Chapter 7
Sampling Distributions

Section 7.1

7.1 The probability distribution of the population data is called the **population distribution**. Table 7.2 in the text provides an example of such a distribution. The probability distribution of a sample statistic is called its **sampling distribution**. Table 7.5 in the text provides an example of the sampling distribution of the sample mean.

7.3 **Nonsampling errors** are errors that may occur in the collection, recording, and tabulation of data. Example 7–1 in the text exhibits nonsampling error. Nonsampling errors can occur in both a sample survey and a census.

7.5

x	$P(x)$	$xP(x)$	x^2	$x^2P(x)$
70	0.20	14.00	4900	980.00
78	0.20	15.60	6084	1216.80
80	0.40	32.00	6400	2560.00
95	0.20	19.00	9025	1805.00
		$\Sigma xP(x) = 80.60$		$\Sigma x^2P(x) = 6561.80$

$\mu = \Sigma xP(x) = 80.60$

$\sigma = \sqrt{\sum x^2 P(x) - \mu^2} = \sqrt{6561.80 - (80.60)^2} = 8.09$

7.6 a.

x	$P(x)$
15	1/6 = 0.167
21	1/6= 0.167
25	1/6= 0.167
28	1/6= 0.167
53	1/6= 0.167
55	1/6= 0.167

b.

Sample	$\bar{x}$	$\bar{x}$	$P(\bar{x})$
55, 53, 28, 25	40.25	22.25	1/15 = 0.067
55, 53, 28, 21	39.25	28.50	1/15 = 0.067
55, 53, 28, 15	37.75	29.00	1/15 = 0.067
55, 53, 25, 21	38.50	29.25	1/15 = 0.067
55, 53, 25, 15	37.00	29.75	1/15 = 0.067
55, 53, 21, 15	36.00	30.25	1/15 = 0.067
55, 28, 25, 21	32.25	30.75	1/15 = 0.067
55, 28, 25, 15	30.75	31.75	1/15 = 0.067
55, 28, 21, 15	29.75	32.25	1/15 = 0.067
55, 25, 21, 15	29.00	36.00	1/15 = 0.067
53, 28, 25, 21	31.75	37.00	1/15 = 0.067
53, 28, 25, 15	30.25	37.75	1/15 = 0.067
53, 28, 21, 15	29.25	38.50	1/15 = 0.067
53, 25, 21, 15	28.50	39.25	1/15 = 0.067
28, 25, 21, 15	22.25	40.25	1/15 = 0.067

c. Answers will vary. Sample answer is given.
The mean for the population data is $\mu = (55 + 53 + 28 + 25 + 21 + 15)/6 = 197/6 = 32.83$
Suppose the random sample of four family members includes the observations: 55, 53, 28, and 21.
The mean for this sample is $\bar{x} = (55 + 53 + 28 + 21)/4 = 157/4 = 39.25$
Sampling error = $\bar{x} - \mu = 39.25 - 32.83 = 6.42$

Section 7.2

7.7 **a.** Mean of $\bar{x} = \mu_{\bar{x}} = \mu$

b. Standard deviation of $\bar{x} = \sigma_{\bar{x}} = \sigma/\sqrt{n}$ where σ is the population standard deviation and n is the sample size.

7.9 An estimator is **consistent** when its standard deviation decreases as the sample size is increased. The sample mean $\bar{x}$ is a consistent estimator of μ because its standard deviation decreases as the sample size increases. As n increases, $\sqrt{n}$ increases, and, consequently, the value of $\sigma_{\bar{x}} = \sigma/\sqrt{n}$ decreases.
Since $\sigma_{\bar{x}} = \sigma/\sqrt{n}$, as n increases, $\sigma_{\bar{x}}$ decreases.

7.11 $\mu = 60$ and $\sigma = 10$

a. $\mu_{\bar{x}} = \mu = 60$ and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 10/\sqrt{18} = 2.357$

b. $\mu_{\bar{x}} = \mu = 60$ and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 10/\sqrt{90} = 1.054$

7.13 $\mu = 125$ and $\sigma = 36$

a. Since $\sigma_{\bar{x}} = \sigma/\sqrt{n}$, $n = (\sigma/\sigma_{\bar{x}})^2 = (36/3.6)^2 = 100$

b. $n = (\sigma/\sigma_{\bar{x}})^2 = (36/2.25)^2 = 256$

7.15 $\mu = \$482{,}000$, $\sigma = \$90{,}000$, and $n = 200$

$\mu_{\bar{x}} = \mu = \$482{,}000$ and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 90{,}000/\sqrt{200} \approx \$6363.96$

7.17 a.

$\bar{x}$	$P(\bar{x})$	$\bar{x}P(\bar{x})$
76.00	0.20	15.200
76.67	0.10	7.667
79.33	0.10	7.933
81.00	0.10	8.100
81.67	0.20	16.334
84.33	0.20	16.866
85.00	0.10	8.500
		$\sum \bar{x}P(\bar{x}) = 80.60$

$\sum \bar{x}P(\bar{x}) = 80.60$ is the same value found in Exercise 7.6 for μ.

b.

$\bar{x}$	$P(\bar{x})$	$\bar{x}P(\bar{x})$	$\bar{x}^2$	$\bar{x}^2P(\bar{x})$
76.00	0.20	15.200	5776.0000	1155.200
76.67	0.10	7.667	5878.2889	587.829
79.33	0.10	7.933	6293.2489	629.325
81.00	0.10	8.100	6561.0000	656.100
81.67	0.20	16.334	6669.9889	1333.998
84.33	0.20	16.866	7111.5489	1422.310
85.00	0.10	8.500	7225.0000	722.500
		$\sum \bar{x}P(\bar{x}) = 80.60$		$\sum \bar{x}^2P(\bar{x}) = 6507.262$

$$\sigma_{\bar{x}} = \sqrt{\sum \bar{x}^2 P(\bar{x}) - \mu_{\bar{x}}^2} = \sqrt{6507.262 - (80.60)^2} = 3.302$$

c. $\sigma/\sqrt{n} = 8.09/\sqrt{3} = 4.67$ is not equal to $\sigma_{\bar{x}} = 3.30$ in this case because $n/N = 3/5 = 0.60 > 0.05$.

d. $$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}} = \frac{8.09}{\sqrt{3}}\sqrt{\frac{5-3}{5-1}} = 3.302$$

Section 7.3

7.19 The **central limit theorem** states that for a large sample, the sampling distribution of the sample mean is approximately normal, irrespective of the shape of the population distribution. Furthermore, $\mu_{\bar{x}} = \mu$ and $\sigma_{\bar{x}} = \sigma/\sqrt{n}$, where μ and σ are the population mean and standard deviation, respectively. A sample size of 30 or more is considered large enough to apply the central limit theorem to $\bar{x}$.

7.21 **a.** Slightly skewed to the right because $n < 30$ and the central limit theorem does not apply.

b. Approximately normal because $n \geq 30$ and the central limit theorem applies.

c. Close to normal with a slight skew to the right because $n < 30$ and the central limit theorem does not apply.

7.23 $\mu = 7.7$ minutes, $\sigma = 2.1$ minutes, and $n = 16$

$\mu_{\bar{x}} = \mu = 7.7$ minutes and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 2.1/\sqrt{16} = 0.525$ minute

The sampling distribution of $\bar{x}$ is normal because the population is normally distributed.

7.25 $\mu = 3.02$, $\sigma = 0.29$, $N = 5540$ and $n = 48$

$\mu_{\bar{x}} = \mu = 3.02$

Since $n/N = 48/5540 = 0.009 < 0.05$, $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 0.29/\sqrt{48} = 0.042$.

The sampling distribution of $\bar{x}$ is approximately normal because the population is approximately normally distributed.

7.27 $\mu = 170.4$ min, $\sigma = 20$ min, and $n = 400$

$\mu_{\bar{x}} = \mu = 170.4$ min and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 20/\sqrt{400} = 1$ min

The sampling distribution of $\bar{x}$ is approximately normal. We do not need to know the shape of the population distribution in order to make this conclusion because the sample size is large ($n \geq 30$) and the central limit theorem applies.

Section 7.4

7.29 $P(\mu - 1.50\sigma_{\bar{x}} \leq \bar{x} \leq \mu + 1.50\sigma_{\bar{x}}) = P(-1.50 \leq z \leq 1.50) = P(z \leq 1.50) - P(z \leq -1.50)$

$= 0.9332 - 0.0668 = 0.8664$ or 86.64%

7.31 $\mu = 75$, $\sigma = 14$, and $n = 20$

$\mu_{\bar{x}} = \mu = 75$ and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 14/\sqrt{20} = 3.13049517$

a. For $\bar{x} = 68.5$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (68.5 - 75)/3.13049517 = -2.08$

For $\bar{x} = 77.3$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (77.3 – 75)/3.13049517 = 0.73

$P(68.5 < \bar{x} < 77.3) = P(-2.08 < z < 0.73) = P(z < 0.73) - P(z < -2.08) = 0.7673 - 0.0188 = 0.7485$

b. For $\bar{x} = 72.4$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (72.4 – 75)/3.13049517 = –0.83

$P(\bar{x} < 72.4) = P(z < -0.83) = 0.2033$

7.33 $\mu = 90$, $\sigma = 18$, and $n = 64$

$\mu_{\bar{x}} = \mu = 90$ and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 18/\sqrt{64} = 2.25$

a. For $\bar{x} = 82.3$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (82.3 – 90)/2.25 = –3.42

$P(\bar{x} < 82.3) = P(z < -3.42) = 0.0003$

b. For $\bar{x} = 86.7$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (86.7 – 90)/2.25 = –1.47

$P(\bar{x} > 86.7) = P(z > -1.47) = 1 - P(z \le -1.47) = 1 - 0.0708 = 0.9292$

7.35 μ = \$100,990, σ = \$30,000, and $n = 400$

$\mu_{\bar{x}} = \mu = \$100{,}990$ and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 30{,}000/\sqrt{400} = \$1500$

a. For $\bar{x}$ = \$103,090: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (103{,}090 - 100{,}990)/1500 = 1.40$

$P(\bar{x} > \$103{,}090) = P(z \ge 1.40) = 1 - P(z \le 1.40) = 1 - 0.9192 = 0.0808$

b. For $\bar{x}$ = \$98,500: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (98,500 – 100,990)/15,000 = –1.66

$P(\bar{x} < \$98{,}500) = P(z < -1.66) = 0.0485$

c. For $\bar{x}$ = \$99,780: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (99,780 – 100,990)/15,000 = –0.81

For $\bar{x}$ = \$103,830: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (103,830 – 100,990)/15,000 = 1.89

$P(\$99{,}780 \le \bar{x} \le \$103{,}830) = P(-0.81 \le z \le 1.89) = P(z \le 1.89) - P(z \le -0.81) = 0.9706 - 0.2090 = 0.7616$

7.37 μ = 170.4 min, σ = 20 min, and $n = 400$

$\mu_{\bar{x}} = \mu = 170.4$ min and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 20/\sqrt{400} = 1$ min

a. For $\bar{x} = 171$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (171 – 170.4)/1= 0.6

For $\bar{x} = 173$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (173 – 170.4)/1= 2.6

$P(171 < \bar{x} < 173) = P(0.6 < z < 2.6) = P(z < 2.6) - P(z < 0.6) = 0.9953 - 0.7257 = 0.2696$

b. For $\bar{x} = 172$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (172 – 170.4)/1 = 1.6

$P(\bar{x} > 172) = P(z > 1.6) = 1 - P(z \le 1.6) = 1 - 0.9452 = 0.0548$

c. For $\bar{x} = 168$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (168 – 170.4)/1 = –2.4

$P(\bar{x} \le 168) = P(z \le -2.4) = 0.0082$

7.39 μ = 202,284, σ = 36,000, and $n = 144$

$\mu_{\bar{x}} = \mu = 202{,}284$ and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 36{,}000/\sqrt{144} = 3000$

a. For $\bar{x} = 198{,}000$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (198,000 – 202,284)/3,000 = –1.43
$P(\bar{x} < 198{,}000) = P(z < -1.43) = 0.0764$

b. For $\bar{x} = 206{,}000$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (206,000 – 202,284)/3,000 = 1.24
$(\bar{x} > 206{,}000) = P(z > 1.24) = 1 - P(z \le 1.24) = 1 - 0.8925 = 0.1075$

c. For $\bar{x} = 200{,}000$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (200,000 – 202,284)/3,000 = –0.76
For $\bar{x} = 205{,}000$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (205,000 – 202,284)/3,000 = 0.91
$P(200{,}000 \le \bar{x} \le 205{,}000) = P(-0.76 \le z \le 0.91) = P(z \le 0.91) - P(z \le -0.76) =$
$0.8186 - 0.2236 = 0.5950$

7.41 $\mu = 3$ inches, $\sigma = 0.1$ inch, and $n = 25$
$\mu_{\bar{x}} = \mu = 3$ inches and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 0.1/\sqrt{25} = 0.02$ inch
For $\bar{x} = 2.95$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (2.95 – 3)/0.02 = –2.50
For $\bar{x} = 3.05$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (3.05 – 3)/0.02 = 2.50
$P(x < 2.95) + P(x > 3.05) = 1 - [P(2.95 \le x \le 3.05)] = 1 - [P(-2.50 \le z \le 2.50)] = 1 -$
$[P(z \le 2.50) - P(z \le -2.50)] = 1 - [0.9938 - 0.0062] = 1 - 0.9876 = 0.0124$

Section7.5

7.43 $p = 600/5000 = 0.12$ and $\hat{p} = 18/120 = 0.15$

7.45 **a.** $\mu_{\hat{p}} = p$

b. $\sigma_{\hat{p}} = \sqrt{pq/n}$

c. The sampling distribution of $\hat{p}$ is approximately normal if $np > 5$ and $nq > 5$.

7.47 Sampling error = $\hat{p} - p = 0.33 - 0.29 = 0.04$

7.49 The sample proportion $\hat{p}$ is a consistent estimator of p, since $\sigma_{\hat{p}}$ decreases as the sample size increases.

7.51 $p = 0.21$ and $q = 1 - p = 1 - 0.21 = 0.79$

a. $n = 400$, $\mu_{\hat{p}} = p = 0.21$, and $\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{(0.21)(0.79)/400} = 0.020$

b. $n = 750$, $\mu_{\hat{p}} = p = 0.21$, and $\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{(0.21)(0.79)/750} = 0.015$

7.53 A sample is considered large enough to apply the central limit theorem if $np > 5$ and $nq > 5$.

7.55 **a.** $np = (20)(0.45) = 9$ and
$nq = (20)(0.55) = 11$
Since $np > 5$ and $nq > 5$, the central limit theorem applies.

b. $np = (75)(0.22) = 16.5$ and
$nq = (75)(0.78) = 58.5$
Since $np > 5$ and $nq > 5$, the central limit theorem applies.

c. $np = (350)(0.01) = 3.5$; since $np < 5$, the central limit theorem does not apply.

d. $np = (200)(0.022) = 4.4$; since $np < 5$, the central limit theorem does not apply.

7.57 $p = 0.51$, $q = 1 - p = 1 - 0.51 = 0.49$, and $n = 200$
$\mu_{\hat{p}} = p = 0.51$, and $\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{(0.51)(0.49)/200} = 0.353$
$np = (200)(0.51) = 102$ and $nq = (200)(0.49) = 98$
Since $np > 5$ and $nq > 5$, the sampling distribution of $\hat{p}$ is approximately normal.

7.59 $p = 0.55$, $q = 1 - p = 1 - 0.55 = 0.45$, and $n = 900$
$\mu_{\hat{p}} = p = 0.55$, and $\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{(0.55)(0.45)/900} = 0.0166$
$np = (900)(0.55) = 495$ and $nq = (900)(0.45) = 405$
Since $np > 5$ and $nq > 5$, the sampling distribution of $\hat{p}$ is approximately normal.

Section 7.6

7.61 $P(p - 3.0\sigma_{\hat{p}} \le \hat{p} \le p + 3.0\sigma_{\hat{p}}) = P(-3.00 \le z \le 3.00) = P(z \le 3.00) - P(z \le -3.00)$
$= 0.9987 - 0.0013 = 0.9974 \text{ } or \text{ } 99.74\%$

7.63 $p = 0.64$, $q = 1 - p = 1 - 0.64 = 0.36$, and $n = 50$
$\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{(0.64)(0.36)/50} = 0.06788225$

a. For $\hat{p} = 0.54$: $z = (\hat{p} - p)/\sigma_{\hat{p}} = (0.54 - 0.64)/0.06788225 = -1.47$
For $\hat{p} = 0.61$: $z = (\hat{p} - p)/\sigma_{\hat{p}} = (0.61 - 0.64)/0.06788225 = -0.44$
$P(0.54 < \hat{p} < 0.61) = P(-1.47 < z < -0.44) = P(z < -0.44) - P(z < -1.47) = 0.3300 - 0.0708 = 0.2592$

b. For $\hat{p} = 0.71$: $z = (\hat{p} - p)/\sigma_{\hat{p}} = (0.71 - 0.64)/.06788225 = 1.03$
$P(\hat{p} > 0.71) = P(z > 1.03) = 1 - P(z \le 1.03) = 1 - 0.8485 = 0.1515$

7.65 $p = 0.06$, $q = 1 - p = 1 - 0.06 = 0.94$, and $n = 150$
$\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{(0.06)(0.94)/150} = 0.0193907194$

For $\hat{p} = 0.08$: $z = (\hat{p} - p) / \sigma_{\hat{p}}$ = (0.08 – 0.06)/0.0193907194 = 1.03
$P(\hat{p} \geq 0.08) = P(z \geq 1.03) = 1 - P(z \leq 1.03) = 1 - 0.8485 = 0.1515$

Supplementary Exercises

7.67 μ = 24,966 hours, σ = 2000 hours, and n = 25
$\mu_{\bar{x}} = \mu = 24{,}966$ hours and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 2000/\sqrt{25} = 400$ hours
The sampling distribution of $\bar{x}$ is normal because the population is normally distributed.

7.69 μ = 50 mpg, σ = 5.9 mpg, and n = 38
$\mu_{\bar{x}} = \mu = 50$ mpg and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 5.9/\sqrt{38} = 0.9571063847$ mpg

a. For $\bar{x}$ = 51.5: $z = (\bar{x} - \mu) / \sigma_{\bar{x}}$ = (51.5 – 50)/0.9571063847 = 1.57
$P(\bar{x} > 51.5) = P(z > 1.57) = 1 - P(z \leq 1.57) = 1 - 0.9418 = 0.0582$

b. For $\bar{x}$ = 48: $z = (\bar{x} - \mu) / \sigma_{\bar{x}}$ = (48 – 50)/0.9571063847 = –2.09
For $\bar{x}$ = 51: $z = (\bar{x} - \mu) / \sigma_{\bar{x}}$ = (51 – 50)/0.9571063847 = 1.04
$P(48 < \bar{x} < 51) = P(-2.09 < z < 1.04) = P(z < 1.04) - P(z < -2.09) = 0.8508 - 0.0183 = 0.8325$

c. For $\bar{x}$ = 53: $z = (\bar{x} - \mu) / \sigma_{\bar{x}}$ = (53 – 50)/0.9571063847 = 3.13
$P(\bar{x} < 53) = P(z < 3.13) = 0.9991$

d. $P(\bar{x}$ greater than μ by 2.5 miles or more) = $P(\bar{x} \geq 52.5)$
For $\bar{x}$ = 52.5: $z = (\bar{x} - \mu) / \sigma_{\bar{x}}$ = (52.5 – 50)/0.9571063847 = 2.61
$P(\bar{x} \geq 52.5) = P(z \geq 2.61) = 1 - P(z < 2.61) = 1 - 0.9955 = 0.0045$

7.71 $p = 0.88$, $q = 1 - p = 1 - 0.88 = 0.12$, and $n = 80$
$\mu_{\hat{p}} = p = 0.88$, and $\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{(0.88)(0.12)/80} = 0.03633180$
$np = (80)(0.88) = 70.4 > 5$, $nq = (80)(0.12) = 9.6 > 5$
Since np and nq are both greater than 5, the sampling distribution of $\hat{p}$ is approximately normal due to central limit theorem.

7.73 $p = 0.113$, $q = 1 - p = 1 - 0.113 = 0.887$, and $n = 500$
$\mu_{\hat{p}} = p = 0.113$, and $\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{(0.113)(0.887)/500} = 0.141584604$

a. **i.** For $\hat{p} = 0.10$: $z = (\hat{p} - p) / \sigma_{\hat{p}} = (0.10 - 0.113)/0.141584604 = -0.92$
$P(\hat{p} < 0.10) = P(z < -0.92) = 0.1788$

ii. For $\hat{p} = 0.13$: $z = (\hat{p} - p) / \sigma_{\hat{p}} = (0.13 - 0.113)/0.141584604 = 1.20$
$P(\hat{p} > 0.13) = P(z > 1.20) = 1 - P(z < 1.20) = 1 - 0.8849 = 0.1151$

b. $P(\hat{p}$ within 0.025 of $p) = P(0.088 \le \hat{p} \le 0.138)$

For $\hat{p} = 0.088$: $z = (\hat{p} - p)/\sigma_{\hat{p}} = (0.088 - 0.113)/0.141584604 = -1.77$

For $\hat{p} = 0.138$: $z = (\hat{p} - p)/\sigma_{\hat{p}} = (0.138 - 0.113)/0.141584604 = 1.77$

$P(0.088 \le \hat{p} \le 0.138) = P(-1.77 \le z \le 1.77) = P(z \le 1.77) - P(z \le -1.77) = 0.9616 - 0.0384 = 0.9232$

c. $P(\hat{p}$ greater than p by 0.03 or more$) = P(\hat{p} \ge 0.143)$

For $\hat{p} = 0.143$: $z = (\hat{p} - p)/\sigma_{\hat{p}} = (0.143 - 0.113)/0.141584604 = 2.12$

$P(\hat{p} \ge 0.143) = P(z \ge 2.12) = 1 - P(z < 2.12) = 1 - 0.9830 = 0.0170$

7.75 **a.** $p = 0.60$, $q = 1 - p = 1 - 0.60 = 0.40$, and $n = 25$

$$\hat{p} = \frac{13}{52} = 0.52,\ \mu_{\hat{p}} = p = 0.60,\ \sigma_{\hat{p}} = \sqrt{\frac{(0.60)(1-0.60)}{25}} = 0.09798$$

For $\hat{p} = 0.52$: $z = \dfrac{0.52 - 0.60}{0.09798} = 0.82$

$P(\hat{p} \ge 0.52) = P(z < 0.82) = 0.7939$

b. For 0.95 or higher, $z = -1.65$. Now,

$z = (\hat{p} - p)/\sigma_{\hat{p}}$, so $\sigma_{\hat{p}} = \dfrac{\hat{p} - p}{z} = \dfrac{0.5 - 0.6}{-1.65} = 0.0606$. Then, since $\sigma_{\hat{p}} = \sqrt{pq/n}$,

$n = pq/(\sigma_{\hat{p}})^2 = 0.60(0.40)/(0.0606)^2 = 65.35.$

The reporter should take a sample of at least 66 voters.

7.77

Sample	Scores	Sample Median
ABC	70, 78, 80	78
ABD	70, 78, 80	78
ABE	70, 78, 95	78
ACD	70, 80, 80	80
ACE	70, 80, 95	80
ADE	70, 80, 95	80
BCD	78, 80, 80	80
BCE	78, 80, 95	80
BDE	78, 80, 95	80
CDE	80, 80, 95	80

Mean of the sample medians: (78 + 78 + 78 + 80 + 80 + 80 + 80 + 80 + 80 + 80)/10 = 79.4. This is not equal to the population mean of 80.6. We could change the 78 to 80 and change the 95 to 90. Then, each of the sample medians would be equal to 80, and therefore have a mean of 80. The population mean would become μ = (70 + 80 + 80 + 80 + 90)/5 = 80.

Self – Review Test

1. b **3.** a **5.** b

7. c **9.** a **11.** c

13. According to the central limit theorem, for a large sample size, the sampling distribution of the sample mean is approximately normal irrespective of the shape of the population distribution. The mean and standard deviation of the sampling distribution of the sample mean are $\mu_{\bar{x}} = \mu$ and $\sigma_{\bar{x}} = \sigma/\sqrt{n}$ for $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}}, \dfrac{n}{N}$ must be ≤ 0.05, respectively. The sample size is usually considered to be large if $n \geq 30$. From the same theorem, the sampling distribution of $\hat{p}$ is approximately normal for sufficiently large samples. In the case of proportion, the sample is sufficiently large if $np > 5$ and $nq > 5$.

15. $\mu = \$88{,}190$ and $\sigma = \$20{,}000$

a. $\mu_{\bar{x}} = \mu = \$88{,}190$ and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 20{,}000/\sqrt{20} = \$4472.14$
Since the population distribution is skewed to the right, and $n < 30$, the shape of the sampling distribution of $\bar{x}$ is also skewed to the right.

b. $\mu_{\bar{x}} = \mu = \$88{,}190$ and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 20{,}000/\sqrt{100} = \$2{,}000$
Since $n \geq 30$, the sampling distribution of $\bar{x}$ is approximately normal.

c. $\mu_{\bar{x}} = \mu = \$88{,}190$ and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 20{,}000/\sqrt{800} = \$707.11$
Since $n \geq 30$, the sampling distribution of $\bar{x}$ is approximately normal.

17. $\mu = 16$ ounces, $\sigma = 0.18$ ounce, and $n = 16$
$\mu_{\bar{x}} = \mu = 16$ ounces and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 0.18/\sqrt{16} = 0.045$ ounce

a. **i.** For $\bar{x} = 15.90$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (15.90 - 16)/0.045 = -2.22$
For $\bar{x} = 15.95$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (15.95 - 16)/0.045 = -1.11$
$P(15.90 < \bar{x} < 15.95) = P(-2.22 < z < -1.11) = P(z < -1.11) - P(z < -2.22) = 0.1335 - 0.0132 = 0.1203$

ii. For $\bar{x} = 15.95$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (15.95 - 16)/0.045 = -1.11$
$P(\bar{x} < 15.95) = P(z < -1.11) = 0.1335$

iii. For $\bar{x} = 15.97$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (15.97 - 16)/0.045 = -0.67$
$P(\bar{x} > 15.97) = P(z > -0.67) = 1 - P(z \leq -0.67) = 1 - 0.2514 = 0.7486$

b. $P(\bar{x}$ within 0.10 ounce of $\mu) = P(15.90 \leq \bar{x} \leq 16.10)$
For $\bar{x} = 15.90$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (15.90 - 16)/.045 = -2.22$
For $\bar{x} = 16.10$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (16.10 - 16)/.045 = 2.22$

$P(15.90 \le \bar{x} \le 16.10) = P(-2.22 \le z \le 2.22) = P(z \le 2.22) - P(z \le -2.22) = 0.9868 - 0.0132 = 0.9736$

c. $P(\bar{x}$ is less than μ by 0.135 ounce or more$) = P(\bar{x} < 15.865)$
For $\bar{x} = 15.865$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (15.865 - 16)/0.045 = -3.00$
$P(\bar{x} < 15.865) = P(z < -3.00) = 0.0013$

19. $p = 0.34$, $q = 1 - p = 1 - 0.34 = 0.66$, and $n = 1150$

a. $\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{(0.34)(0.66)/1150} = 0.0139689096$

i. For $\hat{p} = 0.35$: $z = (\hat{p} - p)/\sigma_{\hat{p}} = (0.35 - 0.34)/0.0139689096 = 0.72$
$P(\hat{p} > 0.35) = P(z > 0.72) = 1 - P(z \le 0.72) = 1 - 0.7642 = 0.2358$

ii. For $\hat{p} = 0.32$: $z = (\hat{p} - p)/\sigma_{\hat{p}} = (0.32 - 0.34)/0.0139689096 = -1.43$
For $\hat{p} = 0.37$: $z = (\hat{p} - p)/\sigma_{\hat{p}} = (0.37 - 0.34)/0.0139689096 = 2.15$
$P(0.32 < \hat{p} < 0.37) = P(-1.43 < z < 2.15) = P(z < 2.15) - P(z < -1.43) = 0.9842 - 0.0764 = 0.9078$

iii. For $\hat{p} = 0.31$: $z = (\hat{p} - p)/\sigma_{\hat{p}} = (0.31 - 0.34)/0.0139689096 = -2.15$
$P(\hat{p} < 0.31) = P(z < -2.15) = 0.0158$

iv. For $\hat{p} = 0.31$: $z = (\hat{p} - p)/\sigma_{\hat{p}} = (0.31 - 0.34)/0.0139689096 = -2.15$
For $\hat{p} = 0.36$: $z = (\hat{p} - p)/\sigma_{\hat{p}} = (0.36 - 0.34)/0.0139689096 = 1.43$
$P(0.31 < \hat{p} < 0.36) = P(-2.15 < z < 1.43) = P(z < 1.43) - P(z < -2.15) = 0.9236 - 0.0158 = 0.9078$

v. For $\hat{p} = 0.36$: $z = (\hat{p} - p)/\sigma_{\hat{p}} = (0.36 - 0.34)/0.0139689096 = 1.43$
$P(\hat{p} < 0.36) = P(z < 1.43) = 0.9236$

vi. For $\hat{p} = 0.32$: $z = (\hat{p} - p)/\sigma_{\hat{p}} = (0.32 - 0.34)/0.0139689096 = -1.43$
$P(\hat{p} > 0.32) = P(z > -1.43) = 1 - P(z \le -1.43) = 1 - 0.0764 = 0.9236$

b. $P(\hat{p}$ within 0.025 of $p) = P(0.315 \le \hat{p} \le 0.365)$
For $\hat{p} = 0.315$: $z = (\hat{p} - p)/\sigma_{\hat{p}} = (0.315 - 0.34)/0.0139689096 = -1.79$
For $\hat{p} = 0.365$: $z = (\hat{p} - p)/\sigma_{\hat{p}} = (0.365 - 0.34)/0.01396890964 = 1.79$
$P(0.315 \le \hat{p} \le 0.365) = P(-1.79 \le z \le 1.79) = P(z \le 1.79) - P(z \le -1.79) = 0.9633 - 0.0367 = 0.9266$

c. $P(\hat{p}$ within 0.03 of $p) = P(0.31 \le \hat{p} \le 0.37)$
For $\hat{p} = 0.31$: $z = (\hat{p} - p)/\sigma_{\hat{p}} = (0.31 - 0.34)/0.0139689096 = -2.15$
For $\hat{p} = 0.37$: $z = (\hat{p} - p)/\sigma_{\hat{p}} = (0.47 - 0.34)/0.0139689096 = 2.15$
$P(0.31 < \hat{p} < 0.37) = P(-2.15 < z < 2.15) = P(z < 2.15) - P(z < -2.15) = 0.9842 - 0.0158 = 0.9684$

d. $P(\hat{p}$ greater than p by 0.02 or more) = $P(\hat{p} \geq .36)$

For $\hat{p} = 0.36$: $z = (\hat{p} - p)/\sigma_{\hat{p}}$ = (0.36 – 0.34)/0.0139689096 = 1.43

$P(\hat{p} \geq 0.36) = P(z \geq 1.43) = 1 - P(z \leq 1.43)$ = 1 – 0.9236 = 0.0764