CS 230 : Discrete Computational Structures

**Spring Semester, 2023**

ASSIGNMENT #6

**Due Date:** Thursday, March 23

**Suggested Reading:** Rosen Section 5.1 - 5.2; Lehman et al. Chapter 5.1 - 5.3

These are the problems that you need to turn in. For more practice, you are encouraged to work on the other problems. **Always explain your answers and show your reasoning.**

For Problems 1-4, prove the statements by mathematical induction. Clearly state your basis step and prove it. What is your inductive hypothesis? Prove the inductive step and show clearly where you used the inductive hypothesis.

1. [**5 Pts**] $1 \cdot 1! + 2 \cdot 2! + \cdots + n \cdot n! = (n+1)! - 1$, for all positive integers $n$.
2. [**5 Pts**] $2 - 2 \cdot 7 + 2 \cdot 7^2 - \cdots + 2(-7)^n = (1 - (-7)^{n+1})/4$, for all non-negative integers $n$.
3. [**5 Pts**] $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \cdots + \frac{1}{n\cdot(n+1)} = \frac{n}{n+1}$, for all positive integers $n$.
4. [**5 Pts**] 3 divides $n^3 + 3n^2 + 2n$, for all positive integers $n$.
5. [**5 Pts**] LLM p.160: Problem 5.12
6. [**9 Pts**] Let $P(n)$ be the statement that $n$-cent postage can be formed using just 3-cent and 8-cent stamps. Prove that $P(n)$ is true for all $n \geq 14$, using the steps below.
   (a) First, prove $P(n)$ by regular induction. State your basis step and inductive step clearly and prove them.
   (b) Now, prove $P(n)$ by strong induction. Again, state and prove your basis step and inductive step. Your basis step should have multiple cases.
7. [**6 Pts**] Suppose $P(n)$ is true for an infinite number of positive integers $n$. Also, suppose that $P(k+1) \rightarrow P(k)$ for all positive integers $k$. Now, prove that $P(n)$ is true for all positive integers. This is the *reverse induction principle.*

For more practice, you are encouraged to work on other problems in Rosen Sections 5.1 and 5.2 and in LLM Chapter 5.