CS 230 : Discrete Computational Structures
Spring Semester, 2023
ASSIGNMENT #7
Due Date: Thursday, Mar 30

Suggested Reading: Rosen Sections 5.2 - 5.3; Lehman et al. Chapters 5, 6.1 - 6.3

For the problems below, explain your answers and show your reasoning.

1. [**6 Pts**] Prove that $f_1 + f_3 + \cdots + f_{2n-1} = f_{2n}$ where f_i are the Fibonacci numbers.

2. [**8 Pts**] Consider the following state machine with six states, labeled 0, 1, 2, 3, 4 and 5. The start state is 0. The transitions are $0 \to 1$, $0 \to 2$, $1 \to 3$, $2 \to 4$, $3 \to 5$, $4 \to 5$, and $5 \to 0$.

   Prove that if we take n steps in the state machine we will end up in state 0 if and only if n is divisible by 4. Argue that to prove the statement above by induction, we first have to *strengthen the induction hypothesis*. State the strengthened hypothesis and prove it.

3. [**8 Pts**] Suppose you have a stack of n bricks. In a sequence of moves, you divide the stack of n bricks into n stacks of 1 brick each. In each move, you take a stack and divide it into two non-empty stacks. In each move, if you divide a stack of $a + b$ bricks into two stacks of a bricks and b bricks, you get ab points.

   Prove by strong induction that the total score will be $n(n - 1)/2$ regardless of the order in which the bricks are split.

4. [**10 Pts**] A robot wanders around a 2-dimensional grid. He starts out at (0,0) and can take the following steps: (+1,+3), (+1,-1), (-4,0) and (0,+4). Define a state machine for this problem. Then, define a Preserved Invariant and prove that the robot will never get to (1,1).

5. [**8 Pts**] Show that if a predicate $P(n)$ can be proven true for all positive integers n by strong induction, then it can be proven true also by regular induction, once you strengthen the inductive hypothesis. In other words, Strong Induction isn't really stronger than Regular Induction. *Hint: Given any statement $P(n)$, define a new (stronger) statement $Q(n)$ so that proving $P(n)$ by strong induction is similar to proving $Q(n)$ by regular induction.*

For more practice, you are encouraged to work on other problems in Rosen Sections 5.2 and 5.3 and in LLM Chapter 5, 6.1 - 6.3.