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LAMINAR BOUNDARY LAYERS

9.1 INTRODUCTION

The boundary layer of a flowing fluid is the thin layer close to the wall. In a flow field, viscous stresses are very prominent within this layer. Although the layer is thin, it is very important to know the details of flow within it. The main-flow velocity within this layer tends to zero while approaching the wall. Also the gradient of this velocity component in a direction normal to the surface is large as compared to the gradient of this component in the streamwise direction.

9.2 BOUNDARY LAYER EQUATIONS

In 1904, Ludwig Prandtl, the well-known German scientist, introduced the concept of boundary layer [1] and derived the equations for boundary layer flow by correct reduction of the Navier–Stokes equations. He hypothesised that for fluids having a relatively small viscosity, the effect of internal fictitious in the fluid is significant only in a narrow region surrounding the solid boundaries or bodies over which the fluid flows. Thus, close to the body is the boundary layer where shear stresses exert an increasingly larger effect on the fluid as one moves from free stream towards the solid boundary. However, outside the boundary layer where the effect of the shear stresses on the flow is small compared to values inside the boundary layer (since the velocity gradient $\partial u/\partial y$ is negligible), the fluid particles experience no vorticity, and therefore, the flow is similar to a potential flow. Hence, the *surface* at the boundary layer interface is a rather fictitious one dividing rotational and irrotational flow. Prandtl's model regarding the boundary layer flow is shown in Fig. 9.1. Hence with the exception of the immediate vicinity of the surface, the flow is frictionless (inviscid) and the velocity is U. In the region very near to the surface (in the thin layer), there is friction in the flow which signifies that the fluid is retarded until it adheres to the surface. The transition of the mainstream velocity from zero at the surface to full magnitude takes place across the boundary layer. Its thickness is δ which is a function of the coordinate direction x. The thickness is considered to be very small compared to the characteristic length L of the domain. In the normal direction, within the thin layer, the gradient $\partial u/\partial y$ is very large compared to the gradient in the flow direction $\partial u/\partial x$. The next step is to simplify the Navier–Stokes equations for steady two-