National University of Computer and Emerging Sciences, Lahore Campus

Course Name:	Probability & Statistics	Course Code:	MT2005/2009
Degree Program:	BS CS/SE	Semester:	Spring 2022
Exam Duration:	3 Hours	Total Marks:	100
Paper Date:	16-06-2022	Weight	50
Section:	ALL	Page(s):	6
Exam Type:	Final Term	Time:	1:00pm - 4:00pm

Student : Name: _____ Roll No. _____ Section: _____

Instruction/Notes:
1. Attempt all the questions on the answer book and show proper working.
2. Use of Scientific calculator is allowed but Exchange of calculators or use of programmable calculators is not allowed.
3. Students are not allowed to write anything on the question paper except roll number.
4. If you have any ambiguity in the data then do not ask anything from invigilator, just make an assumption and continue your paper.

Q 1.

(a) **Points (05)**

The following distribution shows Kilowatt-Hours of Electricity used in one month by 62 residential consumers in a certain locality of Lahore:

Consumption in KWH	5-24	25-44	45-64	65-84	85-104
No. of consumers	4	6	11	22	14

Estimate Median consumption in KWH.

(b) **Points (10)**

Two candidates X and Y in BS exam obtained the following marks in six subjects. By using coefficient of variation, check that which candidate perform more consistently?

Paper	I	II	III	IV	V	VI
X	58	49	76	80	47	72
Y	39	38	86	72	75	69

(c) **Points (10)**

In an experiment to measure the stiffness of a spring, the length of the spring under different loads was measured as follows:

X = Load (lb)	3	5	6	9	10	12
Y = Length (in)	10	12	15	17	19	22

Find the least square regression equation for predicting the weight (load), given the length of the spring.

Q 2.

(a) **Points (10)**

Due to an Internet configuration error, packets sent from New York to Los Angeles are routed through El Paso with probability 3/4. Given that a packet is routed through El Paso, suppose it has conditional probability 1/3 of being dropped. Given that a packet is not routed through El Paso, suppose it has conditional probability 1/4 of being dropped. Find the following probabilities:

(i) The probability that a packet is dropped.

(ii) Find the conditional probability that a packet is routed through El Paso given that it is not dropped.

(b) **Points (05)**

Is it possible for A and B to be independent events yet satisfy $A = B$? Justify your answer.
Hint: Apply the definition of independence of two events to get the required path.

(a) Points

The number of hits at a Web site in any time interval is a Poisson random variable. A particular sit has on average $\lambda = 2$ hits per second. What is the probability that there are no hits in an interval of 0.25 seconds? What is the probability that there are no more than two hits in an interval of one second?

(b) Points (5)

Let N be the number of times a computer polls a terminal until the terminal has a message ready for transmission. If we suppose that the terminal produces messages according to a sequence of independent Bernoulli trials, the N has a geometric distribution. Find the mean of N.

(c) Points (10)

Continuous random variable X has $E[X] = 3$ and $Var[X] = 9$. Find probability density function (PDF) $f_X(x)$, if X has a uniform PDF.

Q 4.

(a) Points (10)

For a random variable X, let $Y = aX + b$. Show that if $a > 0$ then $\rho_{X,Y} = 1$. Also show that if $a < 0$, then $\rho_{X,Y} = -1$.

Hint: (i) Use basic properties of expectation to get the required result.

(ii) $\rho_{X,Y} = \frac{Cov(X,Y)}{\sqrt{V(X)V(Y)}}$

(b) Points (10)

Random variable X and Y have the joint PMF

$$P_{X,Y}(x,y) = \begin{cases} cxy & x = 1, 2, 4; \quad y = 1, 3, \\ 0 & \text{otherwise.} \end{cases}$$

(a) What is the value of the constant c?

(b) What is $P[Y < X]$?

Q 5.

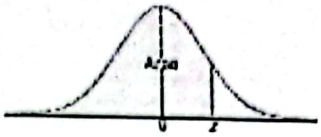
(a) Points (10)

A publishing company has just published a new college textbook. Before the company decides the price at which to sell this textbook, it wants to know the average price of all such textbooks in the market. The research department at the company took a sample of 25 comparable textbooks and collected information on their prices. This information produced a mean price of \$145 and standard deviation is \$35 for this sample. It is known that the population of such prices is normal. Is it reasonable to conclude that mean price of all such college textbooks is at most \$150? Draw your conclusions at 1% level of significance.

(b) Points (10)

Suppose we want to compare the average yearly income in Providence and Boston, two neighboring cities in New England. It is known from experience that the variance of yearly incomes in Providence is \$40000 and the variance for yearly incomes in Boston is \$90000. A random sample of 40 families was taken in Providence, yielding a mean yearly income of \$47000, while a random sample of 50 families was taken in Boston, yielding a mean yearly income of \$52000. At the $\alpha = .01$ significance level, test whether or not there is a significant different in average yearly income between the two cities.

Table A.3 Areas under the Normal Curve



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641