 different outcomes for two dice: (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2),…, (6, 6).

$P(sum = 2) = P(sum = 12) = \dfrac{1}{36}$ $\quad P(sum = 3) = P(sum = 11) = \dfrac{2}{36}$

$P(sum = 4) = P(sum = 10) = \dfrac{3}{36}$ $\quad P(sum = 9) = \dfrac{4}{36}$

$P(x = 20) = P(\text{you win})$
$= P(sum = 2) + P(sum = 3) + P(sum = 4) + P(sum = 9) + P(sum = 10) + P(sum = 11) + P(sum = 12)$
$= \dfrac{1}{36} + \dfrac{2}{36} + \dfrac{3}{36} + \dfrac{4}{36} + \dfrac{3}{36} + \dfrac{2}{36} + \dfrac{1}{36} = \dfrac{16}{36} = 0.4444$

$P(x = -20) = P(\text{you lose}) = 1 - P(\text{you win}) = 1 - \dfrac{16}{36} = \dfrac{20}{36} = 0.5556$

x	$P(x)$	$xP(x)$
20	0.4444	8.89
–20	0.5556	–11.11
		$\sum xP(x) = –2.22$

The value of $\sum xP(x) = -2.22$ indicates that your expected "gain" is –\$2.22, so you should not accept this offer. This game is not fair to you since you are expected to lose an average of \$2.22 per play.

5.73 Let x denote the number of bearings in a random sample of 15 that do not meet the required specifications. Then x is a binomial random variable with $n = 15$ and $p = 0.10$.

a. $P(\text{production suspended}) = P(x > 2) = 1 - P(x \le 2) = 1 - [P(0) + P(1) + P(2)]$
$= 1 - (0.2059 + 0.3432 + 0.2669) = 0.1841$

b. The 15 bearings are sampled without replacement, which normally requires the use of the hypergeometric distribution. We are assuming that the population from which the sample is drawn is so large that each time a bearing is selected, the probability of being defective is 0.10. Thus, the sampling of 15 bearings constitutes 15 independent trials, so that the distribution of x is approximately binomial.

5.75 **a.** It's easier to do part b first.

b. Let x be a random variable that denotes the score after guessing on one multiple choice question.

x	$P(x)$	$xP(x)$
1	0.25	0.250
$-\frac{1}{2}$	0.75	-0.375
		$\sum xP(x) = -0.125$

So a student decreases his expected score by guessing on a question, if he has no idea what the correct answer is.
Back to part a.: Guessing on 12 questions lowers the expected score by (12)(–0.125) = –1.5, so the expected score for 38 correct answers and 12 random guesses is (38)(1) + (12)(–0.125) = 36.5.

c. If the student can eliminate one of the wrong answers, then we get the following:

x	$P(x)$	$xP(x)$
1	$\frac{1}{3}$	$\frac{1}{3}$
$-\frac{1}{2}$	$\frac{2}{3}$	$-\frac{1}{3}$
s		$\sum xP(x) = 0$

So in this case guessing does not affect the expected score.

5.77 For each game, let x = amount you win

Game I:

Outcome	x	$P(x)$	$xP(x)$	$x^2P(x)$
Head	3	0.50	1.50	4.50
Tail	–1	0.50	–0.50	0.50
			$\sum xP(x) = 1.00$	$\sum x^2P(x) = 5.00$

$$\mu = \sum xP(x) = \$1.00$$
$$\sigma = \sqrt{\sum x^2P(x) - \mu^2} = \sqrt{5-(1)^2} = \$2.00$$

Game II:

Outcome	x	$P(x)$	$xP(x)$	$x^2P(x)$
First ticket	300	1/500	0.60	180
Second ticket	150	1/500	0.30	45
Neither	0	498/500	0.00	0
			$\sum xP(x) = 0.90$	$\sum x^2P(x) = 225$

$$\mu = \sum xP(x) = \$.90$$
$$\sigma = \sqrt{\sum x^2P(x) - \mu^2} = \sqrt{225-(.90)^2} = \$14.97$$

Game III:

Outcome	x	$P(x)$	$xP(x)$	$x^2P(x)$
Head	1,000,002	0.50	500,001	$5 \ x\ 10^{11}$
Tail	–1,000,000	0.50	–500,000	$5 \ x\ 10^{11}$
			$\sum xP(x) = 1.00$	$\sum x^2P(x) = 10^{12}$

$$\mu = \sum xP(x) = \$1.00$$
$$\sigma = \sqrt{\sum x^2P(x) - \mu^2} = \sqrt{10^{12}-(1)^2} = \$1,000,000$$

Game I is preferable to Game II because the mean for Game I is greater than the mean for Game II. Although the mean for Game III is the same as Game I, the standard deviation for Game III is extremely high, making it very unattractive to a risk-adverse person. Thus, for most people, Game I is the best and, probably, Game III is the worst (due to its very high standard deviation).

5.79 There are a total of 27 outcomes for the game which can be determined utilizing a tree diagram. Three outcomes are favorable to Player A, 18 outcomes are favorable to Player B, and 6 outcomes are favorable to Player C. Player B's expected winnings are $\sum xP(x) = (0)(9/27) + (1)(18/27) = 0.67$ or 67¢. Player C's expected winnings are also 67¢: $\sum xP(x) = (0)(21/27) + (3)(6/27) = 0.67$. Since Player A has a probability of winning of 3/27, this player should be paid \$6 for winning so that $\sum xP(x) = (0)(24/27) + (6)(3/27) = 0.67$ or 67¢.

Self-Review Test

1. A variable whose value is determined by the outcome of a random experiment is called a **random variable**. A random variable that assumes countable values is called a **discrete random variable**. The number of cars owned by a randomly selected individual is an example of a discrete random variable. A random variable that can assume any value contained in one or more intervals is called a **continuous random variable**. An example of a continuous random variable is the amount of time taken by a randomly selected student to complete a statistics exam.

3. a

5. An experiment that satisfies the following four conditions is called a **binomial experiment**.

i. There are n identical trials. In other words, the given experiment is repeated n times, where n is a positive integer. All these repetitions are performed under identical conditions.

ii. Each trial has two and only two outcomes. These outcomes are usually called a *success* and a *failure*.

iii. The probability of success is denoted by p and that of failure by q, and $p + q = 1$. The probability of p and q remain constant for each trial.

iv. The trials are independent. In other words, the outcome of one trial does not affect the outcome of another trial.

An example of a binomial experiment is flipping a coin many times and observing whether the outcome of each flip is a head or a tail.

7. a

9. a

11. A hypergeometric probability distribution is used to find probabilities for the number of successes in a fixed number of trials, when the trials are not independent (such as sampling without replacement from a finite population.) Example 5-16 is an example of a hypergeometric probability distribution.

13. The following three conditions must be satisfied to apply the Poisson probability distribution.
1) x is a discrete random variable.
2) The occurrences are random.
3) The occurrences are independent.

15. $n = 12$ and $p = 0.60$

a. i. $P(\text{exactly } 8) = P(8) = {}_nC_x p^x q^{n-x} = {}_{12}C_8(0.60)^8(0.40)^4 =$
$(495)(0.01679616)(0.0256) = 0.2128$

ii. $P(\text{at least } 6) = P(x \geq 6)$
$= P(6) + P(7) + P(8) + P(9) + P(10) + P(11) + P(12)$
$= 0.1766 + 0.2270 + 0.2128 + 0.1419 + 0.0639 + 0.0174 + 0.0022$
$= 0.8418$

iii. $P(\text{less than } 4) = P(x < 4) = P(0) + P(1) + P(2) + P(3)$
$= 0.0000 + 0.0003 + 0.0025 + 0.0125 = 0.0153$

b.

x	$P(x)$	x	$P(x)$
0	0.0000	7	0.2270
1	0.0003	8	0.2128
2	0.0025	9	0.1419
3	0.0125	10	0.0639
4	0.0420	11	0.0174
5	0.1009	12	0.0022
6	0.1766		

$\mu = np = 12(0.60) = 7.2$ adults
$\sigma = \sqrt{npq} = \sqrt{12(0.60)(0.40)} = 1.697$ adults

17. $\lambda = 10$ red light runners are caught per day.
Let x = number of drivers caught during rush hour on a given weekday.

a. i. $P(14) = \dfrac{\lambda^x e^{-\lambda}}{x!} = \dfrac{(10)^{14} e^{-10}}{14!} = \dfrac{(100{,}000{,}000{,}000{,}000)(0.0000453999)}{87{,}178{,}291{,}200} = 0.0521$

ii. $P(\text{at most } 7) = P(x \leq 7)$
$= P(0) + P(1) + P(2) + P(3) + P(4) + P(5) + P(6) + P(7) = 0.0000 + 0.0005 +$
$0.0023 + 0.0076 + 0.0189 + 0.0378 + 0.0631 + 0.0901 = 0.2203$

iii. $P(13 \text{ to } 18) = P(13 \le x \le 18)$
$= P(13) + P(14) + P(15) + P(16) + P(17) + P(18)$
$= 0.0729 + 0.0521 + 0.0347 + 0.0217 + 0.0128 + 0.0071 = 0.2013$

b.

x	$P(x)$	x	$P(x)$	x	$P(x)$
0	0.0000	9	0.1251	17	0.0128
1	0.0005	10	0.1251	18	0.0071
2	0.0023	11	0.1137	19	0.0037
3	0.0076	12	0.0948	20	0.0019
4	0.0189	13	0.0729	21	0.0009
5	0.0378	14	0.0521	22	0.0004
6	0.0631	15	0.0347	23	0.0002
7	0.0901	16	0.0217	24	0.0001
8	0.1126				