Mathematical Induction:

২ টা স্টেপ কে প্রমাণিত করতে হবে

[1] Basis: যেই শর্ত দেওয়া থাকবে তার প্রথম টাই

[2] Induction: $(P(k) \to P(k+1))$ where $k \in \mathbb{Z}^+$

* prove the summation formula of the following series for all positive integers n:

~~Let p(n) =~~ $1+2+3+\cdots+n = \frac{n(n+1)}{2}$

Let $p(n)$: $1+2+3+\cdots+n = \frac{n(n+1)}{2}$

Basis: $P(1)$

$L.H.S = 1$

$$R.H.S = \frac{n(n+1)}{2}$$

$$= \frac{1(1+1)}{2}$$

$$= 1$$

$\therefore P(1)$ is true

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Induction: $\forall k\,(P(k) \longrightarrow P(k+1))$

Let $P(k)$ be true, where $k \in Z^+$

$\therefore 1+2+3+\cdots+k = \frac{k(k+1)}{2}$

we have to prove $P(k+1)$ that is

$1+2+3+\cdots+(k+1) = \frac{(k+1)(k+2)}{2}$

Adding $(k+1)$ to both sides of $P(k)$

$$1+2+3+\cdots+k+(k+1) = \frac{k(k+1)}{2}+(k+1)$$

$$= \frac{k(k+1)+(k+1)2}{2}$$

$$= \frac{(k+1)(k+2)}{2}$$

$\therefore \forall k\,(P(k) \rightarrow P(k+1))$ is true

$\therefore P(n)$ is true for all $n \in Z^+$

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* prove that summation formula of the following series for all nonnegative Integers n:

$$1+2+2^2+\dots\dots+2^n = 2^{n+1}-1$$

Let $P(n) : 1+2+2^2+\dots+2^n = 2^{n+1}-1$

Basis: $P(0)$

$L.H.S = 1^0 = 1$

$R.H.S = 2^{0+1}-1$

$= 1$

$\therefore P(0)$ is true

Induction:

$\forall k \, (P(k) \rightarrow P(k+1))$

Let $P(k)$ is true, where $k \in N$

$\therefore 1+2+2^2+\dots 2^k = 2^{k+1}-1$

we have to prove $P(k+1)$ that is

$$1+2+2^2+\dots+2^{k+1} = 2^{k+2}-1$$

Adding 2^{k+1} to both sides of $P(k)$

$$1+2+2^2+\cdots+2^k+2^{k+1} = 2^{k+1}-1+2^{k+1}$$

$$= 2 \cdot 2^{k+1} - 1$$

$$= 2^{k+1+1} - 1$$

$$= 2^{k+2} - 1$$

∴ Induction is true

∴ $P(n)$ is true for all $n \in N$

* prove the summation formula of the following series for all positive integers n,

$$1^2 - 2^2 + 3^2 + \cdots + (-1)^{n-1} n^2 = (-1)^{n-1} \frac{n(n+1)}{2}$$

Let $P(n)$: $1^2 - 2^2 + \cdots + (-1)^{n-1} n^2 = (-1)^{n-1} \frac{n(n+1)}{2}$

Basis: $P(1)$

∴ $L.H.S = 1^2 = 1$

$R.H.S = (-1)^{1-1} \frac{1(1+1)}{2}$

$= 1 \cdot 1$

$= 1$

$\therefore$ P(1) is true

Induction

$\forall k\,(p(k) \to p(k+1))$

Let $p(k)$ be true, where $k \in \mathbb{Z}^+$

$$\therefore\ 1^2 - 2^2 + \cdots + (-1)^{k-1} k^2 = (-1)^{k-1} \frac{k(k+1)}{2}$$

we have to prove $p(k+1)$ that is

$$1^2 - 2^2 + \cdots + (-1)^{k-1+1} (k+1)^2 = (-1)^{k-1+1} \frac{(k+1)(k+2)}{2}$$

$$1^2 - 2^2 + \cdots + (-1)^k (k+1)^2 = (-1)^k \frac{(k+1)(k+2)}{2}$$

Adding $(-1)^k (k+1)^2$ to both sides of $P(k)$

$$1^2 - 2^2 + \cdots + (-1)^2 k^2 + (-1)^k (k+1)^2 = (-1)^{k-1} \frac{k(k+1)}{2} + (-1)^k \frac{(k+1)(k+2)}{2}$$

$$= \frac{-1^k}{-1} \frac{k(k+1)}{2} + (-1)^k \frac{(k+1)(k+2)}{2}$$

$$= -(-1)^k \frac{k(k+1)}{2} + (-1)^k \frac{(k+1)(k+2)}{2}$$

Adding

$$1^2 - 2^2 + \cdots + (-1)^2 k^2 + (-1)^2 (k+1)^2 = (-1)^{k-1} \frac{k(k+1)}{2} + (-1)^k (k+1)^2$$

$$= \frac{-1^k}{-1} \frac{k(k+1)}{2} + (-1)^k (k+1)^2$$

$$= -(-1)^k \cdot \frac{k(k+1)}{2} + (-1)^k \cdot (k+1)^2$$

$$= (-1)^k \left((k+1)^2 - \frac{k(k+1)}{2} \right)$$

$$= (-1)^k \left(\frac{2(k+1)^2 - k(k+1)}{2} \right)$$

$$= (-1)^k \left(\frac{(k+1)(2k+2-k)}{2} \right)$$

$$= (-1)^k \frac{(k+1)(k+2)}{2}$$

Induction is true

$\therefore$ $P(n)$ is true for all $n \in \mathbb{Z}^+$

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* prove that for all positive integers n, n^3-n is divisible by 3

Basis: $p(1)$

~~L.H.S~~

1^3-1

$=0$

wich divisible $\frac{0}{3}=0$

$\therefore$ so basis is true.

Induction:

$\forall k\,(p(k) \longrightarrow p(k+1))$

Let $p(k)$ be true, where $k \in \mathbb{Z}^+$

k^3-k is divisible by 3

We have to prove $p(k+1)$ that is

$(k+1)^3-(k+1)$ is divisible by 3

Let $k^3-k=3m$, where m is an integer

$\therefore (k+1)^3 - (k+1)$

$= k^3 + 3k^2 + 3k + 1^3 - k - 1$

$= k^3 - k + 3k^2 + 3k$

$= 3m + 3k^2 + 3k$

$= 3(m + k^2 + k)$

Since k and m are integer, $m + k^2 + k$ is an integer.

$\therefore 3(m+k^2+k)$ must be divisible by 3

$\therefore (k+1)^3 - (k+1)$ is divisible by 3

* prove that for all nong negative integers n:

$n^3 - n$ is divisible by 6

Basis: $P(0)$

$0^n - 0 = 0$

which divisible by 6 $= \frac{0}{6} = 0$

$\therefore$ Basis is true

Induction:

$\forall k \, (P(k) \to P(k+1))$

Let $P(k)$ be true, where $k \in N$

$k^3 - k$ is divisible by 6

We have to prove $P(k+1)$ that is

$(k+1)^3 - (k+1)$ is divisible by 6

Let $k^3 - k = 6m$, where m is an integers

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$\therefore (k+1)^3 - (k+1)$

$= k^3 + 3k^2 + 3k + 1 - k - 1$

$= k^3 - k + 3k^2 + 3k$

$= 6m + 3k^2 + 3k$

$= 6m + 3k(k+1)$

$\therefore$ Since ~~8m~~ m is an integer.

6m must be divisible by 6.

since k is an integer.

3k(k+1) must be divisible by 3

Furthermore since k and k+1 are consecutive integer. one of them must be even.

$k = 7 \quad k+1 = 8$

$7 \times 8 = 56$

$= 56 \times 3$

$= \frac{168}{6}$

$= 28$

$\therefore 3k(k+1)$ must be even

$\therefore 3k(k+1)$ is divisible by 6

$\therefore (k+1)^3 - (k+1)$ is divisible by 6

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