Models for Synchrophasor with Step Discontinuities in Magnitude and Phase: Estimation and Performance

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***Abstract* — This work proposes an alternative method to assess the calibration of phasor measurements units (PMUs) under conditions of step discontinuities in magnitude or phase. Two parametric mathematical models, which are proposed to represent these signals, are fitted to signal samples via an iterative numerical method. The proposed approach does not require any time adjustment of the analysis window to skip the discontinuity. The estimated parameters can be used to calculate frequency, magnitude, and phase during magnitude or phase steps. During the step transient, we propose reference phasors with an appropriate definition. Moreover, accurate estimates of the location and magnitude of the discontinuities are provided.**

***Index Terms* — Calibration, dynamic tests, phasor measurement units, synchrophasor, uncertainty.**

I. Introduction

The dynamic behavior of modern electric grid demands testing the performance of phasor measurement units (PMUs) under magnitude steps and phase steps, as prescribed in the IEEE standard [1], along with its amendment [2]. The accuracy of those measurements depends on the reference values provided by PMU calibration systems. Recent developments towards the calibration of PMUs for distribution grids demand lower uncertainty levels than the current systems, which were designed for the context of transmission grids [3]. The calibration process depends on generating and sampling synchronized waveforms, from which reference phasors are compared to the values provided by the PMU under test. A synchrophasor is the phasor whose phase is the angle relative to an ideal cosine function at the nominal system frequency with a time base centered in the Universal Time Code (UTC) second [1]. PMUs also provide estimates of frequency and its rate of change (ROCOF) at a given report rate.

For the estimation of synchrophasors, a steady state sinusoidal function can be fitted to a stationary phasor waveform with sufficient accuracy [3]. Methods to estimate parameters of signal models with slowly varying frequency, magnitude or phase, for PMU calibration purposes, are also presented in [3] and [4]. Typical variations in parameters or nonlinearities can be modeled by low order Taylor series expansion. Then, iterative procedures are used to estimate the model parameters.

The usual approach in previous works to deal with transient signals is to ignore the measurements obtained in windows containing disturbances [5] [6], identified by a transient detector [7-9]. This approach can become inappropriate with the evolution of electric grid. One example is a recent report of measurement errors during fast transients caused the protection system of a solar power plant to cut it off the grid [10]. Occurrences like this point out the need for a deeper investigation on measurement errors during transients.

In the specific case of an observed phasor disturbed by a step discontinuity in magnitude or phase, the estimation using an underlying steady state model is inappropriate and does not guarantee convergence nor accuracy [11][12]. Besides, there is a lack of definition of what the reference phasor should be. To overcome this difficulty, the method used in [3] adjusts the timestamp and position of the analysis window to skip the discontinuity and sets the phasor estimates where the discontinuity occurs as those of obtained from the previous or following window. That way, it avoids the mathematical modelling of a step discontinuity and considers the reference value coming from the steady state.

The mentioned procedures are designed to calibrate PMUs, and may be not accurate enough to evaluate the performance of calibration systems. Methods for a more detailed analysis of calibration systems under step conditions are proposed in [13]. The authors use a pointwise root mean squared error for the performance evaluation of the investigated phasor estimators.

We propose an alternative approach to assess the performance of PMU calibration systems under step tests. The time instant of the step is the first useful information to be observed. Besides, we can obtain more information from parametric models that incorporate magnitude or phase steps, once the model parameters are appropriately estimated. Other important observation is the underlying frequency during the transition of one steady state phasor to the next steady state condition. Furthermore, for testing purposes, during a magnitude step, the phase is designed to remain constant and can be measured apart from the magnitude transient. Likewise, during a phase step, the magnitude stability should be

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evaluated. In addition, the estimated parameters can be used to estimate intermediate phasors that can be used as reference values during a PMU calibration.

This paper extends the preliminary investigation reported in [17], and aims at having accurate phasor estimates under transient conditions. In this context, the following contributions are offered:

1) Models that account for step discontinuities in magnitude or phase, incorporating representations of both types of transients and the underlying stationary phasor.

2) Ways to estimate the model parameters, by:

a) using the instantaneous frequency (IF), as provided by the Hilbert transform, to estimate the instant of the step discontinuity;

b) using a nonlinear least-square method (NL-LS) to estimate the other model parameters (see section II).

3) Proposition of a model-based single phasor estimate for transient situations, which can be used as reference values for PMU calibrations and easily implemented in the existing systems, in place of traditional estimation schemes.

4) A preliminary evaluation of a laboratory system intended to be a PMU calibrator based on the aforementioned methods.

In order to assess the contribution of the estimators to the overall uncertainty of the calibration system, we run simulations to obtain the errors of each numerical computation method. In addition to this introduction, the paper is organized as follows: in section II, we present the mathematical background related to the proposed models for dynamic signals with magnitude or phase steps; the intermediary phasor definitions; and the basic concepts of the Hilbert transform and of the NL-LS method. In section III, we describe the Monte Carlo simulations run to analyze the numerical errors of each model estimation method. In section IV, we detail the lab measurements devised to evaluate the use of the proposed system for PMU calibration. Finally, in section V, we discuss the attained results and draw conclusions on the reported investigation.

II. Mathematical Background

A. Mathematical models for dynamic signals

A pure sinusoidal waveform with one magnitude step, located at $t = \tau$, can be modeled in continuous time by

$$y(t) = x_1[1 + x_2 u(t-\tau)]\cos(\omega t + \varphi) + \eta(t), \quad (1)$$

where $u(t)$ is the step function. A similar model for the phasor waveform with one phase step is

$$y(t) = x_1 \cos\big(\omega t + \varphi + x_3 u(t-\tau)\big) + \eta(t), \quad (2)$$

where $u(t-\tau)$ is used as an idealization of a fast step-like transient in magnitude or phase occurring at the instant τ, x_1 is the signal nominal magnitude, x_2 is a decimal value representing the magnitude change, x_3 is the amplitude of the phase step, ω is the angular frequency, φ is the initial phase, and $\eta(t)$ represents interfering noise. Provided a sufficiently accurate estimate of τ, the set of parameters $\mathcal{P} = \{x_1, x_2, x_3, \omega, \varphi\}$ can then be adjusted to obtain a waveform that best fits the data received by the calibration system sampler. Given a prescribed signal to noise ratio (SNR) in dB, for a zero mean gaussian white noise, the noise variance is

$$\eta_0 = \left(\frac{\sigma_y}{10^{\frac{SNR}{20}}}\right)^2, \quad (3)$$

where σ_y is the standard deviation of the signal $y(t)$.

B. Reference phasor values

After one estimates the model parameters, the problem of obtaining one phasor that represents the waveform arises. Instead of considering the values estimated from the analysis windows adjacent to the transient, our proposal is to employ a model-based method to obtain an intermediate value for the phasor magnitude or phase, from the transient-perturbed waveform. The concept is illustrated in Fig. 1, where the phasor V_1 represents the waveform during an initial steady state, V_e is a phasor that could be possibly representative of an intermediate state during the occurrence of a magnitude or phase step, and V_2 represents the signal in the final steady state condition. (In Fig. 1-a, V_e is depicted off-axis only for visualization purposes.)

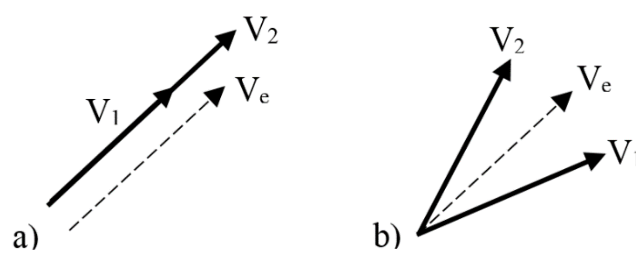

Fig. 1 Transitioning phasors for a) magnitude step, b) phase step

Intermediate phasor estimates can be obtained, for example, using a weighted means out of the estimated model parameters. Denoting $\hat{x}$ as an estimate of x, for any $\tau \in [0, T]$, for a model with magnitude step described by (1), the estimated intermediate phasor is

$$\widehat{V_e} = \widehat{X_e}\angle\hat{\varphi} = \frac{\hat{x}_1\hat{\tau} + \hat{x}_1(1+\hat{x}_2)(T-\hat{\tau})}{T}\angle\hat{\varphi}\ ; \quad (5)$$

and for a model with a phase step test described by (2), the intermediate phasor is

$$\widehat{V_e} = \hat{X}\angle\hat{\varphi}_e = \hat{X}\angle\frac{\hat{\varphi}\hat{\tau} + (\hat{\varphi}+\hat{x}_3)(T-\hat{\tau})}{T}\ . \quad (6)$$

C. Instantaneous frequency via Hilbert transform

Hilbert transform has been used to estimate instantaneous frequency (IF) of narrowband monocomponent signals, which is the case of ideal electric network phasor components. There are various applications of IF estimation reported in the literature, e.g., characterization of electric disturbances [18] and detection of edits in audio signals that bear the electric network frequency. Anomalous perturbations on the IF can flag the occurrence of discontinuities in the signal. The time instant they happened can be estimated via appropriate amplitude threshold schemes [19].

Given a real narrowband monocomponent signal $x(t)$, $-\infty < t < \infty$, let $z(t)$ be called the analytic signal associated to $x(t)$, defined as

$$z(t) = x(t) + j\tilde{x}(t), \tag{7}$$

where

$$\tilde{x}(t) = H\big(x(t)\big) = \int_{-\infty}^{\infty} \frac{x(u)}{\pi(t-u)} du, \tag{8}$$

is the Hilbert transform of $x(t)$. If $z(t)$ is expressed in the polar form

$$z(t) = A(t)e^{j\theta(t)}, \tag{9}$$

$$A(t) = \sqrt{x^2(t) + \tilde{x}^2(t)}, \tag{10}$$

$$\theta(t) = \tan^{-1}(x(t)/\tilde{x}(t)), \tag{11}$$

the instantaneous frequency (IF) can be defined as

$$f_i(t) = \frac{1}{2\pi}\left(\frac{d\theta(t)}{dt}\right). \tag{12}$$

The time discretization of $x(t)$, represented by $x[n]$, gives the discrete version of the analytic signal

$$z[n] = x[n] + jH(x[n]). \tag{13}$$

With discrete version of $H(x[n])$ obtained with the Fast Fourier Transform (FFT) [20], the discrete IF can be estimated by the numerical derivation with respect to time of the discrete instantaneous phase angle $\theta[n]$, obtained as by (11). Let $d[n]$ be a detection signal, given by

$$d[n] = |f_i[n]| - m(f_i[n]), \tag{14}$$

where $m(f_i[n])$ is the median value of the sequence $f_i[n]$. As demonstrated in [19], a waveform discontinuity in $x[n]$ manifests as anomalous variations in $d[n]$. Its time location can be detected via appropriate threshold schemes applied to $d[n]$. Here, a similar detection scheme will be used to estimate the parameter τ for the models (1) and (2). Two examples of the detection signal time aligned with the real signal are shown in Fig. 2 and Fig. 3, respectively for magnitude and phase steps, where the circle highlights the transient.

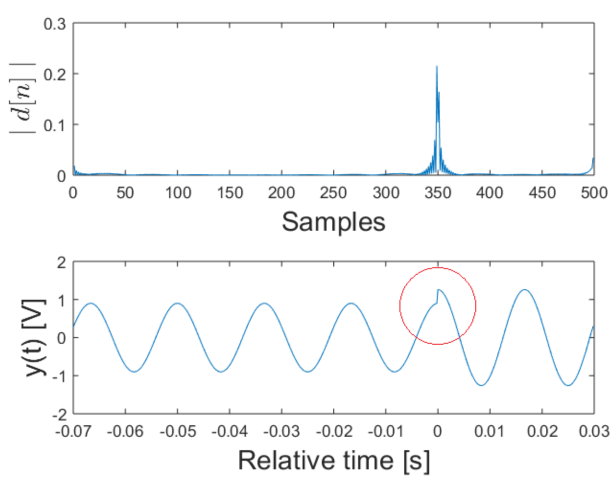

Fig. 2 Detection signal $d[n]$ (top plot) associated with a phasor waveform with magnitude step (bottom plot). (τ = 70% of window duration).

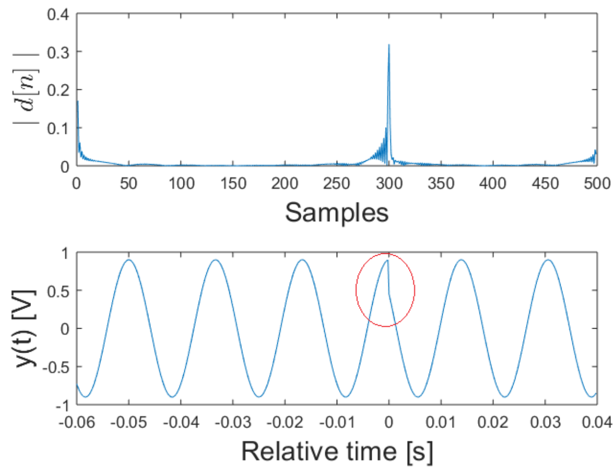

Fig. 3 Detection signal $d[n]$ (top plot) associated with a phasor waveform with phase step (bottom plot). (τ = 60% of window duration).

For the sake of simplicity, in the simulations and experiments, we will consider that a discontinuity step is always present. Therefore, τ will be estimated simply as the instant at which the global maximum of $d[n]$ occurs, after discarding the transients at the beginning and end of $d[n]$. Detecting whether there is or not a discontinuity in the input signal is not the purpose here. Nevertheless, it could be accomplished by using appropriate thresholding schemes [19].

D. *Model Parameters Estimation via Levenberg-Marquardt*

Consider N samples from a sequence $y(k)$, which can be either generated via computational simulation or sampled from measured real phenomenon, with uniform sampling period Δt. One wishes to fit the models (see Section II) with parameters $\mathcal{P}$ to $y(k)$. For that, one can define the error cost function

$$\varepsilon(\mathcal{P}) = \frac{1}{2}\sum_{k=1}^{N}\big(y(k) - \hat{y}(k\Delta t)\big)^2, \tag{4}$$

and try to solve the minimization problem $\min_{\mathcal{P}} \varepsilon(\mathcal{P})$.

The estimation of phasor parameters considering variations in frequency within the model requires dealing with a non-linear function. Existing calibration systems solve this problem for steady-state signals through low order Taylor linearization [3], or using directly some non-linear minimization algorithm, e.g., Levenberg-Marquardt (LM) [13][14].

The Levenberg-Marquardt (LM) algorithm [15][16] is an iterative technique for nonlinear minimization problems. It combines the Gauss-Newton method and the steepest descent, being very useful when the size of the descendant step cannot be obtained in a closed form. Instead, numerical approximations of the Jacobian matrix are used to estimate the gradient of the cost function and establish an optimal direction. Such NL-LS methods can reach local minima and need a convex cost function, provided by (4).

III. Numerical Simulations

We performed Monte Carlo simulations with 1000 runs for each setup, to measure the errors between the reference values (of the signal generating model) and those obtained by the computer-based estimators. The errors will be considered as the contribution of the numerical computations to the uncertainty of the calibration system.

For each run, the input signals were digitally generated, with all nominal parameters prescribed in the standard [1] and random values representative of expected uncertainties in each parameter. The signals were created based on (1) and (2) with a 5 kHz sampling frequency, and with a duration of 0.1 s. The nominal values are summarized in Table I, with their respective uncertainties used in simulations.

TABLE I
NOMINAL VALUES AND UNCERTAINTIES USED IN THE SIMULATIONS

Parameter	x_1	x_2	x_3	$\omega/2\pi$	φ
Nominal	1 V_p	± 0.1	± 10°	60 Hz	360°, ±120°
U[%]	1	1	1	0.05	1

A. *Estimation of τ via Hilbert Analytical Signal*

If we use an ideal signal without uncertainties in the parameters of the signal generating model and nominal frequency, for a total duration of the window T, $\tau \in [0.05T, 0.95T]$, and $SNR > 40dB$, (with additive white noise drawn from a uniform distribution), the maximum absolute errors are not greater than $1\Delta t$.

In a second simulation, designed to represent a more realistic situation, we allowed 500 $\mu Hz/Hz$ variation in frequency and 1% variation in the other parameters, all under a uniform distribution. The maximum errors obtained are not greater than $2\Delta t$ for a $SNR > 75dB$, with additive zero mean white Gaussian noise. The distribution of τ errors (in Δt units) for positive magnitude step of 10% is shown in a histogram in Fig. 4. Similar histograms were obtained for negative magnitude steps and phase steps of ± 10°, with the same results.

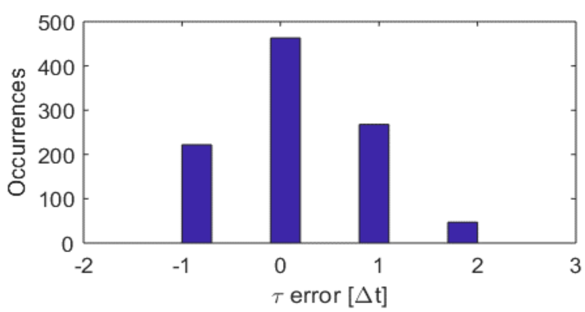

Fig. 4 Histogram of errors in step instant estimation.

B. *Parameter estimation with non-linear least squares*

For the uncertainty analysis reported in this subsection, at each Monte Carlo run, the signal $y(k)$ is generated with uncertainties added to the parameters, drawn from a uniform distribution centered in the nominal values, as shown in Table I, along with additive white Gaussian noise at different SNR levels. Maximum errors of $\pm 2\Delta t$ in the estimation of τ are also simulated. The intermediate phasor magnitude or phase are calculated by (5) and (6). Each estimated quantity is compared to a reference value (nominal value with a perturbation) to obtain magnitude and phase numerical errors.

In the iterative LM algorithm, the model parameters are initiated at the nominal values, and the optimization procedure seeks for the minimum point of $\varepsilon(\mathcal{P})$, which would be reached at the reference values of the parameters. In practice, for this implementation, iterations are stopped when the convergence criterion $\|\nabla\varepsilon(\mathcal{P})\| < 10^{-10}$ is reached.

The final estimates have significantly lower errors than the initial values, despite having some sensitivity to noise. Results for parameters of interest are listed in Table II for magnitude steps, and Table III for phase steps.

TABLE II
STANDARD DEVIATION OF NUMERICAL ERRORS FOR MAGNITUDE STEPS

SNR [dB]	90	93	97
Frequency [$\mu Hz/Hz$]	0.14	0.1	0.06
Magnitude [$\mu V/V$]	1.5	1.0	0.6
Phase [m °]	0.4	0.1	0.06

TABLE III
STANDARD DEVIATION OF NUMERICAL ERRORS FOR PHASE STEPS

SNR [dB]	90	93	97
Frequency [$\mu Hz/Hz$]	0.26	0.19	0.1
Magnitude [$\mu V/V$]	1.5	1.0	0.7
Phase [m °]	0.17	0.11	0.07

IV. LABORATORY MEASUREMENTS

Aiming at validating the proposed method with real-world signals, several measurements were made using one digital sampling voltmeter (DSVM) and one arbitrary waveform generator (AWG), controlled by a personal computer (PC) via GPIB. The connections are shown in the block diagram of Fig. 5. The system connections and synchronization are inspired by [21], from which we can consider that the uncertainties inserted by the DSVM can be neglected for this preliminary analysis, and the most part of deviations are due to the AWG.

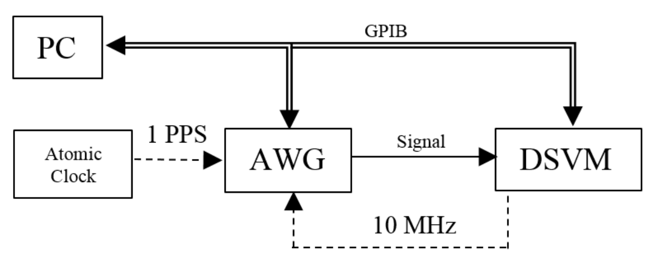

Fig. 5 Diagram of the prototype system.

The same waveforms used in the simulations are set to be reproduced by the AWG, with a nominal output of 1 V_p, and sampled by the DSVM. Both are triggered with a 1 PPS (pulse per second) signal, coming from an atomic clock, so we can control the initial phase. The internal clock from the DSVM is used as an external 10 MHz reference signal for the generator. 5000 samples are taken during 1 s and stored in the DSVM´s internal memory.

The standard [1] establishes that the synchrophasors must be obtained relative to the center of an analysis window, coherent with the UTC second and its decimal fractions. If we set 500 samples/window, we have 10 windows per second. The first window should have 250 samples before the UTC second, but as it is not viable, the analysis starts at the next window. The center of the first complete window will be coherent with the first 0.1 s after the UTC second, as shown in Fig. 6. Then, we have 9 complete windows containing 6 cycles of 60 Hz each. The steps of magnitude or phase occur in the 5th window. According to the procedure for equivalent sampling, the instants of occurrence of the steps are a set of equally spaced intervals $\tau_k = 0.1kT, k = 1 \dots 9$, where T is the duration of a single analysis window, in seconds.

For the windows with steady state waveforms, the same fitting algorithm reported in [3] is used to obtain the synchrophasors and frequency estimates.

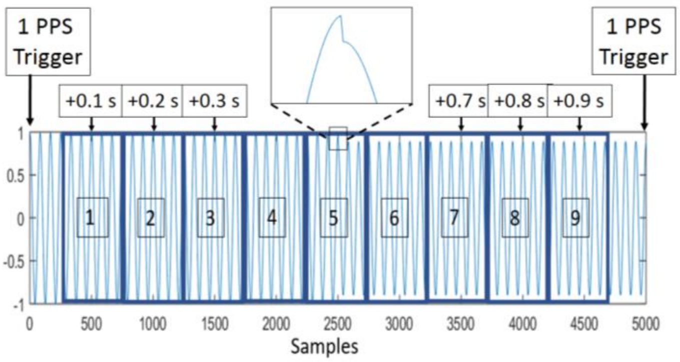

Fig. 6 Magnitude step occurrence in the 5th window.

For the 5th window, the intermediate phasors were calculated using (5) or (6), from the estimated model parameters of (1) or (2), respectively (see Section II.C and II.D).

The estimates of step instant were not greater than $2\Delta t$, inside the expected uncertainty. The other parameters require a more detailed analysis.

A. Intermediate magnitude and phase

The system is capable of providing intermediate values for magnitude and phase, as can be seen in Fig. 7 and Fig. 8, respectively. Each series of data shows different step instant

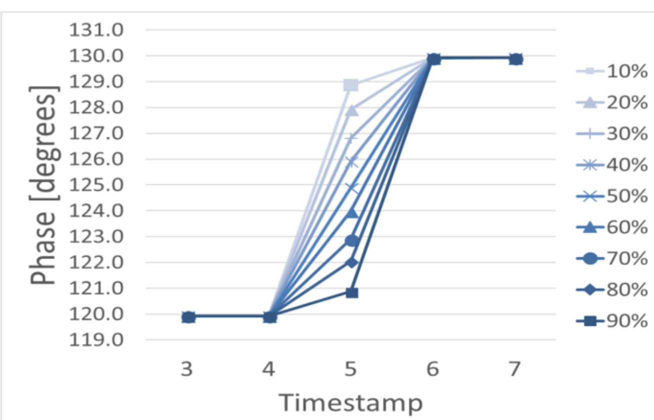

Fig. 7 Intermediate phase. Phase positive step of 10 degrees, starting at 120 degrees.

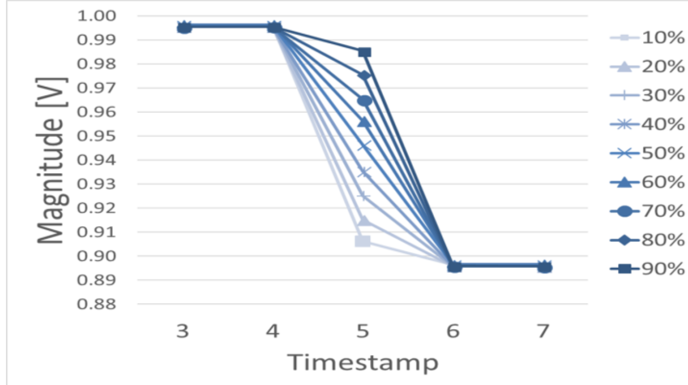

Fig. 8 Intermediate magnitude. Negative step of 10%.

occurrences in the 5th timestamp, in percentage of the window period T.

B. Frequency

Frequency estimates for the analysis windows with steady state signals have standard deviation no greater than 9 $\mu Hz/Hz$ ($0.5\ mHz$) from the nominal value. When submitted to magnitude steps, the estimates provided by system have standard deviations from the nominal value of about 70 $\mu Hz/Hz$ (4 mHz), as can be seen in Fig. 9, where the values for each plot have been measured for the shown locations of τ in percentage of T.

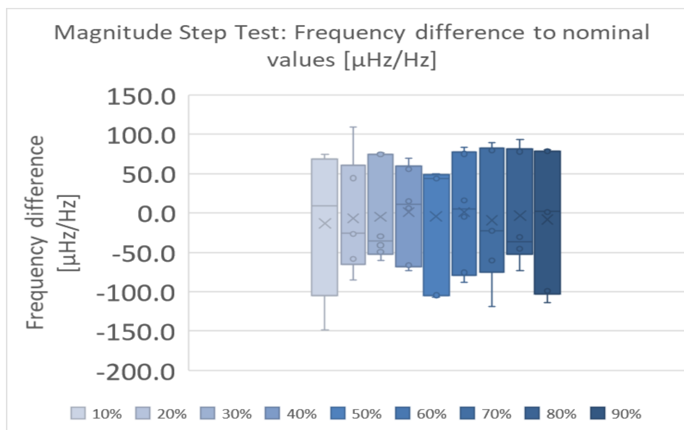

Fig. 9 Frequency difference to nominal value during magnitude tests as function of the step instant, in percentage of T.

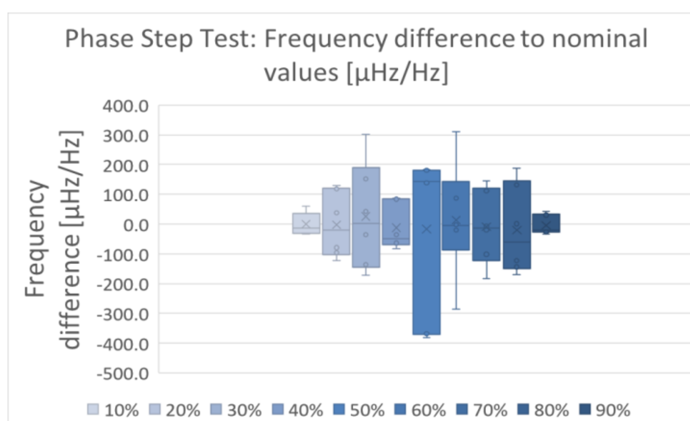

Fig. 10 Frequency difference to nominal value during phase steps as function of the step instant, in percentage of T.

The largest frequency variations are observed when the system is required to generate and measure phase steps. As shown in Fig. 10, standard deviations of the frequency errors of about 40 $\mu Hz/Hz$ (2 mHz) have been measured when the phase step occurrence is near the boudaries of the window, and up to 280 $\mu Hz/Hz$ (17 mHz), if the phase step is in the middle of the window.

C. *Magnitude during phase step*

During steady state conditions, the magnitude estimates have a standard deviation no greater than 160 $\mu V/V$. When the system is set to generate and measure a phasor having only a phase step, perturbations in the measured phasor magnitude have been also observed, with standard deviation of about 200 $\mu V/V$ with respect to the average magnitude value, as measured from the windows with steady-state phasors, see Fig. 11.

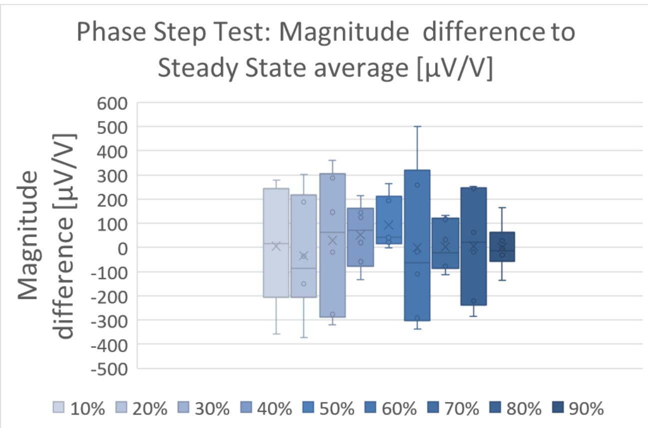

Fig. 11 Magnitude differences to steady state average during phase steps as function of the step instant of occurrence.

D. *Phase during magnitude step*

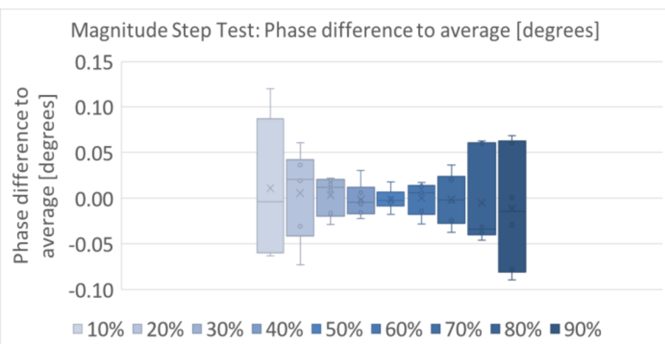

Fig. 12 Phase difference to average during magnitude steps as function of the step instant of occurrence.

During steady state conditions, the phase estimates have standard deviations of about 1.7 $m°$. The estimates of phase during magnitude step tests present higher variations. As can be seen in Fig. 12, there is a tendency for the standard deviation to increase as the location of the step discontinuity is set near the extremities of the analysis window. The standard deviations have a minimum value of about 10 m^o, going up to about 70 m^o. Possibly, the higher errors found during transients are due to limitations of the AWG.

V. Conclusion

Models for phasor signals disturbed by magnitude and phase step discontinuity were proposed, in the context of assessment of PMU calibration systems in transient conditions. We present a model-based method to estimate phasor frequency during magnitude or phase step tests. The method allows measuring the phasor phase during a magnitude step test, as measuring the phasor magnitude during phase step tests. Furthermore, intermediate values for transitioning phasor magnitude or phase can be estimated, to be used as reference values for PMU calibrations.

Estimation of the step location via Hilbert's instantaneous frequency analysis provided accurate results, which allowed the estimation of the model parameters via a nonlinear least-squares method. The proposed approach tackles the estimation of the step discontinuities in the phasor signal observed within an analysis window, instead of dodging the problem.

The estimation accuracy of each parameter and intermediary phasor has been measured via computer-simulated setups under different noise conditions and uncertainties forced upon the model used to generate the test signals. Within the limits reported, the proposed method can give reliable and accurate results to assess PMU calibration systems.

A preliminary analysis of a laboratory system intended to calibrate PMUs has been performed. The experimental results show that higher standard deviations of parameters occur during transient conditions than those observed during steady state. Significant deviations of frequency, magnitude and phase during magnitude or phase tests were detected, and the accuracy limits of the prototype system could be outlined. Those results could be improved using a more stable signal source.

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