T2. e) Расчёт плотности распределения медианы выборки:

$$q_2 = \begin{cases} n=2k+1: & x_{(k+1)} \\ n=2k: & \dfrac{x_{(k+1)}+x_{(k)}}{2} \end{cases}$$

$$n=25 \Rightarrow q_2 = x_{(k+1)}$$

$$x_{(k+1)} \sim \sum_{i=k+1}^{n} C_n^i \cdot (F(x))^i \cdot (1-F(x))^{n-i}$$

$F(x) \equiv F$

$$f_{med} = \left(\sum_{i=k+1}^{n} C_n^i \cdot (F(x))^i \cdot (1-F(x))^{n-i} \right)' =$$

$f(x) = \begin{cases} e^{-x}, & x \geq 0 \\ 0, & x<0 \end{cases}$

$$= \sum_{i=k+1}^{n} C_n^i \cdot \left(i \cdot F^{i-1} \cdot \underbrace{F'}_{=f} \cdot (1-F)^{n-i} + F^i \cdot (1-F)^{n-i-1} \cdot (n-i) \cdot (-\underbrace{F'}_{f(x)\equiv f}) \right) = \sum_{i=k+1}^{n} \Big(C_n^i \cdot i \cdot f \cdot F^{i-1} \cdot (1-F)^{n-i} -$$

$$- C_n^i \cdot (n-i) \cdot f \cdot F^i (1-F)^{n-i-1} \Big) = f \cdot \left[\sum_{i=k+1}^{n} \left(\frac{n! \cdot i}{i! \cdot (n-i)!} \cdot F^{i-1} \cdot \right.\right.$$

$$\left.\left. \cdot (1-F)^{n-i} - \frac{n!}{i! \cdot (n-i)!} \cdot (n-i) \cdot F^i (1-F)^{n-i-1} \right) \right] =$$

$$= f \cdot \left[\sum_{i=k+1}^{n} \left(\frac{n!}{(i-1)!(n-i)!} \cdot F^{i-1} \cdot (1-F)^{n-i} - \right.\right.$$

$$- \frac{n!}{i!\,(n-i-1)!} \cdot F^{i} \cdot (1-F)^{n-i-1} \Big) = f \cdot \Big(\frac{n!}{k!\,(n-k-1)!} \cdot$$

$$\cdot F^{k} (1-F)^{n-k-1} - \cancel{\frac{n!}{(k+1)!\,(n-k-2)!} \cdot F^{k+1} \cdot (1-F)^{n-k-2}} +$$

$$+ \cancel{\frac{n!}{(k+1)!\,(n-k-2)!} \cdot F^{k+1} \cdot (1-F)^{n-k-2}} -$$

$$- \cancel{\frac{n!}{(k+2)!\,(n-k-3)!} \cdot F^{k+2} \cdot (1-F)^{n-k-3}} +$$

$$+ \cancel{\frac{n!}{(k+2)!\,(n-k-3)!} \cdot F^{k+2} \cdot (1-F)^{n-k-3}} -$$

$$- \dots - \frac{n!}{n!\,(n-n)!} \cdot (n-n) \cdot F^{n} \cdot (1-F)^{-1} \Big) =$$

$$= f \cdot \left(\frac{n!}{k!\,(n-k-1)!} \cdot F^{k} \cdot (1-F)^{n-k-1} - 0 \right) =$$

$$= f(x) \cdot \frac{n!}{k!\,(n-k-1)!} \cdot F(x)^{k} \cdot (1-F(x))^{n-k-1} =$$

$$= \left\{ \begin{array}{l} n = 2k+1 \\ k = \frac{n-1}{2} \end{array} \right\} = f(x) \cdot \frac{n!}{\left(\frac{n-1}{2}\right)! \cdot \left(\frac{n+1}{2}\right)!} \cdot F_{(x)}^{\frac{n-1}{2}} \cdot \left(1 - F(x)\right)^{\frac{n+1}{2}}$$

$$f_{med}$$