National University of Computer and Emerging Sciences, Lahore Campus				
	Course:	Numerical Computing	Course Code:	CS 325
	Program:	BS CS	Semester:	Fall 2020
	Duration:	3 hours	Total Marks:	100
	Paper Date:	February 20; 2021	Weight	50%
	Section:	All	Page(s):	02
	Exam:	Final Term		
Instruction/Notes:	Attempt All Questions.			

Q1. Probability distribution function values of a normal distribution are given below: **Points (15)**

x	0.2	0.5	1.0
$f(x)$	0.39104	0.33322	0.24197

Use Newton's Divided Difference table to find the interpolating polynomial based on the given data. Use obtained polynomial to find $f(1.5)$. Also, use obtained polynomial to find local numerical Max/Min if any.

Q2. The arc length of the curve $y = f(x)$ over the interval $a \le x \le b$ is $Length = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$.

Use the function $f(x) = e^{-x}$ for $0 \le x \le 1$ to approximate the arc length using Composite Trapezoidal rule with $N = 5$. **Points (10)**

Q3. Approximation to the integral $\int_0^1 e^{-x^2}\, dx$ having exact solution 0.6321205588, are **Points (5)**

$I_{0.25} = 0.632134175$, $I_{0.125} = 0.632121415$. Use Romberg Integration based on Simpson's rule to find $O(h^6)$ approximation. Compare your results with exact solution.

Note: Throughout the computation take at least 8 decimal results.

Q4. Using Newton Raphson's method, establish the formula $x_{n+1} = \frac{1}{3}\left(2x_n + \frac{N}{x_n^2}\right)$ to calculate the cube root of N. Hence find cube root of 13 correct to three decimals places. **Points (10)**

Q5. The concentration of salt y in a homemade soap maker is given as a function of time by

$$\frac{dy}{dt} = 1 - 0.5\,y - 0.1\,y\,t$$

At the initial time $t = 0$, the salt concentration in the tank is 5 g/L. Use Taylor series method with a step size $h = 1.5$ to find the salt concentration after 1.5 minutes? **Points (15)**

Note: In series solution you must have at least 5 terms.

Q6. Use Picard method of successive approximation to find the second approximate solution (general series solution) of the following differential equation. Use obtained series solution to find residual function for the given differential equation. Moreover, find residual error at $x = 0.1$ and 0.2, and make conclusion on the basis of obtained results.

$$\frac{dy}{dx} = e^x + y^2,\ y(0) = 0$$

Points (15)

Note: You can find residual function by putting back obtained series solution to given differential equation.

Q7. Use Runge-Kutta Method of order 3 to find the solution of the second order ODE at $x = 0.1$.

$$\frac{d^2y}{dx^2} = y^3, \quad y(0) = 10, y'(0) = 5$$

Points (15)

Q8. Solve the following Boundary Value Problem by the finite difference method. **Points (15)**

$$x\frac{d^2y}{dx^2} + y = 0, y(1) = 1, y(2) = 2 \text{ with } h = 0.25$$