3. Remainder & Quetient :- divison divident :- (Number Systems)

$P = Kq + r$

divident → P, divison → K, quetient → q, remainder → r

$K \overset{q}{\big|} P$

r

$\Rightarrow P = Kq + r$

3) in a divison, the divisor is 3 times of quetient & 2 times the remainder. If remainder is 30 then divident is _____.

$\Rightarrow K = 3q \overset{q}{\big|} P$, $r = 1.5q$

$\Rightarrow 1.5q = 30$

$\Rightarrow q = 20$

$P \Rightarrow Kq + 1.5q$

$\Rightarrow 3 \times 20 + 1.5 \times 20$

$P \Rightarrow 1230$ (Ans)

- $x \equiv y \bmod n$ means $x \bmod n = y \bmod n$

x & y when divided by n produces same remainder.

$52 \equiv 8 \bmod 9 \rightarrow \underset{\downarrow 8}{52 \,\%\, 9} = \underset{\downarrow 8}{8 \,\%\, 9}$

$25 \equiv 4 \bmod 7 \rightarrow \underset{\downarrow 4}{25 \,\%\, 7} = \underset{\downarrow 4}{4 \,\%\, 7}$

$60 \equiv 42 \bmod 9 \rightarrow \underset{\downarrow 6}{60 \,\%\, 9} = \underset{\downarrow 6}{42 \,\%\, 9}$

- **Theorem:**

(i) $x \equiv y \bmod m \Leftrightarrow x - y \equiv 0 \bmod m$

Proff:-

Let $\Rightarrow x \Rightarrow m \times a + b$

$y \Rightarrow m \times c + b$

$a \overset{x}{\big|} m$, b

remainder b will be same as congiand to each other

$\Rightarrow x - y \Rightarrow (m \times a + b) - (m \times c + b)$

$\Rightarrow m \times a + b - m \times c - b$

$\Rightarrow m \times a - m \times c$

$\Rightarrow x - y \Rightarrow m(a - c)$

So, the difference will be multiple of m

means, $x - y \equiv 0 \bmod m$

(ii) **Theorem:-**

$x \equiv y \bmod m \iff (x \pm m \times a) \equiv (y \pm m \times b) \bmod m$

Proff:-

let, $53 \equiv 8 \bmod 9$

let, $\Rightarrow (9 \times 5 + 8) \equiv (9 \times 0 + 8) \bmod 9$

$\Rightarrow (9 \times 5 + 8 + 9 \times a) \equiv (9 \times 0 + 8 + 9 \times b) \bmod 9$

(in both size we added a no multiple of 9)

~~$\Rightarrow (9 \times (5+a) + 8)$~~

$\Rightarrow (9 \times 5 + 8 + 9 \times a) \equiv (9 \times 0 + 8 + 9 \times b) \bmod 9$

$\Rightarrow (9 \times (5+a) + 8) \equiv (9(0+b) + 8) \bmod 9$

So, that is multiple of 9 so, when we mod then remainder will be same as previous.

$53 \equiv 8 \bmod 9$

$53 \equiv -1 \bmod 9$

we, subtract -9 multiple of 9, so remainder will be same by theorem.

by normally,

$9 \overline{)-1} (-1$

$+\underline{-9}$

8 ← remainder same as previous 8.

(iii) **Theorem:-**

$x \equiv y \bmod m \Rightarrow k \times x \equiv k \times y \bmod m$

Proff:- $x \equiv y \bmod m$

$\downarrow \qquad \downarrow$

$ma + b \equiv mc + b$

let, think like that $x = b$, $y = b$ let, x, y is remainder b, remainder same, so, if we

So, $k \times b \equiv k \times b \bmod m$ mul constant both same side

$k \times x \equiv k \times y \bmod m$. will be same.

(iv) **Theorem:-**

$a \equiv b \bmod c$

$d \equiv e \bmod c$

$f \equiv g \bmod c$

$(a \times d \times f) \equiv (b \times e \times g) \bmod c$

$a \equiv b \bmod c$

$d \equiv e \bmod c$

$f \equiv g \bmod c$

$(a + d + f) \equiv (b + e + g) \bmod c$

$a \equiv b \bmod c$
$d \equiv e \bmod c$
$f \equiv g \bmod c$

$((a-d)-f) \equiv ((b-e)-g) \bmod c$

Proff :-

$a \equiv b \bmod c$ → a_1
$d \equiv e \bmod c$ → b_1
$f \equiv g \bmod c$ → c_1

Let, think the remainders are a_1, b_1, c_1 they are same
so, the no are if same then we, do addition, MUL, div all be same.
Let, remainder is c_1 remainder will same, so, add, mul, div, same.

$\Rightarrow (a_1 * b_1 + c_1) = (a_1 * b_1 + c_1) \bmod c$.

(v) Theorem :-

$a \equiv b \bmod c \iff a^n \equiv b^n \bmod c$

Proff :-

$a \equiv b \bmod c$
$a \equiv b \bmod c$
$a \equiv b \bmod c$
$\vdots$
$a \equiv b \bmod c$
} n times

$\Rightarrow (\underbrace{a \times a \ldots \times a}_{n \text{ times}}) \equiv (\underbrace{b \times b \times b \ldots \times b}_{n \text{ times}}) \bmod c$

$\Rightarrow a^n \equiv b^n \bmod c$.

(vi) Remainder theorem :-

$(a \times b \times c \ldots) \bmod n = (a \bmod n \times b \bmod n \times c \bmod n \ldots) \bmod n$

Proff :-

Let, $(a \times b) \bmod n = (a \bmod n \times b \bmod n) \bmod n$

$\Rightarrow ((n a_1 + a_2) * (n b_1 + a_2 b_2)) \bmod n$

$\Rightarrow ((n a_1 \times n b_1) + (n a_1 \times b_2) + (n a_2 \times b_1) + a_2 b_2) \bmod n$

$\Rightarrow$ those term will cancel out as, multiple of n and we doing mod n

$\Rightarrow (a_2 b_2) \bmod n$

↑ this is nothing but

$(a \bmod n \times b \bmod n) \bmod n$

↓ if remainder $= a_2$ ↓ if remainder b_2

$\Rightarrow (a_2 \times b_2) \bmod n$

Q) Find remainder of $61 \times 72 \times 87$ when divided by 7

$\Rightarrow$ by theorem $\Rightarrow (61 \% 7 \times 72 \% 7 \times 87 \% 7) \bmod 7$

$\Rightarrow (5 \times 2 \times 3) \bmod 7$

$\Rightarrow (30) \bmod 7$

$\Rightarrow 2$ (Ans)

- **Remainder using binomial theorem :-**

$(x+a)^n \bmod x = ({}^nC_0\, x^n a^0 + {}^nC_1\, x^{n-1} a^1 + {}^nC_2\, x^{n-2} a^2 \dots {}^nC_n\, x^0 a^n) \bmod x$

$\Rightarrow$ if we, mod x the RHS then, all terms are multiple of x so, they are cancel out except the last one

${}^nC_n\, x^0 a^n \Rightarrow 1 \times 1 a^n \Rightarrow a^n$

So, we can say $\Rightarrow$

$$\boxed{(x+a)^n \bmod x \Rightarrow a^n \bmod x} \quad \star\star\star\star\star$$

↑ here also can be multiple of x

Q) $6^{100} \bmod 5$

$\Rightarrow (5+1)^{100} \bmod 5 \Rightarrow 1^{100} \bmod 5 \Rightarrow 1 \bmod 5 \Rightarrow 1$

$(x+a)^n \bmod x \Rightarrow a^n \bmod x$

Q) $3^{150} \bmod 7$

$\Rightarrow$ that no should be greater than this no. otherwise we can not apply binomial.

as, $(x+a)^n \bmod x$

that no is larger than that

$\Rightarrow$ So, we make it, $(3^2)^{75} \bmod 7$

$\Rightarrow (9)^{75} \bmod 7$

$\Rightarrow (8+1)^{75} \bmod 7$

$\Rightarrow {}^{75}C_0\, 8^{75} 1^0 + {}^{75}C_1\, 8^{74} 1^1 + {}^{75}C_2\, 8^{73} 1^2 \dots$

$\Rightarrow (8+9)^{75} \bmod 7$

$\Rightarrow (7+2)^{75} \bmod 7$

$\Rightarrow (2)^{75} \bmod 7$

=> again make x+a greater than x

=> $(2^3)^{25} \bmod 7$

=> $(8)^{25} \bmod 7$

=> $(7+1)^{25} \bmod 7$

=> $(^{25}C_0\, 7^{25} 1^0 + {}^{25}C_1\, 7^{24} 1^1 + {}^{25}C_2\, 7^{23} 1^2 + \ldots\ldots + {}^{25}C_{24}\, 7^1 1^{24} + {}^{25}C_{25}\, 7^0 1^{25}) \bmod 7$

So, all term are multiple of 7 so, all are cancel out except last one => $1^{25} \bmod 7$

=> $1 \bmod 7$

=> 1 (Ans)

Q) $(3)^{250} \bmod 7$

=> grater no than 7 then we can apply binomial

=> $(3^2)^{125} \bmod 7$

=> $(9)^{125} \bmod 7$

=> $(8+2)^{125} \bmod 7$

=> $(2)^{125} \bmod 7$

=> greater no than 7

=> $(2)^{123} \times 2^2 \bmod 7$

=> $(2^3)^{41} \times 2^2 \bmod 7$

=> $(8)^{41} \times 2^2 \bmod 7$

=> $(7+1)^{41} \times 4 \bmod 7$

=> $1^{41} \times 4 \bmod 7$

=> 4 mod => 4 (Ans)

Q) $2^{600} \bmod 15$

=> greater no than 15

=> $(2^4)^{150} \bmod 15$

=> $(16)^{150} \bmod 15$

=> $(15+1)^{150} \bmod 15$

=> $1^{150} \bmod 15$ => 1 mod 15 => 1 (Ans)

9) $2^{600} \bmod 17$ general method

$\Rightarrow$ greater than 17

$\Rightarrow (2^5)^{120} \bmod 17$

$\Rightarrow (32)^{120} \bmod 17$

$\Rightarrow (17+15)^{120} \bmod 17$

$\Rightarrow (15)^{120} \bmod 17$

lengthy process ✗

trick $\Rightarrow$

$\Rightarrow 2^{600} \bmod 17$

$\Rightarrow (2^4)^{150} \bmod 17$

$\Rightarrow (16)^{150} \bmod 17$

$\Rightarrow 16 \equiv 16 \bmod 17$

$\Rightarrow 16 \equiv -1 \bmod 17$ ← by theorem 2

$\Rightarrow (16)^{150} \equiv (-1)^{150} \bmod 17$ ← by theorem 5.

$\Rightarrow (-1)^{150}$ even $\Rightarrow 1$ (Ans)

9) $5^{625} \bmod 7$

$\Rightarrow (5^2)^{312} \times 5^1 \bmod 7$

$\Rightarrow (5^2)^{312} \times 5^1 \bmod 7$

$\Rightarrow (25)^{312} \times 5^1 \bmod 7$

$\Rightarrow (21+4)^{312} \times 5^1 \bmod 7$

↑ multiple of 7 so, that will be cancel out if we do mod 7

$\Rightarrow (4)^{312} \times 5^1 \bmod 7$

$\Rightarrow (4^2)^{156} \times 5^1 \bmod 7$

$\Rightarrow (16)^{156} \times 5^1 \bmod 7$

$\Rightarrow (14+2)^{156} \times 5^1 \bmod 7$

$\Rightarrow (2)^{156} \times 5^1 \bmod 7$

$\Rightarrow (2^3)^{52} \times 5^1 \bmod 7$

$\Rightarrow (8)^{52} \times 5^1 \bmod 7$

$\Rightarrow (7+1)^{52} \times 5^1 \bmod 7$

$\Rightarrow 1^{52} \times 5^1 \bmod 7$

$\Rightarrow 5 \bmod 7 \Rightarrow 5$ (Ans)

9) $6^{100} \bmod 15$

$\Rightarrow (6^2)^{50} \bmod 15$

$\Rightarrow (36)^{50} \bmod 15$

$\Rightarrow (30+6)^{50} \bmod 15$

$\Rightarrow (6)^{50} \bmod 15$ ← multiple of 15

$\Rightarrow (6^2)^{25} \bmod 15$

$\Rightarrow (30+6)^{25} \bmod 15$

$\Rightarrow (6)^{25} \bmod 15$

$= 6^{24} \times 6^{1} \times \text{mod } 15$

$= (6^2)^{12} \times 6^1 \times \text{mod } 15$

$\Rightarrow (30+6)^{12} \times 6^1 \text{ mod } 15$

$\Rightarrow$ multiple of 15

$\Rightarrow (6)^{12} \times 6^1 \text{ mod } 15$

$\Rightarrow (6^2)^6 \times 6^1 \text{ mod } 15$

$\Rightarrow (6)^6 \times 6^1 \text{ mod } 15$

$\Rightarrow (6^2)^3 \times 6^1 \text{ mod } 15$

$\Rightarrow (6)^3 \times 6^1 \text{ mod } 15$

$\Rightarrow$ ~~$(6)^2 \times 6^1 \times 6^1 \text{ mod } 15$~~

$\Rightarrow 6^4 \text{ mod } 15$

$\Rightarrow (6^2)^2 \text{ mod } 15$

$\Rightarrow (30+6)^2 \text{ mod } 15$

$\Rightarrow 6^2 \text{ mod } 15$

$\Rightarrow 36 \text{ mod } 15$

$\Rightarrow 6$.

- **shortcut :-**

$a^{\phi(n)} \equiv 1 \text{ mod } n$ as n must be coprime.

$\phi(n)$: Euler totient

GCD $(a, n, 1) = 1$

it is used to find less than how many no. coprime to that no.

How to calculate $\phi(n)$??

$\Rightarrow \phi(n) \Rightarrow$ 1st step prime factorization

$n \Rightarrow p_1^{n_1} \times p_2^{n_2} \times p_3^{n_3} \cdots$

~~$\phi(n) \times (p_1^{n_1} - p_1^{n_1})$~~

$\Rightarrow \phi(n) = (p_1^{n_1} - p_1^{n_1-1}) \times (p_2^{n_2} - p_2^{n_2-1}) \cdots$

Ex :- $\phi(15) \Rightarrow 3^1 \times 5^1$

$\Rightarrow (3^1 - 3^{1-1}) \times (5^1 - 5^{1-1})$

$\Rightarrow (3^1 - 3^0) \times (5^1 - 5^0)$

$\Rightarrow (2) \times (4) \Rightarrow 8$

Ex :- $6^{100} \text{ mod } 5$

(6 and 5 are coprime to each other)

$\Rightarrow a^{\phi(n)} \equiv 1 \text{ mod } n$

$\Rightarrow 6^4 \equiv 1 \text{ mod } 5$

$\Rightarrow (6^4)^{25} \equiv 1^{25} \text{ mod } 5$

$\Rightarrow 6^{100} \equiv 1^{25} \text{ mod } 5$

$\Rightarrow 1 \text{ mod } 5 \Rightarrow 1$ (Ans)

$\phi(5) \Rightarrow (5^1 - 5^0)$

$\Rightarrow 4$

$\phi(p) \Rightarrow (p-1)$

for prime no

3) $3^{150} \bmod 7$

(coprime)

$\phi(7) \Rightarrow 7^1 - 7^0$

$\Rightarrow 6$

$\Rightarrow a^{\phi(n)} \equiv 1 \bmod n$

$\Rightarrow 3^{\phi(7)} \equiv 1 \bmod 7$

$\Rightarrow 3^6 \equiv 1 \bmod 7$

$\Rightarrow (3^6)^{25} \equiv 1 \bmod 7$

$\Rightarrow 3^{150} \equiv 1 \bmod 7$

$\Rightarrow 1 \bmod 7 \Rightarrow 1$ (Ans).

3) $3^{250} \bmod 7$

$\Rightarrow 3^{\phi(7)} \equiv 1 \bmod 7$

$\Rightarrow 3^6 \equiv 1 \bmod 7$

$\Rightarrow (3^6)^{41} \times 3^4 \equiv 1 \bmod 7$

$\Rightarrow 3^{6 \times 41} \times 3^4 \equiv 1 \bmod 7$

$\Rightarrow 3^{\phi(7) \times 41} \times 3^4 \bmod 7$

$\Rightarrow 1 \times 3^4 \bmod 7$

$\Rightarrow 81 \bmod 7 \Rightarrow 4$ (Ans)

we, know that

$a^{\phi(n)} \equiv 1 \bmod n$

$\Rightarrow a^n \equiv b^n \bmod m$ ← we know that

so, $a^{k \times \phi(n)} \equiv 1^k \bmod n$

$\Rightarrow a^{k \times \phi(n)} \equiv 1 \bmod n$

$2^{600} \bmod 15$

(coprime)

$\phi(15) \Rightarrow 3^1 - 3^0 \times 5^1 - 5^0$

$\Rightarrow 2 \times 4$

$\Rightarrow 8$

$\Rightarrow 2^{\phi(15)} \equiv 1 \bmod 15$

$\Rightarrow 2^8 \equiv 1 \bmod 15$

$\Rightarrow (2^8)^{75} \equiv 1 \bmod 15$

$\Rightarrow (2)^{600} \equiv 1 \bmod 15$

$\Rightarrow 1 \bmod 15 \Rightarrow 1$ (Ans)

g) $2^{600} \bmod 17$ (coprime)

$\phi(17) \Rightarrow 17^1 - 17^0 \Rightarrow 16$

$\Rightarrow (2)^{\phi(17)} \equiv 1 \bmod 17$

$\Rightarrow 2^{16} \equiv 1 \bmod 17$

$\Rightarrow (2^{16})^{37} \times 2^8 \bmod 17$

$\Rightarrow 2^{\phi(17) \times 37} \times 2^8 \bmod 17$

[$a^{\phi(n) \times k} \equiv 1^k \bmod n$]

$\Rightarrow 1 \times 2^8 \bmod 17$

$\Rightarrow 256 \bmod 17$

$\Rightarrow 1$ (Ans)

g) $5^{625} \bmod 7$ (coprime)

$\phi(7) \Rightarrow 7^1 - 7^0 \Rightarrow 6$

$\Rightarrow 5^{\phi(7)} \equiv 1 \bmod 7$

$\Rightarrow 5^6 \equiv 1 \bmod 7$

$\Rightarrow (5^6)^{104} \times 5^1 \bmod 7$

$\Rightarrow 5^{\phi(7) \times 104} \times 5^1 \bmod 7$

[$a^{\phi(n) \times k} \equiv 1 \bmod n$
$a^{\phi(n) \times k} = 1 \bmod n$]

$\Rightarrow 1 \times 5^1 \bmod 7$

$\Rightarrow 5$ (Ans)

g) $6^{100} \bmod 15$ (not coprime $\Rightarrow$ GCD $\Rightarrow 3$)

$\Rightarrow$ we have to use general method

$\Rightarrow (6^2)^{25} \bmod 15$

$\Rightarrow (30+6)^{25} \bmod 15$

$\Rightarrow (6)^{25} \bmod 15$

$\Rightarrow (6)^{12} \times 6^1 \bmod 15$

$\Rightarrow (6)^{2 \times 6} \times 6^1 \bmod 15$

$\Rightarrow (6)^3 \times 6^1 \bmod 15$

$\Rightarrow (6)^4 \bmod 15$

$\Rightarrow (6)^2 \bmod 15$

$\Rightarrow 6^1 \bmod 15 \Rightarrow 6$ (Ans)

9. Calender Problems:- ☆☆☆

Basic concepts:-

1. Multiple of 4 is leap year [4, 8, 12, 16 ....]
2. Every Century year is not a leap year [100, 200, 300...]
3. 4th century year is a leap year [400, 800, 1200 ....]

Note:-

Among normal years all years multiple of 4 are leap years & among Century years every 4th Century year is leap year.

Not Century % 4 = 0 => leap year

Century % 400 = 0 => leap year

Q) Why every century year is not leap year?

=> 1 year is 365 days, 5 hrs, 48 min & 11sec

↓ not 6 hrs.

Q) How many weeks in ordinary year & leap year?

=> Non leap year => $\lfloor \frac{365}{7} \rfloor$ => 52 week & 1 day

=> leap year => $\lfloor \frac{366}{7} \rfloor$ => 52 week & 2 day

Q) How many odd days in ordinary year & leap year?

=> odd days means extra days. from with week

OY => 1 odd day ☆☆

LY => 2 odd day ☆☆

Q) How many leap year from 1st to 100th year?

1 Jan 0001

1 Jan 0002

⋮ $\lfloor \frac{100}{4} \rfloor$ => 25 - 1 => 24

↑ as the 100th year is not leap year

Ex:- 1st to 8th year

=> (1 2 3 4) (5 6 7 8) $\lfloor \frac{8}{4} \rfloor$ => 2 (ans)

1st to 100th

=> (1 2 3 4) (5 6 7 8) 96 (97 98 99 100)

1st 2nd 25th

100 is not a leap year

=> $\lfloor \frac{100}{4} \rfloor$ => 25 - 1 => 24

Ex :- 1st to 100th

=> $\lfloor \frac{10}{4} \rfloor$ => 2

=> (1 2 3 4) (5 6 7 8) (9 10)

Q) How many odd days from 1 to 100th year?

=> Method 1 :-

ordinary year => 1 odd day

Leap " => 2 " "

1st to 100th => 100 extra day

Leap year => 24 extra days

∴ total => 124 day

now, those are extra day ordinary days should group by week

So, again group them. in week

124 % 7 => 5 ODD days (AS)

```
   17
7) 124
  -7
  ---
   54
  -49
  ---
    5
```

means 17 weeks 5 odd days

Method 2 :-

Ex :- 1 to 8th year ODD days?

=> (1 2 3 4) (5 6 7 8)

=> 3 + 2 total => 10 extra day

10 % 7 => 3 odd day

One extra as they are leap year

in, 1 to 3rd How many years ?

=> upper - lower + 1

=> (3 - 1 + 1) => 3 (AS)

Or, count 1st year, 2nd year, 3rd year => total 3.

Method 3 :-

24 LY + (100 - 24) OY

=> 24 × 2 + (76) × 1

=> 124 extra day

- Summary :- Extra days of years %. 7 => 0 dd days.

=> According to Gregorian calender 1st Jan 0001 A.D was

A mo Domini

BC → Before Christ

AD → After Christ.

=> Monday

hence, 1st odd will be Monday.

2nd " " " Tuesday

3rd " " " Wednesday

4th " " " Thusday

:

7th " " " Sunday

8th " " " 1st odd day i.e Monday.

1 → Mon

2 → Tue

3 → Wed

:

7 → Sun

8 → Mon 8 % 7 => 1

[cycle of 7]

in, 200 years odd day => ?

1 → 200 => 200 years

=> leap year => $\lfloor \frac{200}{4} \rfloor$ => 50 - 1 - 1 => 48

(100 is not leap year; 200 is not leap year)

=> ODY (200 - 48) + 48 LY

=> 152 × 1 + 48 × 2

=> 152 + 96 => 248 Extra days

∴ ODD days => (248)% 7 => 3 ODD day.

Easy method :->

200 yrs => 200 Extra day

48 LY => +48 "

248 "

(248 % 7) => 3 ODD day

In, 300 years odd days? (1st to 300th)

LY => $\lfloor \frac{300}{4} \rfloor$ => 75 - 1 - 1 - 1

=> 72 (for 100, 200, 300)

=> 300 years => 300 extra day

72 LY => +72 " "

372 " "

ODD day => (372 % 7) => 1 ODD day.

Q) in 1st to 400th odd days?

=> 400 years => 400 extra day

=> 97 LY => 497 "

497 "

∴ ODD day => 497 % 7

=> 0.

LY => $\lfloor \frac{400}{4} \rfloor$ = 100 - 1 - 1 - 1

=> 97 (for 100, 200, 300)

Note :- for 400 we will not subtract 1 as it is Leap year

- Summary :-

100 Y → 5 odd days

200 Y → (5 × 2) => 10 % 7 => 3 odd days

300 Y → (5 × 3) => 15 % 7 => 1 odd days

400 Y → (5 × 4 + 1) => 21 % 7 => 0 odd days (400 is leap year)

500 Y → 5 ODD

600 Y → 3 ODD

=> Hence, 400, 800, 1200, 1600, 2000 ..... all produce

=> 0 odd day.

years =>	100	200	300	400	500	600	700	800	900	1000	1100	1200
ODD pay =>	5	3	1	0	5	3	1	0	5	3	1	0

∴ (century % 400) / 100 => 1 => 5 odd day
2 => 3 odd day
3 => 1 odd day
0 => 0 odd day

Q) Day on 15th Aug 1947.

⇒ step 1 :- find the century

=> (1900 % 400) / 100

=> 3 => 1 odd day -(i)

step 2 :- go to year less than 1 of that

=> $\lfloor \frac{46}{4} \rfloor$ => 11

11 LY + (46 - 11) OY

~~=> 22 ED~~

=> (11 × 2) ED + (35 × 1) ED

=> 22 ED + 35 ED

=> 1 ED 0 ED

=> 1 odd days -(ii)

Or, another method :-

46 yr → 46 ED

11 LY → 11 ED

57

57 ED => 57 % 7 => 1 odd day

∴ total => 1+1 => 2 odd day by century and year to include that day

step 3 :- then go that year month by moth

Jan	Feb	Mar	April	May	Jun
3 odd	0 odd day	3 odd day	2 odd	3 odd	2 odd

Jul	Aug
3 odd	15 % 7 => 1 odd Day

∴ total Extra day month by moth => ~~1 % 7 =>~~ => 17

∴ odd day => 17 % 7 => 3 odd day

∴ total odd by century + year + moth => 1 + 1 + 3

=> 5 odd day.

∴ 5 odd Day => Friday (Ans)

1 ED => Monday

if, 21 ED % 7 => 0 ← Sunday

• Summary :-

Step 1 :- Reach century

Step 2 :- 1 less than year

Step 3 :- in actual year go month by month and include the given date to calculate OD | ED

• Summary :-

Step 1 : calculate odd days for century

like 1700 → 5 ODD days
1900 → 1 " "
2000 → 0 " "

Step 2 : 1 less than actual year calculate odd days like for 1947 year calculate ODD till 1946

Step 3 : finally month by month for actual year including date.

Q) Day on 22 April 2022.

⇒ Step 1 :- Century ⇒ 2000 ⇒ 0 ODD day

Step 2 :- 2001 to 2021 ⇒ 21 yrs

⇒ $\left\lfloor \frac{21}{4} \right\rfloor$ ⇒ 5 LY

(21 − 5) × 1

⇒ 5 × 2 + 16

⇒ 10 + 16

⇒ 3 2

⇒ 5 ODD days till 2021 ⇒ 0 + 5 ⇒ 5 odd day

Step 3 :- 2022 ⇒

Jan	Feb	March	April
3	0	3	22 % 7 ⇒ 1

⇒ total ⇒ 7 % 7 ⇒ 0

∴ total ⇒ 5 + 0

⇒ 5 ODD day ⇒ Friday.

8) If 15th Aug 1947 was Friday then 26th Jan 1950 was _____ ? ★★★★

=> if one day is given then go with Method 2.

2 methods → ① Directly 26th Jan 1950 day ✗
→ ② Relative Calculation ✓

Step 1 :- month by month given year

Aug	Sep	Oct	Nov	Dec
16/7 => 2	2 odd	3 odd	2 odd	3 odd

(actual)

Step 2 :- then asked year one previous by year

1948 → 2 odd
1949 → 1 odd
~~1950~~ (actual)

Step 3 :- then that year month by month

=> 26/7 => 5 odd.

∴ total => 1 + 5 => 6 odd.

Friday
1 Sat
2 Sun
3 Mon
4 Tues
5 Wed
6 Thus

∴ Thusday (ans)

like, Day given

15 th Aug 1947 → Friday (given)
+1 16 " " " → Sat
+2 17 " " " → Sun
+3 19 " " " → Mon
+4 19 " " " → Tues
+5 20 " " " → ~~Tues~~ Wed
+6 21 " " " → Thus
+7 22 " " " → Friday

When, 7 days => then Same day. (cycle)

if, 15 Aug → 1947 given Fri

the 31 Aug will be :- (31 - 15) => 16

16/7 => 2

So, Fri
+1 Sat
+2 Sunday (ans)

Q) if 31 Aug 1947 was sunday
5 Jul 1949 will be ?

=> Step 1 :- that year moth by moth

=> Sep 2 | Oct 3 | Nov 2 | Dec 3

Step 2 :- till previous year by year

1948 → 2 Odd (leap year)

that year → 1949 →

Step3 :- Jan 3 | Feb 0 | March 3 | April 2 | May 3 | Jun 2 | Jul 5 % 7 => 5

=> 9 % 7 => 2 ODD

Starting was => Sun

So, +2 => Tuesday (Ans)

Q) 30 Aug 1947 was Sat 5th Jul 1949 ?

=> Aug [(31-30) => 1 % 7] => 1 | Sept 2 | Oct 3 | Nov 2 | Dec 3

1948 → 2

1949 → Jan 3 | Feb 0 | Mar 3 | Apr 2 | May 3 | Jun 2 | Jul 5 % 7 => 5

=> 1+3+3+3 % 7 => 3 odd day

Starting was => Sat

+3 => Tues day.

Q) Which year has same calendar as 1991 year ?

=> if given year => Ordinary year

then find year => ordinary year

(as, in LY 1st day may match but after 28th Feb there will be 29th Feb then all calculation will be wrong)

So, given year => Leap year

find year => Leap year

Q) Which year has same calender as 1991 year?

=> if, 1st Jan 1991 → no.day
1 odd day

1992 2 " - (leap year) → Tues
93 1 " - (Ordinary year) → Thus
94 1 " " → Fri
95 1 " " → Sat
96 2 " " → Sun (OLY)
97 1 " " → Tues
98 1
99 1
2000 2
2001 1 → Monday

1st Jan 2002

=> 2002 (Ans).

Q) 1995 => ??

1995 => 1 if Monday
96 => 2
97 => 1
98 => 1
99 → 1
2000 → 2
2001 → 1
2002 → 1
2003 → 1
2004 → 2
2005 → 1
2006 → Monday

=> so we are try to finding odd days of multiple of 7 if, multiple of 7 come then the calender repeat, but if given is ordinary if getting leap year then we have to go furthur if, we get multiple of 7 and ordinary year then that will ans, next year will repeat the calender of multiple of 7.

Ans => 2006

Q) 1996 => ?

have to find Leap year

4 (1996 => 2
1997 => 1
98 => 1
99 => 1) 5 if monday

2000 => 2

2001 => monday 5

=> 2001 ⇐ ~~Monday~~ (2000) but 2001 is not leap year
so, it can not repeat => 1

4 (2002 => 1
03 => 1)

04 => 2

4 year (06 ⇒ 1, 01 ⇒ 2) 5 odd day

08 ⇒ 2

So, 4 year (1996, 2000) 5 odd day

4 (2004) 5

4 (2008) 5

4 (2012) 5

4 (2016) 5

4 (2020) 5

4 (2024) 5

So, we are jumping 4 year to find leap year and we need multiple of 7 in ~~loop~~ odd day

So, each step giving 5 odd day

So, at 35 we will get multiple of 7 by jumping 5 odd days

So, we have to jump 7 times to get 35

So, in year ⇒ 4 × 7 ⇒ 28.

Shortcut :- leap year repeats calender after ⇒ 28 years

So, 1996 (given)
+28
2024 (Ans)

③ Algebra :-

11. Logarithims Basic : Part 1 :-

⇒ Log is nothing but a function

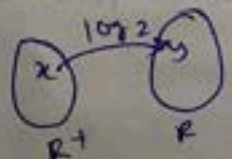
⇒ Log is a function from positive real no to real no

$f \log_a : R^+ \to R$

($a \in R^+$)

⇒ base can be only positive real no. except 1

⇒



$\Rightarrow y = \log_2 x$

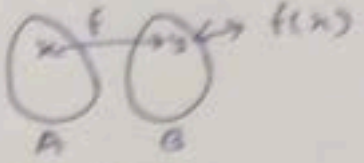
$\Rightarrow x = 2^y$

$\Rightarrow 2^y = x$

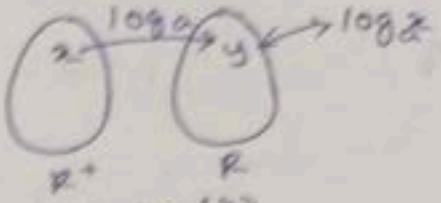
$\Rightarrow y = \log_2 x$

⇒ $\log_a$ is a function from $R^+ \to R$

$f : A \to B$



$y = f(x)$



$y = \log_a(x)$

$\Rightarrow x = a^y$

⇒ $a \in R^+ - \{1\}$ i.e

base of log is any +ve real no except 1.

Let, say base ⇒ -4

⇒ $\log_{-4} x \Rightarrow y$ ∴ Base can not -ve

⇒ $x = -4^y$

Let, $x = 2$

⇒ $2 = (-4)^y$

$(-4)^y$ can not equal to 2 if y is a real no so, for this value this function can not work so, that is not a valid function.

⇒ Let, base = 1

$\log_a x \Rightarrow y$

$\Rightarrow x = a^y$ ∴ base can not be 1

$\Rightarrow x = 1^y$ [a=1]

⇒ Let, say $x = 2$

⇒ $2 = 1^y$

⇒ we can not make $1^y \Rightarrow 2$ so, also that is not valid

Let, say base = 0.5

$\log_{0.5} x \Rightarrow y$

$\Rightarrow x = 0.5^y$ ∴ base can be fraction

Let, say $x = 2$

⇒ $2 = 0.5^y$

if, we put ⇒ $y = -1$ then we will get 2

⇒ $(0.5)^{-1} \Rightarrow \left(\frac{1}{2}\right)^{-1} \Rightarrow 2$ so, it is valid.

⇒ most commonly used base are 2, 10, e

(i) $\log_{10}(x)$ ⇒ Decimal or common logarithim (Science & eng.)

(ii) $\log_e(x)$ ⇒ natural Logarithim [math & physics]

$e = 2.71\ldots$

(iii) $\log_2(x)$ ⇒ Binary logarithim (c.s)

All questions are properties try to remember

Q) $a^x = y$ find x

$\Rightarrow$ ~~log~~ $x \Rightarrow \log_a y$

Q) $\log_a a = \underline{\quad 1 \quad}$.

let, say $\Rightarrow \log_a a \Rightarrow x$

$\Rightarrow a = a^x$

$\Rightarrow a^1 = a^x$

$\therefore x \Rightarrow 1$

$a^x = y$

$\Rightarrow x \Rightarrow \log_a y$

$\therefore \Rightarrow \log_a y \Rightarrow x$

$\Rightarrow y = a^x$

Q) $\log_a a^x = \underline{\quad y \quad}$

let say,

$\Rightarrow \log_a a^x \Rightarrow y$

$\Rightarrow a^x \Rightarrow a^y$

$\Rightarrow x = y$

Ex:- $\log_2 16 \Rightarrow \log_2^4 \Rightarrow 4$

$\log_3 9 \Rightarrow \log_3^2 \Rightarrow 2$

$\log_5 625 \Rightarrow \log_5^4 \Rightarrow 4$

$\log 10^1 \Rightarrow \log_{10}^{10^0} \Rightarrow 0$

Q) $\log 1_a \Rightarrow \underline{\quad 0 \quad}$ ★★

$\Rightarrow$ if, $a \Rightarrow 5$

$\Rightarrow \log 1_5 \Rightarrow \log_5^{5^0} \Rightarrow 0.$

$\Rightarrow \log_2^{(0.5)} \Rightarrow \log_2^{(2)^{-1}} \Rightarrow -1$

Ex:- $\log_2^{16} \Rightarrow 4$

$\frac{16}{2} \Rightarrow 8$

$\frac{8}{2} \Rightarrow 4$

$\frac{4}{2} \Rightarrow 2$

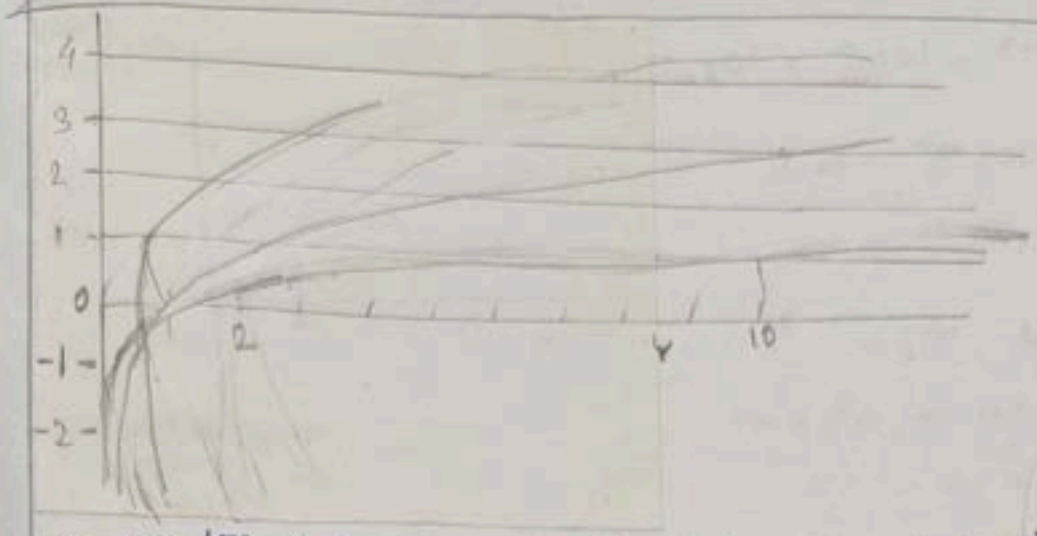
$\frac{2}{2} \Rightarrow 1$

how, many times we have to divide 16 by 2 untill we get 1 is log here, we have to divide 4 time so, $\log_1^{16} \Rightarrow 4$

Q) $\log_2 8 \Rightarrow \underline{\quad 3 \quad}$

$\Rightarrow \log_2^8 = 3$ means no of times 2 can divided by 8 to get 1

(OR) $\Rightarrow$ keep on dividing 8 by 2 till we get 1 & count no of divison.



$\Rightarrow \log_{10}^{100} \Rightarrow 2$

$\Rightarrow \log_{2}^{100} \Rightarrow 6\ldots\ldots$

$\Rightarrow \log_{e}^{100} \Rightarrow$

So, if we increase e base then value decrease

So, $\log_{2}^{x}$ outer middle $\log_{e}^{x}$ inner $\log_{10}^{x}$

$\Rightarrow \log_{a}^{1} \Rightarrow 0$ (x, y)

$\Rightarrow \log_{10}^{.00000001}$

$\Rightarrow \log_{10}^{(10)^{-8}}$

$\Rightarrow -8$

so, any how it can not be 0 so graph will not touch 0.

$\Rightarrow$ in b/w 0 and 1 value

$\Rightarrow \log_{2}^{0.5}$

$\Rightarrow -1 \leftarrow$ value will go to 0 but not negative.

$\Rightarrow \log_{2}^{x}$ for $x \Rightarrow 2$

$\log_{e}^{e}$ if $x \Rightarrow e$ $\Rightarrow 1$

$\Rightarrow \log_{2}^{2} \Rightarrow 1 \quad \therefore y \Rightarrow 1$

$\Rightarrow \log_{2}^{x} \Rightarrow$ for $x \Rightarrow 1$

$\Rightarrow \log_{2}^{1} \Rightarrow \log_{2}^{2^0} \Rightarrow 0 \leftarrow y$

for base 10

$\log_{10}^{1} \Rightarrow \log_{10}^{10^0}$

$\Rightarrow 0 \leftarrow y$

for, base e

$\Rightarrow \log_{e}^{1} \Rightarrow 0 \leftarrow y$

So, for all base it will touch at $\Rightarrow 1$

$\Rightarrow$ So, all base meet at $x=1$ there y value $\Rightarrow 0$

$\Rightarrow$ and for small value it will be $-ve$ but not touch 0

$\Rightarrow$ for $\log_{e}^{e} \Rightarrow 1$ $\log_{2}^{2} \Rightarrow 1$ $\log_{10}^{10} \Rightarrow 1$ for $x =$ that base will touch $y \Rightarrow 1$

$\Rightarrow$ higher base smaller no, smaller base higher no.

Q) $\log_a^{(m\times n)} = \log_a^m + \log_a^n$

$\Rightarrow$ Let, $\log_a^m \Rightarrow x \quad \Rightarrow m \Rightarrow a^x$

$\log_a^n \Rightarrow y \quad \Rightarrow n \Rightarrow a^y$

we need $\log_a^{(m\times n)}$ so, multiply $m \times n$

$\Rightarrow m \times n \Rightarrow a^x \times a^y$

$\Rightarrow m \times n \Rightarrow a^{x+y}$

$\Rightarrow \log_a^{m\times n} \Rightarrow \log_a^{a^{x+y}}$ (taking log both side)

$\Rightarrow \log_a^{m\times n} \Rightarrow x + y$

$\Rightarrow \log_a^{m\times n} \Rightarrow \log_a^m + \log_a^n$ (putting value of x and y)

$\therefore$ (Proved)

- Generalize :-

$\log_a^{(m\times n\times p\ldots)} \Rightarrow \log_a^m + \log_a^n + \log_a^p \ldots$

Q) $\log_a^{m^n} \Rightarrow n \times \log_a^m$

$m^n \Rightarrow \underbrace{m\times m\times m\times m}_{n \text{ times}}$

$\Rightarrow \log_a^{\overbrace{(m\times m\times m\times m\ldots)}^{n \text{ times}}}$

$\Rightarrow \underbrace{\log_a^m + \log_a^m + \log_a^m \ldots}_{n \text{ times}}$

$\Rightarrow n \times \log_a^m$ (proved)

$\Rightarrow \boxed{\log_a^{m^n} \Rightarrow n \times \log_a^m}$

like, $\log_{10}^{81} \Rightarrow \log_{10}^{3^4} \Rightarrow 4 \times \log_{10}^3$

Ex :- $\log_2 n^4 \Rightarrow 4 \times \log_2^n$

Q) $\log_a^{(\frac{1}{n})} = -\log_a^n$

$\Rightarrow \log_a^{(n)^{-1}} \Rightarrow -1 \times \log_a^n$

$\Rightarrow \boxed{\log_a^{(\frac{1}{n})} \Rightarrow -\log_a^n}$

Ex :- $\log_3^{0.5} \Rightarrow \log_3^{2^{-1}}$

$\Rightarrow -\log_3^2$

Ex :- $\log_2^{(0.25)}$

$\Rightarrow \log_2^{(\frac{1}{4})}$

$\Rightarrow \log_2^{(4)^{-1}}$

$\Rightarrow -\log_2^4$

$\Rightarrow -2$

9) $\log_{a^n}^m = \frac{1}{n} \log a^m$

$\Rightarrow$ in RHS Part we can not do some thing, in LHS we can so, take is as x, (try to solve ~~solveab~~ solveable part)

$\Rightarrow x \Rightarrow \log_{a^n}^m$

$\Rightarrow m \Rightarrow (a^n)^x$

$\Rightarrow m \Rightarrow a^{nx}$

$\Rightarrow$ we need $\log_a^m$ so, taking $\log_a$ both side

$\Rightarrow \log_a^m \Rightarrow \log_a^{a^{nx}}$

$\Rightarrow \log_a^m \Rightarrow nx$

$\Rightarrow x \Rightarrow \frac{1}{n} \times \log_a^m$

$\Rightarrow \boxed{\log_{a^n}^m \Rightarrow \frac{1}{n} \times \log_a^m}$ (Putting value of n) (Proved)

Ex :- $\log_8^2$

$\Rightarrow \log_{2^3}^2$

$\log_{a^n}^m$ here, $a=2$, $n \Rightarrow 3$, $m=2$

$\hookrightarrow \frac{1}{n} \log a^m$

$\Rightarrow \frac{1}{3} \log_2^2 \Rightarrow \frac{1}{3}$ (Ans)

12. Logarithms : Part : 2 :-

9) $\log_a(\frac{m}{n}) = \log_a^m - \log_a^n$

$\Rightarrow$ let say, $\log_a^m \Rightarrow x \Rightarrow m \Rightarrow a^x$ ...(i)

$\log_a^n \Rightarrow y \Rightarrow n \Rightarrow a^y$ ..(ii)

$\Rightarrow$ (i) ÷ (ii)

$\Rightarrow \frac{m}{n} = \frac{a^x}{a^y}$

$\Rightarrow \log_a\left(\frac{m}{n}\right) \Rightarrow a^{x-y}$

$\Rightarrow$ taking $\log_a$ both side

$\Rightarrow \log_a\left(\frac{m}{n}\right) \Rightarrow \log_a a^{x-y}$

$\Rightarrow \log_a\left(\frac{m}{n}\right) \Rightarrow x-y$

$\Rightarrow \boxed{\log_a \frac{m}{n} \Rightarrow \log_a^m - \log_a^n}$ (putting value of x and y)

(proved)

8) $\log_a^x \Rightarrow \frac{1}{\log_x^a}$

$\Rightarrow$ Let, $\log_a^x \Rightarrow y$

$\Rightarrow x = a^y$

$\Rightarrow$ taking $\log_x$ both side

$\Rightarrow \log_x^x = \log_x^{a^y}$

$\Rightarrow \log_x^{a^y} = \log_x^x$

$\Rightarrow y \log_x^a = 1$

$\Rightarrow \log_x^a \Rightarrow \frac{1}{y}$

$\Rightarrow \log_x^a = \frac{1}{\log_a^x}$ (putting value)

$\Rightarrow \boxed{\log_a^n \Rightarrow \frac{1}{\log_x^a}}$ (proved)

9) $\log_a^b \times \log_b^a = \underline{1}$?

$\Rightarrow \log_a^b \Rightarrow \frac{1}{\log_b^a}$ (by identity)

$\Rightarrow \frac{1}{\log_b^a} \times \log_b^a \Rightarrow 1$ (Ans)

$\therefore \boxed{\log_a^b \times \log_b^a = 1}$

8) $a^{\log_b c} = c^{\log_b a}$

⇒ let, $a^{\log_b c} = x$ | let, $\log_b^a \Rightarrow y$

$\Rightarrow c = b^x$ | $\Rightarrow a \Rightarrow b^y$

So, now we need to Prove ⇒ $a^x \Rightarrow b^y$

$\Rightarrow a = b^y$

$\Rightarrow a^x = (b)^x$

$\Rightarrow a^x = b^{yx}$

$\Rightarrow a^x = (b^x)^y$

$\Rightarrow a^x = c^y$ (putting value of b^x)

$\Rightarrow$ $\boxed{a^{\log_b c} = c^{\log_b a}}$ (Proved)

9) $\log_a^b = \frac{\log_n^b}{\log_n^a}$ (Base change formula) ★★★

-> let, $\log_a^b \Rightarrow x$

$\Rightarrow b = a^x$

$\Rightarrow$ taking log n both side

$\Rightarrow \log_n^b = \log_n^{a^x}$

$\Rightarrow \log_n^b = x \log_n^a$

$\Rightarrow x = \frac{\log_n^b}{\log_n^a}$

$\Rightarrow$ $\boxed{\log_a^b = \frac{\log_n^b}{\log_n^a}}$ (Proved) (Putting value of x)

Summary :- (Properties) ✱✱

1. $\log_a^a = 1$
2. $\log_a^{(m \times n)} = \log_a^m + \log_a^n$
3. $\log_a^{m^n} = n \times \log_a^m$
4. $\log_a^{\frac{1}{n}} \Rightarrow -\log_a^n$
5. $\log_a^{\frac{m}{n}} \Rightarrow \log_a^m - \log_a^n$
6. $\log_{a^n}^m \Rightarrow \frac{1}{n} \times \log_a^m$
7. $\log_a^b \Rightarrow \frac{\log_n^b}{\log_n^a}$
8. $\log_a^1 \Rightarrow 0$
9. $\log_a^x \Rightarrow \frac{1}{\log_x^a}$
10. $\log_b^a \times \log_a^b = 1$
11. $a^{\log_b^c} \Rightarrow c^{\log_b^a}$

Grade 8) $\frac{1}{\log_2^{30}} + \frac{1}{\log_5^{30}} + \frac{1}{\log_3^{30}} = \text{——}$? $\quad \log_y^x \Rightarrow \frac{1}{\log_x^y}$

$\Rightarrow \log_{30}^2 + \log_{30}^5 + \log_{30}^3$

$\Rightarrow \log_{30}^{2 \times 3 \times 5} \Rightarrow \log_{30}^{30} \Rightarrow 1$

9) $\log_{10}^3 + \log_{10}^{(4x+2)} = \log_{10}^{(x+1)} + 1$

$\Rightarrow \log_{10}^{12x+6} = \log_{10}^{x+1} + \log_{10}^{10}$

$\Rightarrow \log_{10}^{12x+6} = \log_{10}^{10x+10}$

$\Rightarrow 12x + 6 = 10x + 10$

$\Rightarrow 2x = 4$

$\Rightarrow x = 2$ (Ans)

9) $\frac{1}{\log_{(\frac{a}{b})}^x} + \frac{1}{\log_{(\frac{b}{c})}^x} + \frac{1}{\log_{(\frac{c}{a})}^x}$

$\Rightarrow \log_x^{(\frac{a}{b})} + \log_x^{(\frac{b}{c})} + \log_x^{(\frac{c}{a})}$

$\Rightarrow \log_x^{(\frac{a}{b} \times \frac{b}{c} \times \frac{c}{a})} \Rightarrow \log_x^1 \Rightarrow \log_x^{x^0} \Rightarrow 0$ (Ans)

8) if $\log(P) = \frac{1}{2}\log(Q) = \frac{1}{3}\log(R)$ then, True?

a) $P^2 = Q^3R^2$ b) $Q^2 = PR$ c) $Q^2 = R^3$ d) $R = P^2Q^2$

$\Rightarrow \log(P) = \log(Q)^{\frac{1}{2}} = \log(R)^{\frac{1}{3}}$

$\Rightarrow P = \sqrt{Q} = \sqrt[3]{R}$

$\Rightarrow P = Q^{\frac{1}{2}} = R^{\frac{1}{3}}$ $\Rightarrow P^4 = Q^2 = R^{2/3}$

$\Rightarrow P^4 = Q^2$

$\Rightarrow Q^2 = P^3 \cdot P$

$\Rightarrow Q^2 = P^3 \cdot P^3$

$\Rightarrow Q^2 = PR$ $\therefore$ (option b) true

$P^2 = Q = R$

$\therefore$ (a) false

$P^3 = Q^{\frac{3}{2}} = R$

$\therefore \boxed{P^3 = R}$

$\Rightarrow P = Q^{\frac{1}{2}} = P^{1/3}$

$\Rightarrow P^3 = Q^{3/2} = R$

$\Rightarrow R = P^3$

$\Rightarrow R = P \cdot P^2$

$\Rightarrow R = P^2 Q^{1/2}$ $\therefore$ option (d) false.

9) $\log \tan 1^\circ + \log \tan 2^\circ + \log \tan 3^\circ + \ldots + \log \tan 89^\circ = \_\_\_$

$\Rightarrow \log \tan 1^\circ \times \tan 2^\circ \times \tan 3^\circ \ldots \tan 89^\circ$

$\Rightarrow \tan\theta = \cot(90^\circ - \theta)$ $\quad \tan\theta \times \cot\theta = 1$

$\tan 1^\circ = \cot 89^\circ$

$\log \tan 1^\circ \times \tan 2^\circ \ldots \times \cot 2^\circ \times \cot 1^\circ$

$1 \times 2 \times 3 \ldots 45 \ldots 3 \times 2 \times 1$

$\Rightarrow \tan 45^\circ \Rightarrow 1$

$\Rightarrow \log 1$

$\Rightarrow 0$ (Ans)

9) if $\frac{\log P}{y-z} = \frac{\log Q}{z-x} = \frac{\log R}{x-y} = 10$ & $x \neq y \neq z$ find $P \times Q \times R = ??$

$\Rightarrow \log P = 10(y-z)$ ..(i)

$\Rightarrow \log Q = 10(z-x)$ ..(ii)

$\Rightarrow \log R = 10(x-y)$ ..(iii)

(i) + (ii) + (iii)

$\Rightarrow \log(P \times Q \times R) = 10(y - z + z - x + x - y)$

$\Rightarrow \log P \times Q \times R \Rightarrow 10 \times 0$

$\Rightarrow \log P \times Q \times R \Rightarrow 0$

$\Rightarrow P \times Q \times R = 1$ (as $\log 1 = 0$)

9) $\frac{1}{1+\log_u^{wv}} + \frac{1}{1+\log_v^{wu}} + \frac{1}{1+\log_w^{vu}} = ??$

$\log_a^x = \frac{\log_e^a}{\log_e^x}$

$\Rightarrow \frac{1}{1+\log_u^{wv}} + \frac{1}{1+\log_v^{wu}} + \frac{1}{1+\log_w^{vu}}$

$\Rightarrow \frac{1}{1+\frac{\log wv}{\log u}} + \frac{1}{1+\frac{\log wu}{\log v}} + \frac{1}{1+\frac{\log vu}{\log w}}$

$\Rightarrow \frac{\log u}{\log u + \log wv} + \frac{\log v}{\log v + \log wu} + \frac{\log w}{\log w + \log vu}$

$\Rightarrow$ 9

$\Rightarrow \frac{\log u}{\log uwv} + \frac{\log v}{\log uvw} + \frac{\log w}{\log uvw}$

$\Rightarrow \frac{\log uvw}{\log uvw} \Rightarrow 1$

13) Arithmetic progression and Geometric progation :-

=> AP & GP

Seqvence :- collection of elements is ordered in such a way that it has an identified 1st number, 2nd number.

Example of seqvence :-

① Population of humans in a city from 2000 to 2020 :-

100, 200, 300, 400 . . .

we can not swap them.

② Amount of money deposited in bank from 2000 to 2020

$10^1, 10^2, 10^3 \ldots$

we can not swap them

in seqvence the order is fixed => can not change order

=> if seqvence follows particular / specific pattern then it is known as progression.

Ex ⇒ 1, 2, 3, 4, 5 ..... 2-1 = 3-2 = 4-3 = 5-4
2, 4, 6, 8, 10 ..... 4-2 = 6-4 = 8-6 = 10-8
AP 3, 6, 9, 12, 15 ..... 6-3 = 9-6 = 12-9 = 15-12
(difference same)

2, 4, 8, 16, 32 ..... $\frac{4}{2} = \frac{8}{4} = \frac{16}{8} = \frac{32}{16}$
3, 9, 27, 81 ..... $\frac{9}{3} = \frac{27}{9} = \frac{81}{27}$
GP (ratio same)

AP :-

$a, a+d, a+2d, a+3d, a+4d$ ..... $a+(n-1)d$
(1st, 2nd, 3rd, 4th, 5th, nth)

∴ for 1st ⇒ d = 0
2nd ⇒ d = 1d
3rd ⇒ d = 2d
4th ⇒ d = 3d
nth ⇒ d = (n-1)d

d = common difference

∴ nth term of AP = $a + (n-1)d$

Series :-
if a_1, a_2, a_3, a_4 ..... a_n be the sequence then the expression $a_1 + a_2 + a_3 + a_4 + \dots + a_n$ is called Series associated with sequence.

Note :-
series is $a_1 + a_2 + a_3 \dots + a_n$
& sum of series is a number which is result of adding all no in series

1, 2, 3, 4 ← series ✗ sequence ✓
↓
1+2+3+4 ✓ series
↓
10 ← sum of series

⇒ Sum of n terms of AP Series :-

$S_n = a + a+d + a+2d + a+3d \dots + a+(n-1)d$
last term ⇒ $a + (n-1)d = l$ (let)
2nd " " ⇒ $a + (n-2)d = l - d$
3rd " " ⇒ $a + (n-3)d = l - 2d$
4th " " ⇒ $a + (n-4)d = l - 3d$

$\Rightarrow S_n = a + (a+d) + (a+2d) \ldots\ldots (l-3d) + (l-2d) + (l-d) + l \quad \ldots (i)$

$\Rightarrow S_n = (l-3d) + (l-2d) + (l-d) + l \ldots\ldots + a + (a+d) + (a+2d) \quad \ldots (ii)$

$\Rightarrow S_n = l + (l-d) + (l-2d) + (l-3d) \ldots\ldots + a + (a+d)$

(i) + (ii)

$\Rightarrow S_n = a + (a+d) + (a+2d) + (a+3d) \ldots\ldots (l-3d) + (l-2d) + (l-d) + l \quad \ldots (i)$

$\Rightarrow S_n = l + (l-d) + (l-2d) + (l-3d) \ldots\ldots + (a+3d) + (a+2d) + (a+d) + a \quad \ldots (ii)$

(i) + (ii)

$\Rightarrow 2 \times S_n = (a+l) + (a+l) + (a+l) \ldots\ldots + (a+l)$ (n times)

$\Rightarrow 2 \times S_n = n \times (a+l)$

$\Rightarrow S_n = \frac{n \times (a+l)}{2}$

$\Rightarrow S_n = \frac{n \times (a + a + (n-1)d)}{2}$

$\Rightarrow \boxed{S_n = \frac{n}{2}(2a + (n-1)d)}$

- **GP :-**

$a, ar, ar^2, ar^3 \ldots\ldots ar^{n-1}$

(1st term, 2nd term, 3rd term, 4th term, ..., nth)

$\Rightarrow$ nth term of GP $\Rightarrow ar^{n-1}$

$\Rightarrow$ **Sum of GP series :**

$S_n = a + ar^1 + ar^2 + ar^3 + ar^4 \ldots\ldots + ar^{n-1} \quad \ldots (i)$

(1st, 2nd, 3rd, 4th, 5th, ..., last)

now, $r \times S_n = ar^1 + ar^2 + ar^3 + ar^4 \ldots\ldots + ar^{n-1} + ar^n \quad \ldots (ii)$

now, (i) − (ii) $\Rightarrow (1-r)S_n = a - ar^n$

$\Rightarrow \boxed{S_n = \frac{a(1-r^n)}{1-r} \text{ or } \frac{a(r^n-1)}{(r-1)}}$

if, $n = \infty$ and $0 < r < 1$

$$S_\infty = \frac{a[1-0]}{1-0r}$$

$$\boxed{S_\infty = \frac{a}{1-r}} \quad ★★★$$

$(0.2)^2 \Rightarrow 0.04$
if, we Power smaller no then 1 it goes smaller
so, if r is small then, $r^n \Rightarrow$ smaller these too

Q) $1+2+3+4 \ldots +n$ (sum of n natural no)

$$\Rightarrow S_n = \frac{n}{2}(2a + (n-1)d)$$

$$\Rightarrow S_n = \frac{n}{2}(2 + n-1 \times 1)$$

$$\Rightarrow S_n = \frac{n}{2}(n+1)$$

$$\Rightarrow \boxed{S_n = \frac{n[n+1]}{2}}$$ ← sum of n natural no $\Rightarrow \sum_{i=1}^{n} i$

Or, $1+2+3+4 \ldots\ldots +n$ (n is odd)

$\Rightarrow 1+2+3+4 \ldots\ldots +(n-3)+(n-2)+(n-1)+n$

$\Rightarrow \frac{n}{2} \times n+1$ n/2 pairs each sum $\Rightarrow n+1$

When n is odd

$\Rightarrow 1+2+3+\ldots+n$

Let say,

$1+2+3+4+5+6+7$

$\Rightarrow (7+1)+(7+1)+(7+1)+\left(\frac{7+1}{2}\right)$

$\Rightarrow (7+1)\left(1+1+1+\frac{1}{2}\right)$

$\Rightarrow (7+1)\left(\frac{7}{2}\right)$

$\Rightarrow \frac{n}{2}(n+1)$ (Proved)

Q) $1+3+5+7 \ldots$ (sum of odd natural numbers)

$$\Rightarrow S_n = \frac{n}{2}(2\times1 + (n-1)\times2)$$

$$\Rightarrow S_n = \frac{n}{2}(2+2n-2)$$

$\Rightarrow S_n = n^2$

$1+3+5+7$
$\Rightarrow 16$
$4^2 = 16$

Q) $2+4+6+8+10 \ldots\ldots$ (Sum of even natural numbers)

$$\Rightarrow S_n = \frac{n}{2}(2a+(n-1)d)$$

$$\Rightarrow S_n = \frac{n}{2}(4+(n-1)2)$$

$$\Rightarrow S_n = \frac{n}{2}(4+2n-2)$$

$$\Rightarrow S_n = \frac{n}{2}(2+2n)$$

$\Rightarrow S_n = n(n+1)$ (Ans)
$= n^2+1$

Starting from 0,

$0 + 2 + 4 + 6 \ldots + n$

$\Rightarrow S_n = \frac{n}{2}\left(2\times 0 + \frac{(n-1)\,2}{}\right)$

$\Rightarrow S_n = n(n-1)$.

3) Sum of squares of natural numbers

$1^2 + 2^2 + 3^2 + 4^2 + \ldots + n^2$

$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$\Rightarrow$ we know that,

$(x+1)^3 - x^3 = x^3 + 1 + 3x(x+1) - x^3$

$\Rightarrow \boxed{(x+1)^3 - x^3 = 3x^2 + 3x + 1}$

if, $x = 1$

$\Rightarrow 2^3 - 1^3 = 3\times 1^2 + 3\times 1 + 1$

$x = 2$

$\Rightarrow 3^3 - 2^3 = 3\times 2^2 + 2\times 2 + 1$

$x = 3$

$\Rightarrow 4^3 - 3^3 = 3\times 3^2 + 3\times 3 + 1$

$\vdots$

$x = n$

$\Rightarrow (n+1)^3 - n^3 = 3n^2 + 3\times n + 1$

adding all

$\Rightarrow (n+1)^3 - 1^3 = 3(1^2 + 2^2 + 3^2 + \ldots + n^2) + 3(1+2+3+\ldots+n) + 1\times n$

$\Rightarrow (n+1)^3 - 1^3 = 3\times \Sigma n^2 + 3\times\frac{n(n+1)}{2} + n$

$\Rightarrow 3\times \Sigma n^2 = (n^3+1)^3 - 1 - 3\,\frac{n(n+1)}{2} - n$

$\Rightarrow 3\times \Sigma n^2 = (n+1)\left((n+1)^2 - \frac{3n}{2}\right) - (n+1)$

$\Rightarrow 3\times \Sigma n^2 = (n+1)\left((n+1) - \frac{3n}{2} - 1\right)$

$\Rightarrow 3\times \Sigma n^2 = (n+1)\left(n^2 + 1 + 2n - \frac{3n}{2} - 1\right)$

$\Rightarrow 3\times \Sigma n^2 = (n+1)\left(n^2 + \frac{n}{2}\right)$

$\Rightarrow 3\times \Sigma n^2 = (n+1)(n)\left(n + \frac{1}{2}\right)$

$\Rightarrow 3\times \Sigma n^2 = (n+1)(n)\left(\frac{2n+1}{2}\right)$

$\Rightarrow \boxed{\Sigma n^2 = \frac{n(n+1)(2n+1)}{6}}$ ***

Q) $1^3+2^3+3^3+4^3 \ldots\ldots +n^3$ [Sum of cubes of natural no]

=> we know that (binomial theorem)

$\Rightarrow (x+1)^4 - x^4 = {}^4C_0 x^0 + {}^4C_1 x^1 + {}^4C_2 x^2 + {}^4C_3 x^3 + {}^4C_4 x^4 - x^4$

$\Rightarrow (x+1)^4 - x^4 = 1 + 4x + 6x^2 + 4x^3 + x^4 - x^4$

$\Rightarrow (x+1)^4 - x^4 = 1 + 4x + 6x^2 + 4x^3$

$\Rightarrow (x+1)^4 - x^4 = 4x^3 + 6x^2 + 4x + 1$

=> Let $x = 1$

$\Rightarrow 2^4 - 1^4 = 4\times1^3 + 6\times1^2 + 4\times1 + 1$ +

if, $x = 2$

$\Rightarrow 3^4 - 2^4 = 4\times2^3 + 6\times2^2 + 4\times2 + 1$ +

if, $x = 3$

$\Rightarrow 4^4 - 3^4 = 4\times3^3 + 6\times3^2 + 4\times3 + 1$ +

⋮

if, $x = n$

$\Rightarrow (n+1)^3 - n^3 = 4\times n^3 + 6\times n^2 + 4\times n + 1$

adding all of them

$\Rightarrow (n+1)^4 - 1 = 4[1^3+2^3+3^3+4^3+\ldots\ldots+n^3] + 6[1^2+2^2+3^2 \ldots\ldots+n^2] + 4[1+2+3\ldots n] + n$

$\Rightarrow (n+1)^4 - 1 = 4\Sigma n^3 + \frac{6(n(n+1)(2n+1)}{6} + 4\frac{n(n+1)}{2} + n$

$\Rightarrow 4\Sigma n^3 = (n+1)^4 - 1 - n(n+1)(2n+1) - 2n(n+1) - n$

$\Rightarrow 4\Sigma n^3 = (n+1)[(n+1)^3 - 2n(2n+1) - 2n - 1]$

$= (n+1)[n^3 + 1 + 3n(n+1) - 2n^2 - n - 2n - 1]$

$= (n+1)(n^3 + 3n^2 + 3n - 2n^2 - 3n)$

$= (n+1)\, n^2(n+1)$

$= (n+1)^2 n^2$

$\Rightarrow \Sigma n^3 = \frac{n^2(n+1)^2}{4}$

$\Rightarrow \boxed{\Sigma n^3 = \left(\frac{n(n+1)}{2}\right)^2}$ ★★★★

Q) In an AP 7th term is 40 & 21st term is 124 find 31st

$n^{th} \rightarrow a + (n-1)d$

$a + 6d = 40 \quad -(i)$

$a + 20d = 124 \quad -(ii)$

$14d = 84$

$\Rightarrow d = 6$

$a + 6 \times 6 = 40$

$a = 4$

31st $\Rightarrow a + 30d$

$\Rightarrow 4 + 30 \times 6$

$\Rightarrow 184$ (Ans)

Q) Sum of series

$10 + 84 + 734 + \dots + n$ terms

$\Rightarrow (9+1) + (9^2 + 3) + (9^3 + 5) + \dots + n$

$\Rightarrow 9^1 + 9^2 + 9^3 \dots n \rightarrow 9^n + 1 + 3 + 5 \dots + n$

$\Rightarrow \frac{9 \times (9^n - 1)}{9 - 1} + n^2$ (sum of odd no)

$\Rightarrow \frac{9^{n+1} - 9}{8} + n^2$

$\Rightarrow \frac{9^{n+1} - 9 + 8n^2}{8}$ (Ans)

Q) $4 + 44 + 444 + \dots$ n terms

$\Rightarrow 4 (1 + 11 + 111 + \dots +)$

$\Rightarrow \frac{4}{9} (9 + 99 + 999 + \dots)$

$\Rightarrow \frac{4}{9} [(10^1 - 1) + (10^2 - 1) + (10^3 - 1) \dots]$

$\Rightarrow \frac{4}{9} [10^1 + 10^2 + 10^3 \dots 10^n + \dots + 10^n - n]$

$\Rightarrow \frac{4}{9} \times \left[10 \times \frac{10^n - 1}{10 - 1} - n\right]$

$\Rightarrow \frac{4}{9} \times \left[\frac{10^{n+1} - 10}{9} - n\right]$

$\Rightarrow \frac{4}{81} [10^{n+1} - 10 - 9n]$ (Ans)

Q) $2 + 2^2 + 222 + \dots$ n terms

$\Rightarrow 2[1 + 11 + 111 + \dots n]$

$\Rightarrow \frac{2}{9} [9 + 99 + 999 + \dots n]$

9) $5 + 55 + 555 + \dots n$

$\Rightarrow 5(1 + 11 + 111 + \dots n)$

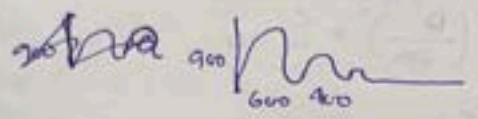
$\Rightarrow \frac{5}{9}[9 + 99 + 999 + \dots n]$

8) A ball is dropped from 900 m & everytime it bounce to 2/3rd of its height. find total distance covered by ball before coming to rest.

$\Rightarrow 900 + \frac{2}{3} \times 900 + \left(\frac{2}{3}\right)^2 \times 900 + \left(\frac{2}{3}\right)^3 \times 900 \dots + \infty$

✗ wrong.

$\Rightarrow$ there we are considering only fall but we have to consider UP + fall both after fall each time the ball goes UP.



$\frac{2}{3} \times 900 \Rightarrow 600$

$\frac{2}{3} \times 600 \Rightarrow 400$

$\Rightarrow 900 + 2 \times \frac{2}{3} \times 900 + 2 \times \left(\frac{2}{3}\right)^2 \times 900 + 2 \times \left(\frac{2}{3}\right)^3 \times 900 \dots \infty$

$\Rightarrow$ first it UP $\frac{2}{3}$ height then down $\frac{2}{3}$ height like that for this 2 times

$\Rightarrow 900\left[1 + 2 \times \frac{2}{3} + 2 \times \left(\frac{2}{3}\right)^2 + 2 \times \left(\frac{2}{3}\right)^3 + \dots \infty\right]$

$\Rightarrow 900\left[1 + 2\left[\frac{1}{3} + \left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^3 + \dots \infty\right]\right]$

$\Rightarrow 900\left[1 + 2\left[\frac{2/3}{1 - 2/3}\right]\right]$ (Sum of infinite term where, $0 < a < 1$ sum $\Rightarrow \frac{1}{1-r}$)

$\Rightarrow 900\left[1 + 2\left[\frac{2/3}{1/3}\right]\right]$

$\Rightarrow 900 \times 5 \Rightarrow 4500$ (Ans)

16) Quardatic eq :- Part 1

③ quardratic equations :- ★★★

$Ax^2 + bx + c = 0$ where $A \neq 0$

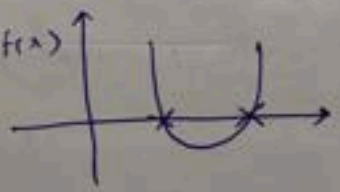
Root of quardratic equation :-

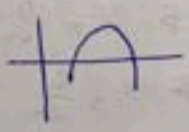
↳ Those values of x which satisfy above equation

- Actual Definition :-

Roots of quardratic equation are points of intersection of curve with x axis.

$\Rightarrow y = f(x) = Ax^2 + bx + c = 0$





$\Rightarrow ax^2 + bx + c = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Sum of roots $= -\frac{b}{a}$

Product of roots $= \frac{c}{a}$

Proff:- $ax^2 + bx + c = 0$

$\Rightarrow x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

$\Rightarrow x^2 + \frac{2b}{2a}x + \frac{c}{a} = 0$

$\Rightarrow (x)^2 + 2 \cdot (x) \cdot \left(\frac{b}{2a}\right) + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 + \frac{c}{a} = 0$

$\Rightarrow \left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$

$\Rightarrow \left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$

$\Rightarrow \left(x + \frac{b}{2a}\right) = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$

$\Rightarrow x = \pm \frac{\sqrt{b^2 - 4ac}}{2a} - \frac{b}{2a}$

$\Rightarrow \boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$ (proved)

$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$

$\beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$

$\therefore \alpha + \beta = \frac{-b + \sqrt{b^2 - 4ac}}{2a} + \frac{-b - \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-2b}{2a}$

$\boxed{\alpha + \beta = -\frac{b}{a}}$ (proved)

$\Rightarrow \alpha \cdot \beta = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \times \frac{-b - \sqrt{b^2 - 4ac}}{2a}$

$\boxed{\alpha \cdot \beta = + \frac{c}{a}}$ (Ans)

- General rule is alternate $-, +$:-

$ax^2 + bx + c = 0$ — quadratic

$- \quad +$

$\Rightarrow \alpha + \beta \Rightarrow -\frac{b}{a}$

$\Rightarrow \alpha \cdot \beta \Rightarrow +\frac{c}{a}$

$(x - y)^2 = z$

$x - y = \pm z$

$x = \pm z + y$

$(x + y)^2 = x^2 + 2xy + y^2$

$x^2 + \frac{bx}{a} + \frac{b^2}{a^2}$

$\left(x + \frac{b}{a}\right)^2$

Cubic :-

$$ax^3 + bx^2 + cx + d = 0$$

(− + −)

$\Rightarrow \alpha + \beta + \gamma = -\frac{b}{a}$ ★★★ (question in gate)

$\Rightarrow \alpha\beta + \beta\gamma + \alpha\gamma = +\frac{c}{a}$

$\Rightarrow \alpha\beta\gamma = -\frac{d}{a}$

- Discriminant $(D) = b^2 - 4ac$ $\quad x = \frac{-b \pm \sqrt{D}}{2a}$

$D > 0$ $\quad D < 0$ $\quad D = 0$

(i) $D > 0$:-

- Real & distinct Root
- Curve will cut x axis at 2 Places
- If roots are irrational & one root is $P + \sqrt{q}$ then other root will be $P - \sqrt{q}$

$\Rightarrow$ irrational means can not be expressed in $\frac{p}{q}$ form

Rational " can be " " " "

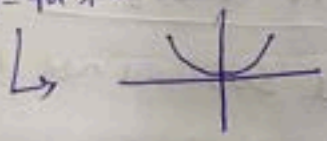
$\Rightarrow$ Ex $\Rightarrow$ $D = 3$ is a irrational no

$$\frac{-b \pm \sqrt{3}}{2a} \rightarrow \frac{-b+\sqrt{3}}{2a}, \quad \frac{-b-\sqrt{3}}{2a}$$

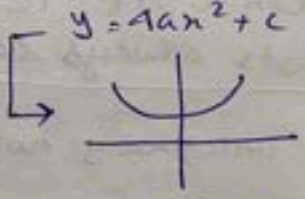
$\Rightarrow$ two distinct root $\therefore$ cut two places.

Extra :-

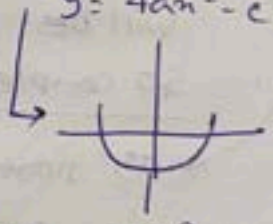
$y = 4ax^2$



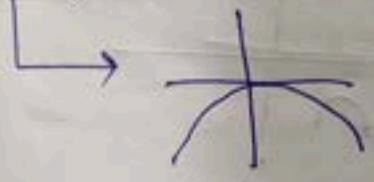
$y = 4ax^2 + c$



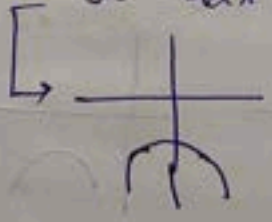
$y = 4ax^2 - c$



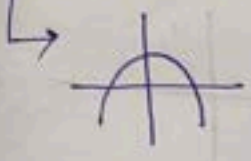
$y = -4ax^2$



$y = -4ax^2 - c$

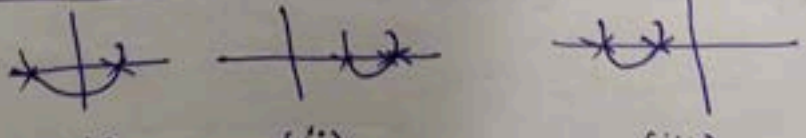


$y = -4ax^2 + c$



$\Rightarrow$ if, $a > 0$ and $D > 0$

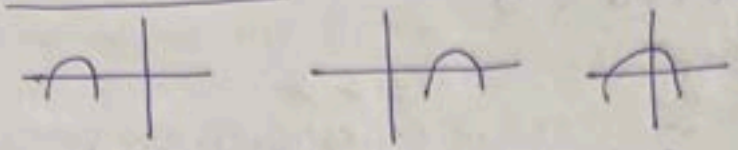
then cases :-



(i) (ii) (iii)

Value of a can be < 0 or > 0 not $= 0$ as then not quadratic eq.

if, $a<0$, $D>0$



(i) (ii) (iii)

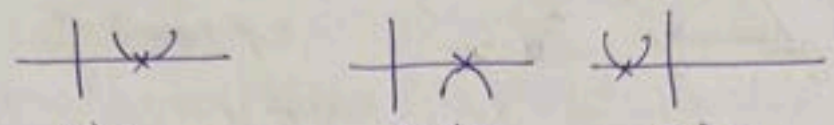
(ii) $D=0$:-

→ Real & equal roots

→ curve will x axis at 1 place

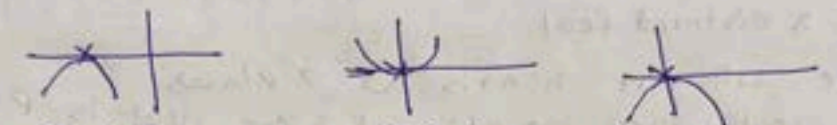
$$x = \frac{-b \pm \sqrt{0}}{2a}$$

=>



$a>0$ $a<0$ $a>0$

=>



$a<0$ $a>0$ $a<0$

=> curve will depend on a and D here, $D=0$ so, only depend on a and only cut at one place as equal root.

(iii) $D<0$:-

→ imaginary root & Distinct

→ curve will not cut x axis i.e curve is either completely above x axis or completely below x axis

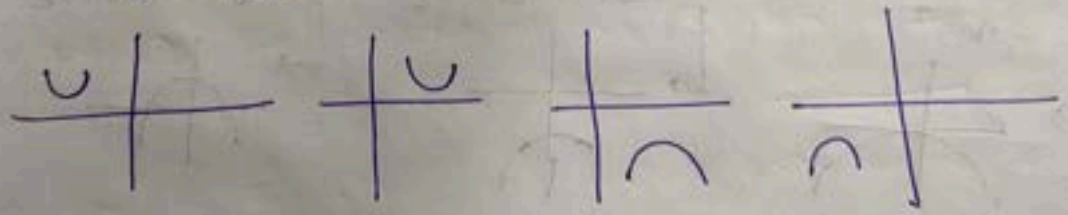
→ if one root is $p+iq$ then root will be its conjugate ie $p-iq$

so complex roots always comes in conjugate.

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

if, D = -ve then we know $\sqrt{-ve}$ will be imaginary not real

→ as root are imaginary not real so, will not touch x axis.



(i) (ii) (iii) (iv)

By Quardratic Equation Part: 2 :-

Q) Consider a function $f(x) = x^2 - 5x + 6$ curve of $f(x)$ will cut x axis at ___ & ___ ?

$\Rightarrow x^2 - 5x + 6 = 0$

$\Rightarrow (x-3)(x-2) = 0$

$\Rightarrow x = 3, 2$ (Ans)

Q) Consider a function $f(x) = x^2 - 6x + 9$. Curve of $f(x)$ will cut 'x' axis at a) 1 Point b) 2 point c) 3 Point d) 4 Point

$\Rightarrow D > 0$ or $D < 0$
$D \neq 0$
$D \Rightarrow 0 \Rightarrow 1$

$\Rightarrow 2$

$b^2 - 4ac$
$\Rightarrow 36 - 4 \times 9$
$\Rightarrow 0 \quad \therefore D = 0$

$\therefore$ 1 Point (Ans)

Q) Consider a function $f(x) = x^2 - 6x + 10$.
Curve of $f(x)$ will cut x axis at a) 1 Point b) 2 Point c) 3 Point d) 4 Point e) 0 Point.

$\Rightarrow D = b^2 - 4ac$
$= 36 - 4 \times 1 \times 10$
$D = -4$

So, roots will be imaginary & Distinct as, roots are not real so it will not touch x axis so. 0 Point (Ans)

Q) $x^3 - 0.6x^2 - 1.84x + 1.344 = 0$ one of the root is 0.8, other roots? a) 1.1, -1.4 b) -1.2, 1.4 c) 1.2, -1.4 d) -1.1, 1.4

$\Rightarrow \alpha + \beta + \gamma = -\frac{b}{a}$

$\Rightarrow \alpha + \beta + \gamma = +0.6$

$\Rightarrow \alpha = 0.8$ (Given)

$\Rightarrow 0.8 + \beta + \gamma = +0.6$

$\Rightarrow \beta + \gamma = 0.2$

Sum of the other root is 0.2 only (c) option satisfies. (Ans).

Q) $x^3 - 8x^2 + 37x - 50 = 0$ one of the root is $(3+4i)$, what is the real root?

$\Rightarrow$ we, know that imaginary root always come with pair
$\Rightarrow$ conjugate $\Rightarrow \therefore$ if one is $3+4i$ another one $3-4i$
let say other one is x $\quad \alpha + \beta + \gamma = -\frac{b}{a}$

$\Rightarrow 3+4i + 3-4i + x = 8$
$\Rightarrow x = 2$ (Ans)

9) Gate 2022 Question :-

Let r be a root of the equation $x^2+2x+6=0$. Then the value of the expression $(r+2)(r+3)(r+4)(r+5)$ is :-

a) 51 b) -51 c) 126 d) -126

$\Rightarrow x^2+2x+6=0$

$r = \frac{-2 \pm \sqrt{4-4\times6}}{2}$ lengthy process

$r = \frac{-2 \pm \sqrt{-20}}{2}$

$x^2+2x+6=0$
r is root so it satisfy the eq $\Rightarrow r^2+2r+6=0$

$\Rightarrow (r+2)(r+3)(r+4)(r+5)$

$\Rightarrow (r^2+5r+6)(r^2+4r+20)$

$\Rightarrow 3r \times (7r+14)$ (as $r^2+2r+6=0$)

$\Rightarrow 21r^2+42r$

$\Rightarrow 21(r^2+2r+6)-2\times16$

$\Rightarrow 21\times0-3162$

$\Rightarrow -162$ (Ans)

9) $(x-\frac{1}{2})^2-(x-\frac{3}{2})^2 = x+2$ x ?

a) 2 b) 4 c) 6 d) 8

$\Rightarrow a^2-b^2 = x+2$

$\Rightarrow (a+b)(a-b) = x+2$

$\Rightarrow \left(x-\frac{1}{2}+x-\frac{3}{2}\right)\left(x-\frac{1}{2}-x+\frac{3}{2}\right) = x+2$

$\Rightarrow (2x-2)\times1 = x+2$

$\Rightarrow x = 4$ (Ans)

9) Consider a quadratic equation $x^2-13x+36=0$ with coefficients in base b. The solution of this equation in the same base b are $x=5$ and $x=6$ then b = _____

we, can solve quadratic eq in base 10 not in b so Convert it to base b

$\Rightarrow 1\times x^2-13x+36=0$

$\Rightarrow x^2-(b+3)x+(3b+6)=0$

now, $x=(5)_b$ $x=(6)_b$ given

$\Rightarrow x = 5\times b^0$ $x = 6\times b^0$

$= 5$ $= 6$

(1) $_b \Rightarrow \frac{1}{b^0}$
$\Rightarrow 1\times b^0$
$\Rightarrow 1$

$\frac{1}{b}\frac{0}{0}$
$(13)_b \Rightarrow 1\times b^1+3\times b^0$
$\Rightarrow b+3$

$\frac{b^1}{3}\frac{b^0}{6}$
$(36) \Rightarrow 3\times b^1+6\times b^0$
$\Rightarrow 3b+6$

We, know, $\alpha + \beta = -\frac{b}{a}$

$\Rightarrow 5+6 = b+3$

$\Rightarrow b = 8$ (Ans)

Q) a, b, c are real numbers, The quadratic equation $ax^2 + bx + c = 0$ has equal roots, which is β then a) $\alpha\beta = b/a$

b) $\beta^2 = ac$ c) $\beta^3 = bc/2a^2$ D) $b^2 \neq 4ac$

$\Rightarrow$ equal roots

So, $D = 0$

$\Rightarrow x = \frac{-b \pm \sqrt{D}}{2a}$

$\Rightarrow x = +\frac{b}{2a}$ (as $\Rightarrow D = 0$)

option (D) :-

$D = 0$

$\Rightarrow (-b)^2 - 4ac = 0$

$\Rightarrow b^2 = 4ac$

$\beta^2 \neq b^2$

$x = +\frac{b}{2a}$ as β is a root

$\beta = +\frac{b}{2a}$

$\Rightarrow \beta^2 = \frac{b^2}{4a^2}$

by eq. $\Rightarrow \beta \times \beta \Rightarrow \frac{c}{a}$

$\Rightarrow \beta^2 = \frac{c}{a}$

$\therefore b^2$ and β^2 not equal

$\therefore b^2 \neq \beta^2$

$\therefore$ option (D) is correct

$b^2 = 4ac$ and $\beta^2 \Rightarrow \frac{b^2}{4a^2}$ or $\frac{c}{a}$

So, clearly we can say $\Rightarrow b^2 \neq \beta^2$

or, $\beta^2 = \frac{b^2}{4a^2}$ or $\beta^2 = \frac{c}{a}$

$\therefore \frac{b^2}{4a^2} = \frac{c}{a}$

$\Rightarrow c = \frac{b}{4a}$

$b^2 = 4ac$ not $\beta^2 = 4ac$

$\Rightarrow \therefore \beta^2 \Rightarrow \frac{b}{4ac}$ 4ac (as per option)

$\Rightarrow \beta^2 \Rightarrow 4/a \times \frac{b^2}{4a}$ and $\beta^2 \neq b^2$. $\beta^2 \neq 4ac$

option (a) :-

$\beta = +\frac{b}{2a}$ not b/a

$\therefore$ (a) wrong.

option (b) :-

$\beta^2 = \frac{b^2}{4a^2}$ or $\beta^2 = \frac{c}{a}$ (by formula)

$\beta^2 =$ not ac $\therefore$ option (b) wrong.

option (c) :-

$\beta^3 = \beta^2 \times \beta$ $\beta^2 \Rightarrow \frac{c}{a}$ by formula

$= \frac{b^3}{8a^3}$ or, $\frac{c}{a} \times \frac{b}{2a}$

$\beta^2 \times \beta \Rightarrow \frac{bc}{2a^2}$

$\Rightarrow \beta^3 = bc/2a^2$ $\therefore$ option (c) correct.

18. Practice questions on Aptitude :- (gate questions)

9) $\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} \cdots \frac{1}{\sqrt{80}+\sqrt{81}}$

$\Rightarrow \frac{1}{\sqrt{1}+\sqrt{2}} \times \frac{\sqrt{1}-\sqrt{2}}{\sqrt{1}-\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} \times \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} \times \frac{\sqrt{3}-\sqrt{4}}{\sqrt{3}-\sqrt{4}}$

$+ \cdots + \frac{1}{\sqrt{80}+\sqrt{81}} \times \frac{\sqrt{80}-\sqrt{81}}{\sqrt{80}-\sqrt{81}}$

$\Rightarrow \frac{\sqrt{1}-\sqrt{2}}{(\sqrt{1})^2-(\sqrt{2})^2} + \frac{\sqrt{2}-\sqrt{3}}{(\sqrt{2})^2-(\sqrt{3})^2} + \frac{\sqrt{3}-\sqrt{4}}{(\sqrt{3})^2-(\sqrt{4})^2}$

$\Rightarrow \frac{1}{\sqrt{2}+\sqrt{1}} \times \frac{\sqrt{2}-\sqrt{1}}{\sqrt{2}-\sqrt{1}} + \frac{1}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

$+ \frac{1}{\sqrt{3}+\sqrt{4}} \times \frac{\sqrt{4}-\sqrt{3}}{\sqrt{4}-\sqrt{3}} + \cdots + \frac{1}{\sqrt{80}+\sqrt{81}} \times \frac{\sqrt{81}-\sqrt{80}}{\sqrt{81}-\sqrt{80}}$

$\Rightarrow \frac{\sqrt{2}-\sqrt{1}}{(\sqrt{2})^2-(\sqrt{1})^2} + \frac{\sqrt{3}-\sqrt{2}}{(\sqrt{3})^2-(\sqrt{2})^2} + \frac{\sqrt{4}-\sqrt{3}}{(\sqrt{4})^2-(\sqrt{3})^2} + \cdots + \frac{\sqrt{81}-\sqrt{80}}{(\sqrt{81})^2-(\sqrt{80})^2}$

$\Rightarrow \sqrt{2}-\sqrt{1}+\sqrt{3}-\sqrt{2}+\sqrt{4}-\sqrt{3}+\cdots+\sqrt{81}-\sqrt{80}$

$\Rightarrow \sqrt{81}-\sqrt{1}$ (when, consigutive root then factorization)

$\Rightarrow 9-1$

$\Rightarrow 8$ (Ans)

9) $\frac{1}{3} + \frac{1}{15} + \frac{1}{35} + \cdots \frac{1}{399}$

$\Rightarrow \frac{1}{1\times3} + \frac{1}{3\times5} + \frac{1}{5\times7} + \cdots \frac{1}{19\times21}$

$\Rightarrow \frac{1}{2}\left[\left(\frac{1}{1}-\frac{1}{3}\right) + \left[\frac{1}{3}-\frac{1}{5}\right] + \left(\frac{1}{5}-\frac{1}{7}\right) + \cdots \left(\frac{1}{19}-\frac{1}{21}\right)\right]$ (when multiplication shortcut this)

$\Rightarrow \frac{1}{2}\left[1-\frac{1}{21}\right]$

$\Rightarrow \frac{1}{2}\times\frac{20}{21} \Rightarrow \frac{10}{21} \Rightarrow 2\cdot1$ (Ans)

Q Three friends P, S and T shared toffees from a bowl. P took 1/3 of the toffee but returned four to bowl. S took 1/4 of the what was left but returned three toffee to the bowl. T took half of the remainder but returned two back into the bowl. If the bowl has 17 toffees left, How many toffees were originally there in the bowl?
a) 38 b) 31 c) 48 d) 91

⇒ Keep an eye on option ⇒ P took 1/3 of the toffee so should multiple of 3 as if not then fraction and toffee can not divided into fraction ⇒ so only option is 48 (c).

OR, Solve :-
let, x toffees in bowl
remain ⇒ $\frac{2x}{3}+4$ ⇒ take $\frac{1}{4}$ rd so, remain $\frac{3}{4}$

S → $\frac{3}{4}\left[\frac{2x}{3}+4\right]+3$ ⇒ take $\frac{1}{2}$ th so, remain will be $\frac{3}{4}$

T → $\frac{1}{2}\left[\frac{3}{4}\left[\frac{2x}{3}+4\right]+3\right]+2=17$

$\frac{1}{2}\left[\frac{x}{2}+3+3\right]+2=17$

$\Rightarrow \frac{x}{4}+3+2=17$

$\Rightarrow \frac{x}{4}++5=17$

$\Rightarrow x = 12 \times 4$

$\Rightarrow x = 48$ (Ans)

Q) x and Y are two positive real numbers such that $2x+y \leq 6$ and $x+2y \leq 8$. For which of the following values of (x,y) the function $f(x,y) = 3x+6y$ will give maximum value?

A) $\left(\frac{4}{3}, \frac{10}{3}\right)$ B) $\left(\frac{8}{3}, \frac{20}{3}\right)$ C) $\left(\frac{8}{3}, \frac{10}{3}\right)$ d) $\left(\frac{4}{3}, \frac{20}{3}\right)$

⇒ Keep an eye on option :-

⇒ in option (b) ⇒ if we put ⇒ $y = \frac{20}{3}$ ⇒ 6. something

So, we put in $2x+y \leq 6$ value of y then should be -ve but it is given those are +ve real no

∴ option (b) ✗

for same reason (d) ✗

option (c) ⇒ $x = \frac{8}{3}$, $y = \frac{10}{3}$ ∴ option (c) ✗

$\Rightarrow 2\times\frac{8}{3}+\frac{10}{3}$

$\Rightarrow \frac{26}{3} \Rightarrow 8.\text{something} \leq 6$ ✗

option (a) ⇒ $x = \frac{4}{3}$, $y = \frac{10}{3}$ ⇒ $2\times\frac{4}{3}+\frac{10}{3}$ option (a) ✓

$\Rightarrow \frac{18}{3} \Rightarrow 6 \leq 6$ ✓

Q) $\sqrt{12+\sqrt{12+\sqrt{12}\ldots}}\ \infty$ $\quad \infty - 1 \Rightarrow \infty$

$y = \sqrt{12+\sqrt{12+\sqrt{12}}}\ \ldots \infty$

$y = \sqrt{12+y}$ (as ∞, so one less $\sqrt{12}$ nothing much happened)

$\Rightarrow y^2 = 12+y$

$\Rightarrow y^2 - y - 12 = 0$

$\Rightarrow (y-4)(y+3) = 0$

$\Rightarrow y = 4$ (Ans)

-3 x then $\sqrt{-3}$ imaginary root

- **Short cut :-** root of two consigutive no multiple then ans will be larger no. in case of + case

here, $\sqrt{12\sqrt{12+\sqrt{12}}}\ldots\infty$

$\Rightarrow 12 \Rightarrow 3\times 4 \Rightarrow$ Ans (4).

Q) $\sqrt{72+\sqrt{72+\sqrt{72}}}\ldots\infty$

$\Rightarrow y = \sqrt{72+y}$

$\Rightarrow y^2 - y - 72 = 0$

$\Rightarrow (y-9)(y+8) = 0$

$\Rightarrow y = 9$ ✓, -8 ✗

$9\times 8 = 72$

Ans :- 9

Q) $\sqrt{42+\sqrt{42+\sqrt{42+}}}\ldots\infty$

$\Rightarrow 42 \Rightarrow 6\times 7 \Rightarrow 7$ (Ans)

Q) $\sqrt{132+\sqrt{132+\sqrt{132+}}}\ldots\infty$

$\Rightarrow 11\times 12 \Rightarrow 12$ (Ans).

Shortcut :- if consigutive multiple
− ⇒ small no.
+ ⇒ large no.

Q) $\sqrt{72+\sqrt{72+\sqrt{72+}}}\infty$

$y = \sqrt{72+y}$

$\Rightarrow y^2 - y - 72 = 0$

$\Rightarrow (y+9)(y-8) = 0$

$\Rightarrow y = 8$ ✓, -9 ✗ (Smaller no)

Q) if 40 is the Perimeter of a rectangle then among all possible rectangles which will have max area?

$\Rightarrow 2(L+B) = 40$

$\Rightarrow L+B = 20$

1 19
2 19
3 17
⋮
9 11
10 10
11 10
⋮

$\therefore$ max $L \times B$ will be of $\Rightarrow 10 \times 10 \rightarrow 100$ (Ans)

$\Rightarrow$ means $L = B$ so, area max when it a Square.

$\times$ $L \times B$ = max?

- Shortcut:- Product of two no is max when no are equal when $x+y$ is given.

Ex:- $x+y = 10$ $x \times y$ when max?

1 9
2 8
3 7
4 6
5 5
6 4 ← same came so we stop
⋮

$x \times y \Rightarrow 5 \times 5 \rightarrow 25$ (max) (same)

$x + y = 16$ x, y are +ve real no then $x \times y$ can not be?

a) 65 b) 64 c) 64 d) 60

$\Rightarrow$ max $x+y \Rightarrow$ when x and y same

$\Rightarrow 8 \times 8 \Rightarrow 64$

$\therefore$ max 64

$\therefore$ a) 65 $\times$ b) 64 $\times$ c) 64 ✓ b) 60 possible — 10×6

$x+y = 16$
$\Rightarrow 8\ 8$

Q) if 100 is the area of recangle then among all possible rectangles which will have min perimeter?

$\Rightarrow$ $L \times B = 100$

101 ← $1 \times 100 = 100$
52 ← $2 \times 50 = 100$
29 ← $4 \times 25 = 100$
⋮
20 ← $10 \times 10 = 100$

$L = 10, B = 10$

min Perimeter $\Rightarrow 2(10+10) \Rightarrow 40$ (Ans)

$\therefore$ ~~min Perimeter $\Rightarrow 2(10+10)$~~
$\Rightarrow 40$

$\therefore$ min Perimeter when $L = B$ means Square.

- **Short cut:** Sum of two no is min when no are equal when $x \times y$ given / product given.

$x \times y = 100$
$x + y = \min \Rightarrow x = y \Rightarrow$
$x^2 \Rightarrow 100$
$\Rightarrow x = 10$
$\Rightarrow 10 + 10 = 20$ is the min sum.

if, $x + y = 20$
$x \times y \Rightarrow \max \Rightarrow x = y$
$x + x = 20$
$\Rightarrow x = 10$
$10 \times 10 \Rightarrow \max$ and mul $\Rightarrow 100$

Q) $\sqrt{m \times n} = 10$ then $m+n$ can not be? a) 25 b) 52 c) 29 d) 19

$\Rightarrow m \times n = 100$ $\quad m+n \Rightarrow 100$

When, ~~$m+n \Rightarrow 100$~~ min
$m = n$
$\Rightarrow m^2 = 100$
$\Rightarrow m = 10$

min $\Rightarrow 10 + 10 \Rightarrow 20$ So, a) 25 ✗ b) 52 ✗ c) 29 ✗ d) 19 ✓

19) **Introduction to averages in Aptitude :-**

$\Rightarrow$ Average $(\bar{x}) = \frac{\Sigma x_i}{n}$ ← sum of all values / ← no of values

↑ Arithmetic min.

Ex:- 5, 7, 3, 10, 6, 8, 2 $\Rightarrow$ 6.1

$\frac{43}{7} = 6.1$

$\Rightarrow$ Average of 'n' values is equal distribution of sum of n values over all values.

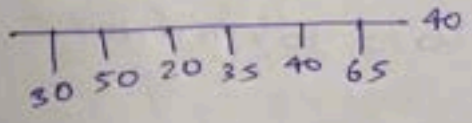
$\Rightarrow$ 30, 50, 20, 35, 40, 65

$\frac{240}{6} = 40$

$(30+10) + (50-10) + (20+20) + (35+5) + (40+0) + (65-35)$

Average $= \frac{\Sigma n_i}{n}$

Average $\times n = \Sigma n_i$



- **properties of Average :-**

(i) **min value ≤ average ≤ maximum value**

When, all same 5, 5, 5, 5, 5 the all max, min, avg same

$\Rightarrow \frac{25}{5} \Rightarrow 5$ (AS)

(ii) **If every element is increased by x, the average is also increased by x & vice versa.**

Proff :- $\underbrace{a_1 a_2 a_3 \ldots a_n}_{\hookrightarrow n \text{ values}}$

$$A_{old} \Rightarrow \frac{a_1 + a_2 + a_3 \ldots + a_n}{n}$$

$$A_{new} \Rightarrow \frac{a_1 + x + a_2 + x + a_3 + x \ldots + a_n + x}{n}$$

$$A_{new} \Rightarrow \frac{a_1 + a_2 + a_3 \ldots + a_n}{n} + \frac{xn}{n}$$

$\boxed{A_{new} \Rightarrow A_{old} + x}$ (Proved)

Also, true for decreased by x

(iii) **If every element is multiplied by x, the average is also multiplied by x & vice versa.**

Proff :-

$\underbrace{a_1 \; a_2 \; a_3 \ldots a_n}_{\hookrightarrow n \text{ values}}$

$$A_{old} = \frac{a_1 + a_2 + a_3 \ldots a_n}{n}$$

$$A_{new} = \frac{a_1 \times x + a_2 \times x + a_3 \times x \ldots a_n \times x}{n}$$

$$A_{new} = x \times \left[\frac{a_1 + a_2 + a_3 \ldots a_n}{n}\right]$$

$\boxed{A_{new} = x \times A_{old}}$

Also, true for division.

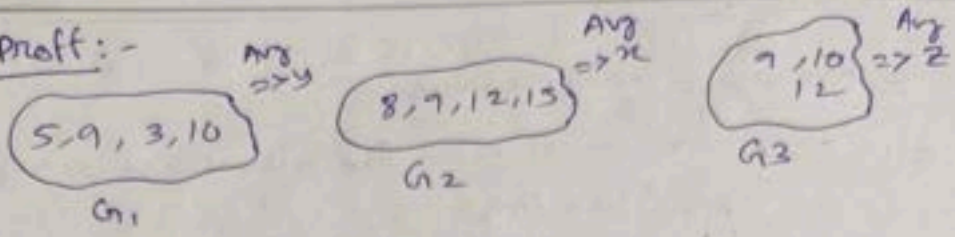
(iv) **Average of n groups :-**

~~no of elements in G1 × Average of~~ ~~no of elements in grp 1 × no.~~

no of elements in group 1 × Average of grp 1 + no of elements in grp 2 × Avg of grp 2 ...

total no of elements in all groups

Proff:-



$$\frac{4xy + 2x4 + 3x3}{11} \text{ (Ans)}$$

Mathematical Proff:-

$$A = \frac{a_1 + a_2 + a_3 + \dots + a_n}{n}$$

$$A = \frac{k \times \frac{a_1 + a_2 + a_3 + \dots + a_k}{k} + (n-k) \times \frac{a_{k+1} + a_{k+2} \dots a_n}{(n-k)}}{n}$$

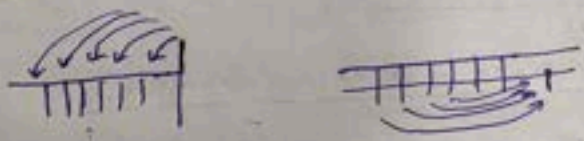
$$A = \frac{k \times A_k + (n-k) \times A_{n-k}}{n} \text{ (Proved)}$$

[A_k = Avg of k elements, A_{n-k} = Avg of $n-k$ elements]

(v) if new element is ~~longer than~~ added say a_{n+1} if $a_{n+1} > A$ then A ↑ else A ↓

$$A_{new} = A_{old} + \frac{a_{n+1} - A_{old}}{n+1}$$

=> Proff:-



20. Averages Part 2:-

Q) Average weight of 11 players is 60 kg & if one player leaves then Average weight is increased 0.5 kg. Find weight of that Player

=> $A_{11} = 60kg$, $A_{10} = 60.5 Kg$

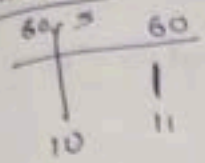
$$A_{11} = A_{10} + \frac{x - A_{10}}{11}$$

$$\Rightarrow 60 = 60.5 + \frac{x - 60.5}{11}$$

$$\Rightarrow x = -5.5 + 60.5$$

$$\Rightarrow x = 55 kg \text{ (Ans)}$$

Method 2 :-



$\Rightarrow 60 - 5 \Rightarrow 55$ (Ans).

Method 3 :-

$660 - 605 \Rightarrow 55\,kg$

Method 4 :-

$$\frac{A_0 + A_1 + \dots + A_{11}}{11} = 60$$

$$\frac{A_0 + A_1 + \dots + A_{10}}{10} = 60.5$$

$\Rightarrow A_0 + A_1 + \dots + A_{10} = 605$

$$\Rightarrow \frac{605 + A_{11}}{11} = 60$$

$\Rightarrow A_{11} = 55$ (Ans)

8) Average weight of 8 person is increased by 3kg if one person of 70kg is replaced by new person. what is his weight?

$$\Rightarrow \frac{A_1 + A_2 + \dots + 70}{8} = x \quad \dots (i)$$

$$\Rightarrow \frac{A_1 + A_2 + \dots + y}{8} = x + 3 \quad \dots (ii)$$

(ii) - (i)

$$\Rightarrow \frac{y}{8} - \frac{70}{8} = x + 3 - x$$

$$\Rightarrow \frac{y - 70}{8} = 3$$

$\Rightarrow y = 8 \times 3 + 70$

$\Rightarrow y = 94$

- Method 2 :-

$8 \times 3 \Rightarrow 24$ Extra

$70 + 24 \Rightarrow 94$ (Ans)

- method 3 :-

$$A_8 = A_7 + \frac{70 - A_7}{8} \quad \dots (i)$$

$$\Rightarrow A_8 + 3 = A_7 + \frac{x - A_8}{8} \quad \dots (ii)$$

(ii) - (i)

$$\Rightarrow 3 = \frac{x - A_7}{8} - \frac{70 - A_7}{8}$$

$\Rightarrow x = 24 + 70$

$\Rightarrow x = 94$ (Ans)

Q) Average age of 12 persons is increased by 1 year if 35 & 43 years men are replaced by 2 woman what is the average age of both women?

=>
$$\frac{A_1+A_2+\dots+A_{10}+A_{11}+A_{12}}{12} = A+1 \quad \text{--(i)}$$

=>
$$\frac{A_1+A_2+\dots+A_{10}+35+43}{12} = A \quad \text{--(ii)}$$

(i) -(ii)

=>
$$-\frac{35+43}{12} + \frac{A_{11}+A_{12}}{12} = 1$$

=> $-78 + A_{11}+A_{12} \Rightarrow 12$

=> $A_{11}+A_{12} \Rightarrow 90$

* Sum => 90

Avg => $\frac{90}{2} = 45$

- Method :- 2

12 + 35 + 43 ← weight of them

=> 90

$\frac{90}{2} = 45$

Avg age increased by 1 means $12 \times 1 \Rightarrow 12$ age increased

Q) Average of 10 number is 30 & one of the number is wrongly typed as 36 but actual is 63. What is actual Avg?

=> $A_{10} = 30$

36 → 63

=> 63 − 36 => 27

$\frac{27}{10} \Rightarrow 2.7$ ∴ new Avg => 30 + 2.7 => 32.7 (Ans)

- Method 2 :-

$$\frac{A_1+A_2+\dots+A_9+36}{10} = 30 \quad \text{--(i)}$$

$$\frac{A_1+A_2+\dots+A_9+63}{10} = x \quad \text{--(ii)}$$

(ii) -(i)

$\Rightarrow x - 30 \Rightarrow 6.3 - 3.6$

$\Rightarrow x = 32.7$ (Ans)

Q) Average of 5 consigutive even no is 44 find highest no.

=> a, a+2, a+4, a+6, a+8 if, a = even

even + even => even

=> $\frac{5a+20}{5}$ => a+4

now, $\frac{a+4}{}$ => 44

=> a = 40

∴ highest no => 40+8 => 48.

~~if odd no a, a+2, a+4,~~

Q) Average of 5 consigutive odd no is 59 find sum of next 5 odd no.

odd + even = odd

=> a, a+2, a+4, a+6, a+8

=> a + A = 59

=> a = 55 donot need

now, a+10, a+12, a+14, a+16, a+18

So, all no are increased by 10 So, avg will be also increased by 10

∴ Avg => old +10

=> 59+10

=> 69

$\frac{\Sigma x_i}{n}$ => 69

=> Σx_i => 69 × 5

Sum of all 5 no. => 345

Q) Average temparature of week is 27°c but one day instead of 16°c the temparature was recorded as 16°c, what is avg tem of the week?

=> $A_7 = 27$ diff $\frac{27}{-16}$ $\frac{}{7}$

So, sum reduced by 7 So, avg will be reduced by = $\frac{7}{7}$ => 1

∴ new avg => 27-1 => 26 (Ans)

• Method 2:-

$$\frac{D_1+D_2+D_3 \cdots D_6+16}{7} = 27 \quad \cdots (i)$$

$$\frac{D_1+D_2+D_3+\cdots D_6+16}{7} \Rightarrow x \quad \cdots (ii)$$

(ii) - (i)

=> x = 26 (Ans)

21. Averages Part 3 :-

- Deviation = value of term − mean of all terms of term

Q) Find mean & deviation of 3 numbers => 1, 3, 5

=> mean is nothing but avg

=> mean => $\frac{1+3+5}{3}$ => 3

=> Deviation of 1 => 1 − 3 => −2

" " 3 => 3 − 3 => 0

" " 5 => 5 − 3 => 2

=> **Sum of deviations of all term = 0** ★★

- Standard deviation = $\frac{\sqrt{d_1^2 + d_2^2 + d_3^2 + \cdots + d_n^2}}{n}$

Q) Find Standard deviation of 3 numbers => 1, 3, 5.

=> Step 1 => find mean

=> mean => 3

=> Step 2 => find deviation

deviation of 1 => −2

3 => 0

5 => 2

=> Step 3 => find Standard deviation

=> $\frac{\sqrt{(-2)^2 + (0)^2 + (2)^2}}{3}$

=> $\frac{\sqrt{8}}{3}$ => $\frac{2\sqrt{2}}{3}$ (Ans)

Q) What is SD indicates?

=> if SD is less than all values in data are close to $\bar{x}$ (mean) hence data is predictable.

=> But if SD is more than values are far from mean & hence data is less predictable.

Unpredictable: 

1, 2, 3, 5, 15, 150, 1000

range => 1000 − 1 => 999

then SD ↑↑

Predictable: 

1, 2, 3, 5, 7, 9, 8, 12, 15

range => 15 − 1 => 14 then SD ↓↓

- Gate ques:-

In the summer of 2012. in New Delhi, the mean temperature of Monday to wednesday was 41°c and of Tuesday to Thursday was 43°c. If the temperature on thrusday was 15% higher than that of Monday. then the temperature in °c on thrusday was A> 40 B> 43 C> 46 D> 49.

$\Rightarrow M + T + W = 41 \times 3 = 123°C$ ...(i)

$T + W + Th = 43 \times 3 = 129°C$ ...(ii)

(ii) - (i)

$Th - M = 6°C$

$1.15M - M = 6°C$

$\Rightarrow 0.15M = 6°C$

$\Rightarrow M = \frac{6}{15} \times 100$

$= 40°C$

$Th \Rightarrow 1.15 \times 40°C \Rightarrow 46°C$ (Ans)

if, M => monday temp

Thusday temp => $M + M \times \frac{15}{100}$

$\Rightarrow 1.15M$.

- Mean, Mode & Median:-

1, 3, 2, 7, 3, 11, 5, 4, 3, 6, 9

Mean → Average = $\frac{\Sigma x_i}{n}$

Mode → most freqvent values

Median → ASC/DES => mid value

=> ASC => (1, 2, 3, 3, 3, 4, 5, 6, 7, 9, 11)

Mean => $\frac{54}{11}$

Mode => 3

Median => 4

Q> 1, 3, 2, 7, 3, 11, 5, 4, 3, 6, 9, 15

=> (1, 2, 3, 3, 3, 4, 5, 6, 7, 9, 11, 15)

=> if total no are even then we will get two middle value then we have to take mean of those two nos.

=> Median => $\frac{5+4}{2} \Rightarrow 4.5$

Mean => $\frac{69}{12}$

Mode => 3

Gate

8> in a company with 100 employees, 45 earn 20,000 per month, 25 earn RS 30000, 20 earn RS 40,000, 8 earn RS 60000 and 2 earn RS 150,000. The median of the salaries is A> 20,000/- B> 30,000/- C> 32,3000/- D> 40,000/-

total 100

=> 20,000 ... 20,000 (45) 30,0000 ... 30,0000 (25)

49 th → 30,000
50 th → 30,000
51 th → 30,000

(49) (50, 51) (49) ✓ => $\frac{30,000 + 30,000}{2}$ => 30,000 median

(48) (49, 50) (50) ✗

Mode => 20,000

Mean => $\frac{45 \times 20,000 + 25 \times 30,000 + 20 \times 40,000 + 8 \times 60,000 + 2 \times 150,000}{100}$

9> The following sequence of numbers is arranged in increasing order 1, x, x, x, Y, Y, 9, 16, 18 Given that the mean and median are equal, and are also equal to twice the mode, the value of y is A> 5 B> 6 C> 7 D> 8

=> mean => $\frac{1 + 3x + 2y + 9 + 16 + 18}{9}$

=> median => 1, x, x, x, (Y) Y, 9, 16, 18

=> Y

=> acc to question => $Y = \frac{1 + 3x + 2y}{9}$

and $Y = 2x$

=> $x = \frac{Y}{2}$

$y = \frac{44 + \frac{3y}{2} + 2y}{9}$

=> $18Y = 88 + 3y + 4y$

=> $11Y = 88$

=> $Y = 7$ (A)

Percentages Part 1 :-

Percentage :-

Cent => means 100

Per 100

90 % => 90 out of 100

50 % => 50 out of 100

% ⟷ fraction

90 % => $\frac{90}{100}$ => 0.9

50 % => $\frac{50}{100}$ => $\frac{1}{2}$

75 % => $\frac{75}{100}$ => $\frac{3}{4}$

=> $\frac{9}{10} \times 100$ => 90 %.

=> $\frac{1}{2} \times 100$ => 50 %.

=> out of 10 → 9

1 → $\frac{9}{10}$

100 → $\frac{9}{10} \times 100$ => 90 %.

- ### Percentage :-

=> Percent means Per 100

70 % means $\frac{70}{100}$ i.e 70 out of 100

80 % means $\frac{80}{100}$ i.e 80 out of 100

=> Every Percent can be converted into fraction

75 % => $\frac{75}{100}$ => $\frac{3}{4}$

20 % => $\frac{20}{100}$ => $\frac{1}{5}$

=> Every fraction can be Converted to Percent

$\frac{3}{4}$ => $\frac{3 \times 25}{4 \times 25}$ => $\frac{75}{100}$ => 75 %.

$\frac{1}{5}$ => $\frac{1 \times 20}{5 \times 20}$ => $\frac{20}{100}$ => 20 %.

Another way : can be directly multiply fraction by 100

$\frac{3}{4}$ => $\frac{3}{4} \times 100$ => 75 %.

$\frac{1}{5}$ => $\frac{1}{5} \times 100$ => 20 %.

Proff :- unitree method

out of 4 3

out of 1 $\frac{3}{4}$

out of 100 $\frac{3}{4} \times 100$ => 75 %.

- note :- Every Percentage can be converted to fraction & vicevers

Q) 50 % of 12 is equal to 12 % of 50 T/F?

$\Rightarrow \frac{50 \times 12}{100} \Rightarrow 12 \times \frac{50}{100}$ T ✓

$\Rightarrow$ x % of y is same as y % of x

$$\frac{x \times y}{100} = \frac{y}{100} \times x$$

$$\Rightarrow \frac{x \times y}{100} = \frac{y}{100}$$

as, multiplication is comutative.

Ex :- 8 % of 50 ?

$\longleftrightarrow$ 50 % of 8 $\Rightarrow \frac{50}{100} \times 8 \Rightarrow 4$

Ex :- 12 % of 25 ?

$\Rightarrow$ 25 % of 12 $\Rightarrow \frac{25}{100} \times 12 \Rightarrow 3$

Q) 40 % of x is 60 find x ?

$\Rightarrow \frac{40}{100} \times x \Rightarrow 60$

$\Rightarrow x = 60 \times \frac{100}{40} 25$

$= 150$ (Ans)

Q) if x is increased by k % then find new value of x

$\Rightarrow x + \frac{k}{100} \times x \Rightarrow \boxed{x + \frac{kx}{100}}$

Q) if x is decreased by k % then find new value of x

$\Rightarrow \boxed{x - \frac{kx}{100}}$

Q) if A salary is 100,000 & it is increased by 40 % find new Salary.

$\Rightarrow 140\,000$

$x + \frac{kx}{100}$

$\Rightarrow 100,000 + \frac{40 \times 100,000}{100}$

$\Rightarrow 140,000$ (Ans)

Q) If A Spend 20% on HRA, 30% on food, 10% on TA 25% miscelaneous & remaining amount is 4500 find actual salary?

⇒ Let, actual salary is = x

expendinces ⇒ 85%

Saving ⇒ 15%

$\frac{15}{100} \times x \Rightarrow 4500$

$\Rightarrow x = 30,000$ ∴ Actual Salary ⇒ 30,000

HRA ⇒ $\frac{20}{100} \times 30,000 \Rightarrow 6000$

Food ⇒ $\frac{30}{100} \times 30,000 \Rightarrow 9000$

TA ⇒ $\frac{10}{100} \times 30,000 \Rightarrow 3000$

M ⇒ $\frac{25}{100} \times 30,000 \Rightarrow 7500$

Saving ⇒ 4500

∴ total got ⇒ 30,000 (correct).

Q) Aman gave 40% of amount to Rohan. Rohan gave $\frac{1}{4}$th to Sahil After Paying 200 to Taxi Sahil now have 600. How much amount did aman have?

⇒ Let aman has x

R → .4 x

S → $\frac{1}{4} \times .4x$

$\frac{1}{4} \times .4x - 200 \Rightarrow 600$

$\Rightarrow x = 8000$ (Ans)

Q) a% of b% of k = b% of a% of k T/F?

$\frac{a}{100} \times \left(\frac{b}{100} \times k\right) = \frac{b}{100} \times \left(\frac{a}{100} \times k\right)$

$\Rightarrow \frac{a \times b \times}{10000} \times k = \frac{b \times a}{10000} \times k$

∴ a% of b% of k = b% of a% of k

$\Rightarrow \frac{ab}{100}\% k = \frac{ab}{100}\% k$ ∴ (T)

Ex:- 70% of 30% of 100? ⟷ $\frac{30}{100} \times 100 \Rightarrow 30$

$\Rightarrow \frac{70 \times 30}{100}\% \Rightarrow 21\%$

70% of 30 ⇒ $\frac{70}{100} \times 30$

⇒ 21%

Q) Find 30 % of 70 % of 2000 ?

=> 21 % of 2000

420 (Ans)

| => $\frac{21}{100} \times 2000$

=> 420 (Ans)

Generalize :-

=> $\frac{xy}{100}$ % of z

Q) 40 % of 20 % of x is 160. what is 80 % of number?

8 % of x is 160

~~$\frac{8}{100} \times x$~~

=> 80 % of x is 1600 (Ans)

(Best way)

| $\frac{40}{100}\left(\frac{20}{100} \times x\right)$ => $\frac{8}{100} \times x = 160$

=> $\frac{80}{100} \times x = 1600$

=> 80 % x => 1600 (Ans)

Or, ~~40~~ $\frac{40}{100} \times \left(\frac{20}{100} \times x\right) \Rightarrow 160$

$\Rightarrow x = 160 \times \frac{100}{8}$

$\Rightarrow x = 2000$

$\frac{80}{100} \times 2000 \Rightarrow 1600$ (Ans)

Effective % => x % of y % of z

$\frac{xy}{100}$ % of z

Q) A want to donate 10 % of salary to charity but he changed his mind & donate 2100. which is 70 % of what he decided earlier. what is his actual salary?

Salary => x

Donate => 0.1 x x

$0.7 \times 0.1 \times x \Rightarrow 2100$

$\Rightarrow x = \frac{2100}{0.07}$

$\Rightarrow x \Rightarrow 30,000$ (Ans)

verify => 30,000 x 0.1 x 0.7 => 2100 fine. Getting Proved

Method 2 :-

70% of 10% of x

$\Rightarrow$ 7% of x

$\frac{7}{100} x \Rightarrow 21000$

$\Rightarrow x \Rightarrow 30,000$

23) Percentage Part x^2 :-

Q) In an election b/w A & B, A get 76% votes & win election by 312 votes. find total no of votes?

$\Rightarrow$ A $\rightarrow$ 76%

Let total $\Rightarrow x$

B $\rightarrow$ 24%

% of extra votes to A $\Rightarrow$ (76 - 24)%

$\Rightarrow$ 52 %

$\frac{52}{100} \times x = 312$

$\Rightarrow x = 312 \times \frac{100}{52}$

$\Rightarrow x = 600$ (Ans)

Live

100

A $\rightarrow$ 76% $\Rightarrow$ 76

B $\rightarrow$ 24% $\Rightarrow$ 24

extra B have 76 - 24 $\Rightarrow$ 52

Q) In a college election 1st candidate got 55% of total valid votes. 10% of votes found invalid if total votes were 8000. No of valid votes to other candidate?

$\Rightarrow$ total votes $\Rightarrow$ 8000

10% invalid $\Rightarrow$ 800

valid $\Rightarrow$ 8000 - 800 $\Rightarrow$ 7200

55% | 45%

$\frac{45}{100} \times 7200$

$\Rightarrow 45 \times 72$

(Ans)

15% of 90% of 8000

$\Rightarrow \frac{15}{100} \times \frac{90}{100} \times 8000$

$\Rightarrow$ (Ans)

Q) in a college election 10% of votes were invalid, winner got 60% of valid votes & defeat the loser by 1800 votes. How many votes were totally casted?

$\Rightarrow$ total $\Rightarrow x$

valid $\rightarrow 0.9x$

winner $\Rightarrow 0.6 \times 0.9 \times x$

looser $\Rightarrow 0.4 \times 0.9 \times x$

winner - loser $\Rightarrow$ 1800

W $\rightarrow$ 60%

L $\rightarrow$ 40%

win has extra $\Rightarrow$ 20%

$0.2 \times 0.9 \times x \Rightarrow 1800$

$\Rightarrow x = 10,000$

$0.2 \times 0.9 \times x = 1800$

$\Rightarrow x = 10,000$

total vote $\Rightarrow 10,000$

invalid $\Rightarrow 10,000 \times \frac{10}{100}$

$\Rightarrow 1000$

$\therefore$ casted $\Rightarrow 10,000 - 1000 = 9000$ (ans)

Case:1:- if casted were invalid $\Rightarrow$ then 10,000 (ans)

Case:- in question only given invalid $\Rightarrow$ so, ans 9000 (Ans)
casted word missing

- x is what % of $y \Rightarrow \frac{x}{y} \times 100$
  What % of y is $x \Rightarrow \frac{x}{y} \times 100$
  What % is x of $y \Rightarrow \frac{x}{y} \times 100$

Profit:-
From y parts x part taken
1 " $\frac{x}{y}$ " "
100 " $\frac{x}{y} \times 100$ "

★ **Rule:- After of in denominator**

9) if x is 90 % of y, what % is of x is y?

$\Rightarrow \frac{x}{y} \times 100 = 90 \Rightarrow \frac{y}{x} = \frac{100}{90}$

$\Rightarrow \frac{y}{x} \times 100 = ?$

$\Rightarrow \frac{100}{90} \times 100 \Rightarrow 111.1\%$

9) in an exam A scored 20% less than B & e scored 30% less than B. c score is what % of A & score?

$\Rightarrow$ Let B Score x

A score $x - \frac{x}{5} \Rightarrow \frac{4x}{5}$

C B score $x - \frac{3x}{10} \Rightarrow \frac{7x}{10}$

$\Rightarrow \frac{\frac{7x}{10}}{\frac{4x}{5}} \times 100$

$\Rightarrow \frac{7x}{10} \times \frac{5}{4x} \times 100$

in, this type of question take 100

B → 100
A → 80
C → 70

$\frac{70}{80} \times 100$

Or, normal method take x

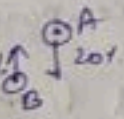
B → x
A → 0.8x
C → 0.7x

$\frac{0.7x}{0.8x} \times 100$

$\Rightarrow \frac{7}{8} \times 100$

Q) if A salary is 25% more than B, then by what % B's salary is less than A?

$\Rightarrow$ B → 100 (let)
A → 125
B → x % less than A



$A - \frac{x}{100} \times A = B$

$\Rightarrow 125 - \frac{x}{100} \times 125 = 100$

$\Rightarrow \frac{x}{100} \times 125 = 25$

$\Rightarrow x = 25 \times \frac{100}{125}$

Shortcut :-
in Denominator that is often than in numerator the percentage

$\frac{25}{125} \times 100$

3) if a salary is 25% less than B, then by what % B's salary is more than A?

B → 100
A → 75

$\Rightarrow \frac{25}{75} \times 100 \Rightarrow 33.3\%$

29) Percentage Part 3 :-

3) In an exam S_1 scored 9 marks more than S_2 & S_1 marks are 56% of sum of marks of S_1 & S_2. What are their marks.

$\rightarrow S_2 \rightarrow x$ marks
$S_1 \rightarrow x + 9$ marks

$(x+9) = \frac{56}{100} \times (2x+9)$

2 $\Rightarrow$

Q) Diffence of 2 numbers is 1800. If 5% of one number is 8% of other. find numbers ?

$\Rightarrow x - y = 1800$

$\Rightarrow \frac{5}{100} \times x = \frac{8}{100} \times y$

$\Rightarrow 5x = 8y$

$\therefore x = 9800$

$\Rightarrow 8y - y = 1800$

$\Rightarrow y = 3000$

$\Rightarrow 5x - 5y = 9000$

$\Rightarrow 8y - 5y = 9000$

$\Rightarrow y = 3000$

- **Effective % change** $\Rightarrow \left(\frac{FV - IV}{IV}\right) \times 100$ $\Rightarrow$ +ve $\Rightarrow FV > IV$, −ve $\Rightarrow FV < IV$

Note :- change is always compared to IV

Q) Population of city is increased from 12 lakh to 15 lakh in a year. Find % increase in that year

$\Rightarrow$ IV $\rightarrow$ 12, FV $\rightarrow$ 15

$\Rightarrow \frac{FV - IV}{IV} \times 100$

$\Rightarrow \frac{15-12}{12} \times 100 \Rightarrow 25\% \uparrow$

Or, W/o formula Proff :-

$12 + \frac{x}{100} \times 12 = 15$

$\Rightarrow \frac{x}{100} \times 12 = 15 - 12$

$\Rightarrow \frac{x}{100} = \frac{15-12}{12}$

$\Rightarrow x = \left(\frac{15-12}{12}\right) \times 100$

$\uparrow$ $\frac{FV - IV}{IV} \times 100$ (Proved)

Q) Present Profit of company is 18 lakh but 2 years ago it was 24 lakh. What is the % change in Profit ?

$\Rightarrow$ 24 $\downarrow$ 18

$\frac{18-24}{24} \times 100$

$\Rightarrow -25\%$

Or, by intution :- x%.

$24 - \frac{x}{100} \times 24 = 18$

$\Rightarrow x = \left(\frac{18-24}{24}\right) \times 100$

Revenue of shop is ~~40,000~~ 40,000 shopkepper gave 30% discount on every item s as a result his sales go up by 20%. Find his new revenue.

=> Revenue => no of item's sold * price of item

x y

=> 40000 $x \times y$

$y - 0.3y$

$x + 0.2x$

now, => $1.2x \times 0.7y$

=> $1.2 \times 0.7 \times x \times y$

=> $1.2 \times 0.7 \times 40000$

=> 33600 (Ans)

method 2 :-

if, $z = x * y$ => then short cut to find z is is => $\left(a + b + \frac{ab}{100}\right)\%$

$a\% \uparrow$ $b\% \uparrow$

Proff :- $z = \left(x \uparrow + \frac{ax}{100}\right) * \left(y \uparrow + \frac{by}{100}\right)$

$= xy + \frac{bxy}{100} + \frac{axy}{100} + \frac{abxy}{10000}$

$= z_{old} + \left(b + a + \frac{ab}{100}\right)\%$

∴ Proved.

$a\%$ of z

$\Rightarrow \frac{a}{100} \times z$

like that

$\Rightarrow \frac{a}{100} \times xy$

=> if $z = x \times y$

$a\% \uparrow$ $b\% \downarrow$

$= \left(a - b - \frac{ab}{100}\right)\%$

So, ans of question by this method

=> $\left(20 - 30 - \frac{20 \times 30}{100}\right)\%$

=> -16%.

So, $40,000 - \frac{16}{100} \times 40000$ => 33600 (Ans)

So, this method better when question about % of change in z.

=> and when need value of z then previous one is better.

9) if L ↓ by 20% & B ↑ 40%. find % change of rectangle.

→ Area => L × B

=> $\left(40 - 20 - \frac{40 \times 20}{100}\right)\%$

=> (20 − 8)%

=> 12 %.

∴ 12% ↑

Or, w/o formula:-

A => L × B

↓ ↓

·8L × 1.0 B

=> ·8 × 1.4 × L × B

=> 1.12 × L B (previous was L B)

So, ~~P200~~ now 1.12 × L B

∴ increased 12%.

9) Price of onion ↑ by 30%. by what % should family ↓ the consumption of onion so that their budget remain unchanged.

=> Budget => Price of onion × no of onion

B => x ↑ × y ↓

B => 1.3x × z (let say new no of onion is z)

=> x × y = 1.3x × z

=> y = 1.3z

~~y is 30% than~~

=> y is 30% more than z

=> z is ← less than of y?

30 ↑ y ↓ / z

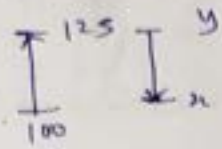
130 y ↓ / 100 ↑ z

=> $\frac{30}{130} \times 100$ (Ans)

Short cut of the question:-

130 / 30% ↑ y / x / 100 I ↓ => $\frac{30 \times 100}{130}$ (Ans)

8) Price → 25% ↑ Consumption ↓ ?? that Budget remain same.

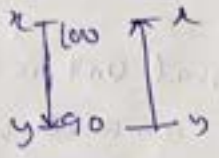
=> 

=> have to find x is ___ less than of y ??

=> $\frac{25}{125} \times 100$

=> 20% (Ans)

9) Price of petrol ↓ by 10% by what % family ↑ Bike ride. to Budget remain unchange.



new, x is how much ~~greater~~ greater than of y.

=> $\frac{10}{90} \times 100$

=> 11% (Ans)

=> 90% is 10% less than 100

100 is ___% more than 90

- Examples :-

1) if A is 10% more than B then B is 9.09% less than A.

9.09 ↓ A : B ↑ 10

2) If A is 20% more than B then B is 16.6 % less than A.

16.6̄ ↓ A : B ↑ 20

3) if A is 25 % more than B then B is 20% less than A.

20 ↓ A : B ↑ 25

4) if A is 33.3̄ % more than B then B is 25 % less than A

25 ↓ A : B ↑ 33.3̄

5) if A is 50% more than B then B is 33.3̄ % less than A

33.3̄ ↓ A : B ↑ 50

- Proff of if, A is 25 % greater than B then B is 20 % less A.

$\Rightarrow \frac{25}{125} \times 100 \Rightarrow 20\ \%$

or, $125 \rightarrow 100$

Let x % lesser than A

$125 - \frac{x}{100} \times 125 \Rightarrow 100$

$\Rightarrow x = \frac{25}{125} \times 100$

$= 20\ \%$ (Proved)

25) Ratio and Proportion :-

- **Ratio :-**

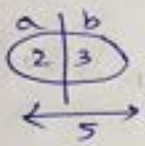
→ comparison b/w quantities of equal unit is called is ratio.

→ here unit means height must be compared with height, weight must be compared with weight etc

→ it is same as fraction i.e

$a/b \leftrightarrow a : b$ (a: antecedent, b: consequent)

→ if a : b = 2 : 3 means from 5 parts a is 2 & b is 3 parts



- **Proportion :-**

→ whenever 2 ratios are equal we call it as Proportion

→ for eg. if A : B is same as C : D then its a Proportion

A : B :: C : D (1st Proportinate, 2nd Proportinate, 3rd Proportinate, 4th Proportinate)

A : B :: C : D (extrems, means) $\Leftrightarrow \frac{a}{b} = \frac{c}{d} \Leftrightarrow a \times d = b \times c$

- **continued Proportion :-**

→ a, b, c are in continious Proportion means $\frac{a}{b} = \frac{b}{c}$

$\Rightarrow ac = b^2 \Rightarrow b^2 = ac \Rightarrow b = \sqrt{ac}$ & here b is called geometric mean or mean Proportion b/w a & c

D.P:- Time ↑ Distance ↑

$T \propto D$

if $a \propto b$

$\frac{a}{b} = k$ [Divison constant]

$\frac{a_1}{b_1} = \frac{a_2}{b_2} \Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2}$

T	D
3h	600 km
6h	1200 km

$\frac{T_1}{D_1} = \frac{T_2}{D_2}$ or, $\frac{T_1}{T_2} = \frac{D_1}{D_2}$

$\Rightarrow \frac{3}{600} = \frac{6}{1200}$

- 3 characteristics:-

① $a \propto b \Rightarrow a\uparrow, b\uparrow$ a=double b=double

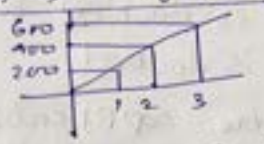
a → halfed
b → halfed

② Unitree method:- in only DP

3h → 600km
1 → $\frac{600}{3}$ km

5 → 200 × 5 ⇒ 100 km

③ Graph ⇒ Straight line from origin ⇒ $y = mx$



- Direct proportion :- (used in Time & Distance etc)

$a \propto b$ [a is directly proportional to b]

$a = k \times b$

$\frac{a}{b} = k$ [∴ Directly Proportional means divison constant]

$\frac{a_1}{b_1} = \frac{a_2}{b_2} \leftrightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2}$

- 3 characteristics :-

① if a ↑ then b ↑ i.e if a is doubled then b is doubled, a is halfed then b is halfed.

② Unitary method is applicable only in direct proportion.
↳ Find the cost of 1 unit & whatever is applicable for 1 Unit is applicable to any no of units.

③ Graph of DP looks like line passing through origin.
i.e ⇒ $y = mx$.

I.P :- (Speed ∝ $\frac{1}{\text{Distance}}$)

(Speed ∝ $\frac{1}{\text{time}}$)

M D
10 → 50
20 → 25

$a \propto \frac{1}{b}$

$a = \frac{k}{b}$

$\Rightarrow ab = k$ [IP = product is constant]

$a_1 \times b_1 = a_2 \times b_2$

$\Rightarrow 10 \times 50 = 20 \times x$

$\Rightarrow x = 25$

$T \propto D$ ← in DP

$\frac{T_1}{D_1} = \frac{T_2}{D_2}$

$T_1 \times T_2 = D_1 \times D_2$ ← in IP

- Inverse proportion :- (used in Time & Speed etc)

$a \propto \frac{1}{b}$ [a is inversely proportional to b]

$a = \frac{k}{b}$

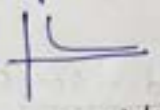
$a \times b = k$ [Directly proportional means product is constant]

- 3 characteristic :-

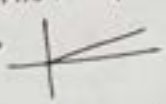
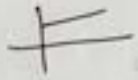
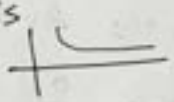
① If a ↑ then b ↓ i.e if a is doubled then b is halfed, a is halfed then b is doubled

② Unitary method is never applicable.

③ Graph of IP looks like rectangular hyperabola



Q) Relationship b/w men & days is

a)  b)  c)  d) 

⇒ option c

Q) 30 m → 180 d
10 m → ??

⇒ men is $\frac{1}{3}$ then day $\frac{1}{3}$ in 3 times increase

⇒ ∴ 180 × 3 ⇒ 540 days

or, can do like this ⇒ $30 \times 180 = 10 \times x$

$\Rightarrow x = \frac{30 \times 180}{10}$

$\Rightarrow 540.$

- Chain rule :- ☆☆☆☆☆

=> When few quantities are DP & few are IP then apply this rule.

men	Day	Lagth	breath
20	15	45	60
30	x	40	90

=> Write left side which need to be calculated :-

D	men	Lagth	b
15	20	45	60
x	30	40	90

$D \propto \frac{1}{M}$ => $\therefore d_1 \times M_1 = d_2 \times M_2$

$D \propto L$ => $\therefore \frac{d_1}{L_1} = \frac{d_2}{L_2}$

$D \propto B$ => $\therefore \frac{d_1}{b_1} = \frac{d_2}{b_2}$

So, which are DP then write them in Divsion type

" " IP " " " " multiplication "

=> $\frac{d_1 \times m_1}{l_1 \times b_1} \Rightarrow \frac{d_2 \times m_2}{l_2 \times b_2}$

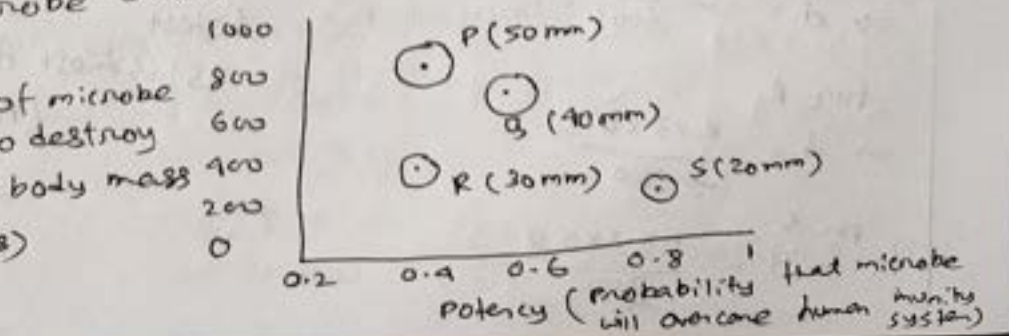
=> $\frac{15 \times 20}{45 \times 60} = \frac{x \times 30}{40 \times 90}$

=> $x = \frac{40}{3}$ days (Ans)

26) Ratio and Proportion practice question Part:1

- Gate 2011:- ☆☆☆

P, Q, R and S are four types of dangerous microbes recently found in a human habitat. The area of each circle with its diameter printed in brackets represents the growth of a single microbe surviving human immunity system with 24 hours of entering the body. The danger to human beings varies proportionaly with the toxicity, potency and growth attributed to a microbe shown in the figure below.



A Pharmatical company is contemplating the development of a vaccine against the most dangerous micorbe. which microbe should the company target in its first attempt ?

A> P B> Q C> R D> S.

=> indirectly we need to find most dangerous virus.

So, write d in left

d | t P g

=> $d \propto \frac{1}{t}$ as, less miligram virus is most dangerous
so, less amount of a virus attack human
large " "

So, less amount of virus is more degerous than large

∴ inversely proportional.

=> $d \propto P$ as, Potency increase then the chance will be high to over come the human being.

So, high potency virus is more dangerous than small

∴ Direct proportional

=> $d \propto g$ as, if growth large amount then that is more dangerous than ~~small~~ less growth.

∴ Direct proportional

So, $\frac{d_1 \times t_1}{P_1 \times g_1} = \frac{d_2 \times t_2}{P_2 \times g_2}$

as, here one d_1 value not given so can not apply this formula

So, we can say, $d \propto \frac{P \times g}{t}$ as, $d \propto P$, $d \propto g$, $d \propto \frac{1}{t}$

$d = \frac{k \times P \times g}{t}$

So, for P

=> $d = \frac{k \times 5 \times 0.9}{800}$

for Q

=> $d = \frac{k \times 4 \times 0.5}{600}$

for R

=> $d = \frac{k \times 3 \times 0.9}{300}$

for S

=> $d = \frac{k \times 2 \times 0.8}{200}$

So, S has lowest denominator has highest numerator

∴ S's d value will be highest

∴ S is most dangerous (Ans).

Q) Interview => appeared : selected = 9 : 1 if No of Candidate Selected is 10 then no of Candidate appeared?

$\Rightarrow \frac{A}{S} = \frac{9}{1}$

$\Rightarrow \frac{9 \times 10}{1 \times 10}$ => So, 90 is ~~selected~~ appeared

OR $\frac{A_1}{A_2} = \frac{S_1}{S_2} \Rightarrow A_2 = \frac{S_2 \times A_1}{S_1}$

$= \frac{10 \times 9}{1}$

=> 90

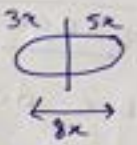
OR if, selected is 10 then appeared should be 90 to maintain same ratio.

=> (A.S) 90 is appeared.

Q) 2 numbers in ratio 3 : 5 if 9 is subtracted from each the new ratio is 12 : 23. find smaller no.

Let, 2 no's are => 3x, & 5x

$\Rightarrow \frac{3x - 9}{5x - 9} = \frac{12}{23}$



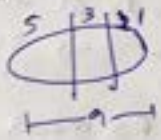
$\Rightarrow 69x - 207 = 60x - 108$

$\Rightarrow 9x = 99$

$\Rightarrow x = 11$

∴ smaller no is => $3 \times 11 = 33$.

Q) land of 900,00 is Shared b/w 3 persons A, B & C in ratio 5 : 3 : 1 Diff b/w Share of B & C.



So, B has => 300,000 (by Common Sense)

C " => 100, 000

∴ B - C => 200,000

OR :- A's Share => $\frac{5x}{9x} \times 900,000$ => 500,000

B " => $\frac{3x}{9x} \times 900,000$ => 300,000

C " => $\frac{x}{9x} \times 900,000$ => 100,000

Q) RS 782 divided into 3 parts, proportional to 1/2 : 2/3 : 3/4 find 1st Part.

So, multiply by Lcm to get Natural no (So, calculation easy)

$\Rightarrow \frac{1}{12} \times 12$ $\Rightarrow \frac{2}{3} \times 12 \Rightarrow \frac{3}{4} \times 12$

=> 6 => 8 => 9

∴ 6 : 8 : 9

=> 1st part => $\frac{6}{23} \times 782$ => 360.92 (Ans)

Gate :-

Q) Bag has 25P, 10P, 5P coins in ratio of 1:2:3. If there are 30 RS in all. How many 5P coins are there?

=> Let, there are x coins

25P) x 10P) 2x 5P) 3x 30RS => 30,000

=> 25×x + 10×2x + 5×3x => 3000

=> x = 50

So, 5P coins there is 3x

=> 3 × 50 => 150 coins of 5P (Ans)

So, 750
1000
1250
3000 P.

Q) A, B, C divide RS 3000 among themselves in ratio 7:6:2. if, RS 200 is added to each of their share. What will be new ratio of their share?

A → $\frac{7}{15}$ × 3000 => 1400

B → $\frac{6}{15}$ × 3000 => 1200

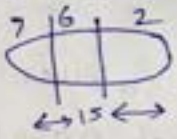
C → $\frac{2}{15}$ × 3000 => 400

So, each share 200 is added

=> 1600 : 1400 : 600

=> 8 : 7 : 3 (Ans)

Or, Shortcut :-



So, 15 → 30

15 parts of 3000 RS

1 " => $\frac{3000}{15}$ => 200 RS

So, 200 RS means 1 Part

So, 1 part increased in each ∴ (7+1) : (6+1) : (2+1)

=> 8 : 7 : 3 (Ans)

27. Ratio and proportion Practic question part :2 :-

Q) Ratio of No of students joining colleges x, y & z is 5:4:6. If No of Studet joining these college ↑ by 20%, 25% & 25% respectively what is new ratio?

=> $5 + \frac{20}{100} \times 5 : 4 + \frac{4 \times 25}{100} : 6 + \frac{6 \times 25}{100}$

=> 6 : 5 : 7.5

or, 12 : 10 : 15 (Ans)

5) if, two quantity has ratio and increased by same % then ratio will remain same.

Ex :- 2 : 3
if, both increased 50%

⇒ 3 : 4.5
⇒ 6 : 9
⇒ 2 : 3 ∴ (remain same)

• Note :- If same % ↑ then ratio same.

Q) A : B = 1 : 5 & B : C = 3 : 4 Find A : C.

⇒ $\frac{A}{B} = \frac{1}{5}$ ..(i) (i)x(ii) ⇒ $\frac{A}{B} \times \frac{B}{C}$ ⇒ $\frac{1}{5} \times \frac{3}{4}$

⇒ $\frac{B}{C} = \frac{3}{4}$ ..(ii) ⇒ $\frac{A}{C} = \frac{3}{20}$

∴ A : C = 3 : 20 (Ans)

Q) Ratio of earning of A & B is 4 : 7. If A earning ↑ by 50%. & B earning ↑ by 25%. New ratio of their earning become 8 : 7 Find A income?

⇒ $\frac{A}{B} = \frac{4}{7}$ new will be ⇒ $4 + \frac{50}{100} \times 4 : 7 + \frac{7 \times 25}{100}$ ⇒ 6 : 8.75

but in question given that 8 : 7, so, after increasing How Sal (A) > Sal (B)? Should be Sal (A) < Sal (B)

∴ in Adequate data. (Ans)

Q) mean Properties of 9 & 4 is _____ ?

$\sqrt{9 \times 4}$ ⇒ 6 (Ans)

Or, $\frac{9}{b} = \frac{b}{4}$ ⇒ $b^2 = 36$
∴ b ⇒ 6 (Ans)

Q) Find fourth proportionate of 2 3 3 4

⇒ 2 : 3 :: 4 : x

⇒ $\frac{2}{3} = \frac{4}{x}$ ⇒ x ⇒ 6 (Ans)

8) Find 3rd proportinate of 2 & 6?

⇒ We, know ⇒ 2 : 6 :: 6 : x

⇒ $\frac{2}{6} \Rightarrow \frac{6}{x}$

⇒ x = 12 (Ans)

28) Profit and Loss Part 1 :-

⇒ CP ⇒ cost Price ⇒ Price of Purchase

⇒ SP ⇒ sell Price ⇒ Price of sell

⇒ Profit ⇒ SP > CP ⇒ SP − CP

⇒ Loss ⇒ CP > SP ⇒ CP − SP

No Profit ✗, no loss ✗ ⇒ SP = CP

⇒ Profit Percentage ⇒ P% ⇒ $\frac{SP-CP}{CP} \times 100$ ⇒ $SP = CP + \frac{P \times CP}{100}$

⇒ Loss Percentage ⇒ L% ⇒ $\frac{CP-SP}{CP} \times 100$ ⇒ $SP = CP - \frac{L \times CP}{100}$

⇒ Always Percentage calculated acc to CP So, always in denominator CP.

⇒ but, we don't use those formula's we will solve by intution.

Q) CP = 100 SP = 140 P% ??

⇒ 40 %.

P% ⇒ $\frac{SP-CP}{CP} \times 100$ ⇒ $\frac{140-100}{100} \times 100$ ⇒ 40%.

Q) CP = 1000, SP = 700 L% ??

⇒ 30%.

L% ⇒ $\frac{CP-SP}{CP} \times 100$ ⇒ $\frac{1000-700}{100} \times 100$ ⇒ 30%.

Q) 100 eggs @ 450 & sold @ 98 per dozen L% ??

⇒ Rule :- if CP and SP no of items are diff then find one item Price of SP " " " " " " CP " then check Profit or Loss??

100 → 450

1 → 4.5

CP ⇒ 4.5

12 → 98

1 → 4

SP ⇒ 4

L% ⇒ $\frac{CP-SP}{CP} \times 100$

⇒ $\frac{4.5-4}{4.5} \times 100$

⇒ 11.1% (Ans)

Q) If SP = 3000 then P = 10%. but if item sold @ 2700 find % Pert?

=> SP = 3000, P = 10%. → CP = 0.9 SP ✗

=> SP = 1.1 CP

Always cal SP dont calculate CP.

I ↕ 100 / 10%.

=> 3000 = 1.1 CP

CP = 2727.27

now, SP = 2700

∴ L% => $\frac{2727 - 2700}{2727}$

=> $\frac{27}{2727} \times 100$

=> $\frac{100}{11}$ => 9. %

Q) Eggs are bought @ 7 eggs for 1 $. If shopkeeper wants to make profit of 40%. how many eggs should he sell for 1 $.

=> 1 $ → 7 egg

7 egg → 1 $

1 egg → $\frac{1}{7}$ $

∴ CP of 1 egg => $\frac{1}{7}$ $

SP → 1.4 × CP => $1.4 \times \frac{1}{7}$ => 0.2 $

∴ 0.2 $ is the SP of one egg

∴ 0.2 $ → 1 egg

1 $ → $\frac{1}{0.2}$ => 5 egg (ans)

Q) 80 items of 12 RS each are mixed with 120 items of 16 RS each. What must be SP of each item to get profit of 10%?

=> find avg cost of 1 item

80 @ 12 => 960

120 @ 16 => 1920

200 item => 2880 cost

1 " => $\frac{2880}{200}$ => 14.4 cost of one item CP

SP => 1.4 × CP

=> 1.4 × 14.4

=> 15.84

or, 14.4 / 1.44 ← as 10% increased / 15.84

9) 2 Hats 1st sold @ 1300 & 2nd sold @ 700. If %. of Profit earned while selling 1st hat = %. of loss for Selling other find %. P or L?

⇒ %, net Profit / Loss = 0 (as same profit and Loss will be)

⇒ SP_1 ⇒ 1300 CP = x ↔ both case. (CP same)
⇒ SP_2 ⇒ 700 CP = x

$SP_1 = CP + \frac{P \times CP}{100}$ ... (i) 1st case

$SP_2 = CP - \frac{P \times CP}{100}$ ... (ii) 2nd case

(i) + (ii)

⇒ 700 + 1300 = 2CP

⇒ CP = 1000

∴ P%. or L% ⇒ $\frac{SP-CP}{CP} \times 100 \Rightarrow \frac{1300-1000}{1000} \times 100$

⇒ 30%. (Ans)

Profit or loss same ⇒ 30%.

29) Profit or loss Part: 2 :-

Rule :- CP of 2 Items same

if, $CP_1 = CP_2$ ⇒ then net profit is 0
$P_1\% = L_2\%$.

⇒ if, SP is same for 2 items & 1 item 10% P & another 1% L or 10% loss.

then, cost price of two item is different must be.

I_2 CP — 10%.
⇒ SP — net loss
10%.
CP

9) x sells 2 jars @ 990 each to y. On one x got 10% Profit while on other 10% loss. Find net Profit or Loss

⇒ Normal method :-

$SP_1 + SP_2 = 990 + 990 \Rightarrow 1980$

$SP_1 \Rightarrow 990 \Rightarrow SP_1 \Rightarrow 1.1 CP_1$

$CP_1 \Rightarrow \frac{9900}{11} \Rightarrow 900$

$SP_2 \Rightarrow 990 \Rightarrow SP_2 = 0.9 CP_2$

$\Rightarrow 990 = 0.9 CP_2$

$\Rightarrow CP_2 = \frac{9900}{09}$ 1100

$\therefore CP_1 + CP_2 \Rightarrow 1100 + 900 = 2000$

$\%L \Rightarrow \frac{2000 - 1980}{2000} \times 100 \Rightarrow 1\%$

Method 2 :-

Same SP of 2 items — 1st x% profit, 2nd x% Loss

$\therefore$ net will be loss

9.09% | 10%

10%

$SP_1 = CP_1 + \frac{x}{100} \times CP_1$

$SP_2 = CP_2 - \frac{x}{100} \times CP_2$

$\therefore$ Net loss $\Rightarrow \frac{CP - SP}{CP} \times 100 \Rightarrow$

$CP_1 + CP_2 - \left[CP_1 + \frac{x}{100} CP_1 - CP_2 \times CP\right]$

As, SP_1 and SP_2 is same

$\Rightarrow CP_1 + \frac{x}{100} CP_1 = CP_2 - \frac{x}{100} \times CP_2$

$\Rightarrow \frac{CP_2 - CP_1}{CP_2 + CP_1} = \frac{x}{100}$

$\Rightarrow SP_1 = CP_1 + \frac{x}{100} \times CP_1$

$\Rightarrow SP_2 = CP_2 - \frac{x}{100} \times CP_2$

$\therefore$ net loss $\Rightarrow \frac{CP - SP}{CP} \times 100$

$\Rightarrow \frac{CP_1 + CP_2 - \left[CP_1 + \frac{x}{100} CP_1 + CP_2 - \frac{x}{100} CP_2\right]}{CP_1 + CP_2} \times 100$

$\Rightarrow \frac{\frac{x}{100}\left[CP_2 - CP_1\right]}{CP_1 + CP_2} \times 100$

$\Rightarrow \frac{x}{100} \times \frac{1}{100} \times 100$ (putting value)

$\Rightarrow \frac{x^2}{100}\%$

net loss $\Rightarrow \boxed{\Rightarrow \left(\frac{x}{10}\right)^2\%}$ $\Rightarrow$ (when same SP same profit and loss)

$\therefore$ as of the qves will be $\Rightarrow \left(\frac{10}{10}\right)^2\% \Rightarrow 1\%$

$\Rightarrow$ when same CP same profit and loss then net profit $\Rightarrow 0$

Q) A man gain SP of 11 bananas what is %. gain?

=> SP → x SP of 33 => 33x
LP → y CP of 33 => 33y

Profit => SP - CP

=> 11x = 33x - 33y

=> 22x = 33y

%P => $\frac{SP-CP}{CP} \times 100$

=> $\frac{x-y}{y} \times 100$

=> $\frac{22x-22y}{22y} \times 100$

=> $\frac{33y-22y}{22y} \times 100$

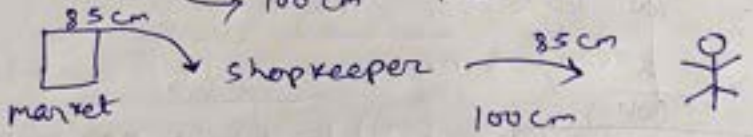
=> $\frac{11y}{22y} \times 100$

=> 50% (Ans)

30) Profit and Loss Part 3:-

Q) Dishonest shopkeeper uses 85 cm scale instead of 1m to measure cloth length & says SP = CP %??

=> shopkeeper → 85 cm → 100 cm i.e cost taken is of 100 cm



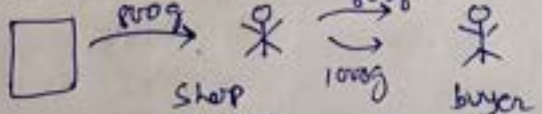
CP => 85
SP => 100

%P => $\frac{100-85}{85} \times 100$

=> $\frac{15}{85} \times 100$ => $\frac{300}{17}$

Q) Dishonest shopkeeper use false weight of 800g instead of 1kg & claims to sell goods @ CP find P%.

=>



CP => 800
SP => 1000
P% => $\frac{1000-800}{800} \times 100$
=> 25% (Ans)

⭐⭐ ⇒ the thing which you bought from market and sell to the customer is CP

⇒ the money that you taking from the customer is SP.

8) on selling 36 mangoes, a shopkeeper recovers CP of 33 mangoes. find loss??

⇒ SP → x SP of 36 mangoes ⇒ 36x
CP → y CP of 33 mangoes ⇒ 33y

$36x = 33y$

Loss % ⇒ $\frac{y-x}{y} \times 100$

⇒ $\frac{33y - 33x}{33y} \times 100$

⇒ $\frac{36x - 33x}{33x} \times 100$

⇒ 8.3 %.

9) 2000 Robots @ 25 lakh each were sold to govt if 5% fail even then 25% Profit. But 50% were rejected find loss.

⇒ SP of 2000 → 25 * 2000 L ⇒ 50,000 L

$\frac{5}{100} \times 2000 \Rightarrow 100$

2500 L

5% fail ⇒ SP ⇒ (50000 − 2500) L ⇒ ₹47500 L of 2000 R

SP ⇒ 1.25 CP

CP of 2000 R ⇒ $\frac{47500}{1.25}$

⇒ 38000 L

50% were rejected ⇒ 1000 taken

SP of 1000 R ⇒ 25000 L

Loss ⇒ (38000 − 25000) L ⇒ 13000 L

- Successive Discount :- (70% + 30%) off

8) [80% + 40% + 5%*] off on Rs 1000 shirt find the price after discount?

* → axis bank card holders

=> 80% Discount → $1000 - \frac{80}{100} \times 1000$

=> 200

=> $200 - \frac{40}{100} \times 200^{2}$

=> 120

=> Extra 5% => $120 - \frac{120}{100} \times 5$

=> 114 (Ans)

like $\frac{x}{1000} - 0.8 \times \frac{2}{1000} \cdot 2 \times 1000 \Rightarrow 200$

Direct method :-

$(((1000) \times 0.2) 0.6)0.5$

=> 120 × 0.5

=> 114 (Ans)

9) What is single effective discount that is eavivalent to 2 Surcesive discounts of [80% + 40%] ?

=> $(1000 \times 0.2) \times 0.6$

After .: Discount, have to pay => $0.2 \times 0.6 \Rightarrow 0.12$

Effective discount => $1 - 0.12 \Rightarrow 0.88$

∴ 88%.

10) What is single effective discount that is eavivalent to 2 successive discounts of [70% + 30%] ?

=> $0.3 \times 0.7 \Rightarrow 0.21$ %.

$1 - 0.21 \Rightarrow 0.79$

79% (Ans)

100 → 70% → 30 → 30% → 21

∴ 79 %

8) [80% + 40% + 5%] Effective Discount ?

=> $0.2 \times 0.6 \times 0.95$

$1 - (0.2 \times 0.6 \times 0.95)$

=> 88.6%.

100 → 80% → 20 → 40% → 12 → 5% → 11.4

88.6% (Ans)

31. Profit and Loss : Part 4 :-

- **Marked Price :-** ☆☆☆

→ Price Printed on article

→ Also called MRP, List Price, Labeled Price, Printed Price, tagged Price etc.

- Note :-

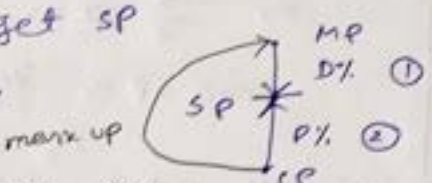
→ By how % MP is more than CP is mark up %.

→ mark up is always done on CP like Profit, loss always on CP.

→ Discount is given on M.P. & we get SP so if MP is 100 then 20% discount means SP is 80.

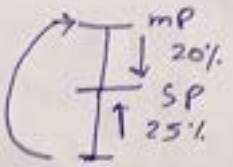
→ Profit is always on CP & we get SP

→ Hence we can get SP in 2 ways.



Q) on offering a discount of 20%, a shopkeeper still manages to make a Profit of 25%. By how much % is MP above CP??

⇒



CP = 100

SP = 125

let, MP is x

20% discount

⇒ $x - \frac{20}{100} \times x$

⇒ $0.8x$

$0.8x = 125$

$\Rightarrow x = 156.25$

∴ % of MP ⇒ [illegible] ⇒ [illegible]

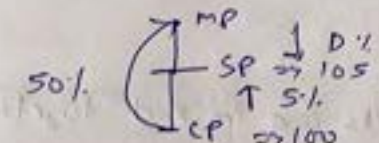
⇒ 56.25 % (Ans)

★★★

When ever see, MP?? CP?? in question make a Diagram.

Q) If Price is marked 50% over CP & after giving discount he make 5% Profit. find % of discount offered on MP??

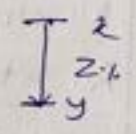
⇒



$D = \frac{MP - SP}{MP} \times 100$

$= \frac{150 - 105}{150} \times 100$

⇒ 30%



Q) By selling article @ 1500 shopkeeper loses 20%. What is gain or loss if sell the same @ 1900.

⇒ SP ⇒ 1500

SP ⇒ 0.8 CP

$\frac{1500}{0.8}$ = CP ⇒ [illegible] 1875

Gain → 1900 − 1875 ⇒ 25 RS

Q) A man buys an article @ 7000 & sell at a profit of $\frac{2}{7}$ of SP Find SP?

=> CP => 7000

$P = SP - CP$

$\frac{2}{7} \times SP = SP - 7000$

$\Rightarrow \frac{5}{7} SP \Rightarrow 7000$

$\Rightarrow SP \Rightarrow \frac{7000 \times 7}{5} \Rightarrow 9800$

Q) 'A' sell article to B at gain of 10% & B sell it to C at loss of 20%. If C pay 1000 to B. What is CP of article to A.

=> CP_A SP_A

$SP_A = 1.1 CP_A$

$\downarrow$

CP_B

$\Rightarrow SP_B = 0.8 \times CP_B$

$\Rightarrow SP_B = 0.8 \times 1.1 CP_A$

$\Rightarrow SP_B = 0.88 CP_A$

$\Rightarrow \downarrow 1000$

$CP_A \Rightarrow \frac{1000 \times 100}{88}$ (As)

32 Alligation and Mixture Part 3 : 1

8 Alligation & Mixtures :-

Mixture : When 2 or more elements are mixed together in certain ratio we get mixture.

Alligation : Rule which enable us to find ratio in which 2 or more elements are mixed to get desired mixtures is called alligation

Q) class A has 40 students & avg marks is 60 class B has 50 Students & avg marks is 70 what is average marks of all Students taken together ?

$\Rightarrow A = \frac{P_1 \times A_1 + N_2 \times A_2}{N_1 + N_2}$

$= \frac{40 \times 60 + 50 * 70}{40 + 50} \Rightarrow \frac{2400 + 3500}{900} \Rightarrow \frac{5900}{900}$

Students

Q) class A has 40 ~~subjects~~ & Avg mark is 60 [assume all students have same marks] class B has 50 students & avg marks is 70 (assume all students have same marks). In what ratio we can take students from class A & B to get average as 67 marks.

=> let, take in N_1, N_2

$N_1 \rightarrow A$
$N_2 \rightarrow B$

$$\Rightarrow 67 = \frac{N_1 \times 60 + N_2 \times 70}{N_1 + N_2}$$

$$\Rightarrow 67N_1 + 67N_2 = 60N_1 + 70N_2$$

$$\Rightarrow \frac{N_1}{N_2} = \frac{3}{7}$$

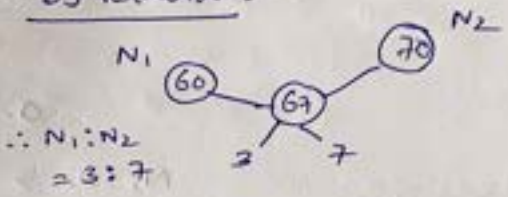
$$\Rightarrow N_1 : N_2 = 3:7 \text{ (Ans)}$$

Or, 6:14
9:21
:
30:70
} can be Ans

Proff :-

$$\frac{3 \times 60 + 7 \times 70}{7+3} \Rightarrow \frac{670}{10} \Rightarrow 67$$

By formula :-



$\therefore N_1 : N_2 = 3:7$

Other way Proff :

$$A = \frac{N_1A_1 + N_2A_2}{N_1 + N_2}$$

$$A = \left(\frac{N_1}{N_1+N_2}\right) \times A_1 + \left(\frac{N_2}{N_1+N_2}\right) A_2$$

$$\Rightarrow 67 = \left(\frac{3}{3+7}\right) \times 60 + \left(\frac{7}{3+7}\right) \times 70$$

If, we need Average close to A_1 we have to take N_1 greater
" " " " " " A_2 " " " N_2 "

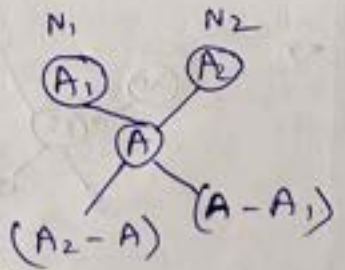
=> So, if we want to find ratio :-

$$A = \frac{N_1A_1 + N_2A_2}{N_1 + N_2}$$

$$\Rightarrow AN_1 + AN_2 = N_1A_1 + N_2A_2$$

$$\Rightarrow N_1(A - A_1) = N_2(A_2 - A)$$

$$\Rightarrow \boxed{\frac{N_1}{N_2} = \frac{A_2 - A}{A - A_1}} \star\star\star \leftarrow \text{Alligation}$$



=> try to keep it Positive

9) In what ratio ~~[illegible]~~ 20 /- kg & 30 /- kg sugar must be mixed to ~~[illegible]~~ get 27 /- kg sugar.

Visualise:

⇒ class A → 1000 g → 20

1 g → 0.02

↙

1 crystal

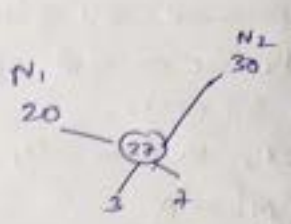
⇒ class B → 1000 g → 30

1 g → 0.03

↓

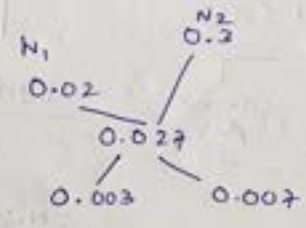
1 crystal

0.027



N1 : N2 ⇒ 3 : 7 (Ans)

Or, Directly :-



N1 : N2 ⇒ 0.003 : 0.007

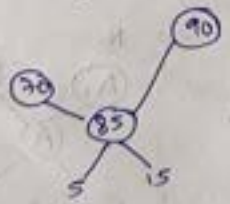
⇒ 3 : 7 (Ans)

- Summary :-

Mixture is given and avg is ~~[illegible]~~ given and asked about Ratio ?? ⇒ then we will apply aligation.

9) In what ratio 70 /- kg & 90 /- kg Rice must be mixed to get 85 /- kg Rice.

⇒



5 : 15

⇒ 1 : 3

Proff :-

$$\frac{70 \times 1 + 90 \times 3}{4}$$

$$\Rightarrow \frac{340}{4}$$

⇒ 85

if, question was have to take 10 kg Rice then ratio ??

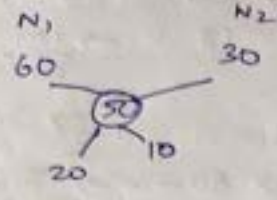
⇒ 4x = 10

⇒ x = 2.5

2.5 : 7.5 (Ans)

8) In what ratio 60/per litre & 30/per litre milk must be mixed to get 50/- per litre milk.

⇒



$N_1 : N_2 = 60 : 30$
$= 2 : 1$

Proff:

$\frac{60 \times 2 + 30 \times 1}{3}$

$\Rightarrow \frac{120 + 30}{3}$

$\Rightarrow 50$

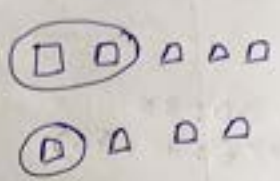
⇒ visualize:-

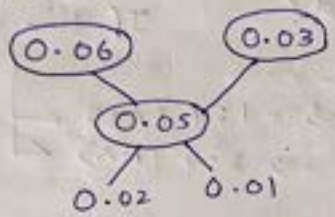
1000 ml ⇒ 60

1 ml ⇒ 0.06

1000 ml ⇒ 30

1 ml ⇒ 0.03



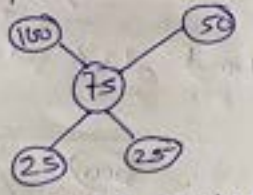


⇒ 0.02 : 0.01

33. Alligation and Mixtures Part 2:

8) D_1 distance @ 100 km/hr & D_2 distance @ 50 km/hr what must be the ratio of distance travelled if avg speed is 75 km/hr

⇒



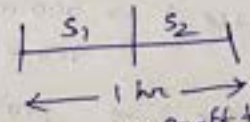
25 : 25 ⇒ 1 : 1

∴ $\frac{1}{2}$ hr go with 100 km/hr ⇒ 50

$\frac{1}{2}$ hr " " 50 km/hr ⇒ 25

⇒ 50 : 25

∴ $D_1 : D_2 \Rightarrow 2 : 1$ (Ans)



9) if we need avg speed 65.

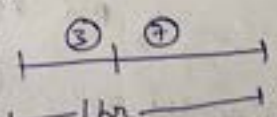
(by intution we proff that ratio is correct as 50 is more close to 65 so, we go large part in that)

So, in 1 hr 3 part S_1 7 part S_2

⇒

⇒ 15 : 35

⇒ 3 : 7



⇒ So, in 1 hr S_1 ⇒ $\frac{3}{10} \times 60$ min ⇒ 18 min

in 1 hr S_2 ⇒ $\frac{7}{10} \times 60$ ⇒ 42 min

So, in 1 hr speed S_1 travel 18 min
" " " S_2 " 42 min

∴ Distance covered in 1 hr by S_1 ⇒ 60 min → 100 km
18 " → $\frac{100}{60} \times 18$
⇒ 30 km

Distance covered in 1 hr by S_2 ⇒ 60 min → 50 km
42 " → $\frac{50 \times 42}{60}$
⇒ 35 km

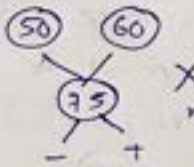
∴ $D_1 : D_2$ ⇒ 30 : 35
⇒ 6 : 7 (Ans)

Q) What if ratio of time asked?
⇒ So, same as like speed
for first question ans ⇒ 1 : 1 (Ans)
for second " " ⇒ 3 : 7 (Ans)

If time asked

- **Note :-**
Same formula will work for absolute values [as seen till now] Percentages & fractions.

Proff :-
Bottle 1 → 60 % Alchol
Bottle 2 → 80 % Alchol
Bottle 3 → initially empty & we want 75% alchohol.

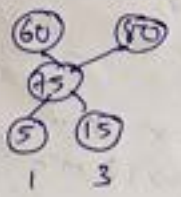


X that not possible

⇒ Bottle 1 → 600 ml Alchohol
400 ml water
Bottle 2 → 800 ml Alchohol
200 ml water

⇒ Per 100 here
⇒ Per kg (Previously

Or, Directly ⇒



60%
1 : 3 < 80%

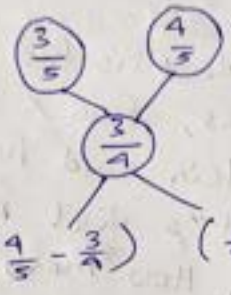
∴ Work in Percentage (Proved)

Now,

60% $\leftrightarrow \frac{3}{5}$ ⇒ Every fraction can be converted to % & vice versa.

80% $\leftrightarrow \frac{4}{5}$

75% $\leftrightarrow \frac{3}{4}$



$\frac{4\times4}{5\times4}$ $\frac{3\times5}{4\times5}$

$\Rightarrow \frac{16}{20}$ $\frac{15}{20}$

by that we can understand which one is bigger.

∴ work in fractions.

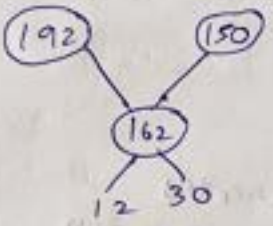
Fraction ⇒ $\frac{3}{7}$ $\frac{4}{7}$

Ratio ⇒ 3 : 4

A W

9) In what ratio 192 per kg & 150 per kg Pulses must be mixed to get SP of 194.4 per kg in which 20% is profit

⇒



SP = 1.2 CP

⇒ 194.4 = 1.2 CP

⇒ CP = 162

12 : 30 ⇒ 2 : 5

$N_1 : N_2$ ⇒ 2 : 5 (Ans)

192 150

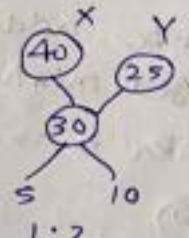
8) Seed mixture X is 40 percent ryegrass and 60 percent bluegrass by ~~weight~~ weight seed mixture Y is 25 percent ryegrass and 75 % fescue. If a mixture of X and Y contains 30% ryegrass, what percent of the weight of the mixture is X? a) 10% b) 33.33% c) 40% d) 50%.

⇒ X → 40% RG

~~60% BG~~

Y → 25% RG

~~75% F~~



$N_1 : N_2$ = 1 : 2

↑ X ↑ Y

X → $\frac{1}{3} \times 100$

~~=33.3%~~ ⇒ 33.3% (Ans)

Y → $\frac{2}{3}$

⇒ Just ignore the other's weights

like this ⇒ 3 → 1
1 → $\frac{1}{3}$
100 → $\frac{1}{3} \times 100$
⇒ 33.3 %

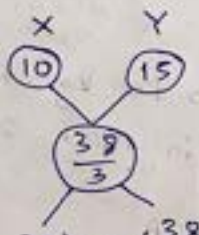
Per Cent
Per 100 ??

34. Alligation and Mixture : Part 3 :-

Q) A Rabbit on a controlled diet is fed daily 300 grams of a mixture of two foods, food x and food Y. Food x contains 10 % protein and food Y contains 15 % Protein. If the rabbit's diet provides exactly 38 grams of Protein daily, How many grams of food x are in the mixture.

A) 100 B) 140 C) 150 D) 160 E) 200

⇒



$\frac{38}{300} \times 100$
⇒ $\frac{38}{3}$

$\left(15 - \frac{38}{3}\right) : \left(\frac{38}{3} - 10\right)$

$\frac{7}{3} : \frac{8}{3}$

7 : 8

x ⇒ $\frac{7}{15} \times 300$
⇒ 140

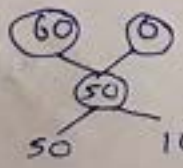
or, Directly
15 × 20 = 300
7 × 20 = 140
8 × 20 = 160

or, 15 → 7
1 → $\frac{7}{15}$
300 → $\frac{7}{15} \times 300$
⇒ 140

or, $7x + 8x = 300$
$\Rightarrow x = \frac{300}{15}$
$\Rightarrow x = 20$
∴ 7 × 20 ⇒ 140

Q) How much water must be added to 10 litre of milk costing 60 / lit to get mixture @ 50 / lit.

⇒ let, say cost of water is ⇒ 0

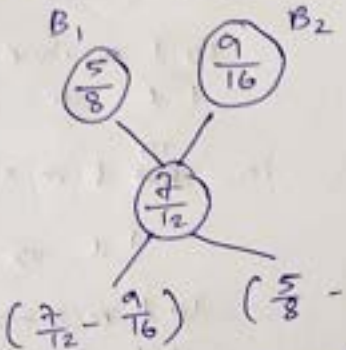


5 : 1
↓ ↓
M W
5 lit → 1 lit
10 lit → $\frac{10 \times 1}{5}$ ⇒ 2 lit (Ans)

Q) Alchol : water in Bottle 1 & Bottle 2 is 5:3 & 9:7 respectively. In what ratio mixture from bottole 1 & 2 must be taken to get alchol : water as 7:5?

⇒ our formula does not work in ratio

⇒ so, convert into Fraction



$(\frac{7}{12} - \frac{9}{16})$ $(\frac{5}{8} - \frac{7}{12})$

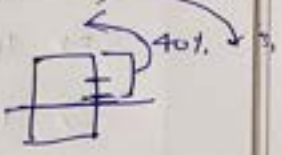
$\Rightarrow \frac{28-27}{48}$ $\Rightarrow \frac{15-14}{24}$

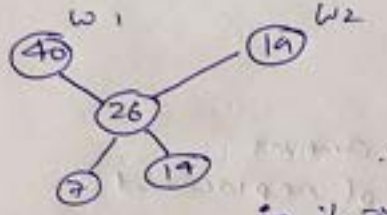
$\Rightarrow \frac{1}{48} : \frac{1}{24}$

$\Rightarrow 1:2$ (Ans)

Q) Jar full of whisky (40% alchol). A part of it is replaced by another containing 19% alcohol. final mixture is 26% alchol. What % of whisky is replaced.

⇒ mixture, % ⇒ alligation





40% 19%

$W_1 : W_2 \Rightarrow 1:2$

∴ 2 part is replaced from 3

in % ⇒ $\frac{2}{3} \times 100$

$\Rightarrow 66.66\%$

3 → 2

100 → $\frac{2 \times 100}{3}$

☆☆

Q) Gate 2021 :-

A container originally contains 10 liters of pure spirit. From this container 1 litre of spirit is replaced with 1 litre of water. Subsequently, 1 litre of the mixture is again replaced with 1 litre of water and this process is repeated one more time. How much spirit is now left in the container?

A) 7.58 lit B) 7.84 lit C) 7 lit D) 7.29 lit

	Spirit	water
	10	0
1st →	−1	+1
	9 :	1
2nd →	−0.9	−0.1 +1 ← added
	8.1	1.9
3rd	−0.81	−0.19 +1 ← added
	7.29	2.71

∴ Ans ⇒ 7.29

10 l

9l 1l

9 : 1

10l mixture → 9 lt Spirit

1 Lit " → 0.9 "

10 l mixture → 1 lt water

1 l " → 0.1 " "

in, 10 l mix → 8.1 l Spirit

1 l → 0.81 l Spirit

10 l mix → 1.9 lit water

1 l " → 0.19 lit water

- **Formula :-**

$\Rightarrow \left(10 - \frac{10\times10}{100}\right) - \frac{10}{100}\left[100 - \frac{10\times10}{100}\right]$ ← again 10% of it

↑ so, each time we are replacing 10% of mixture

$\Rightarrow \left[10 - \frac{10\times10}{100}\right]\left[1 - \frac{10}{100}\right]$

$\Rightarrow 10\left[1 - \frac{10}{100}\right]\left[1 - \frac{10}{100}\right]$

$\Rightarrow 10\left[1 - \frac{10}{100}\right]^2$

$\Rightarrow \boxed{x\left[1 - \frac{y}{x}\right]^n}$

x = Initial quantity y = replace amount n = no of replaced.

⇒ So, by formula ⇒ $10\left[1 - \frac{1}{10}\right]^3$

⇒ 7.29 (Ans)

35) **Alligation and Mixture : Part 4 :-**

Q) 8 lit drawn from 16 lit. Wine & replaced with water same process repeated 3 more times Amount of Wine remaining ?

⇒ replaced, same process repeated ⇒ have to use formula

⇒ $x\left(1 - \frac{y}{x}\right)^n$ ⇒ $16\left[1 - \frac{8}{16}\right]^4$ n ⇒ 4 not 3

⇒ 1 lit

as, done then repeated 3 time 1 + 3 like that.

8) 8 lit drawn from pure wine & replaced with water same process repeated 3 more times. Ratio of wine present now & before is 16:81. Amount of wine originally taken?

let, z remaining wine

$\Rightarrow x\left[1-\frac{y}{x}\right]^{n} = z$

(x: initial quantity; replace y; n: no of time)

$\Rightarrow \left(1-\frac{y}{x}\right)^{n} = \frac{z}{x}$ ← $\frac{\text{now}}{\text{before}}$

$\Rightarrow \left(1-\frac{8}{x}\right)^{4} = \frac{16}{81}$

$\Rightarrow 1-\frac{8}{x} = \frac{2}{3}$

$\Rightarrow x = 24$ lit (Ans)

36) Simple interest & compound interest part 1 :

- Simple interest :

interest paid on Principle only & this Principle remain constant.

=> Principle meas initial value.

100,000 → SI => 1·1 /per month as interest

$\frac{1}{100} \times 100000 \Rightarrow 1000$

at end of 1st month => 1000

" " " 2nd " => 1000

" " " 3rd " => 1000

:

6th " => 1000

6000 => SI

After 6 month?

SI → 6000

↓

1000 × 6

$SI = \frac{100,000 \times 1 \times 6}{100}$

=> Principle doesn't chage

=> pay after all at once after time no change in Principle

$$SI = \frac{P \times R \times T}{100}$$

Ex: 100,000 @ 1% per month for 1 year

SI?

P → 100000

R → 1 % per month

T → 1 year ↔ 12 month

$SI \Rightarrow \frac{100000 \times 1 \times 12}{100} \Rightarrow 12000$

=> qvarter meas => 3 month.

Keep eye on Units :

Ex: R → 3 % per qvater

T → 1 year => 12 month => 4 qvater

P = 1000

$SI \Rightarrow \frac{1000 \times 3 \times 4}{100}$

$\Rightarrow 120$

Ex:

R → 6 % half yearly

T → 2 year => 4 half year

P = 1000

$\Rightarrow SI \Rightarrow \frac{1000 \times 6 \times 4}{100}$

$\Rightarrow 240$

Ex: R → 12 % PA

T → 36 months ⇒ $\frac{36}{12}$ ⇒ 3 year

P = 1000

$SI \Rightarrow \frac{P \times R \times T}{100} \Rightarrow \frac{1000 \times 12 \times 3}{100} \Rightarrow 360$

Ex: R → 12 % PA ← Per Anum

T ⇒ 6 month ⇒ $\frac{6}{12} \Rightarrow \frac{1}{2}$

P = 1000

$SI \Rightarrow \frac{1000 \times 12 \times \frac{1}{2}}{100}$

$= \frac{1000 \times 12 \times 1}{2 \times 100} \Rightarrow 60$

Method 2 :-

R → 12 % PA → 1 % Pm

⇒ 12 m → 12 %

1 m → $\frac{12}{12}$ ⇒ 1 %

T → 6m

$SI \Rightarrow \frac{P \times R \times T}{100} \Rightarrow \frac{1000 \times 1 \times 6}{100} \Rightarrow 60$

Ex: R = 3 % per quater

T = 6 months ⇒ 2 quater

P = 1000

$SI \Rightarrow \frac{1000 \times 3 \times 2}{100} \Rightarrow 60$ (Ans)

OR:

3m → 3 %

1m → 1 %.

$SI \Rightarrow \frac{1000 \times 1 \times 6}{100} \Rightarrow 60$ (Ans)

Ex: 100,000 @ 1.1 Per month Per 1 year SI ? So, Amount to Pay ?

$SI = \frac{100,000 \times 1 \times 12}{100} \Rightarrow 12000$

$SI = \frac{100,000 \times 12 \times 1)}{1000} \Rightarrow 12000$ ← Another way

Amount = 100000 + 12000 ⇒ 112000 (Ans)

$A = P + SI$

$A = P + \frac{P \times R \times T}{100}$

$\boxed{A = P\left(1 + \frac{R\,T}{100}\right)}$

Part 2 :-

37)

Q) What is Amount Paid if man borrowed 10,000 for 5 years @ 12 % PA.

$\Rightarrow A = P + SI$

$= P + \frac{P \times R \times T}{100}$

$= P\left[1 + \frac{RT}{100}\right]$

$= 10000\left[1 + \frac{12 \times 5}{100}\right]$

$= 16000$ Ans

Q) SI, Principle ?? @ 8 % PA, amount Paid = 3720 after 3 yrs.

$\Rightarrow 3720 = P\left[1 + \frac{RT}{100}\right]$

$\Rightarrow 3720 = P\left[1 + \frac{8 \times 3}{100}\right]$

$\Rightarrow P = \frac{372000}{1124}$

Q) If P becomes 2P in 5 years then find R

$\Rightarrow A = P\left[1 + \frac{RT}{100}\right]$

$\Rightarrow 2P = P\left[1 + \frac{RT}{100}\right]$

$\Rightarrow 2 = \left[1 + \frac{R \times 5}{100}\right]$

$\Rightarrow R = 20\%$ PA

Q) A certain money amounts to 800 in 2 years & 1100 in 7 years at SI. what is Sum?

$\Rightarrow$ 800 @ 2 years

1100 @ 7 years — 5 years — SI $\Rightarrow$ 300

in 5 years SI 300

1 ~ ~ $\frac{300}{5}$ 60

$\Rightarrow 60$

2 years $\rightarrow 60 \times 2$

$\Rightarrow 120$

$A = P + SI$

$\Rightarrow 800 = P + 120$

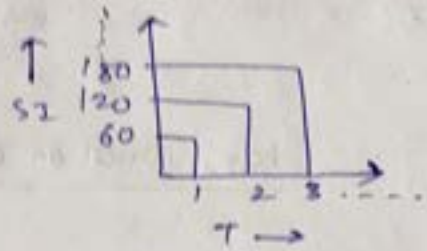
$\Rightarrow P = 680$

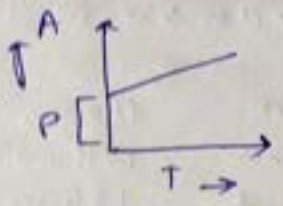
Ans $\Rightarrow 680$

$SI = \frac{P \times R \times T}{100}$

$SI = C \times T$

$SI \propto T$





as, at $T = 0$

⇒ Principle must be Paid

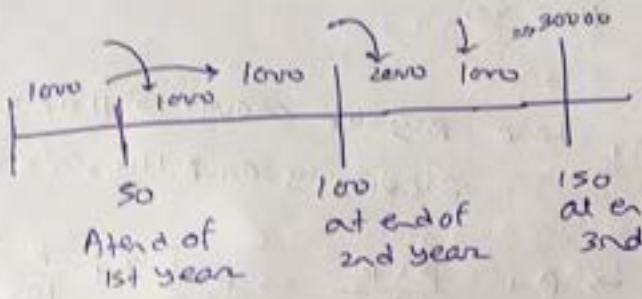
$A = P + \frac{PRT}{100}$

$A = C_1 + C_2 \times T$

$y = mx + c$

Q) A lends 1000 every year to B @ 5% p.a find the amount Paid back by B after 3 years.

Ans:



⇒ Amount Paid ⇒ A ⇒ P + SI ⇒ 50 + 100 + 150 + 3000

⇒ 3300

⇒ $SI = \frac{P \times R \times T}{100}$

⇒ $SI = C \times P$

$SI \propto P$

$\uparrow \quad \uparrow$

$2 \times SI \quad 2 \times P$

⇒ Part 3 :-

- Compound Interest :

Banks → CI compounded Annually

10%

100,000

1st year → 110000 → 100,000 + 10,000 → Interest on original → 10,000

2nd year → 121000 → interest on interest → 1000

3rd year → 133100

4th year → 146410

5th year → 161051

here, interest on original + interest

If it was in SI 12%

1st	112000
2nd	124000
3rd	136000
4th	148000
5th	160000

here, interest on only on original

now, 10% 3 500 Rs voucher compounded half yearly

1000,000

6 month ⇒ 105,000

Another 6 month ⇒ 110250

after 1 year here payable amount ⇒ 110250

previously annually case ⇒ 112000

⇒ After 1 year → $P + \frac{P \times R}{100} \Rightarrow P\left[1 + \frac{R}{100}\right] \Rightarrow 110000$

After 2nd year ⇒ $P\left[1+\frac{R}{100}\right] + P\left[\frac{1+R}{100}\right] \times \frac{R}{100} \Rightarrow 121000$

here that is the principal

$\Rightarrow P\left[1+\frac{R}{100}\right]\left[1+\frac{R}{100}\right] = P\left[1+\frac{R}{100}\right]^2$

After 3rd year ⇒ $P\left[1+\frac{R}{100}\right]^2 + P\left[1+\frac{R}{100}\right]^2 \times \frac{R}{100}$

$\Rightarrow P\left[1+\frac{R}{100}\right]^2\left[1+\frac{R}{100}\right]$ ⇒ 133100

$\Rightarrow P\left[1+\frac{R}{100}\right]^3$

SI → $\frac{P \times R \times T}{100}$ ← interest ★★★

Amount Payable → $P + SI \Rightarrow P + \frac{P \times R \times T}{100} \Rightarrow P\left[1 + \frac{RT}{100}\right]$

CI → ~~$\frac{P \times R \times T}{100}$~~ $P\left(1+\frac{R}{100}\right)^T$ ← Amount Payable

interest ⇒ Amount payable − Principal

$\Rightarrow P\left(1+\frac{R}{100}\right)^T - P$

=> 100,000 | 5 year | 10 % PA compounded Anually,

Anually compounded :- $100,000 \times \left[1 + \frac{10}{100}\right]^5$

=> 161051

compounded Half yearly :- $100000 \times \left(1 + \frac{5}{100}\right)^{10}$

=> 162889.463

12 m → 10 %
1 m → $\frac{10}{12}$ %
3 m → $\frac{10}{12} \times 3$
$\frac{10}{4}$
=> 2.5

So, extra 1838.463 /- than Anually

compounded Qvaterly : $100000 \times \left(1 + \frac{2.5}{100}\right)^{20}$

=> 163861.644

Compounded monthly : $100000 \times \left[1 + \frac{10}{12 \times 100}\right]^{60}$

=> 164530.861

=> Key words for Compound interest :

- → Compounded annually
- → population growth
- → Birth rate
- → Death rate
- → GDP
- → GNP
- → Industrial growth
- → per capita income
- → Industrial consumption
- → Any govt. data

=> Otherwise SI

Sol:

Eg> The population of a new city is 5 million and is growing at 20 % annually. How many years would it take to double at this growth rate?

a> 3-4 years b> 4-5 years c> 5-6 years d> 6-7 years

=> Amount payable => $P\left[1 + \frac{R}{100}\right]^n$

=> $2P = P\left[1 + \frac{20}{100}\right]^n$

=> putting value of n from option

$(1.2)^3 \Rightarrow 1.728$

$(1.4)^4 \Rightarrow 2.0736$

∴ option (a)

Q) In How may years money doubles @ 28.1 PA. Compounded anually $[\log_2 100 = 6.64]$

$\Rightarrow 2P = P\left[1+\frac{R}{100}\right]^T$

$\Rightarrow 2 = \left[1+\frac{28}{100}\right]^T$

$\Rightarrow 2 = (1.28)^T$

$\Rightarrow \log_2 2 = T \times \log_2 (1.28)$

$\Rightarrow 1 = T \times \log_2 \left(\frac{128}{100}\right)$

$\Rightarrow 1 = T \times \left(\log_2 128 - \log_2 100\right)$

$\Rightarrow 1 = T \times [7 - 6.64]$

$\Rightarrow T = 2.8$ years

Q) Population of town is 2400 3 years ago & Now 3600. What will be population 3 year hence

$\Rightarrow 3600 = 2400\left[1+\frac{R}{100}\right]^3 \quad \ldots (i)$ ← acc to 2019

2022 ← 3600

2019 ← 2400

$\Rightarrow x = 3600\left[1+\frac{R}{100}\right]^3 \quad \ldots (ii)$

(i) ÷ (ii)

$\Rightarrow \frac{3600}{x} = \frac{2400}{3600}$

$\Rightarrow x = 5400.$

Q) 1000 lended @ 10% Compunded quarterly. What is the CI for 9 months?

$\Rightarrow A = P\left[1+\frac{R}{100}\right]^T$

$\Rightarrow CI = A - P$

$\Rightarrow CI = P\left[1+\frac{R}{100}\right]^T - P$

$\Rightarrow CI = 1000\left(1+\frac{10}{100}\right)^3 - 1000$

$\Rightarrow 331$ (Ans)

Q) A sum of money amounts to 4840 in 2 years and 5324 in 3 years at CI. What is the rate Percent?

a) 9% b) 9% c) 9% d) 10%.

$\Rightarrow 4840 = P\left[1+\frac{R}{100}\right]^2 \ldots (ii)$

$\Rightarrow 5324 = P\left[1+\frac{R}{100}\right]^3 \ldots (i)$

(i) ÷ (ii)

$\Rightarrow \frac{5324}{4840} = 1+\frac{R}{100}$

$\Rightarrow \frac{R}{100} = \frac{5324-4840}{4840}$

$\Rightarrow \frac{R}{100} = \frac{484}{4840}$

$\Rightarrow R = 10\%$ (Ans)

Q) A certain amount becomes 500,000 in 5 yrs @ 10% compounded annually. Find the amount?

$\Rightarrow x \rightarrow 500{,}000$ @ 10%.

$A = P\left[1+\frac{R}{100}\right]^5$

$\Rightarrow 500000 = P\left[1+\frac{10}{100}\right]^5$

$\Rightarrow 500000 = P(1.1)^5$

$\Rightarrow P = \frac{500000}{(1.1)^5}$

$\Rightarrow P = 310460.662$ (Ans)

Q) Certain sum of money doubles itself in 5 yrs @ CI in how many years will it become 8 times.

$\Rightarrow A = P\left[1+\frac{R}{100}\right]^T$

$\Rightarrow 2P = P\left[1+\frac{R}{100}\right]^5$

$\Rightarrow 2 = \left[1+\frac{R}{100}\right]^5$

$\Rightarrow 2^3 = \left[\left(1+\frac{R}{100}\right)^5\right]^3$ (cube each side.

$\Rightarrow 8P = P\left[1+\frac{R}{100}\right]^{15}$ (multiply each side P)

$2 \rightarrow 5$	
8 → ??	⇒ here multiply
here, power ⇒ 2^3	$3 \times 5 = 15$

So, P 8 times in 15 years by this eqn. Ans: 15

Q) 3 times in 7 years, 81 times?

3 → 7

4 ↓ 7×4 ⇒ 28 years ans.

81

↑

Q) Find difference b/w CI & SI for 2 years @ R%.

⇒ CI − SI

$\Rightarrow \left[P\left[1+\frac{R}{100}\right]^2 - P\right] - \frac{P \times R \times T}{100}$

$\Rightarrow P\left[1+\left(\frac{R}{100}\right)^2 + \frac{2R}{100}\right] - P - \frac{2PR}{100}$

$\Rightarrow P + P\left(\frac{R}{100}\right)^2 + \frac{2PR}{100} - P - \frac{2PR}{100}$

$\Rightarrow P\left(\frac{R}{100}\right)^2$ (Ans)

Q) Find difference b/w CI & SI for 3 years

$\Rightarrow P\left[1+\frac{R}{100}\right]^3 - P - \frac{3PR}{100}$

$\Rightarrow P\left[1+\left(\frac{R}{100}\right)^3 + \frac{3R}{100}\left[1+\frac{R}{100}\right]\right] - P - \frac{3PR}{100}$

$\Rightarrow P + P\left(\frac{R}{100}\right)^3 + \frac{3PR}{100} + \frac{3PR^2}{(100)^2} - P - \frac{3PR}{100}$

$\Rightarrow P\left(\frac{R}{100}\right)^3 + 3P\left(\frac{R}{100}\right)^2$

Q) Sum of money becomes 16 times of itself in 4 years. Find R if interest compounded annually.

$\Rightarrow 16P = P\left[1+\frac{R}{100}\right]^4$

$\Rightarrow 2^4 = \left(1+\frac{R}{100}\right)^4$

$\Rightarrow 2 = 1+\frac{R}{100}$

$\Rightarrow R = 100\%$

Blood relations:

Father's or mother's father = Grand father
Father's or mother's mother = Grand mother
Father's or mother's Brother = uncle
Father's or mother's Sister = Aunt

Uncle 's or Aunt child = cousin

Sibling's son = Nephew
Sibling's daughter = Niece

Son or ~~dau~~ daughter's children = Grand children

Husband or wife father = father in law
Husband or wife mother = mother in law
Husband or wife brother = brother in law
Husband or wife sister = sister in law

Sister's Husband = Brother in law
Brother's wife = Sister in law
Son's wife = Daughter in law
Daughter's Husband = son in law

=> in law means second family.

=> sibling means don't specified gender can be male or can be female

=> Parental Grand father / Grand father → father's father
Maternal " " / " " → mother's "
Parental " mother / " mother → father's mother
Maternal " " / " " → mother's "

level 1

level 2 — Father, mother, Parental / Maternal uncle, Parental / maternal Aunt

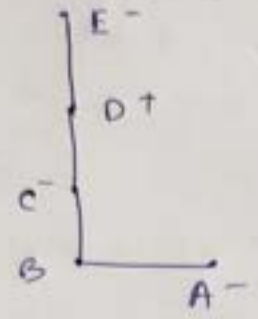
level 3 — Self, Brother, sister, Sibling, cousins, Sister in law, Brother in law,

level 4 — Daughter, son, nephew, Niece, Daughter in law, son in law

level 5 — Grand child

Male → +
Female → −

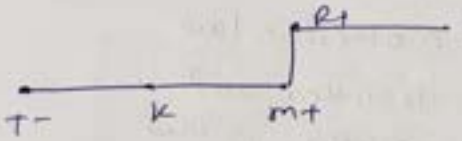
9) A is B's sister, C is B's mother, D is C's father, E is D's mother, How is A related to D?

⇒ 

⇒ Don't assume gender

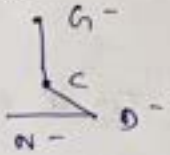
⇒ Grand child (AS)

9) M is brother of K, T is sister of K, R is father of M, J is wife of R, How many sons does J have?



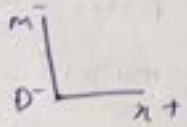
⇒ Data inadequate as K's gender not given

9) Pointing to a girl Nidhi said, She is daughter of my grand mother's only child. How girl is related to Nidhi.



Sister

9) Pointing to photo Maya said "He is the brother of the wife of my Husband", How is man related to maya?

⇒ 

⇒ Son

B of D of Maya

- Coded relation:

Study the info given below & answer the question

A+B means A is father of B
A−B " " − brother " "
A×B " " − sister " "
A/B " " − mother " "

9) if, N−P×L/E÷M, then N is M's

⇒ Maternal uncle

9) Which one of the following means g is grandfather of P?

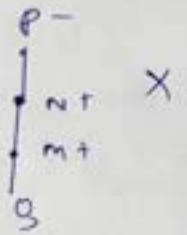
a) P/N + M+g

b) g+N+M/P

c) g - M - N + P

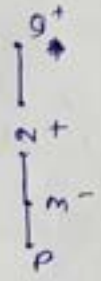
d) g+M/P

a)



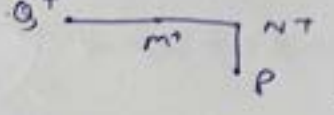
X

b)



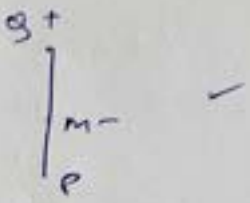
X

c)



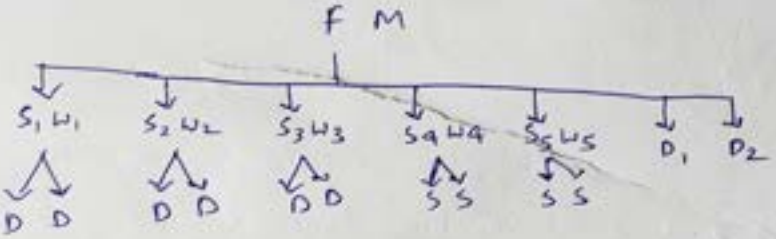
X

d)



✓

10) In a joint family there are father, mother, 5 married sons & 2 unmarried daughters. 3 sons have 2 daughter each & 2 have 2 sons each. How many female in family.



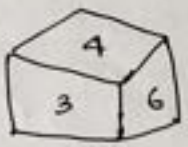
∴ total => 6

41) **Reasoning** => collection of topics from various subject which need logic

- verbal
  - General Mental Ability
    - Blood relation
    - Direction test
    - Seating arrangement ☆☆
    - Clock
    - Calender
    - Ranking
    - Syllogism
    - Coding / Decoding
    - No / Letter series ☆☆
  - Analytical
    - Arguments
- Non verbal
  - mirror image
  - paper image
  - odd one out

42) **Dice questions :-**

(I)



(II)

(III)

Which number is just opposite to 6?
a> 1 b> 4 c> 2 d> 5

Which number is opposite to 5?
a> 3 b> 2 c> 1 d> 6

Method 1 : Left ☆ rotate 2 times (I) dice

So, clearly can see
opposite of 6 is 1
" 3 is 6
∴ opposite of 4 is 2

Key Method :- (Repeat) which are same no write them in clocck wise ↻

acc to 4 ⇒ 4 1 5 (I)
4 6 3 (II)

So, by that also can say
opposite of 5 is 3
" " 1 " 6
" " 4 " 2

Also, can write ~~rotate~~ acc to 6

6 3 4 (ii)
6 5 2 (iii)

⇒ opposite of 4 is 2
" " 3 " 5
" " 6 " 1 (remaining)

Also, can write acc to 5

5 2 6 (iii)
5 4 1 (i)

⇒ opposite of 6 is 1
" " 2 " 4
" " 5 " 3

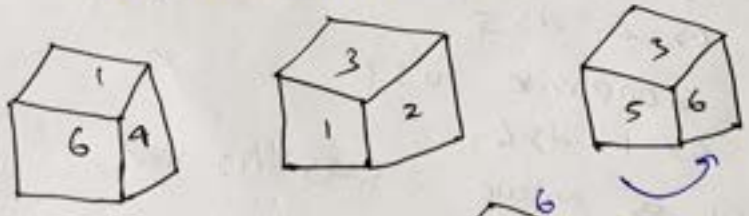
Q) Three view of a dice following a particular motion are given below. Which letter is opposite to A?

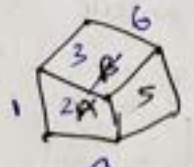
a) H b) P
c) B d) m

⇒ B A K (i)
B H B (iii)

So, opposite of A is H

Q)





1 4 6
1 3 2

6 ↦ 2
4 ↦ 3
1 ↦ 5

So, A = 2
B = 3

9)

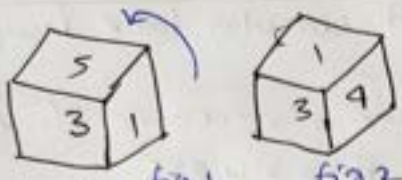


fig 1 fig 2

opposite of 4 ?

we, know that by fig 2

So, opposite of 5 is 4

Rule :- 2 no common then un common no will be opposite of each other about common no can't say,

Here, 1 & 3 common so, 4 & 5 opposite of each other

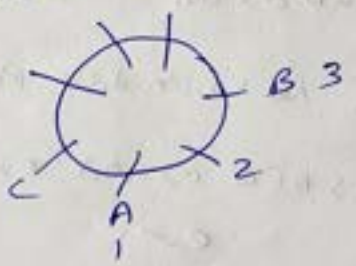
Rule :- one no common Rotate clock wise or Anti clock wise

43) Circular Arrangement :-

Background :-

A is 2nd to left of B & is 3rd to the right of C

→ A is 2nd to left of B who is 3rd to the right of C



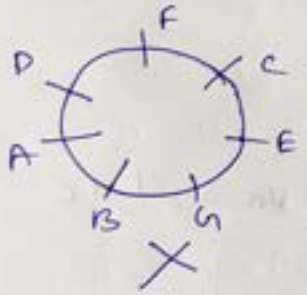
9) ABCD EF

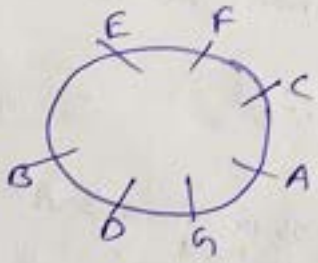
9) A B C D E F & G are sitting around circular table facing at center
G is 2nd to the left of C, who is immediate left of F. A is 3rd to the left of E, B is b/w D & E

⇒

AND ⇒ 1st Person
who ⇒ 2nd Person

⇒ first make slots of element
⇒ if multiple posibilities draw both of them ⇒ dillema draw both
⇒ then apply 3 & cross out by condition
⇒ between means immediate b/w.



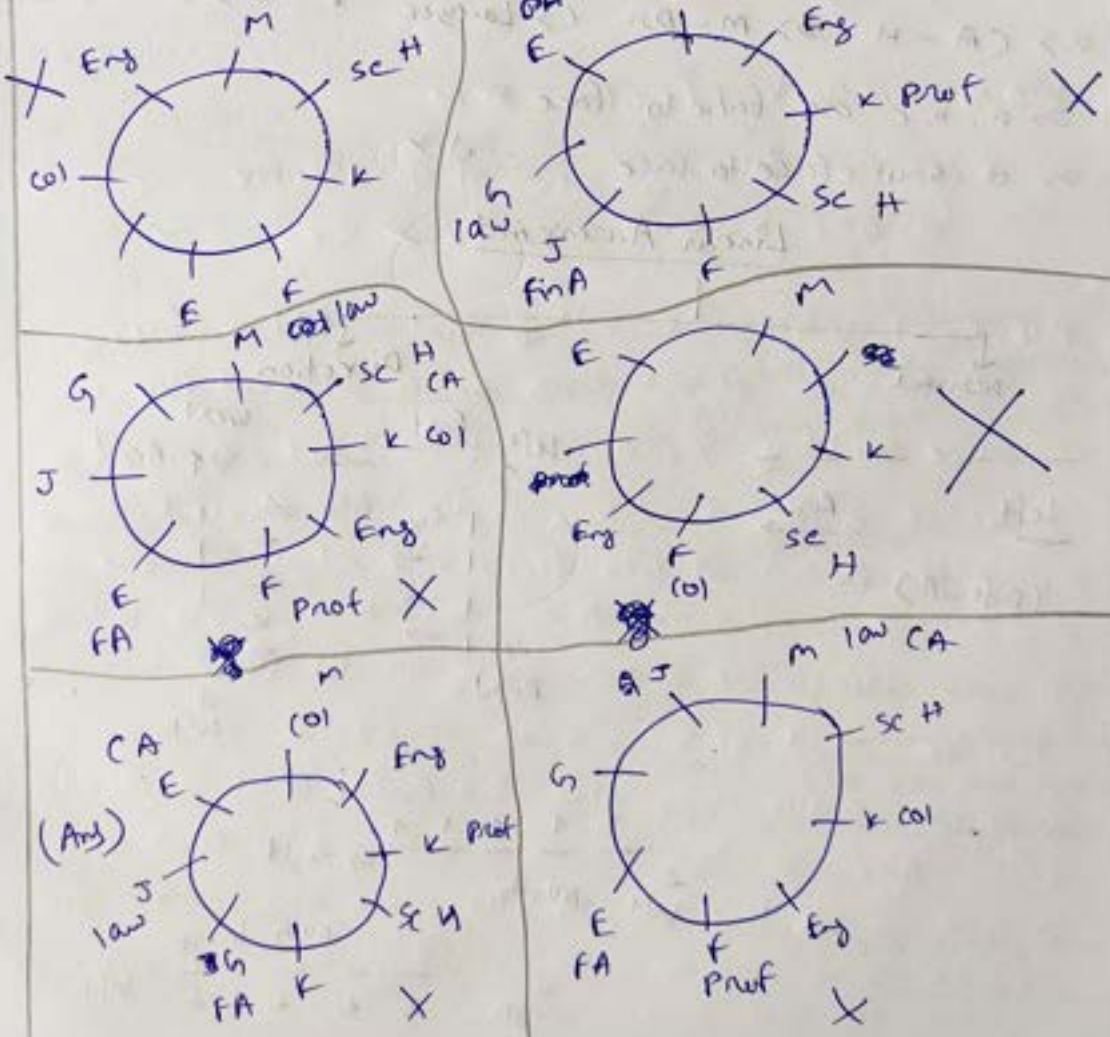


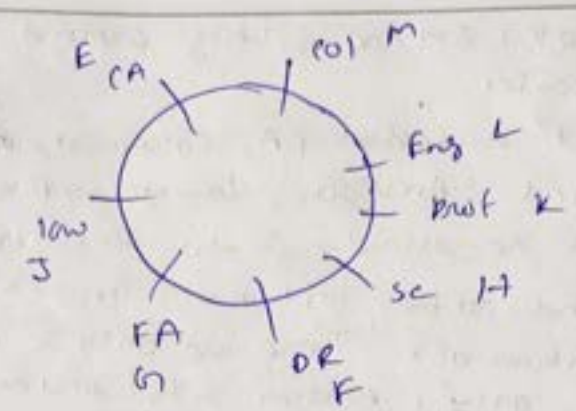
9) Who is 4th to the right of D ⇒ F

9) Which of the following pairs has the 1st person sitting to the immediate left of 2nd person
a) BE b) CA c) GD d) DG ⇒ d ✓

9) Which of the following has the middle person sitting b/w the remaining two
a) FCE b) EFB c) DEB d) GDA e) None
⇒ e ✓

9) 8 persons E F G H J K L & M are sitting around circular table facing the center. Each has different profession (A, Columnist, DR, engineer, financial Advisor, Lawyer, Professor & scientist but not necessarily in the same order. F is sitting second to the left of K. The scientist is immidiate neighbour of K. There are only 3 persons b/w scientist & E. only 1 person sits b/w engineer & E. The columnist is to the immediate right of the engineer. M is second to the right of K. H is the scientist G & J are immediate neighbours of each other neither G nor J is an engineer. The PA is to the immediate left of F. The lawyer is 2nd to the right of columnist. The Professor is an immediate neighbour of engineer. G is 2nd to the right of the CA.





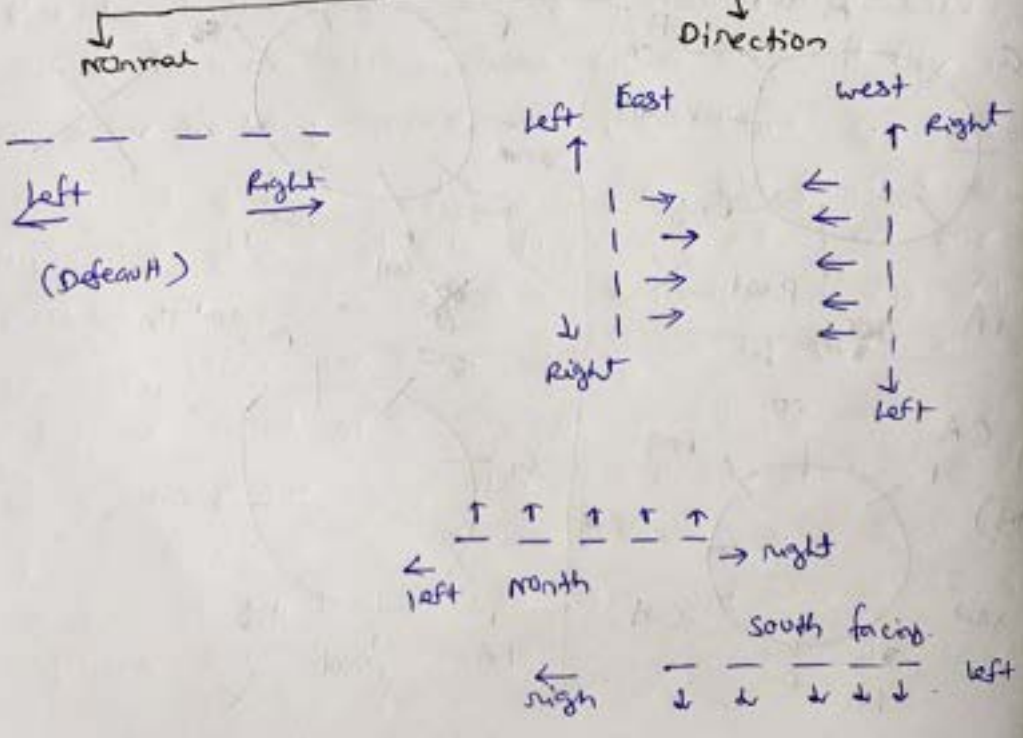
Q) Who is sitting 2nd to the right of E?
⇒ G

Q) Who is Professor?
⇒ K

Q) What is position of L w.r.t Scientist?
⇒ 2nd right

Q) Find odd one out
a) CA - H b) M - Dr c) Lawyer - K d) J - engineer
⇒ a, b, c are face to face
⇒ d is not face to face ∴ d ✗

Linear Arrangement



Q) A B C D E F G are standing in line. G is to the right of D & to the left of B. A is on the right of C. A & D have one person b/w them. E & B have 2 persons b/w them. F is immediate right of B.

Q) Who are on the extrems? ⇒ C & F

Q) Who is exactly in middle? ⇒ D

Q) Who is on extreme left? ⇒ C

Q) Who is on extreme right? ⇒ F

D G B _ _ _ X

_ D G B _ _ X

A C D G B F _ X

C A E D G B F ✓

_ _ A E D G B X

Q) A B C D E F G are sitting on a wall & all of them are facing east. C is on immediate right of D. B is at one extreme end & has E as his neighbour. G is b/w E & F. D is sitting 3rd from south end.

left

B
E
G
F
D
C
A

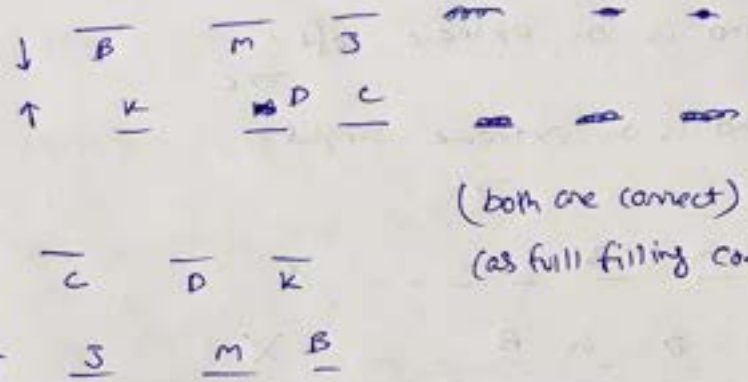
→ Right

Q) Who are at extreme ends
A, B

Q) Who change Place with e such that he get 3rd place from north end?
G

Q) D is sitting b/w
F & C

9) 6 children B, C, D, M, S & K are split into 2 groups of 3 each & are made to stand in 2 rows in such a way that a child in one row is exactly facing a child in another row. M is not at ends of any row S is to the right of J, who is facing C, K is to the left of D, who is facing M.

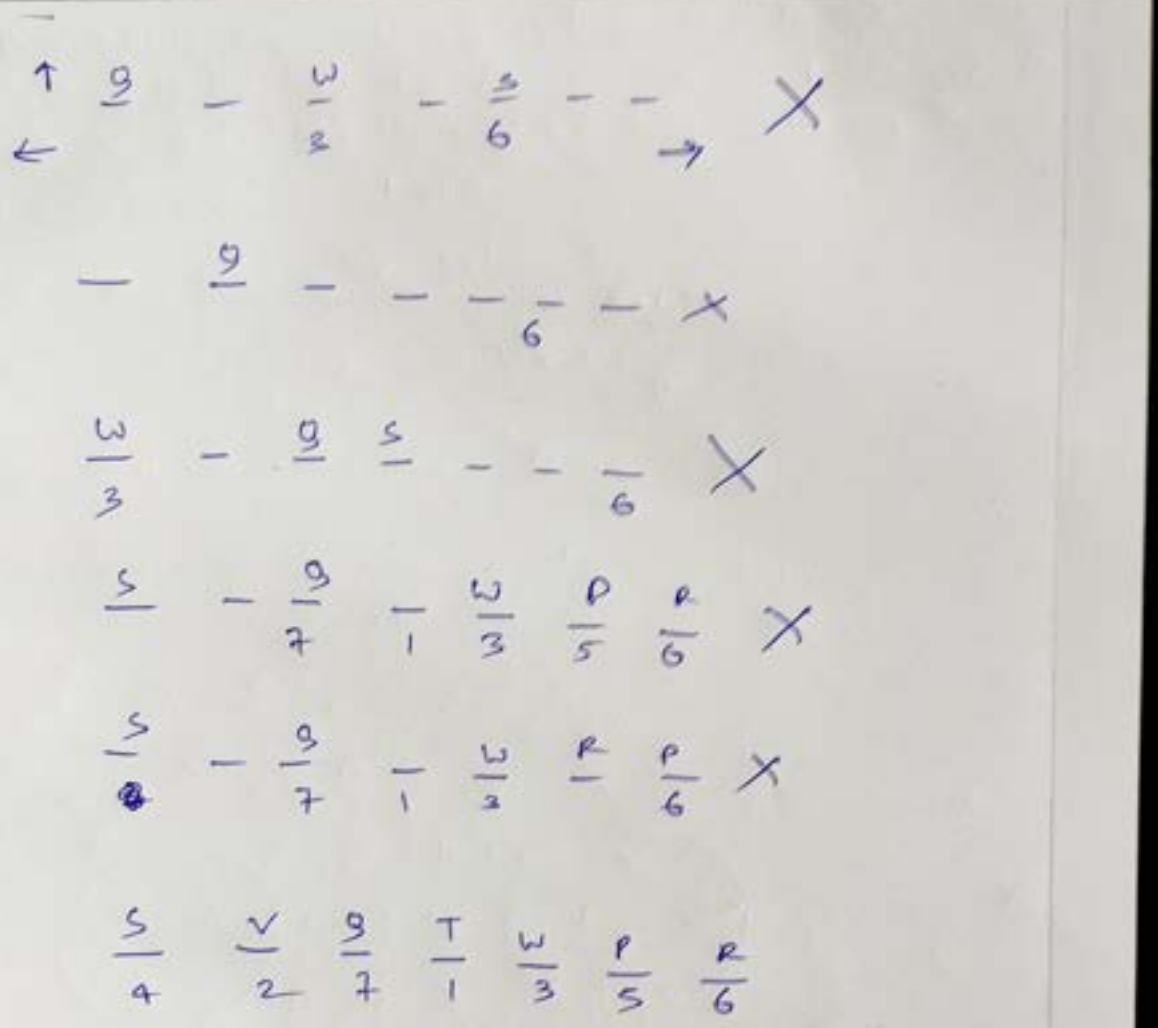


(both are correct)
(as full filling conditions)

9) Who is immediate left of B ~~&~~ M

which of the following groups of Children are in the same row a) BDC b) BMD c) MJK d) none

9) P, Q, R, S T, V & W are sitting in straight line facing North. Each one of them lives on a different floor in a same building numbered from 1 to 7, Q sits 4th to left of 6 person living on the 6th floor. Either Q or the person of 6th floor sits at the extreme end of the only one person sits b/w Q & W. W lives on 3rd floor. The person living on 1st floor sits 3rd to the right of S. S is not immediate neighbour of V. Only one person sits b/w T & person who lives on 2nd floor. P & R are immediate neighbours of each other. P does not ~~live~~ live on 6th floor. One who lives on 5th floor sits 3rd to the right of the one who lives on the 7th floor.



9) How many floors one b/w the floors on which we live

(2)

Q) Odd one out

a) S/4 b) V/2 c) R/6 d) P/5 ✓(d)

Q) Odd one out

a) 4 S - 3 +1 b) 2 V - 1 +1 c) 3 W - 2 +1 d) 6 R - 7 -1