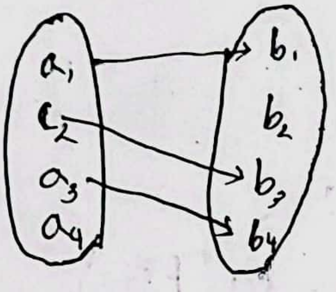
function

$f(x) = x^2$

$f(x) \neq \pm\sqrt{x}$ এটা function না কারণ output ২টা



↓

নহে a function

Domain set of all inputs $\{a_1, a_2, a_3, a_4\}$

co-domain → set of all potential outputs $\{b_1, b_2, b_3, b_4\}$

Range → set of all actual outputs $\{b_1, b_3, b_4\}$

$f: R \to R, \; f(x) = x^2$

Range → R^+ (সব positive numbers} [x^2 output]

$f: R \to R, \; f(x) = \sqrt{x}$ {not a function}

$fog(n) = f(g(n))$

$gof(n) = g(f(n))$

$f(n) = n^2$

$g(n) = n+1$

$\therefore fog(n) = f(g(n))$

$= f(n+1)$

$= (n+1)^2$

$\therefore gof(n) = g(f(n))$

$= g(n^2)$

$= n^2+1$

$f(n) = e^n$

$g(n) = n^4$

$\therefore fog = f(g(n))$

$= f(n^4)$

$= e^{n^4}$

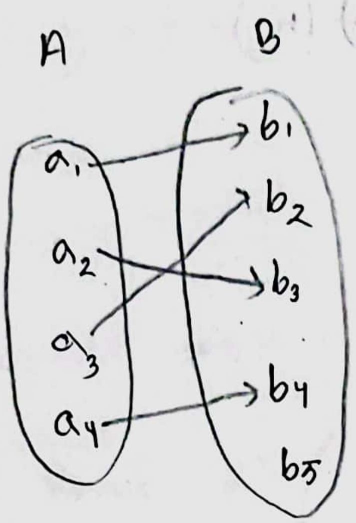
$gof = g(f(n))$

$= g(e^n)$

$= (e^n)^4$

$= e^{4n}$

one to one function / injection



[must have unique output]

* $f: R \to R$; $f(x) = x^2$ not one to one function

Here, $f(1) = f(-1) = 1^2 = 1$

$\therefore$ f is not one to one function

* $f: R \to R$, $f(x) = (x+2)^2$

Here, $f(-3) = f(-1) = (-3+2)^2 = 1$

$\therefore$ f is not one to one function

* $f: R \to R,\ f(x) = x+5$

Let $x_1, x_2 \in R$ and $f(x_1) = f(x_2)$

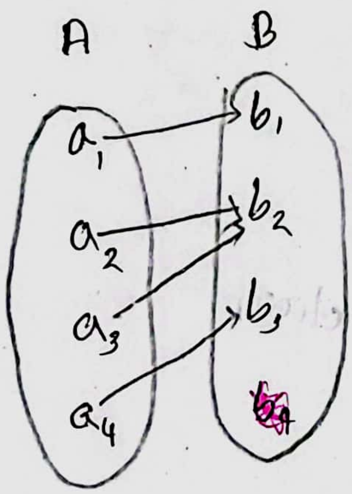
$\therefore\ x_1 + 5 = x_2 + 5$

$\Rightarrow x_1 = x_2$

$\therefore f$ is one to one

$$\left[\begin{array}{l} (x_1 \neq x_2) \to f(x_1) \neq f(x_2) \\ f(x_1) = f(x_2) \longrightarrow x_1 = x_2 \end{array}\right]$$

⊞ onto function / surjection



co-domain এর প্রত্যেকটাকে কারো না কারোর output হতে হবে।

onto function

[co-domain = Range]

codomain ≠ ~~Range~~ Range

Just (+)

* $f: R \to R,\ f(x) = x^2$ not onto [codomain ≠ Range]

Let $y = f(x)$

$\Rightarrow y = x^2$

$\Rightarrow x = \pm\sqrt{y}$

For all ~~x∈R~~ $x \in R$, y must be non negative

$\therefore$ Range $= R^+$ / $\{x \in R \mid x \geq 0\}$

Since codomain ≠ Range

$\therefore f$ is not onto

* $f: R \to R,\ f(x) = x+5$

Let $y = f(x)$

$\Rightarrow y = x+5$

$\Rightarrow x = y-5$

$\therefore$ for all $x \in R$, $y \in R$

Range = codomain / Range = R

$\therefore f$ is onto

one to one

$f: R \to R,\ f(x) = x^2$

Let $x_1, x_2 \in R$ and $f(x_1) = f(x_2)$

$\therefore x_1^2 = x_2^2$

$\Rightarrow x_1 = \pm x_2 \implies x_1 = x_2$ (squar output কে passitiv [illegible] ফেলবে।

$\therefore f$ is not one to one

$\boxed{+}$ $f: R^+ \to R\ \ f(x) = (x)^2$ one to one

$f: R \to R^+\ \ f(x) = x^2$ one to one

$f: R^+ \to R^+\ \ f(x) = x^2$ Bijection

$f: N \to N,\ f(x) = x^2$ one to one

$N = \{0, 1, 2, 3 \cdots\}$

$Z^+ = \{1, 2, 3 \cdots\}$

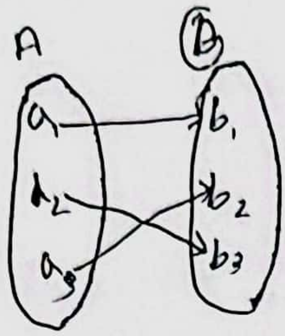
$f: N \to N,\ f(x) = x^2$

$f(0) = 0$

$f(1) = 1 \longrightarrow 2, 3\ ?$

$f(2) = 4$

Bijection (injection + surjection)



This set injection + surjection

∴ it's a Bijection

inverse

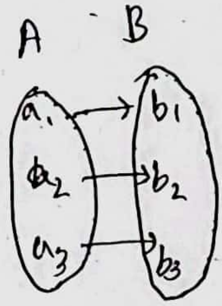
$f: R \to R$. $f(x) = x+5$

① find out if f is a bijection

② If Bijection

$x = $ (terms of y)

$\Rightarrow f^{-1}(y) = (\cdots)$



$f: A \to B$

$f^{-1}: B \to A$

$\therefore f(x) = x+5$

~~x = y - 5~~

Let $y = f(x)$. $y = x+5 \Rightarrow x = y-5$ $\therefore f^{-1}(y) = y-5$

$f^{-1}(x) = x-5$