CS 230 : Discrete Computational Structures

Spring Semester, 2021

HOMEWORK ASSIGNMENT #5

Due Date: Thursday, March 9

Suggested Reading: Rosen Sections 9.1 and 9.5; Lehman et al. Chapter 10.5, 10.6 and 10.10

For the problems below, explain your answers and show your reasoning.

1. **[10 Pts]** For each of these relations decide whether it is reflexive, anti-reflexive, symmetric, anti-symmetric and transitive. Justify your answers. R_1 and R_2 are over the set of real numbers.

   (a) $(x, y) \in R_1$ if and only if $xy \geq 0$

   (b) $(x, y) \in R_2$ if and only if $x = 2y$

2. **[8 Pts]** Let R_3 be the relation on $\mathcal{Z}^+ \times \mathcal{Z}^+$ where $((a, b), (c, d)) \in R_3$ if and only if $ad = bc$.

   (a) Prove that R_3 is an equivalence relation.

   (b) Define a function f such that $f(a, b) = f(c, d)$ if and only if $((a, b), (c, d)) \in R_3$.

   (c) Define the equivalence class containing $(1, 1)$.

   (d) Describe the equivalence classes. How many classes are there and how many elements in each class?

3. **[8 Pts]** Are these relations on the set of 5 digit numbers equivalence relations? If so, prove the properties satisfied, describe the equivalence classes and describe a new equivalence relation which is a refinement of the relation given. If not, describe which properties are violated.

   (a) $(a, b) \in R_4$ if and only if a and b start with the same two digits

   (b) $(a, b) \in R_5$ if and only if a and b have the same kth digit, where k is a number from 1 to 5

4. **[12 Pts]** Prove that these relations on the set of all functions from $\mathcal{Z}$ to $\mathcal{Z}$ are equivalence relations. Describe the equivalence classes.

   (a) $R_6 = \{(f, g) \mid f(0) = g(0)\}$

   (b) $R_7 = \{(f, g) \mid \exists C \in \mathcal{Z}, \forall x \in \mathcal{Z}, f(x) - g(x) = C\}$

5. **[12 Pts]** Consider the following relations on the set of positive real numbers. One is an equivalence relation and the other is a partial order. Which is which? For the equivalence relation, describe the equivalence classes. What is the equivalence class of 2? of π? Justify your answers.

(a) $(x, y) \in R_8$ if and only if $x/y \in \mathcal{Z}$

(b) $(x, y) \in R_9$ if and only if $x - y \in \mathcal{Z}$