CS230-HW4Sol

1. **Jacob 6 pts**

   We will prove if $A \neq B$, then $A \times B \neq B \times A$ by using a proof by contrapositive: if $A \times B = B \times A$, then $A = B$. To prove $A = B$, then if $x \in A$, then $x \in B$ and if $y \in B$, then $y \in A$. Have $x \in A$ and $y \in B$. Then $(x, y) \in A \times B$ by definition of Cartesian product. Since $A \times B = B \times A$ and $(x, y) \in A \times B$, then $(x, y) \in B \times A$. So $x \in B$ and $y \in A$. Thus $A = B$. Therefore, if $A \neq B$, then $A \times B \neq B \times A$.

2. **Haniyeh 4 pts**

   We will prove $(A \cup B) - C = (A - C) \cup (B - C)$ using a series of equivalences.

$$
\begin{aligned}
x \in (A \cup B) - C \text{ iff } & x \in A \cup B \wedge x \notin C && \text{definition of set difference} \\
\text{iff } & (x \in A \vee x \in B) \wedge x \notin C && \text{definition of union} \\
\text{iff } & (x \in A \wedge x \notin C) \vee (x \in B \wedge x \notin C) && \text{distributive law of logic} \\
\text{iff } & (x \in A - C) \vee (x \in B - C) && \text{definition of set difference} \\
\text{iff } & x \in (A - C) \cup (B - C) && \text{definition of set union}
\end{aligned}
$$

3. **Haniyeh 8 pts**

   a) Consider $A = \{1\}, B = \{2\}$, and $C = \{1, 2\}$. Then $A \cup C = B \cup C = C = \{1, 2\}$, so $A \cup C \subseteq B \cup C$, but A is not a subset of B.

   b) Consider $A = \{1\}, B = \{2\}$, and $C = \emptyset$. Then $A \cap C = B \cap C = C = \emptyset$, so $A \cap C \subseteq B \cap C$, but A is not a subset of B since $1 \in A$ but $1 \notin B$.

4. **Haniyeh 8 pts**

   Let $A \cup C \subseteq B \cup C$ and $A \cap C \subseteq B \cap C$. Assume, for contradiction, A is not a subset of B. Now divide the problem into two cases:

   case1 Let $x \in A$, Assume, for contradiction $x \notin B$ and $x \in C$. By definition of $\cap$, $x \in A \cap C$. Use $A \cap C \subseteq B \cap C$ to get $x \in B \cap C$. You will get a contradiction where you get to $x \in B$ with your initial assumption.

   case2 Let $x \in A$, $x \notin B$ and $x \notin C$. By addition, $x \in A \cup C$. Use $A \cup C \subseteq B \cup C$ to get $x \in B \cup C$. This contradicts the assumption that $x \notin B$ and $x \notin C$.

5. **Jacob 8 pts**

   To prove $(A \cup B) - (A \cap B) = (A - B) \cup (B - A)$, we must show that $(A \cup B) - (A \cap B) \subseteq (A - B) \cup (B - A)$ and $(A - B) \cup (B - A) \subseteq (A \cup B) - (A \cap B)$.

   This problem is solved in exam 1 review. You can check the solutions of that under Exam module on Canvas.

6. **Jacob 4 pts**

   Also proved in exam 1 review. Can find sample solutions under Exam module.

7. **Nathan 4 pts**

   For a function to be onto, everything in the co-domain must be mapped to by some element in the domain under the function. In this problem, for any $c \in \mathbb{Z}$, then $(0, c)$ would map to c using the function f.

   To show that f is not one to one, we need to find two different inputs that map to the same place under f. We can consider $(0,2)$ and $(2,0)$.

8. **Nathan 8 pts**

   a) Let x and y be two elements in the domain of g. Prove that $g(x) = g(y) \Rightarrow x = y$. Since $f \circ g$ is one to one and $f(g(x)) = f(g(y))$, then $x = y$. Thus $g(x) = g(y) \Rightarrow f(g(x)) = f(g(y)) \Rightarrow x = y$. Therefore g is one to one.

   b) g does not have to be onto. Let $A = \{a\}, B = \{1,2\}$, and $C = \{c\}$. Have $f(1) = f(2) = c$ and $g(a) = 1$. Then $f \circ g$ is onto but g is not onto because no element in A maps to $2 \in B$.