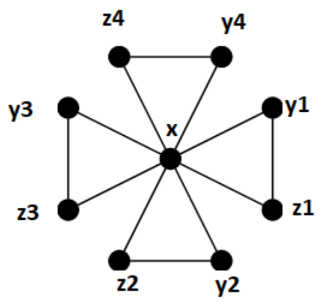
Explain your steps. The calculations and answers should be written neatly on paper which is attached as a single pdf. Submits the solution on GoogleClassroom with in dua date.

Problem 1

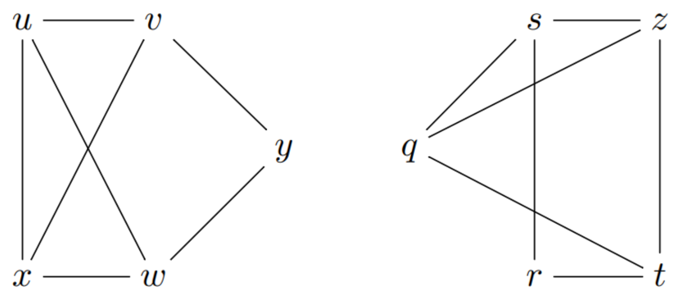
The friendship graph F_n has $2n+1$ vertices $x, y_1, \cdots, y_n$, and $z_1, \cdots, z_n$. The vertex x is adjacent to all other vertices; also, vertices y_i and z_i are adjacent for $i = 1, \cdots, n$. There are no other edges. Draw a diagram of the friendship graph F_4.

Solution:



Problem 2

Find a vertex-bijection that specifies and isomorphism between the two graphs:



Solution: The vertex-bijection is

V(left graph)	u	v	w	x	y
V(right graph)	z	s	t	q	r

The edge-bijection is

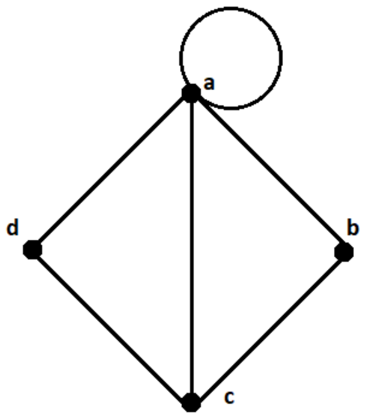
E(left graph)	uv	uw	ux	vx	vy	wx	wy
E(right graph)	sz	tz	qz	qs	rs	qt	rt

Problem 3

For the graph with the adjacency matrix A. (The vertices are in the order a, b, c, d)

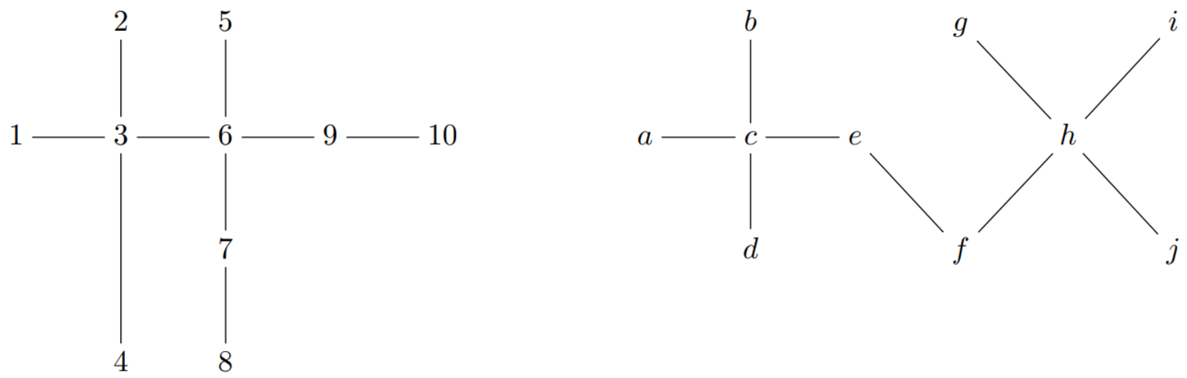
$$A = \begin{pmatrix} 2 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}$$

Solution:



Problem 4

Isomorphic? produce isomorphism. Non-isomorphic? Explain



Solution:

In the left graph, vertices 3 and 6 have four degree and adjacent, whereas, in the right graph, vertices c and h have four degree but are not adjacent. Hence given graphs are not isomorphic.

Problem 5

A 20-vertex graph has 62-edges. Every vertex has degree 3 or 7. How many vertices have degree 3?

Solution: First we tag 3 degree vertex as X and 7 degree vetrex as $20 - X$. There degree sum is $3X + 7(20 - X) = 2(124)$ implies $X = 4$

Problem 6

Either draw a 3-regular 7-vertix graph or prove that none exits.

Solution:

First we compute degree sum
$3 \times 7 = 21$ which is an odd number and contraduct with Hand-Shaking lemma.

Problem 7

Determine whether or not the sequence $6, 5, 4, 4, 3, 2$ is graphical. If it is, draw a graph with that degree sequence.

Solution:

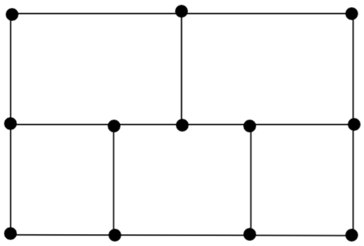
By applying Havel-Hakimi theorem, we have:

$$\begin{array}{cccccc} 6 & 5 & 4 & 4 & 3 & 2 \\ & 4 & 3 & 3 & 2 & 1 \\ & & 2 & 2 & 1 & 0 \\ & & & 1 & 0 & 0 \\ & & & & -1 & 0 \end{array}$$

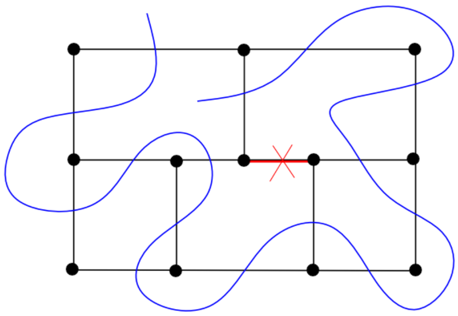
As the last sequence contains -1 , which is a contradiction so given sequence is not graphical.

Problem 8

Consider the following blueprint for a house. Each box represents a room, and the lines connecting two dots denote walls separating adjoining rooms (so there are 16 walls in all).



The problem is to draw a curve in the plane that passes through each wall exactly once. The curve may start at any point in the plane and end at any other point (the start and end points don't have to be the same). Here is an example curve that misses a wall



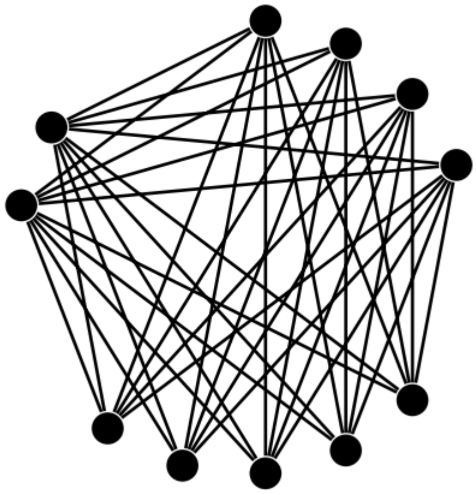
Also, one cannot *kill two walls with one pass*, meaning that you cannot go through a corner and claim that you covered two (or three) walls at the same time. Either find such a curve or show that none exists.

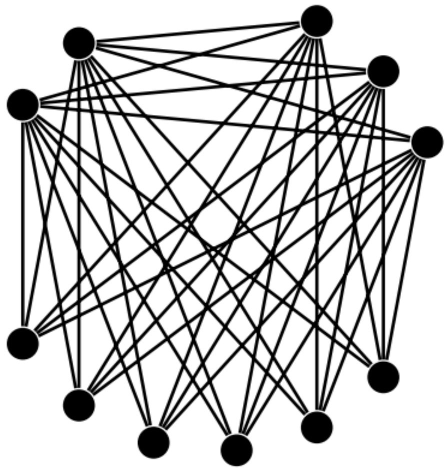
Solution:

From the statement of problem, we conclude that we have to find semi-Eulerian trail in the given graph, but there are more than two vertices of odd degree. Hence there is no such path exist in the given graph.

Problem 9

Below are diagrams of $K_{2,4,5}$ (left) and $K_{2,3,6}$ (right).

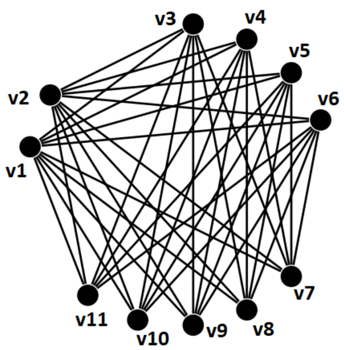




One of these graphs is Hamiltonian. Find a Hamiltonian cycle in that graph.

Solution:

The graph $K_{2,4,5}$ is complete, so it has a Hamiltonian cycle. First we label the given graph as



The Hamiltonian cycle is $v_1 v_3 v_7 v_4 v_8 v_5 v_9 v_6 v_{10} v_2 v_{11} v_1$