11-2-23

start for 2nd CT

predicates and quantifiers

predicates

$P(x, y): x + y = 10$

প্রেডিকেট মানে কোন মান দেওয়া থাকবে না তাই

$x + y = 10$ সত্য বলা সম্ভব না।

Type of predicates

* ① unary → 1 variable

"x is greater than 3" → $P(x)$

* ② binary → 2 variable

"x is greater than y" → $Q(x, y)$

* ③ Ternary → 3 variable

"x is greater than y and z" → $R(x, y, z)$

"$\underbrace{x}_{\text{variable/ subject}}$ $\underbrace{\text{is greater than 3}}_{\text{predicates}}$"

$P(x) = x > 3$

$x = 4$

$P(4) = 4 > 3$ True

$x = 2$

$P(2) = 2 > 3$ False

*** Let $P(x)$ denote the statement "$x > 3$" what are the truth value of $P(4)$ and $P(2)$?

$P(x) = x > 3$

$P(4) = 4 > 3 \longrightarrow$ Truth

$P(2) = 2 > 3 \longrightarrow$ False

*** Let $Q(x,y)$ denot the statement "$x = y+3$" what are the truth value of the propositions $Q(1,2)$, $Q(3,0)$?

$\therefore Q(x,y) \Rightarrow x = y+3$

$Q(1,2) \Rightarrow 1 = 2+3$ False

$Q(3,0) \Rightarrow 3 = 0+3$ True

*** Let $k(x)$ be "x is a knight"

"Bahubali is a knight" $k(\text{bahubali}) \longrightarrow$ True

" Ballaladeve is a knight" $k(\text{Ballaladeve}) \longrightarrow$ False

Quantifier

$\forall x$ = Universal quantifier

$\exists x$ = Existential quantifier

** $\forall x$ সবার জন্য for all x সত্য

** $\exists x$ কিছু সংখ্যক / at least one person/things

** ** যা থাকলে আমরা $\forall x$ / $\exists x$ বুঝবো

$\forall x$ Universal

for all, for every, for each, for any, all of, given any, Every

** ** $\exists x$ Existential

The exists, for some, for at least one, There is

$\forall x$ = সবার কাছে আছে

$\neg \forall x$ = সবার কাছে আছে এমন না কারো কাছে থাকতে পারে

$\exists x$ = কারো কাছে আছে সবার কাছে না

$\neg \exists x$ = কারোর কাছেই নাই

Example

There is a student who can speak Russian

$\exists x$ — $P(x)$

$\therefore \exists x P(x)$

Let $Q(x)$ be the statement "$x < 2$;" what is ~~the~~ a truth value of the quantification $\forall x Q(x)$, where is the domon consists of all real numbers

$Q(x) = x < 2$

$\forall x Q(x)$ False

$Q(1) = 1 < 2$

$Q(2) = 2 < 2$

$Q(3) = 3 < 2$

∀x মানে সব সত্য যেহেতু x(3) মানে নাই তাই

What is the truth value of $\exists x P(x)$, when $P(x)$ is the statement "$x^2 > 10$" and the domain consists of the possitive integers not exceeding 4?

$\{0, 1, 2, 3, 4\}$

$\exists x P(x) = x^2 > 10$
$= 1^2 > 10$
$= 2^2 > 10$
$= 3^2 > 10$
$= 4^2 > 10$

যেহেতু অন্তত একটি সত্য তাই এই statement True.

■ English to logical

* $\underbrace{\text{Every}}_{\forall x}$ student in the $\underbrace{\text{class has studied calculus}}_{c(x)}$

$\therefore \forall x \; c(x)$

* $\underbrace{\text{Every}}_{\forall x}$ person x, if person x $\underbrace{\text{is a student in the class}}_{s(x)}$ then x $\underbrace{\text{has studied calculus}}_{c(x)}$

domain $\rightarrow$ person

$\therefore \forall x \left(s(x) \rightarrow c(x) \right)$

* $\underbrace{\text{some}}_{\exists x}$ student in this class $\underbrace{\text{has visited mexico}}_{m(x)}$

$\therefore \exists x \; m(x)$

* There is a person x, having the properties that x is a student in the class and x has visited Mexico

$\exists x$ — There is; $S(x)$ — x is a student in the class; $\wedge$ — and; $M(x)$ — x has visited Mexico

$\therefore \exists x \left(S(x) \wedge M(x) \right)$

* For every person x, if x is a student in the class then x has visited Mexico or x has visited Canada

$\forall x$ — For every person; $\rightarrow$ — if … then; $S(x)$ — x is a student in the class; $M(x)$ — x has visited Mexico; $C(x)$ — x has visited Canada

$\therefore \forall x \left(S(x) \rightarrow M(x) \vee C(x) \right)$

* Let $P(x)$ be the statement "x can speak Russian and Let $Q(x)$ be the statement "x knows the computer language C++"

a) There is a student at your school who can speak Russian and who knows C++"

$P(x)$ — can speak Russian; $\wedge$ — and; $Q(x)$ — who knows C++

$\therefore \exists x \left(P(x) \wedge Q(x) \right)$

b/ There is a student at your school who can speak Russian but who doesn't know c++

$\underbrace{\text{There is a student}}_{\exists n}$ ... $\underbrace{\text{speak Russian}}_{P(n)}$ $\underbrace{\text{but}}_{\wedge}$ $\underbrace{\text{who doesn't know c++}}_{\neg Q(n)}$

$\exists n \left(P(n) \wedge \neg Q(n) \right)$

c/ No student at your school can speak Russian or knows c++

$\underbrace{\text{No student}}_{\exists n \neg}$ ... $\underbrace{\text{can speak Russian}}_{P(n)}$ $\underbrace{\text{or}}_{\vee}$ $\underbrace{\text{knows c++}}_{Q(n)}$

$\therefore \exists n \neg \left(P(n) \vee Q(n) \right)$ একজনও না তাই

[($\forall n \neg$) না হওয়ার কারণে ভেতরে negation দেবে না বরং অন্তত"]

d/ Every student at your school either can speak or c++

$\underbrace{\text{Every}}_{\forall n}$ ... $\underbrace{\text{can speak}}_{P(n)}$ $\underbrace{\text{or}}_{\vee}$ $\underbrace{\text{c++}}_{Q(n)}$

$\therefore \forall n \left(P(n) \vee Q(n) \right)$

Logical to English

$C(x)$; is "x is a comedian"

$F(x)$; is "x is a funny"

domain → all people

a/ $\forall x\,(C(x) \rightarrow F(x))$

For all x, if x is a comedian then x is a funny people [Every comedian is funny]

b/ $\forall x\,(C(x) \wedge F(x))$

For all x, x is a comedian and x is a funny

[Every person is funny comedian]

c/ $\exists x\,(C(x) \vee F(x))$

some x, x is a comedian or x is a funny

[some person is comedian or funny]

Law

Logical Equivalence Involving Quantifiers

$\forall x\,(P(x) \wedge Q(x)) \equiv (\forall x\, P(x)) \wedge (\forall x\, Q(x))$

$\exists x\,(P(x) \vee Q(x)) \equiv (\exists x\, P(x)) \vee (\exists x\, Q(x))$

Negating Quantifiers Expressions

De Morgan's Law for Quantifier

$\neg(\forall x\, P(x)) \equiv \exists x\, \neg P(x)$

$\neg(\exists x\, P(x)) \equiv \forall x\, \neg P(x)$

* De Morgan's ~~wh~~ without negation ($\neg$)

$\neg \exists x\,(x^2 = 2)$

$\equiv \forall x\, \neg (x^2 = 2)$

$\equiv \forall x\,(x^2 \neq 2)$

$\neg \forall x\,(x^2 > x)$

$\equiv \exists x\, \neg (x^2 > x)$

$\equiv \exists x\,(x^2 \leq x)$

যেটা থাকবে সেটা
উল্টিয়ে দিলে বাকি
সব কাজে লাগবে
যেমন > এর পরিবর্তে
≤ হয়েছে

* Every student in your class has taken a course in calculus.

$\forall x \qquad\qquad\qquad\qquad\qquad\qquad C(x)$

$\therefore \quad \forall x C(x)$

$\neg(\forall x C(x))$

$\equiv \exists x \neg C(x)$

$\downarrow$

There is a student in your class who hasn't taken a couse in calulus.

* There is an honest politician."

$\exists x \qquad\qquad\qquad H(x)$

$\therefore \quad \exists x H(x)$

$= \neg(\exists x H(x))$

$= \forall x \neg H(x)$

$\downarrow$

Every politician is ~~dis~~ not honest

not all polician are honest

Nested quantifier

$\forall x(\exists x\, P(x,y))$.

** Statement to English

$\forall x \forall y\, ((x>0) \wedge (y<0) \longrightarrow xy<0)$

For every real number x and y. If x is positive and y is negative then xy is negative

এখানে product negative means, $xy<0$ হতে হলে উক্ত শর্ত গুলো sufficiant তাই Implies ($\rightarrow$) হয়েছে

** English to Logical

The some of two possitive integers is always possitive.

$\cancel{\forall x \forall y}$ $\exists x \exists y\, ((x>0) \wedge (y>0) \rightarrow (x+y)>0)$

** For every nonzero value of x, There is a corresponding real number y such that $xy = 1$

$\forall x \left((x \neq 0) \rightarrow \exists y \, (xy = 1) \right)$

⊞ Implies : হওয়াবে অর্থ

যে থাকবে তার আগে বসাবার

{যে --- সে} — এভাবে অনুবাদ হবে

⊞ Order of quantifiers

$\forall x \forall y \, (x+y = y+x)$
$\forall y \forall x \, (x+y = y+x)$
[note দুইটা সেম অপারেটর থাকলে চেঞ্জ করা যাবে] **

$\exists x \exists y \, (xy = 6)$
$\exists y \exists x \, (xy = 6)$

$\forall x \exists y \, (xy = 0) \longrightarrow$ possible নয় কেননা
$\exists y \forall x \, (xy = 0)$ এক fact অর্থ বুঝাতে হবে অপরিবর্তিত থাকতে হবে।

Exercise

Given predicat:

$Q(x,y)$: x has been a contestant on the game show y.

Domain

x: set ~~fo~~ of students in your ~~class~~ school

y: set of all television quiz shows

a/ There is a student at your school who has been a contestant on a television quiz show.

$\exists x \exists y \; Q(x,y)$

b/ No student at your school has ever been a contestant on a tv quiz show

$\neg \exists x \neg \forall y \; Q(x,y)$

c/ There is a student at your school who has been a contestant on Jeopardy! and on Wheel of Fortune.

$\exists x \, (Q(x, \text{Jeopardy}) \wedge Q(x, \text{Fortune}))$

d/ Every tv quiz show has had a student from your school as a contestant.

$\cancel{\forall y}$ $\forall y \exists x\, Q(x, y)$

$\cancel{\exists x_1 \exists x_2 ((x_1 \neq x_2) \wedge Q(x, J)}$

~~Exercize~~

e/ At Least two students from your school have been contestant on Jeopardy!

$\cancel{\exists x \forall y\, Q(x, Jeopardy)}$ $\exists x_1 \exists x_2 ((x_1 \neq x_2) \wedge Q(x_1, J) \wedge Q(x_2, J))$

Exercise → 2

Given predica~~te~~te

$I(x)$: x ha an internet connection

$C(x, y)$: x and y have chatted over the internet

Domain: set of all students in your class

a/ Jerry does not have an internet connection

$\neg I(\text{Jerry})$

b/ no one in the class has chatted with Bob.

$\neg \exists x\, C(x, Bob)$

c/ sanJay has chatted with everyone expect Joseph.

$\forall x\, ((x \neq Josseph) \rightarrow C(x, sanJay))$

d/ not everyone in your class has an internet connection

$\neg \forall x I(x)$

e/ Everyone in you class with an Intepnet connection has chatted over the Internet with at Least one othen student in your class.

$\forall x\, (I(x) \rightarrow \exists y C(x, y))$

Exercise → 3

Given predicate

$I(x)$: x has an Internet connection

$C(x, y)$; x and y have chatted over the internet

Domain: (set of all) students in your class

a/ Exactly one student in your class has an Internet connection

Exactly

$$\exists x \big(I(x) \wedge \neg \underbrace{\exists y \big((y \neq x) \wedge I(y) \big)}_{\text{Exactly}} \big)$$

b/ someone in your class has an Internet connection but has not chatting with anyone else in your class.

$$\exists x \big(I(x) \wedge \forall y \neg C(x, y) \big)$$

c/ There are two students in your class who have not chatted with each other over the Internet.

$\cancel{\exists x \exists y (I(x)}$ $\exists x \exists y \neg C(x, y)$

d/ There are at least two students in your class who have not chatted with the same person in your class.

$$\exists x \exists y \left((x \neq y) \wedge \neg \exists z \, C(z, x) \wedge C(z, y) \right)$$

e/ There are two students in your class who between them have chatted with everyone else in the class.

$$\exists x \exists y \left(\forall z \left((z \neq x) \wedge (z \neq y) \right) \rightarrow C(x, z) \wedge C(y, z) \right)$$