

Server Restart

$$\begin{aligned}
 \text{ServerRestart}(s) &\triangleq \\
 &\text{LET } \text{currentState} \triangleq \text{serverStates}[s] \text{ IN} \\
 &\text{LET } \text{terminationTime} \triangleq (\text{currentState.start} + 1) \text{ IN} \\
 &\wedge \text{currentState.state} \neq \text{"waiting"} \quad \text{Server must be active} \\
 &\quad \text{This is the only state a server can reach if past termination time} \\
 &\wedge \text{time} \Rightarrow \text{terminationTime} \\
 &\wedge \text{serverStates}' = [\text{serverStates} \text{ EXCEPT} \\
 &\quad \text{!}[s].\text{state} = \text{"waiting"}, \\
 &\quad \text{!}[s].\text{userId} = \text{"UNSET"}, \\
 &\quad \text{!}[s].\text{metadata} = \text{"UNSET"}, \\
 &\quad \text{!}[s].\text{image} = \text{"UNSET"}, \\
 &\quad \text{!}[s].\text{imageId} = \text{"UNSET"}] \\
 &\wedge \text{UNCHANGED } \langle \text{databaseState}, \text{blobStoreState}, \text{operations} \rangle \\
 &\wedge \text{UNCHANGED } \text{cleanerVars} \\
 &\wedge \text{UNCHANGED } \text{time}
 \end{aligned}$$

Server Writes

$$\begin{aligned}
 \text{ServerStartWrite}(s) &\triangleq \\
 &\wedge \text{serverStates}[s].\text{state} = \text{"waiting"} \\
 &\wedge \exists u \in \text{USERIDS}, m \in \text{METADATAS}, i \in \text{IMAGES} : \\
 &\quad \wedge \text{serverStates}' = [\text{serverStates} \text{ EXCEPT} \\
 &\quad \quad \text{!}[s].\text{state} = \text{"started\_write"}, \\
 &\quad \quad \text{!}[s].\text{userId} = u, \\
 &\quad \quad \text{!}[s].\text{metadata} = m, \\
 &\quad \quad \text{!}[s].\text{image} = i, \\
 &\quad \quad \text{The time a write request starts} \\
 &\quad \quad \text{!}[s].\text{start} = \text{time} \\
 &\quad ] \\
 &\wedge \text{operations}' = \text{Append}(\text{operations}, \\
 &\quad [ \\
 &\quad \quad \text{type} \mapsto \text{"WRITE"}, \\
 &\quad \quad \text{userId} \mapsto u, \\
 &\quad \quad \text{metadata} \mapsto m, \\
 &\quad \quad \text{image} \mapsto i \\
 &\quad ]) \\
 &\quad \text{Cleaner state needs to be added as unchanged for all server operations} \\
 &\wedge \text{UNCHANGED } \langle \text{databaseState}, \text{blobStoreState}, \text{cleanerStates} \rangle \\
 &\wedge \text{UNCHANGED } \text{time}
 \end{aligned}$$

$$\begin{aligned}
 \text{ServerWriteBlob}(s) &\triangleq \\
 &\text{LET } \text{currentState} \triangleq \text{serverStates}[s] \text{ IN}
 \end{aligned}$$

$\text{LET } \text{terminationTime} \triangleq (\text{currentState.start} + 1)\text{IN}$   
 $\wedge \text{time} < \text{terminationTime}$  Can only start this state if server is live  
 $\wedge \text{currentState.state} = \text{"started\_write"}$   
 $\wedge \exists id \in \text{UUIDS} :$   
 $\quad \wedge \text{blobStoreState}[id] = [\text{status} \mapsto \text{"UNSET"}, \text{image} \mapsto \text{"UNSET"}]$   
 $\quad \wedge \text{blobStoreState}' = [\text{blobStoreState} \text{ EXCEPT}$   
 $\quad \quad \quad \text{![id]} = [$   
 $\quad \quad \quad \quad \text{image} \mapsto \text{currentState.image},$   
 $\quad \quad \quad \quad \text{created} \mapsto \text{time}$   
 $\quad \quad \quad \quad ]]$   
 $\quad \wedge \text{serverStates}' = [\text{serverStates} \text{ EXCEPT}$   
 $\quad \quad \quad \text{![s].state} = \text{"wrote\_blob"},$   
 $\quad \quad \quad \text{![s].imageId} = id]$   
 $\wedge \text{UNCHANGED } \langle \text{databaseState}, \text{operations} \rangle$   
 $\wedge \text{UNCHANGED } \text{cleanerVars}$   
 $\wedge \text{UNCHANGED } \text{time}$

$\text{ServerWriteMetadataAndReturn}(s) \triangleq$   
 $\text{LET } \text{currentState} \triangleq \text{serverStates}[s]\text{IN}$   
 $\text{LET } \text{terminationTime} \triangleq (\text{currentState.start} + 1)\text{IN}$   
 $\wedge \text{time} < \text{terminationTime}$  Can only start this state if server is live  
 $\wedge \text{currentState.state} = \text{"wrote\_blob"}$   
 $\wedge \text{databaseState}' = [\text{databaseState} \text{ EXCEPT}$   
 $\quad \quad \quad \text{![currentState.userId]} = [$   
 $\quad \quad \quad \quad \text{metadata} \mapsto \text{currentState.metadata},$   
 $\quad \quad \quad \quad \text{imageId} \mapsto \text{currentState.imageId}]$   
 $\wedge \text{serverStates}' = [\text{serverStates} \text{ EXCEPT}$   
 $\quad \quad \quad \text{![s].state} = \text{"waiting"},$   
 $\quad \quad \quad \text{![s].userId} = \text{"UNSET"},$   
 $\quad \quad \quad \text{![s].metadata} = \text{"UNSET"},$   
 $\quad \quad \quad \text{![s].image} = \text{"UNSET"},$   
 $\quad \quad \quad \text{![s].imageId} = \text{"UNSET"}]$   
 $\wedge \text{UNCHANGED } \langle \text{blobStoreState}, \text{operations} \rangle$   
 $\wedge \text{UNCHANGED } \text{cleanerVars}$   
 $\wedge \text{UNCHANGED } \text{time}$

$\text{ServerFailWrite}(s) \triangleq$   
 $\text{LET } \text{currentState} \triangleq \text{serverStates}[s]\text{IN}$   
 $\text{LET } \text{terminationTime} \triangleq (\text{currentState.start} + 1)\text{IN}$   
 $\wedge \text{time} < \text{terminationTime}$  Can only start this state if server is live  
 $\wedge \text{serverStates}[s].\text{state} \in \{ \text{"started\_write"}, \text{"wrote\_blob"} \}$   
 $\wedge \text{serverStates}' = [\text{serverStates} \text{ EXCEPT}$   
 $\quad \quad \quad \text{![s].state} = \text{"waiting"},$   
 $\quad \quad \quad \text{![s].userId} = \text{"UNSET"},$

$$\begin{aligned}
& ! [s].\text{metadata} = \text{"UNSET"}, \\
& ! [s].\text{image} = \text{"UNSET"}, \\
& ! [s].\text{imageId} = \text{"UNSET"}] \\
& \wedge \text{UNCHANGED } \langle \text{databaseState}, \text{blobStoreState}, \text{operations} \rangle \\
& \wedge \text{UNCHANGED } \text{cleanerVars} \\
& \wedge \text{UNCHANGED } \text{time}
\end{aligned}$$

#### Server Reads

$$\begin{aligned}
\text{ServerStartRead}(s) & \triangleq \\
& \text{LET } \text{currentState} \triangleq \text{serverStates}[s] \text{ IN} \\
& \wedge \text{serverStates}[s].\text{state} = \text{"waiting"} \\
& \wedge \exists u \in \text{USERIDS} : \\
& \quad \text{serverStates}' = [\text{serverStates} \text{ EXCEPT} \\
& \quad \quad ! [s].\text{state} = \text{"started\_read"}, \\
& \quad \quad ! [s].\text{userId} = u, \\
& \quad \quad \text{The time a read request starts} \\
& \quad \quad ! [s].\text{start} = \text{time} \\
& \quad ] \\
& \wedge \text{UNCHANGED } \langle \text{databaseState}, \text{blobStoreState} \rangle \\
& \wedge \text{UNCHANGED } \text{operations} \\
& \wedge \text{UNCHANGED } \text{cleanerVars} \\
& \wedge \text{UNCHANGED } \text{time} \\
\\
\text{ServerReadMetadata}(s) & \triangleq \\
& \text{LET } \text{currentState} \triangleq \text{serverStates}[s] \text{ IN} \\
& \text{LET } \text{terminationTime} \triangleq (\text{currentState.start} + 1) \text{ IN} \\
& \wedge \text{time} < \text{terminationTime} \quad \text{Can only start this state if server is live} \\
& \wedge \text{currentState.state} = \text{"started\_read"} \\
& \wedge \text{databaseState}[\text{currentState.userId}].\text{metadata} \neq \text{"UNSET"} \\
& \wedge \text{serverStates}' = \\
& \quad [\text{serverStates} \text{ EXCEPT} \\
& \quad \quad ! [s].\text{state} = \text{"read\_metadata"}, \\
& \quad \quad ! [s].\text{metadata} = \text{databaseState}[\text{currentState.userId}].\text{metadata}, \\
& \quad \quad ! [s].\text{imageId} = \text{databaseState}[\text{currentState.userId}].\text{imageId}] \\
& \wedge \text{UNCHANGED } \langle \text{databaseState}, \text{blobStoreState} \rangle \\
& \wedge \text{UNCHANGED } \text{operations} \\
& \wedge \text{UNCHANGED } \text{cleanerVars} \\
& \wedge \text{UNCHANGED } \text{time} \\
\\
\text{ServerReadMetadataAndReturnEmpty}(s) & \triangleq \\
& \text{LET } \text{currentState} \triangleq \text{serverStates}[s] \text{ IN} \\
& \text{LET } \text{terminationTime} \triangleq (\text{currentState.start} + 1) \text{ IN}
\end{aligned}$$

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 $\wedge time < terminationTime$  Can only start this state if server is live
 $\wedge currentState.state = \text{"started\_read"}$ 
 $\wedge databaseState[currentState.userId].metadata = \text{"UNSET"}$ 
 $\wedge serverStates' = [serverStates \text{ EXCEPT}$ 
     $! [s].state = \text{"waiting"},$ 
     $! [s].userId = \text{"UNSET"},$ 
     $! [s].metadata = \text{"UNSET"},$ 
     $! [s].image = \text{"UNSET"},$ 
     $! [s].imageId = \text{"UNSET"}]$ 

 $\wedge operations' = Append(operations,$ 
     $[$ 
     $type \mapsto \text{"READ"},$ 
     $userId \mapsto currentState.userId,$ 
     $metadata \mapsto \text{"UNSET"},$ 
     $image \mapsto \text{"UNSET"}$ 
     $])$ 
 $\wedge \text{UNCHANGED } \langle databaseState, blobStoreState \rangle$ 
 $\wedge \text{UNCHANGED } cleanerVars$ 
 $\wedge \text{UNCHANGED } time$ 

 $ServerReadBlobAndReturn(s) \triangleq$ 
     $LET\ currentState \triangleq serverStates[s]IN$ 
     $LET\ terminationTime \triangleq (currentState.start + 1)IN$ 
     $\wedge time < terminationTime$  Can only start this state if server is live
     $\wedge currentState.state = \text{"read\_metadata"}$ 
     $\wedge operations' = Append(operations,$ 
     $[$ 
     $type \mapsto \text{"READ"},$ 
     $userId \mapsto currentState.userId,$ 
     $metadata \mapsto currentState.metadata,$ 
     $image \mapsto blobStoreState[currentState.imageId].image$ 
     $])$ 
     $\wedge serverStates' = [serverStates \text{ EXCEPT}$ 
     $! [s].state = \text{"waiting"},$ 
     $! [s].userId = \text{"UNSET"},$ 
     $! [s].metadata = \text{"UNSET"},$ 
     $! [s].image = \text{"UNSET"},$ 
     $! [s].imageId = \text{"UNSET"}]$ 
     $\wedge \text{UNCHANGED } \langle databaseState, blobStoreState \rangle$ 
     $\wedge \text{UNCHANGED } cleanerVars$ 
     $\wedge \text{UNCHANGED } time$ 

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