National University
of computer and emerging sciences

Foundation for Advancement of Science and Technology

Graph Theory (MT-3001)

Lecturer

USAMA ANTULEY

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LECTURE # 01

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Course Details

➢ Text Book:

1. Saoub, Karin R. Graph Theory: An Introduction to Proofs, Algorithms, and Applications. CRC Press, 2021.
2. Graph theory: undergraduate mathematics / by Khee Meng Koh, Fengming Dong, Kah Loon Ng, Eng Guan Tay. Bondy, John Adrian, and Uppaluri Siva Ramachandra Murty

➢Reference Book (s):

1. Bondy, John Adrian, and Uppaluri Siva Ramachandra Murty. *Graph theory with applications*. Vol. 290. London: Macmillan, 1976.
2. West, Douglas Brent. *Introduction to graph theory*. Vol. 2. Upper Saddle River: Prentice Hall, 2001.

Marking Division

S. No	Particulars	% Marks
1.	**Assignment (at least 2)**	05
2.	**Quizzes (at least 2) + Presentation/Project**	15
3.	**Sessional 1 & 2**	30
4.	**Final Exam**	50
	Total	**100**

Airlines Graph

Consider a small country with five cities: A, B, C, D, E.

There are six flights:

$A - B, A - C, A - E,$

$B - D, C - D, C - E.$

Is there a direct flight from A to D?
With one stop?
With two stops?

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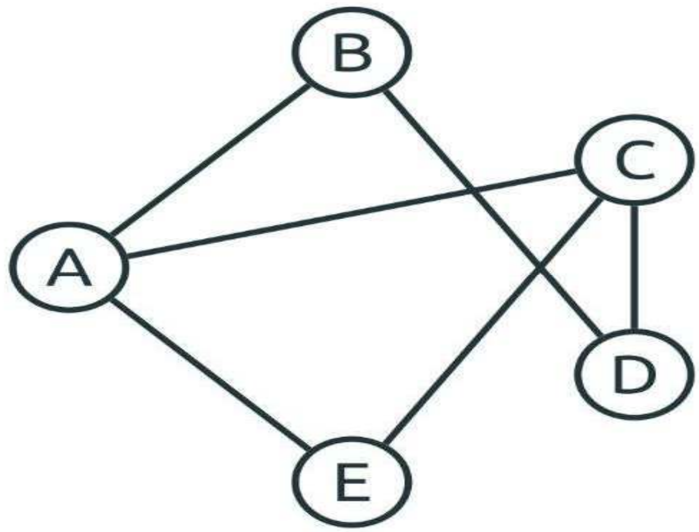
$B - D, C - D, C - E.$

Is there a direct flight from A to D? With one stop? With two stops?



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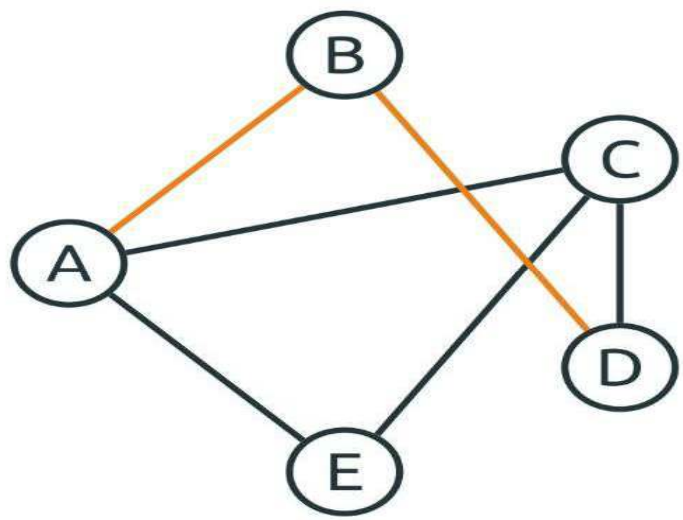

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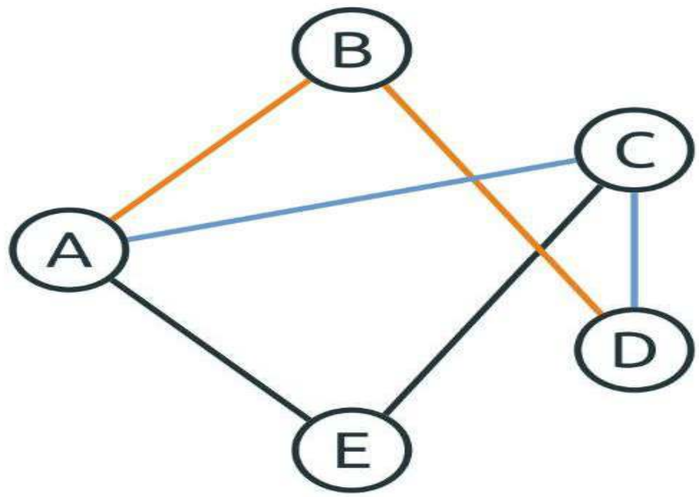

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Outline

What is a Graph?

Graph Examples

Graph Applications

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Graphs

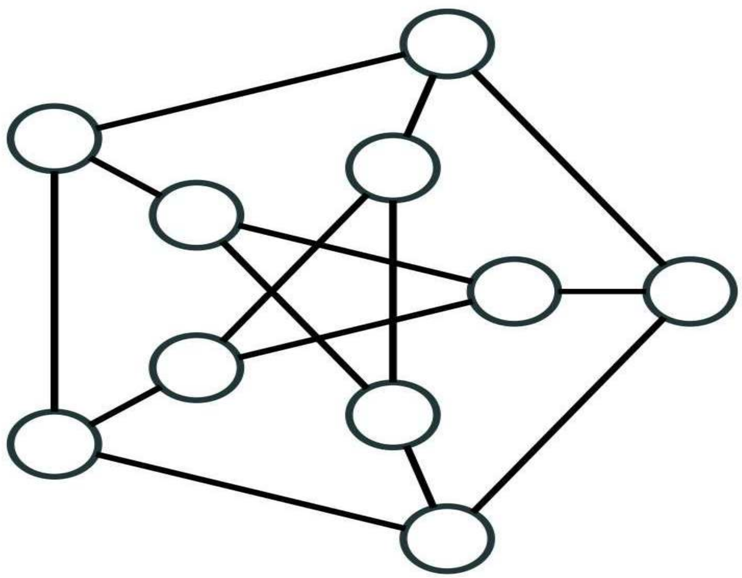
- A set of Objects

Graphs

- A set of Objects
- Relations between pairs of objects

Graphs

- A set of Objects
- Relations between pairs of objects



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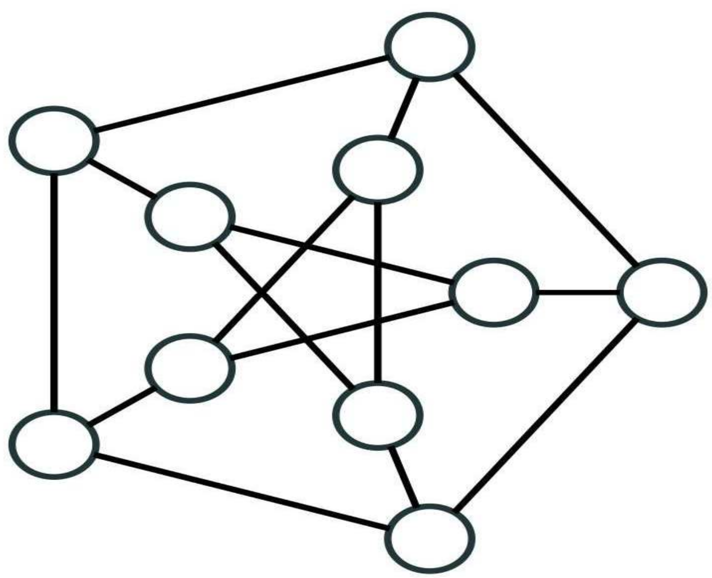
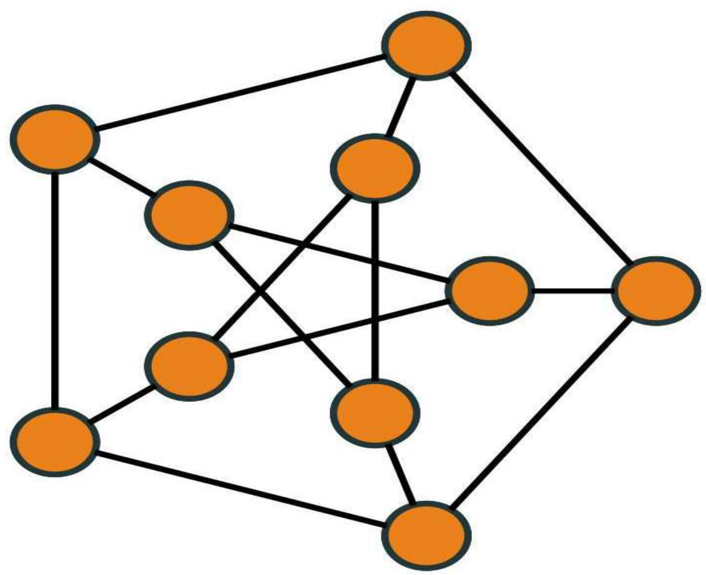
Definition

- A Graph $G = (V, E)$



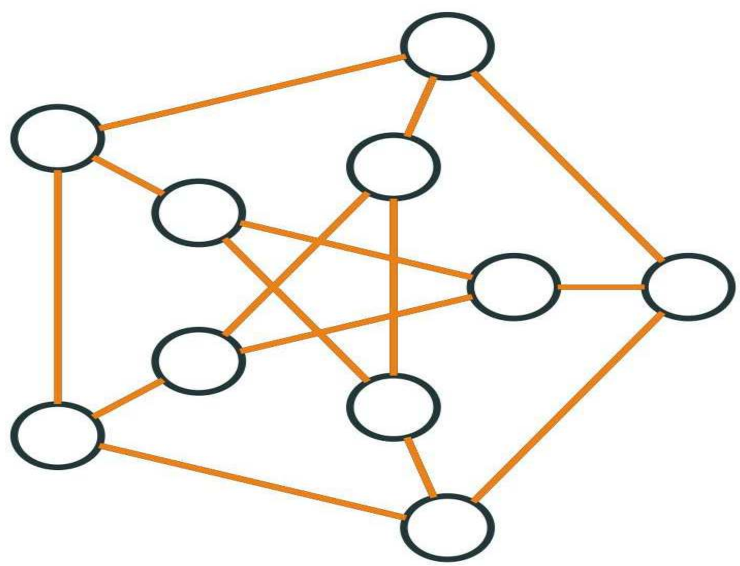
Definition

- A Graph $G = (V, E)$
- A set V of Vertices/Nodes



Definition

- A Graph $G = (V, E)$
- A set V of Vertices/Nodes
- A set E of Edges



Vocabulary

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- We can name individual vertices and edges

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- *u* and *v* are **End Points** of *e*

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- u and v are End Points of e
- u and e are Incident

Vocabulary

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- *e* Connects *u* and *v*
- *u* and *v* are End Points of *e*
- *u* and *e* are Incident
- *u* and *v* are Adjacent

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Vocabulary

- We can name individual vertices and edges
- e Connects u and v
- u and v are End Points of e
- u and e are Incident
- u and v are Adjacent
- u and v are Neighbors

Drawing a Graph

Objects: {A,B,C,D}

Relations: {{A,C},{D,A},{B,D},{C,B}}

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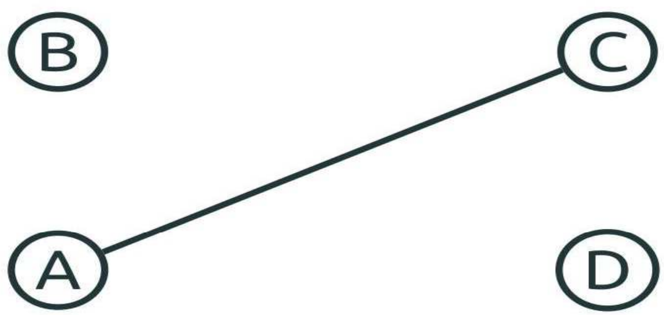

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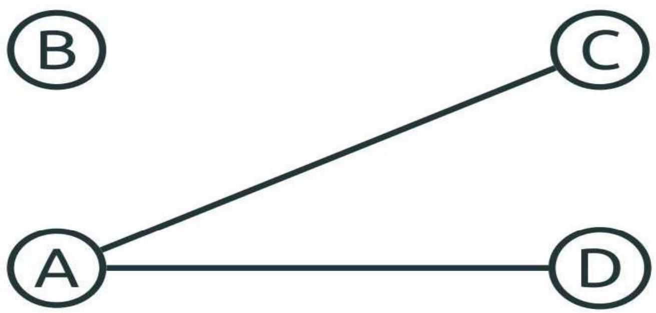

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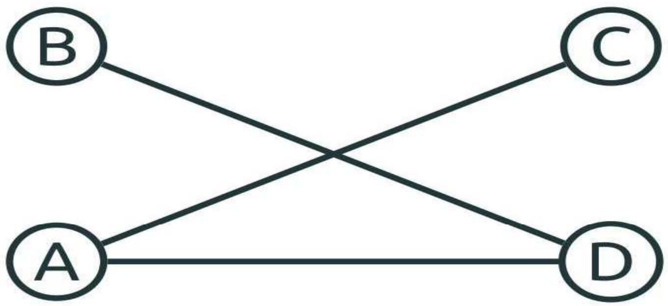

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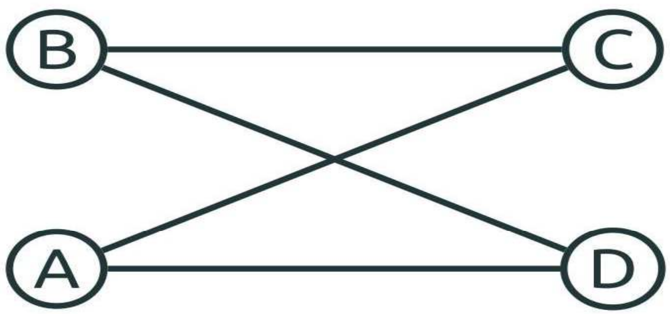

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Directed Graph

- It is often convenient to consider Directed Edges (Arcs)

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- It is often convenient to consider Directed Edges (Arcs)
- They describe asymmetric relations
- There is a flight from A to B, but not the other way around
- Such a graph is called Directed

Drawing a Directed Graph

Objects: {A,B,C,D}

Relations: {(A,C),(D,A),(B,D),(C,B)}

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Drawing a Directed Graph

Objects: {A,B,C,D}

Relations: {(A,C),(D,A),(B,D),(C,B)}

Drawing a Directed Graph

Objects: {A,B,C,D}

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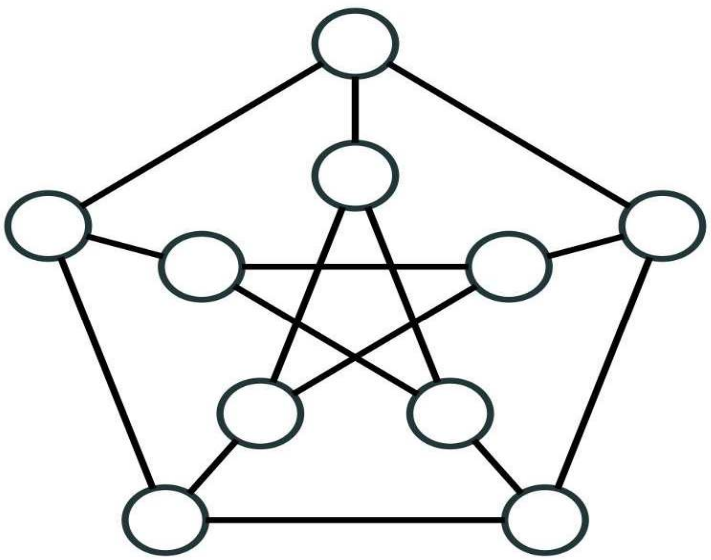
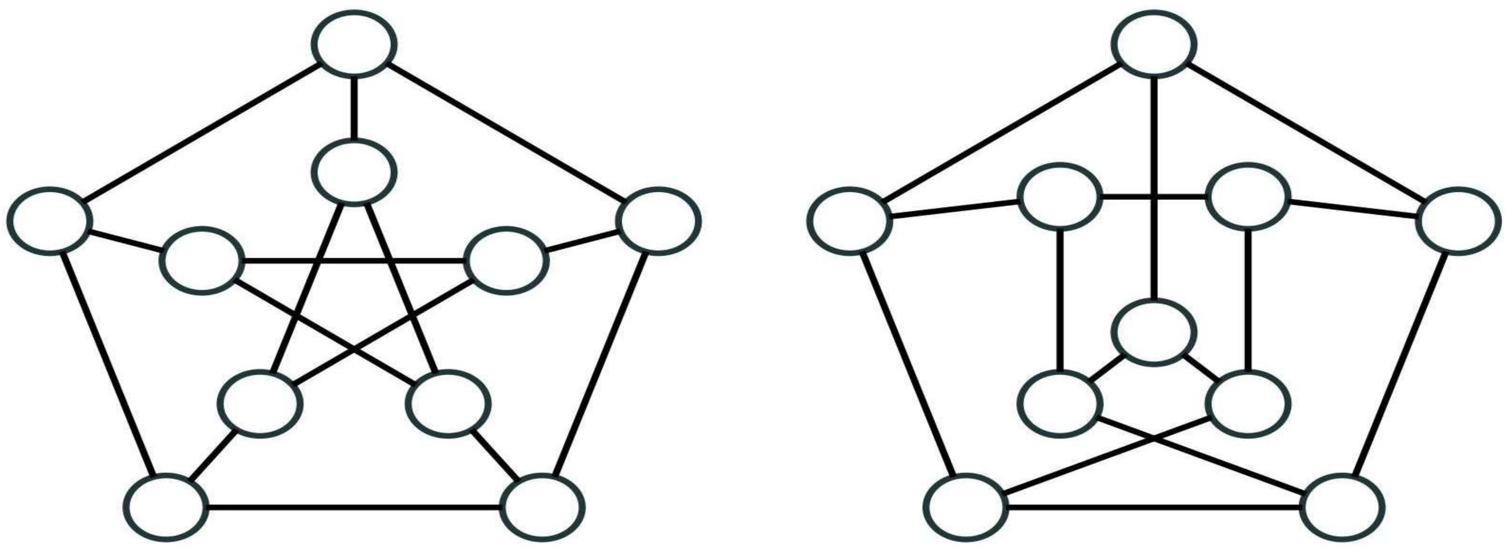
Many Ways to Draw a Graph



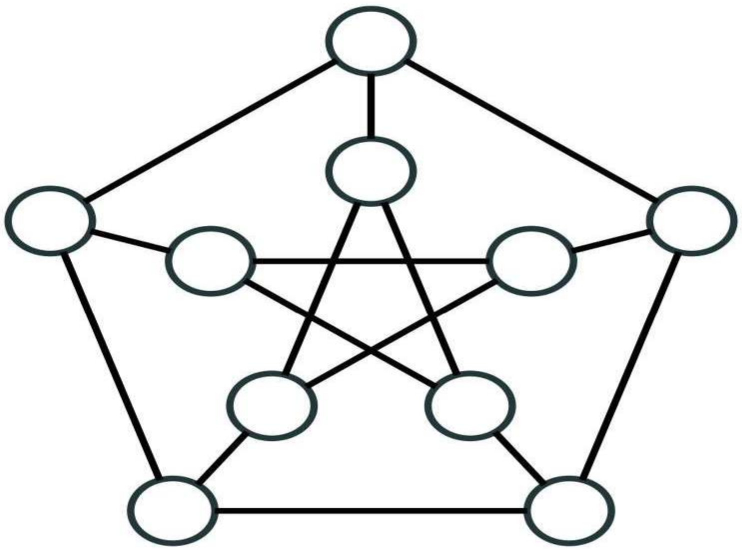
Many Ways to Draw a Graph

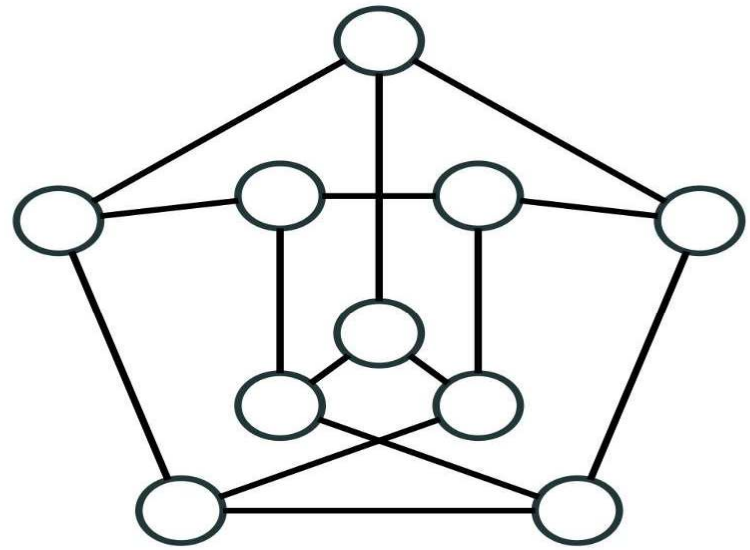
Are these graphs the same?



Many Ways to Draw a Graph

10 vertices and 15 edges?





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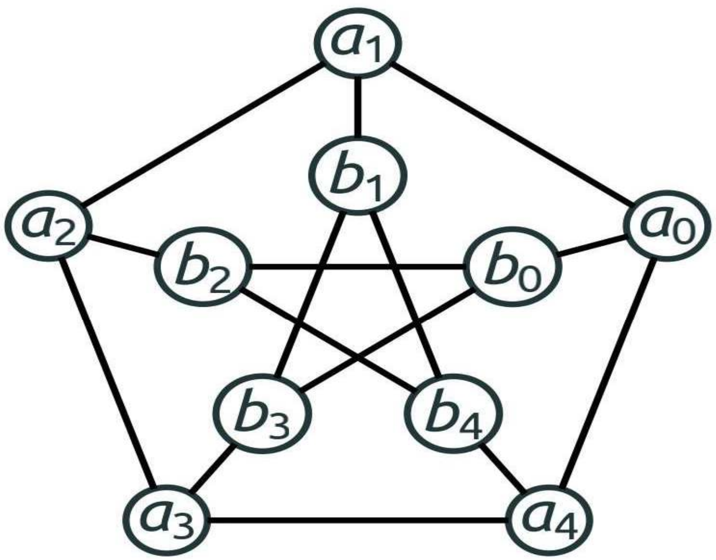

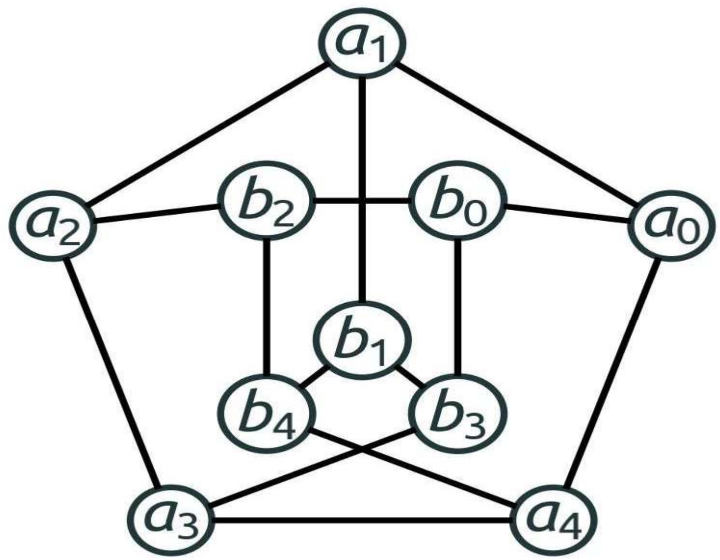

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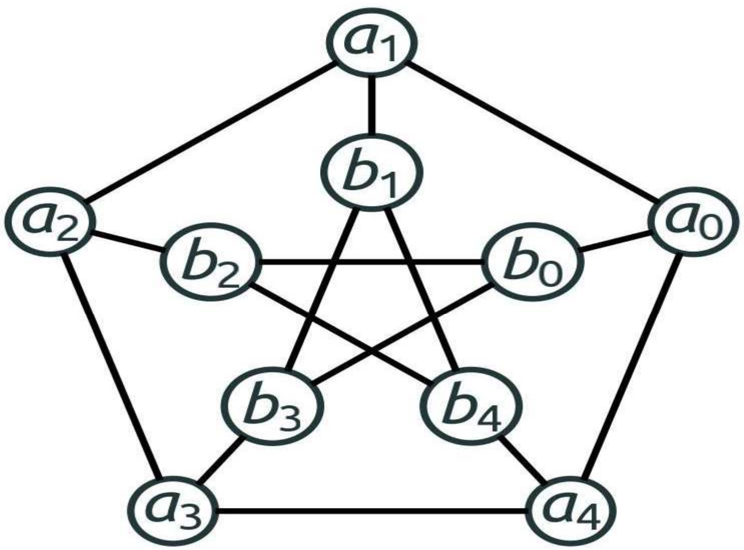

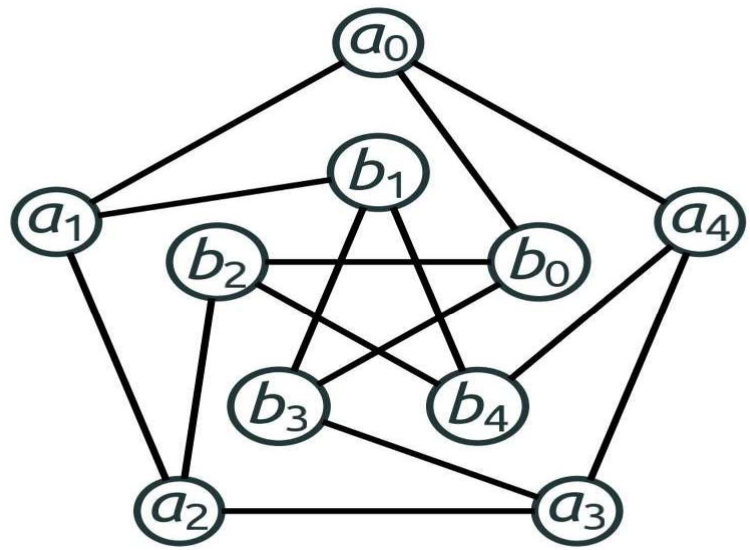

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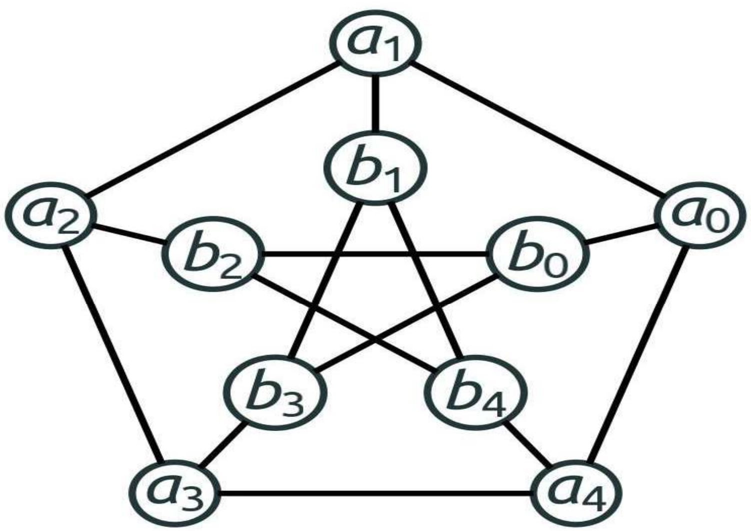

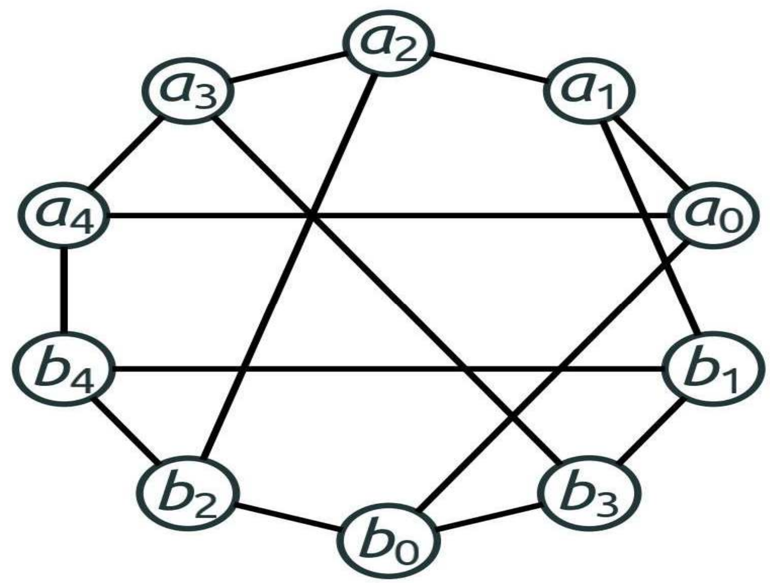

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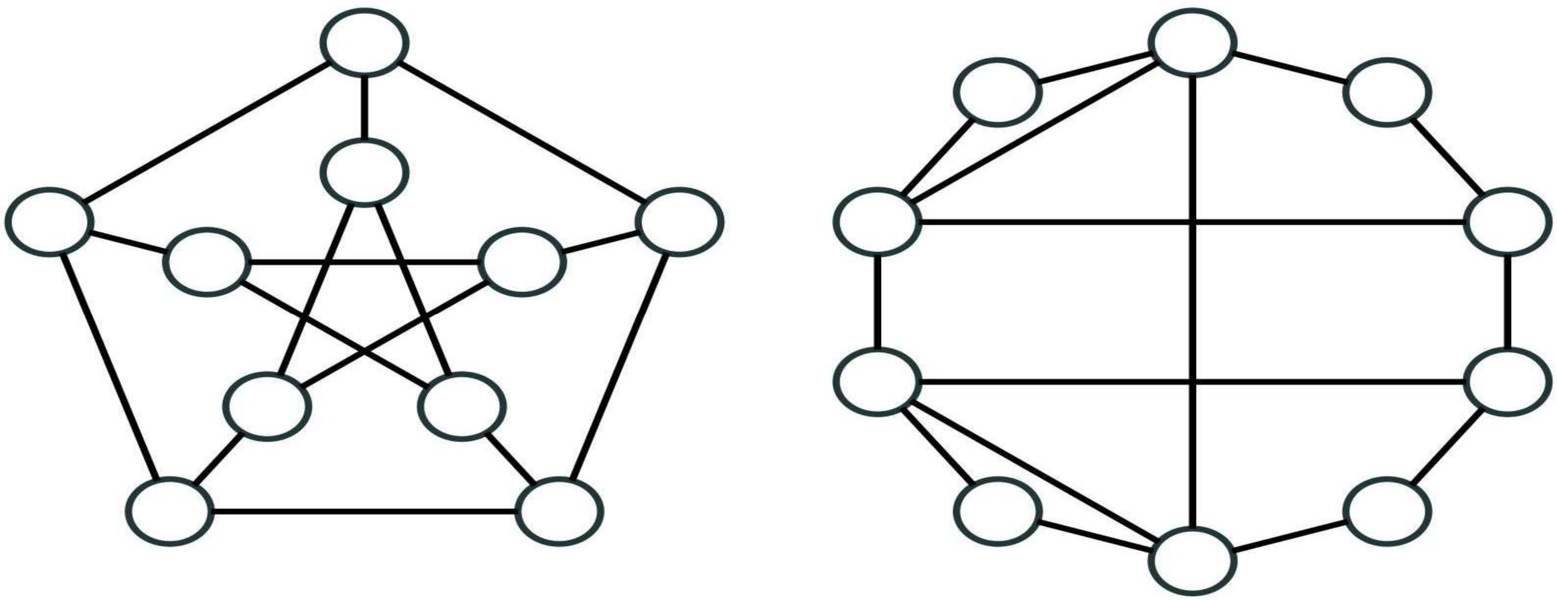

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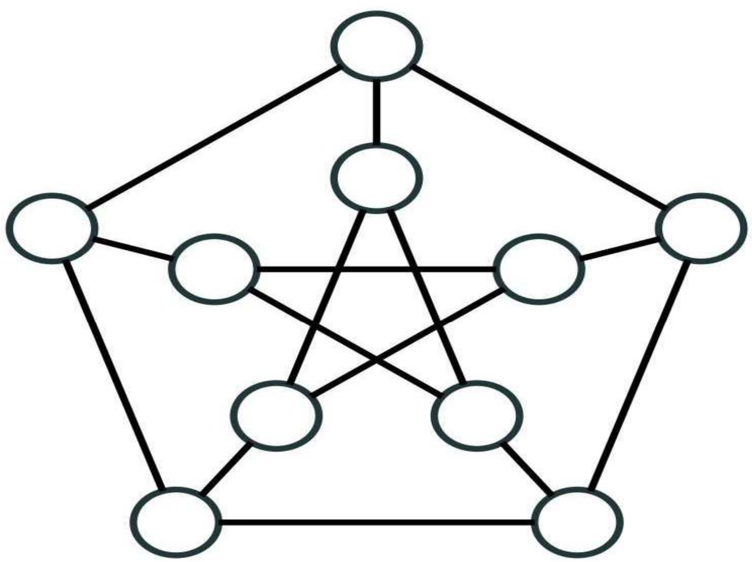

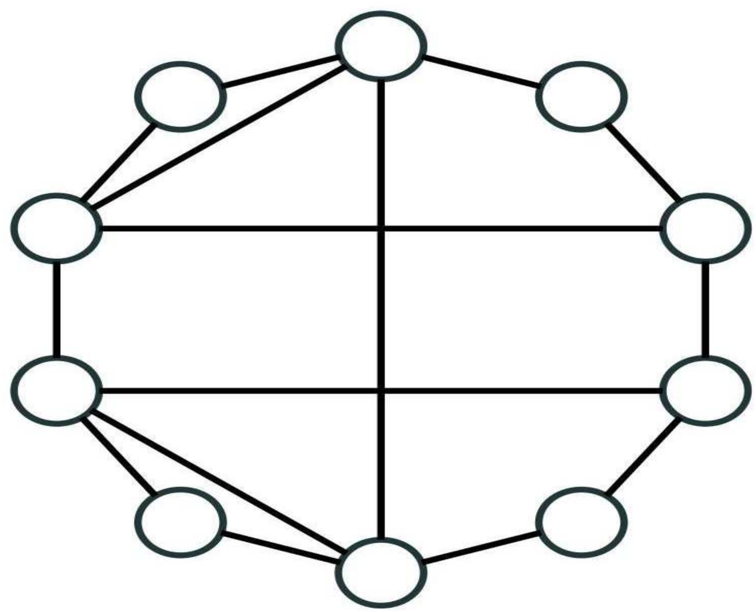

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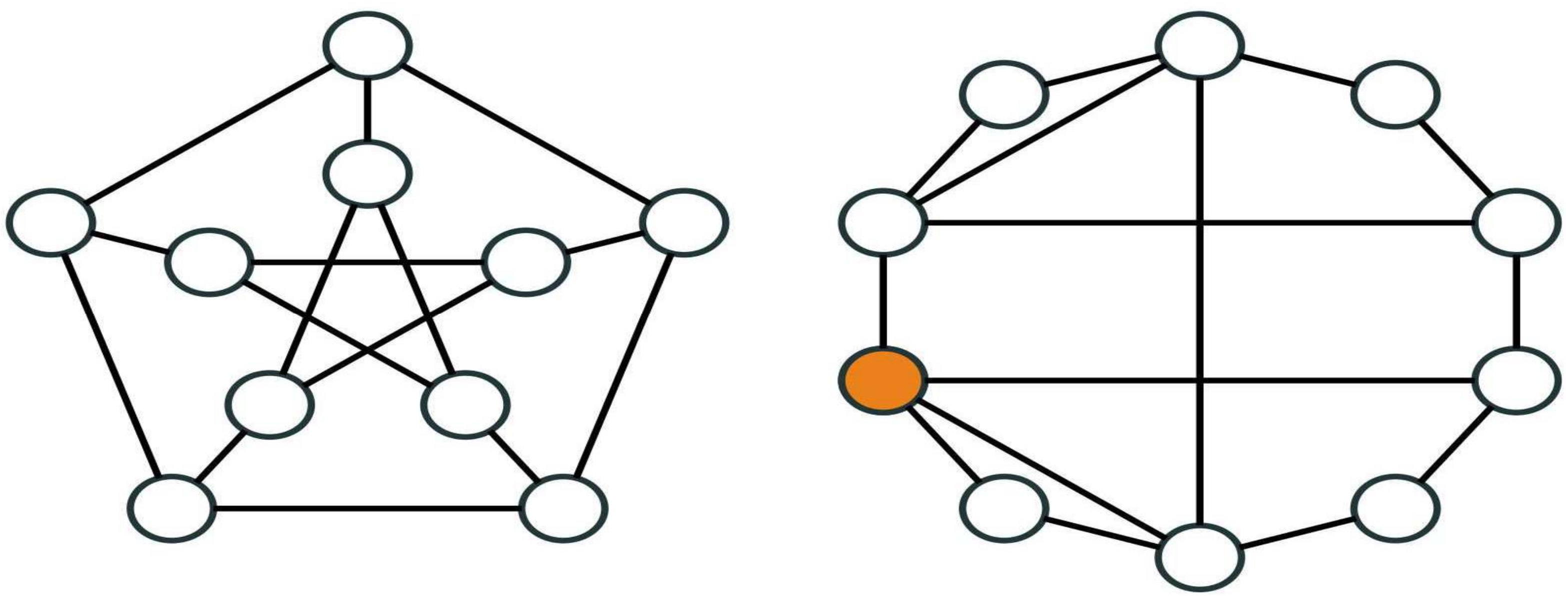

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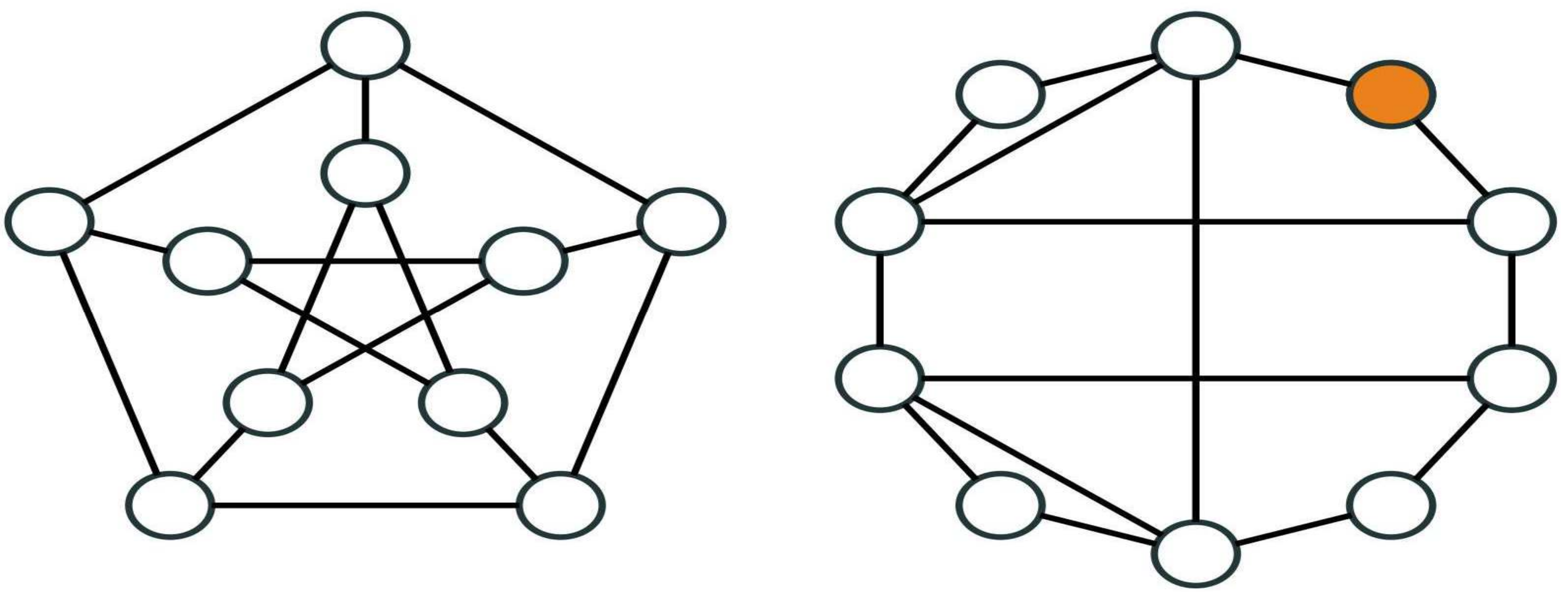


Graph Drawing is Beautiful!

Donald E. Knuth

Graph drawing is the best possible field I can think of: It merges aesthetics, mathematical beauty and wonderful algorithms. It therefore provides a harmonic balance between the left and right brain parts.

Outline

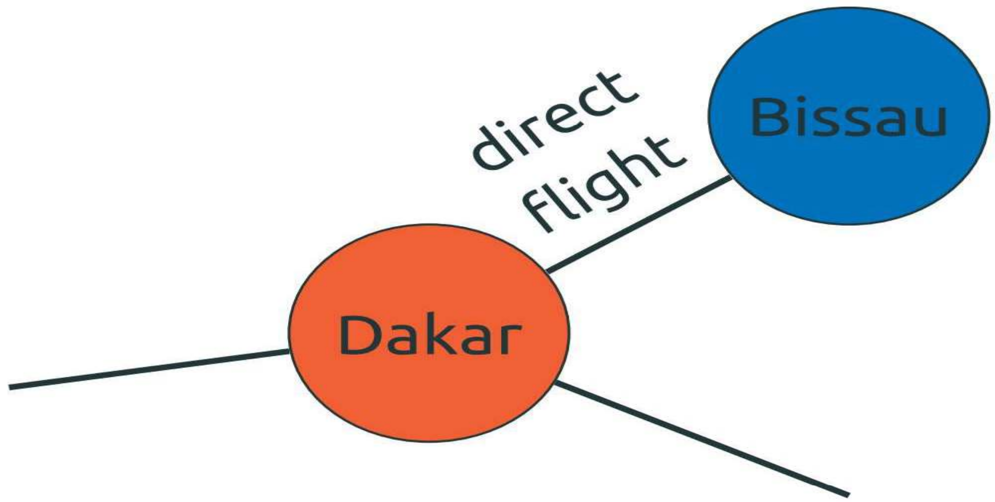
What is a Graph?

Graph Examples

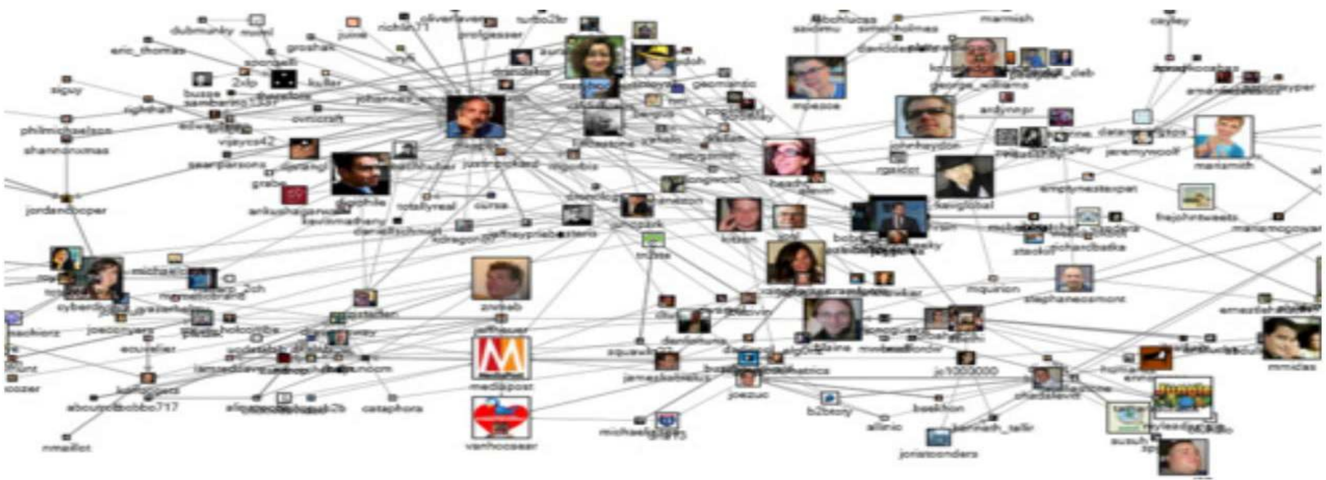
Graph Applications

Airlines Graph

Airlines Graph



Facebook Graph



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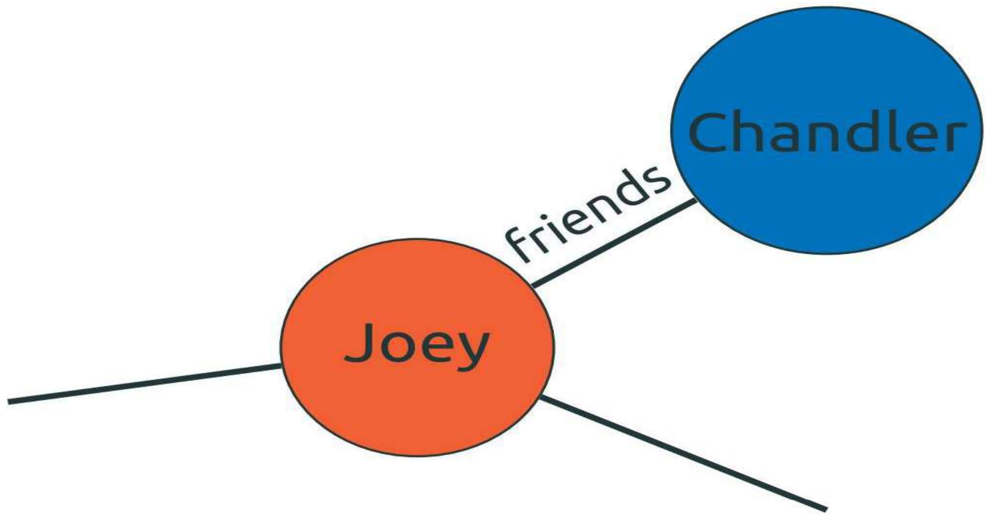
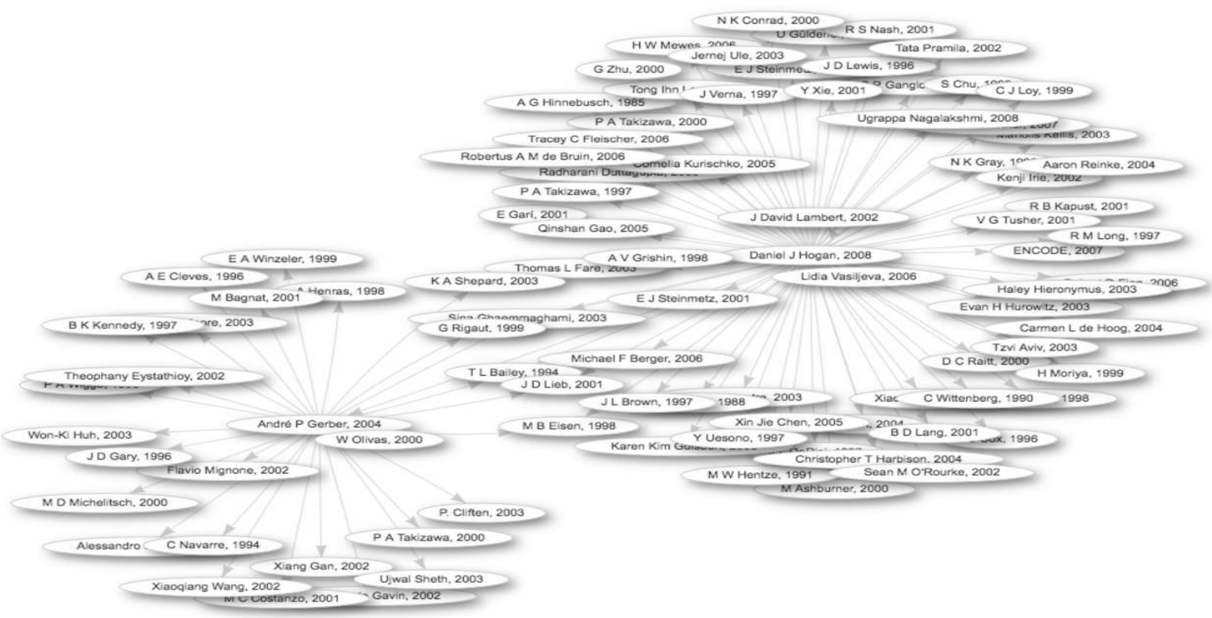


Citation Graph For a Paper



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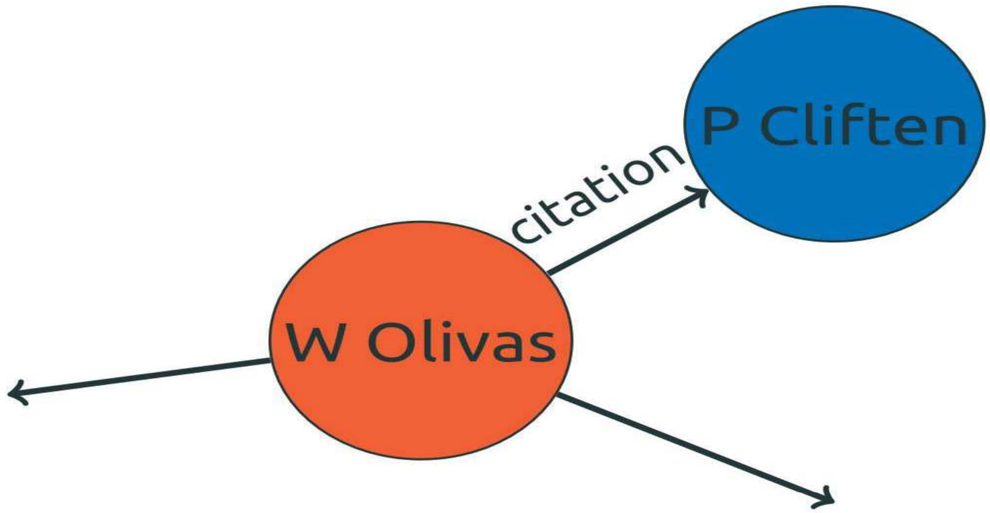
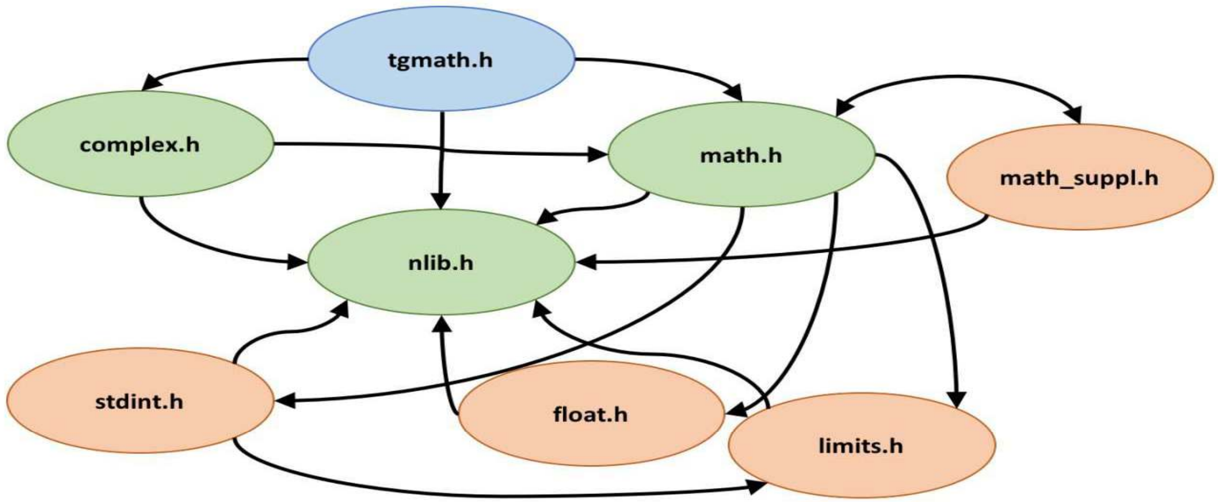


Dependency Graph



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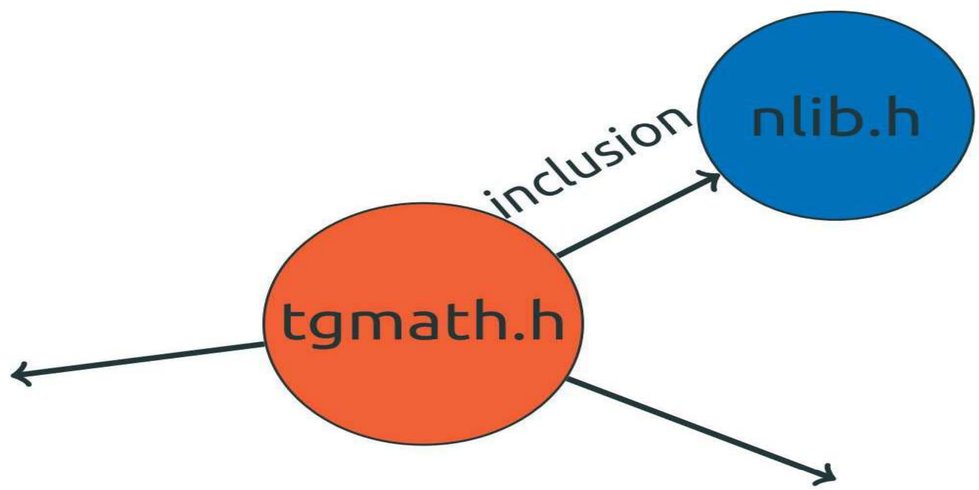
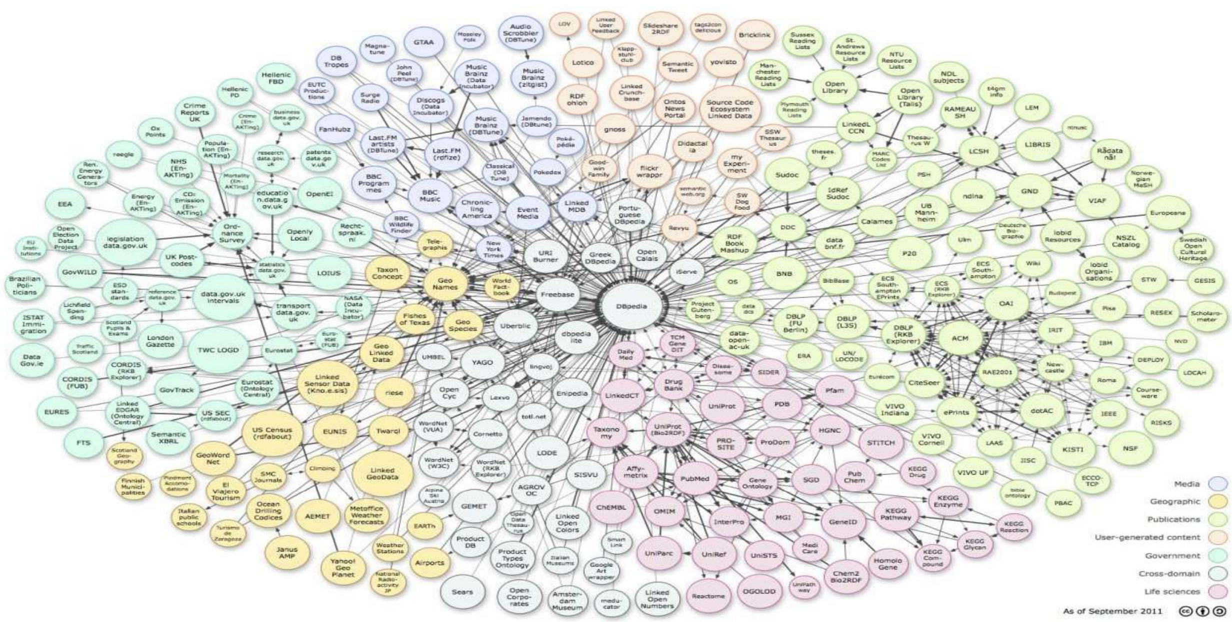
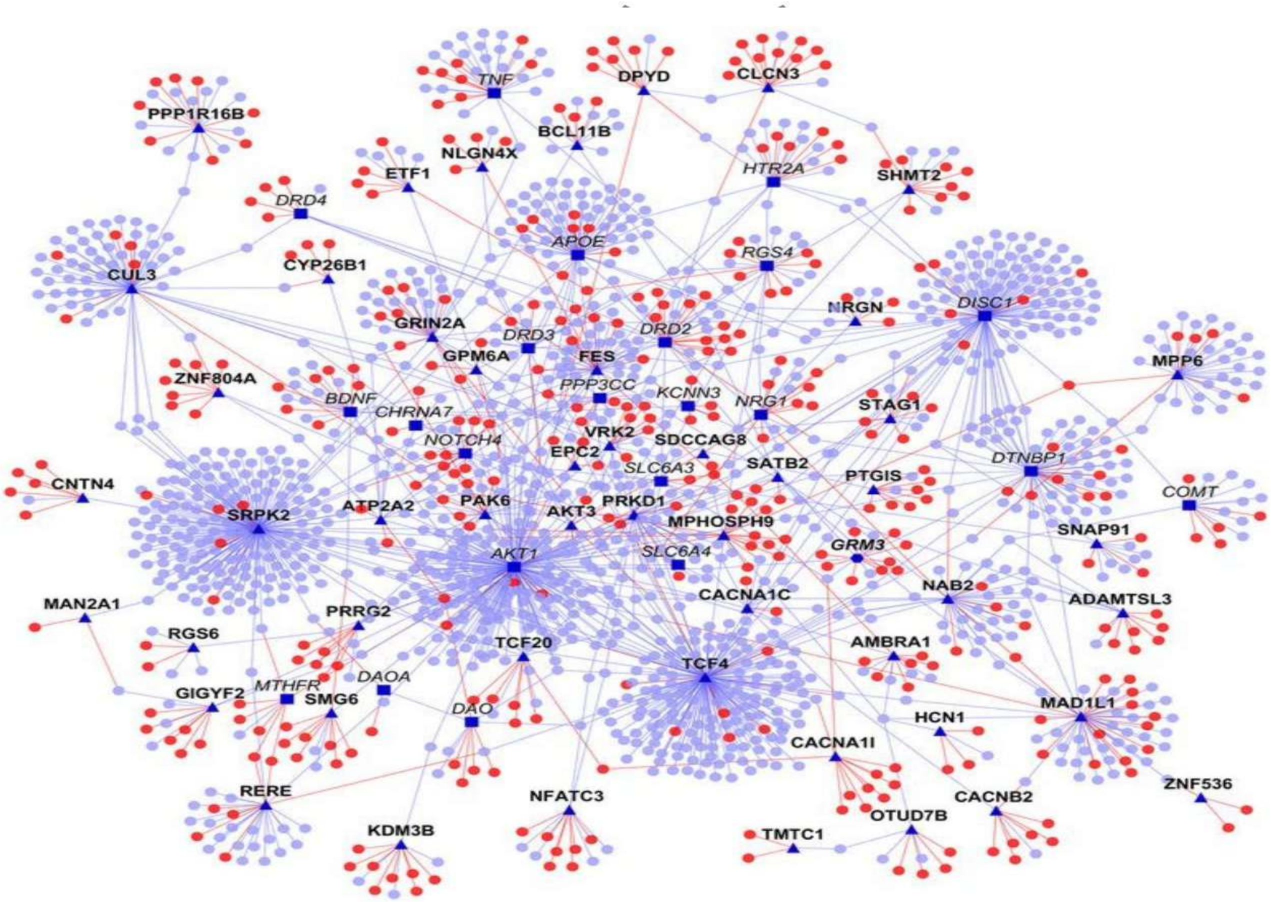


Linked Open Data Diagram



Schizophrenia Protein–Protein Interaction



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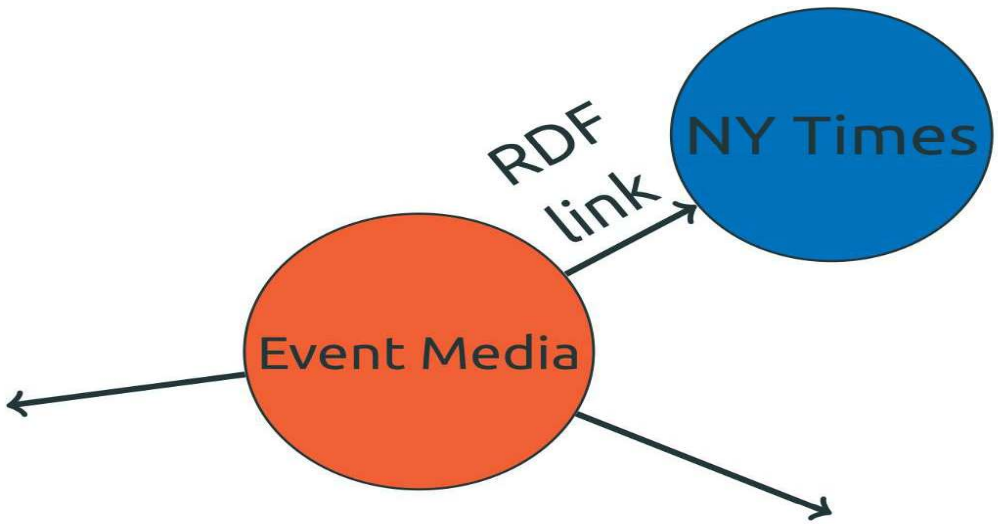

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Outline

What is a Graph?

Graph Examples

Graph Applications

Navigation

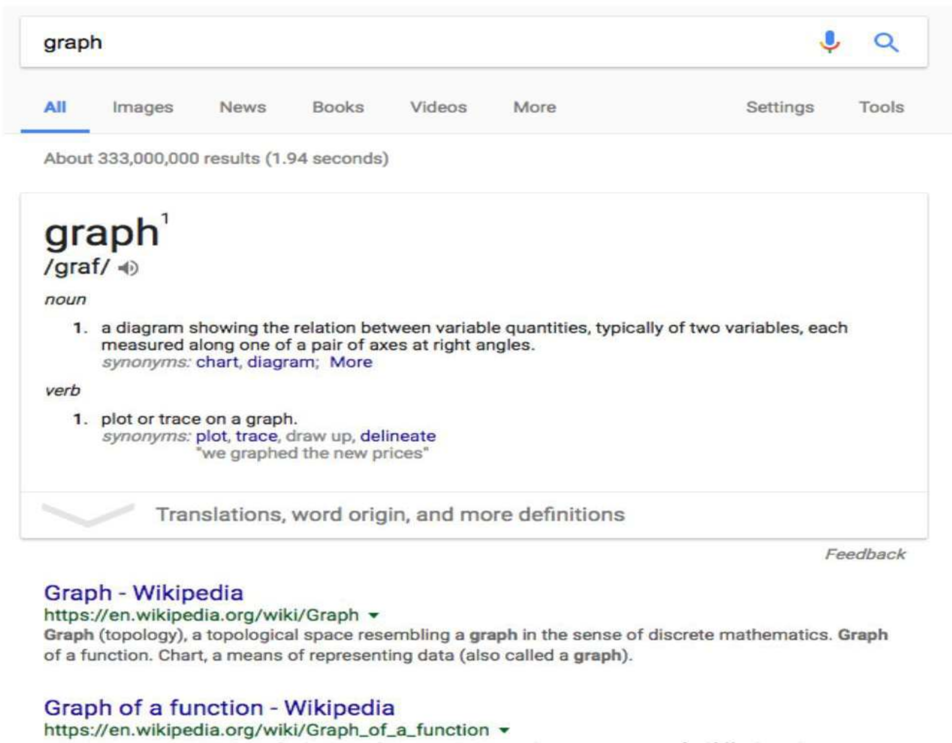

Central Park
Central Park W
A
B
Penn Plz, 4
Grand Central Terminal
Pennsylvania Station-New York
E 66th St
E 59th St
E 57th St
E 79th
1st Ave
FDR Dr
5th Ave
Broadway

Navigation

PageRank


Gmail
Images
Sign in
Google
Google Search
I'm Feeling Lucky
Advertising
Business
About
Privacy
Terms
Settings

PageRank


graph
All
Images
News
Books
Videos
More
Settings
Tools
About 333,000,000 results (1.94 seconds)
graph 1
/graf/
noun
1. a diagram showing the relation between variable quantities, typically of two variables, each measured along one of a pair of axes at right angles.
synonyms: chart, diagram; More
verb
1. plot or trace on a graph.
synonyms: plot, trace, draw up, delineate
"we graphed the new prices"
Translations, word origin, and more definitions
Feedback
Graph - Wikipedia
https://en.wikipedia.org/wiki/Graph
Graph (topology), a topological space resembling a graph in the sense of discrete mathematics. Graph of a function. Chart, a means of representing data (also called a graph).
Graph of a function - Wikipedia
https://en.wikipedia.org/wiki/Graph_of_a_function

PageRank

Graph TV
graphtv.kevinformatics.com/
Graph Ratings of Your Favorite TV Shows. Visualize IMDb ratings and trends of TV shows by episode. Have you seen Mad Men, Breaking Bad, or Battlestar ...

< Goooooooooogle >
Previous 1 2 3 4 5 6 7 8 9 10 Next

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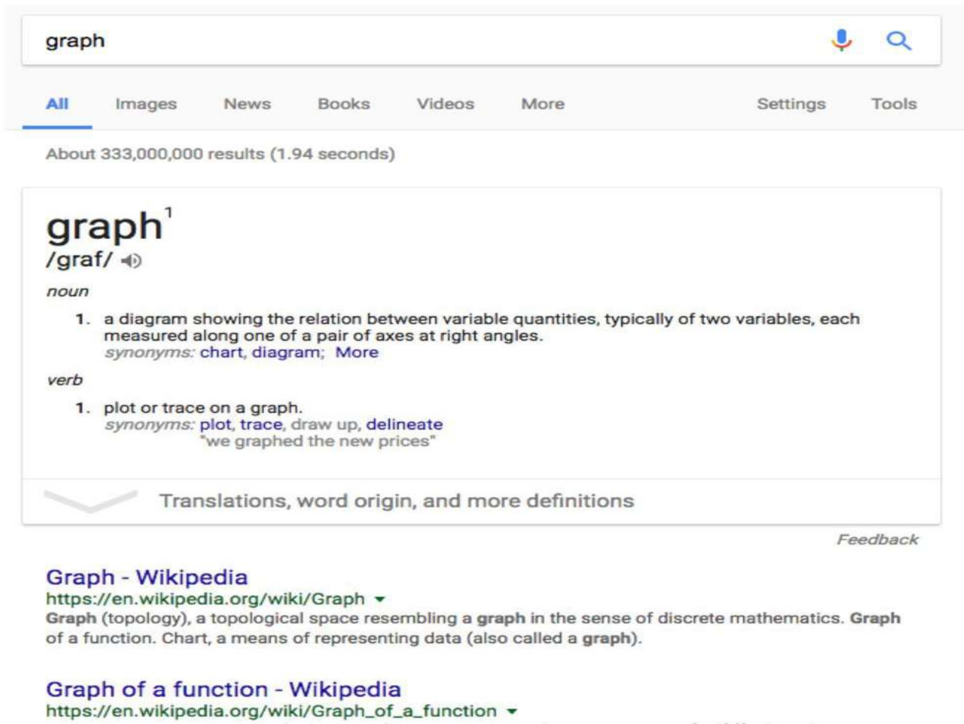
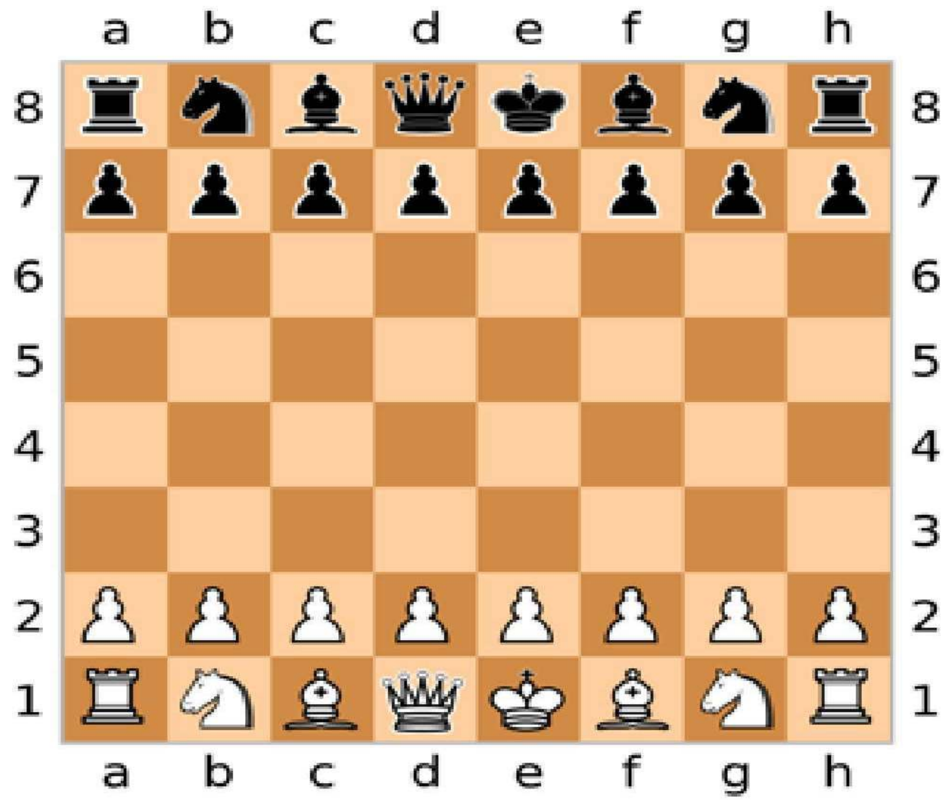
PageRank



Game Strategies



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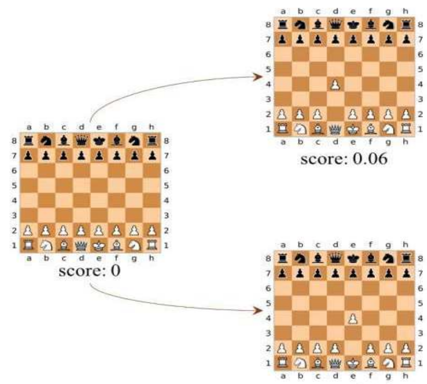

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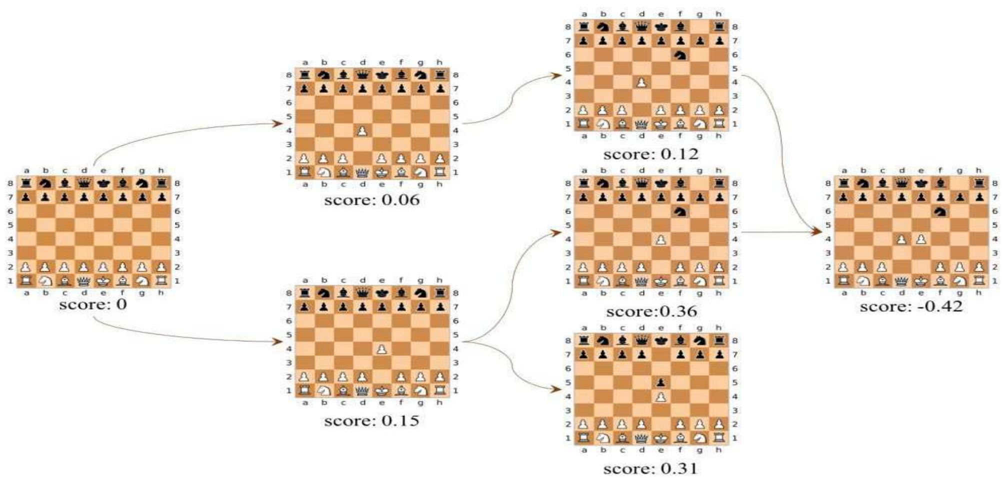
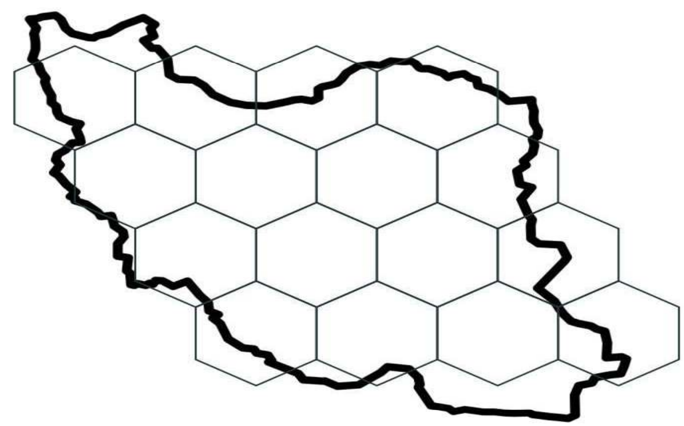
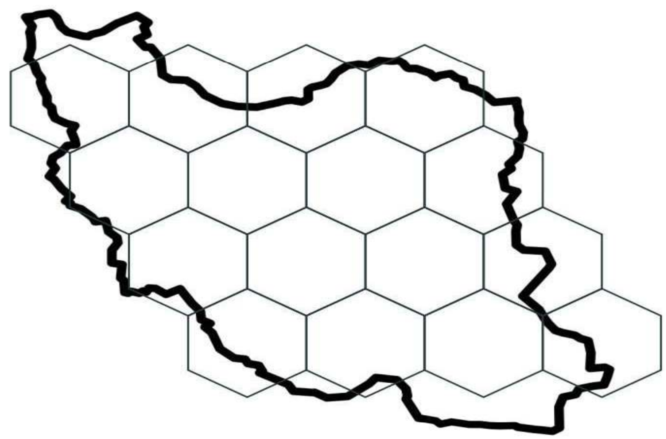


GSM



GSM

4 Frequency
Ranges



GSM

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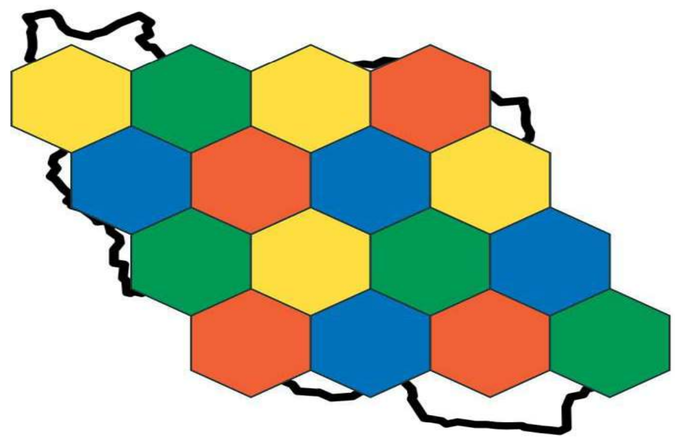
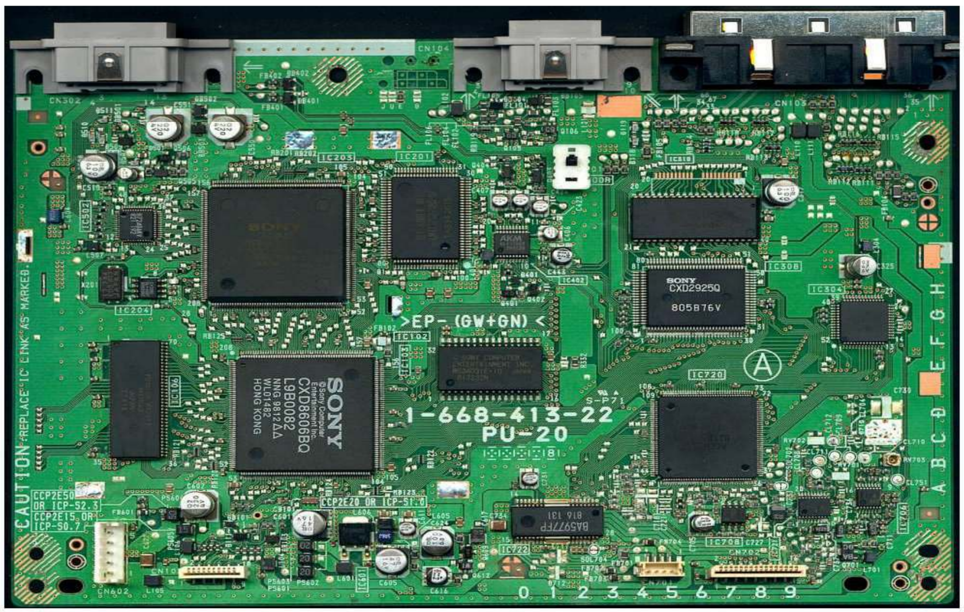


Computer Chips



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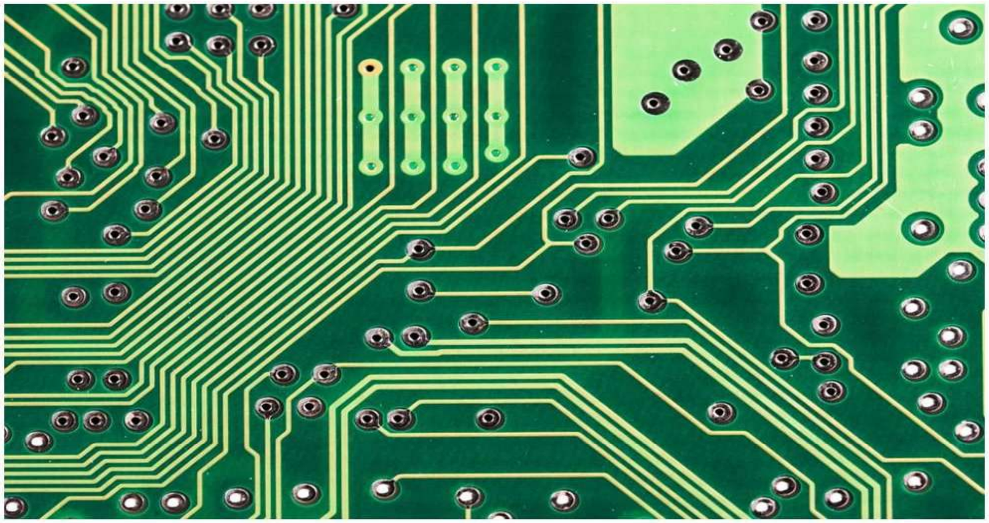

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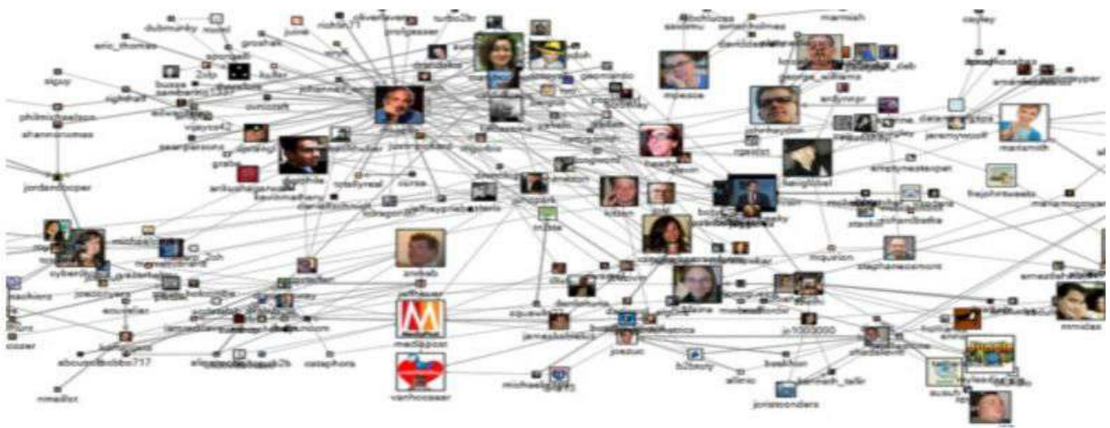
LECTURE # 02

Some Basic Definitions

The Degree of a Vertex

The number of friends



The Degree of a Vertex

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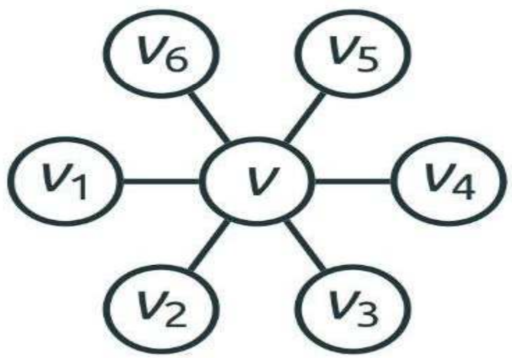
- The Degree of a vertex is the number of its incident edges
- I.e., the Degree of a vertex is the number of its neighbors
- The degree of a vertex v is denoted by deg(v)
- The degree of a graph is the maximum degree of its vertices

The Degree of a Vertex: Examples

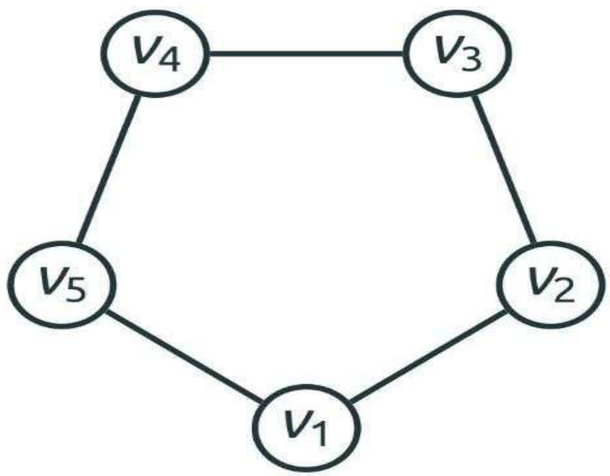
The degree of v is 6: $\deg(v) = 6$

The degree of v_6 is 1: $\deg(v_6) = 1$

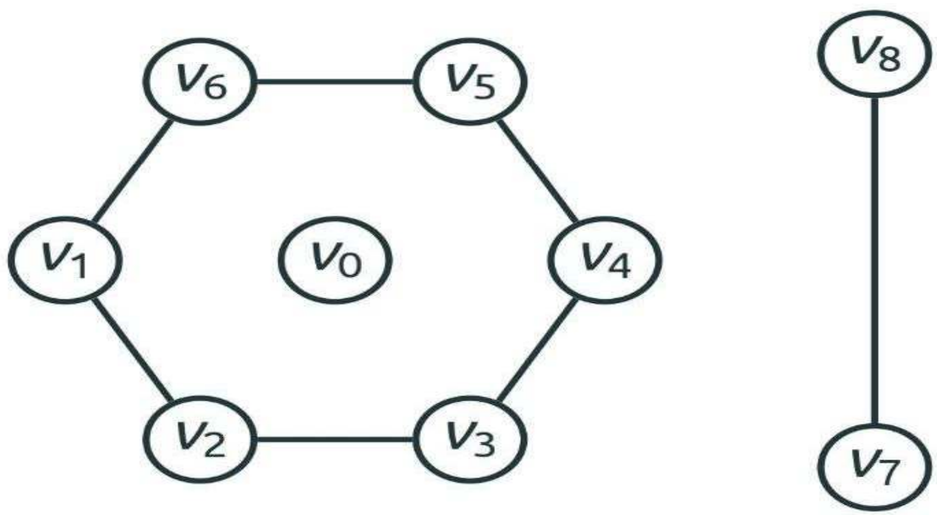


The Degree of a Vertex: Examples

The degree of every vertex is 2: $\forall i, \deg(v_i) = 2$

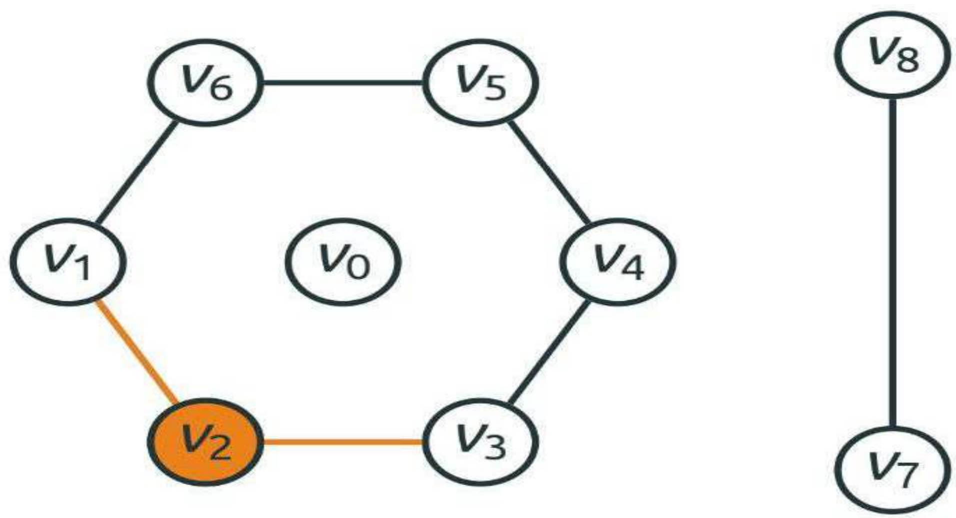


Isolated Vertices



Isolated Vertices

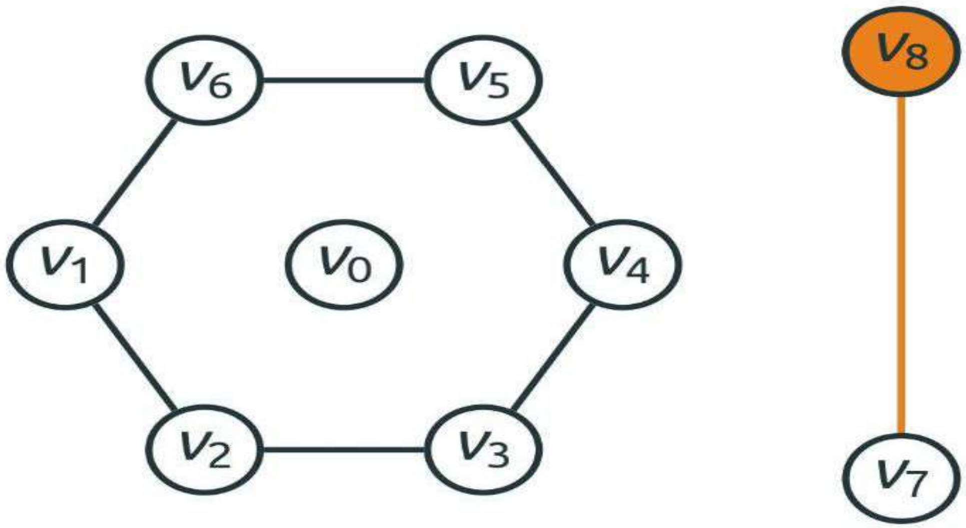
$\deg(v_2) = 2$



Isolated Vertices

$\deg(v_2) = 2$

$\deg(v_8) = 1$

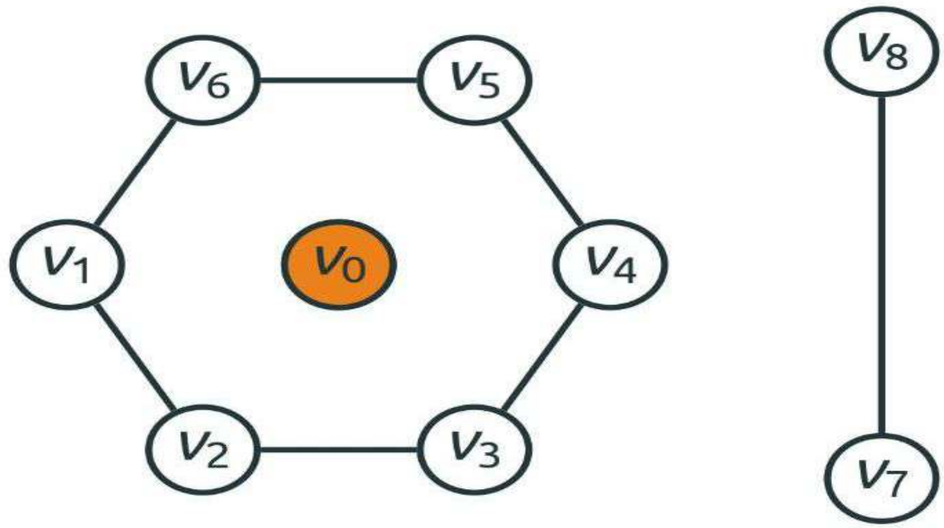


Isolated Vertices

$\deg(v_2) = 2$

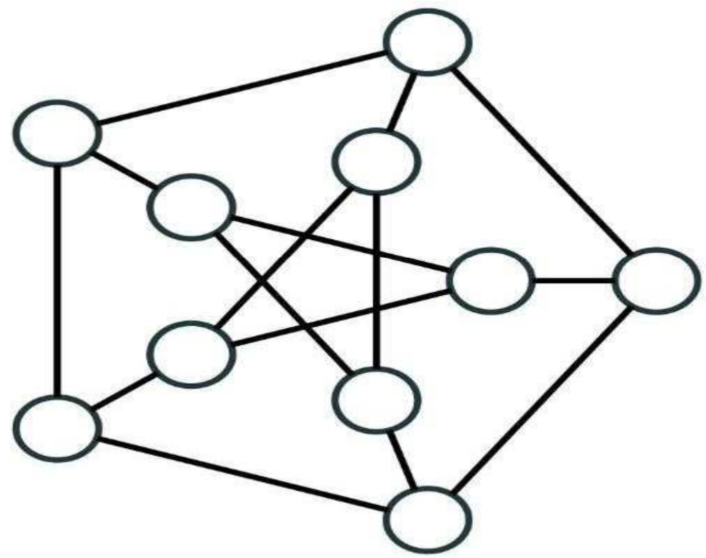
$\deg(v_8) = 1$

$\deg(v_0) = 0$. v_0 is an Isolated Vertex



Regular Graphs

A Regular graph is a graph where each vertex has the same degree



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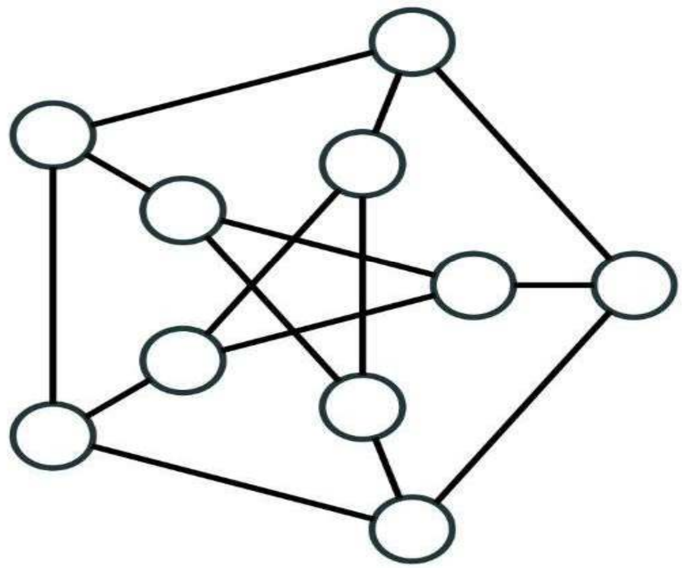
A regular graph of degree *k* is also called *k*-Regular



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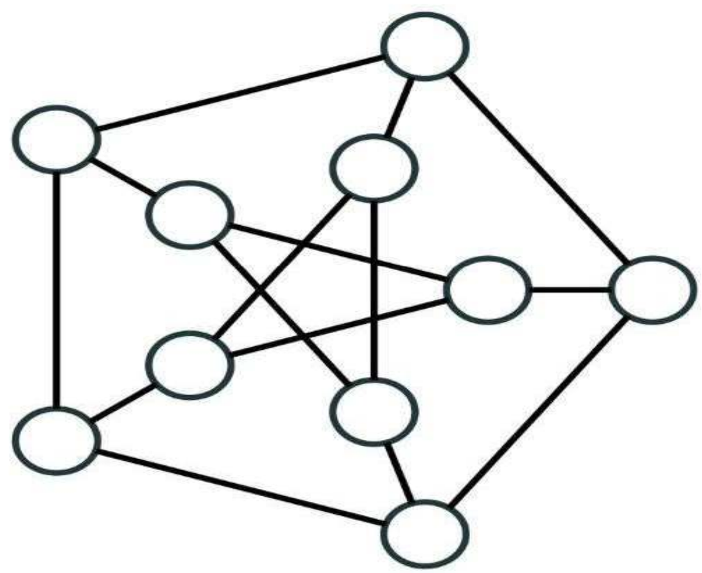
E.g., this graph is 3-Regular



Complement Graph

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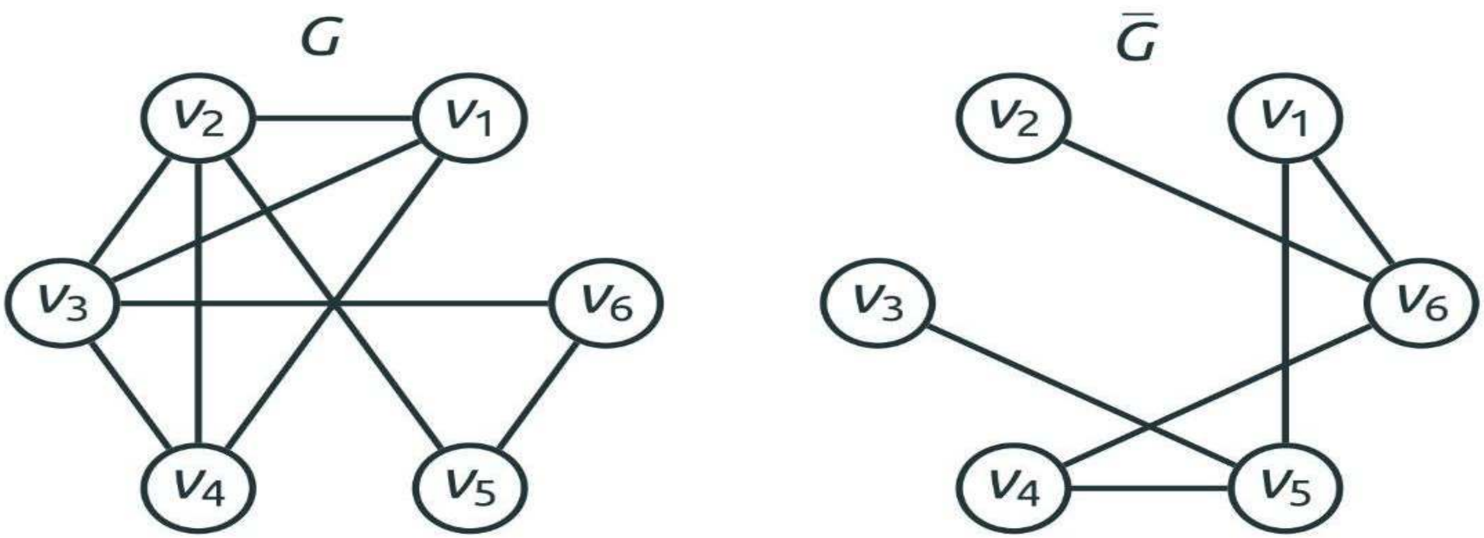
- I.e., $(u, v) \in \bar{E}$ if and only if $(u, v) \notin E$

Complement Graph



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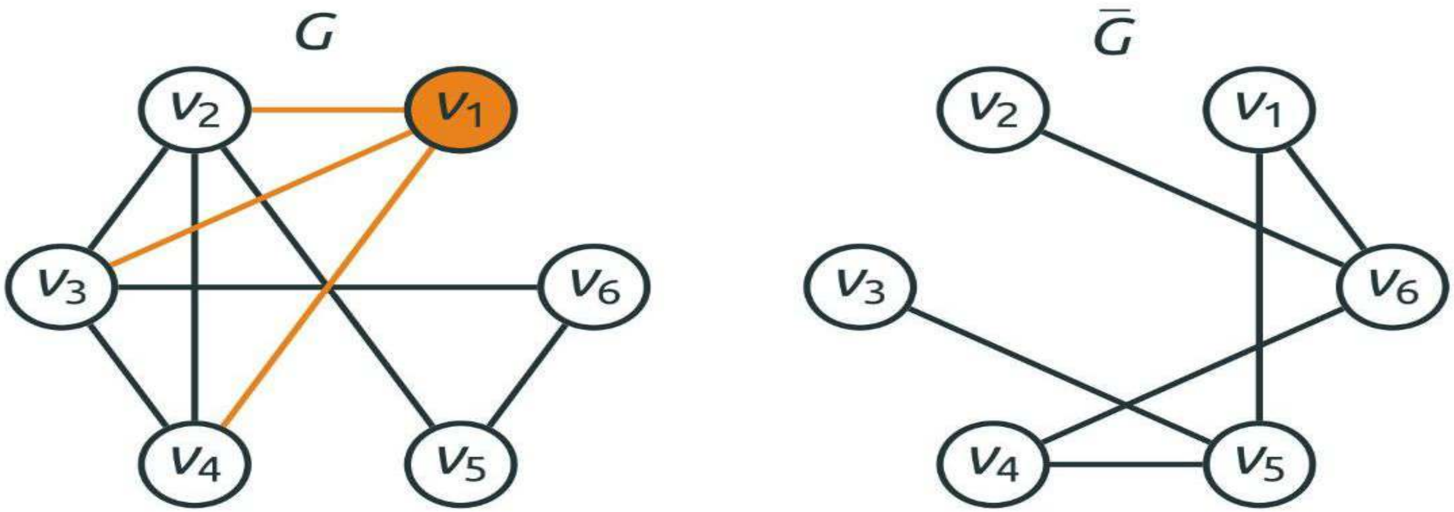

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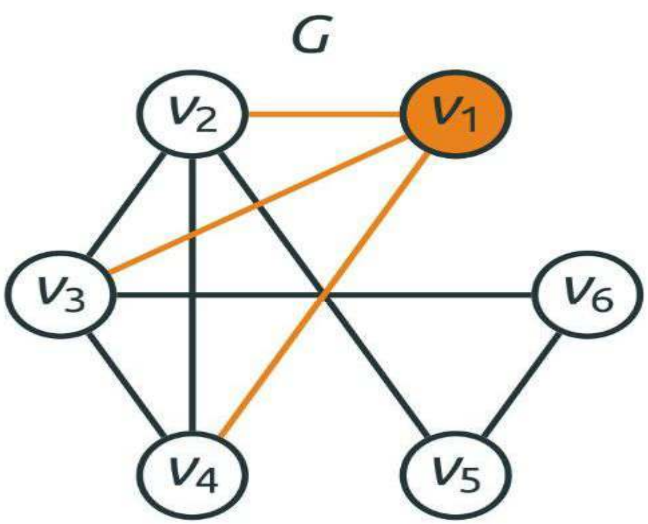

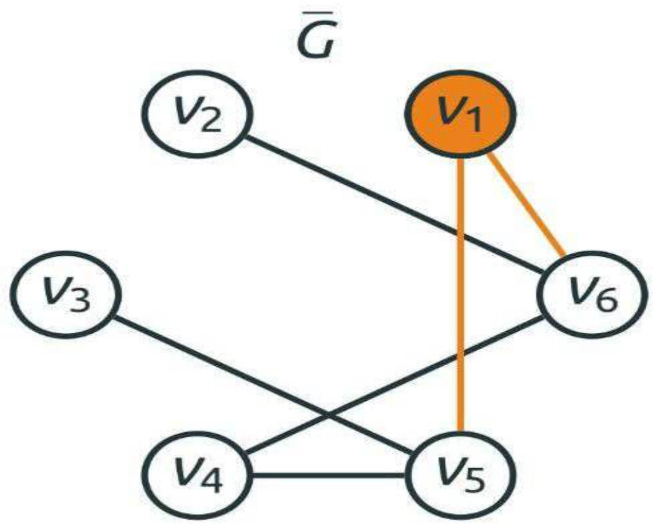

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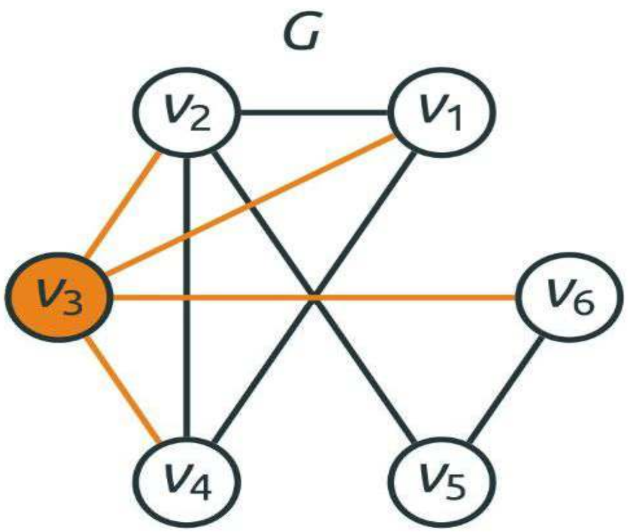

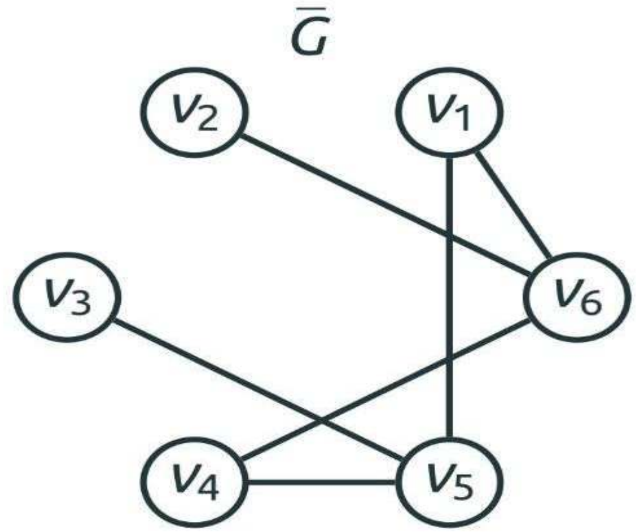

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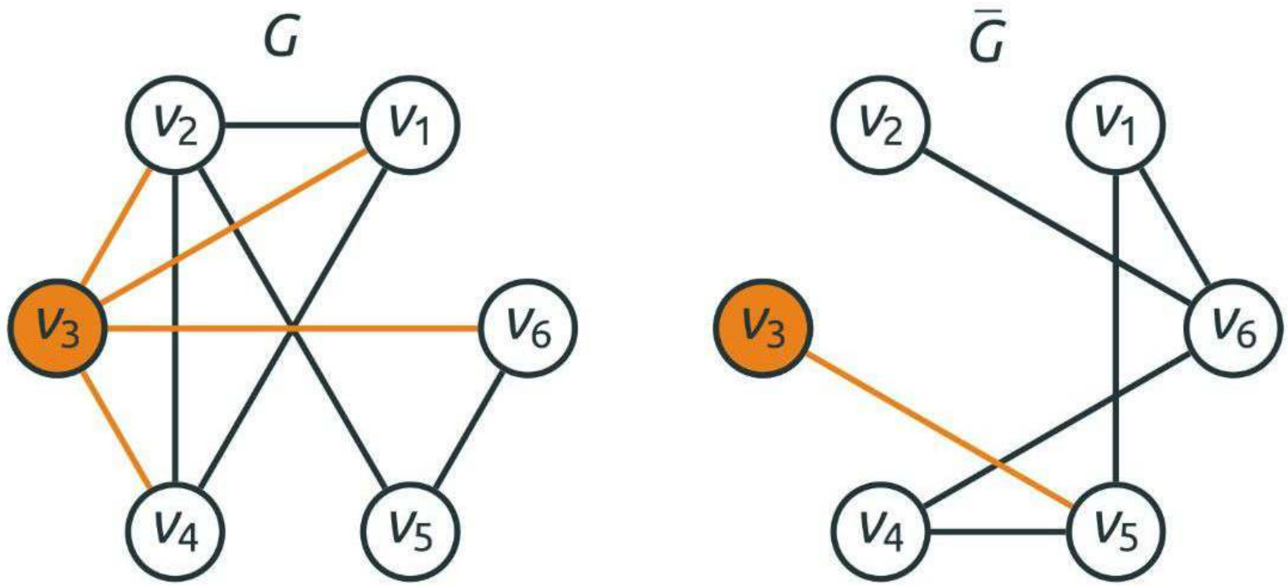

Paths

Is there a path from one point to another?

Walks

- A Walk in a graph is a sequence of edges, such that each edge (except for the first one) starts with a vertex where the previous edge ended

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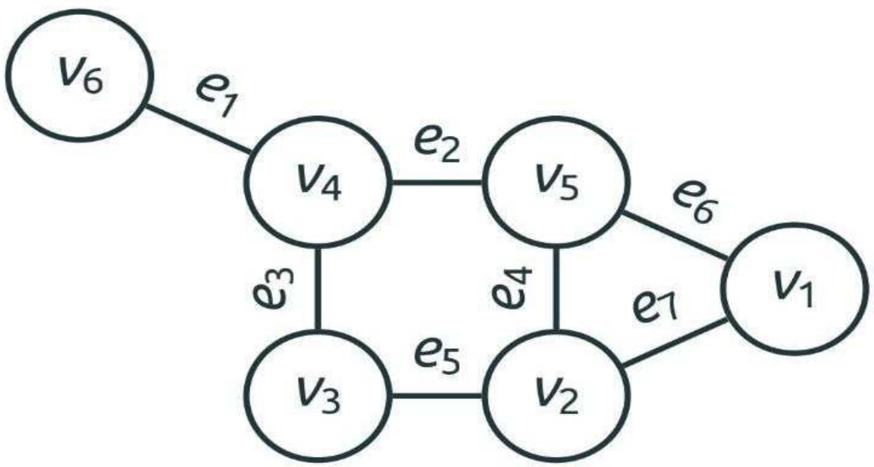
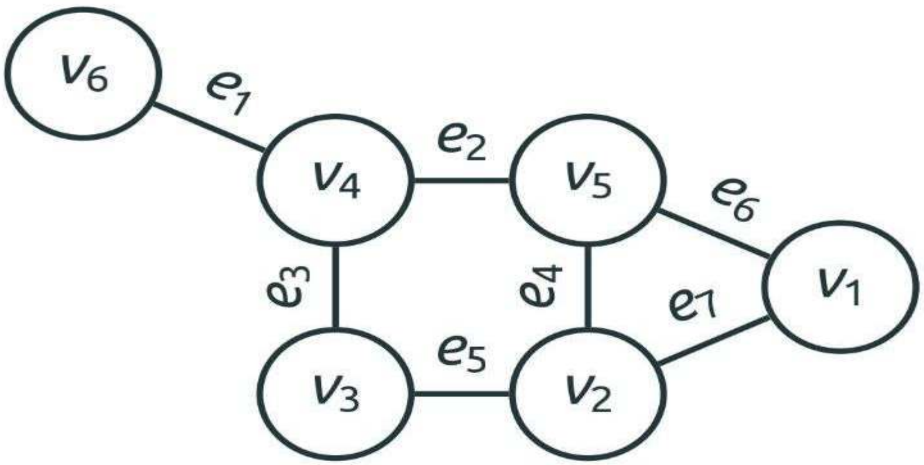
- The Length of a walk is the number of edges in it
- A Path is a walk where all edges are distinct
- A Simple Path is a walk where all vertices are distinct

Walks: Examples



Walks: Examples

A walk of length 6: $(e_1, e_2, e_4, e_5, e_3, e_1)$



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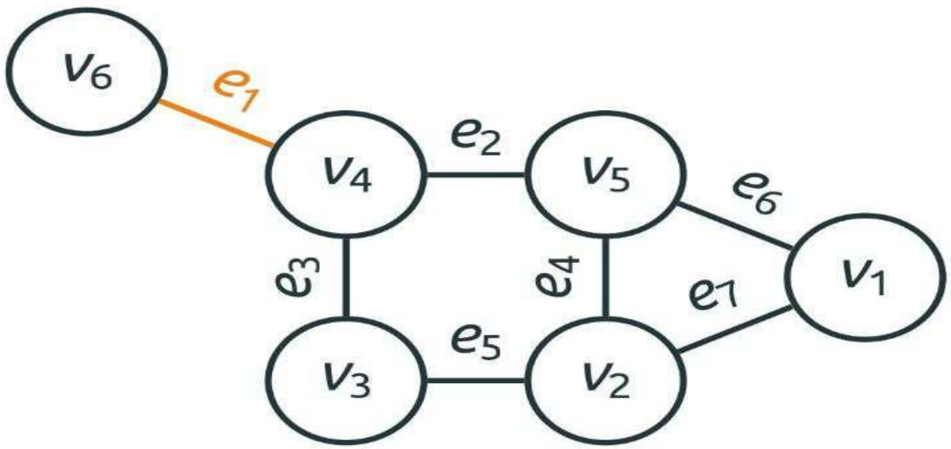

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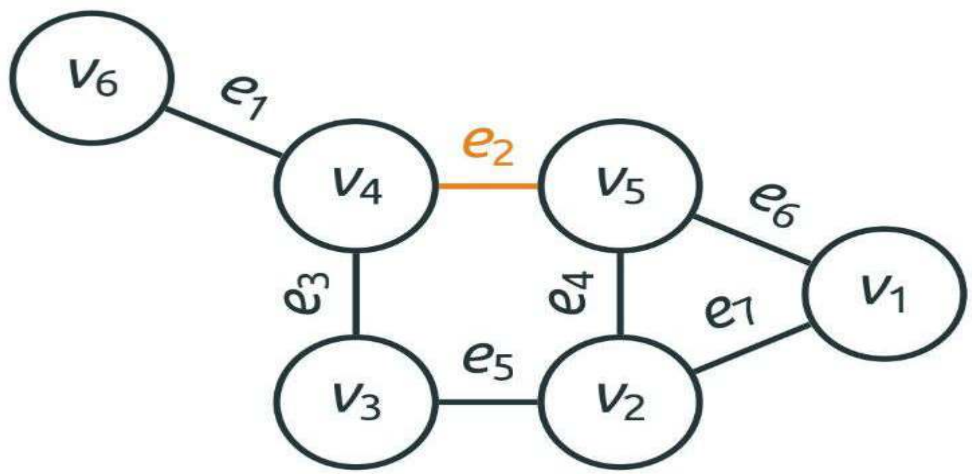

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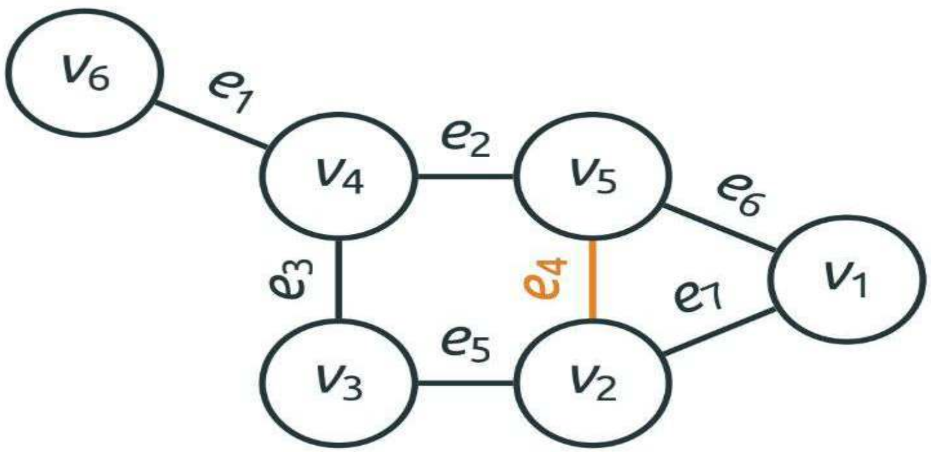

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Walks: Examples

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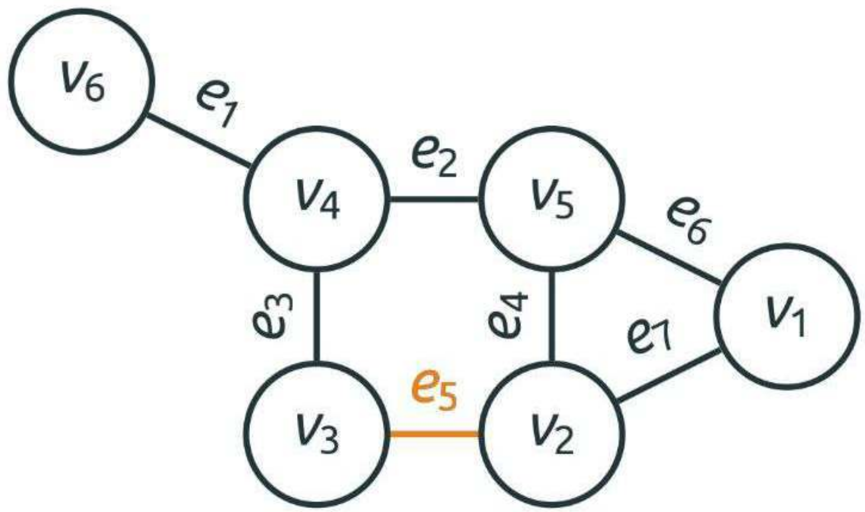

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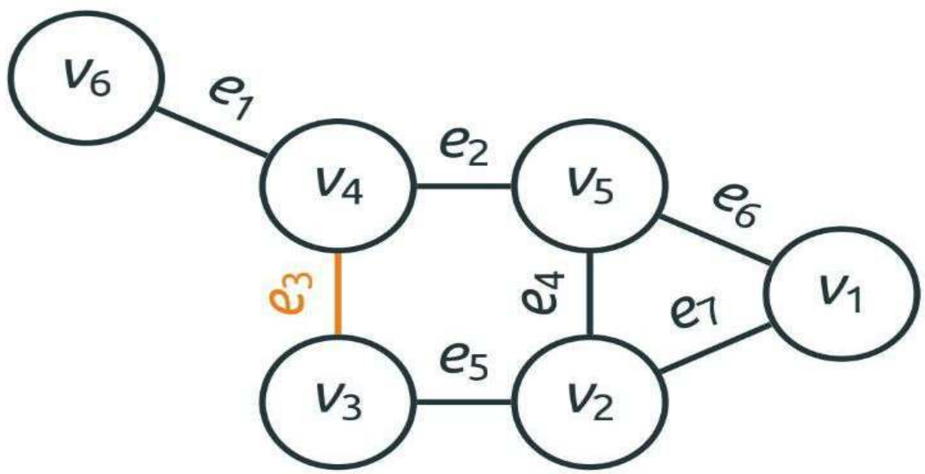

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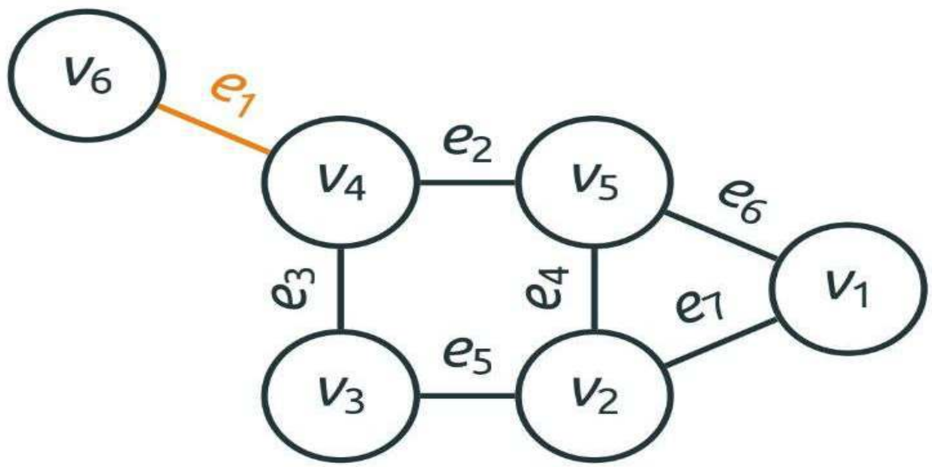
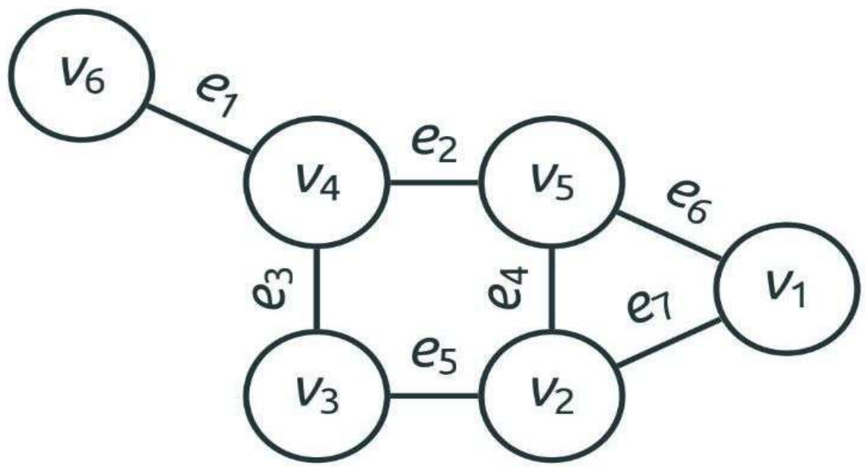


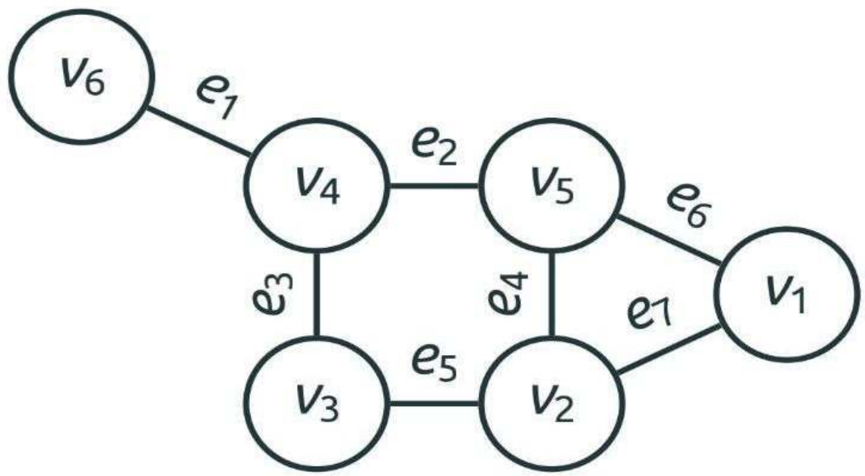
Walks: Examples

A walk of length 6: $(e_1, e_2, e_4, e_5, e_3, e_1)$
Not a path: uses e_1 twice



Walks: Examples

A path of length 4: (e_7, e_6, e_4, e_5)



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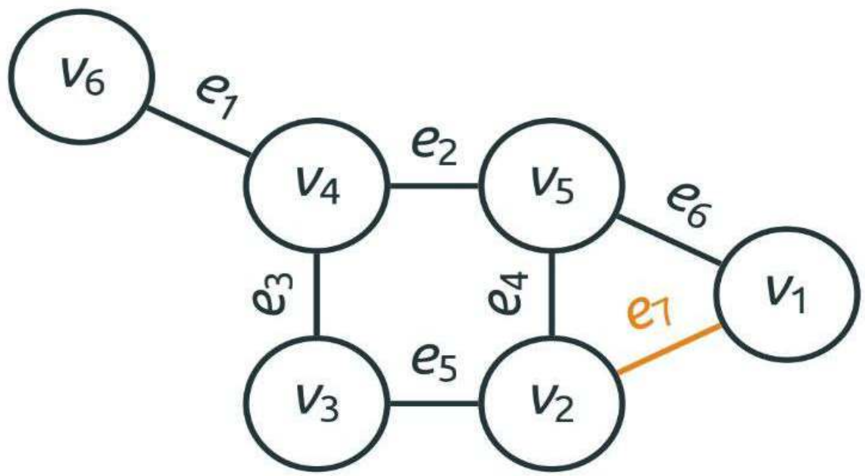

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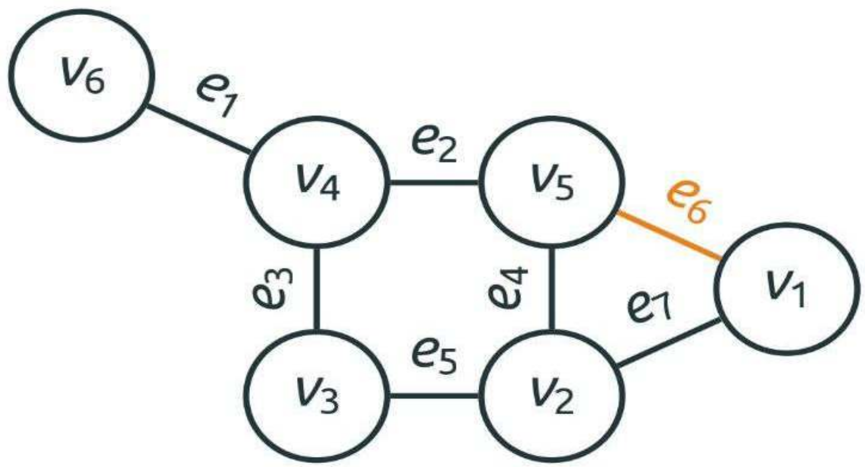

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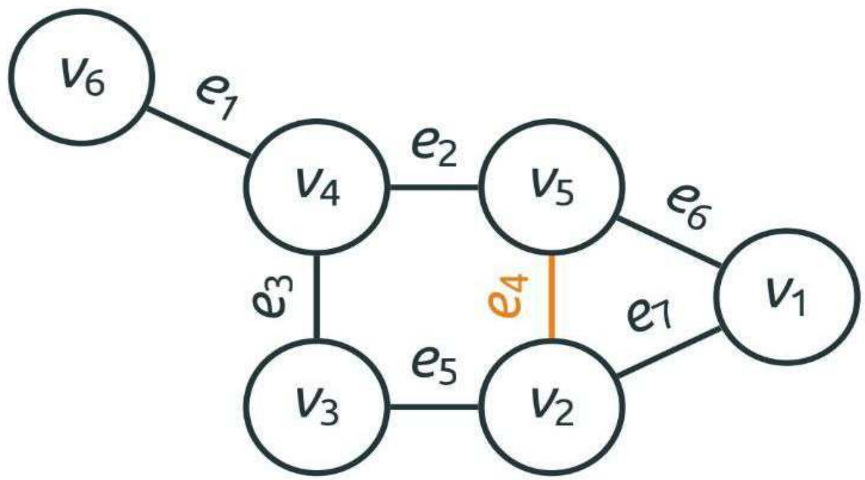

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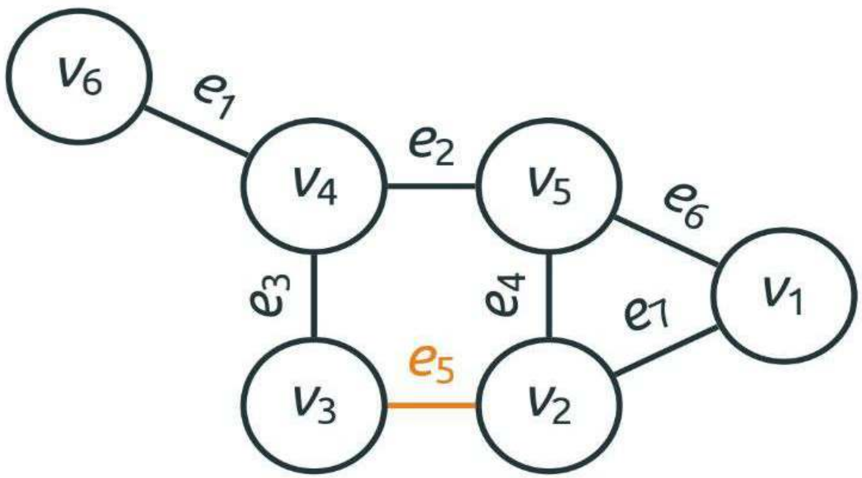

Walks: Examples

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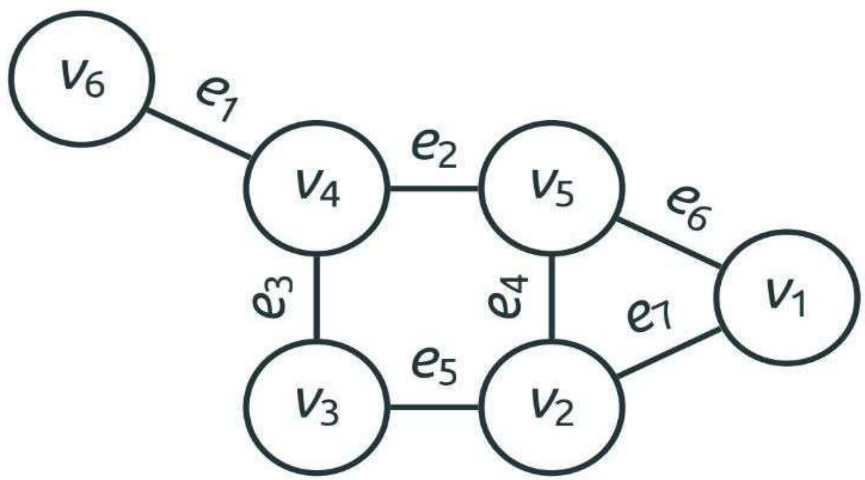
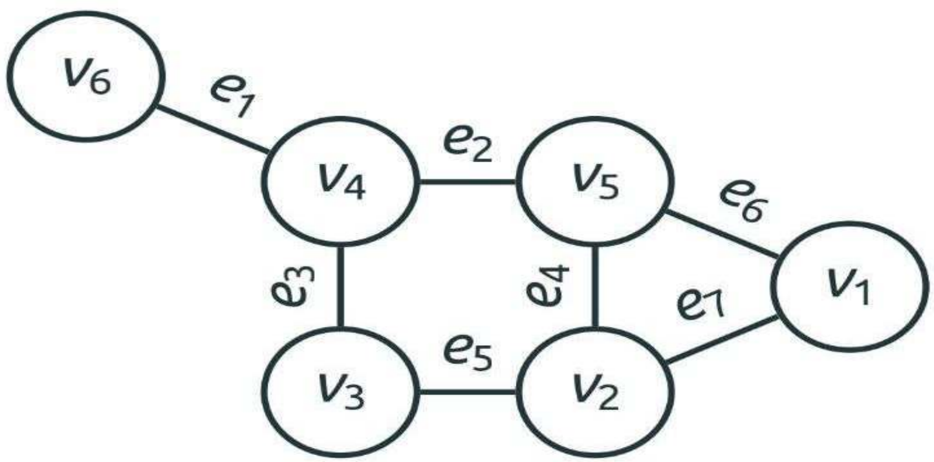
Not a simple path: visits v_2 twice



Walks: Examples

A simple path of length 4: (e_7, e_6, e_2, e_3)



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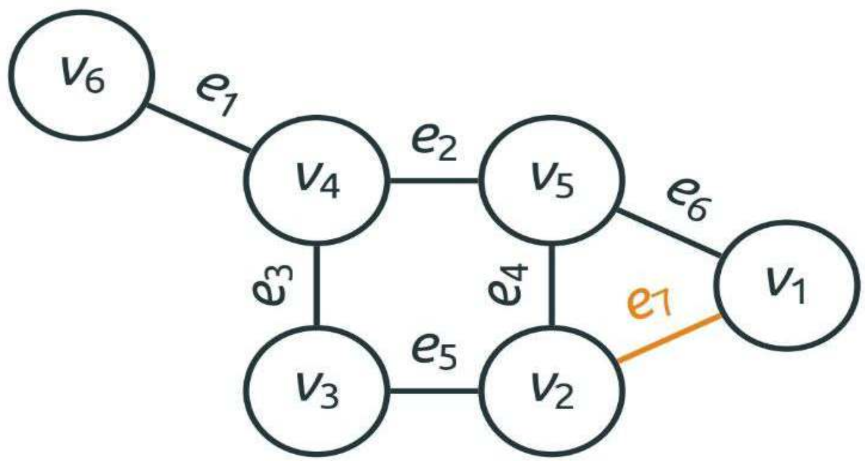

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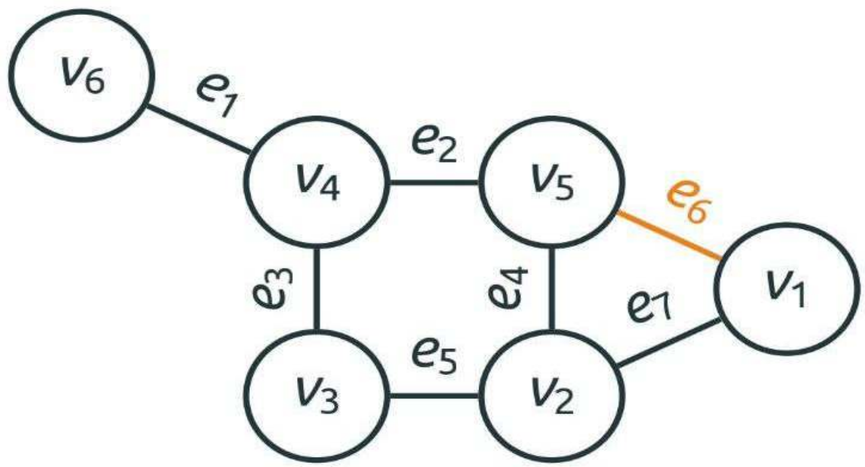

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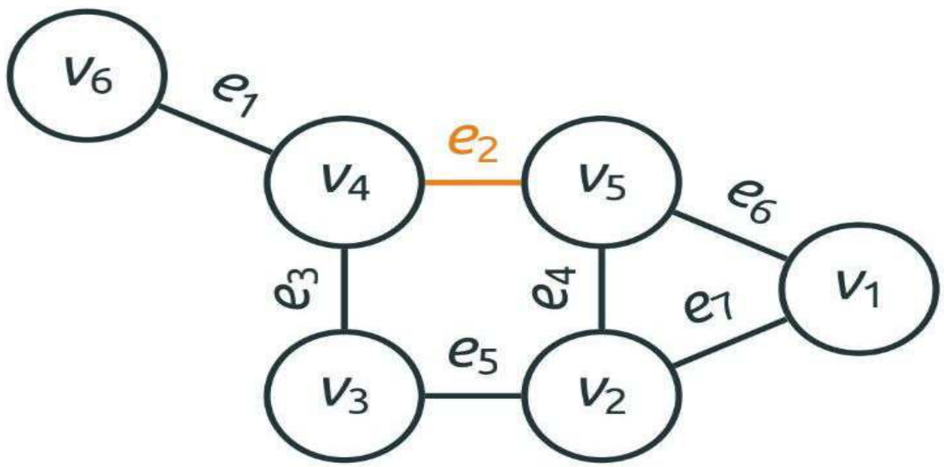

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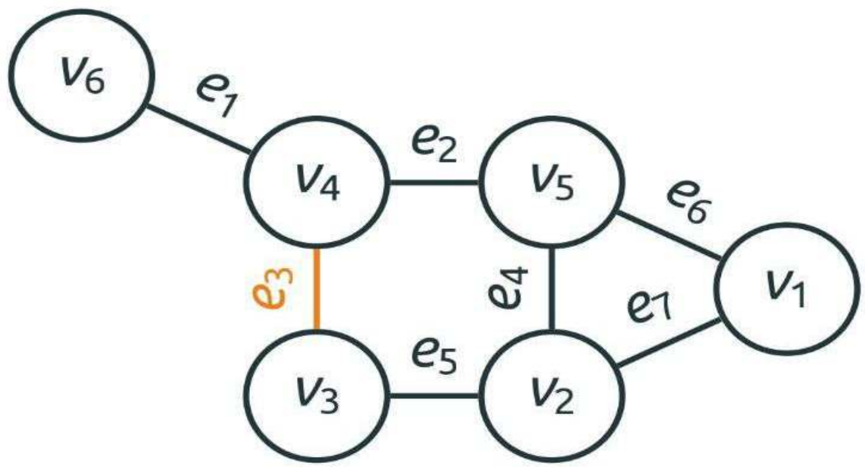
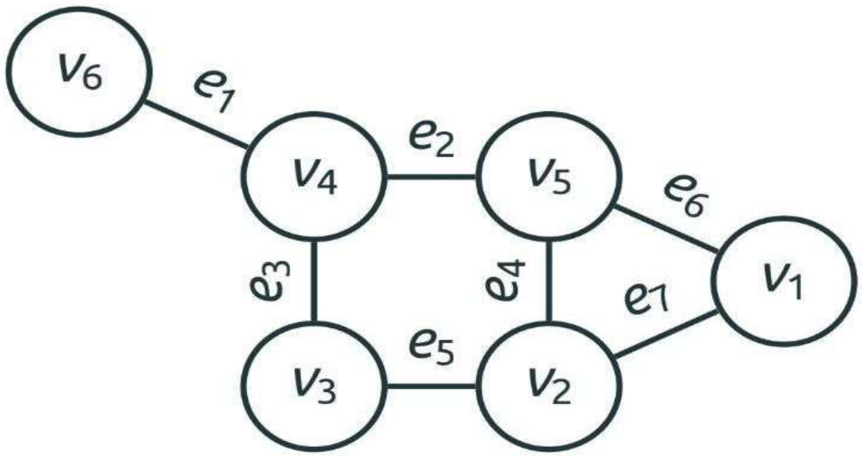


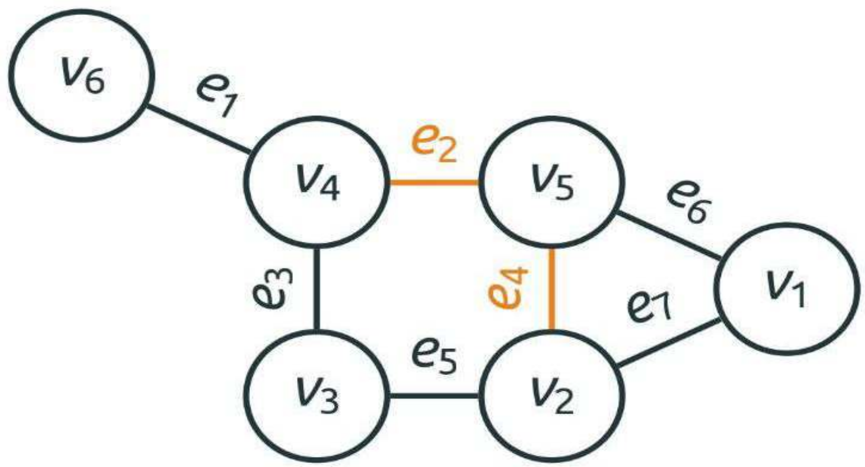
Walks: Examples

It is sometimes convenient to specify a path (walk) by a list of its vertices



Walks: Examples

(v_4, v_5, v_2) is a path of length 2



Cycles

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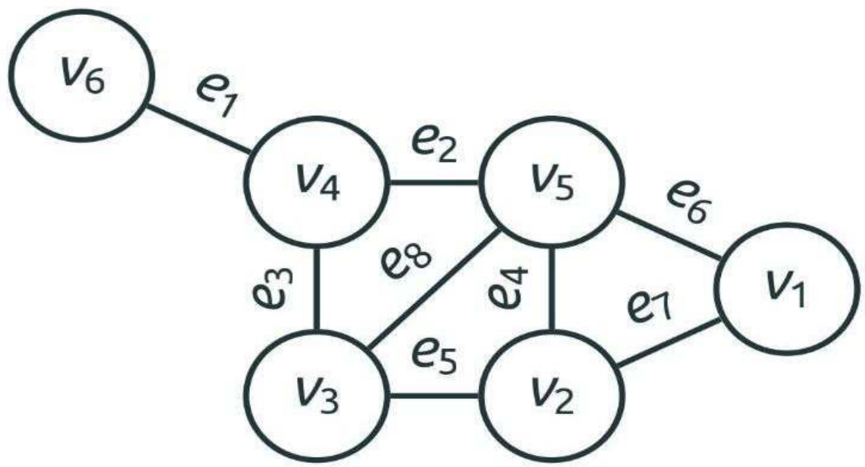
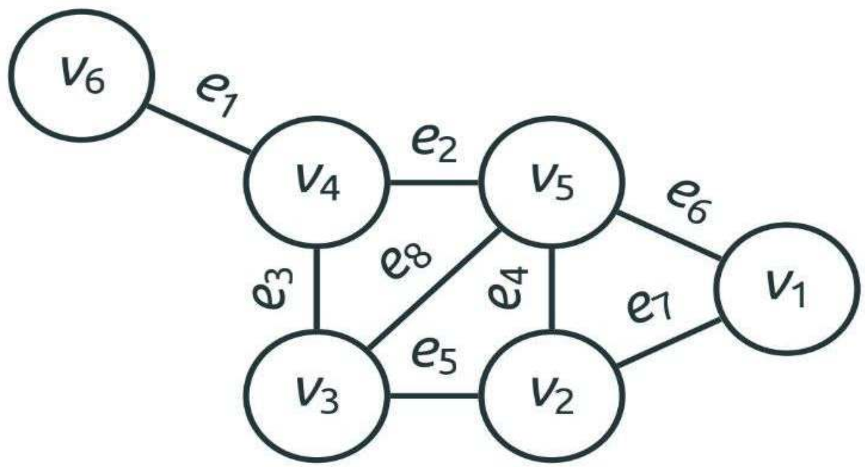
- In particular, all the edges in a Cycle are distinct
- A Simple Cycle is a cycle where all vertices except for the first one are distinct. (And there first vertex is taken twice)

Cycles: Examples



Cycles: Examples

A cycle of length 6: $(e_2, e_3, e_8, e_4, e_7, e_6)$



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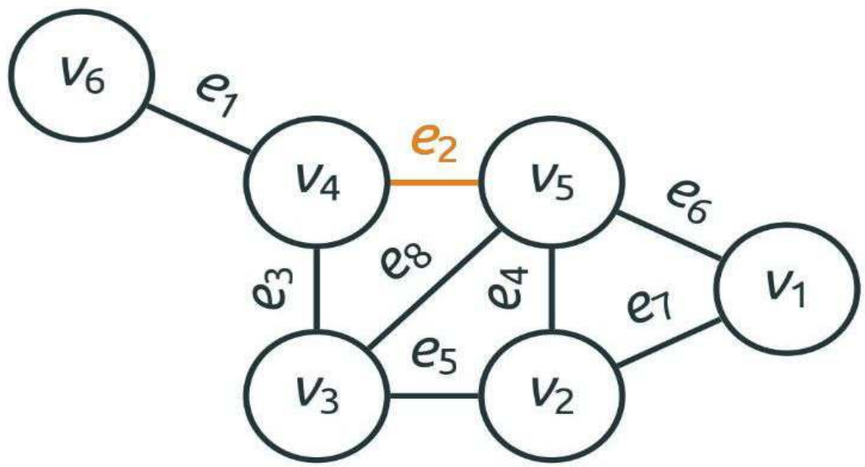

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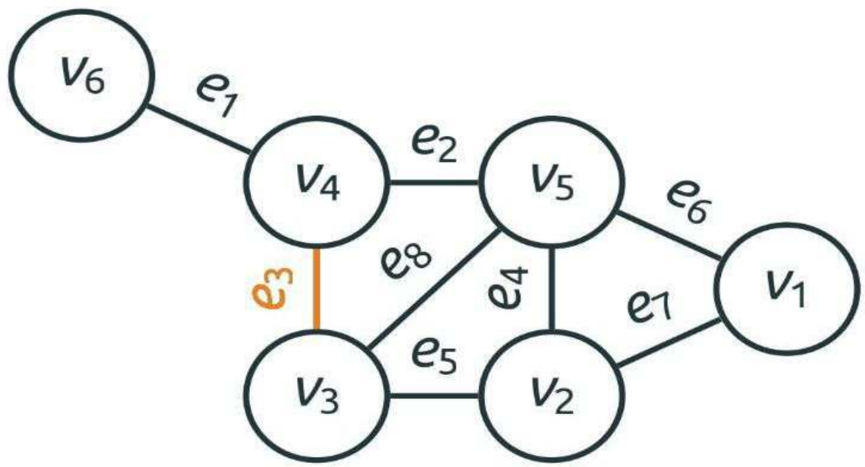

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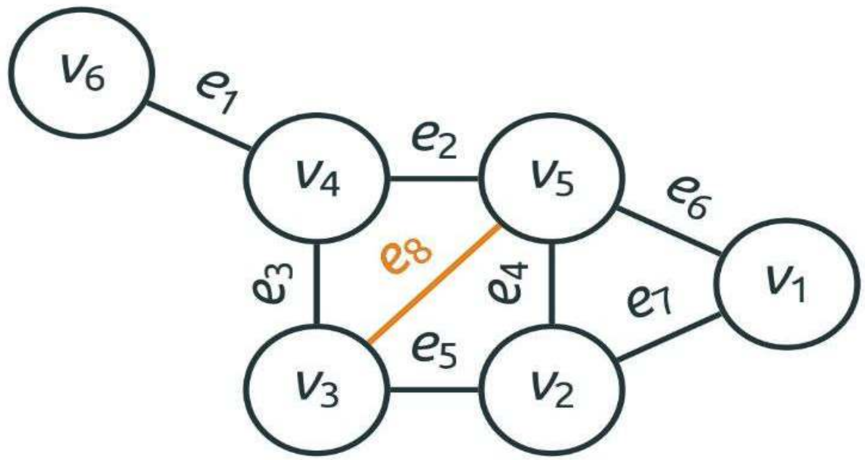

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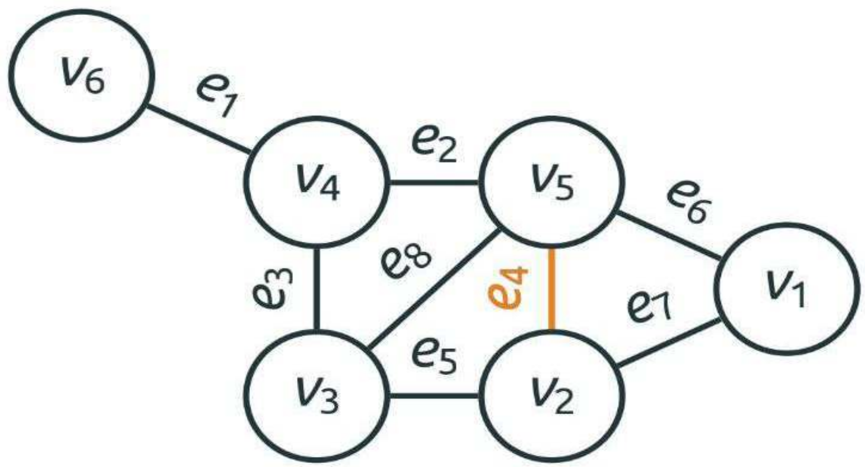

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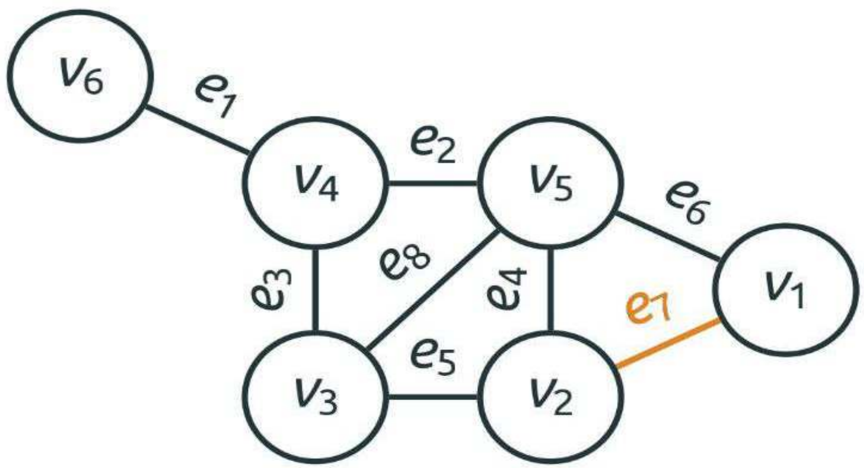

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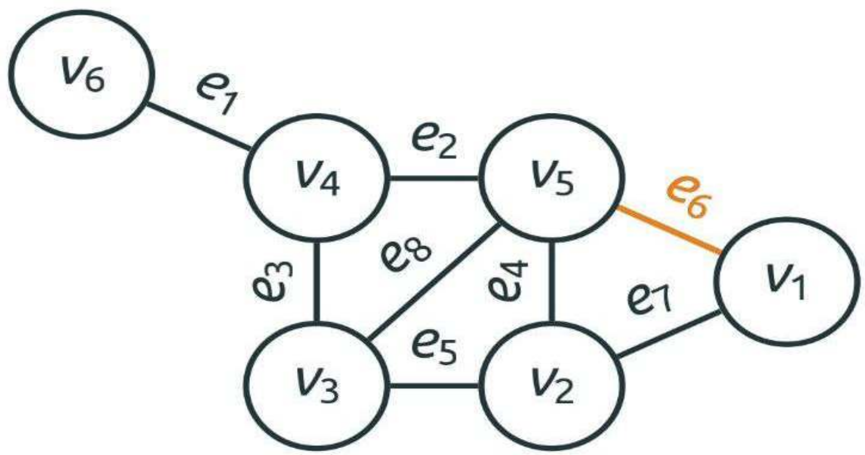
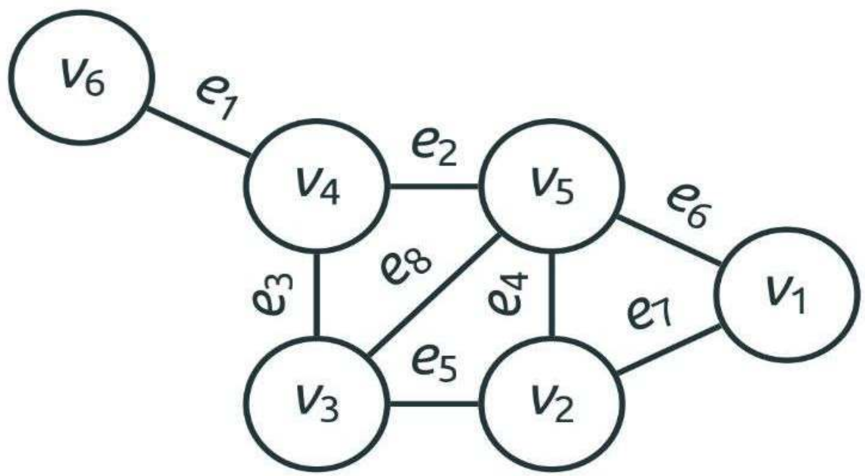


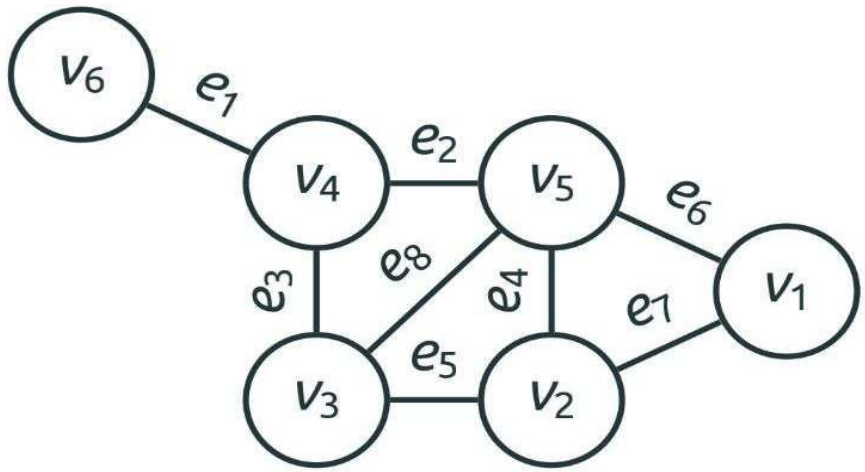
Cycles: Examples

A cycle of length 6: $(e_2, e_3, e_8, e_4, e_7, e_6)$
Not a simple cycle: visits v_5 three times



Cycles: Examples

A simple cycle of length 4: (e_5, e_4, e_2, e_3)



Cycles: Examples

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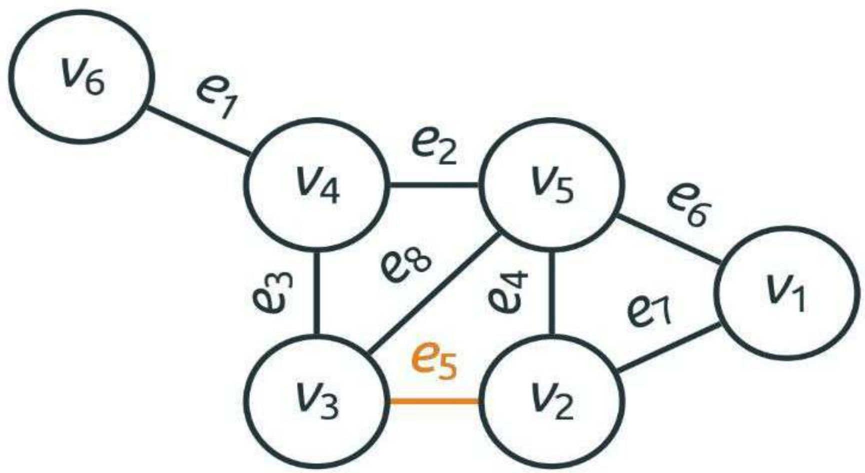

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Cycles: Examples

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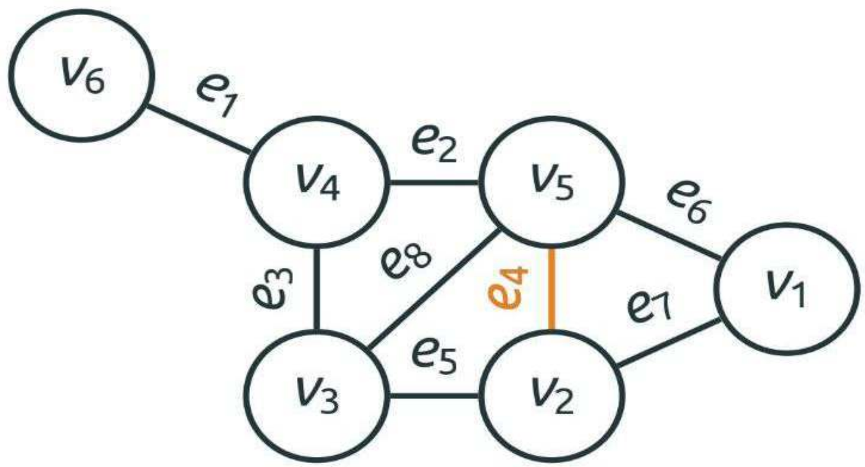

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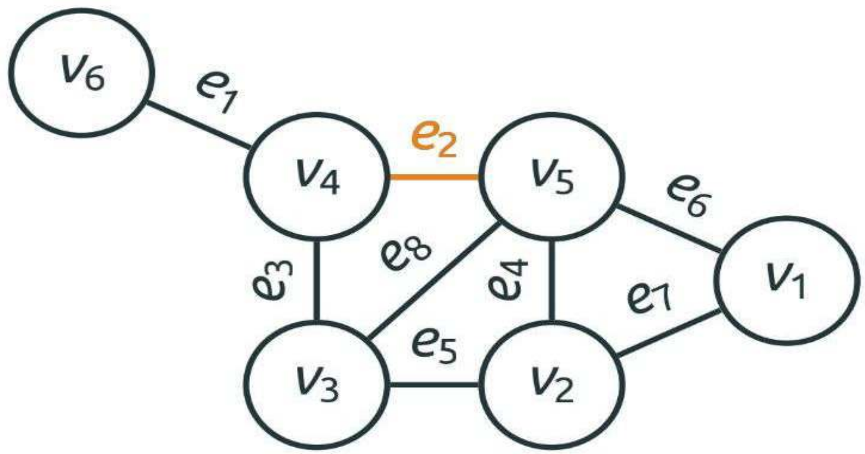

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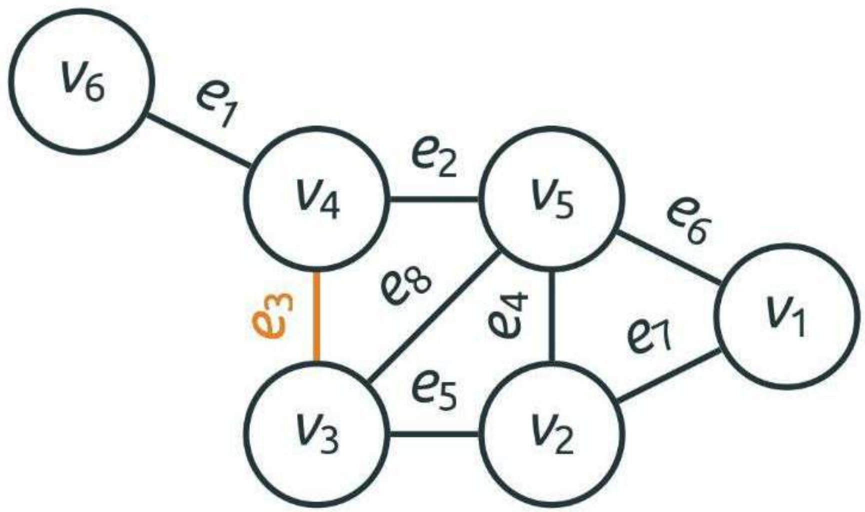
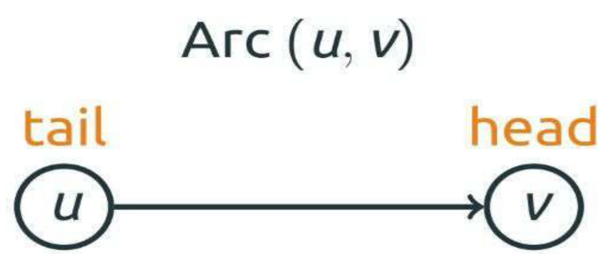


Directed Edge (Arc)



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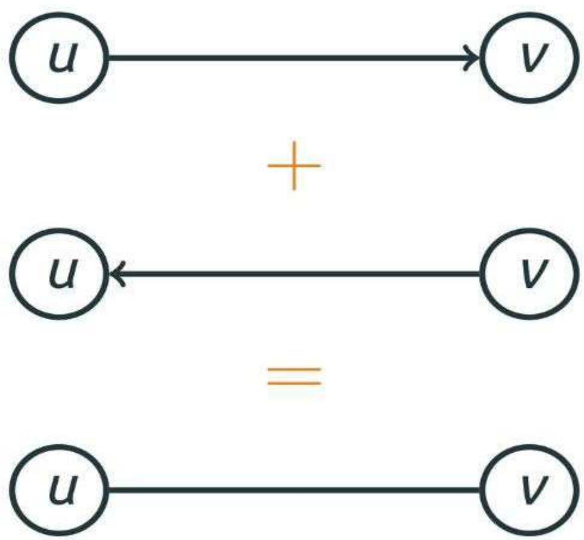
Directed Edge (Arc)

Arc (u, v)

Directed Edge (Arc)

Arc (u, v)



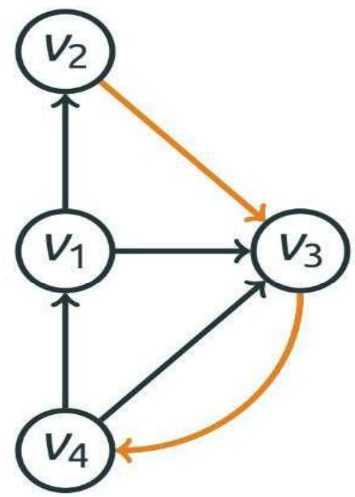
The Degree of a Vertex

- The Indegree of a vertex *v* is the number of edges ending at *v*

- The Outdegree of a vertex *v* is the number of edges leaving *v*

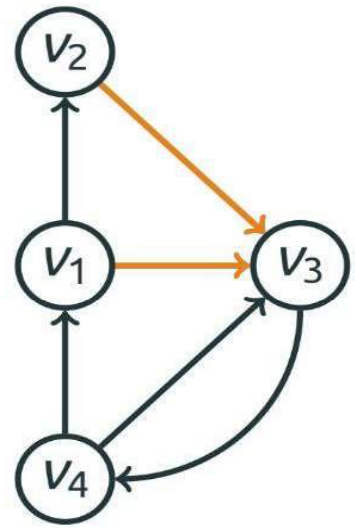
Directed Paths

(v_2, v_3, v_4) is a
Path of length 2



Directed Paths

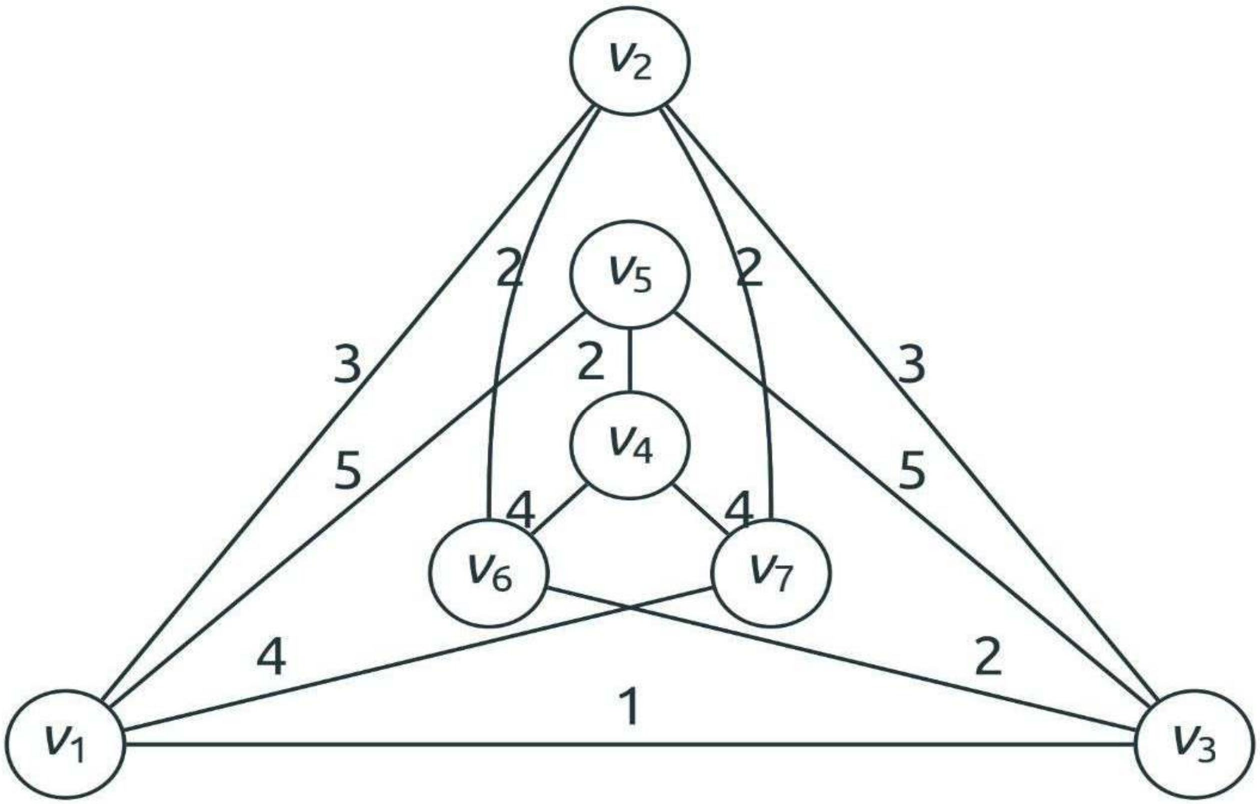
(v_1, v_3, v_2) is not a Path



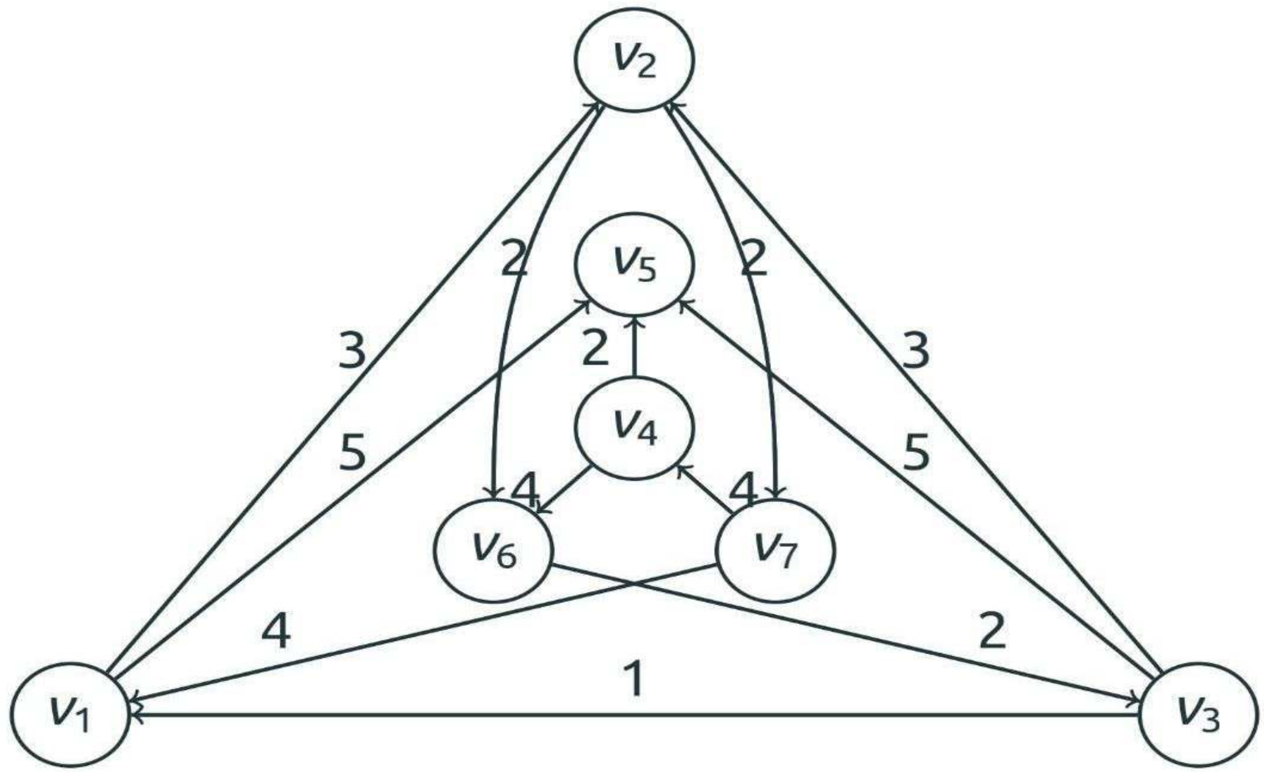
Weighted Graphs

Distance, Driving Time, etc.

Weighted Graphs: Examples



Weighted Graphs: Examples

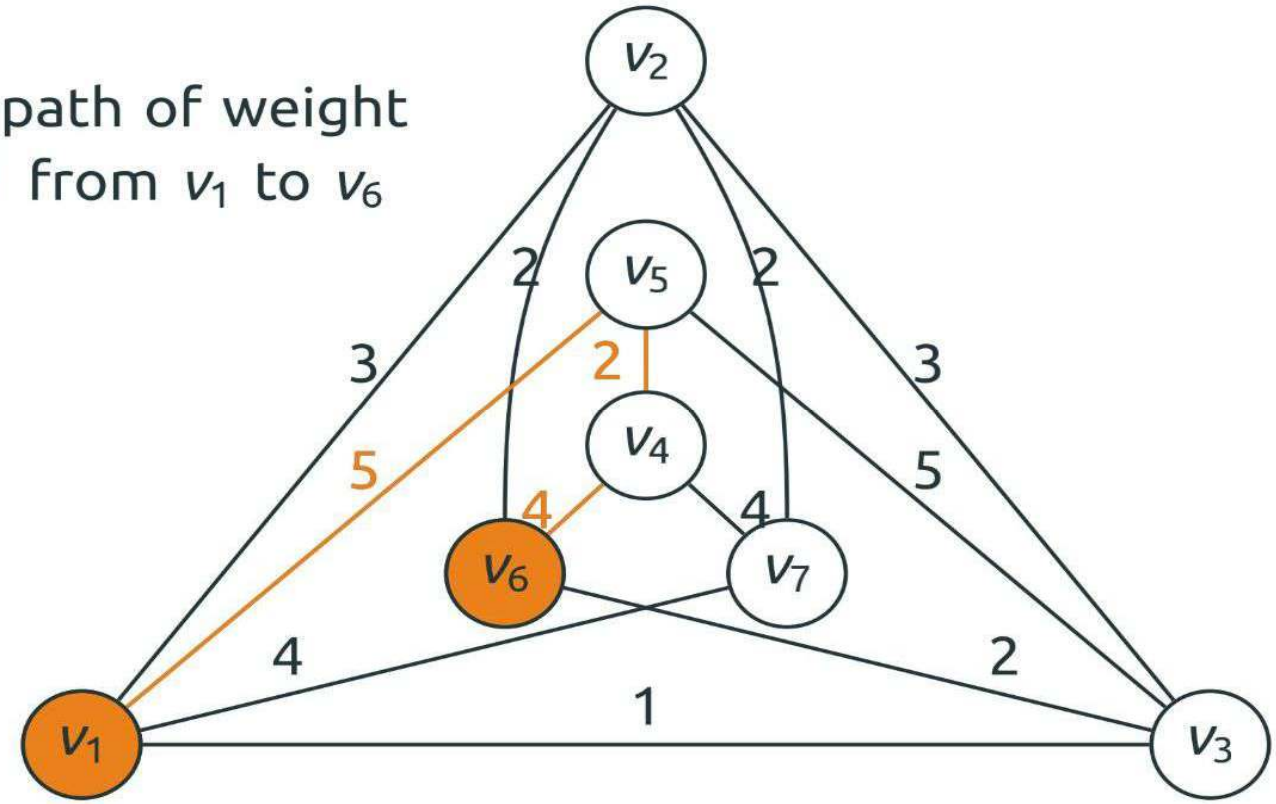


Weighted Paths

- A Weighted Graph associates a weight with every edge
- The Weight of a path is the sum of the weights of its edges
- A Shortest Path between two vertices is a path of the minimum weight
- The Distance between two vertices is the length of a shortest path between them

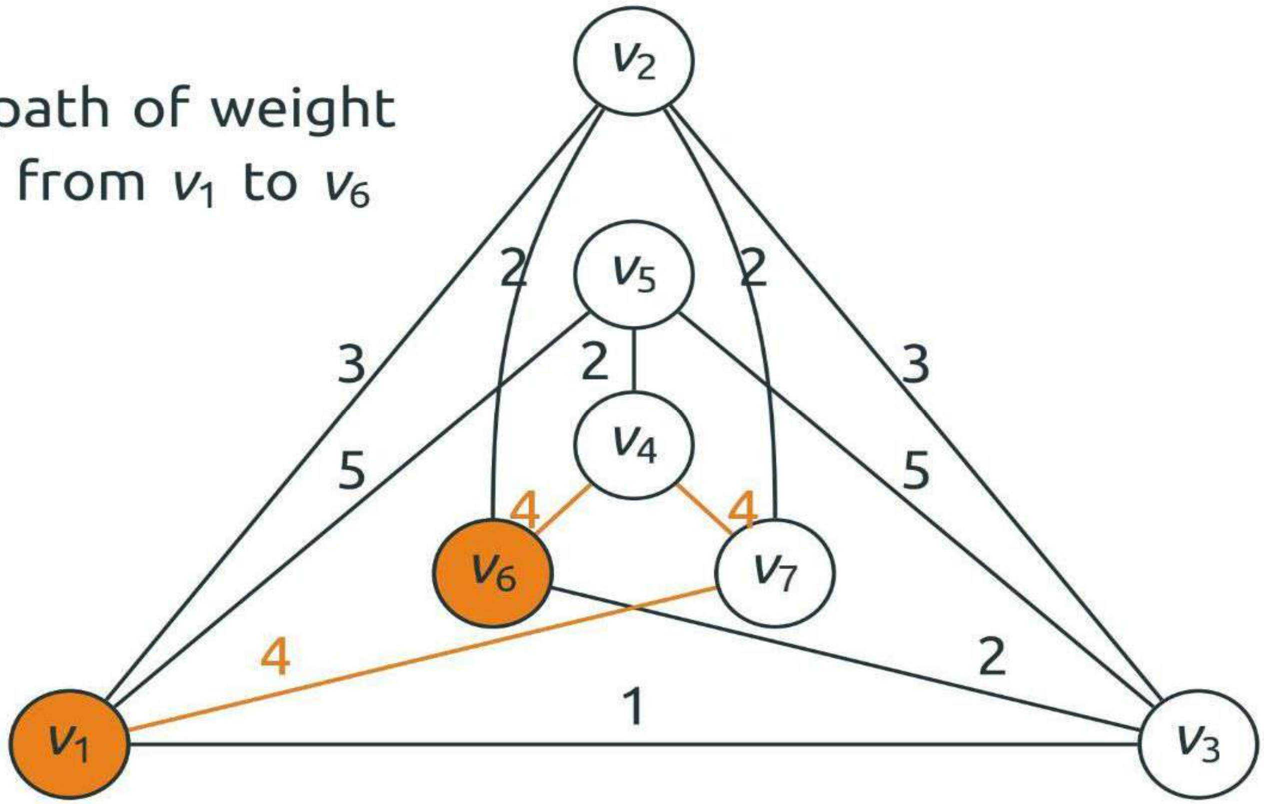
Weighted Paths: Examples

A path of weight 11 from v_1 to v_6



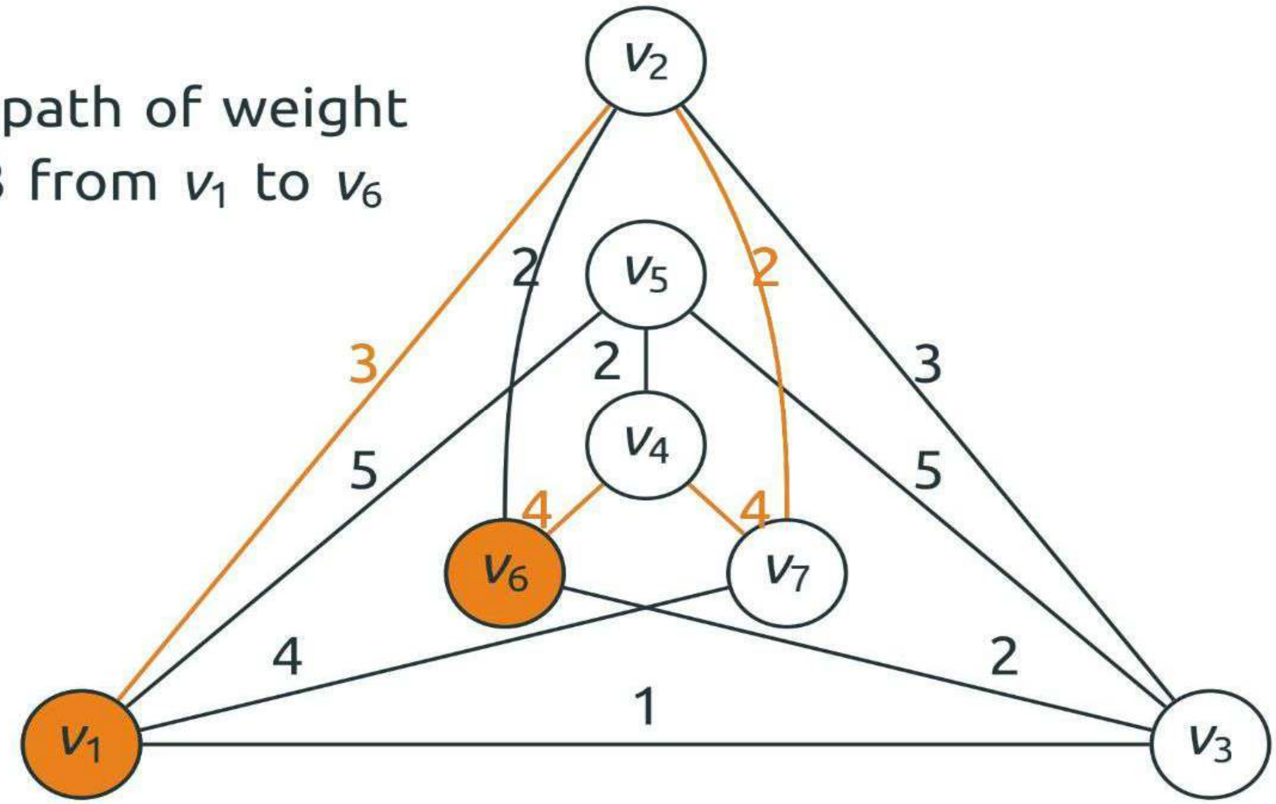
Weighted Paths: Examples

A path of weight 12 from v_1 to v_6



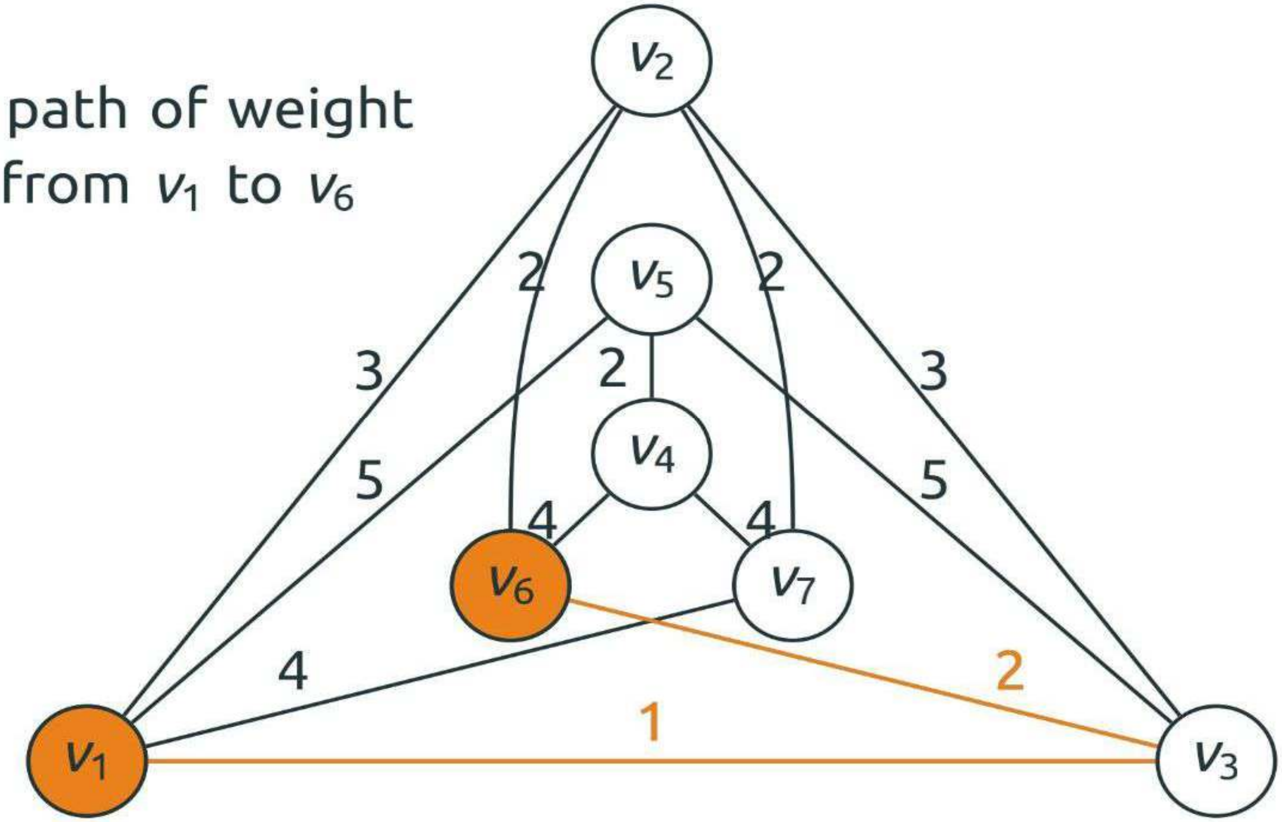
Weighted Paths: Examples

A path of weight 13 from v_1 to v_6



Weighted Paths: Examples

A path of weight
3 from v_1 to v_6



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Weighted Paths: Examples

The distance between v_1 and v_6 is 3