

□ set

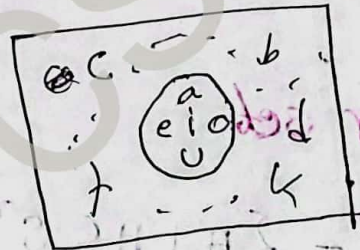
$$V = \{a, e, i, o, u\}$$

$a \in V$  means  $a$  is a member of  $V$

\*  $U$  is the set of all English letter

$V$  is the set of all vowels

$$V \subseteq U$$



□ Write down the following set in roster method

$T = \{u \mid u \text{ is a member of the original Avengers}\}$

$T = \{\text{Ironman, Captain America, Hulk, Black Widow, Hawkeye}\}$

$$N \in \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$$

$$N = \{0, 1, 2, 3, \dots\}$$

$$\mathbb{Z} = \{-2, -1, 0, 1, 2, \dots\}$$

$$\mathbb{Z}^+ = \{1, 2, 3, \dots\}$$

$$\mathbb{Q} = \left\{ \frac{-7}{2}, \frac{8}{2}, \dots \right\}$$

$$\mathbb{Q}^+ = \left\{ \frac{7}{2}, \frac{8}{2}, \dots \right\}$$

$$\mathbb{R} = \{-0.1111, 1.0125, \dots\}$$

▣ Cardinality of finite set

$$V = \{a, e, t, o, u\}$$

cardinality of  $V, |V| = 5$

Empty set

$$|\emptyset| = 0$$

$$|P(S)| = 2^{|S|} = 2^n$$

▣ power set

$$P(S) = \{A \mid A \subseteq S\}$$

$$P(\{1, 2\}) = \{\{1\}, \{2\}, \{1, 2\}, \emptyset\}$$

$$S = \{1, 2, 3\}$$

$$P(S) = \{\{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}, \emptyset\}$$

{ cardinality মাত্র চারটির নয় }



## set

### □ cartesian product

\*  $A \times B = \{ (x, y) \mid x \in A \wedge y \in B \}$  { order pair }

$A = \{ 1, 2, 3 \}$   $B = \{ a, b \}$

$\{ 1, a \} = \{ a, 1 \}$   
 $\{ 1, a \} \neq \{ a, 1 \}$

$\therefore A \times B = \{ (1, a), (1, b), (2, a), (2, b), (3, a), (3, b) \}$

cardinality =  $3 \times 2$

= 6

$A = \{ 1, 2, 3 \}$

$B = \{ a, b \}$

\*  $A = \{ 1, 2 \}$ ,  $B = \{ a, b \}$ ,  $C = \{ \alpha, \beta \}$

$\therefore A \times B \times C = \{ (1, a, \alpha), (1, a, \beta), (1, b, \alpha), (1, b, \beta), (2, a, \alpha), (2, a, \beta), (2, b, \alpha), (2, b, \beta) \}$

[অবস্থান অনুযায়ী ২য় অংশ]

\*  $A = \{ a, b \}$ ,  $B = \{ c, d \}$ ,  $C = \{ e, f \}$

$A \times B \times C = \{ (a, c, e), (a, c, f), (a, d, e), (a, d, f), (b, c, e), (b, c, f), (b, d, e), (b, d, f) \}$

Restricting quantifiers with sets

$$\forall n \in \mathbb{R} (n > 0) \rightarrow \text{True.}$$

$$\exists n \in \mathbb{Z} (n = 1) \rightarrow \text{True.}$$

\*  $P(n) \equiv (n > 1)$

For which values of  $n$  will  $P(n)$  be true?

① Any <sup>real</sup> number larger than 1

②  $n$  real  $n$  less than 0

$$P(n) \equiv |n| = 1 \rightarrow \{n \in \mathbb{R} \mid |n| = 1\} = \{1, -1\}$$

$$Q(n) \equiv |n| = n \quad \mathbb{R}^+ / [0, \infty] / 0 \leq n < \infty$$

$$\{n \in \mathbb{R} \mid n \geq 0\}$$

$$R(n) \equiv n+1 > n \rightarrow \{\mathbb{R}\}$$

$$\# P(n) \equiv n > n \quad \{n \in \mathbb{R} \mid (n > 1) \vee (n < 0)\} / \mathbb{R} - [0, 1]$$

$$Q(n) \equiv n+1 \leq n \quad \{\emptyset\}$$

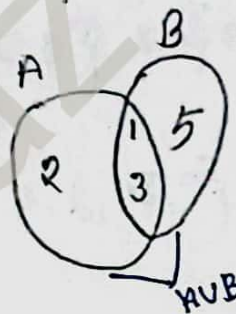
$$R(n) \equiv n = 4 \quad \{-2, 2\}$$

Union set

$$A = \{1, 2, 3\}, B = \{1, 3, 5\}$$

$$A \cup B = \{n \mid n \in A \vee n \in B\}$$

$$A \cup B = \{1, 2, 3, 5\}$$

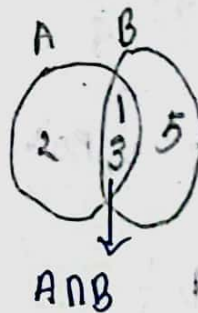


Intersection set

$$A = \{1, 2, 3\}, B = \{1, 3, 5\}$$

$$A \cap B = \{n \mid (n \in A) \wedge (n \in B)\}$$

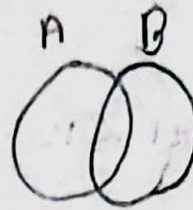
$$A \cap B = \{1, 3\}$$





Cardinality of union set

$$|A \cup B| = |A| + |B| - |A \cap B|$$



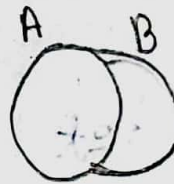
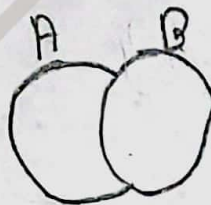
$$= |A| + 0 + 0 + |B| - |A \cap B|$$

Difference

$$A = \{1, 2, 3\} \quad B = \{1, 3, 5\} \quad A - B = \{x \mid (x \in A) \wedge x \notin B\}$$

$$A - B = \{2\}$$

$$B - A = \{5\}$$



$$A = \{1, 3, 9, 10, 12\} \quad B = \{1, 5, 12\}$$

$$\Rightarrow A - B = \{3, 9, 10\}$$

$$B - A = \{5\}$$

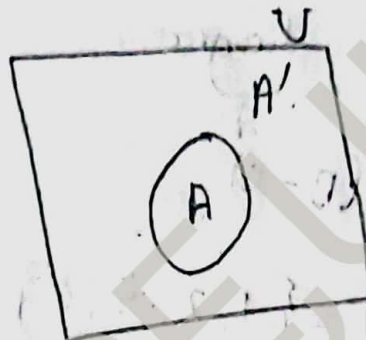
□ complement of a set

$$A' = U - A$$

$$U = \{1, 2, 3, 4, 5, 6\}$$

$$A = \{2, 5, 6\}$$

$$A' = U - A \\ = \{1, 3, 4\}$$



□ Given that  $A = \{0, 2, 4, 6, 8\}$ ,  $B = \{0, 1, 2, 3, 4\}$ ,  $C = \{0, 3, 6, 9\}$

$$A \cup B \cup C = \{0, 1, 2, 3, 4, 6, 8, 9\}$$

$$A \cap B \cap C = \{0\}$$

$$U = \{0, 1, 2, 3, 4, 6, 8, 9\}$$

$$A - (B \cap C)'$$

$$\therefore B \cap C = \{0, 3\}$$

$$\therefore (B \cap C)' = U - (B \cap C) \\ = \{1, 2, 4, 6, 8, 9\}$$

$$\therefore A - (B \cap C)' \\ = \{0, 2, 4, 6, 8\} - \{1, 2, 4, 6, 8, 9\} \\ = \{0\}$$

problem

Draw diagram

$$A' \cup (B - C)$$

$$A = \{1, 2, 4, 5\}$$

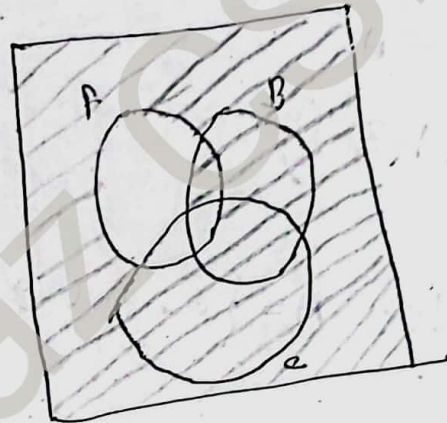
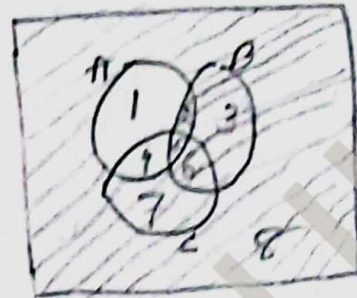
$$B = \{2, 3, 5, 6\}$$

$$C = \{4, 5, 6, 7\}$$

$$\begin{aligned} \therefore A' &= U - A \\ &= \{3, 6, 7, 8\} \end{aligned}$$

$$B - C = \{2, 3\}$$

$$A' \cup (B - C) = \{2, 3, 6, 7, 8\}$$





## Generalized Union and Intersection

$$\sum_{i=1}^n a_i = a_1 + a_2 + \dots + a_n$$

$$\prod_{i=1}^n a_i = a_1 a_2 a_3 \dots a_n$$

$$A \cup \bigcup_{i=1}^n A_i = A \cup A_1 \cup A_2 \cup \dots \cup A_n$$

$$\bigcap_{i=1}^n A_i = A_1 \cap A_2 \cap \dots \cap A_n$$

$$* A_1 = \{1, 2, 3, 4\}$$

$$A_2 = \{2, 4, 6, 8\}$$

$$A_3 = \{4, 8, 12, 16\}$$

$$\bigcup_{i=1}^3 A_i = A_1 \cup A_2 \cup A_3 = \{1, 2, 3, 4, 6, 8, 12, 16\}$$

$$\bigcap_{i=1}^3 A_i = A_1 \cap A_2 \cap A_3 = \{4\}$$

### problem

A Let  $A_i = \{i, i+1, i+2, \dots\}$

Find out,  $\bigcup_{i=1}^n A_i$  and  $\bigcap_{i=1}^n A_i$

$$A_1 = \{1, 2, 3, 4, 5, \dots\}$$

$$A_2 = \{2, 3, 4, 5, \dots\}$$

$$A_3 = \{3, 4, 5, \dots\}$$

$$\bigcup_{i=1}^n A_i = A_1$$

$$\bigcap_{i=1}^4 A_i = \{4, 5, \dots\}$$

$$\bigcap_{i=1}^n A_i = \{A_n\}$$

$$A_2 \subseteq A_1, A_3 \subseteq A_1$$

### problem

Let  $A_i = \{1, 2, 3, \dots, i\}$

$$\bigcup_{i=1}^{\infty} A_i = \overbrace{A_4 = \{1, 2, 3, 4\}}^{A_2}$$

$$\bigcap_{i=1}^{\infty} A_i = \{1\} = A_1$$

$$A_1 = \{1\}$$

$$A_2 = \{1, 2\}$$

$$A_3 = \{1, 2, 3\}$$

$$A_4 = \{1, 2, 3, 4\}$$