CS 230 : Discrete Computational Structures
Spring Semester, 2023
ASSIGNMENT #8
Due Date: Thursday, April 6

Suggested Reading: Rosen Sections 5.3; Lehman et al. Chapter 7

For the problems below, explain your answers and show your reasoning.

1. **[12 Pts]** Let S defined recursively by (1) $5 \in S$ and (2) if $s \in S$ and $t \in S$, then $st \in S$. Let $A = \{5^i \mid i \in \mathbf{Z}^+\}$. Prove that

   (a) **[6 Pts]** $A \subseteq S$ by mathematical induction.

   (b) **[6 Pts]** $S \subseteq A$ by structural induction.

2. **[5 Pts]** Give an inductive definition of the set of palindromes over the alphabet $\{a, b, c\}$. You do not need to prove that your construction is correct. *Note*: a, b, c, aa, cc, aba are all palindromes.

3. **[5 Pts]** Define the set $S = \{2^k 3^m \mid k, m \in \mathcal{Z}^+\}$ inductively. You do not need to prove that your construction is correct.

4. **[8 Pts]** Given the inductive definition of full binary trees (FBTs), define $n(T)$, the number of vertices in tree T, and $\ell(T)$, the number of leaves in tree T, inductively. Then, use structural induction to prove that for all FBTs T, $n(T) = 2\ell(T) - 1$.

5. **[15 Pts]** Let $L = \{(a, b) \mid a, b \in \mathcal{Z}, (a - b) \bmod 3 = 0\}$. We want to program a robot that can get to each point $(x, y) \in L$ starting at $(0, 0)$.

   (a) **[5 Pts]** Give an inductive definition of L. This will describe the steps you want the robot to take to get to points in L starting at $(0, 0)$. Let L' be the set obtained by your inductive definition.

   (b) **[5 Pts]** Prove inductively that $L' \subseteq L$, i.e., every point that the robot can get to is in L.

   (c) **[5 Pts] Extra Credit** Prove that $L \subseteq L'$, i.e., the robot can get to every point in L. To prove this, you need to give the path the robot would take to get to every point in L from $(0, 0)$, following the steps defined by your inductive rules.

For more practice, you are encouraged to work on other problems in Rosen Sections 5.3 and in LLM Chapter 7.