Arrays

College of Computer Science, CQU

Outline

- Array ADT
- Matrix
- Symmetric Matrix
- Triangular Matrix
- Symmetric Band Matrix
- Sparse Matrix

    Representation， Transposing

Arrays

- Array:

  a set of pairs (index and value)

- data structure:

  For each index, there is a value associated with that index.

- representation (possible):

  implemented by using consecutive memory.

The Array ADT

- **Objects:** A set of pairs <index, value> where for each value of index there is a value from the set item. Index is a finite ordered set of one or more dimensions, for example, {0, … , n-1} for one dimension, {(0,0),(0,1),(0,2),(1,0),(1,1),(1,2),(2,0),(2,1),(2,2)} for two dimensions, etc.

  Methods:

  for all A ∈ Array, i ∈ index, x ∈ item, j, size ∈ integer

  Array Create(j, list)

  // **return** an array of j dimensions where list is a j-tuple whose kth element is the

  //size of the kth dimension. Items are undefined.

  Item Retrieve(A, i)

  // **if** (i ∈ index) **return** the item associated with index value i in array A

  // **else return** error

  Array Store(A, i, x)

  // **if (**i in index) **return** an array that is identical to array A except the new pair

  //<i, x> has been inserted **else return** error

Matrices

- Two-dimensional arrays are a particularly common representation for matrices.
- A matrix, also referred to as a general matrix, is an m by n ordered collection of numbers. It is represented symbolically as:

$$A = \begin{bmatrix} a_{11} & . \quad . & . \, a_{1n} \\ . & & . \\ . & & . \\ . & & . \\ a_{m1} & . \quad . & . \, a_{mn} \end{bmatrix}$$

- where the matrix is named **A** and has m rows and n columns. And a_{ij} is the element in ith row and jth column of matrix A.

Matrices

- A matrix appears as two-dimensional,but physically it is stored in a linear fashion. How to represent this two-dimensional array？

$$A=\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Matrices

- Common ways to index into multi-dimensional arrays include:
- Row-major order:

  The elements of each row are stored in order.

1	2	3	4	5	6	7	8	9

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

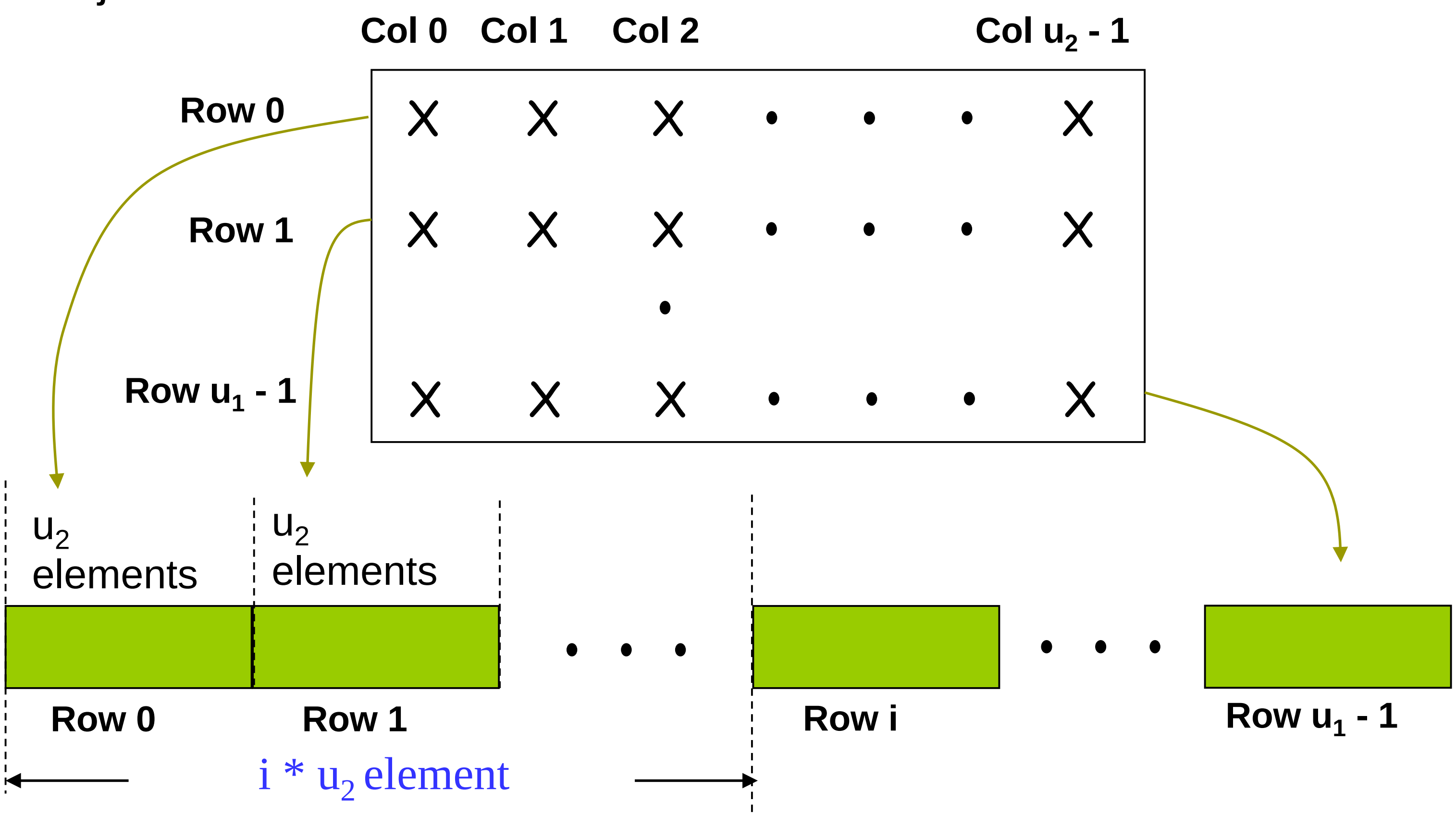
- Column-major order:

  The elements of each column are stored in order.

1	4	7	2	5	8	3	6	9

Matrices

Row-major order:



Matrices

- So,in order to map logical view to physical structure,we need indexing formula.

  - Row-major order:Assume that the base address is at M,the address of a_{ij} will be obtained as

    $Address(a_{ij})=M+((i-1)*n+j-1)*d$

  - Column-major order:Considering the base address at M,the formula will stand as

    $Address(a_{ij})=M+((j-1)*m+i-1)*d$

$$A = \begin{bmatrix} a_{11} & . & . & . & a_{1n} \\ . & & & & . \\ . & & & & . \\ . & & & & . \\ a_{m1} & . & . & . & a_{mn} \end{bmatrix}$$

按行序为主序存放

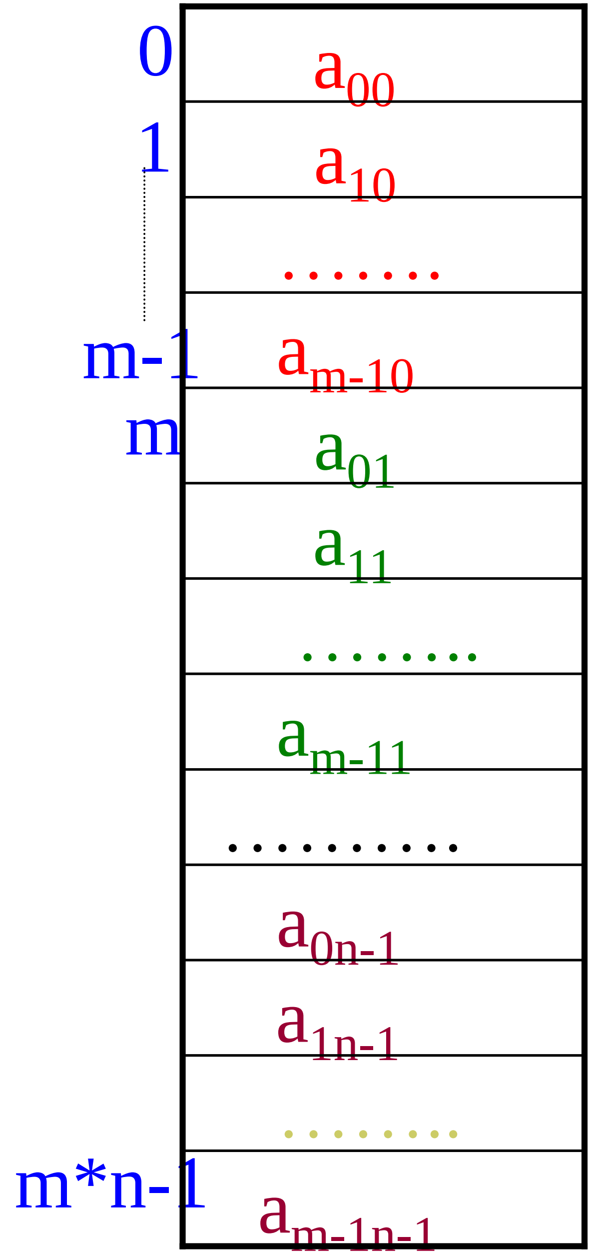
$$
\begin{bmatrix}
a_{00} & a_{01} & \cdots\cdots & a_{0n-1} \\
a_{10} & a_{11} & \cdots\cdots & a_{1n-1} \\
 & \cdots\cdots\cdots\cdots & \cdots\cdots\cdots\cdots & \\
a_{m-10} & a_{m-11} & \cdots & a_{m-1n-1}
\end{bmatrix}
$$

$$LOC(i,j) = LOC(0,0) + (n\times i + j)\times d$$

0	a_{00}
1	a_{01}
	……..
n-1	a_{0n-1}
n	a_{10}
	a_{11}
	……..
	a_{1n-1}
	……….
	a_{m-10}
	a_{m-11}
	……..
m*n-1	a_{m-1n-1}

按列序为主序存放

$$\begin{bmatrix} a_{00} & a_{01} & \cdots\cdots\cdots & a_{0n-1} \\ a_{10} & a_{11} & \cdots\cdots\cdots & a_{1n-1} \\ & \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots & & \\ a_{m-10} & a_{m-11} & \cdots & a_{m-1n-1} \end{bmatrix}$$



LOC(i,j) = LOC(0,0) + (m×j+i)×d

推广到一般情况
n维数组的行序为主序存储地址计算公式

$b_1, b_2, ..., b_n$是n维的维界，数组元素$A(j_1, j_2, ..., j_n)$的存储位置为

$$
\begin{aligned}
LOC[j_1, j_2, ... j_n] = LOC[0,0,...,0] + (&j_1 * b_2 * b_3 * ... * b_n \\
+ &j_2 * b_3 * ... * b_n \\
+ &...... \\
+ &j_{n-1} * b_n \\
+ &j_n \ ) * d
\end{aligned}
$$

$$= LOC[0,0,...,0] + \left(\sum_{i=1}^{n-1} j_i \prod_{k=i+1}^{n} b_k + j_n \right) * d$$

练习

假设有二维数组A $_{6\times8}$，每个元素用相邻的6个字节存储，存储器按字节编址。已知A的起始存储位置（基地址）为1000，计算：

（1）数组A的体积（即存储量）；

（2）数组A的最后一个元素a_{57}的第一个字节的地址；

（3）按行存储时，元素a_{14}的第一个字节的地址；

（4）按列存储时，元素a_{47}的第一个字节的地址。

Symmetric Matrix

- The matrix **A** is symmetric if it has the property **A** equal to $\mathbf{A}^T$, which means:

  - It has the same number of rows as it has columns; that is, it has n rows and n columns.

  - The value of every element a_{ij} on one side of the main diagonal equals its mirror image a_{ji} on the other side: a_{ij} equal to a_{ji}.

  - $\mathbf{A} == \mathbf{A}^T$

  - $a_{ij} == a_{ji}$

Symmetric Matrix

- The following matrix illustrates a symmetric matrix of order n; that is, it has n rows and n columns. The subscripts on each side of the diagonal appear the same to show which elements are equal:

$$A = \begin{bmatrix} a_{11} & a_{21} & a_{31} & . & . & . & a_{n1} \\ a_{21} & a_{22} & a_{32} & & & & . \\ a_{31} & a_{32} & a_{33} & & & & . \\ . & & & . & & & . \\ . & & & & . & & . \\ . & & & & & . & . \\ a_{n1} & . & . & . & . & . & a_{nn} \end{bmatrix}$$

Symmetric Matrix

- When a symmetric matrix is stored in lower-packed storage mode, the lower triangular part of the symmetric matrix is stored, including the diagonal, in a one-dimensional array.
- The lower triangle can be packed by row or columns. The matrix is packed sequentially row by row (column by column) in n(n+1)/2 elements of a one-dimensional array.
- When the matrix is packed sequentially row by row ,to calculate the location k of element a_{ij} of matrix **A** in an array, use the following formula:

$$k=i*(i-1)/2+j-1 \qquad i>=j, \text{ lower triangular part}$$

$$k=j*(j-1)/2+i-1 \qquad i<j, \text{ upper triangular part}$$

$$
\begin{bmatrix}
a_{11} & a_{12} & \cdots & \cdots\ \cdots & a_{1n} \\
a_{21} & a_{22} & \cdots & \cdots & a_{2n} \\
 & & \cdots\cdots\cdots\cdots & & \\
a_{n1} & a_{n2} & \cdots & & a_{nn}
\end{bmatrix}
$$

按行序为主序：

a11	a21	a22	a31	a32	……	an1	……	ann
k=0	1	2	3	4		n(n-1)/2		n(n+1)/2-1

$$
k = \begin{cases} i(i-1)/2 + j - 1, & i \geq j \\ j(j-1)/2 + i - 1, & i < j \end{cases}
$$

Triangular Matrix

A matrix of the form

$$
\begin{bmatrix}
a_{0.0} & & & & & \\
a_{1.0} & a_{1.1} & & & & \\
a_{2.0} & a_{2.1} & a_{2.2} & & & \\
\vdots & & & & & \\
a_{n-1.0} & a_{n-1.1} & & \cdots & a_{n-1.n-1} & \\
a_{n.0} & a_{n.1} & a_{n.2} & \cdots & a_{n.n-1} & a_{n.n}
\end{bmatrix}
$$

is called a **triangular matrix**.

Triangular Matrix

- There are two types of triangular matrices: upper triangular matrix and lower triangular matrix. Triangular matrices have the same number of rows as they have columns; that is, they have n rows and n columns.
- A matrix **U** is an **upper triangular matrix** if its nonzero elements are found only in the upper triangle of the matrix, including the main diagonal; that is: u_{ij} equal to 0（or constant C） if i greater than j
- A matrix **L** is an **lower triangular matrix** if its nonzero elements are found only in the lower triangle of the matrix, including the main diagonal; that is: l_{ij} equal to 0 （or constant C） if i less than j

Triangular Matrix

- The following matrices, **U** and **L**, illustrate upper and lower triangular matrices of order n, respectively:

$$U = \begin{bmatrix} u_{11} & u_{12} & u_{13} & . & . & . & u_{1n} \\ 0 & u_{22} & u_{23} & & & & . \\ 0 & 0 & u_{33} & & & & . \\ . & & & . & & & . \\ . & & & & . & & . \\ . & & & & & . & . \\ 0 & . & . & . & . & 0 & u_{nn} \end{bmatrix} \qquad L = \begin{bmatrix} l_{11} & 0 & 0 & . & . & . & 0 \\ l_{21} & l_{22} & 0 & & & & . \\ l_{31} & l_{32} & l_{33} & & & & . \\ . & & & . & & & . \\ . & & & & . & & . \\ . & & & & & . & 0 \\ l_{n1} & . & . & . & . & . & l_{nn} \end{bmatrix}$$

Triangular Matrix

- When a lower-triangular matrix is stored in lower-triangular-packed storage mode, the lower triangle of the matrix is stored, including the diagonal, in a one-dimensional array. The lower triangle is packed by row or by columns. The elements are packed sequentially, row by row (column by column), in $n(n+1)/2$ elements of a one-dimensional array. To calculate the location of each element of the triangular matrix in the array, use the technique described in Symmetric Matrix.
- When an upper-triangular matrix is stored in upper-triangular-packed storage mode, the upper triangle of the matrix is stored, including the diagonal, in a one-dimensional array.

Symmetric Band Matrix

- A symmetric band matrix is a symmetric matrix whose nonzero elements are arranged uniformly near the diagonal, such that: a_{ij} equal to 0   if |i-j| greater than k，where k is the half band width.

Symmetric Band Matrix

- The following matrix illustrates a symmetric band matrix of order n, where the half band width k equal to q-1:

$$
\left| \leftarrow \quad k \quad \rightarrow \right|
$$

$$
A = \begin{bmatrix}
a_{11} & a_{21} & a_{31} & \cdot & \cdot & a_{q1} & 0 & \cdot & \cdot & 0 \\
a_{21} & a_{22} & a_{32} & & & & & 0 & & \cdot \\
a_{31} & a_{32} & a_{33} & & & & & & 0 & \cdot \\
\cdot & & & \cdot & & & & & & 0 \\
\cdot & & & & \cdot & & & & & \cdot \\
a_{q1} & & & & & \cdot & & & & \cdot \\
0 & & & & & & \cdot & & & \cdot \\
\cdot & 0 & & & & & & \cdot & & \cdot \\
\cdot & & 0 & & & & & & \cdot & \\
0 & \cdot & \cdot & 0 & \cdot & \cdot & \cdot & & & a_{nn}
\end{bmatrix}
$$

- Only the band elements of the symmetric band matrix are stored.

★对角矩阵

$$\begin{bmatrix} a_{11} & a_{12} & 0 & \cdots\cdots\cdots\cdots\cdots\cdots. & & 0 \\ a_{21} & a_{22} & a_{23} & 0 \cdots\cdots\cdots\cdots\cdots\cdots & & 0 \\ 0 & a_{32} & a_{33} & a_{34} & 0 \cdots\cdots\cdots & 0 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots & & & & & \\ 0 & 0 \ldots & a_{n-1,n-2} & & a_{n-1,n-1} & a_{n-1,n} \\ 0 & 0 & \ldots & \ldots a_{n,n-1} & & a_{nn} \end{bmatrix}$$

按行序为主序：

a11	a12	a21	a22	a23	…… ……	ann-1	ann
k=0	1	2	3	4			3*n-3

$$Loc(a_{ij})=Loc(a_{11})+[3(i-1)+(j-i)]*d$$

Sparse Matrix

A sparse matrix is a matrix having a relatively small number of nonzero elements.

$$\begin{array}{cc} & \begin{array}{ccc} \text{col 1} & \text{col 2} & \text{col 3} \end{array} \\ \begin{array}{c} \text{row 1} \\ \text{row 2} \\ \text{row 3} \\ \text{row 4} \\ \text{row 5} \end{array} & \begin{bmatrix} -27 & 3 & 4 \\ 6 & 82 & -2 \\ 109 & -64 & 11 \\ 12 & 8 & 9 \\ 48 & 27 & 47 \end{bmatrix} \end{array}$$

15/15

$$\begin{array}{cc} & \begin{array}{cccccc} \text{col1} & \text{col2} & \text{col3} & \text{col4} & \text{col5} & \text{col6} \end{array} \\ \begin{array}{c} \text{row0} \\ \text{row1} \\ \text{row2} \\ \text{row3} \\ \text{row4} \\ \text{row5} \end{array} & \begin{bmatrix} 15 & 0 & 0 & 22 & 0 & -15 \\ 0 & 11 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & -6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 91 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 28 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

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sparse matrix

data structure?

稀疏矩阵

假设 m 行 n 列的矩阵含 t 个非零元素，则称

$$\delta = \frac{t}{m \times n}$$

为稀疏因子。

通常认为 $\delta \le 0.05$ 的矩阵为稀疏矩阵。

Sparse Matrix Representation

- The standard representation of a matrix is a two dimensional array defined as

  $a[MAX\_ROWS][MAX\_COLS]$

  - We can locate quickly any element by writing $a[i][j]$
- Sparse matrix wastes space
  - We must consider alternate forms of representation.
  - Our representation of sparse matrices should store only nonzero elements.
  - Each element is characterized by <row, col, value>.

Sparse Matrix Representation

- Figure shows how the sparse matrix is represented in the array *a*.
  - Represented by a two-dimensional array.
  - Each element is characterized by <row, col, value>.

$$
\begin{array}{c|cccccc|}
 & \text{col1} & \text{col2} & \text{col3} & \text{col4} & \text{col5} & \text{col6} \\
\text{row0} & 15 & 0 & 0 & 22 & 0 & -15 \\
\text{row1} & 0 & 11 & 3 & 0 & 0 & 0 \\
\text{row2} & 0 & 0 & 0 & -6 & 0 & 0 \\
\text{row3} & 0 & 0 & 0 & 0 & 0 & 0 \\
\text{row4} & 91 & 0 & 0 & 0 & 0 & 0 \\
\text{row5} & 0 & 0 & 28 & 0 & 0 & 0
\end{array}
$$

row, column in ascending order

nonzero terms

rows columns

transpose

	row	col	value
a[0]	6	6	8
[1]	0	0	15
[2]	0	3	22
[3]	0	5	−15
[4]	1	1	11
[5]	1	2	3
[6]	2	3	−6
[7]	4	0	91
[8]	5	2	28

(a)

	row	col	value
b[0]	6	6	8
[1]	0	0	15
[2]	0	4	91
[3]	1	1	11
[4]	2	1	3
[5]	2	5	28
[6]	3	0	22
[7]	3	2	−6
[8]	5	0	−15

(b)

Transposing A Matrix

- Transpose a Matrix
  - For each row i
    - take element <i, j, value> and store it in element <j, i, value> of the transpose.
    - difficulty: where to put <j, i, value>
      (0, 0, 15) ====> (0, 0, 15)
      (0, 3, 22) ====> (3, 0, 22)
      (0, 5, -15) ====> (5, 0, -15)
      (1, 1, 11) ====> (1, 1, 11)
      Move elements down very often.
  - For all elements in column j,
    - place element <i, j, value> in element <j, i, value>

Transposing A Matrix

Assign
A[i][j] to B[j][i]

place element <i, j, value>
in element <j, i, value>

For all columns i

For all elements in column j

Scan the array “columns” times.The array has “elements” elements.

```
void transpose(term a[], term b[])
/* b is set to the transpose of a */
{
   int n,i,j, currentb;
   n = a[0].value;        /* total number of elements */
   b[0].row = a[0].col; /* rows in b = columns in a */
   b[0].col = a[0].row; /* columns in b = rows in a */
   b[0].value = n;
   if (n > 0 )  { /* non zero matrix */
     currentb = 1;
     for (i = 0; i < a[0].col; i++)
     /* transpose by the columns in a */
       for (j = 1; j <= n; j++)
       /* find elements from the current column */
         if (a[j].col == i) {
         /* element is in current column, add it to b */
           b[currentb].row = a[j].col;
           b[currentb].col = a[j].row;
           b[currentb].value = a[j].value;
           currentb++;
         }
   }
}
```

==> O(columns*elements)

EX: A[6][6] transpose to B[6][6]

i=1 j=8
a[i].col = 2 != i

Matrix A

	Row	Col	Value
a[0]	6	6	8
[1]	0	0	15
[2]	0	3	22
[3]	0	5	−15
[4]	1	1	11
[5]	1	2	3
[6]	2	3	−6
[7]	4	0	91
[8]	5	2	28

	Row	Col	Value
0	6	6	8
1	0	0	15
2	0	4	91
3	1	1	11

```
void transpose(term a[], term b[])
/* b is set to the transpose of a */
{
  int n,i,j, currentb;
  n = a[0].value;          /* total number of elements */
  b[0].row = a[0].col; /* rows in b = columns in a */
  b[0].col = a[0].row; /* columns in b = rows in a */
  b[0].value = n;
  if (n > 0 )  { /* non zero matrix */
    currentb = 1;
    for (i = 0; i < a[0].col; i++)
    /* transpose by the columns in a */
      for (j = 1; j <= n; j++)
      /* find elements from the current column */
        if (a[j].col == i) {
        /* element is in current column, add it to b */
          b[currentb].row = a[j].col;
          b[currentb].col = a[j].row;
          b[currentb].value = a[j].value;
          currentb++;
        }
  }
}
```

Set Up row & column in B[6][6]

And So on...

Transposing A Matrix

- Discussion: compared with 2-D array representation
  - O(columns*elements) vs. O(columns*rows)
  - elements --> columns * rows when non-sparse,
    $O(\text{columns}^2 * \text{rows})$
- Problem: Scan the array "columns" times.
  - In fact, we can transpose a matrix represented as a sequence of triples in O(columns + elements) time.
- Solution:
  - First, determine the number of elements in each column of the original matrix.
  - Second, determine the starting positions of each row in the transpose matrix.

col	1	2	3	4	5	6	7
Row-terms[col]	**2**	**2**	**2**	**1**	**0**	**1**	**0**
Starting-pos[col]	**1**	**3**	**5**	**7**	**8**	**8**	**9**
	2	4	6			9	
	3	5	7				

	i	j	e
0	6	7	8
p→1	1	2	12
p→2	1	3	9
p→3	3	1	-3
p→4	3	6	14
p→5	4	3	24
p→6	5	2	18
p→7	6	1	15
p→8	6	4	-7

M.data

	i	j	e
0	7	6	8
1	1	3	-3
2	1	6	15
3	2	1	12
4	2	5	18
5	3	1	9
6	3	4	24
7	4	6	-7
8	6	3	14

T.data

Fast Matrix Transposing

Cost:
Additional row_terms and starting_pos arrays are required.
Let the two arrays row_terms and starting_pos be shared.

Buildup row_term & starting_pos

For columns

For elements

For columns

transpose

For elements

```
void fast_transpose(term a[], term b[])
{
/* the transpose of a is placed in b */
   int row_terms[MAX_COL], starting_pos[MAX_COL];
   int i,j, num_cols = a[0].col, num_terms = a[0].value;
   b[0].row = num_cols;  b[0].col = a[0].row;
   b[0].value = num_terms;
   if (num_terms > 0) { /* nonzero matrix */
      for (i = 0; i < num_cols; i++)
         row_terms[i] = 0;
      for (i = 1; i <= num_terms; i++)
         row_terms[a[i].col]++;
      starting_pos[0] = 1;
      for (i = 1; i < num_cols; i++)
         starting_pos[i] =
                  starting_pos[i-1] + row_terms[i-1];
      for (i = 1; i <= num_terms; i++) {
         j = starting_pos[a[i].col]++;
         b[j].row = a[i].col;   b[j].col = a[i].row;
         b[j].value = a[i].value;
      }
   }
}
```

Fast Matrix Transposing

- After the execution of the third **for** loop, the values of *row_terms* and *starting_pos* are:

	[0]	[1]	[2]	[3]	[4]	[5]
row_terms =	2	1	2	2	0	1
starting_pos =	1	3	4	6	8	8

	row	col	value
a[0]	6	6	8
[1]	0	0	15
[2]	0	3	22
[3]	0	5	–15
[4]	1	1	11
[5]	1	2	3
[6]	2	3	–6
[7]	4	0	91
[8]	5	2	28

(a)

transpose $\longrightarrow$

	row	col	value
b[0]	6	6	8
[1]	0	0	15
[2]	0	4	91
[3]	1	1	11
[4]	2	1	3
[5]	2	5	28
[6]	3	0	22
[7]	3	2	–6
[8]	5	0	–15

(b)

	[0]	[1]	[2]	[3]	[4]	[5]
row_terms	2	1	2	2	0	1
starting_pos	1	3	4	6	8	8

#col = 6
#term = 6

Matrix A

	Row	Col	Value
a[0]	6	6	8
[1]	0	0	15
[2]	0	3	22
[3]	0	5	−15
[4]	1	1	11
[5]	1	2	3
[6]	2	3	−6
[7]	4	0	91
[8]	5	2	28

```
void fast_transpose(term a[], term b[])
{
/* the transpose of a is placed in b */
   int row_terms[MAX_COL], starting_pos[MAX_COL];
   int i,j, num_cols = a[0].col, num_terms = a[0].value;
   b[0].row = num_cols;  b[0].col = a[0].row;
   b[0].value = num_terms;
   if (num_terms > 0) { /* nonzero matrix */
      for (i = 0; i < num_cols; i++)
         row_terms[i] = 0;
      for (i = 1; i <= num_terms; i++)
         row_terms[a[i].col]++;
      starting_pos[0] = 1;
      for (i = 1; i < num_cols; i++)
         starting_pos[i] =
                     starting_pos[i-1] + row_terms[i-1];
      for (i = 1; i <= num_terms; i++) {
         j = starting_pos[a[i].col]++;
         b[j].row = a[i].col;   b[j].col = a[i].row;
         b[j].value = a[i].value;
      }
   }
}
```

I = 8

	[0]	[1]	[2]	[3]	[4]	[5]
row_terms =	2	1	2	2	0	1
starting_pos =	3	4	6	8	8	9

Matrix A

	Row	Col	Value
a[0]	6	6	8
[1]	0	0	15
[2]	0	3	22
[3]	0	5	−15
[4]	1	1	11
[5]	1	2	3
[6]	2	3	−6
[7]	4	0	91
[8]	5	2	28

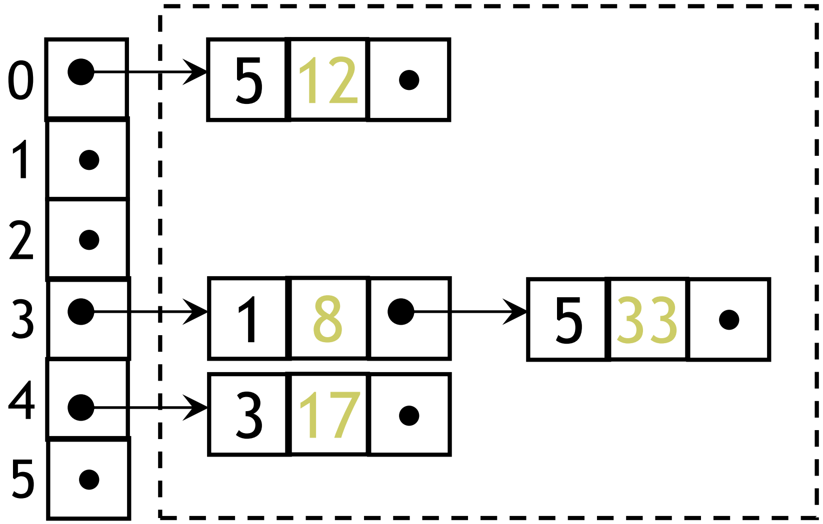
	Row	Col	Value
0	6	6	8
1	0	0	15
2	0	4	91
3	1	1	11
4	2	1	3
5	2	5	28
6	3	0	22
7	3	2	-6
8	5	0	-15

```
void fast_transpose(term a[], term b[])
{
/* the transpose of a is placed in b */
   int row_terms[MAX_COL], starting_pos[MAX_COL];
   int i,j, num_cols = a[0].col, num_terms = a[0].value;
   b[0].row = num_cols;  b[0].col = a[0].row;
   b[0].value = num_terms;
   if (num_terms > 0) { /* nonzero matrix */
      for (i = 0; i < num_cols; i++)
         row_terms[i] = 0;
      for (i = 1; i <= num_terms; i++)
         row_terms[a[i].col]++;
      starting_pos[0] = 1;
      for (i = 1; i < num_cols; i++)
         starting_pos[i] =
                      starting_pos[i-1] + row_terms[i-1];
      for (i = 1; i <= num_terms; i++) {
         j = starting_pos[a[i].col]++;
         b[j].row = a[i].col;   b[j].col = a[i].row;
         b[j].value = a[i].value;
      }
   }
}
```

Represented as an *array* of linked lists

- Here is an example of a sparse two-dimensional array, and how it can be represented as an *array* of linked lists:

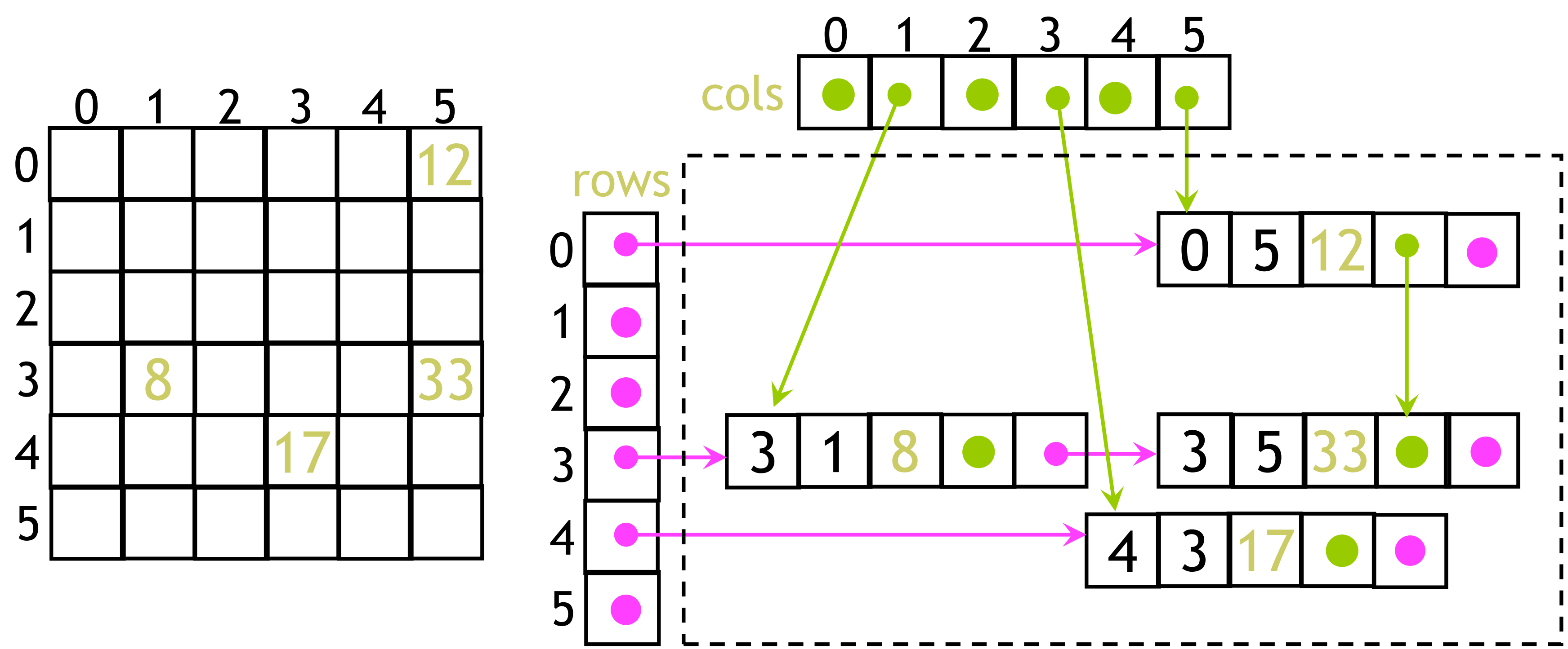
	0	1	2	3	4	5
0						12
1						
2						
3		8				33
4				17		
5						



- With this representation,
  - It is efficient to step through all the elements of a *row*
  - It is expensive to step through all the elements of a *column*
  - Clearly, we could link columns instead of rows
  - Why not both?

Another implementation

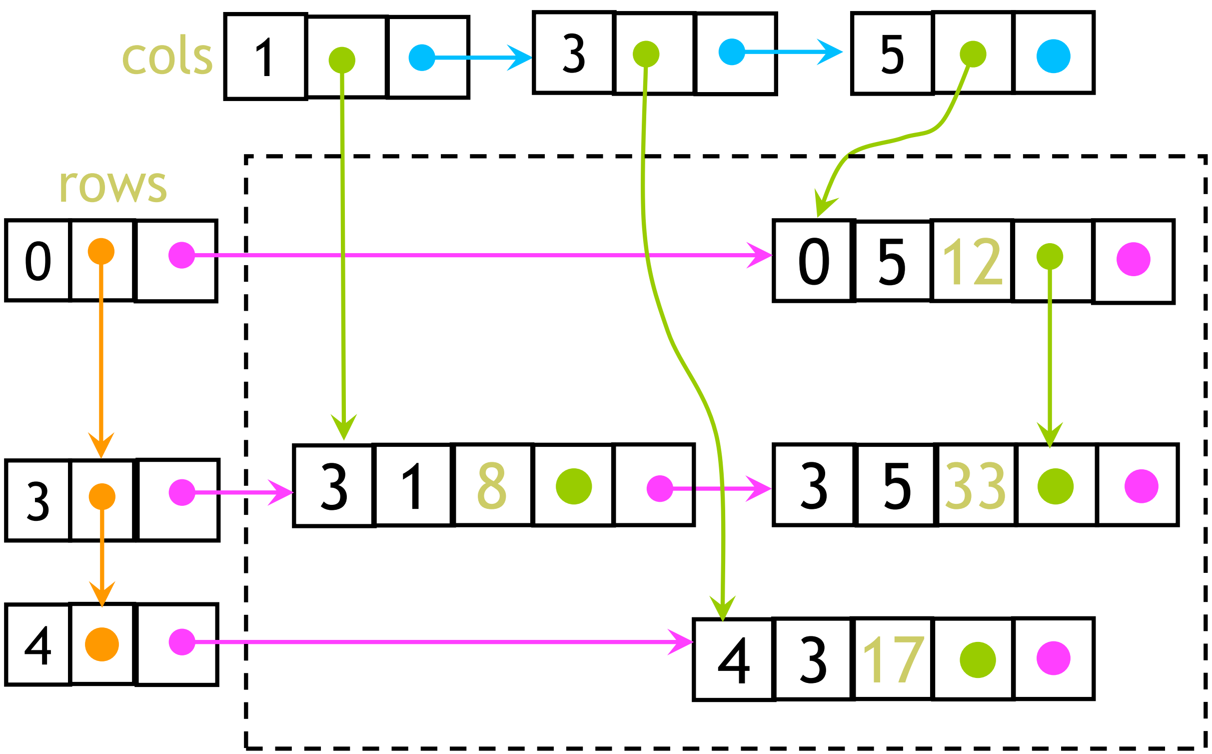
- If we want efficient access to both rows and columns, we need another array and additional data in each node



- Do we really need the row and column number in each node?

Yet another implementation

- Instead of arrays of pointers to rows and columns, you can use linked lists:



	0	1	2	3	4	5
0						12
1						
2						
3		8				33
4				17		
5						

- Would this be a good data structure?
- This may be the best implementation if most rows and most columns are totally empty

Reference

- 《数据结构（ C 语言版）》，严蔚敏，吴伟民编著，清华大学出版社，1997年第1版,P91-99