CS 230 : Discrete Computational Structures
Spring Semester, 2023
ASSIGNMENT #9
Due Date: Tuesday, Apr 18

Suggested Reading: Rosen Section 2.5; LLM Chapter 7.1

These are the problems that you need to turn in. For more practice, you are encouraged to work on the other problems. **Always explain your answers and show your reasoning.**

1. **[12 Pts]** Show that the following sets are countably infinite, by defining a bijection between $\mathbb{N}$ (or $\mathbb{Z}^+$) and that set. You do not need to prove that your function is bijective.
   (a) **[6 Pts]** the set of integers divisible by 6
   (b) **[6 Pts]** $A \times \mathbb{Z}^+$ where $A = \{0, 1, 2, 3\}$
2. **[7 Pts]** Prove that the set of functions from $\mathbb{N}$ to $\mathbb{N}$ is uncountable, by using a diagonalization argument.
3. **[12 Pts]** Determine whether the following sets are countable or uncountable. Prove your answer. To prove countable, describe your enumeration precisely, using dovetailing. There is no need to define a bijection.
   (a) **[6 Pts]** the set of real numbers with decimal representation consisting of all 9's (9.99 and 99.999... are such numbers).
   (b) **[6 Pts]** the set of real numbers with decimal representation consisting of 7's, 8's and 9's.
4. **[9 Pts]** Give an example of two uncountable sets A and B (along with a justification) such that $A \cap B$ is (a) finite (b) countably infinite (c) uncountably infinite

For more practice, you are encouraged to work on other problems like the ones below. You can find more problems in the textbook.

1. Argue that the set of all finite strings over the alphabet Σ, where $\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, /\}$, is countable. Use this to argue that the set of positive rationals is countable.
   Hint: Represent any positive rational as a finite string.
2. Argue that if $A \subseteq B$, and A is uncountable then B is uncountable.