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Parallel Computing xxx (2004) xxx–xxx
PARALLEL COMPUTING
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# Cellular automata computations and secret key cryptography

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Received 10 November 2003; accepted 15 December 2003

**Abstract**

In this paper, cellular automata (CAs) are used to design a symmetric key cryptography system based on Vernam cipher. CAs are applied to generate a pseudo-random numbers sequence (PNS) which is used during the encryption process. The quality of PNSs highly depends on the set of applied CA rules. Rules of radius $r = 1$ and 2 for nonuniform one-dimensional CAs have been considered. A new set of rules has been discovered using an evolutionary technique called cellular programming. This set provides very high quality encryption, and the system is very resistant to attempts of breaking the cryptography key.


*Keywords:* Cellular automata; Cellular programming; Random number generators; Symmetric key cryptography; Vernam cipher

## 1. Introduction

Confidentiality is mandatory for a majority of network applications including, for example, commercial uses of the Internet. Two classes of algorithms exist on the

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0167-8191/$ - see front matter © 2004 Published by Elsevier B.V.
doi:10.1016/j.parco.2003.12.014

market for data encryption: secret key systems and public key systems. An extensive overview of currently known or emerging cryptography techniques used in both types of systems can be found in [14]. One of such a promising cryptography techniques are cellular automata. Cellular automata are highly parallel and distributed systems which are able to perform complex computations. While it is still difficult to use universally this non-von Neumann style of computation to solve problems, new perspectives in this area have been opened when evolutionary techniques appeared and have been used to design automatically CA-based systems. An overview on current state of research on CAs and their application can be found in [13].

CAs were proposed for public-key cryptosystems by Guan [1] and Kari [5]. In such systems two keys are required: one key is used for encryption and the other for decryption, and one of them is held in private, the other is published. However, the main concern of this paper are secret key cryptosystems. In such systems the same key is used for encryption and decryption. The encryption process is based on the generation of pseudorandom bit sequences, and CAs can be effectively used for this purpose. In the context of symmetric key systems, CAs were first studied by Wolfram [17], and later by Habutsu et al. [3], Nandi et al. [11] and Gutowitz [2]. Recently they were a subject of study by Tomassini and his colleagues (see, e.g. [16]). This paper extends these recent studies and describes the application of one-dimensional (1D) CAs for the secret key cryptography.

The paper is organized as follows. The following section presents the idea of an encryption process based on Vernam cipher and used in CA-based secret key cryptosystems. Section 3 outlines the main concepts of CAs, overviews current state of applications of CAs in secret key cryptography and states the problem considered in the paper. Section 4 outlines evolutionary technique called cellular programming and Section 5 shows how this technique is used to discover new CA rules suitable for encryption process. Section 6 contains the analysis of results and Section 7 concludes the paper.

## 2. Vernam cipher and secret key cryptography

Let $P$ be a plain-text message consisting of $m$ bits $p_1p_2 \ldots p_m$, and $k_1k_2 \ldots k_m$ be a bit stream of a key $k$. Let $c_i$ be the $i$th bit of a cipher-text obtained by applying a XOR (exclusive-or) enciphering operation:

$$c_i = p_i \,\mathrm{XOR}\, k_i.$$

The original bit $p_i$ of a message can be recovered by applying the same operation XOR on $c_i$ with use of the same bit stream key $k$:

$$p_i = c_i \,\mathrm{XOR}\, k_i.$$

The enciphering algorithm called Vernam cipher is known to be [9,14] perfectly safe if the key stream is truly unpredictable and is used only one time. From practical point of view it means that one must find answers on the following questions: (a) how to provide a pure randomness of a key bit stream and unpredictability of ran-

dom bits, (b) how to obtain such a key with a length enough to encrypt practical amounts of data, and (c) how to pass safely the key from the sender to receiver and protect the key.

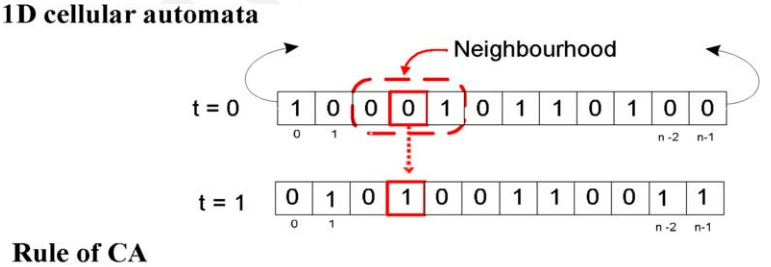
In this paper we address questions (a) and (b). We will apply CAs to generate high quality pseudorandom number sequences (PNSs) and a safe secret key.

## 3. Cellular automata and cryptography

One-dimensional CA is in a simplest case a collection of two-state elementary automata arranged in a lattice of the length $N$, and locally interacted in a discrete time $t$. For each cell $i$ called a central cell, a neighborhood of a radius $r$ is defined, consisting of $n_i = 2r + 1$ cells, including the cell $i$. When considering a finite size of CAs a cyclic boundary condition is applied, resulting in a circle grid (cf. Fig. 1).

It is assumed that a state $q_i^{t+1}$ of a cell $i$ at the time $t + 1$ depends only on states of its neighborhood at the time $t$, i.e. $q_i^{t+1} = f(q_i^t, q_{i1}^t, q_{i2}^t, \ldots, q_{in}^t)$, and a transition function $f$, called a *rule*, which defines a rule of updating a cell $i$. A length $L$ of a rule and a number of neighborhood states for a binary uniform CAs is $L = 2^n$, where $n = n_i$ is a number of cells of a given neighborhood, and a number of such rules can be expressed as $2^L$. For CAs with e.g. $r = 2$ the length of a rule is equal to $L = 32$, and a number of such rules is $2^{32}$ and grows very fast with $L$. When the same rule is applied to update cells of CAs, such CAs are called uniform CAs, in contrast with nonuniform CAs when different rules are assigned to cells and used to update them.

Wolfram was the first to apply CAs to generate PNSs [17]. He used uniform, 1D CAs with $r = 1$, and rule 30. Hortensius et al. [4] and Nandi et al. [11] used nonuniform CAs with two rules 90 and 150, and it was found that the quality of generated PNSs was better than the quality of the Wolfram system. Recently Tomassini and Perrenoud [16] proposed to use nonuniform, 1D CAs with $r = 1$ and four rules




**Rule of CA**

Neighbourhood radius r = 1, rule $01011010_2 = 90_{10}$

| Number | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |
|---|---|---|---|---|---|---|---|---|
| Neighbourhood | 111 | 110 | 101 | 100 | 011 | 010 | 001 | 000 |
| Rule result | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 0 |

Fig. 1. 1D cellular automata with neighborhood = 1.

90, 105, 150 and 165, which provide high quality PNSs and a huge space of possible secret keys which is difficult for cryptanalysis. Instead to design rules for CAs they used evolutionary technique called cellular programming (CP) to search for them.

In this study we continue this line of research. We will use finite, 1D, nonuniform CAs. However, we extend the potential space of rules by consideration of two sizes of rule neighborhood, namely neighborhood of radius $r = 1$ and 2. To discover appropriate rules in this huge space of rules we will use CP.

## 4. Cellular programming environment

### 4.1. Cellular programming

CP is an evolutionary computation technique similar to the diffusion model of parallel genetic algorithms and introduced [15] to discover rules for nonuniform CAs. Fig. 2 shows a CP system implemented [10] to discover such rules. In contrast with the CP used in [16] the system has a possibility to evaluate nonuniform rules of two types. The system consists of a population of $N$ rules (left) and each rule is assigned to a single cell of CAs (right). After initiating states of each cell, i.e. setting an initial configuration, the CAs start to evolve according to assigned rules during a predefined number of time steps. Each cell produces a stream of bits, creating this way a PNS.

After stopping evolving CAs all PNSs are evaluated. The entropy $E_h$ is used to evaluate the statistical quality of each PNS. To calculate a value of the entropy each PNS is divided into subsequences of a size $h$. In all experiments the value $h = 4$ was used. Let $l$ be the number of values which can take each element of a sequence (in

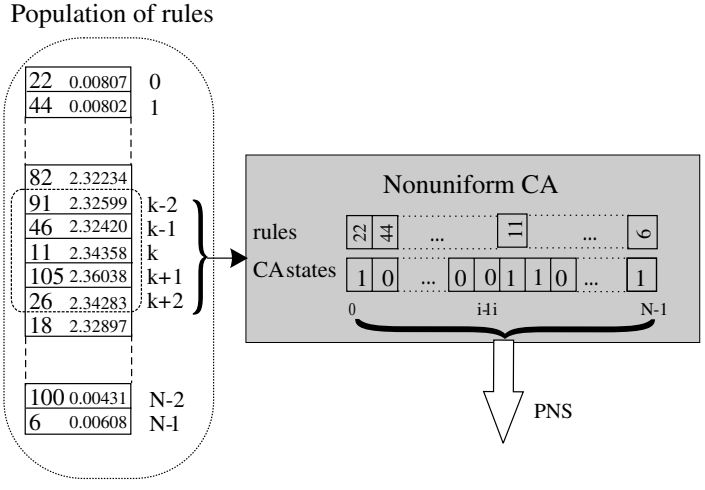



Fig. 2. CP environment for evolution of rules of nonuniform CAs.

our case of binary values of all elements $l = 2$) and $l^h$ a number of possible states of each sequence ($l^h = 16$). $E_h$ can be calculated in the following way:

$$E_h = -\sum_{j=1}^{l^h} p_{h_j} \log_2 p_{h_j},$$

where $p_{h_j}$ is a measured probability of occurrence of a sequence $h_j$ in a PNS. The entropy achieves its maximal value $E_h = h$ when the probabilities of the $k_h$ possible sequences of the length $h$ are equal to $1/l^h$. The entropy will be used as a fitness function of CP.

A single PNS is produced by a CA cell according to assigned rules and depends on a configuration $c_i$ of states of CAs. To evaluate statistically reliable value of the entropy, CAs run with the same set of rules $C$ times for different configurations $c_i$, and finally the average value of entropy is calculated and serves as a fitness function of each rule from the population of rules.

After evaluation of a fitness function of all rules of the population genetic operators of selection, crossover and mutation are locally performed on rules. The evolutionary algorithm stops after some predefined number of generations of CP. The algorithm can be summarized in the following way:

1. initiate randomly *population* of $N$ rules of type 1 ($r = 1$) or type 2 ($r = 2$), or both types, and create CAs consisting of $N$ cells
2. assign $k$th rule from the CP population to $k$th cell of CAs
3. **for** $i = 1 \ldots C$ **do**
   { create randomly configuration $c_i$ of CAs
     evolve CAs during $M$ time steps
     evaluate *entropy* of each PNS }
4. evaluate *fitness function* of each rule
5. apply locally to rules in a specified sequence genetic operators of *selection, crossover* and *mutation*
6. if STOP condition is not satisfied return to 2.

### 4.2. Genetic operators

In contrast with the standard genetic algorithm population, rules-individuals of CP population occupy specific place in the population and have strictly defined neighborhood. For example, the rule 11 (see, Fig. 2) (also indexed by $k$) corresponds to $k$th cell of CAs, and rules 46 and 105 are its immediate neighbors. All rules shown in this figure belong to the first type of rules with $r = 1$, i.e. a transition function of the rule depends on 3 cells, a given cell and two cell-neighbors. However, in more general case considered in the paper, we assume that rules are either of type 1 ($r = 1$, short rules) or of type 2 ($r = 2$, long rules).

Additionally to a neighborhood associated with two types of rules we use also an evolutionary neighborhood, i.e. the neighborhood of rules which are considered for mating when genetic operators are locally applied to a given rule. The size and pattern

of this neighborhood may differ from the neighborhood associated with types of rules. Fig. 2 shows an example of the evolutionary neighborhood for the rule $k$ which is created by rules $k-2$, $k-1$, $k$, $k+1$, $k+2$. It is assumed that the pattern of such a neighborhood is the same for all rules and is a predefined parameter of an experiment.

A sequence of genetic operators performed locally on a given rule depends on values of fitness function of rules (numbers on the right side of rule names, see Fig. 2) from the evolutionary neighborhood of this rule. Genetic operators are applied in the following way:

(1) if the $k$th rule is the best (the highest value of the fitness function) in its evolutionary neighborhood then the rule survives (selection) and remains unchanged for the next generation; no other genetic operators are performed;
(2) if in the evolutionary neighborhood of the rule $k$ only one rule exists which is better than considered rule then the rule $k$ is replaced by the better rule (selection) only if both rules are of the same type, and next mutation on this rule is performed; the rule remains unchanged if the better rule is of the other type;
(3) if two rules better than the rule $k$ exist in the neighborhood then a crossover on the pair of better rules is performed; a randomly selected child from a pair of children replaces rule $k$, and additionally a mutation is performed;
(4) if more than two rules better than the rule $k$ exist in the neighborhood then two randomly selected better rules create (crossover) a pair of children; on a randomly selected child a mutation is performed, and the child replaces the rule $k$.

Two types of rules existing in a CP population can be considered as two species of a coevolutionary algorithm. Therefore to perform a crossover between rules special regulations are required. It is assumed that two parental rules of the same species create a single child rule of the same species, which can replace either the first type of a rule or the second type of the rule. If rules of different types take part in the mating then a species of a child depends on species of a replaced rule, and is the same as a species of a rule to be replaced. Fig. 3 shows a crossover between a short rule 156 ($r = 1$) and a long rule 617528021 ($r = 2$), and the result of crossover—a short rule 154.

The short rule P1 taking part in crossover consists of 8 genes ($n = 0, \ldots, 7$) which values correspond to values of transition function defined on 8 neighborhood states {000, 001, ..., 111} existing for $r = 1$. The long rule P2 consists of 32 genes, each corresponding to values of transition function defined on 32 neighborhood states existing for $r = 2$. The long rule is folded because there is a strict relation between a state order number which corresponds to $j$th gene of P1 and states' order numbers corresponding to genes $2j$, $2j+1$ and $2j+16$, $2j+17$ of P2. These order numbers of states of P2 are just an extension of corresponding order number of a gene from P1. For example, the gene $n = 7$ of P1 corresponds to the neighborhood state {**111**}, and genes 15, 14 and 31, 30 of P2 correspond to states respectively {0**111**1, 0**111**0} and {1**111**1, 1**111**0} containing the state of P1 (marked in bold).

As Fig. 3 shows both rules P1 and P2 are crossed between genes 2 and 3 and a child Ch corresponding to a short rule ($r = 1$) is created. For this purpose the left part of the short rule is copied to the left part of the child. The right part of Ch is

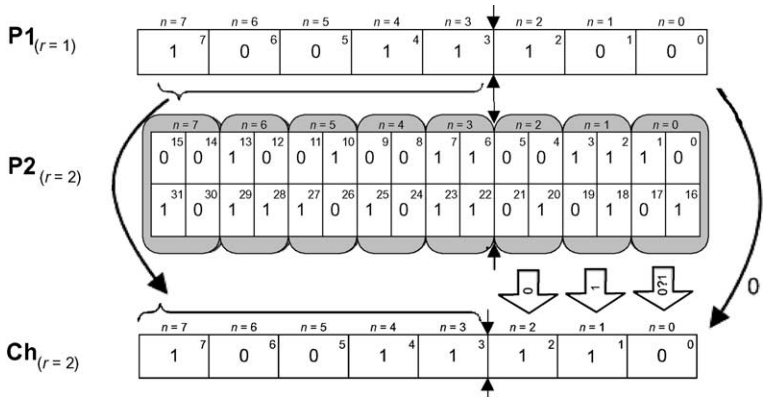



Fig. 3. Example of crossover resulting in a short child rule.

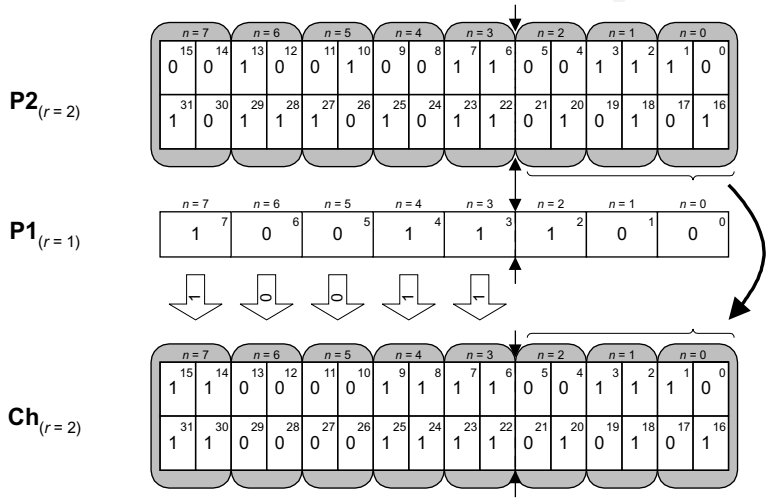



Fig. 4. Example of crossover resulting in a long child rule.

created according to the right part of P2 on the basis of majority of 0s or 1s in the corresponding genes. Fig. 4 shows the crossover of two rules resulting in a long child rule.

Last genetic operator is a flip-bit mutation performed with the probability $p_m - 0.001$.

## 5. Discovery of rules in 1D, nonuniform CAs

In all conducted experiments a population of CP and the size of nonuniform CAs were equal to 50 and the population was processing during 50 generations. The CAs

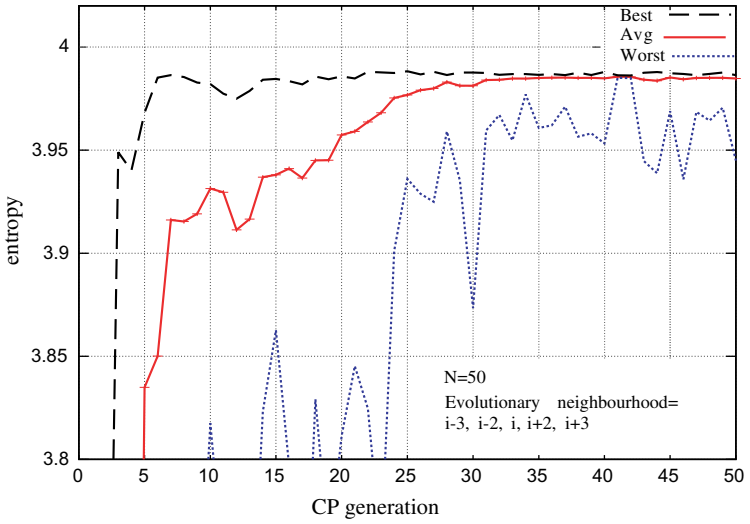



Fig. 5. A single run of CP evolutionary process.

with initial random configuration of states and a set of assigned rules evolved during $M = 4096$ time steps. Running CAs with a given set of rules was repeated for $C = 300$ initial configurations. Fig. 5 shows an example of running CP for the evolutionary neighborhood $i-3, i-2, i, i+2, i+3$. One can see that whole CAs is able to produce very good PNSs after about 40 generations (see, the average value avg of the entropy close to 4).

A typical result of a single run of an evolutionary process starting with a random rules assigned to cells of CAs is discovering by CP a small set of good rules which divide the cellular space of CAs into domains-areas where the same rules, short ($r = 1$) or long ($r = 2$), live together (see Table 1). Evolutionary process is continued on borders of domains where different rules live. This process may result in increasing domains of rules which are only slightly better than neighboring rules, which domains will decrease and finally disappear. This happens in particular when two neighboring domains are occupied respectively by the same short rules and the same long rules. The search space of short rules is much smaller than the search space of the long rules. Therefore better short rules are discovered faster than better long rules, and for this reason long rules are gradually replaced by short rules. To limit this premature convergence of short rules, the short and long rules are initially randomly assigned to cells in the proportion of 1:3 in all subsequent experiments.

The purpose of the experiments which followed was to discover an enlarged set of rules (to enlarge the key space of cryptography system) which working collectively would produce very high quality PNSs. It was noticed that in a single run of CP the evolutionary algorithm produces typically a small set of rules with a very high value of the entropy. In the result of evolutionary searching process a set of 8 short rules (including 5 rules found by [16]) and a set of 39 long rules was found.

Table 1
Domains in a final population of a evolutionary process

| Rule | Rule name | Fitness value |
|---|---|---|
| *Generation 50* | | |
| 01011010 | 90 | 3.98924 |
| 01011010 | 90 | 3.98943 |
| 01011010 | 90 | 3.98920 |
| 01011010 | 90 | 3.98981 |
| 00110011110011000011001111001100 | 869020620 | 3.98924 |
| 00110011110011000011001111001100 | 869020620 | 3.98959 |
| 00110011110011000011001111001100 | 869020620 | 3.98940 |
| 00110011110011000011001111001100 | 869020620 | 3.98906 |
| 00110011110011000011001011001100 | 869020364 | 3.94157 |
| 00110011110011000011001111001100 | 869020620 | 3.98960 |
| 00110011110011000011001111001100 | 869020620 | 3.98952 |
| 00110011110011000011001111001100 | 869020620 | 3.98929 |
| 00110011110011000011001111001100 | 869020620 | 3.98931 |
| 00110011110011000011001111001100 | 869020620 | 3.98933 |
| 00110011110011000011001111001100 | 869020620 | 3.98955 |
| 00110011110011000011001111001100 | 869020620 | 3.98964 |
| 00110011110011000011001111001100 | 869020620 | 3.98911 |
| 00110011110011000011001111001100 | 869020620 | 3.98941 |
| 00110011110011000011001111001100 | 869020620 | 3.98952 |
| 00110011110011000011001111001100 | 869020620 | 3.98933 |
| 00110011110011000011001111011100 | 869020636 | 3.97190 |
| 00110011110011000011001111001100 | 869020620 | 3.98981 |
| 00110011110011000011001111001100 | 869020620 | 3.98940 |
| 00110011110011000011001111001100 | 869020620 | 3.98930 |
| 00110011110011000011001111001100 | 869020620 | 3.98978 |
| 00110011110011000011001111001100 | 869020620 | 3.98922 |
| 00110011110011000011001111001100 | 869020620 | 3.98922 |
| 00110011110011000011001111001100 | 869020620 | 3.98957 |
| 01011010 | 90 | 3.98977 |
| 00110011110011000011001111001100 | 869020620 | 3.98949 |
| 01011010 | 90 | 3.98971 |
| 01011010 | 90 | 3.98988 |
| 01011010 | 90 | 3.98950 |
| 01011010 | 90 | 3.98945 |
| 01011010 | 90 | 3.98934 |
| 01011010 | 90 | 3.98935 |
| 01011010 | 90 | 3.98897 |
| 01011010 | 90 | 3.98942 |
| 01011010 | 90 | 3.98961 |
| 01011010 | 90 | 3.98962 |
| 01011010 | 90 | 3.98960 |
| 01011010 | 90 | 3.98970 |
| 01011010 | 90 | 3.98962 |
| 01011010 | 90 | 3.98933 |
| 01011010 | 90 | 3.98943 |
| 01011010 | 90 | 3.98955 |
| 01011010 | 90 | 3.98927 |
| 01011010 | 90 | 3.98925 |
| 01011010 | 90 | 3.98935 |
| 01011010 | 90 | 3.98948 |
| Global fitness of automata: 3.99976 | | |

## 6. Analysis and comparison of results

The entropy used as the fitness function for evolution CA rules producing high quality PNSs is only one of existing statistical tests of PNSs. None of them is enough strong to claim statistical randomness of a PNS in the case of passing a given test. For this purpose uniform CAs consisting of 50 cells evolved during 65536 time steps with each single discovered rule. Each PNS produced by CAs was divided into 4-bit words and tested on general statistical tests such as the entropy, $\chi^2$ test, serial correlation test [6] (some weaker rules after this testing were removed), and next on a number of statistical tests required by the FIPS 140-2 standard [12], such as monobit test, poker test, runs test, and long runs test.

Table 2 presents the results of testing rules against the FIPS 140-2 standard. The best scores were achieved by rules 30, 86, 101, 153 and by 8 long rules. Rules 90, 105, 150 and 65 [16] working separately in uniform CAs obtained good results in test of entropy and long runs test, quite good results in serial correlation test and monobit test but were weak in $\chi^2$ test, poker test and runs test. However this set of rules working collectively in nonuniform CAs achieves good results (see, Table 3).

For this reason only 10 rules were removed from discovered set of rules which have passed the FIPS 140-2 standard testing. These rules were worse than Tomassini and Perrenoud rules. However passing all statistical tests does not exclude a possibility that the PNS is not suitable for cryptographic purposes. Before a PNS is accepted it should pass special cryptographic tests.

Therefore rules which passed tests were next submitted to a set of Marsaglia tests [7]—a set of 23 very strong tests of randomness implemented in the Diehard program. Only 11 rules passed all 23 Marsaglia tests. These are short rules 30, 86, 101, and long rules 869020563, 1047380370, 1436194405, 1436965290, 1705400746, 1815843780, 2084275140 and 2592765285.

The purpose of the last set of experiments was a selection of a small set of short and long rules for nonuniform CAs which working collectively would provide a generation of very high quality PNSs suitable for the secret key cryptography. Simple combination of different rules which passed all Marsaglia tests in nonuniform CAs have shown that resulting PNSs may have worse statistical characteristic than PNSs obtained using uniform CAs. On the other hand, experiments with Tomassini and Perrenoud rules show that rules that separately are working worse can provide better quality working collectively. For these reasons rules 153 and some long rules which obtained very good results in general tests but not passed all Marsaglia tests were also accepted for the set of rules to search a final set of rules.

In the result of combining rules into sets of rules and testing collective behavior of these sets working in nonuniform CAs the following set of rules has been selected: 86, 90, 101, 105, 150, 153, 165 ($r = 1$), and 1436194405 ($r = 2$; Fig. 6). Among the rules are 4 rules discovered in [16]. The set of found rules have been tested again on statistical and cryptographic tests using nonuniform CAs with random assignment of rules to CA cells. Table 3 presents the results of testing this new set of rules and compares the results with ones obtained for Tomassini and Perrenoud rules. One can see that results of testing both sets on general tests and FIPS 140-2 tests are sim-

Table 2
Testing rules against the FIPS 140-2 standard

| No. | Rules | Monobit test | Poker test | Run test | Long run test |
|---|---|---|---|---|---|
| *Rules found by Tomassini and Perrenoud* | | | | | |
| 1 | 30 | 50 | 50 | 50 | 50 |
| 2 | 90 | 46 | 0 | 23 | 50 |
| 3 | 105 | 41 | 0 | 4 | 50 |
| 4 | 150 | 45 | 0 | 12 | 50 |
| 5 | 165 | 46 | 0 | 14 | 50 |
| *Rules of radius r = 1 (No 6, 7, 8) and r = 2* | | | | | |
| 6 | 86 | 50 | 50 | 50 | 50 |
| 7 | 101 | 50 | 50 | 50 | 50 |
| 8 | 153 | 50 | 50 | 50 | 50 |
| 11 | 728094296 | 50 | 5 | 17 | 50 |
| 12 | 869020563 | 50 | 50 | 49 | 50 |
| 13 | 892695978 | 50 | 2 | 9 | 50 |
| 14 | 898995801 | 50 | 0 | 4 | 50 |
| 15 | 988725525 | 50 | 11 | 16 | 50 |
| 17 | 1042531548 | 38 | 0 | 12 | 50 |
| 18 | 1047380370 | 50 | 50 | 47 | 50 |
| 19 | 1367311530 | 50 | 5 | 5 | 50 |
| 20 | 1378666419 | 42 | 3 | 16 | 50 |
| 21 | 1386720346 | 50 | 20 | 32 | 50 |
| 22 | 1403823445 | 50 | 19 | 32 | 50 |
| 23 | 1427564201 | 46 | 1 | 27 | 50 |
| 24 | 1436194405 | 50 | 50 | 50 | 50 |
| 25 | 1436965290 | 50 | 50 | 50 | 50 |
| 27 | 1457138022 | 50 | 0 | 0 | 50 |
| 28 | 1470671190 | 50 | 50 | 49 | 50 |
| 29 | 1521202561 | 40 | 0 | 3 | 50 |
| 30 | 1537843542 | 50 | 48 | 37 | 50 |
| 31 | 1588175194 | 50 | 21 | 27 | 50 |
| 32 | 1704302169 | 50 | 50 | 50 | 50 |
| 33 | 1705400746 | 50 | 50 | 50 | 50 |
| 35 | 1721277285 | 49 | 1 | 4 | 50 |
| 37 | 1721325161 | 50 | 50 | 50 | 50 |
| 38 | 1746646725 | 49 | 6 | 2 | 50 |
| 39 | 1755030679 | 50 | 49 | 34 | 50 |
| 43 | 1815843780 | 50 | 50 | 50 | 50 |
| 45 | 2036803240 | 50 | 0 | 0 | 50 |
| 46 | 2084275140 | 50 | 50 | 50 | 50 |
| 47 | 2592765285 | 50 | 50 | 49 | 50 |

ilar. However, the main difference between these results can be observed in passing Marsaglia tests: while the new discovered set of rules passes all 23 Marsaglia tests, the Tomassini and Perrenoud set of rules passes only 11 tests. Fig. 7 shows a space-time diagram of both set of rules working collectively.

The secret key $K$ which should be exchanged between two users of considered CA-based cryptosystem consists of a pair of randomly created vectors: the vector $R_i$

Table 3
Comparison of rules found by Tomassini and Perrenoud [16] and new set of discovered rules

| Test | Tomassini and Perrenoud rules (90, 105, 150, 165) | Discovered rules (86, 90, 101, 105, 150, 153, 165, 1436194405) |
|---|---|---|
| Min entropy | 3.9988 | 3.9987 |
| Max entropy | 3.9998 | 3.9997 |
| Min $\chi^2$ | 5.0254 | 6.998 |
| Max $\chi^2$ | 26.396 | 30.805 |
| Min correlation | 0.00007 | −0.00006 |
| Max correlation | 0.02553 | 0.01675 |
| Monobit test | 50 | 50 |
| Poker test | 50 | 50 |
| Run test | 50 | 50 |
| Long run test | 50 | 50 |
| Number of passed Marsaglia tests | 11 | 23 |



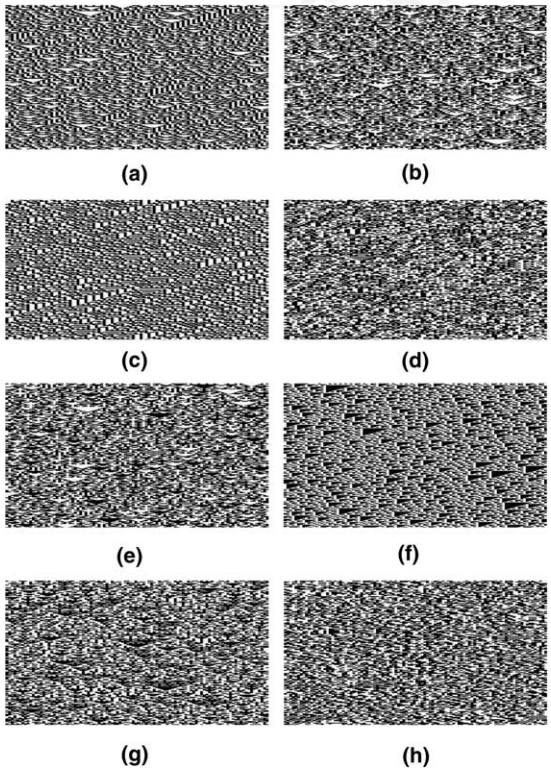



Fig. 6. Space-time diagram of CAs with $N = 100$ and $M = 200$ time steps working with (a) rule 86, (b) rule 90, (c) rule 101, (d) rule 105, (e) rule 150, (f) rule 153, (g) rule 165 and, (g) rule 1436194405.

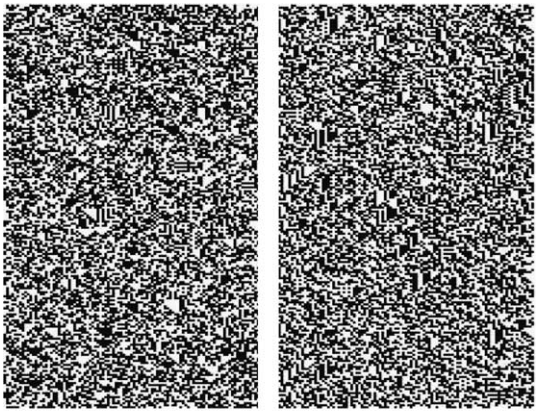



Fig. 7. Space-time diagram of CAs with $N = 100$ and $M = 200$ time steps working collectively with (a) randomly assigned Tomassini and Perrenoud [16] rules, and (b) with new set of discovered rules.

informing about assigning 8 rules to $N$ cells of CAs and the vector $C(0)$ describing an initial binary state of CA cells. The whole key space has therefore the size $8^N \times 2^N$. The key space is much larger than the key space ($4^N \times 2^N$) of 1D CA-based system [16]. Therefore the proposed system is much more resistant for cryptographic attacks.

## 7. Conclusions

In the paper we have reported results of the study on applying CAs to the secret key cryptography. The purpose of the study was to discover a set of CA rules which produce PNSs of a very high statistical quality for a CA-based cryptosystem which is resistant on breaking a cryptography key. The main assumption of our approach was to consider nonuniform 1D CAs operating with two types of rules. Evolutionary approach called CP was used to discover suitable rules. After discovery of a set of rules they were carefully selected using a number of strong statistical and cryptographic tests. Finally, the set consisting of 8 rules has been selected. Results of experiments have shown that discovered rules working collectively are able to produce PNSs of a very high quality outperforming the quality of known 1D CA-based secret key cryptosystems, which also are much more resistant for breaking cryptography keys that known systems.

## 8. Uncited reference

[8].

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