CS230-HW5Sol

1. **[10 Pts] Jacob**

   (a) Reflexive.
   Not Anti-reflexive: $(0, 0) \in R_1$.
   Symmetric.
   Not Anti-symmetric: $(1, 2), (2, 1) \in R_1$.
   Not Transitive: $(1, 0), (0, -1) \in R_1$, but $1 * -1 < 0$ and $(1, -1) \notin R_1$.

   (b) Not Reflexive: $(1, 1) \notin R_2$.
   Not anti-reflexive: $(0, 0) \in R_2$. Thus R_2.
   Not Symmetric: $(2, 1) \in R_2$. Then $x = 2, y = 1$, and $x = 2y$, but $y \neq 2x \Rightarrow 1 = 2 * 2$.
   Anti-symmetric.
   Not Transitive: $(4, 2), (2, 1) \in R_2$ but $(4, 1) \notin R_2$.

2. **[8 Pts] Nathan**

   (a) Prove the relation to be reflexive, symmetric and transitive.

   (b) $f(x, y) = x/y$.

   (c) $[(1, 1)] = \{(x, x) | x \in \mathbb{Z}^+\}$.

   (d) $\{(x, y) | f(x, y) = q, x \in \mathbb{Z}^+, y \in \mathbb{Z}^+\} = \{(x, y) | x/y = q, x \in \mathbb{Z}^+, y \in \mathbb{Z}^+\}$.
   We can also describe the classes as follows. By definition of rationals, for each $q \in \mathbb{Q}^+$, there exist $m, n \in \mathbb{Z}^+$ such that $q = m/n$, where m and n have no common factors. So, for each such pair m, n there is a unique equivalence class $[m, n] = \{(am, an) | a \in \mathbb{Z}^+\}$.
   The number of equivalence classes is therefore countably infinite, and each class contains a countably infinite number of elements.

3. **[8 Pts] Jacob**

   (a) R_4 is an equivalence relation, which makes it reflexive, symmetric, and transitive (should provide proof).
   Equivalence classes: $[a] = \{a_1a_2d_3d_4d_5 | d_i \in \{0, 1, \ldots, 9\}\}$.
   Refinement: Consider $(a, b) \in S$ iff the first three digits of a and b are the same. S is clearly an equivalence relation following similar logic to R_4. For the partitions of defined by S to be a refinement of the partitions of R_4, each partition of S is a subset of a partition of R_4. $[a]_S = \{a_1a_2a_3d_4d_5 | d_i \in \{0, 1, \ldots, 9\}\} \subseteq [a]_{R_4}$.

   (b) R_5 is not an equivalence relation because R_5 is not transitive. Consider $12345, 16789$, and 56789. $(12345, 16789) \in R_5$ since the two numbers have the same first digit. $(16789, 56789) \in R_5$ since the two numbers have the second digit in common. But $(12345, 56789) \notin R_5$ since neither have the same digit in the same place. Thus R_5 is not transitive.

4. **[12 Pts] Nathan**

   (a) Prove the relation to be reflexive, symmetric and transitive.
   Equivalence Classes: $\{g | g(0) = n, \forall n \in \mathbb{Z}\}$, to use [] notation we could say $f_n = f_n(0) = n \ \ \forall n \in \mathbb{Z}$, so, $[f_n] = \{g | g(0) = n\}$.

   (b) Prove the relation to be reflexive, symmetric and transitive.
   Equivalence Classes: $[f] = \{f(x) + C | C \in \mathbb{Z}\}$

5. **[12 Pts] Haniyeh**

(a) This is a partial order. So prove to be reflexive, anti-symmetric and transitive.

(b) This is an equivalence class. So prove to be reflexive, symmetric and transitive.
Equivalence Classes:
$[2] = \mathbb{Z}$
$[\pi] = \{\pi + k | k \in \mathbb{Z}\}$
$[a] = \{a + k | k \in \mathbb{Z}\}$