BITS F464 : Machine Learning
Assignment - 1
Fisher's Linear Discriminant

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Introduction

In Fisher's linear discriminant model we maximise a function that will give large separation between the projected class means while also giving minimum variance between each class. We try to analyse the performance of Fisher's linear discriminant on the given dataset by implementing it using Jupyter Notebooks, NumPy, Pandas and Matplotlib.

Approach

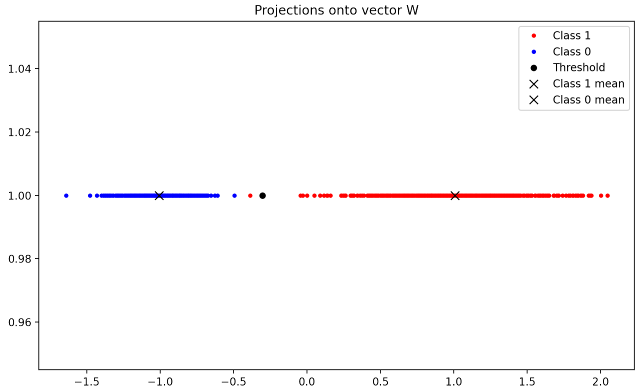
1. First we split the data into 2 classes (positive and negative) according to the target variable.
2. We calculate means of both classes i.e. $M_0 = \frac{1}{N_0} \sum_{n \in \mathcal{C}_0} x_n$ and $M_1 = \frac{1}{N_1} \sum_{n \in \mathcal{C}_1} x_n$, negative and positive respectively.
3. We calculate the total within class covariance matrix S_w which can be obtained as the sum of the covariance matrices of both classes, i.e. $S_w = S_0 + S_1$.
4. The unit vector onto which the points are projected can be found out by $\overrightarrow{w} = {S_w}^{-1}(M_1 - M_0)$ and $\hat{w} = \frac{\overrightarrow{w}}{\|\overrightarrow{w}\|}$.
5. Now we project the data points in the direction of the unit vector. Positive points have red colors and Negative points have blue color as shown in the below figures.
6. With the help of the reduced 1D features as inputs , we plot the Gaussian distributions for both the classes and find the intersection point (α) by solving the following quadratic equation -

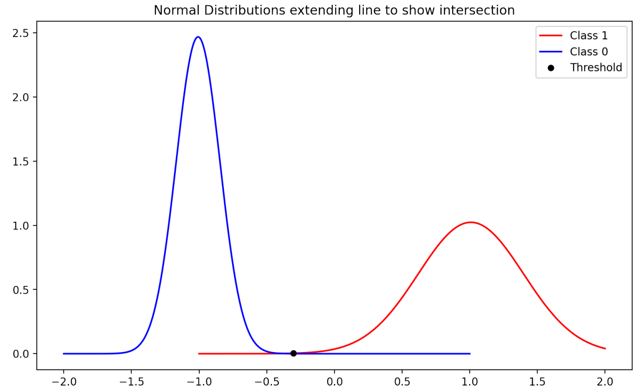
$$\left(\frac{1}{\sigma_1^2} - \frac{1}{\sigma_0^2}\right) x^2 + 2\left(\frac{\mu_1}{\sigma_1^2} - \frac{\mu_0}{\sigma_0^2}\right) x + \frac{\mu_0^2}{\sigma_0^2} - \frac{\mu_1^2}{\sigma_1^2} + \log\left(\frac{\mu_0}{\mu_1}\right)$$

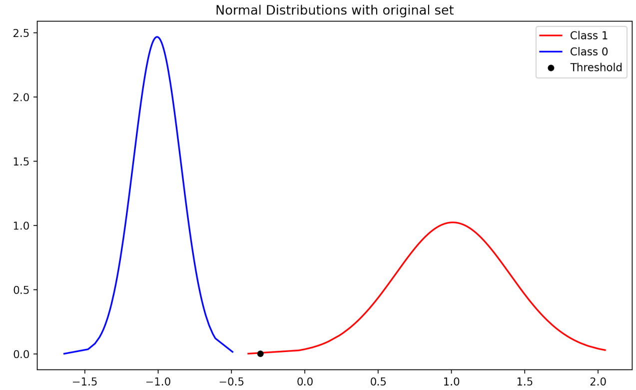
   where μ_0, μ_1 are means of negative and positive classes & σ_0^2, σ_1^2 are variances of negative and positve classes respectively.
7. The point α is the discriminating point (Threshold) in 1D space. The equation for decision boundary is found in the original feature space using $w^T x = \alpha$. Therefore if $w^T x > \alpha$ then x will be classified as positive point, else it will be a negative point.
8. Finally we calculate the accuracy of the Model which was 99.9% on the given data set.

Results

1D space







Original Feature space