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Parameter Estimation Assignment
[102103035-Aniket]

(Q1) Let $(x_1, x_2 \ldots)$ be a random sample of size n taken from a Normal Population with parameters: mean = θ1 and variance = θ2. Find the Maximum Likelihood Estimates of these two parameters.

→ $L(\theta_1, \theta_2 \mid x_1, x_2, x_3 \ldots x_n) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x_i-\theta_1)^2}{2\theta_2}}$

→ Taking Log likelihood:

$\ln L(\theta_1, \theta_2 \mid x_1, x_2, x_3 \ldots x_n) = -\frac{n}{2}\ln(2\pi\theta_2) - \frac{1}{2\theta_2}\sum_{i=1}^{n}(x_i - \theta_1)^2$

MLE for θ1:

$\frac{\partial}{\partial\theta_1} \ln L(\theta_1, \theta_2 \mid x_1 \ldots x_n) = \frac{1}{\theta_2}\sum_{i=1}^{n}(x_i - \theta_1) = 0$

$\Rightarrow \sum_{i=1}^{n}(x_i) - n\theta_1 = 0$

$\Rightarrow \theta_1 = \frac{1}{n}\sum_{i=1}^{n} x_i \rightarrow \bar{x}$

MLE for θ2:

$\frac{\partial}{\partial\theta_2} \ln(\theta_1, \theta_2 \mid x_1 \ldots x_n) = \frac{-n}{2\theta_2} + \frac{1}{2\theta_2^2}\sum_{i=1}^{n}(x_i - \theta_1)^2 = 0$

$\Rightarrow -n + \frac{1}{\theta_2}\sum_{i=1}^{n}(x_i - \theta_1)^2 = 0$

$\Rightarrow \theta_2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \theta_1)^2 = s^2$

Ans: i MLE for θ1 : $\bar{x}$ (mean)
MLE for θ2 : s^2 (variance)

(Q3) Let $X_1, X_2 \ldots X_n$ be random sample from $B(m, \theta)$ distribution, where $\theta \in \Theta = (0, 1)$ is unknown and 'm' is a known +ve integer. Compute value of θ using M.L.E

→ $L(\theta \mid x_1, x_2 \ldots x_n) = \sum_{i=1}^{n} \left[\ln\binom{m}{x_i} + x_i \ln(\theta) + (m - x_i)\ln(1-\theta) \right]$

→ MLE for θ:

$$\frac{\partial}{\partial\theta} \ln L(\theta \mid x_1, x_2 \ldots x_n) = \sum_{i=1}^{n} \left[\frac{x_i}{\theta} - \frac{m - x_i}{1-\theta} \right] = 0$$

$$\Rightarrow \sum_{i=1}^{n} \left[\frac{x(1-\theta) - \theta(m - x_i)}{\theta(1-\theta)} \right] = 0$$

$$\Rightarrow \hat{\theta} = \left(\sum_{i=1}^{n} x_i\right) \Big/ \left(mn + \sum_{i=1}^{n} x_i\right)$$

let $\sum_{i=1}^{n} x_i = s$ (success cases)

$$\therefore \hat{\theta} = \frac{s}{nm + s}$$