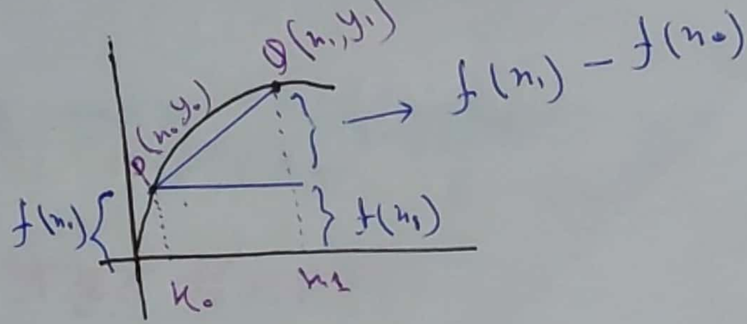
2.1

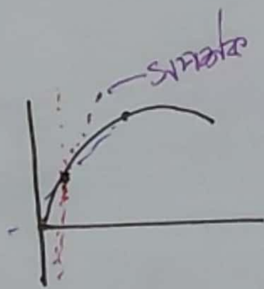
Tangent line and Rates of change



PQ secant line → যদি ছেদক
PQ tangent line → " স্পর্শক

if $P \to Q$

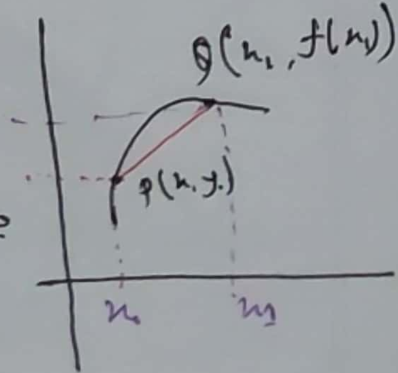
secant → tangent



যখন QP এর কাছাকাছি তখন secant টা tangent হয়ে যাবে।

Average rate of change

মানে এটাকে সময় দিয়ে ভাগ

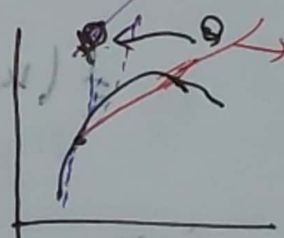


if $y = f(x)$, then the average rate of change of y w.r.to x over the interval $[x_0, x_1]$, is the slope/ gradient of the secant line joining the part $(x_0, f(x_0))$ and $(x_1, f(x_1))$

$$\therefore m_{secant} = \frac{f(x_1) - f(x_0)}{x_1 - x_0} \; ; \; m = \frac{y_1 - y}{x_1 - x}$$

gradient/ slope of secant line is the average rate of change

□ Instentanious Rate of change :-



if $y = f(x)$ then the instentanious of change of y w.r.to x at the point x_0 is the slope/gradient of the tangent line to the graph at the point x_0

$$m_{tan} = \lim_{x_1 \to x_0} \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$\therefore$ The gradient/slope of tangent line is the instentanious Rate of change.

Q: Let $y = x^2 + 3$

a) Find the average rate of change of y w.r.to x over $[3, 5]$

b) Find the instantenious Rate of Change w.r.to x at $x = -4$

a) $f(x) = x^2 + 3$ $\quad \underset{x_0 \; x_1}{[3, 5]}$

$f(x_0) = 3^2 + 3 = 12$

$f(x_1) = (5)^2 + 3 = 28$

$\therefore m_{secant} = \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}$

$= \dfrac{28 - 12}{5 - 3} = +8$

b) $m_{tan} = \lim\limits_{x_1 \to x_0} \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}$ $\quad [x_0 = -4$

$= \lim\limits_{x_1 \to x_0} \dfrac{(x_1^2 + 3) - (x_0^2 + 3)}{x_1 - x_0}$

$= \lim\limits_{x_1 \to x_0} \dfrac{x_1^2 + \cancel{3} - x_0^2 + \cancel{3}}{x_1 - x_0}$

$= \lim\limits_{x_1 \to x_0} \dfrac{(x_1 + x_0)\cancel{(x_1 - x_0)}}{\cancel{(x_1 - x_0)}} = x_0 + x_0$

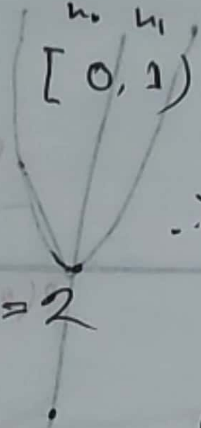
$= 2x_0 = 2 \times -4$

$= -8$

ex:

(17)

$$y = 2x^2 \qquad [x_0, x_1] = [0, 1]$$



$$f(x_0) = 0$$

$$f(x_1) = 2 \times 1^2 = 2$$

$$\therefore P(0,0),\ Q(1,2)$$

a)

$$m_{secant} = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{2-0}{1-0} = 2$$

b)

$$m_{tan} = \lim_{x_1 \to x_0} \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$= \lim_{x_1 \to x_0} \frac{2x_1^2 - 2x_0^2}{x_1 - x_0}$$

$$= \lim_{x_1 \to x_0} \frac{2(x_1^2 - x_0^2)}{x_1 - x_0}$$

$$= \lim_{x_1 \to x_0} \frac{2\cancel{(x_1 - x_0)}(x_1 + x_0)}{\cancel{x_1 - x_0}}$$

$$= \lim_{x_1 \to x_0} 2(x_1 + x_0)$$

$$= 2(x_0 + x_0)$$

$$= 4x_0$$

c

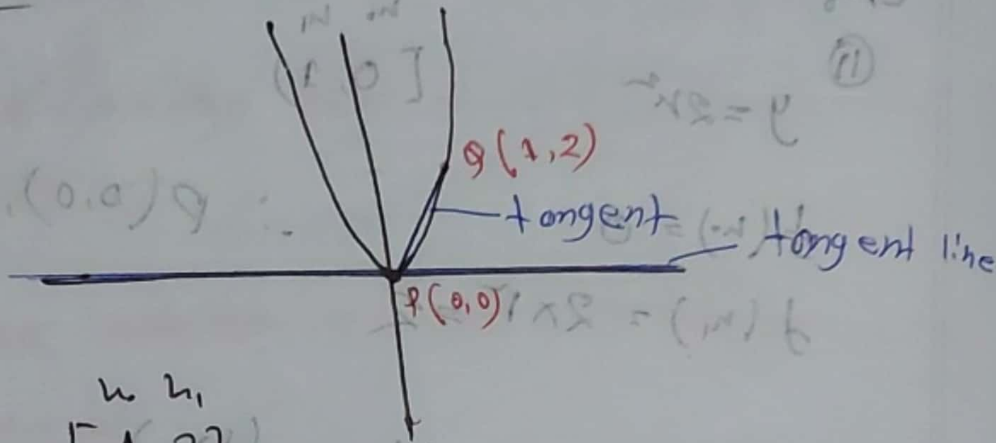
$$; [x_0 = 0$$

b

$$= 4 \times 0$$

$$= 0$$

d) $y = 2x^2$



(12) $y = x^3 \quad [1, 2]$ (x_0 above 1, x_1 above 2)

$f(x_0) = (1)^3 = 1 \qquad P(1,1) \cdot Q(2,8)$

$f(x_1) = 2^3 = 8$

a) $m_{secant} = \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}$

$= \dfrac{8 - 1}{2 - 1}$

$= 7$

b) $m_{tan} = \lim\limits_{x_1 \to x_0} \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}$

$= \lim\limits_{x_1 \to x_0} \dfrac{\cancel{8-7}\ x_1^3 - x_0^3}{x_1 - x_0}$

$= \lim\limits_{x_1 \to x_0} \dfrac{\cancel{(x_1 - x_0)}(\cancel{a^2+ab+b^2})(x_1^2 + x_1 x_0 + x_0^2)}{\cancel{x_1 - x_0}}$

$= x_0^2 + x_0 \cdot x_0 + x_0^2$

$= 3x_0^2$

7

$y = x + \sqrt{x}$

a) $m_{tan} = \lim_{x_1 \to x_0} \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}$; $[x_0 = 1$

$= \lim_{x_1 \to x_0} \dfrac{x_1 + \sqrt{x_1} - x_0 - \sqrt{x_0}}{x_1 - x_0}$

$= \lim_{x_1 \to x_0} \dfrac{x_1 - x_0 + \sqrt{x_1} - \sqrt{x_0}}{x_1 - x_0}$

$= \lim_{x_1 \to x_0} \left(\dfrac{x_1 - x_0}{x_1 - x_0} + \dfrac{\sqrt{x_1} - \sqrt{x_0}}{x_1 - x_0} \right)$

$= \lim_{x_1 \to x_0} \left(1 + \dfrac{\sqrt{x_1} - \sqrt{x_0}}{(\sqrt{x_1})^2 - (\sqrt{x_0})^2} \right)$

$= \lim_{x_1 \to x_0} \left(1 + \dfrac{\cancel{\sqrt{x_1} - \sqrt{x_0}}}{(\sqrt{x_1} + \sqrt{x_0}) \cancel{(\sqrt{x_1} - \sqrt{x_0})}} \right)$

$= \lim_{x_1 \to x_0} \left(1 + \dfrac{1}{\sqrt{x_1} + \sqrt{x_0}} \right)$

$= \left(1 + \dfrac{1}{\sqrt{x_0} + \sqrt{x_0}} \right)$

$= \left(1 + \dfrac{1}{2\sqrt{x_0}} \right)$

b) $m_{tan} = \left(1 + \dfrac{1}{2(\sqrt{1})} \right)$

$= 1 + \dfrac{1}{2}$

$= \dfrac{2+1}{2} = \dfrac{3}{2}$

2.2

Derivative function

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$L.H.D = \lim_{h\to 0^-} \frac{f(x+h) - f(x)}{h}$$

$$R.H.D = \lim_{h\to 0^+} \frac{f(x+h) - f(x)}{h}$$

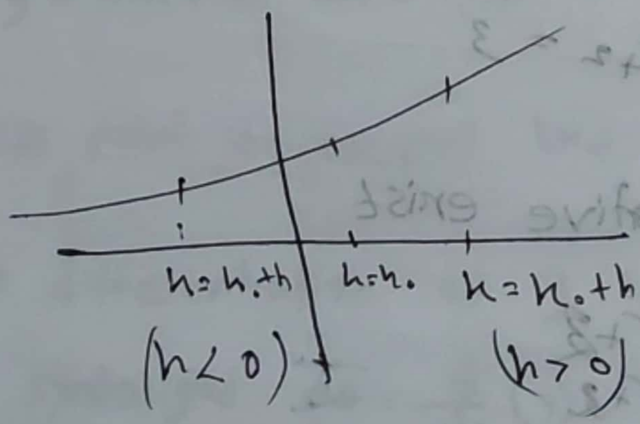
if R.H.D = L.H.D

∴ Derivative exist

if R.H.D ≠ L.H.D

∴ not differentiable

~~(0,0)~~ $h \to 0$ ধরে যদি তার বাম ডান সমান হয় তাহলে exist.



ex:
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$$f(x) = \begin{cases} x^2+2 & ; x \leq 1 \\ x+2 & ; x > 1 \end{cases}$$

$$L.H.D = \lim_{x \to 1^+} f(x)$$

$$= \lim_{x \to 1^+} x+2$$

$$= 1+2 = 3$$

$$R.H.D = \lim_{x \to 1^-} f(x)$$

$$= \lim_{x \to 1^-} (x^2+2)$$

$$= 1^2+2 = 3$$

$$=$$

∴ Derivative exist

$$f(1) = x^2+2$$

$$= 1^2+2$$

$$= 3$$

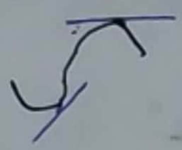
∴ $R.H.D = L.H.D = f(1)$

∴ continuous

Differentiation of a function

$y = f(x) \rightarrow$ y is a function of x

y → dependent variable; x → independent variable



To find the rate of change at any ~~instan~~ point of a curve, we have to draw a tangent at that point. the gradient of the tangent is the rate of chang of the curve at this point

$y = c$ Rate $= 0$

$y = mx + c$ Rate $= m$

যেকোনো point এর tangent line ই differentiation.

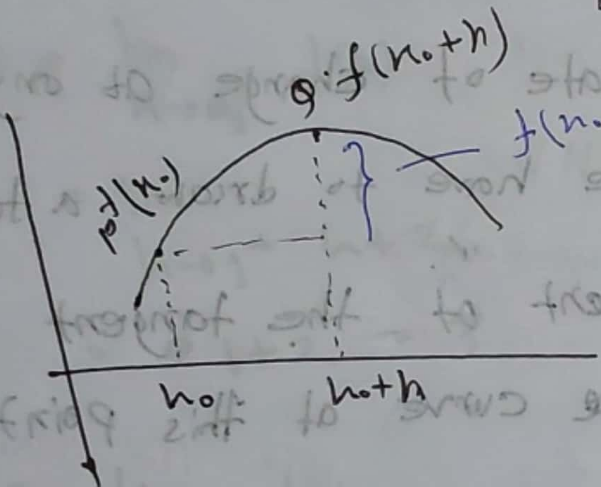
The differentiation of y with respect to x is symboliged as $\frac{d}{dx}(y) = \frac{dy}{dx}$

$\boxed{\begin{array}{c} 5 \text{ divided by } 2 \\ 5 \div 2 \end{array}}$

Defination of differentiation

$y = f(x)$ the differentiation of y w.r.to is denoted by $\frac{dy}{dx}$ or $f'(x)$ and defined as a

$$\frac{dy}{dx} = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$



if x chang by h unit y chage by $f(x+h) - f(x)$

$\therefore$ 1 unit $\frac{f(x_0+h) - f(x_0)}{h}$

$$f'(x) = \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h}$$

মূল নিয়মে অন্তরীকরণ

সূর্য নিয়েছে

$y = x^2$

$$f'(x) = \lim_{h\to 0} \frac{f(x_0+h) - f(x_0)}{h}$$

$$= \lim_{h\to 0} \frac{(x_0+h)^2 - x_0^2}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x_0^2} + 2x_0h + h^2 - \cancel{x_0^2}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(2x_0+h)}{\cancel{h}}$$

$$= 2x_0 + 0$$

$$= 2x_0 = 2x$$

Technique:

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(x^2) = 2x^{2-1}$$
$$= 2x$$

2.6

Technique of Differentiation

① $\frac{d}{dx}(x^n) = nx^{n-1}$; n is a real number

$$\frac{d}{dx}(3x^2) = 6x$$

② $\frac{d}{dx}(x) = 1$

③ $\frac{d}{dx}(c) = 0$

④ $y = [f'(x)]^n$

$\frac{dy}{dx} = n[f'(x)] \cdot \frac{d}{dx}(f(x))$ } chain rule

(4) $\frac{d}{dx}(3x^2-x+1)^5$

$= 5(3x^2-x+1)^{5-1} \cdot \frac{d}{dx}(3x^2-x+1)$ Chain Rule

$= 5(3x^2-x+1)^4 \cdot (6x-1+0)$

$= 5(3x^2-x+1)^4(6x-1)$

(5) product rule

$$\frac{d}{dx}(u \times v) = u\frac{d}{dx}(v) + v\frac{d}{dx}(u)$$

Ex. $\frac{d}{dx}x^2(x^2-1)$

$= x^2\frac{d}{dx}(x^2-1) + (x^2-1)\cdot\frac{d}{dx}(x^2)$

$= x^2 \cdot 2x + ((x^2-1)\cdot 2x)$

$= 2x^3 + 2x^3 - 2x$

$= 4x^3 - 2x$

(6) Quationt rule:

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{-u\frac{d}{dx}(v) + v\frac{d}{dx}(u)}{v^2}$$

$\frac{d}{dx}\left(\frac{u}{v}\right)$

$= \frac{d}{dx}(u + v^{-1})$

$= u \cdot \frac{d}{dx}(v^{-1}) + v^{-1}\frac{d}{dx}(u)$

$= ~~v.~~$

$= u \cdot -1 \times v^{-1-1}\frac{dv}{dx} + v^{-1}\frac{d}{dx}(u)$

$= -\frac{u}{v^2} \cdot \frac{d}{dx}(v) + \frac{1}{v} \cdot \frac{d}{dx}(u)$

$= \dfrac{-u\frac{d}{dx}(v) + v\frac{d}{dx}(u)}{v^2}$

Ex: $\frac{d}{dx}\left(\frac{(2x+1)^2}{2\sqrt{x}}\right)$

$= \frac{d}{dx}\left(\frac{4x^2+4x+1}{2\sqrt{x}}\right)$

যখানে x এর power আলাদা দিতে হবে।

$= \frac{d}{dx}\left(\frac{\cancel{2}(2x^2+2x+\frac{1}{2})}{\cancel{2}\sqrt{x}}\right)$

$= \frac{d}{dx}\left(\frac{2x^2+2x+\frac{1}{2}}{\sqrt{x}}\right)$

$= \frac{d}{dx}\left(2x^2 \cdot x^{-\frac{1}{2}} + 2x \cdot x^{-\frac{1}{2}} + \frac{1}{2}x^{-\frac{1}{2}}\right)$

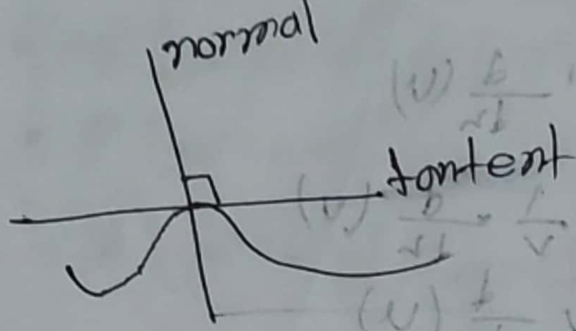
$= \frac{d}{dx}\left(2x^{2-\frac{1}{2}} + 2x^{1-\frac{1}{2}} + \frac{1}{2}x^{-\frac{1}{2}}\right)$

$= \frac{d}{dx}\left(2x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + \frac{1}{2}x^{-\frac{1}{2}}\right)$

$= \cancel{2} \cdot \frac{3}{\cancel{2}}x^{\frac{3}{2}-1} + 2 \cdot \frac{1}{2}x^{\frac{1}{2}-1} + \frac{1}{2} \cdot -\frac{1}{2}x^{-\frac{1}{2}-1}$

$= 3x^{\frac{1}{2}} + x^{-\frac{1}{2}} - \frac{1}{4}x^{-\frac{3}{2}}$

(vii) Equation of tangent and normal



tangent: a line that touch a curve

normal: a line perpendiculer to tangent at the point of contact

$$m = \frac{dy}{dx}$$

tangent: $y - y_1 = m(x - x_1)$

normal: $y - y_1 = -\frac{1}{m}(x - x_1)$

☒ Find the equation of the tangent and normal to the curve $y = x^3 - 3x^2 + 4x + 1$ at $x = 1$

$y = 1^3 - 3(1)^2 + 4(1) + 1$
$= 1 - 3 + 4 + 1$
$= 3$ $\therefore \cancel{(1, 3)}$

$\therefore (1, 3)$

$m = \frac{d}{dx}(x^3 - 3x^2 + 4x + 1)$; $[x = 1$

$= 3x^2 - 6x + 4$

$= 3(1)^2 - 6(1)^2 + 4$

$= 1$

$\therefore$ equation of tangent : $y - 3 = 1(x - 1)$

$\Rightarrow y = x + 2$

equation of normal : $y - 3 = \frac{-1}{1}(x - 1)$

$\Rightarrow y - 3 = -x + 1$

$\Rightarrow y = 4 - x$

⑧ $\frac{d}{dx}(\sec x) = \sec x \cdot \tan x$

⑨ $\frac{d}{dx}(\operatorname{cosec} x) = \operatorname{cosec} x \cdot \cot x$

⑩ $\frac{d}{dx}(\sin x) = \cos x$

⑪ $\frac{d}{dx}(\cos x) = -\sin x$

⑫ $\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$

⑬ $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

⑭ $\frac{d}{dx} \ln x = \frac{1}{x}$

$\frac{d}{dx}(\csc x) = \csc x \cdot \cot x$

2.6

19)

$f(x) = \cos^2(3\sqrt{x}) = \{\cos(3\sqrt{x})\}^2$

$f'(x) = 2\cos(3\sqrt{x}) \cdot \frac{d}{dx}(\cancel{3\sqrt{x}}\ \cos(3\sqrt{x}))$

$= 2\cos(3\sqrt{x}) \cdot -\sin(3\sqrt{x}) \cdot \frac{d}{dx}(3\sqrt{x})$

$= 2\cos(3\sqrt{x}) \cdot -\sin(3\sqrt{x}) \cdot 3 \cdot \frac{1}{2\sqrt{x}}$

13)

$f(x) = \sqrt{4+\sqrt{3x}}$

$f'(x) = \frac{1}{2\sqrt{4+\sqrt{3x}}} \cdot \frac{d}{dx}(4+\sqrt{3x})$

$= \frac{1}{2\sqrt{4+\sqrt{3x}}} \cdot \left\{0 + \frac{1}{2\sqrt{3x}} \cdot \frac{d}{dx}(3x)\right\}$

$= \frac{1}{2\sqrt{4+\sqrt{3x}}} \cdot \left\{0 + \frac{1}{2\sqrt{3x}} \cdot 3 \cdot 1\right\}$

25)

$f(x) = [x + \operatorname{cosec}(x^3+3)]^{-3}$

$= -3[x+\csc(x^3+3)]^{-4} \cdot \frac{d}{dx}(x+\csc(x^3+3))$

$= -3[x+\csc(x^3+3)]^{-4} \cdot \left\{1 + \csc(x^3+3) \cdot \cot(x^3+3) \cdot \frac{d}{dx}(x^3+3)\right\}$

$= -3[x+\csc(x^3+3)]^{-4} \cdot \left\{1 + \csc(x^3+3) \cdot \cot(x^3+3) \cdot 3x^2\right\}$

CamScanner

$\frac{d}{dx}(\sec x) = \sec x \cdot \tan x$

$\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$

27

$y = x^3 \sin^2(5x)$

$= x^3 (\sin(5x))^2$

$\frac{dy}{dx} = x^3 \frac{d}{dx}(\sin(5x))^2 + (\sin(5x))^2 \frac{d}{dx}(x^3)$

$\cancel{= x^3 \cdot 2\sin(5x) \cdot \frac{d}{dx}(5x) + (\sin(5x))^2 \cdot 3x^2}$

$\cancel{= x^3\, 2\sin(5x) \cdot 5 + \sin^2(5x) \cdot 3x^2}$

$= x^3\, 2\sin(5x) \cdot \frac{d}{dx}(\sin(5x)) + \sin^2(5x) \cdot 3x^2$

$= x^3\, 2\sin(5x) \cos(5x) \cdot \frac{d}{dx}(5x) + \sin^2(5x) \cdot 3x^2$

$= x^3\, 2\sin(5x) \cdot \cos(5x) \cdot 5 + \sin^2(5x) \cdot 3x^2$

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$y = x^5 \sec\left(\frac{1}{x}\right)$

$= x^5 \frac{d}{dx}\left(\sec\left(\frac{1}{x}\right)\right) + \left(\sec\left(\frac{1}{x}\right)\right) \cdot \frac{d}{dx}(x)^5$

$\cancel{= x^5 \cdot \frac{d}{dx}}$

$= x^5 \cdot \sec\left(\frac{1}{x}\right) \cdot \tan\left(\frac{1}{x}\right) \cdot \frac{d}{dx}\left(\frac{1}{x}\right) + \sec\left(\frac{1}{x}\right) \cdot 5x^4$

$= x^5 \cdot \sec\left(\frac{1}{x}\right) \cdot \tan\left(\frac{1}{x}\right) \cdot -\frac{1}{x^2} + \sec\left(\frac{1}{x}\right) \cdot 5x^2$

31

$y = \cos(\cos x)$

$\frac{dy}{dx} = -\sin(\cos x) \cdot \frac{d}{dx}(\cos x)$

$= -\sin(\cos x) \cdot -\sin x$

$= \sin(\cos x) \cdot \sin x$

33)

$y = \cos^3(\sin 2x)$

$\frac{dy}{dx} = \{\cos(\sin 2x)\}^3$

$= 3\{\cos(\sin 2x)\}^2 \cdot \frac{d}{dx}(\cos(\sin 2x))$

$= 3\{\cos(\sin 2x)\}^2 \cdot -\sin(\sin 2x) \cdot \frac{d}{dx}(\sin(2x))$

$= 3\{\cos(\sin 2x)\}^2 \cdot -\sin(\sin 2x) \cdot \cos(2x) \cdot \frac{d}{dx}(2x)$

$= 3\{\cos(\sin 2x)\}^2 \cdot -\sin(\sin 2x) \cdot \cos(2x) \cdot 2$

$= -6\cos^2(\sin 2x) \cdot \sin(\sin 2x) \cdot \cos(2x)$

35

$y = (5x+8)^7(1-\sqrt{x})^6$

$\frac{dy}{dx} = (5x+8)^7 \cdot \frac{d}{dx}(1-\sqrt{x})^6 + (1-\sqrt{x})^6 \cdot \frac{d}{dx}(5x+8)^7$

$= (5x+8)^7 \cdot 6(1-\sqrt{x})^5 \cdot \frac{d}{dx}(1-\sqrt{x}) + (1-\sqrt{x})^6 \cdot 7(5x+8)^6 \cdot \frac{d}{dx}(5x+8)$

$$= (5x+8)^7 \cdot 6(1-\sqrt{x})^5 \cdot \left\{0 - \frac{1}{2\sqrt{x}}\right\} + (1-\sqrt{x})^6 \cdot 7(5x+8)^6 \cdot \{5 \cdot 1 + 0\}$$

$$= -\frac{3}{\sqrt{x}}(5x+8)^7 \cdot (1-\sqrt{x})^5 + 35(1-\sqrt{x})^6 (5x+8)^6$$

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$$y = \left(\frac{x-5}{2x+1}\right)^3$$

$$\frac{dy}{dx} = 3\left(\frac{x-5}{2x+1}\right)^2 \cdot \frac{d}{dx}\left(\frac{x-5}{2x+1}\right)$$

$$= 3\left(\frac{x-5}{2x+1}\right)^2 \cdot \frac{-(x-5)\frac{d}{dx}(2x+1) + (2x+1)\frac{d}{dx}(x-5)}{(2x+1)^2}$$

$$= 3\left(\frac{x-5}{2x+1}\right)^2 \cdot \frac{-(x-5)\cdot(2\cdot 1+0) + (2x+1)\cdot 1}{(2x+1)^2}$$

$$= 3\left(\frac{x-5}{2x+1}\right)^2 \cdot \frac{-(2x-10) + (2x+1)}{(2x+1)^2}$$

$$= 3\left(\frac{x-5}{2x+1}\right)^2 \cdot \frac{-\cancel{2x}+10+\cancel{2x}+1}{(2x+1)^2}$$

$$= 3\left(\frac{x-5}{2x+1}\right)^2 \cdot \frac{11}{(2x+1)^2}$$

43.

$[y - y_1 = m_{tan}(x - x_1)]$

$y = x\cos 3x \quad ; x = \pi$

$y = \pi(\cos(3\pi))$
$= \pi \times (-1)$
$= -\pi$

$\therefore (\pi, -\pi)$

$m_{tan} = \frac{d}{dx}(x\cos 3x)$. Derivative = slope of tangent

$= x\frac{d}{dx}(\cos 3x) + \cos 3x \frac{d}{dx}(x)$

$= \cancel{x \frac{d}{}}$

$= x \cdot -\sin 3x \frac{d}{dx}(3x) + \cos 3x . 1$

$= -x\sin 3x \cdot 3 \cdot 1 + \cos 3x . 1$

$= -3x\sin(3x) + \cos 3x$

$= -3 \times \pi \times \underbrace{\sin(3\times\pi)}_{0} + \cos(3\times\pi) \; ; [x = \pi$

$= -0 + (-1)$

$= -1$

$\therefore \cancel{y - (-\pi) = -1(x - \pi)}$

$\therefore \quad y - y_1 = m_{tan}(x - x_1)$

$\Rightarrow y - (-\pi) = -1(x - \pi)$

$\Rightarrow y + \pi = -x + \pi$

$\Rightarrow y = -x$

45 (D)

$y = x^2\sqrt{5-x^2} \quad ; \; x_1 = 1$

$y = 1^2\sqrt{5-1^2}$

$= 2 \qquad \therefore \; (x_1, y_1) = (1, 2)$

$\therefore \; m_{tan} = \frac{d}{dx}\left(x^2\sqrt{5-x^2}\right)$

$= x^2 \frac{d}{dx}\left(\sqrt{5-x^2}\right) + \left(\sqrt{5-x^2}\right)\frac{d}{dx}\left(x^2\right)$

$= x^2 \cdot \frac{1}{2\sqrt{5-x^2}} \cdot \frac{d}{dx}\left(5-x^2\right) + \left(\sqrt{5-x^2}\right) \cdot 2x$

$= \frac{x^2}{\cancel{2}\sqrt{5-x^2}} \cdot -\cancel{2}x + 2x\left(\sqrt{5-x^2}\right)$

$= -\frac{x^3}{\sqrt{5-x^2}} + 2x\left(\sqrt{5-x^2}\right)$

$= -\frac{(1)^3}{\sqrt{5-1^2}} + 2 \times (1)\left(\sqrt{5-x^2}\right)$

$= -\frac{1}{2} + 4,$

$$= \frac{-1+8}{2}$$

$$= \frac{7}{2}$$

$$\therefore y - y_1 = m_{tan}(x - x_1)$$

$$\Rightarrow y - 2 = \frac{7}{2}(x-1)$$

$$\Rightarrow y = \frac{7x-7}{2} + 2$$

$$\Rightarrow y = \frac{7x-7+4}{2}$$

$$= \frac{7x-3}{2}$$

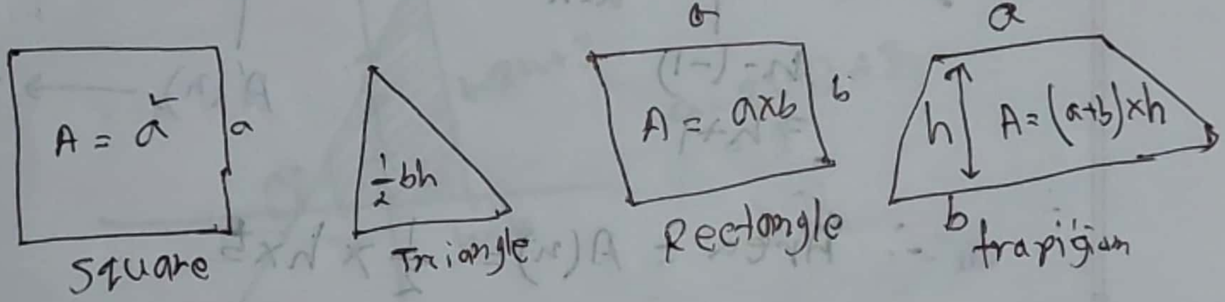
$$\frac{d}{dx}\left(e^{f(x)}\right) = e^{f(x)} \times \frac{d}{dx}(f(x))$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{dx}(\ln f(x)) = \frac{1}{f(x)} \times \frac{d}{dx}(f(x))$$

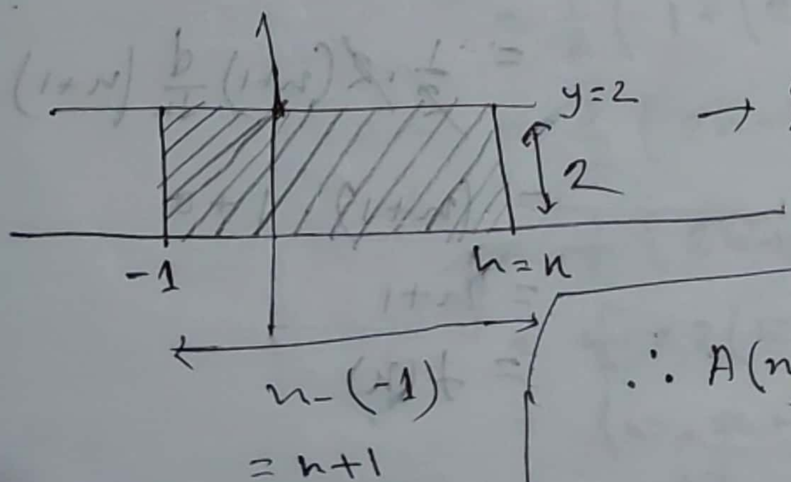
5.1 overview of Area & f anti derivative of Area

question অনুযায়ী graph আঁকতে হবে এর পর Arbhitary A equation বের করতে হবে, এইটাকে 1 বার Differentiate করলে original function আসবে থাকে।



Example 1

a) $f(x) = 2$; $[-1, x]$

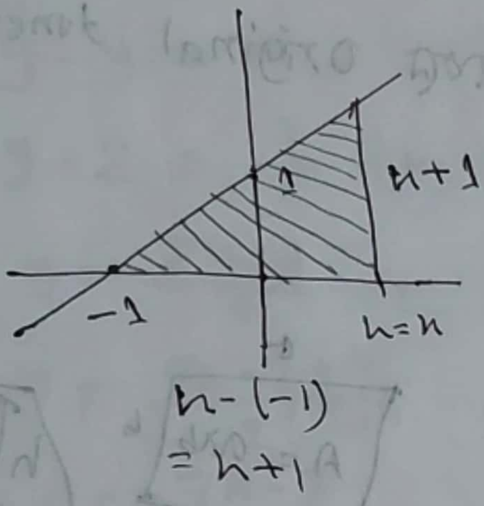


→ Rectangle shape

$\therefore A(x) = a\times b$
$= (x+1)\times 2$
$= 2x+2$

$\therefore A'(x) = \frac{d}{dx}(2x+2)$
$= 2\times 1 + 0 = 2 = f(x)$

b) $f(x) = x+1 \quad ; \quad [-1, x]$



$f(x) \downarrow A(x) \mid A'(x) \longrightarrow f(x)$

$\therefore$ Area $= A(x) = \frac{1}{2} \times h \times b$

$= \frac{1}{2} \times (x+1)(x+1)$

$= \frac{1}{2}(x+1)^2$

Derivative

$\therefore A'(x) = \frac{d}{dx}\left(\frac{1}{2}(x+1)^2\right)$

$= \frac{1}{\cancel{2}} \cdot \cancel{2}(x+1)\frac{d}{dx}(x+1)$

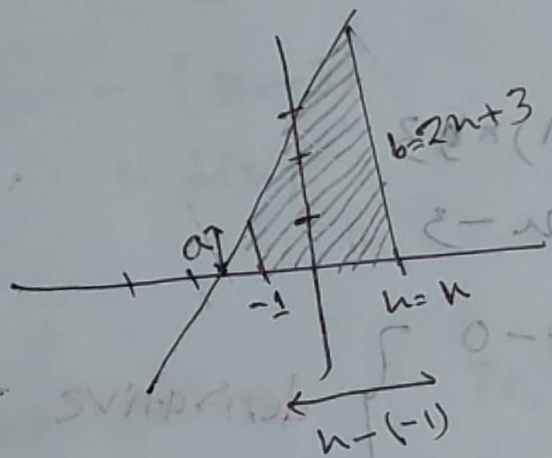
$= (x+1) \cdot 1 + 0$

$= x+1$

$= f(x)$

$$A(x) \underset{\text{Anti-Derivative}}{\overset{\text{Derivative}}{\rightleftarrows}} f(x)$$

c) $f(x) = 2x+3$; $[-1, x]$



$a = 2(-1)+3$
$= 1$

$h = x+1$

$\therefore$ Area $= A(x) = \frac{1}{2}(a+b) \times h$

$= \frac{1}{2}\left(1 + (2x+3)\right) \times (x+1)$

$= \frac{1}{2}(2x+4) \times (x+1)$

$= \frac{1}{2}(2x^2 + 2x + 4x + 4)$

$= \frac{1}{2} \times 2(x^2 + 3x + 2)$

$= (x^2 + 3x + 2)$

$\therefore A'(x) = \cancel{2x+} \frac{d}{dx}(x^2+3x+2)$

$= 2x+3+0 = 2x+3 = f(x)$

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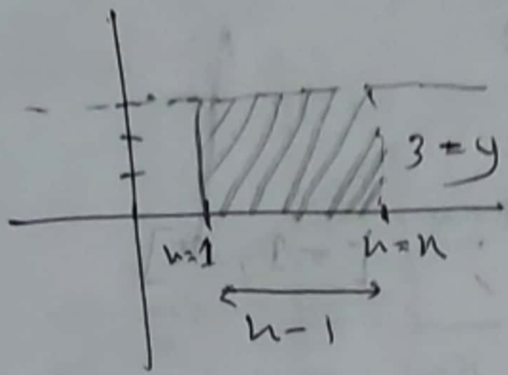
$f(x) = 3, \quad [1, x]$



Area = a×b

$\therefore A(x) = (x-1) \times 3$

$= 3x - 3$

$A'(x) = 3 - 0$

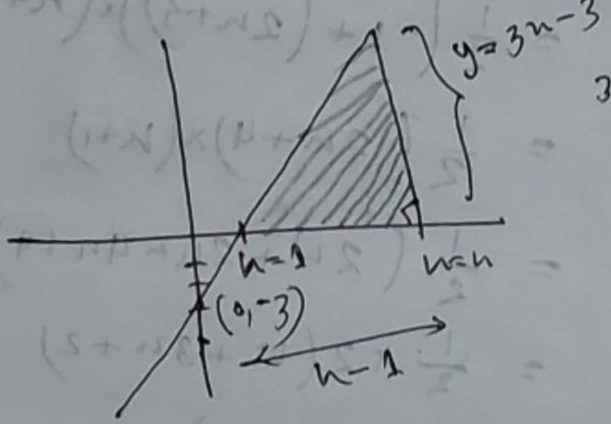
$= 3$

$= f(x)$

derivative

16)

$f(x) = 3x - 3 \; ; \; [1, x]$



$3x - 3 = 0$

$x = 1$

$y = -3$

$A(x) = \frac{1}{2}\, b.h$

$= \frac{1}{2}(x-1) \times (3x-3)$

$= \frac{1}{2}\left(3x^2 - \cancel{3x} - \cancel{3x} + 3\right)$

$= \cancel{\frac{1}{2}\left(3x^2 + 3\right)} = \frac{1}{2}\left(3x^2 - 6x + 3\right)$

$A'(x) = \frac{1}{2}\cdot\left(6x - 6 + 0\right)$

$= \frac{1}{2}\cdot 2\left(3x - 3\right)$

$= (3x - 3)$

$= f(x)$

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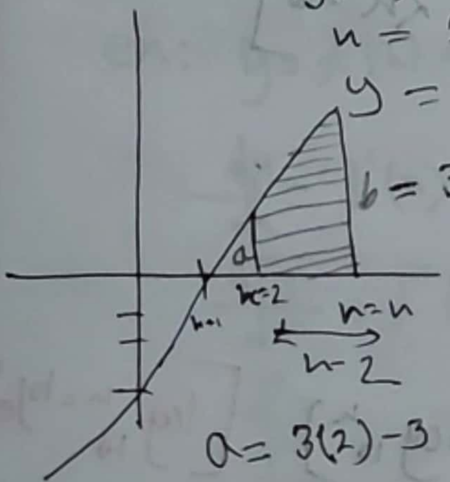
$f(x) = 3x - 3 \quad ; \quad [2, x]$

$3x - 3 = 0$

$x = 1$

$y = -3$



$a = 3(2) - 3$

$= 3$

$\therefore A(x) = \frac{1}{2}(a+b)\times h$

$= \frac{1}{2}\left(\cancel{3} + 3x \cancel{-3}\right)\times(x-2)$

$= \frac{1}{2}\left(3x^2 - 6x\right)$

$A'(x) = \frac{1}{2}\left(6x - 6\right)$

$= 3x - 3$

$= f(x)$

Differentiation example

Ex. $\frac{d}{dx}\left[\ln \sqrt[3]{\frac{2x+1}{x^2+1}}\right]$

$= \frac{d}{dx}\left[\ln \left(\frac{2x+1}{x^2+1}\right)^{\frac{1}{3}}\right]$

$= \frac{d}{dx}\left[\frac{1}{3}\ln\left(\frac{2x+1}{x^2+1}\right)\right]$

$= \frac{1}{3}\frac{d}{dx}\left[\ln(2x+1) - \ln(x^2+1)\right]$ $\left[\ln x/y = \ln x - \ln y\right]$

$= \frac{1}{3}\left[\frac{1}{2x+1}\cdot\frac{d}{dx}(2x+1) - \frac{1}{x^2+1}\frac{d}{dx}(x^2+1)\right]$ $\left[\ln x = \frac{1}{x}\right]$

$= \frac{1}{3}\left[\frac{1}{2x+1}\cdot 2 - \frac{1}{x^2+1}\cdot 2x\right]$

$= \frac{1}{3}\left[\frac{2}{2x+1} - \frac{2x}{x^2+1}\right)$

Ex. $\frac{d}{dx}(\log x) = \frac{d}{dx}\left[\log_{10} x\right]$ $\left[\log_{10} x = \log_e x \times \log_{10} e\right]$

$= \frac{d}{dx}\left[\log_{10} x \times \log_e 10\right]$

$= \frac{d}{dx}\left[\ln x \times \log_{10} e\right]$

$= \frac{1}{x}\cdot \log_{10} e$

$\boxtimes$ $\frac{d}{dx}[\log_a x] = \frac{1}{x}\log_a e$

$\boxtimes$ Higher derivatives and notation

$y = f(x)$

$\frac{d}{dx}(y) = f'(x)$

$\frac{d^2}{dx^2}(y) = \frac{d^2y}{dx^2} = f''(x)$

$\frac{d^3}{dx^3}(y) = \frac{d^3y}{dx^2} = f'''(x)$

ex: $y = f(x) = \cancel{3}x^3 - \cancel{\ }2x^{-1}$

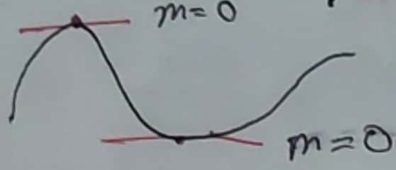
$\frac{dy}{dx} = f'(x) = 3x^2 + 2x^{-2}$

$\frac{d^2y}{dx^2} = f''(x) = 6x - 4x^{-3}$

$\frac{d^3y}{dx^3} = f'''(x) = 6 + 12x^{-4}$

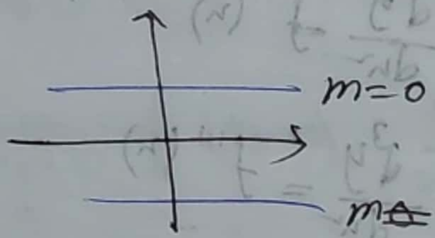
Finding coordinate of turning point

$y = f(x) =$



the gradiant of turning point is 0

$m = \frac{dy}{dx} = 0$



Find the turning point of the line

$y = \frac{1}{3}x^3 - 2x^2 + 3x + 1$

$\frac{dy}{dx} = x^2 - 4x + 3$

at turning point $\frac{dy}{dx} = 0$

$\therefore x^2 - 4x + 3 = 0$

$\Rightarrow (x-1)(x-3) = 0$

$\therefore x = 1, \quad x = 3$

4.1

~~Derivate~~

Main functiot to Derivative function

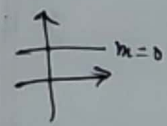
(i) graph increasing হলে derivative এর এ অংশ x-axis এর উপরে থাকবে

(ii) graph decreasing হলে derivative এর এ অংশ x-axis এর নিচে থাকবে।

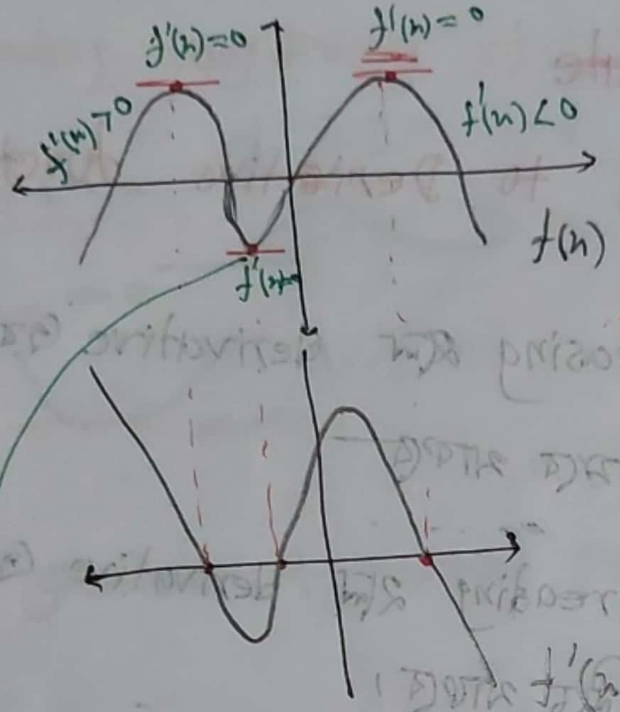
(iii) যেখানে graph এর turning point আছে সেখানে derivative = 0, তাই derivative graph টি বিন্দুতে x-axis কে স্পর্শ করবে।

কারণ Derivativ=0 মানে y=0

(iv) Horizontal line এর gradiant always 0



(v) কোণ থাকলে সেখানে gradiant থাকে না বা এ point diffentiable না।



যদি curve আকারে থাকে তাহলে এই নিয়ম

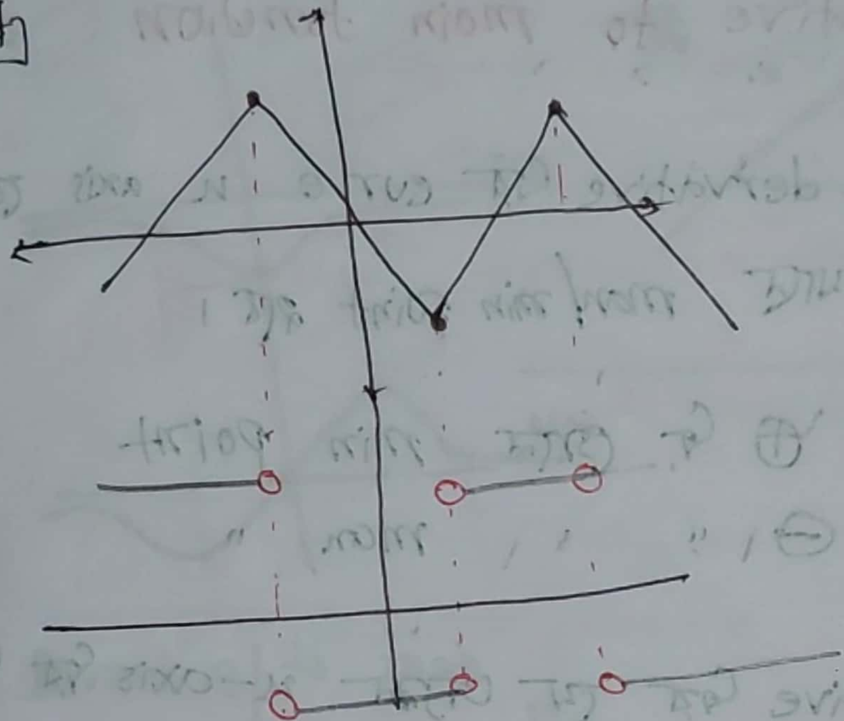
1. Horizontal tangent line
2. $f(x)$ is increasing $\longrightarrow f'(x) > 0$
3. $f(x)$ is decreasing $\longrightarrow f'(x) < 0$

main এই slope point টি x অক্ষকে ছেদ স্পর্শ করবে যার $m=0$ মানে $y=0$

increase part নতুন graph এর x-axis এর উপরে থাকবে

decreasing part নতুন graph এর x-axis এর নিচে থাকবে।

নো



① কর্নার point differentiable নয়

নো এরকম ~~গ্রাফ~~ গ্রাফ থাকলে increase হবে x-axis এর উপরে হবে; Decreasing হবে x-axis এর নিচে আসবে; [কিন্তু যেহেতু কর্নার point গুলো Differentiable না তাই নতুন গ্রাফে ঐ point গুলো ফাঁকা রাখবো।

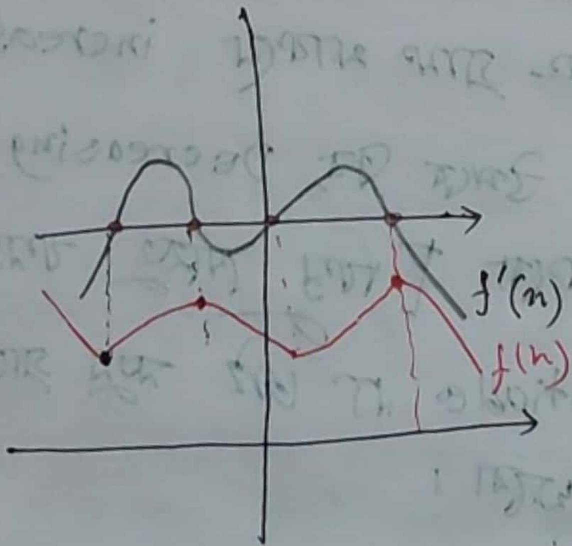
11 Derivative to main function

(i) যেখানে derivative এর curve x axis কে স্পর্শ করছে সেখানে max/min point হবে।

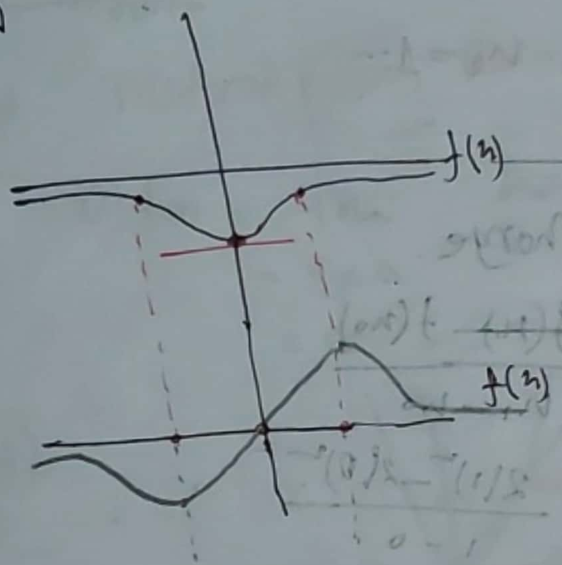
⊖ থেকে ⊕ এ গেলে min point

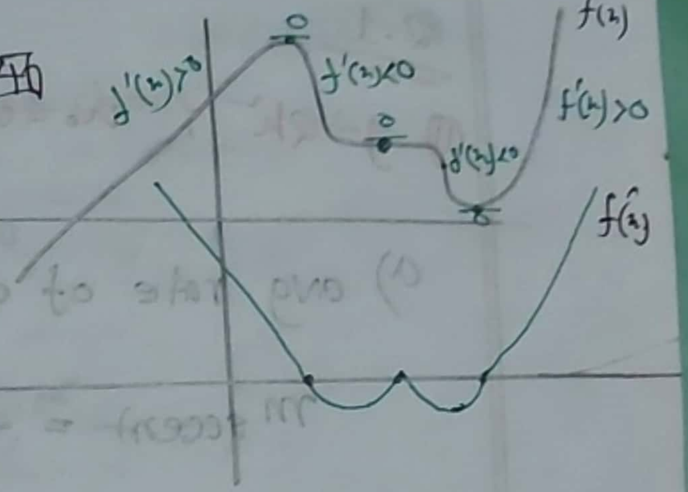
⊕ " " ⊖ " " max "

(ii) derivative এর যে অংশ x-axis এর নিচে থাকবে main graph এর ততটুকু অংশ decreasing করবে। x-axis এর উপরে থাকলে increase হবে

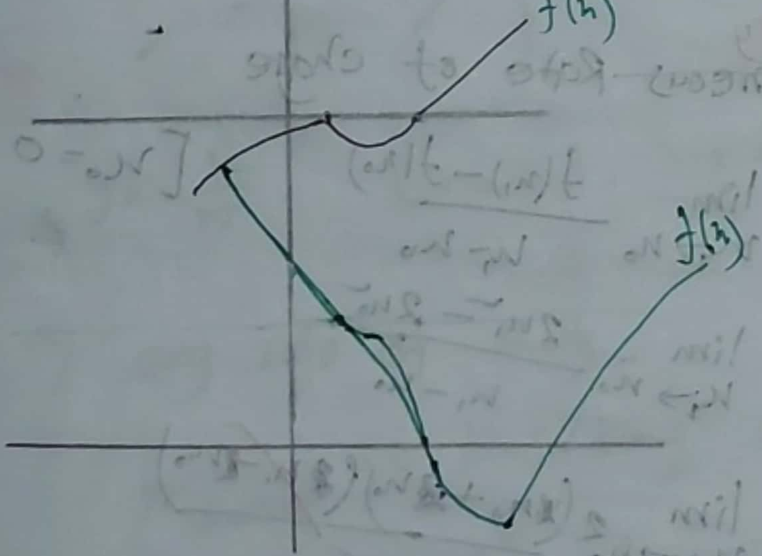


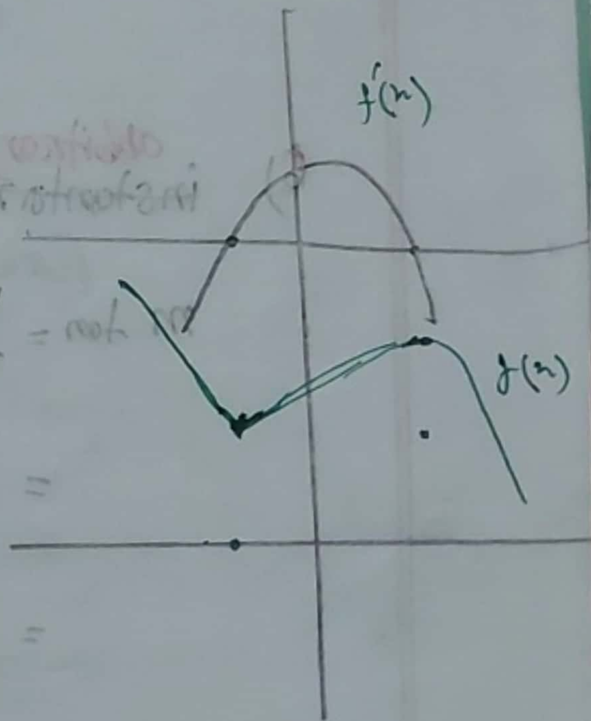
Derivative to main এর সময় x axis এ y=0 যেখানে point বসাতে হবে।





Derivative to main





Book Exercise

2.1

(11) $y = 2x^2$; $x_0 = 0$, $x_1 = 1$

a) avg rate of change

$$m_{secant} = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$= \frac{2(1)^2 - 2(0)^2}{1 - 0}$$

$$= 2$$

b) arbitrary ~~instantaneous~~ Rate of chage

$$m_{tan} = \lim_{x_1 \to x_0} \frac{f(x_1) - f(x_0)}{x_1 - x_0} \quad [x_0 = 0$$

$$= \lim_{x_1 \to x_0} \frac{2x_1^2 - 2x_0^2}{x_1 - x_0}$$

$$= \lim_{x_1 \to x_0} \frac{2(x_1 + x_0)\cancel{(x_1 - x_0)}}{\cancel{x_1 - x_0}}$$

$$\cancel{= \lim_{x_1 \to x_0} \frac{2(2x_1 + 2x_0)\, 2(x_1 - x_0)}{x_1 - x_0}}$$

$$= \lim_{x_1 \to x_0} \cancel{(4x_1 + 4x_0)} = 2(x_0 + x_0)$$

$$\Rightarrow 4x_0 + 4x_0 \qquad = 2(2x_0)$$

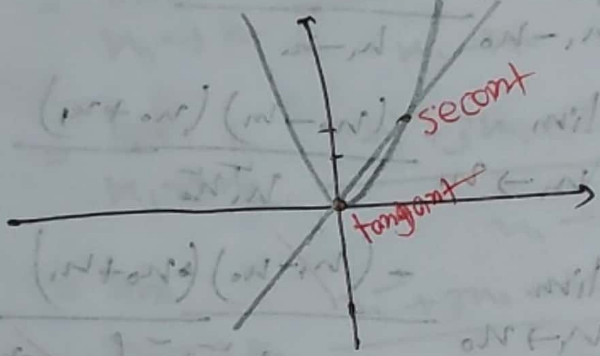
$$\Rightarrow 8x_0 \qquad \boxed{= 4x_0}$$

arbitrary

~~b) $m_{tan}=$~~

b) Instantaneous Rate of change

$$m_{tan} = 4x_0 \qquad x_0 = 0$$
$$= 4\times 0$$
$$= 0$$

d) 

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$$y = \frac{1}{x^2} \qquad [x_0 = 1,\ x_1 = 2]$$

a) avg rate of change

$$m_{secant} = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$
$$= \frac{\frac{1}{x_1^2} - \frac{1}{x_0^2}}{2-1}$$
$$= \frac{\frac{1}{4} - \frac{1}{1}}{1}$$
$$= \frac{1-4}{4}$$
$$= -\frac{3}{4}$$

c) Arbitrary

$$m_{tan} = \lim_{x_1 \to x_0} \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$= \lim_{x_1 \to x_0} \frac{\frac{1}{x_1^2} - \frac{1}{x_0^2}}{x_1 - x_0}$$

$$= \lim_{x_1 \to x_0} \frac{\frac{x_0^2 - x_1^2}{x_1^2 x_0^2}}{x_1 - x_0}$$

$$= \lim_{x_1 \to x_0} \frac{(x_0 - x_1)(x_0 + x_1)}{x_1^2 x_0^2} \times \frac{1}{x_1 - x_0}$$

$$= \lim_{x_1 \to x_0} \frac{-\cancel{(x_1 - x_0)}(x_0 + x_1)}{x_1^2 x_0^2 \cancel{(x_1 - x_0)}}$$

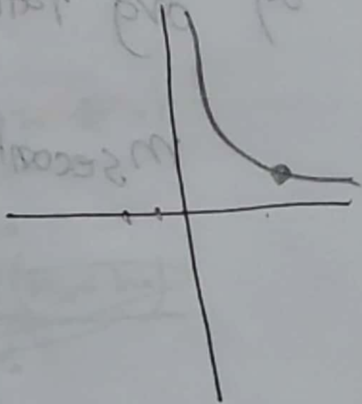
$$= \lim_{x_1 \to x_0} \frac{-(x_0 + x_1)}{x_1^2 x_0^2}$$

$$\Rightarrow \frac{-(x_0 + x_0)}{x_0^2 x_0^2}$$

$$= -\frac{2x_0}{x_0^4}$$

$$= -\frac{2}{x_0^3}$$

d)



b) instanteneous Rate

$$m_{tan} = -\frac{2}{(1)^3}$$

$$= -2$$

$\frac{1}{x^2} \rightarrow y = 1$

$(1,1)$

$m = \frac{3}{4}$ -2

$m_{tan} = \frac{3}{4}$

$y - 1 = -2(x +$

(4)

$f(x) = x^2 + 3x + 2 \; ; \; [x_0 = 2$

a) $m_{tan} = \lim\limits_{x_1 \to x_0} \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}$

$= \lim\limits_{x_1 \to x_0} \dfrac{(x_1^2 + 3x_1 + 2) - (x_0^2 + 3x_0 + 2)}{x_1 - x_0}$

$= \lim\limits_{x_1 \to x_0} \dfrac{x_1^2 + 3x_1 + \cancel{2} - x_0^2 - 3x_0 - \cancel{2}}{x_1 - x_0}$

$= \lim\limits_{x_1 \to x_0} \dfrac{x_1^2 + 3x_1 - x_0^2 - 3x_0}{x_1 - x_0}$

$= \lim\limits_{x_1 \to x_0} \dfrac{(x_1^2 - x_0^2) + 3(x_1 - x_0)}{x_1 - x_0}$

$= \cancel{(x_0^2 - x_0^2) + 3}$

$= \lim\limits_{x_1 \to x_0} \dfrac{(x_1 + x_0)(x_1 - x_0) + 3(x_1 - x_0)}{x_1 - x_0}$

$= \lim\limits_{x_1 \to x_0} \dfrac{\cancel{(x_1 - x_0)}((x_1 + x_0) + 3)}{\cancel{x_1 - x_0}}$

$= (x_0 + x_0) + 3$

$= 2x_0 + 3$

b) $m_{tan} = 2x_0 + 3$ $\quad [x_0 = 2$

$= 2 \times 2 + 3$

$= 7$

2.2

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$$f(x) = \begin{cases} x^2+2, & x \le 1 \\ x+2 & x > 1 \end{cases}$$

$\therefore \lim_{x\to 1^+} f(x) \qquad \lim_{x\to 1^-} f(x) \qquad f(1) = 1^2+2$

$= 3$

$= \lim_{x\to 1^+} x+2 \qquad = \lim_{x\to 1^-} x^2+2$

$= 1+2 \qquad = (1)^2+2$

$= 3 \qquad = 3$

thus L.H.L = R.H.L = f(1), so continuous

L.H.D $= \lim_{h\to 0^-} \dfrac{f(x+h) - f(x)}{h}$; $\boxed{x^2+2}$

$= \lim_{h\to 0^-} \dfrac{(x+h)^2+2 - (x^2+2)}{h}$

$= \lim_{h\to 0^-} \dfrac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2} \cancel{+2} \cancel{-2}}{h}$

$= \lim_{h\to 0^-} \dfrac{\cancel{h}(2x+h)}{\cancel{h}}$

$= (2\times 1 + 0)$

$= 2$

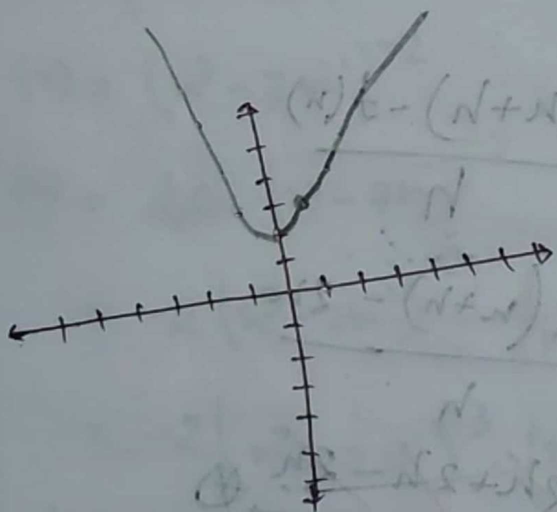
$$R.H.D = \lim_{h\to 0^+} \frac{f(x+h)-f(x)}{h} \qquad x \; \boxed{x+2}$$

$$= \lim_{h\to 0^+} \frac{(x+h)+2-(x+2)}{h}$$

$$= \lim_{h\to 0^+} \frac{\cancel{x}+h+\cancel{2}-\cancel{x}-\cancel{2}}{h}$$

$$= 1$$

$\therefore$ thus L.H.D $\neq$ R.H.D, so not Defferentiable



47 $f(x) = \begin{cases} x^2+1 ; & x \le 1 \\ 2x ; & x > 1 \end{cases} ; x = 1$

$$L.H.L = \lim_{x\to 1^-} f(x) \qquad R.H.L = \lim_{x\to 1^+} f(x)$$

$$= \lim_{x\to 1^-} x^2+1 \qquad = \lim_{x\to 1^+} 2x$$

$$= 1+1 \qquad = 2\cdot 1$$

$$= 2 \qquad = 2$$

$$f(1) = (1)^2+1 = 2$$

Thus LHL = R.H.L = f(1)

so, continuous

$$L \cdot HD = \lim_{h \to 0^-} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0^-} \frac{(x+h)^2+1 - (x^2+1)}{h}$$

$$= \lim_{h \to 0^-} \frac{\cancel{x^2} + 2xh + h^2 \cancel{+1} - \cancel{x^2} \cancel{-1}}{h}$$

$$= \lim_{h \to 0^-} \frac{\cancel{h}(2x+h)}{\cancel{h}}$$

$$= (2 \times 1 + 0)$$

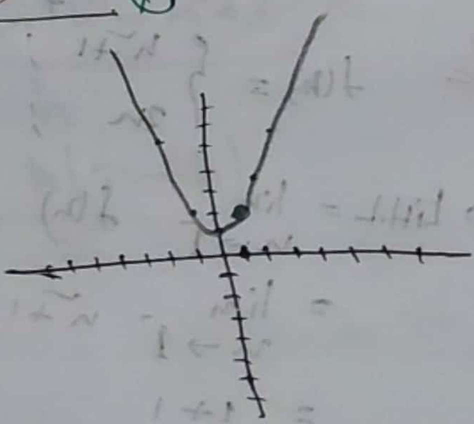
$$= 2$$

$$R.H.D = \lim_{h \to 0^+} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0^+} \frac{2(x+h) - 2x}{h}$$

$$= \lim_{h \to 0^+} \frac{\cancel{2x} + 2\cancel{h} - \cancel{2x}}{\cancel{h}}$$

$$= 2$$



thus $R.H.D = L.H.D$

So Differentiable

3.6

7)

$f(x) = (x^3 + 2x)^{37}$

$f'(x) = \frac{d}{dx}(x^3+2x)^{37}$

$f'(x) = 37(x^3+2x)^{36} \frac{d}{dx}(x^3+2x)$

$= 37(x^3+2x)^{36} \cdot (3x^2+2)$

9)

$f(x) = \left(x^3 - \frac{7}{x}\right)^{-2}$

$f(x) = (x^3 - 7x^{-1})^{-2}$

$f'(x) = d/dx\,(x^3 - 7x^{-1})^{-2}$

$= -2(x^3-7x^{-1})^{-3} \cdot \frac{d}{dx}(x^3-7x^{-1})$

$= -2(x^3-7x^{-1})^{-3} \cdot (3x^2 - 7(-1)x^{-2})$

$= -2(x^3-7x^{-1})^{-3} \cdot (3x^2+7x^{-2})$

11)

$f(x) = \frac{4}{(3x^2-2x+1)^3}$

$f(x) = 4(3x^2-2x+1)^{-3}$

$f'(x) = \frac{d}{dx}\left(4(3x^2-2x+1)^{-3}\right)$

$= -12(3x^2-2x+1)^{-4} \frac{d}{dx}(3x^2-2x+1)$

$= -12(3x^2-2x+1)^{-4} \cdot (6x-2)$

15

$f(x) = \sin\left(\frac{1}{x^2}\right)$

$f(x) = \sin(x^{-2})$

$f'(x) = d/dx\left(\sin(x^{-2})\right)$

$= \cos(x^{-2}) \cdot \frac{d}{dx}(x^{-2})$

$= \cos(x^{-2}) \cdot -2x^{-3}$

17)

$f(x) = 4\cos^5 x$

$f(x) = 4(\cos x)^5$

$f'(x) = \frac{d}{dx}\left(4(\cos x)^5\right)$

$= 20(\cos x)^4 \cdot \frac{d}{dx}(\cos x)$

$= 20(\cos x)^4 \cdot -\sin x$

$= -20(\cos x)^4 \cdot \sin x$

21)

$f(x) = 2\sec^2(x^7)$

$f(x) = 2\left(\sec(x^7)\right)^2$

$f'(x) = d/dx\left(2\sec(x^7)\right)^2$

$= 4\sec(x^7) \cdot \frac{d}{dx}(\sec x^7)$

$= 4\sec(x^7) \cdot \sec x^7 \cdot \tan x^7 \cdot \frac{d}{dx}(x^7)$

$= 4\sec^2 x^7 \cdot \tan x^7 \cdot 7x^6$

23

$f(x) = \sqrt{\cos(5x)}$

$f'(x) = \frac{d}{dx}\left(\sqrt{\cos(5x)}\right)$

$= \frac{1}{2\sqrt{\cos(5x)}} \cdot \frac{d}{dx}\left(\cos(5x)\right)$

$= \frac{1}{2\sqrt{\cos(5x)}} \cdot -\sin(5x) \cdot \frac{d}{dx}(5x)$

$= \frac{1}{2\sqrt{\cos(5x)}} \cdot -\sin(5x) \cdot 5$

39)

$$y = \frac{(2x+3)^3}{(4x^2-1)^8}$$

$$= \frac{-(2x+3)^3 \cdot \frac{d}{dx}((4x^2-1)^8) + (4x^2-1)^8 \frac{d}{dx}((2x+3)^3)}{((4x^2-1)^8)^2}$$

$$= \frac{-(2x+3)^3 \cdot 8(4x^2-1)^7 \cdot \frac{d}{dx}(4x^2-1) + (4x^2-1)^8 \cdot 3(2x+3)^2 \cdot \frac{d}{dx}(2x+3)}{(4x^2-1)^{16}}$$

$$= \frac{-8(2x+3)^3(4x^2-1)^7 \cdot 8x + 3(4x^2-1)^8(2x+3)^2 \cdot 2}{(4x^2-1)^{16}}$$

$$= \frac{-16x(2x+3)^3(4x^2-1)^7 + 6(4x^2-1)^8(2x+3)^2}{(4x^2-1)^{16}}$$

44

$y = \sin(1+x^3)$; $x = -3$

$$y = \sin(1+(-3)^3)$$

$$= \sin(1-27)$$

$$= \sin(-26)$$

$$= -0.44$$

$\cancel{m_{tan}}$

$x = -3$

$$m_{tan} = \frac{d}{dx}\left(\sin(1+x^3)\right)$$

$$= \cos(1+x^3) \cdot \frac{d}{dx}(1+x^3)$$

$$= \cos(1+x^3)(0+3x^2)$$

$$= \cos(1+x^3)(3x^2)$$

$$= \cos(1+(-3)^3)(3\times(-3)^2)$$

$$= \cos(-26)(\cancel{18})\,27$$

$$= 0.898 \times \cancel{18}\,27$$

$$= \cancel{16.17}$$

$$= 24.26$$

$\therefore$

$$y - y_1 = m_{tan}(x - x_1)$$

$x = -3$

$y = -0.44$

$$y - (-0.44) = 24.26(x+3)$$

$$y + 0.44 = 26.24 + 72.8$$

$\tan x = \sec^2 x$

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47

$y = \tan(4x^2)$; $x = \sqrt{\pi}$

$y = \tan\left(4 \times (\sqrt{\pi})^2\right)$

$= \tan(4\pi)$

$= 0$

$m_{\tan} = \frac{d}{dx}\left(\tan(4x^2)\right)$

$= \sec^2 x \frac{d}{dx}(4x^2)$

$= \sec^2 4x^2 \cdot 8x$

$= (\sec 4x^2)^2 \cdot 8x$

$= \left(\sec 4(\sqrt{180})\right)^2 \cdot 8\sqrt{180}$

$= \frac{1}{(\cos 4\sqrt{180})^2} \cdot 8 \cdot \sqrt{180}$

$= \cancel{110.34}$

$= 1 \times 8\sqrt{180}$

$\therefore y - y_1 = m_{\tan}(x - x_1)$

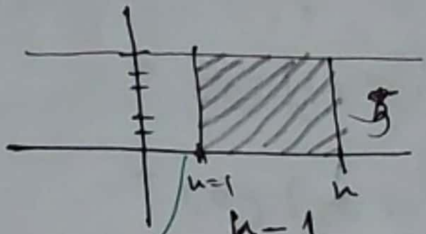
$y - 0 = 8\sqrt{180}(x - \sqrt{\pi})$

$y = 8\sqrt{180}(x - \sqrt{\pi})$ $\longrightarrow$ ~~A~~

5.1

14

$f(x) = y = 5; \quad [a, x] = [1, x]$



Rectangle Area = a×b

$\therefore A(x) = a \times b$

$= (x-1) \times 5$

$= 5x - 5$

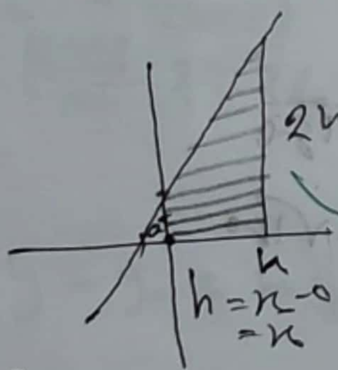
$A'(x) = \frac{d}{dx}(5x-5)$

$= 5.1 - 0$

$= 5$ (proved)

15

$f(x) = 2x+2 \quad ; \quad [0, x]$



Trapezium $= \frac{1}{2}(a+b)h$

$a = 2x+2$

$= 2\times 0+2$

$= 2$

$\therefore A(x) = \frac{1}{2}(a+b)\times h$

$= \frac{1}{2}(2+2x+2)\times x$

$= \frac{1}{2}\cdot 2(x+2)\cdot x$

$= x^2+2x$

$A'(x) = \frac{d}{dx}(x^2+2x)$

$= 2x+2\cdot 1$

$= 2x+2$ proved

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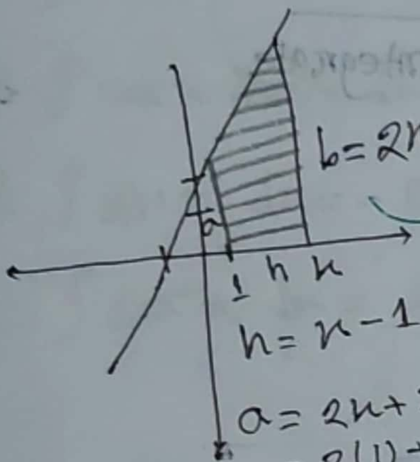
17

$f(x) = 2x+2 \quad ; [1, x]$



$b = 2x+2$

Trapezium Area $= \frac{1}{2}(a+b)h$

$h = x-1$

$a = 2x+2$
$= 2(1)+2$
$= 4$

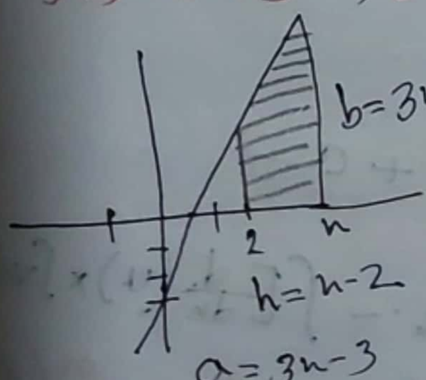
$\therefore A(x) = \frac{1}{2}(A+B)h$
$= \frac{1}{2}(4+2x+2)(x-1)$
$= \frac{1}{2}(4x+2x^2+\cancel{2x}-4-\cancel{2x}-2)$
$= \frac{1}{2}(4x+2x^2-6)$
$= \frac{1}{2}\times 2(2x+x^2-3)$
$= (2x+x^2-3)$

$\cancel{A'(x)}$

$\therefore A'(x) = \frac{d}{dx}(2x+x^2-3)$
$= 2+2x-0$
$= 2x+2$
(proved)

18

$f(x) = 3x-3 \quad ; [2, x]$



$b = 3x-3$

$h = x-2$

$a = 3x-3$
$= 3\times2-3$
$= 3$

$\therefore A(x) = \frac{1}{2}(a+b)\times h$
$= \frac{1}{2}(\cancel{3}+3x-\cancel{3})(x-2)$
$= \frac{1}{2}(3x^2-6x)$
$= \frac{3x^2}{2}-3x$

$\therefore A'(x) = \frac{d}{dx}\left(\frac{3}{2}x^2-3x\right)$
$= \frac{3}{2}\times 2x-3$
$= 3x-3$ (proved)

Integration

$y = x^2$ $\qquad A \xrightarrow{\text{differentiat}} B$

$\frac{dy}{dx} = 2x$ $\qquad \xleftarrow{\text{integrate}}$

$$\int 2x\,dx = 2 \cdot \frac{x^{1+1}}{1+1}$$

$$= 2 \cdot \frac{x^2}{2} = x^2$$

Technique of intigration

① $\int x^n dx = \frac{x^{n+1}}{n+1} + c$ $\boxed{n \neq -1}$

② $\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{(n+1) \times a} + c \longrightarrow$ [if function is linear

③ $\int e^x dx = e^x + c$

④ $\int e^{ax+b} dx = \frac{e^{ax+b}}{a} + c \longrightarrow$

⑤ $\int \frac{1}{x} dx = \ln x + c$

⑥ $\int \frac{1}{ax+b} dx = \frac{\ln(ax+b)}{a} + c$

7) $\int uv\,dx = u \int v\,dx - \int \left[\frac{d}{dx}(u) \times \int v\,dx\right] dx$

order for holding $u \rightarrow \ln x \rightarrow x^n \rightarrow e^x / \sin x$

⑧ $\int \ln x \, dx = x \ln x - x + C$

⑨ $\int \operatorname{cosec}^2 x \, dx = -\cot x + C$

⑩ $\int \sec x \tan x \, dx = \sec x + C$

⑪ $\int \sin x \, dx = -\cos x + C$

⑫ $\int \sec^2 x \, dx = \tan x + C$

Integration

1. $\int 1 \, dx = x + c$
2. $\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$
3. $\int \cos x \, dx = \sin x + c$
4. $\int \sin x \, dx = -\cos x + c$
5. $\int \sec^2 x \, dx = \tan x + c$
6. $\int \operatorname{cosec}^2 x \, dx = -\cot x + c$
7. $\int \sec x . \tan x \, dx = \sec x + c$
8. $\int \operatorname{cosec} x . \cot x \, dx = -\operatorname{cosec} x + c$
9. $\int \tan x \, dx = \ln|\sec x| + c$
10. $\int \frac{1}{x} \, dx = \ln|x| + c$
11. $\int e^x \, dx = e^x + c$
12. $\int a^x \, dx = \frac{a^x}{\ln a} + c$
13. $\int \ln x \, dx = x \ln x - x + c$
14. $\int \frac{1}{\sqrt{1-x^2}} \, dx = \sin^{-1} x + c$
15. $\int -\frac{1}{\sqrt{1-x^2}} \, dx = \cos^{-1} x + c$
16. $\int \frac{1}{1+x^2} \, dx = \tan^{-1} x + c$
17. $\int -\frac{1}{1+x^2} \, dx = \cot^{-1} x + c$
18. $\int \frac{1}{x\sqrt{x^2-1}} \, dx = \sec^{-1} x + c$
19. $\int -\frac{1}{x\sqrt{x^2-1}} \, dx = \operatorname{cosec}^{-1} x + c$

- $\int \cot x \, dx = \ln|\sin x| + c$
- $\int \sec x \, dx = \ln|\sec x + \tan x| + c$
- $\int \operatorname{cosec} x \, dx = \ln|\operatorname{cosec} x - \cot x| + c$
- $\int \tan x \, dx = -\ln|\cos x| + c$

CamScanner

ঘ Hyperbolic Function

1. $\int \sinh x \, dx = \cosh x + c$
2. $\int \cosh x \, dx = \sinh x + c$
3. $\int \operatorname{sech}^2 x \, dx = \tanh x + c$
4. $\int \operatorname{csch}^2 x \, dx = -\coth x + c$
5. $\int \operatorname{sech} x \tanh x \, dx = -\operatorname{sech} x + c$
6. $\int \operatorname{csch} x \coth x \, dx = -\operatorname{csch} x + c$

ঙ Algebric Function $(a>0)$

1. $\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\frac{x}{a} + c \quad (|x|<a)$
2. $\int \frac{1}{a^2+x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$
3. $\int \frac{1}{x\sqrt{x^2-a^2}} dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + c \quad (0<a<|u|)$

⊛ $\int \frac{dx}{\sqrt{a^2+x^2}} = \ln$

Example → 1

1) $\int x^2\,dx$

$= \dfrac{x^{2+1}}{2+1}+c$

$= \dfrac{x^3}{3}+c$

2) $\int \dfrac{1}{x^5}\,dx$

$= \int x^{-5}\,dx$

$= \dfrac{x^{-5+1}}{-5+1}+c$

$= \dfrac{x^{-4}}{-4}+c$

3) $\int 2x\,dx$

$= 2\left(\dfrac{x^{1+1}}{1+1}+c\right)$

$= x^2+c$

4) $\int x^3\,dx = \dfrac{x^{3+1}}{3+1}+c$

$= \dfrac{x^4}{4}+c$

5) $\int \sqrt{x}\,dx$

$= \int x^{\frac{1}{2}}\,dx$

$= \dfrac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}+c$

$= \dfrac{x^{\frac{3}{2}}}{\frac{3}{2}}+c$

5) $\dfrac{d}{dx}\left(\sqrt{x^3+5}\right)$

$= \dfrac{1}{2\sqrt{x^3+5}}\dfrac{d}{dx}(x^3+5)$

$= \dfrac{1}{2\sqrt{x^3+5}}\cdot 3x^2$

So, $\int \dfrac{3x^2}{2\sqrt{x^3+5}}\,dx$ Let $z = x^3+5$

$\dfrac{dz}{dx} = 3x^2$

$dz = 3x^2\,dx$

$\int \dfrac{1}{2\sqrt{x^3+5}}\cdot dz$

$\cancel{\dfrac{1}{2}\cdot\dfrac{}{(\sqrt{x^3+5})}}$

$$\Rightarrow \int \frac{1}{2} \frac{1}{\sqrt{x^3+5}}\, dz$$

$$= \frac{1}{2} \int \frac{1}{\sqrt{z}}\, dz$$

$$= \frac{1}{2} \int z^{-\frac{1}{2}}\, dz$$

$$= \frac{1}{2} \cdot \frac{z^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + c$$

$$= \frac{1}{\cancel{2}} \cdot \frac{z^{+\frac{1}{2}}}{\frac{1}{\cancel{2}}} + c$$

$$= z^{+\frac{1}{2}} + c$$

$$= \sqrt{z} + c$$

$$= \sqrt{x^3+5} + c$$

[প্রথমে differentiation করে পরে integration করতে হবে।

integration করার সময় যদি function ~~টি~~ Direct কোন Law না যায় ~~তাহলে~~ তাহলে কোনো কিছুকে ধরে Differentiate করতে হবে।

7) $\frac{d}{dx}\left(\sin(2\sqrt{x})\right)$

$$\Rightarrow \cos(2\sqrt{x}) \cdot \frac{d}{dx}(2\sqrt{x})$$

$$= \cos(2\sqrt{x}) \cdot \cancel{2} \cdot \frac{1}{\cancel{2}\sqrt{x}}$$

$$= \cos(2\sqrt{x}) \cdot \frac{1}{\sqrt{x}}$$

$$\int \cos(2\sqrt{x}) \cdot \frac{1}{\sqrt{x}}\, dx$$

$$= \int \cos(2\sqrt{x}) \cdot x^{-\frac{1}{2}}\, dx$$

$$= \frac{\sin(2\sqrt{x})}{\cancel{2}} \cdot \frac{x^{+\frac{1}{2}}}{\frac{1}{\cancel{2}}} + c$$

$$= \sin(2\sqrt{x}) \cdot \sqrt{x} + c$$

$$\int \cos(2\sqrt{x}) \cdot \frac{1}{\sqrt{x}}\, dx$$

$z = \cancel{\cos(\sqrt{x})}\ 2\sqrt{x}$

$\frac{dz}{dx} = \cancel{\sin(2\sqrt{x})}\ 2 \cdot \frac{1}{2\sqrt{x}} = \frac{1}{\sqrt{x}}$

$dz = \frac{dx}{\sqrt{x}}$, $\quad dx = dz\,\sqrt{x}$

$$\int \cos(z) \cdot \frac{1}{\cancel{\sqrt{x}}} \cancel{\sqrt{x}}\, dz$$

$$\int \cos(z) \cdot dz$$

$$= \sin z + c = \sin(2\sqrt{x}) + c$$

8

$\frac{d}{dx}[\sin x - x\cos x]$

$= \cancel{\cos x - 1(-\sin x)}$

$= \cancel{\cos x + \sin x}$

$= \cos x - \left\{x\frac{d}{dx}(\cos x) + \cos x\frac{d}{dx}(x)\right\}$

$= \cos x - (x(-\sin x) + \cos x \cdot 1)$

$= \cos x + x\sin x - \cos x$

$= x\sin x$

$\cancel{\int x\,dx}$

$\int x\sin x\,dx$

$= x\int \sin x\,dx - \int\left[\frac{d}{dx}(x)\cdot\int \sin x\,dx\right]dx$

$= x(-\cos x) - \int[1\cdot -\cos x]\,dx$

$= -x\cos x - (-\sin x) + C$

$= -x\cos x + \sin x + C$

$= \sin x - x\cos x + C$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

15

$$\int x(1+x^3)\,dx$$

$$= \int (x + x^4)\,dx$$

$$= \frac{x^{1+1}}{1+1} + \frac{x^{4+1}}{4+1} + c$$

$$= \frac{x^2}{2} + \frac{x^5}{5} + c$$

$$\frac{d}{dx}\left(\frac{x^2}{2} + \frac{x^5}{5} + c\right)$$

$$= \frac{1}{\cancel{2}}\cdot\cancel{2}x + \frac{1}{\cancel{5}}\cdot\cancel{5}x^4 + 0$$

$$= x + x^4$$

$$= x(1+x^3)$$

(proved)

16

$$\int (2+y^2)^2\,dy$$

$$\int (4 + 4y^2 + y^4)\,dy$$

$$= 4y + \frac{4y^{2+1}}{2+1} + \frac{y^{4+1}}{4+1} + c$$

$$= 4y + \frac{4}{3}y^3 + \frac{1}{5}y^5 + c$$

$$\frac{d}{dy}\left(4y + \frac{4}{3}y^3 + \frac{1}{5}y^5 + c\right)$$

$$= 4 + 4\cdot y^2 + \frac{1}{\cancel{5}}\cdot\cancel{5}y^4$$

$$= y + 4y^2 + y^4$$

$\therefore$ proved

17)

$$\int x^{\frac{1}{3}}(2-x)^2\,dx$$

$$= \int x^{\frac{1}{3}}(4 - 4x + x^2)\,dx$$

$$= \int \left(4x^{\frac{1}{3}} - 4x^{\frac{1}{3}+1} + x^{\frac{1}{3}+2}\right)dx$$

$$= \int \left(4x^{\frac{1}{3}} - 4x^{\frac{4}{3}} + x^{\frac{7}{3}}\right)dx$$

$$= \frac{4x^{\frac{1}{3}+1}}{\frac{1}{3}+1} - \frac{4x^{\frac{4}{3}+1}}{\frac{4}{3}+1} + \frac{x^{\frac{7}{3}+1}}{\frac{7}{3}+1} + c$$

$\int \frac{1}{x} dx = \ln|x| + c$ $\int \sin x \, dx = -\cos x + c$

$\int \sec^2 x \, dx = \tan x + c$

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18

$\int (1+x^2)(2-x) \, dx$

$= \int (2 + x + 2x^2 - x^3) \, dx$

$= 2x + \frac{x^{1+1}}{2} + 2 \cdot \frac{x^3}{3} - \frac{x^4}{4} + c$

19

$\int \frac{x^5 + 2x^2 - 1}{x^4} dx$

$= \int \left(\frac{x^5}{x^4} + \frac{2x^2}{x^4} - \frac{1}{x^4}\right) dx$

$= \int (x + 2x^{-2} - x^{-4}) \, dx$

$= \frac{x^2}{2} + 2 \cdot \frac{x^{-1}}{-1} - \frac{x^{-3}}{-3} + c$

$= \frac{x^2}{2} - 2 \cdot x^{-1} + \frac{x^{-3}}{3} + c$

20

$\int \frac{1-2t^3}{t^3} dt$

$= \int \left(\frac{1}{t^3} - \frac{2t^3}{t^3}\right) dx$

$= \int (t^{-3} - 2.1) \, dx$

$= \frac{t^{-2}}{-2} - 2x + c$

21

$\int \left[\frac{2}{x} + 3e^x\right] dx$

$= \int \left[2 \cdot \frac{1}{x} + 3e^x\right] dx$

$= 2 \cdot \ln|x| + 3 \cdot e^x + c$

22

$\int \left[\frac{1}{2t} - \sqrt{2} e^t\right] dt$

$= \int \left[\frac{1}{2} \frac{1}{t} - \sqrt{2} \cdot e^t\right] dt$

$= \frac{1}{2} \cdot \ln|t| - \sqrt{2} \cdot e^t + c$

23

$\int [3\sin x - 2\sec^2 x] \, dx$

$= 3(-\cos x) - 2(\tan x) + c$

$= -3\cos x - 2\tan x + c$

$\int \csc^2 x \, dx = -\cot x + c$

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$\int \sec x \tan x \, dx = \sec x + c$

24

$\int [\csc^2 t - \sec t \tan t] \, dx$

$= -\cot x - \sec x + c$

$= -\cot t - \sec t + c$

25

$\int \sec x (\sec x + \tan x) \, dx$

$= \int (\sec^2 x + \sec x \tan x) \, dx$

$= \tan x + \sec x + c$

26

$\int \csc x (\sin x + \cot x) \, dx$

$= \int (\csc x . \sin x + \csc x . \cot x) \, dx$

$= \int \left(\frac{1}{\sin x} \sin x + \csc x . \cot x\right) dx$

$= \int (1 + \csc x . \cot x) \, dx$

$= x + (-\csc x) + c$

$= x - \csc x + c$

27

$\int \frac{\sec \theta}{\cos \theta} \, d\theta$

$= \int \frac{\frac{1}{\cos \theta}}{\cos \theta} \, d\theta$

$= \int \frac{1}{\cos^2 \theta} \, d\theta$

$= \int \sec^2 \theta \, d\theta$

$= \tan \theta + c$

28

$\int \frac{dy}{\csc y}$

$= \int \frac{1}{\csc y} \, dy$

$= \int \sin y \, dy$

$= -\cos y + c$

29

$\int \frac{\sin x}{\cos^2 x} \, dx$

$= \int \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} \, dx$

$= \int \sec x . \tan x \, dx$

$= \sec x + c$

30

$\int \left[\phi + \frac{2}{\sin^2 \phi}\right] d\phi$

$= \int [\phi + 2 . \csc^2 \phi] \, d\phi$

$= \frac{\phi^2}{2} + 2 . (-\cot \phi) + c$

$= \frac{\phi^2}{2} - 2\cot \phi + c$

5.3

$\cos 3A = 4\cos^3 A - 3\cos A$, $4\cos^3 A = \cos 3A + 3\cos A$

$\cos^3 A = \frac{1}{4}(\cos 3A + 3\cos A)$

$\int \cos^3 A \, dA$

$= \int \frac{1}{4}(\cos 3A + 3\cos A)\, dA$

$= \frac{1}{4}\left[\frac{\sin 3A}{3} + 3\sin A\right] + c$

linear

$\int \cos 3A = \frac{\sin 3A}{3}$

যদি x এর সাথে constant গুন আকারে থাকে তাহলে তা ভাগ হয়ে যাবে।

$\sin 3A = \sin(2A + A)$

$= \sin 2A \cos A + \cos 2A \sin A$

$= 2\sin A \cdot \cos A \cdot \cos A + (1 - 2\sin^2 A)\sin A$

$= 2\sin A \cdot \cos^2 A + \sin A - 2\sin^3 A$

$= 2\sin A(1 - \sin^2 A) + \sin A - 2\sin^3 A$

$= 2\sin A - 2\sin^3 A + \sin A - 2\sin^3 A$

$= 3\sin A - 4\sin^3 A$

$\cancel{= (3\sin A - 4\sin^3 A) - 4\sin^3 A}$

$\sin(A + B)$

$= \sin A \cos B + \cos A \sin B$

$\cos 2A$

$= (1 - 2\sin^2 A)$

$\sin 2A = \cancel{\sin 2A}$

$= 2\sin A \cdot \cos A$

$\cos^2 A = 1 - \sin^2 A$

$\therefore \sin 3A = 3\sin A - 4\sin^3 A$

$\therefore \sin 3A = 3\sin A - 4\sin^3 A$

4 Integration of exponential function

$$\int e^{x}\,dx = e^{x} + c$$

$$\int e^{ax+b}\,dx = \frac{e^{ax+b}}{a} + c$$

Ex: $\int e^{4-3x}\,dx = \frac{e^{4-3x}}{-3} + c$

4 Integration of logarithmic function

$$\int \frac{1}{x}\,dx = \ln x + c$$

$$\int \frac{1}{ax+b}\,dx = \frac{\ln(ax+b)}{a} + c$$

Ex: ① $\int \frac{1}{3x+5}\,dx = \frac{\ln(3x+5)}{3} + c$

② $\int \frac{1}{3-7x}\,dx = \frac{\ln(3-7x)}{-7} + c$

Integration by parts

$$\int uv\,dx = u\int v\,dx - \int\left[\frac{d}{dx}(u)\cdot\int v\,dx\right]dx$$

Ex: $\int x^2 e^{2x}\,dx$

$$= x^2\int e^{2x}\,dx - \int\left[\frac{d}{dx}(x^2)\times\int(e^{2x})\,dx\right]dx$$

$$= x^2\frac{e^{2x}}{2} - \int\left[\cancel{2}x\cdot\frac{e^{2x}}{\cancel{2}}\right]dx$$

$$= \frac{x^2\cdot e^{2x}}{2} - \left[x\int e^{2x}\,dx - \int\left[\frac{d}{dx}(x)\cdot\int e^{2x}\,dx\right]dx\right]$$

$$= \frac{x^2\cdot e^{2x}}{2} - \left[x\cdot\frac{e^{2x}}{2} - \int\left[\frac{e^{2x}}{2}\right]dx\right]$$

$= \cancel{x^2 e}$

$$= \frac{x^2 e^{2x}}{2} - \left[\frac{xe^{2x}}{2} - \int\left[\frac{1}{2}e^{2x}\right]\right]$$

$$= \frac{x^2 e^{2x}}{2} - \left[\frac{xe^{2x}}{2} - \frac{1}{2}\cdot\frac{e^{2x}}{2}\right] + c$$

Ex: $\int x\sin x\,dx$

$$= x\int\sin x\,dx - \int\left[\frac{d}{dx}(x)\int(\sin x)\,dx\right]dx$$

$$= x(-\cos x) - \int[1\times-\cos x]\,dx$$

$$= -x\cos x + \sin x + c$$

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Ex: $\int \underset{v}{x} \underset{u}{\ln x}\, dx$

$\int (\ln x)\, x\, dx$

$= \ln x \int x\, dx - \int \left[\frac{d}{dx}(\ln x) \int x\, dx\right] dx$

$= \ln x \frac{x^2}{2} - \int \left[\frac{1}{x} \cdot \frac{x^2}{2}\right] dx$

$= \ln x \frac{x^2}{2} - \int \left[\frac{1}{2} x\right] dx$

$= \ln x \frac{x^2}{2} - \frac{1}{2} \frac{x^2}{2} + c$

Ex: $\int e^{-x} \sin x\, dx = I$

$I = e^{-x} \int \sin x\, dx - \int \left[\frac{d}{dx}(e^{-x}) \int \sin x\, dx\right] dx$

$= e^{-x}(-\cos x) - \int [-e^{-x} \cdot (-\cos x)]\, dx$

$= -e^{-x} \cos x - \int e^{-x} \cos x\, dx$

$= -e^{-x} \cos x - \left[e^{-x} \int \cos x - \int \left[d/dx(e^{-x}) \int \cos x\, dx\right] dx\right]$

$= -e^{-x} \cos x - \left[e^{-x}(\sin x) - \int [-e^{-x} \sin x]\, dx\right]$

$= -e^{-x} \cos x - e^{-x} \sin x - \int [e^{-x} \sin x]\, dx$

$= -e^{-x} \cos x - e^{-x} \sin x - I$

$2I = -e^{-x} \cos x - e^{-x} \sin x$

$I = \frac{1}{2}(-e^{-x} \cos x - e^{-x} \sin x)$

Integration by using appropriate substitutions

→ যা নিয়ম জানি ও নিয়মে করতে পারি কিনা তা দেখবো প্রথমে

→ compleeate part কে substitu করবো।

→ সেই টা কে substitu করলে ~~একটি~~ dx এর সাথে মিল

→ substitute করার পর তাকে x এর সাপেক্ষে Differentiate করতে হবে

শেষে আসলটি মানে তাতে বসাবো

$\int x e^{x^2+1} dx$

$u = x^2+1$

$\cancel{\frac{d}{dx}(x^2+1)}$

$= \int e^{u} \cdot x\,dx$

$\frac{du}{dx} = \frac{d}{dx}(x^2+1)$

$= \int e^{u} \cdot \frac{1}{2} du$

$\Rightarrow \frac{du}{dx} = 2x$

$= \frac{1}{2} e^{u} + c$

$\Rightarrow x\,dx = \frac{du}{2}$

$= \frac{1}{2} e^{x^2+1} + c$

$\int \frac{\cos x}{\sin x} dx$

$u = \sin x$

$\frac{du}{dx} = \frac{d}{dx}(\sin x)$

$= \int \frac{1}{u} du$

$\frac{du}{dx} = \cos x$

$\rightarrow dx \cos x = du$

$= \ln(u) + c$

$= \ln(\sin x) + c$

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(*) $\int x^2\sqrt{x^3+1}\,dx$

$= \int \sqrt{x^3+1}\cdot x^2\,dx$

$= \int \sqrt{u}\cdot \frac{1}{3}du$

$= \frac{1}{3}\int u^{\frac{1}{2}}du$

$= \frac{1}{3}\left(\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right)+c$

$= \frac{1}{3}\left(\frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right)+c$

$= \frac{2}{9}(x^3+1)^{\frac{3}{2}}+c$

$u = x^3+1$

$\frac{du}{dx} = \frac{d}{dx}(x^3+1)$

$\frac{du}{dx} = 3x^2$

$dx\,3x^2 = du$

$x^2dx = \frac{1}{3}du$

$\int \frac{\ln x}{x}dx$

$= \int \ln x\cdot\frac{1}{x}dx$

$= \int u\,du$

$= \frac{u^{1+1}}{1+1}+c$

$= \frac{1}{2}u^2+c$

$= \frac{1}{2}(\ln x)^2+c$

$u = \ln x$

$\frac{du}{dx} = \frac{d}{dx}(\ln x)$

$\frac{du}{dx} = \frac{1}{x}$

$\Rightarrow \cancel{dx = dux}$

$du = \frac{1}{x}dx$

$(\log x)^2 \neq \log x^2$

15

$$\int (4x-3)^9\, dx$$

Let, $u = 4x-3$

$\Rightarrow \frac{du}{dx} = \frac{d}{dx}(4x-3)$

$\Rightarrow \frac{du}{dx} = 4$

$\Rightarrow dx = \frac{1}{4}du$

$$= \int u^9 \cdot \frac{1}{4} du$$

$$= \int \frac{1}{4} u^9 du$$

$$= \frac{1}{4} \cdot \frac{u^{9+1}}{9+1} + c$$

$$= \frac{1}{4} \cdot \frac{(4x-3)^{10}}{10} + c$$

16

$$\int x^3 \sqrt{5+x^4}\, dx$$

Let, $u = 5+x^4$

$\Rightarrow \frac{du}{dx} = \frac{d}{dx}(5+x^4)$

$\Rightarrow \frac{du}{dx} = 4x^3$

$\Rightarrow x^3 dx = \frac{1}{4} du$

$$= \int \sqrt{5+x^4}\, x^3\, dx$$

$$= \int \sqrt{u} \cdot \frac{1}{4} du$$

$$= \frac{1}{4} \int u^{\frac{1}{2}} du$$

$$= \frac{1}{4} \cdot \frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c$$

17

$$\int \sin 7x\, dx$$

Let $u = 7x$

$\Rightarrow \frac{du}{dx} = \frac{d}{dx}(7x)$

$\Rightarrow \frac{du}{dx} = 7$

$\Rightarrow dx = \frac{1}{7} du$

$$= \int \sin u \, \frac{1}{7} du$$

$$= \frac{1}{7}(-\cos u) + c$$

$$= -\frac{1}{7} \cos(7x) + c$$

18

$\int \cos \frac{x}{3}\, dx$

Let, $u = \frac{x}{3}$

$\frac{du}{dx} = \frac{d}{dx}\left(\frac{1}{3} \cdot x\right)$

$\Rightarrow \frac{du}{dx} = \frac{1}{3}$

$\Rightarrow dx = 3\, du$

$= \int \cos u \cdot 3\, du$

$= 3 \sin u + C$

$= 3 \sin\left(\frac{x}{3}\right) + C$

19

$\int \sec 4x \cdot \tan 4x\, dx$

Let $u = 4x$

$\frac{du}{dx} = 4$

$\Rightarrow dx = \frac{1}{4}\, du$

$= \int \sec u \cdot \tan u \cdot \frac{1}{4}\, du$

$= \frac{1}{4} \cdot \sec u + C$

$= \frac{1}{4} \cdot \sec 4x + C$

20

$\int \sec^2 5x\, dx$

let $u = 5x$

$\frac{du}{dx} = 5$

$dx = \frac{1}{5}\, du$

$= \int \sec^2 u \cdot \frac{1}{5}\, dx$

$= \frac{1}{5} \cdot \tan u + C$

$= \frac{1}{5} \cdot \tan 5x + C$

21

$$\int e^{2x}\,dx \quad \text{let } u = 2x$$

$$\frac{du}{dx} = 2$$

$$dx = \frac{1}{2}du$$

$$= \int e^{u}\cdot\frac{1}{2}\,du$$

$$= e^{u}\cdot\frac{1}{2} + c$$

$$= \frac{1}{2}\cdot e^{2x} + c$$

22

$$\int \frac{1}{2x}\cdot dx \quad \text{let, } u = 2x$$

$$\frac{du}{dx} = 2$$

$$dx = \frac{1}{2}du$$

$$= \int \frac{1}{u}\cdot\frac{1}{2}\,du$$

$$= \frac{1}{2}\ln|u| + c$$

$$= \frac{1}{2}\ln|2x| + c$$

23

$$\int \frac{1}{\sqrt{1-4x^2}}\,dx \quad \text{Let } u = \cancel{4x^2}\ 2x$$

$$\frac{du}{dx} = 2$$

$$\Rightarrow dx = \frac{1}{2}du$$

$$= \int \frac{1}{\sqrt{1-(2x)^2}}\,dx$$

$$= \int \frac{1}{\sqrt{1-u^2}}\cdot\frac{1}{2}\,du$$

$$= \frac{1}{2}\sin^{-1}u + c$$

$$= \frac{1}{2}\sin^{-1}(2x) + c$$

24

$$\int \frac{dx}{1+16x^2}$$

Let ~~4x~~ $u = 4x$

$$\frac{du}{dx} = 4$$

$$dx = \frac{1}{4} du$$

$$= \int \frac{1}{1+(4x)^2} dx$$

$$= \int \frac{1}{1+u^2} \cdot \frac{1}{4} du$$

$$= \frac{1}{4} \cdot \tan^{-1} u + c$$

$$= \frac{1}{4} \cdot \tan^{-1} 4x + c$$

27

$$\int \frac{6}{(1-2x)^3} dx$$

Let $1-2x = u$

$$\frac{du}{dx} = -2$$

$$\Rightarrow dx = -\frac{1}{2} du$$

$$= \int \frac{6}{u^3} \cdot -\frac{1}{2} du$$

$$= \int -\frac{6}{2} \cdot u^{-3} du$$

$$= -3 \cdot \frac{u^{-3+1}}{-3+1} + c$$

$$= -3 \cdot \frac{(1-2x)^{-2}}{-2} + c$$

28

$$\int \frac{x^2+1}{\sqrt{x^3+3x}}\,dx$$

Let, $u = x^3+3x$

$\Rightarrow \frac{du}{dx} = 3x^2+3$

$\Rightarrow \frac{du}{dx} = 3(x^2+1)$

$\Rightarrow \cancel{dx = \frac{1}{3(x^2+1)}}\,du$

$\Rightarrow dx\,(x^2+1) = \frac{1}{3}\,du$

$= \int \frac{\cancel{x^2+1}}{\sqrt{u}} \cdot (x^2+1)\,dx$

$= \int u^{-\frac{1}{2}} \cdot \frac{1}{3}\,du$

$= \frac{1}{3} \cdot \frac{u^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + c$

$= \frac{1}{3} \cdot \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + c$

$= \frac{2}{3} \cdot (x^3+3x)^{\frac{1}{2}} + c$

30

$$\int \frac{\sin\left(\frac{1}{x}\right)}{3x^2}\,dx$$

Let $u = \frac{1}{x}$

$\frac{du}{dx} = -\frac{1}{x^2}$

$\frac{dx}{x^2} = -\,du$

$= \int \frac{1}{3} \cdot \sin(u) \cdot \frac{1}{x^2}\,dx$

$= \int \frac{1}{3} \sin(u) \cdot -\,du$

$= -\frac{1}{3} \cos u + c$

$= -\frac{1}{3} \cos\left(\frac{1}{x}\right) + c$

$= \frac{1}{3} \cos\left(\frac{1}{x}\right) + c$

31

$\int e^{\sin x} \cos x \, dx$

let $u = \sin x$

$\Rightarrow \frac{du}{dx} = \cos x$

$\Rightarrow \cos x \, dx = du$

$= \int e^{u} \cdot du$

$= e^{u} + C$

$= e^{\sin x} + C$

34

$\int \frac{e^{x}+e^{-x}}{e^{x}-e^{-x}} dx$

~~alway~~ always নিজের হাতে substitude করবো

Let $u = e^{x} - e^{-x}$

$\frac{du}{dx} = e^{x} + e^{-x}$

$\Rightarrow dx(e^{x} + e^{-x}) = du$

$= \int \frac{1}{u} \cdot du$

$= \ln|u| + C$

$= \ln|e^{x} - e^{-x}| + C$

35

$\int \frac{e^{x}}{1+e^{2x}} dx$

Let, $e^{x} = u$

$\Rightarrow \frac{du}{dx} = e^{x}$

$\Rightarrow e^{x} dx = du$

$= \int \frac{e^{x}}{1+(e^{x})^2} dx$

$= \int \frac{1}{1+u^2} \cdot du$

$= \tan^{-1} u + C$

$= \tan^{-1}(e^{x}) + C$

38

$$\int \frac{\sec^2(\sqrt{x})}{\sqrt{x}}\,dx$$

Let $u = \sqrt{x}$

$\frac{du}{dx} = \frac{1}{2\sqrt{x}}$

$\Rightarrow dx = 2\sqrt{x}\,du$

$\Rightarrow \frac{dx}{\sqrt{x}} = 2\,du$

$= \int \frac{dx}{\sqrt{x}} \cdot \sec^2 u$

$= \int \sec^2 u \cdot 2\,du$

$= 2\tan u + c$

$= 2\tan(\sqrt{x}) + c$

39

$$\int \cos^4 3t \sin 3t\, dt$$

Let $u = \cos 3t$

$\Rightarrow \frac{du}{dt} = -\sin 3t \cdot \frac{d}{dt}(3t)$

$\Rightarrow \frac{du}{dt} = -3\sin 3t$

$\Rightarrow \sin 3t\, dt = -\frac{1}{3}\,du$

$= \int (\cos 3t)^4 \cdot \sin 3t\, dt$

$= \int (u)^4 \cdot -\frac{1}{3}\,dt$

$= -\frac{1}{3} \cdot \frac{u^5}{5} + C$

$= -\frac{1}{15} \cdot (\cos 3t)^5 + C$

41

$$\int x\sec^2(x^2)\,dx$$

Let $u = x^2$

$\Rightarrow \frac{du}{dx} = 2x$

$\Rightarrow x\,dx = \frac{1}{2}\,du$

$= \int \sec^2(u) \cdot \frac{1}{2}\,du$

$= \frac{1}{2}\tan u + C$

$= \frac{1}{2} \cdot \tan(x^2) + c$

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~~56~~ 42

$$\int \frac{\cos 4\theta}{(1+2\sin 4\theta)^4}\, d\theta$$

Let, $u = 1 + 2\sin 4\theta$

$$\frac{du}{d\theta} = (0 + 2\cos 4\theta)\cdot \frac{d}{d\theta}(4\theta)$$

$$\Rightarrow \frac{du}{d\theta} = 4(2\cos 4\theta)$$

$$\Rightarrow d\theta \cos 4\theta = \frac{1}{8}\, du$$

$$= \int \frac{1}{u^4}\cdot \frac{1}{8}\, du$$

$$= \int (u^{-4})\cdot \frac{1}{8}\, du$$

$$= \frac{1}{8}\cdot \frac{u^{-4+1}}{-4+1} + c$$

$$= -\frac{1}{12}\cdot (1+2\sin 4\theta)^{-3} + c$$

43

$$\int \cos 4\theta \sqrt{2-\sin 4\theta}\, d\theta$$

let, $2 - \sin 4\theta = u$

$$\Rightarrow \frac{du}{d\theta} = (0 - \cos 4\theta)\cdot \frac{d}{d\theta}(4\theta)$$

$$= \frac{du}{d\theta} = 4\cos 4\theta$$

$$= \cos 4\theta\, d\theta = -\frac{1}{4}\, du$$

$$= \int \sqrt{u}\cdot \frac{1}{4}\, du$$

$$= \int u^{\frac{1}{2}}\cdot -\frac{1}{4}\, du$$

$$= -\frac{1}{4}\cdot \frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c$$

$$= -\frac{1}{4}\cdot \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$= -\frac{1}{6}\cdot (2-\sin 4\theta)^{\frac{3}{2}} + c$$

45

$$\int \frac{\sec^2 x\, dx}{\sqrt{1-\tan^2 x}}$$

let $u = \tan x$

$\Rightarrow \frac{du}{dx} = \sec^2 x$

$\Rightarrow du = \sec^2 x\, dx$

$$= \int \frac{du}{\sqrt{1-u^2}}$$

$$= \int \frac{1}{\sqrt{1-u^2}}\, du$$

$$= \sin^{-1} u + c$$

$$= \sin^{-1}(\tan x) + c$$

48

$$\int [\sin(\sin\theta)] \cos\theta \, d\theta$$

Let $u = \sin\theta$

$\Rightarrow \frac{du}{d\theta} = \cos\theta$

$\Rightarrow du = \cos\theta \cdot d\theta$

$$= \int [\sin(u)] \cdot du$$

$$= -\cos u + c$$

$$= -\cos(\sin\theta) + c$$

49

$$\int \frac{1}{e^x} \cdot dx$$

Let $u = -x$

$\frac{du}{dx} = -1$

$\Rightarrow du = -dx$

$\Rightarrow dx = -du$

$$\int e^{-x}\, dx$$

$$= \int e^u \cdot -du$$

$$= -e^u + c$$

$$= -e^{-x} + c$$

52

$$\int \frac{e^{\sqrt{2y+1}}}{\sqrt{2y+1}} dy$$

Let $u = \sqrt{2y+1}$

$$\frac{du}{dy} = \frac{1}{\cancel{2}}(2y+1)^{\frac{1}{2}-1} \cdot \cancel{2}$$

$$\Rightarrow \frac{du}{dy} = (2y+1)^{-\frac{1}{2}}$$

$$\Rightarrow \frac{du}{dy} = \frac{1}{\sqrt{2y+1}}$$

$$\Rightarrow \frac{du}{dy} = \frac{1}{\sqrt{2y+1}}$$

$$\Rightarrow du = \frac{dy}{\sqrt{2y+1}}$$

$$= \int e^u \, du$$

$$= e^u + c$$

$$= e^{\sqrt{2y+1}} + c$$

53

$$\int \frac{y}{\sqrt{2y+1}} dy$$

Let, $\sqrt{2y+1} = u$; $y = \frac{u-1}{2}$

$$\Rightarrow \frac{du}{dy} = \frac{1}{2\sqrt{2y+1}} \cdot \frac{d}{dy}(2y+1)$$

$$\Rightarrow \frac{du}{dy} = \frac{1}{\cancel{2}\sqrt{2y+1}} \cdot \cancel{2}$$

$$\Rightarrow du = \frac{dy}{\sqrt{2y+1}} \quad 2\,dy$$

$$\Rightarrow dy = \frac{1}{2} du$$

$$= \int \frac{y}{\sqrt{u}} \cdot \frac{1}{2} du$$

$$= \int \frac{\frac{u-1}{2}}{\sqrt{u}} \cdot \frac{1}{2} du$$

$$= \int \frac{u-1}{2} \times \frac{1}{\sqrt{u}} \cdot \frac{1}{2} du$$

$$= \int \frac{1}{4} \cdot \frac{1}{\sqrt{u}} (u-1) du$$

$$= \frac{1}{4} \cdot \frac{1}{\sqrt{u}} \cdot \int (u-1) du - \int \left[\frac{d}{du}\left(\frac{1}{\sqrt{u}}\right) \cdot \int (u-1)\right] du$$

$$= \frac{1}{4} \cdot \frac{1}{\sqrt{u}} \cdot \left(\frac{u^2}{2} - u\right) - \int \left[\frac{1}{}\right.$$

54

$$\int x\sqrt{4-x}\,dx$$

let $u = 4-x \Rightarrow x = 4-u$

$\frac{du}{dx} = -1$

$\Rightarrow dx = -du$

$$= \int (4-u)\sqrt{u}\cdot du$$

$$= -\int (4\sqrt{u} - u\sqrt{u})\,du$$

$$= -\int \left(4\cdot u^{\frac{1}{2}} - u^{1+\frac{1}{2}}\right) du$$

$$= -\int \left(4\cdot u^{\frac{1}{2}} - u^{\frac{3}{2}}\right) du$$

$$= 4\cdot\frac{u^{\frac{3}{2}}}{\frac{3}{2}} - \frac{u^{\frac{5}{2}}}{\frac{5}{2}} + c$$

56

$$\int \sec^4 3\theta\,d\theta$$

Let, $u = \tan 3\theta$

$\frac{du}{d\theta} = 3\sec^2 3\theta$

$\Rightarrow \sec^2 3\theta\,d\theta = \frac{1}{3}du$

$$= \int \sec^2 3\theta\cdot\sec^2 3\theta\,d\theta$$

$$= \int (1+\tan^2 3\theta)\cdot\sec^2 3\theta\,d\theta$$

$$= \int (1+u^2)\cdot\frac{1}{3}du$$

$$= \frac{1}{3}\left(u + \frac{u^3}{3}\right) + c$$

$$= \frac{1}{3}\left(\tan 3\theta + \frac{\tan^3 3\theta}{3}\right) + c$$

a) $\int_a^b [c f(x)]\,dx = c\int_a^b f(x)\,dx$

b) $\int_a^b [f(x)+g(x)]\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$

→ যদি বলা থাকে Geometry ~~এর~~ formula use করতে এবং signed তাহলে shape অনুযায়ী formula ~~ab~~ and যার সাপেক্ষে করবো তার axis অনুযায়ী signed হবে।

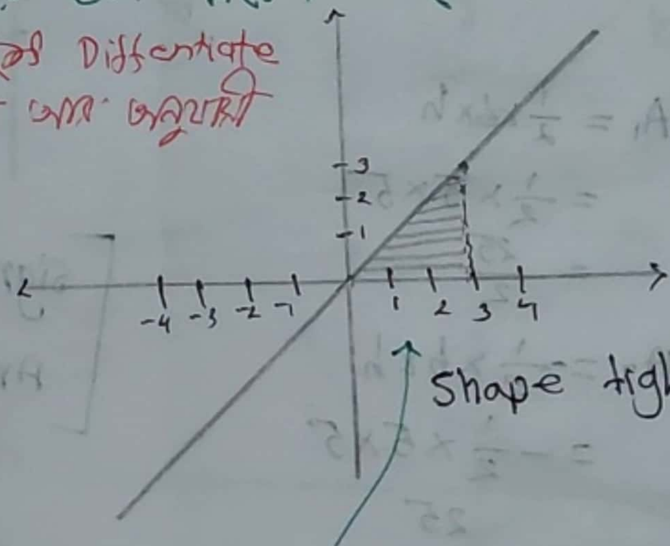
→ function এর ছবি আর্ট করে লিমিট অনুযায়ী Are বের করতে হবে।

→ যদি ~~y~~ x axis এর উভয় দিকে হয় তাহলে (+,-) দিতে হয়
যার সাপেক্ষে Differentiate করতে হবে তার অনুযায়ী

নং 13

a)

$$\int_0^3 x\,dx$$

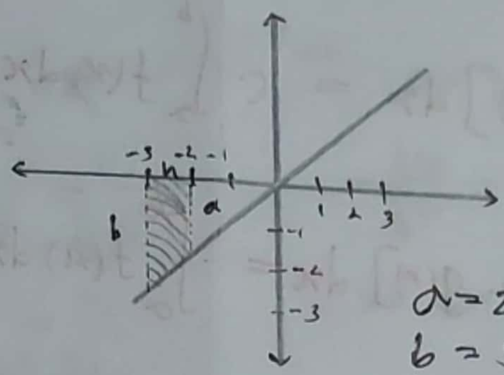


Shape right angle triangle

$$\text{Area} = \frac{1}{2}\times b\times h$$
$$= \frac{1}{2}\times 3\times 3$$
$$= +4.5 \text{ unit}$$

b/

$$\int_{-2}^{-3} x\,dx$$



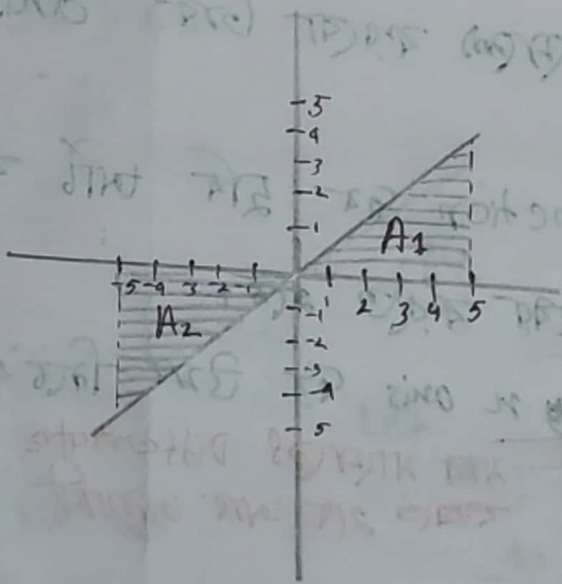
• Trapezium $\frac{1}{2}(a+b)\times h$

$a = 2$
$b = 3$
$h = 1$

$$\text{Area} = -\frac{1}{2}(a+b)\times h$$
$$= -\frac{1}{2}(2+3)\times 1$$
$$= -\frac{5}{2}$$

d

$$\int_{-5}^{5} x\,dx$$



$$A_1 = \frac{1}{2}\times b\times h$$
$$= \frac{1}{2}\times 5\times 5$$
$$= \frac{25}{2}$$

$$A_2 = -\frac{1}{2}\times b\times h$$
$$= -\frac{1}{2}\times 5\times 5$$
$$= -\frac{25}{12}$$

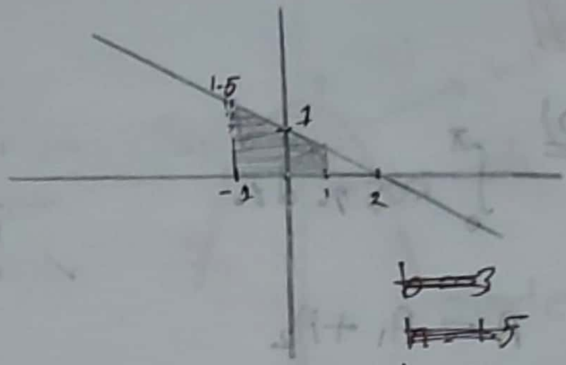
$$\therefore A = A_1 + A_2$$
$$= \frac{25}{2} + \left(-\frac{25}{2}\right)$$
$$= 0$$

[Signed হলে তবে
Area (+ -) দিবে]

14//

b//

$$\int_{-1}^{1} \left(1-\frac{1}{2}x\right) dx$$

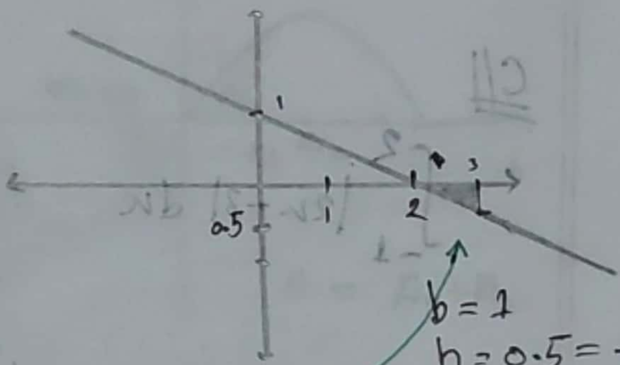


~~b = 3~~
~~h = 1.5~~
h = 2
b = 1.5
a = .5

$$\text{Area} = \frac{1}{2}(a+b)\times h$$
$$= \frac{1}{2}\left(\frac{1}{2}+\frac{3}{2}\right)\times 2$$
$$= \frac{1}{\cancel{2}}\left(\frac{1+3}{2}\right)\times \cancel{2}$$
$$= 2$$

c//

$$\int_{2}^{3} \left(1-\frac{1}{2}x\right) dx$$

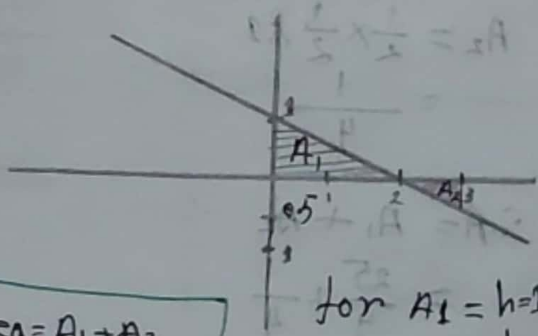


b = 1
$h = 0.5 = \frac{1}{2}$

$$\text{Area} = -\frac{1}{2}\times b\times h$$
$$= -\frac{1}{2}\times 1\times \frac{1}{2}$$
$$= -\frac{1}{4}$$

d//

$$\int_{0}^{3} \left(1-\frac{1}{2}x\right) dx$$



~~Area~~ $A_1 = \frac{1}{2}\times b\times h$
$= \frac{1}{2}\times 2\times 1$
$= 1$

$A_2 = -\frac{1}{2}\times b\times h$
$= -\frac{1}{2}\times 1\times \frac{1}{2}$
$= -\frac{1}{4}$

$$\boxed{\therefore \text{Area} = A_1 + A_2 = 1 - \frac{1}{4} = \frac{3}{4}}$$

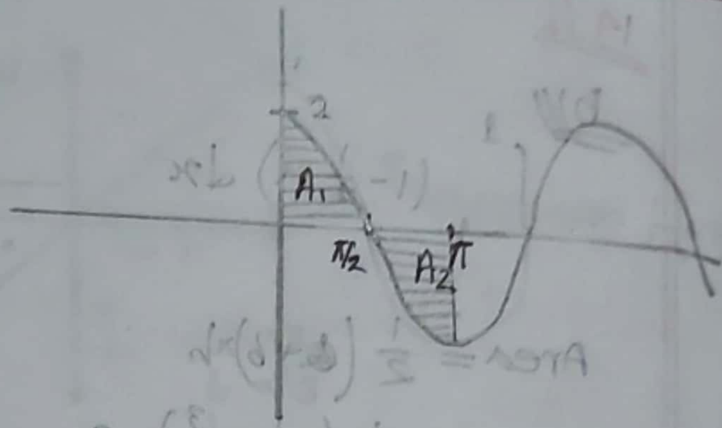
for A_1 = h = 1
b = 2

for A_2 = $h = \frac{1}{2}$
b = 1

15/

b)

$$\int_0^{\pi} \cos x \, dx$$



$A = A_1 + A_2$

$\because A_1 = -A_2$

$A_1 + A_2 = 0$

$A = 0$

c)

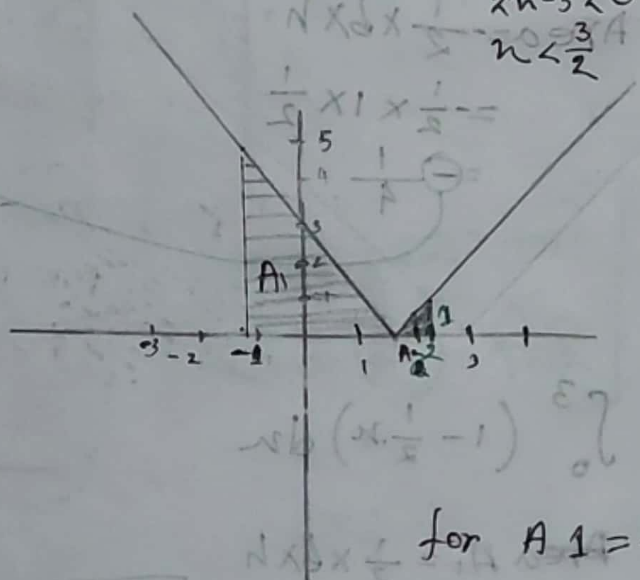
$$\int_{-1}^{2} |2x-3| \, dx$$

$$\begin{cases} 2x-3 & x \geq 0 \\ & 2x-3 \geq 0 \\ & x \geq \frac{3}{2} \\ -(2x-3) & x < 0 \\ & 2x-3 < 0 \\ & x < \frac{3}{2} \end{cases}$$

$A_1 = \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times \frac{5}{2} \times 5$

$= \frac{25}{4}$



$A_2 = \frac{1}{2} \times \frac{1}{2} \times 1$

$= \frac{1}{4}$

for $A_1 =$ $b = \frac{5}{2}$, $h =$

for $A_2 = b = \frac{1}{2}$, $h = 1$

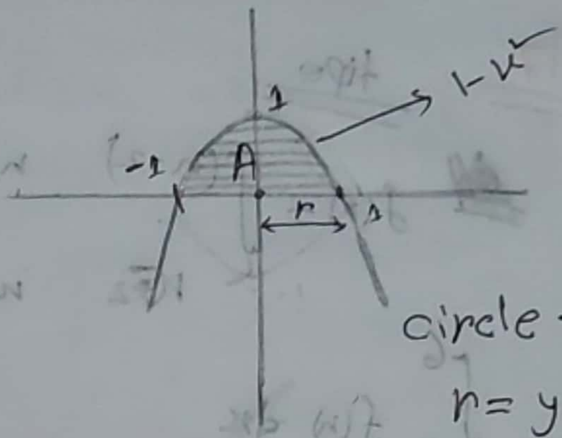
$\therefore A = A_1 + A_2$

$= \frac{25}{4} + \frac{1}{4}$

$= \frac{26}{4}$

d//

$$\int_{-1}^{1} \sqrt{1-x^2}\, dx$$



$$\text{Area} = \pi r^2 \times \frac{1}{2}$$

$$= \pi \times 1^2 \times \frac{1}{2}$$ for Half circle

$$= \frac{\pi}{2}$$

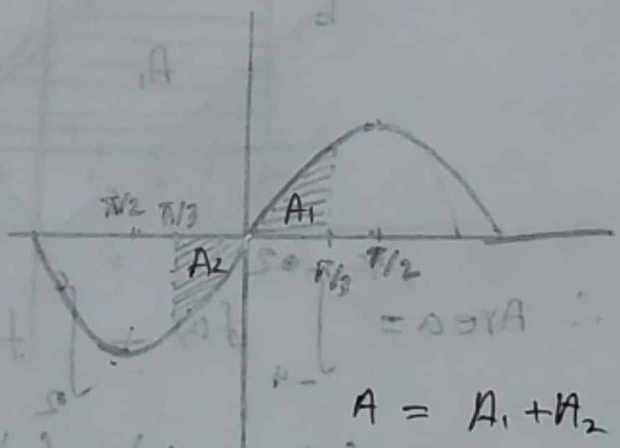
circle $\rightarrow \pi r^2$

$r = y = 1 = x$

16

b//

$$\int_{-\pi/3}^{\pi/3} \sin x \, dx$$



$A = A_1 + A_2$

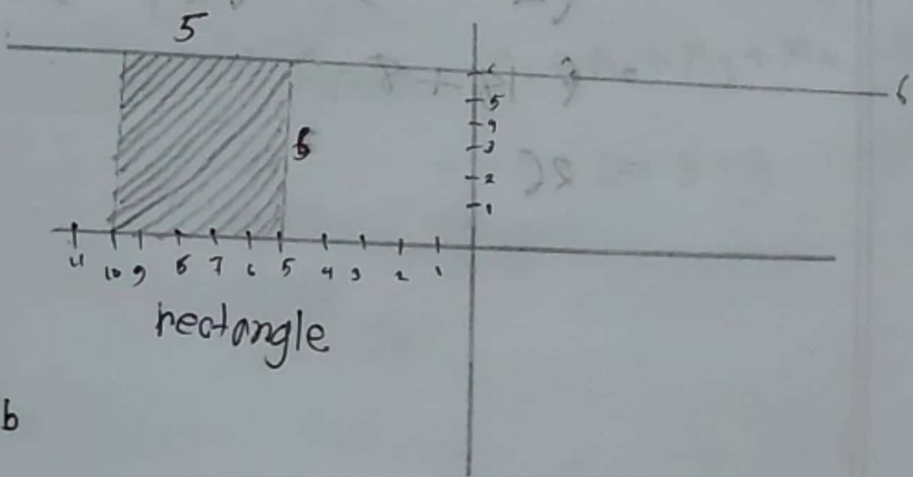
$A_1 = -A_2$

$A_1 + A_2 = 0$

$A = 0$

a//

$$\int_{-10}^{-5} 6\, dx$$



$$\text{Area} = a \times b$$

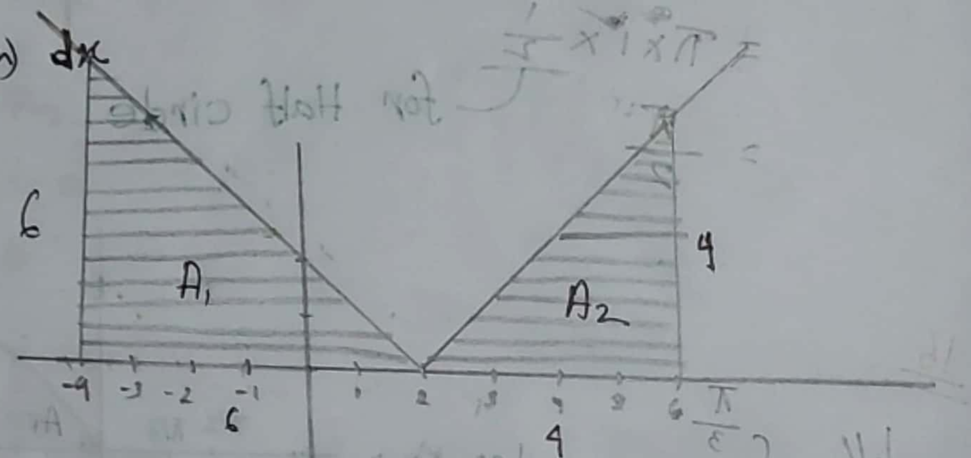
$$= 5 \times 6$$

$$= 30$$

17 tipe

$$f(x) = \begin{cases} (x-2) & x \geq 0 \\ x-2 & x < 0 \end{cases} \qquad f(x) = |x-2|$$

$$\int_{-4}^{6} f(x)\, dx$$



$$\therefore \text{Area} = \int_{-4}^{2} f(x) + \int_{2}^{6} f(x)$$

$$= \left(\frac{1}{2} \times b \times h\right) + \left(\frac{1}{2} \times b \times h\right)$$

$$= \left(\frac{1}{2} \times 6 \times 6\right) + \left(\frac{1}{2} \times 4 \times 4\right)$$

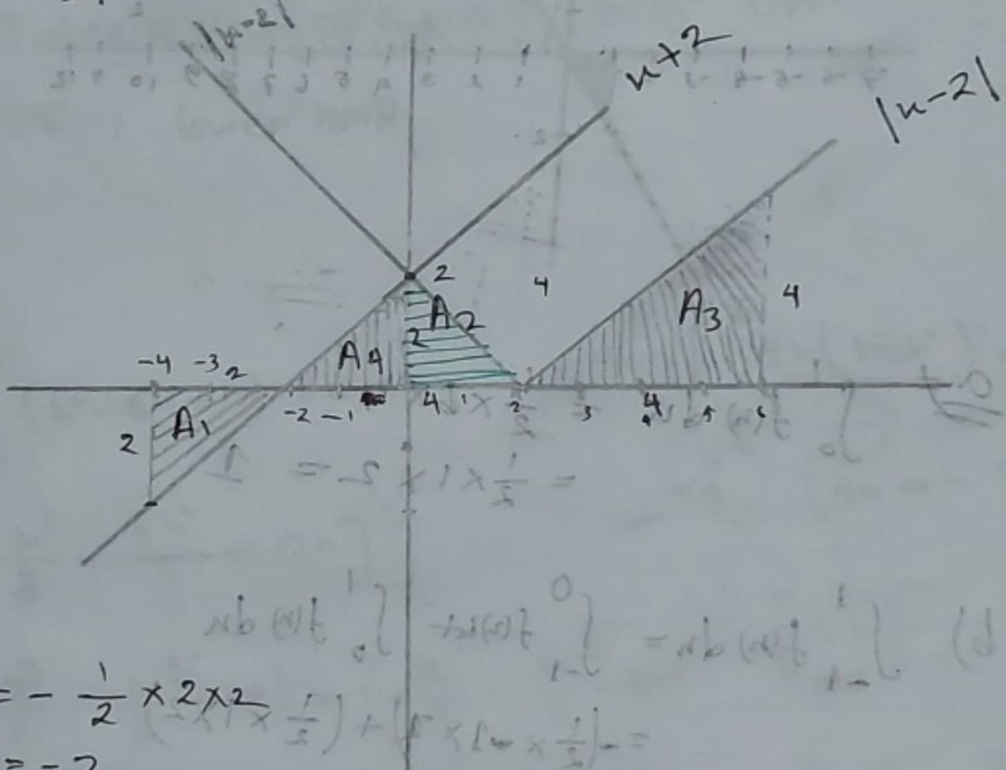
$$= 18 + 8$$

$$= 26$$

d)

$$f(x) = \begin{cases} |x-2|, & x \geq 0 \\ x+2 & x < 0 \end{cases}$$

$$\int_{-4}^{6} f(x)\,dx$$



$A_1 = -\frac{1}{2} \times 2 \times 2$
$= -2$

$A_2 = \frac{1}{2} \times 2 \times 2$
$= 2$

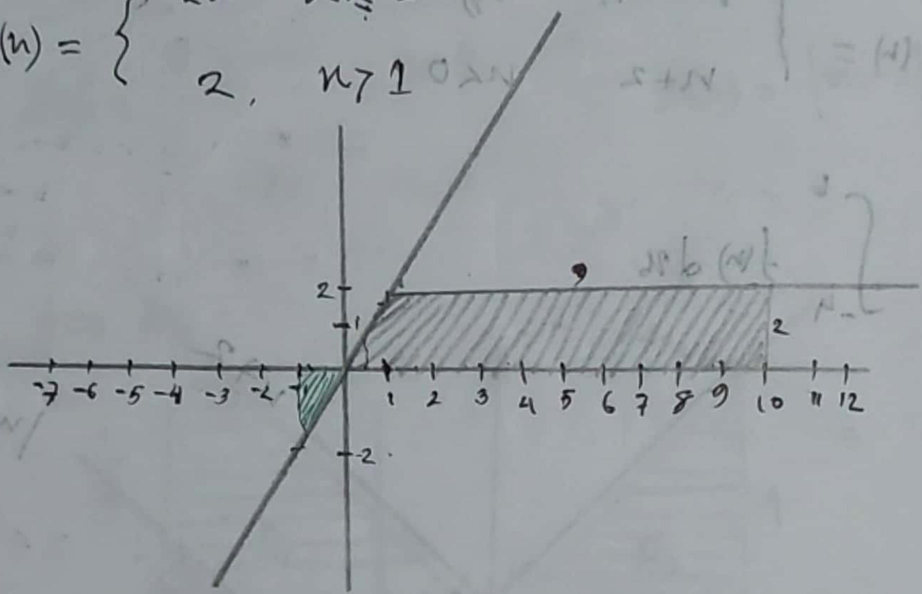
$A_3 = \frac{1}{2} \times b \times h$
$= \frac{1}{2} \times 4 \times 4$
$= 8$

$A_4 = \frac{1}{2} \times b \times h$
$= \frac{1}{2} \times 2 \times 2$
$= 2$

$\therefore$ Area $= A_1 + A_2 + A_3 + A_4$
$= -2 + 2 + 8 + 2$
$= 10$

18

$$f(x) = \begin{cases} 2x & x \leq 1 \\ 2, & x > 1 \end{cases}$$



a) $\int_0^1 f(x)\,dx = \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 1 \times 2 = 1$

b) $\int_{-1}^{1} f(x)\,dx = \int_{-1}^{0} f(x)dx + \int_0^1 f(x)\,dx$

$= -\left(\frac{1}{2} \times -1 \times 2\right) + \left(\frac{1}{2} \times 1 \times 2\right)$

$= 0$

c) $\int_1^{10} f(x)\,dx = a \times b$

$= \cancel{2 \times 10}\ 2 \times 9$

$= 18$

d) $\int_{1/2}^{5} f(x)\,dx = \frac{1}{2}(a+b) \times h$

$\frac{1}{2}(4+4.5) \times 2 = 8.5$

$= \cancel{\frac{1}{2}(1+2+5) \times 9}\ 4.5$

$a = 4$

$b = 4.5$

$h = 2$

$h = 2\ \cancel{4.5}\ 5$

5.6

$\rightarrow \int f(x)\,dx = J(x) + C$

$\rightarrow \int_{x=a}^{x=b} f(x)\,dx = [J(x)]_a^b = J(b) - J(a)$

→ Integration করার সময় প্রথমে upper limit বসাতে হবে, lower limit

13

$\int_{-2}^{1} (x^2 - \cancel{6x} + 12)\,dx$

$= \left[\frac{x^3}{3} - \frac{6x^2}{2} + 12x\right]_{-2}^{1}$

$= \left(\frac{1^3}{3} - \frac{6(1)^2}{2} + 12\right) - \left(\frac{(-2)^3}{3} - \frac{6(-2)^2}{2} + 12(-2)\right)$

$= \left(\frac{1}{3} - \frac{6}{2} + 12\right) - \left(-\frac{8}{3} - \frac{24}{2} - 24\right)$

$= \left(\frac{1}{3} - 3 + 12\right) + \frac{8}{3} + 12 + 24$

$= \frac{1}{3} + \frac{8}{3} + 45$

$= \frac{1 + 8 + 135}{3}$

$= 48$

14

$\int_{-1}^{2} 4x(1 - x^2)\,dx$

$= \int_{-1}^{2} (4x - 4x^3)\,dx$

$= \left[4\frac{x^2}{2} - \frac{4x^4}{4}\right]_{-1}^{2}$

$= \left(4\frac{(2)^2}{2} - \frac{4\times(2)^4}{4}\right) - \left(4\cdot\frac{(-1)^2}{2} - \frac{4(-1)^4}{4}\right)$

$= (8 - 16 - 2 + 1)$

$= -9$

17

$$\int_4^9 2x\sqrt{x}\, dx$$

$$= \int_4^9 2x \cdot x^{\frac{1}{2}}\, dx$$

$$= \int_4^9 2x^{1+\frac{1}{2}}\, dx$$

$$= \int_4^9 2x^{\frac{3}{2}}\, dx$$

$$= 2\left[\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1}\right]_4^9$$

$$= 2\left[\frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right]_4^9$$

$$= 2\left[\frac{9^{\left(\frac{5}{2}\right)}}{\frac{5}{2}} - \frac{4^{\left(\frac{5}{2}\right)}}{\frac{5}{2}}\right]$$

$$= 168.8$$

20

$$\int_0^{\pi/4} \sec^2\theta\, d\theta$$

$$= [\tan\theta]_0^{\pi/4}$$

$$= \tan(\pi/4) - \tan(0)$$

$$= 1 - 0$$

$$= 1$$

23

$$\int_{\ln 2}^{3} 5e^x\, dx$$

$$= \left[5e^x\right]_{\ln 2}^{3}$$

$$= 5\left(e^3 - e^{\ln 2}\right)$$

$$= 90.43$$

25

$$\int_0^{1/\sqrt{2}} \frac{dx}{\sqrt{1-x^2}}$$

$$= \int_0^{1/\sqrt{2}} \frac{1}{\sqrt{1-x^2}}\, dx$$

$$= \left[\sin^{-1} x\right]_0^{1/\sqrt{2}}$$

$$= \left[\sin^{-1}(1/\sqrt{2}) - \sin^{-1}(0)\right]$$

$$= \frac{\pi}{4} - 0 = \frac{\pi}{4}$$

26

$$\int_{-1}^{1} \frac{dx}{1+x^2}$$

$$= \int_{-1}^{1} \frac{1}{1+x^2}\, dx$$

$$= \left[\tan^{-1} x\right]_{-1}^{1}$$

$$= \tan^{-1}(1) - \tan^{-1}(-1)$$

$$= \frac{\pi}{4} - \left(-\frac{\pi}{4}\right)$$

$$\frac{\pi}{4} + \frac{\pi}{4}$$

$$= \frac{2\pi}{4}$$

$$= \frac{\pi}{2}$$

27.

$$\int_{\sqrt{2}}^{2} \frac{dx}{x\sqrt{x^2-1}}$$

$$= \int_{\sqrt{2}}^{2} \frac{1}{x\sqrt{x^2-1}}\, dx$$

$$= \left[\sec^{-1}x\right]_{\sqrt{2}}^{2}$$

$$= \sec^{-1}(2) - \sec^{-1}(\sqrt{2})$$

29//

$$\int_{1}^{4}\left(\frac{1}{\sqrt{t}} - 3\sqrt{t}\right) dt$$

$$= \int_{1}^{4}\left(t^{-\frac{1}{2}} - 3t^{\frac{1}{2}}\right) dt$$

$$= \left[\frac{t^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} - 3\frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]_1^4$$

$$= \left[\frac{t^{\frac{-1+2}{2}}}{\frac{1}{2}} - 3\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^4$$

$$= \left[2t^{\frac{1}{2}} - 2t^{\frac{3}{2}}\right]_1^4$$

$$= \left(2\times(4)^{\frac{1}{2}} - 2\times(4)^{\frac{3}{2}}\right) - \left(2\times(1)^{\frac{1}{2}} - 2(1)^{\frac{3}{2}}\right)$$

$$= 4 - 16 - \cancel{2} + \cancel{2} = -12$$

30//

$$\int_{\pi/6}^{\pi/2}\left(x + \frac{2}{\sin^2 x}\right) dx$$

$$= \int_{\pi/6}^{\pi/2}\left(x + 2\cdot\frac{1}{\sin^2 x}\right) dx$$

$$= \int_{\pi/6}^{\pi/2}\left(x + 2\cdot \operatorname{cosec}^2 x\right) dx$$

$$= \left[\frac{x^2}{2} + 2(-\cot x)\right]_{\pi/6}^{\pi/2}$$

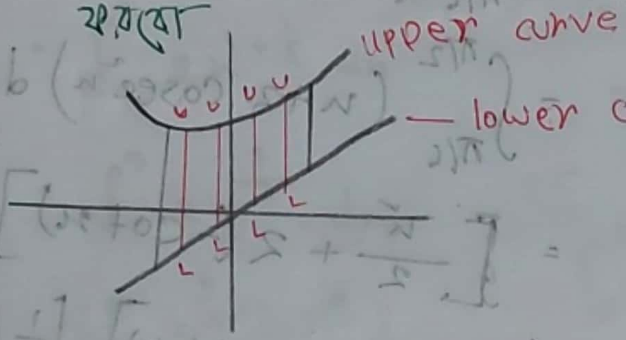
$$= \left[\frac{1}{2}(\pi/2)^2 - 2\cot(\pi/2)\right] - \left[\frac{1}{2}(\pi/6)^2 - 2\cot(\pi/6)\right]$$

6.1

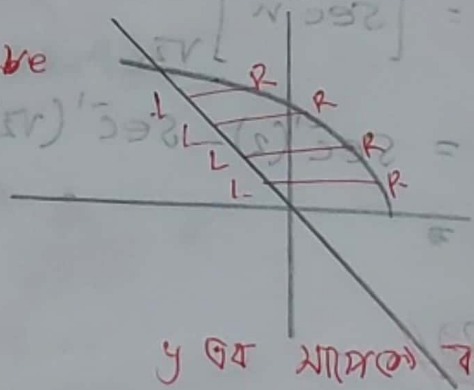
→ x এর সাপেক্ষে করলে vertical line টানবো।

→ y " " সাপেক্ষে Horizontal line "

→ যার সাপেক্ষে করলে সবগুলো (upper - lower) এর; (Right - Left) এই ভিত্তিতে curve কে চেক করে তার সাপেক্ষে করবো



x এর সাপেক্ষে করবো



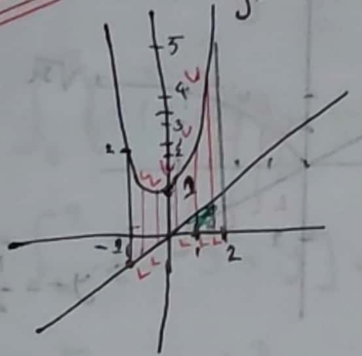
y এর সাপেক্ষে করবো

→ যার সাপেক্ষে করবো limit তাকে দিবো।

$$\left.\begin{aligned}&\int_{x=a}^{x=b} (\text{upper curve} - \text{lower curve})\, dx \\ &\quad [f_1(x) - f_2(x)]\, dx\end{aligned}\right\} x \text{ এর সাপেক্ষে}$$

$$\left.\begin{aligned}&\int_{y=a}^{y=b} (\text{Right curve} - \text{Left curve})\, dy \\ &\quad [f_1(y) - f_2(y)]\, dy\end{aligned}\right\} y \text{ এর সাপেক্ষে}$$

1



Function দেওয়া আছে
y এর চেয়ে x এর সাপেক্ষে করবো

With respect to x

$$\int_{-1}^{2} \left[(x^2+1) - (x)\right] dx$$

$$= \int_{-1}^{2} \left[x^2+1-x\right] dx$$

$$= \left[\frac{x^3}{3} + x - \frac{x^2}{2}\right]_{-1}^{2}$$

$$= \left(\frac{2^3}{3} + 2 - \frac{2^2}{2}\right) - \left(\frac{(-1)^3}{3} + (-1) + \frac{(-1)^2}{2}\right)$$

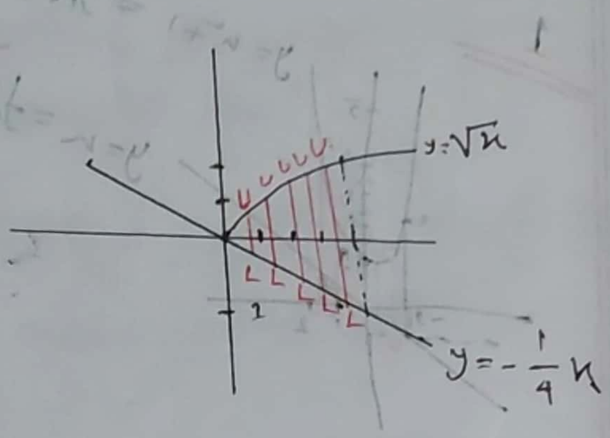
$$= \frac{8}{3} + \cancel{2} - \cancel{2} + \frac{1}{3} + 1 + \frac{1}{2}$$

$$= \frac{8}{3} + 1 - \frac{1}{2} + \frac{1}{3}$$

$$= \frac{16+30-3+2}{12}$$

$$= 4.5$$

2



$$\int_0^4 \left[\sqrt{x} - \left(-\frac{1}{4}x\right)\right] dx$$

$$= \int_0^4 \left[\sqrt{x} + \frac{1}{4}x\right] dx$$

$$= \int_0^4 \left[x^{\frac{1}{2}} + \frac{1}{4}x\right] dx$$

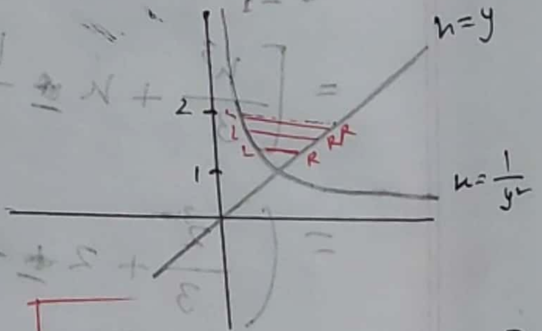
$$= \left[\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{1}{4}\frac{x^2}{2}\right]_0^4$$

$$= \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^2}{8}\right]_0^4$$

$$= \left(\frac{(4)^{\frac{3}{2}}}{\frac{3}{2}} + \frac{4^2}{8}\right) - \left(\frac{0}{\frac{3}{2}} + \frac{0}{8}\right)$$

$$= \frac{22}{3}$$

3



$$\int_1^2 \left[\text{Right curve} - \text{left curve}\right] dy$$

$$\int_1^2 \left[\cancel{\text{Right}}\ y - \frac{1}{y^2}\right] dy$$

$$= \int_1^2 \left[y - y^{-2}\right] dy$$

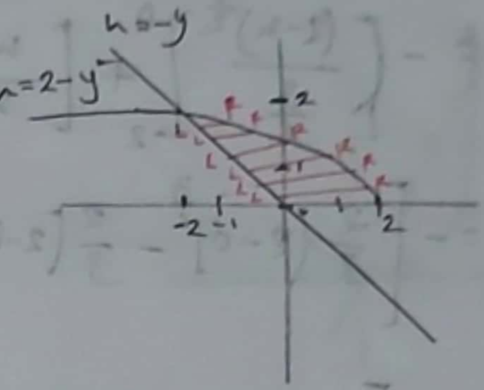
$$= \left[\frac{y^2}{2} - \frac{y^{-1}}{-1}\right]^2$$

$$= \left[\frac{y^2}{2} + y^{-1}\right]_1^2$$

$$= \left(\frac{2^2}{2} + 2^{-1}\right) - \left(\frac{1^2}{2} + (1^{-1})\right)$$

$$= 2 + \frac{1}{2} - \frac{1}{2} - 1 = 1$$

4 with y axis



$$\int_0^2 \left[(2-y^2) - (-y)\right] dy$$

$$= \int_0^2 \left[2 - y^2 + y\right] dy$$

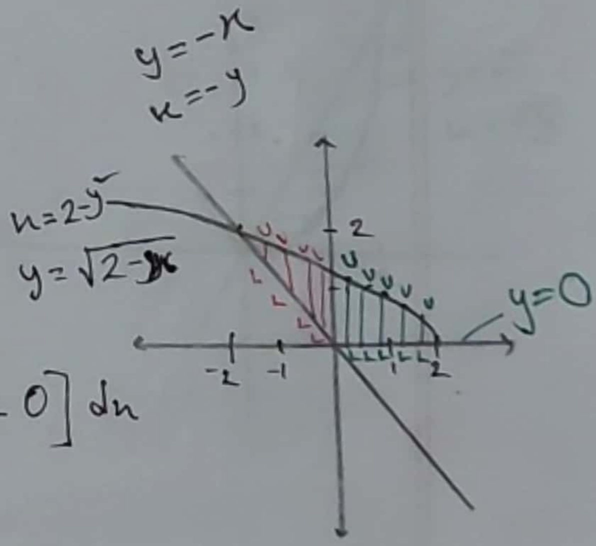
$$= \left[2y - \frac{y^3}{3} + \frac{y^2}{2}\right]_0^2$$

$$= \left(2\times 2 - \frac{2^3}{3} + \frac{2^2}{2}\right) - (0 - 0 + 0)$$

$$= \left(4 - \frac{8}{3} + 2\right)$$

$$= 10/3$$

with respect to 'x'



$$\int_{-2}^{0} \left[\left(\sqrt{2-x}\right) - (-x)\right] dx + \int_0^2 \left[\left(\sqrt{2-x}\right) - 0\right] dx$$

$$= \int_{-2}^{0} \left[\sqrt{2-x} + x\right] dx + \int_0^2 \left[\sqrt{2-x}\right] dx$$

Let, $2 - x = z$

$$\frac{dz}{dx} = -1$$

$$-dz = dx$$

$$= \int_{-2}^{0} \left[\sqrt{2-x}\right] dx + \int_{-2}^{0} x\, dx + \int_0^2 \left[\sqrt{2-x}\right] dx$$

$$= -\int_{-2}^{0} \left[\sqrt{z}\right] dz + \int_{-2}^{0} x\, dx - \int_0^2 \left(\sqrt{z}\right) dz$$

$$= -\left[\frac{z^{\frac{3}{2}}}{\frac{3}{2}}\right]_{-2}^{0} + \left[\frac{x^2}{2}\right]_{-2}^{0} - \left[\frac{z^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^2$$

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$$= -\left[\frac{(2-x)^{\frac{3}{2}}}{\frac{3}{2}}\right]_{-2}^{0} + \left[\frac{x^2}{2}\right]_{-2}^{0} - \left[\frac{(2-x)^{\frac{3}{2}}}{3/2}\right]_{0}^{2}$$

$$= -\left[\frac{2}{3}(2-0)^{\frac{3}{2}} - \frac{2}{3}\left[2-(-2)\right]^{\frac{3}{2}}\right] + \left[\frac{0^2}{2} - \frac{(-2)^2}{2}\right] - \left[\frac{2}{3}\left[2-2\right]^{\frac{3}{2}} - \frac{2}{3}\left[\cancel{2}2-0\right]^{\frac{3}{2}}\right]$$

$$= -\left[1.8856 - 5.3333\right] - \left[0-2\right] - \left[0 - 1.8856\right]$$

$$= \boxed{7.33}$$

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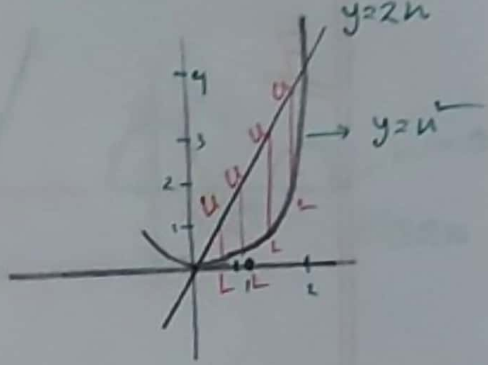
5 With respect to 'x'



$$\int_0^2 [2x - x^2]\, dx$$

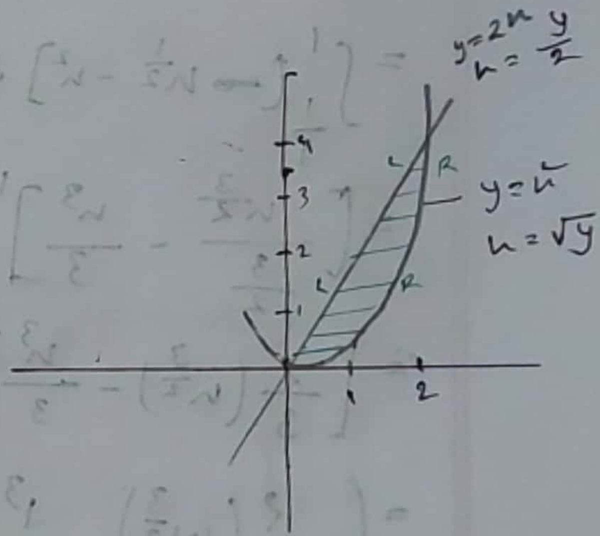
$$= \left[\frac{2x^2}{2} - \frac{x^3}{3}\right]_0^2$$

$$= \left[2^2 - \frac{2^3}{3}\right] - \left[0^2 - \frac{0^3}{3}\right]$$

$$= \left[4 - \frac{8}{3}\right]$$

$$= \frac{12-8}{3} = \frac{4}{3}$$

With respect to 'y'



$$\int_0^4 \left(\sqrt{y} - \frac{1}{2}y\right) dy$$

$$= \int_0^4 \left[y^{\frac{1}{2}} - \frac{1}{2}y\right] dy$$

$$= \left[\frac{y^{\frac{1}{2}+1}}{\frac{1}{2}+1} - \frac{1}{2} \times \frac{y^{1+1}}{1+1}\right]_0^4$$

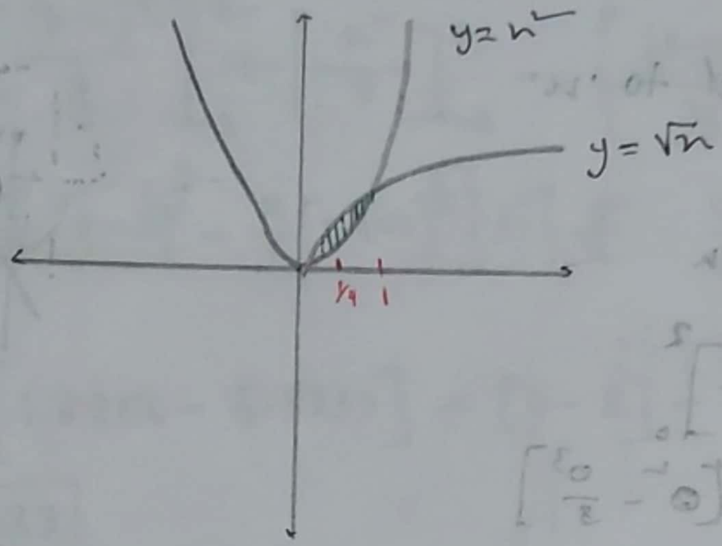
$$= \left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}} - \frac{y^2}{4}\right]_0^4$$

$$= \left[\frac{2}{3}(4)^{\frac{3}{2}} - \frac{4^2}{4}\right] - [0 - 0]$$

$$= \left[\frac{16}{3} - 4\right]$$

$$= \frac{16-12}{3} = \frac{4}{3}$$

7



$$\int_{\frac{1}{4}}^{1} [\sqrt{x} - x^2]\, dx$$

$$= \int_{\frac{1}{4}}^{1} [x^{\frac{1}{2}} - x^2]\, dx$$

$$= \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^3}{3}\right]_{\frac{1}{4}}^{1}$$

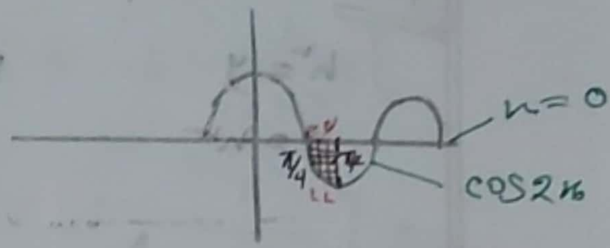
$$= \left[\frac{2}{3}\left(x^{\frac{3}{2}}\right) - \frac{x^3}{3}\right]_{\frac{1}{4}}^{1}$$

$$= \left(\frac{2}{3}\left((1)^{\frac{3}{2}}\right) - \frac{1^3}{3}\right) - \left(\frac{2}{3}\left(\frac{1}{4}\right)^{\frac{1}{4}} - \frac{\left(\frac{1}{4}\right)^3}{3}\right)$$

$$\boxed{= \frac{2}{3} - \frac{1}{3} - \frac{71}{150} + \frac{1}{192} \qquad = \frac{128 - 64 - 90 + 1}{192}}$$

$=$

9



$$\int_{\pi/4}^{\pi/2} [0-\cos 2n]\, dn$$

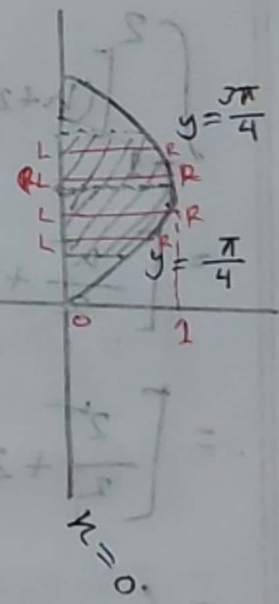
$$= \left[\frac{-\sin 2n}{2}\right]_{\pi/4}^{\pi/2}$$

$$= \left[-\frac{\sin(\pi/2)}{2}\right) - \left(-\frac{\sin(\pi/4)}{2}\right)$$

$$= -\frac{1}{2} + \frac{\frac{\sqrt{2}}{2}}{2}$$

$$= -\frac{1}{2} + \frac{\sqrt{2}}{4}$$

11



$$\int_{\pi/4}^{3\pi/4} [\sin y - 0]\, dy$$

$$= \int_{\pi/4}^{3\pi/4} [\sin y]\, dy$$

$$= [-\cos y]_{\pi/4}^{3\pi/4}$$

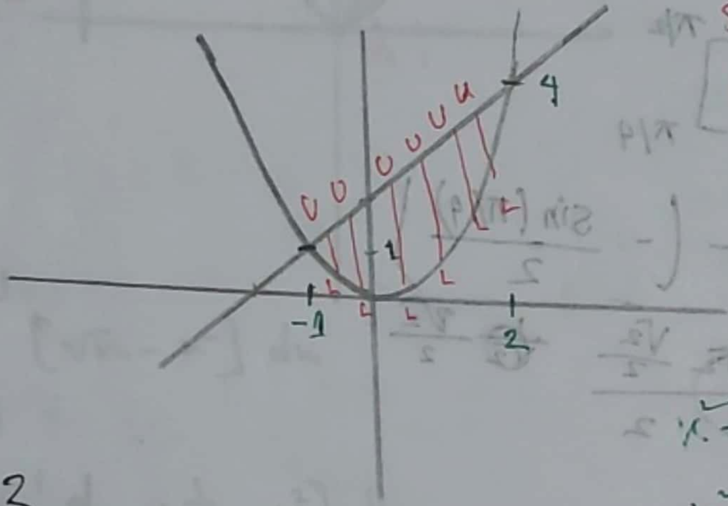
$$= \left[-\cos(3\pi/4) + \cos\left(\frac{\pi}{4}\right)\right]$$

12

$x^2 = y$ $\qquad$ $x = y - 2$

$y = x^2$ $\qquad$ $y = x + 2$

equation 2টো ~~হতে~~ solve করলে intersecting point পাওয়া যাবে।



$x^2 = x + 2$

$\Rightarrow x^2 - x - 2 = 0$

$\Rightarrow x^2 - 2x + x - 2 = 0$

$\Rightarrow x(x-2) + 1(x-2) = 0$

$\Rightarrow (x+1)(x-2) = 0$

$x = -1, 2$

$$\int_{-1}^{2} \left[(x+2) - x^2\right] dx$$

$$= \left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^{2}$$

$$= \left[\frac{2^2}{2} + 2\times 2 - \frac{2^3}{3}\right] - \left[\frac{(-1)^2}{2} + (2\times -1) - \frac{(-1)^3}{3}\right]$$

$$= \left(2 + 4 - \frac{8}{3}\right) - \frac{1}{2} + 2 - \frac{1}{3}$$

$$= 8 - \frac{8}{3} + \frac{1}{2} - \frac{1}{3}$$

$$= \frac{48 - 16 + 3 - 2}{6}$$

$$= \frac{9}{2}$$

7.1

$\cancel{\text{fin}}$ $\sinh x = \dfrac{e^{x}+e^{-x}}{2}$

$\cosh x = \dfrac{e^{x}-e^{-x}}{2}$

1

$\int (4-2x)^3\,dx$ Let $z = 4-2x$

$\frac{dz}{dx} = -2$

$dx = -\frac{1}{2}\,dz$

$= \int z^3 \cdot -\frac{1}{2}\,dz$

$= \frac{1}{2}\cdot\frac{z^4}{4} + c$

$= -\frac{1}{8}(4-2x)^4 + c$

4

$\int 4x\tan(x^2)\,dx$ Let, $z = x^2$

$\frac{dz}{dx} = 2x$

$x\,dx = \frac{1}{2}\,dz$

$= \int 4\cdot\tan z\,\frac{1}{2}\,dz$

$= \cancel{2\sec^2 z + c}$

$= \cancel{2\sec^2(x^2)+c}$

$= 2\ln|\sec z| + c$

$= 2\ln|\sec(x^2)| + c$

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6

$$\int \frac{1}{9+4x^2}\,dx$$

$$= \int \frac{1}{4\left(\frac{9}{4}+x^2\right)}\,dx$$

$$= \int \frac{1}{4}\cdot\frac{1}{\left(\frac{3}{2}\right)^2+x^2}\,dx$$

$$= \cancel{\frac{1}{4}\tan^{-1}}$$

$$= \frac{1}{4}\cdot\frac{1}{\frac{3}{2}}\tan^{-1}\cdot\frac{x}{\frac{3}{2}}+c$$

$$= \frac{1}{6}\tan^{-1}\frac{2x}{3}+c$$

10

$$\int \frac{x}{\sqrt{1-x^4}}\,dx$$

$$= \int \frac{x}{\sqrt{1-(x^2)^2}}\,dx$$

Let $z = x^2$

$\frac{dz}{dx} = 2x$

$= \frac{1}{2}dz - xdx$

$$= \int \frac{1}{\sqrt{1-z^2}}\cdot\frac{1}{2}\,dz$$

$$= \frac{1}{2}\sin^{-1}z+c$$

$$= \frac{1}{2}\sin^{-1}x^2+c$$

7

$$\int e^x \sinh(e^x)\,dx$$

Let,

$z = e^x$

$\frac{dz}{dx} = e^x$

$e^x dx = dz$

$$= \int \sinh(z)\cdot dz$$

$$= \cosh z + c$$

$$= \cosh(e^x)+c$$

8

$$\int \frac{\sec(\ln x)\tan(\ln x)}{x}\,dx$$

Let $z = \ln x$

$\frac{dz}{dx} = \frac{1}{x}$

$\Rightarrow dz = \frac{dx}{x}$

$$= \int \sec z \tan z \,dz$$

$$= \sec z + c$$

$$= \sec(\ln x)+c$$

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12

$$\int \frac{\cos x}{\sin x\sqrt{\sin^2 x+1}}\,dx \qquad \text{Let } z=\sin x$$

$$\frac{dz}{dx} = \cos x$$

$$dz = \cos x\,dx$$

$$= \int \frac{1}{z\sqrt{z^2+1}}\,dz$$

$$= \cancel{\sec^{-1} z + C}$$

$$= \cancel{\sec^{-1}(\sin x) + C}$$

$$= -\ln\left|\frac{a+\sqrt{1+z^2}}{z}\right| + c$$

$$= -\ln\left|\frac{1+\sqrt{1-\sin^2 x}}{\sin x}\right| + C$$

15

$$\int \frac{e^{\sqrt{x-1}}}{\sqrt{x-1}}\,dx \qquad \text{Let}$$

$$z = \sqrt{x-1}$$

$$\frac{dz}{dx} = \frac{1}{2\sqrt{1-x}}\cdot 1$$

$$2\,dz = \frac{dx}{\sqrt{1-x}}\cdot\cancel{2}$$

$$= \int e^z\, 2\,dz$$

$$= 2e^z + C$$

$$= 2e^{\sqrt{x-1}} + C$$

18

$$\int \frac{dx}{x(\ln x)^2} \qquad \text{Let } \ln x = z$$

$$\frac{dz}{dx} = \frac{1}{x}$$

$$dz = \frac{dx}{x}$$

$$= \int \frac{1}{z^2}\cdot dz$$

$$= \frac{z^{-1}}{-1} + C = -(\ln x)^{-1} + C$$

~~19~~

$\int \frac{dx}{\sqrt{x}}$

20

$\int \sec(\sin\theta)\tan\theta\cos\theta\, d\theta$

Let $z = \sin\theta$

$\frac{dz}{d\theta} = \cos\theta$

$\Rightarrow dz = \cos\theta\, d\theta$

$= \int \sec(z)\tan(z)\, dz$

$= \sec z + c$

$= \sec(\sin\theta) + c$

22

$\int \frac{dx}{\sqrt{x^2-4}}$

$= \int \frac{dx}{\sqrt{4\left(\frac{x}{2}\right)^2 - 1}}$

$= \int \frac{1}{2}\frac{dx}{\sqrt{\left(\frac{x}{2}\right)^2-1}}$

30

$\int 2^{\pi x}\, dx$

Let $u = \pi x$

$\frac{du}{dx} = \pi$

$dx = \frac{1}{\pi}\, du$

$\int a^x = \frac{a^x}{\ln a} + c$

$= \int 2^u \cdot \frac{1}{\pi}\, du$

$= \frac{1}{\pi}\frac{2^u}{\ln 2} + c$

$= \frac{2^{\pi x}}{\pi \ln 2} + c$

7.2

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L I A T E priority $L \to I \to A \to T \to E$

বামে priority বেশি

log, ln — inverse trigonometry $\sin^{-1}x$, $\tan^{-1}x$ ....

Algorithmic x^n

Trigonometric $\sin x$, $\cos x$ ....

exponential e^x

$\longrightarrow$ Integration by parts করার সময় উপর priority অনুযায়ী u ধরবো বাকি যা থাকবে তাকে dv ধরবো

$$I = \int \underbrace{x}_{u}\, \underbrace{e^{-2x}\, dx}_{dv}$$

যাকে u ধরবো তাকে Differentiate করবো
যাকে v " Integration করবো

$$\boxed{I = \int u\,dv = uv - \int v\,du}$$

$$\frac{du}{dx} = \frac{d}{dx}(u), \quad u = x$$

$$dv = e^{-2x}\,dx$$

$$\int 1.\,dv = \int e^{-2x}\,dx$$

1.

$$\int \underbrace{x}_{u} \underbrace{e^{-2x} dx}_{dv}$$

$u = x$

$\Rightarrow \frac{du}{dx} = 1$

$\Rightarrow du = dx$

$dv = e^{-2x} dx$

$\int 1 \cdot dv = \int e^{-2x} dx$

$v = \frac{e^{-2x}}{-2}$

$$= \int u \, dv$$

$$= uv - \int v \, du$$

$$= x \cdot \frac{e^{-2x}}{-2} - \int \frac{e^{-2x}}{-2} \cdot dx$$

$$= -\frac{1}{2} x e^{-2x} - \left(\frac{-1}{2}\right) \int e^{-2x} dx$$

$$= -\frac{1}{2} x e^{-2x} + \frac{1}{2} \cdot \frac{e^{-2x}}{-2} + c$$

$$= -\frac{1}{2} x e^{-2x} - \frac{1}{4} e^{-2x} + c$$

2.

$$\int \underbrace{x}_{u} \underbrace{e^{3x} dx}_{dv}$$

$u = x$

$\frac{du}{dx} = 1$

$du = dx$

$dv = e^{3x} dx$

$\int 1 \cdot dv = \int e^{3x} dx$

$v = \frac{e^{3x}}{3}$

$$= \int u \, dv$$

$$= uv - \int v \, du$$

$$= x \cdot \frac{e^{3x}}{3} - \int \frac{e^{3x}}{3} dx$$

$$= x \cdot \frac{e^{3x}}{3} - \frac{1}{3} \cdot \frac{e^{3x}}{3} + c$$

$$= \frac{1}{3} x e^{3x} - \frac{1}{9} e^{3x} + c$$

4

$$\int \underbrace{x^2}_{u} \underbrace{e^{-2x}\,dx}_{dv}$$

$u = x^2$, $\frac{du}{dx} = 2x$ $\Rightarrow du = 2x\,dx$

$dv = e^{-2x}\,dx$, $\int 1 \cdot dv = \int e^{-2x}\,dx$, $v = \frac{e^{-2x}}{-2}$

$$= \int u\,dv$$

$$= uv - \int v\,du$$

$$= x^2 \frac{e^{-2x}}{-2} - \int \frac{-1}{2} \cdot e^{-2x}\, 2x\,dx$$

$$= -\frac{1}{2} x^2 e^{-2x} + \int e^{-2x} x\,dx$$

$u = x$, $\frac{du}{dx} = 1$, $du = dx$

$dv = e^{-2x}\,dx$, $\int 1 \cdot dv = \int e^{-2x}\,dx$, $v = \frac{e^{-2x}}{-2}$

$$= -\frac{1}{2} x^2 e^{-2x} + \left[uv - \int v\,du\right]$$

$$= -\frac{1}{2} x^2 e^{-2x} + \left[x \frac{e^{-2x}}{-2} - \int \frac{1}{-2} e^{-2x}\,dx\right]$$

$$= -\frac{1}{2} x^2 e^{-2x} - \frac{1}{2} x e^{-2x} + \frac{1}{2} \frac{e^{-2x}}{-2}$$

$$= -\frac{1}{2} x^2 e^{-2x} - \frac{1}{2} x e^{-2x} - \frac{1}{4} e^{-2x}$$

6

$$\int \underbrace{x}_{u} \underbrace{\cos 2x\,dx}_{dv}$$

$u = x$, $\frac{du}{dx} = 1$, $\Rightarrow du = dx$

$dv = \cos 2x\,dx$, $\int 1\,dv = \int \cos 2x\,dx$, $v = \frac{\sin 2x}{2}$

$$= uv - \int v\,du$$

$$= x \cdot \frac{\sin 2x}{2} - \int \frac{\sin 2x}{2}\,dx$$

$$= \frac{1}{2} x \sin(2x) - \frac{1}{2}\left(-\frac{\cos 2x}{2}\right) + c$$

7

$$\int \underbrace{x^2}_{u} \underbrace{\cos x \, dx}_{dv}$$

; $u = x^2$ $\quad dv = \cos x \, dx$

$\frac{du}{dx} = 2x$ $\quad \int 1 \cdot dv = \int \cos x \, dx$

$du = 2x \, dx$ $\quad v = \sin x$

$= uv - \int v \, du$

$= x^2 \sin x - \int \sin x \, 2x \, dx$

$= x^2 \sin x - 2\int \underbrace{x}_{u} \underbrace{\sin x \, dx}_{dv}$

; $u = x$ $\quad dv = \sin x \, dx$

$\frac{du}{dx} = 1$ $\quad \Rightarrow \int 1 \, dv = \int \sin x \, dx$

$du = dx$ $\quad \Rightarrow v = -\cos x$

$= x^2 \sin x - 2\left[uv - \int v \, du\right]$

$= x^2 \sin x - 2\left[x(-\cos x) - \int -\cos x \, dx\right]$

$= x^2 \sin x + 2x \cos x - 2\sin x + c$

10

$$\int \sqrt{x} \ln x \, dx$$

; $u = \ln x$ $\quad dv = x^{\frac{1}{2}} \, dx$

$\frac{du}{dx} = \frac{1}{x}$ $\quad \int 1 \, dv = \int x^{\frac{1}{2}} \, dx$

$du = \frac{1}{x} dx$ $\quad v = \frac{x^{\frac{3}{2}}}{\frac{3}{2}}$

$v = \frac{2}{3} x^{\frac{3}{2}}$

$= \int \underbrace{\ln x}_{u} \underbrace{x^{\frac{1}{2}} \, dx}_{dv}$

$= uv - \int v \, du$

$= \ln x \, \frac{2}{3} x^{\frac{3}{2}} - \int \frac{2}{3} x^{\frac{3}{2}} \cdot \frac{1}{x} \, dx$

$= \ln x \, \frac{2}{3} x^{\frac{3}{2}} - \frac{2}{3} \int x^{\frac{3}{2} + (-1)} \, dx$

$= \ln \frac{2}{3} x^{\frac{3}{2}} - \frac{2}{3} \int x^{\frac{1}{2}} \, dx$

$= \ln \frac{2}{3} x^{\frac{3}{2}} - \frac{2}{3} \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$

11

$$\int (\ln x)^2 \, dx$$

$$= \int 1 \, (\ln x)^2 \, dx$$

$$= \int x^0 \, (\ln x)^2 \, dx$$

$$= \int \underbrace{(\ln x)^2}_{u} \underbrace{x^0 dx}_{v}$$

$u = (\ln x)^2$

$\Rightarrow \frac{du}{dx} = 2 \ln x \frac{1}{x}$

$\Rightarrow du = \frac{2 \ln x}{x} dx$

$dv = x^0 \, dx$

$\int dv = x^0 \, dx$

$\Rightarrow v = x$

$$\Rightarrow \int u \, dv = uv - \int v \, du$$

$$= (\ln x)^2 x - \int x \frac{2 \ln x}{x} dx$$

$$= x (\ln x)^2 - 2 \int \ln x \, dx$$

$$= x (\ln x)^2 - 2 [x \ln x - x] + c$$

$$= x (\ln x)^2 - 2x \ln x - 2x + c$$

$$\int \ln x \, dx$$

$$\Rightarrow \int 1 \cdot \ln x \, dx$$

$$\int x^0 \cdot \ln x \, dx$$

$u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$du = \frac{1}{x} dx$

$dv = x^0 \, dx$

$\int 1 \cdot dv = \int x^0 \, dx$

$v = x$

$$\Rightarrow \int u \, dv = uv - \int v \, du$$

$$= \ln x \cdot x - \int x \cdot \frac{1}{x} dx$$

$$\boxed{= x \cdot \ln x - x + c}$$

14

$\int \ln(x^2+4)\,dx$ $\qquad u = \ln(x^2+4)$ $\qquad dv = x^0\,dx$

$= \int \ln(x^2+4)\cdot x^0\,dx$ $\qquad \frac{du}{dx} = \frac{1}{x^2+4}\cdot 2x$ $\qquad \int 1\cdot dv = \int x^0\,dx$

$\qquad v = x$

$\qquad \Rightarrow du = \frac{2x}{x^2+4}\,dx$

$= \int u\,dv$

$= uv - \int v\,du$

$= \ln(x^2+4)\cdot x - \int x\cdot \frac{2x}{x^2+4}\,dx$

$\cancel{= \ln(x^2+4)x - \int 2x^3 \frac{1}{x^3\left(\frac{1}{x^2}+\frac{4}{x^3}\right)}\,dx}$

$\cancel{= \ln(x^2+4)x - 2\int \frac{1}{\left(\frac{1}{x}\right)^2}}$

$= \ln(x^2+4)\cdot x - \int \frac{2x^2}{x^2+2^2}\,dx$

$= \ln(x^2+4)\cdot x - 2\int \frac{x^2+2^2-2^2}{x^2+2^2}\,dx$

$= \ln(x^2+4)\cdot x - 2\int \frac{x^2+2^2}{x^2+2^2} - \frac{2^2}{x^2+2^2}\,dx$

$= \ln(x^2+4)\cdot x - 2\int 1 - 4\frac{1}{2^2+x^2}\,dx$

$= \ln(x^2+4)\cdot x - 2x + 8\cdot\frac{1}{2}\tan^{-1}\frac{x}{2} + c$

$= x\ln(x^2+4) - 2x + 4\tan^{-1}\frac{x}{2} + c$

18

$$\int x \tan^{-1} x \, dx$$

$u = \tan^{-1} x$ $\quad dv = x \, dx$

$\frac{du}{dx} = \frac{1}{1+x^2}$ $\quad \int dv = \int x \, dx$

$du = \frac{1}{1+x^2} dx$ $\quad v = \frac{x^2}{2}$

$$I = \int \tan^{-1} x \, x dx$$

$$= \int u \, dv$$

$$= uv - \int v du$$

$$= \tan^{-1} x \cdot \frac{x^2}{2} - \int \frac{x^2}{2} \cdot \frac{1}{1+x^2} dx$$

$$= \frac{1}{2} \tan^{-1} x \, x^2 - \frac{1}{2} \int \frac{x^2+1-1}{1+x^2} dx$$

$$= \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \int 1 - \frac{1}{1+x^2} dx$$

$$= \frac{x^2}{2} \tan^{-1} x - \frac{1}{2}\left[x - \tan^{-1} x\right] + c$$

$$= \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} x + \frac{1}{2} \tan^{-1} x + c$$

20

$$I = \int e^{3x} \cos 2x \, dx$$

$$= \int \underbrace{\cos 2x}_{u} \underbrace{e^{3x} \, dx}_{dv}$$

$u = \cos 2x$

$\frac{du}{dx} = -2\sin 2x$

$du = -2\sin 2x \, dx$

$$\Rightarrow I = \int u dv$$

$$\Rightarrow I = uv - \int v du$$

$dv = e^{3x} dx$

$\int 1 \, dv = \int e^{3x} dx$

$v = \frac{e^{3x}}{3}$

$= \cos 2x \cdot \frac{e^{3x}}{3} - \int \frac{e^{3x}}{3} \cdot (-2\sin 2x)\, dx$

$u = \sin 2x$

$\frac{du}{dx} = 2\cos 2x$

$du = 2\cos 2x\, dx$

$= \frac{1}{3}\cos 2x\, e^{3x} + \frac{2}{3}\left[\int e^{3x} \sin 2x\, dx\right]$

$= \frac{1}{3}\cos 2x\, e^{3x} + \frac{2}{3}\left[\int \sin 2x\, e^{3x}\, dx\right]$

$dv = e^{3x}\, dx$

$\int dv = \int e^{3x}\, dx$

$v = \frac{e^{3x}}{3}$

$= \frac{1}{3}\cos 2x\, e^{3x} + \frac{2}{3}\left[uv - \int v\, du\right]$

$= \frac{1}{3}\cos 2x\, e^{3x} + \frac{2}{3}\left[\sin 2x \cdot \frac{e^{3x}}{3} - \int \frac{e^{3x}}{3}\, 2\cos 2x\, dx\right]$

$= \frac{1}{3}\cos 2x\, e^{3x} + \frac{2}{9}\sin 2x\, e^{3x} - \frac{2}{3}\left[\frac{2}{3}\int e^{3x} \cos 2x\, dx\right]$

$= \frac{1}{3}\cos 2x\, e^{3x} + \frac{2}{9}\sin 2x\, e^{3x} - \frac{2}{3}\left[\frac{2}{3} I\right] + C$

$\Rightarrow I \Rightarrow \frac{1}{3}\cos 2x\, e^{3x} + \frac{2}{9}\sin 2x\, e^{3x} - \frac{4I}{9} + C$

$\Rightarrow I + \frac{4I}{9} = \frac{1}{3}\cos 2x\, e^{3x} + \frac{2}{9}\sin 2x\, e^{3x} + C$

$\Rightarrow \frac{9I + 4I}{9} = \frac{1}{3}\cos 2x\, e^{3x} + \frac{2}{9}\sin 2x\, e^{3x} + C$

$\Rightarrow \frac{13}{9} I = \frac{1}{3}\cos 2x\, e^{3x} + \frac{2}{9}\sin 2x\, e^{3x} + C$

$\Rightarrow I = \left[\frac{1}{3}\cos 2x\, e^{3x} + \frac{2}{9}\sin 2x\, e^{3x}\right] \times \frac{9}{13} + C$

22

$I = \int \cos(\ln x)\, dx$ Let $u = \cos(\ln x)$

$I = \int u\, dv$ $\frac{du}{dx} = -\sin(\ln x)\frac{1}{x}$

$\cancel{= uv - \int v\, du}$ $du = -\sin(\ln x)\frac{1}{x}\, dx$

$= uv - \int v\, du$

$dv = dx$

$= \cos(\ln x).x - \int \cancel{x}. -\sin(\ln x)\frac{1}{\cancel{x}}\, dx$ $\int 1.\, dv = \int 1.\, dx$

$= \cos(\ln x).x + \int \sin(\ln x).\, dx$ $\Rightarrow v = x$

again

$= \cos(\ln x)x + \left[\int u\, dv\right]$ $u = \sin(\ln x)$

$= \cos(\ln x).x + \left[uv - \int v\, du\right]$ $\frac{du}{dx} = \cos(\ln x).\frac{1}{x}$

$du = \cos(\ln x).\frac{1}{x}\, dx$

$= \cos(\ln x).x + \left[\sin(\ln x).x - \int \cancel{x}\cos(\ln x).\frac{1}{\cancel{x}}\, dx\right]$ $vd = dx$

$= \cos(\ln x).x + \left[\sin(\ln x).x - \int \cos(\ln x).\, dx\right]$ $\int 1.\, vd = \int .1.\, dx$

$\Rightarrow I = \cos(\ln x)x + \sin(\ln x).x - I$ $v = x$

$\Rightarrow \underbrace{I + I}_{\Rightarrow 2I} = \cos(\ln x)x + \sin(\ln x).x$

$\Rightarrow I = \frac{1}{2}\left(\cos(\ln x)x + \sin(\ln x).x\right) + C$

32 Limit এর value বসিয়ে দিতে হবে এবং Integration complete করো

$$I=\int_1^e x^2 \ln x \, dx$$

Let $u=\ln x$

$\frac{du}{dx}=\frac{1}{x}$

$du=\frac{1}{x}dx$

$dv=x^2 dx$

$\int 1\,dv=\int x^2 dx$

$v=\frac{x^3}{3}$

$$=\int_1^e u\,dv$$

$$=[uv]_1^e \int v\,du$$

$$=\left[\ln x\cdot\frac{x^3}{3}\right]_1^e-\int_1^e\frac{x^3}{3}\cdot\frac{1}{x}\,dx$$

$$=\left[\ln e\cdot\frac{e^3}{3}-\underbrace{\ln 1}_{0}\cdot\frac{1^3}{3}\right]-\frac{1}{3}\int_1^e x^2\,dx$$

$$=\left[\ln e\cdot\frac{e^3}{3}-0\right]-\frac{1}{3}\cdot\left[\frac{x^3}{3}\right]_1^e$$

$$=\left[\ln e\cdot\frac{e^3}{x}\right]-\frac{1}{3}\left[\frac{e^3}{3}-\frac{1^3}{3}\right]$$

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$$I=\int_0^{\frac{\sqrt3}{2}}\sin^{-1}x\,dx$$

Let, $u=\sin^{-1}x$

$\frac{du}{dx}=\frac{1}{\sqrt{1-x^2}}$

$du=\frac{1}{\sqrt{1-x^2}}dx$

$dv=dx$

$\int 1.\,dv=\int 1.\,dx$

$\Rightarrow v=x$

$$=\int_0^{\frac{\sqrt3}{2}}u\,dv$$

$$=[uv]_0^{\frac{\sqrt3}{2}}-\int_0^{\frac{\sqrt3}{2}}v\,du$$

$$=\left[\sin^{-1}x\cdot x\right]_0^{\frac{\sqrt3}{2}}-\int_0^{\frac{\sqrt3}{2}}x\cdot\frac{1}{\sqrt{1-x^2}}\,dx$$

limit change করলে সাইন (-) দিতে লাগে তবে $- \cdot - = +$

$$I = \left[\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\cdot\frac{\sqrt{3}}{2} - 0\right] - \int_0^{\frac{\sqrt{3}}{2}} \frac{x}{\sqrt{1-x^2}}\,dx$$

যদি কিছু দিয়ে দেই তাহলে সেই অনুযায়ী Limit এর value change করতে হবে

$$= \left[\frac{\pi}{3}\cdot\frac{\sqrt{3}}{2}\right] - \int_1^{\frac{1}{4}} \frac{1}{\sqrt{z}}\cdot -\frac{1}{2}\,dz$$

$$= \left[\frac{\pi}{3}\cdot\frac{\sqrt{3}}{2}\right] + \left\{-\frac{1}{2}\int_{\frac{1}{4}}^{1} z^{-\frac{1}{2}}\,dz\right\}$$

$$= \left[\frac{\pi}{3}\cdot\frac{\sqrt{3}}{2}\right] + \left(-\frac{1}{2}\right)\left[\frac{z^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\right]_{\frac{1}{4}}^{1}$$

$$= \left[\frac{\pi}{3}\cdot\frac{\sqrt{3}}{2}\right] - \frac{1}{2}\left[\frac{z^{\frac{1}{2}}}{\frac{1}{2}}\right]_{\frac{1}{4}}^{1}$$

$$= \left[\frac{\pi}{3}\cdot\frac{\sqrt{3}}{2}\right] - \frac{1}{2}\left[\frac{(1)^{\frac{1}{2}}}{\frac{1}{2}} - \frac{\left(\frac{1}{4}\right)^{\frac{1}{2}}}{\frac{1}{2}}\right]$$

Let

$$1 - x^2 = z$$

$$\frac{dz}{dx} = -2x$$

$$x\,dx = -\frac{1}{2}\,dz$$

Limit

$$1 - \left(\frac{\sqrt{3}}{2}\right)^2 = z$$

$$= 1 - \frac{3}{4} = z$$

$$= \frac{1}{4} = z$$

$$1 - (0)^2 = z$$

$$\Rightarrow z = 1$$

z = lower limit 1

z = upper Limit $= \frac{1}{4}$

কিন্তু upper ছোট হতে পারবে না তাই Limit ঘুরিয়ে দিয়ে সামনে একটি (-) দিতে হবে।

note for Bonus Assignment

$$u(x) = f(x) \cdot g(x)$$

$$u'(x) = f(x) \cdot g'(x) + g(x) f'(x)$$

Differentiation

1. $\frac{d}{dx}(\sin x) = \cos x$

2. $\frac{d}{dx}(\cos x) = -\sin x$

3. $\frac{d}{dx}(\tan x) = \cancel{\tan x}.\sec^2 x$

4. $\frac{d}{dx}(\sec x) = \sec x \cdot \tan x$

5. $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

6. $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x . \cot x$

7. $\frac{d}{dx}(x^n) = nx^{n-1}$

8. $\frac{d}{dx}(c) = 0$

9. $\frac{d}{dx}[f'(x)]^n = n[f(x)] \cdot \frac{d}{dx}(f(x))$

10. $\frac{d}{dx}(u \times v) = u\frac{d}{dx}(v) + v . \frac{d}{dx}(u)$

11. $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{-u\frac{d}{dx}(v) + v . \frac{d}{dx}(u)}{v^2}$

12. $\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$

13. $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

14. $\frac{d}{dx}(\ln x) = \frac{1}{x} = \frac{d}{dx}(\log_e^x)$

15. $\frac{d}{dx}\left(e^{f(x)}\right) = e^{f(x)} . \frac{d}{dx}(f(x))$

16. $\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$

17. $\frac{d}{dx}(a^x) = a^x . \ln a$

~~20.~~ 18. $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

19. $\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$

20. $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$

21. $\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$

22. $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$

23. $\frac{d}{dx}(\operatorname{cosec}^{-1} x) = \frac{-1}{x\sqrt{x^2-1}}$

United International University

School of Science and Engineering

Final Examination Trimester: Fall-2023

Course Title: Fundamental Calculus

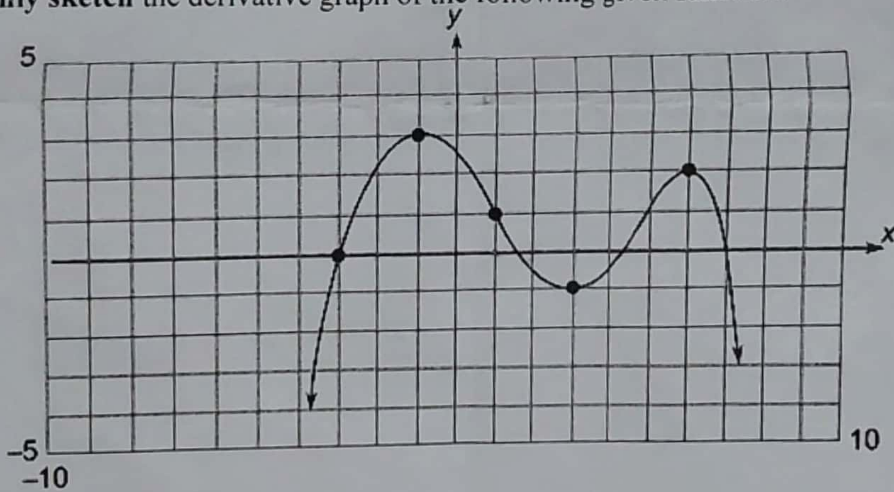
Course Code: Math 1151 Marks: 40 Time: 2 Hours

Answer all the questions. Answer all parts of a question together.

$(-1)^2-2(-1)$
$=1+2$

1. Consider the function $f(x) = x^2 - 2x$; $x_0 = -1$ and $x_1 = 2$.
   (a) **Find** the instantaneous rate of change of $f(x)$ with respect to x at an arbitrary value of x_0. [2]
   (b) Use part **(a)** to **find** the slope of the tangent lines for the value of x_0. [1]
   (c) **Find** the average rate of change of function in the interval $[x_0, x_1]$. [1]
   (d) **Find** the equation of the tangent line to the function $f(x)$ at x_0. [2]
   (e) **Find** the equation of the secant line to the function $f(x)$ on the interval $[x_0, x_1]$. [2]
   (f) **Draw** the graph of $f(x)$ together with the tangent lines and secant lines. [2]

2. (a) **Roughly sketch** the derivative graph of the following given function. [2]



(b) Consider the function [5]

$$f(x) = \begin{cases} 4 - x^2, & x > -1 \\ 2x + 5, & x \le -1 \end{cases}$$

(i) **Sketch** the graph of $f(x)$.
(ii) **Determine** whether the function $f(x)$ is continuous and differentiable at $x = -1$.

(c) **Find** $\frac{dy}{dx}$, where $y = \cot^3 \sqrt{2x - 3\sin x}$. [3]

Please Turn Over

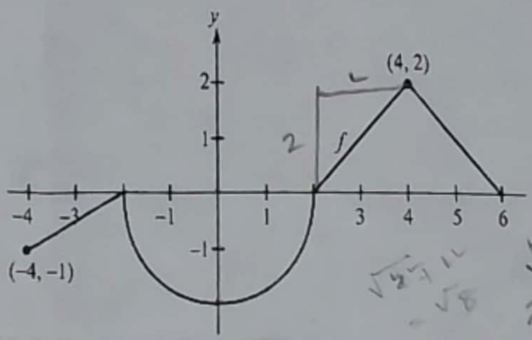
3. (a) Use appropriate formula of geometry to **evaluate** the following integrals: [5]

(i) $\int_{-3}^{3}\{(x+4)-\sqrt{9-x^2}\}dx$ (ii) $\int_{-2}^{3}(5-|x+1|)dx$

(b) The graph of $f(x)$ is shown. **Evaluate** the following definite integrals.

(i) $\int_{-2}^{6} f(x)dx$ [3]

(ii) $\int_{0}^{4} |f(x)|dx$ [2]



4. (a) Evaluate the following integrals: [6]

(i) $\int \frac{t^3-t^2\cos t-5t+1}{t^2}dt$

(ii) $\int \frac{x}{1+x^4}dx$

(iii) $\int x\sin 2x\, dx$

(b) **Find** and **Sketch** the area of the region enclosed by the parabola $y=x^2$ and the line $y=x+2$. [4]

LIATE

BEST OF LUCK!!!