DM → note for mid-term

Propositional logic

A proposition is a declarative sentence.

* shakib all Hasan is a cricketer. → True

- question is not proposition
- order is not proposition

- $x+1=y$ → Predicate

Truth value < False / True

- I am Lying → Paradox
কারন মিথ্যা বলাটা সত্যি কিন্তু যা বলে তা, হয়তো বা মিথ্যা
তাই T → F → T → F cause $\boxed{\text{Paradox}}$

- conjunction → AND → $\wedge$ $(p \wedge q)$

$\underbrace{\text{Today is friday}}_{p}$ (and) $\underbrace{\text{It is raining}}_{q}$

p	q	p ∧ q
T	T	T
T	F	F
F	T	F
F	F	F

$2^n = 2^2 = 4$

- But থাকলেও conjunction
- , ..., ... কমা থাকলেও conjune

* conjunction এ দুইটি value-ই সত্য হতে হবে
that's why output is $\boxed{\text{True}}$
Like AND operation means $\boxed{\text{গুণ}}$

Disjunction → OR → $\vee$ $(p \vee q)$

* At-least one proposition is True

$\underbrace{\text{Today is friday}}_{p}$ (or) $\underbrace{\text{It is raining}}_{q}$

p	q	p∨q
T	T	T
T	F	T
F	T	T
F	F	F

* Let (r) shakib all hasan is a cricketer (or) (q) shakib khan is a cricketer

r	q	r∨q
T	T	T
T	F	T
F	T	T
F	F	F

Negation → $\neg p$

(p) Shakib Al Hasan is a footballer → False

$\neg$(p) Shakib Al Hasan is not a footballer → True

p	¬p
1	0
0	1

→

p	¬p
T	F
F	T

$1+3 \neq 4$

The rules of writing a truth table

* Start ~~to~~ from input of right
* A T and a F will go down along the column like this. → (r)
* 2nd 2T and 2F will go down along the column like this → (q)
* 3rd 4T and 4F will go down along the column like this → (p)

p	q	r
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

Truth table

* input | output → $((p \wedge q) \vee r)$

p	q	r	$p \wedge q$	$(p \wedge q) \vee r$
T	T	T	T	T
T	T	F	T	T
T	F	T	F	T
T	F	F	F	F
F	T	T	F	T
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

(v) * $(p \vee \neg q) \wedge r$

Exange → or → and

p	q	r	$(\neg q)$	$(p \vee \neg q)$	$(p \vee \neg q) \wedge r$
T	T	T	F	T	T
T	T	F	F	T	F
T	F	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	F
F	T	F	F	F	F
F	F	T	T	T	T
F	F	F	T	T	F

Exclusive OR → (⊕)

** Can't do the same work, two or more person or things, same time.

* two wicket keeper can't playin the same match
* two goal keeper can't playing the same match

* $\underbrace{\text{Quinton de kock}}_{p}$ or $\underbrace{\text{Heinrich kulssen}}_{q}$, will play in today's match.

or ... but not both

$\underline{p}$ ⊕ $\underline{q}$

p	q	p ⊕ q
T	T	F
T	F	T
F	T	T
F	F	F

* $\underbrace{\text{Mushfiqur Rahim}}_{r}$ or $\underbrace{\text{Liton Das}}_{s}$ will play in today's match.

r	s	p ⊕ q
T	T	F
T	F	T
F	T	T
F	F	F

* Or..... not both

"students who have taken calculus or computers science, but not both, can take the class"

p ... q

p	q	p⊕q
T	T	F
T	F	T
F	T	T
F	F	F

* coffee or tea comes with dinners.

coffee = p, tea = q, or = ⊕

* Experience with c++ or Java is required

c++ = p, Java = q, or = ∨

* Lunch includes soup or salad

soup = p, salad = q, or = ⊕

* you can pay using us. dollars or euros

us. dollars = p, euros = q, or = ∨

2nd class

Conditional statement

* "If p then q"

p: today is holiday

q: The story is closed

If today is holiday then the story is closed

p	q	p→q
T	T	T
T	F	F
F	T	T
F	F	T

****যদি statement টি T F থাকে তাহলে $p \to q = F$

* If it rains, I will stay home.

we can also say " It is raining implies that I will stay home.

$p \to q$

p	q	p→q
T	T	T
T	F	F
F	T	T
F	F	T

* If p then q
* p implies q
* q if p

* If $\underset{T}{\underline{1+1=2}}$ then $\underset{F}{\underline{2+2=5}}$ F

If $\underset{F}{\underline{1+1=3}}$ then $\underset{T}{\underline{2+2=4}}$ T

If $\underset{F}{\underline{1+1=3}}$ $\underset{F}{\underline{\text{then dog can fly}}}$ T

If $\underset{T}{\underline{1+1=2}}$ then $\underset{F}{\underline{\text{dog can fly}}}$ F

* "If p, q"

"If it is below freezing, it is also snowing"

$p \to q$

যেমন (→) আমাদের লাগবেই তুমার পাশের শর্ত জন্য তারা Freezing sufficient

If p then q

$\Rightarrow$ q if p

q unless $\neg p$

$\underline{\text{maria will } \overset{\neg\text{find}}{\text{get}} \text{ ~~a job~~ good Job}}_{q}$ unless she $\underline{\text{does not Learn discrete mathematics}}_{\neg p}$

$\Rightarrow$ q unless $\neg p$

Items of Implies

* If ..... then
* when

eg if ... p ... q

* "q whenever p"
* "q is necessary for p"
* "q follow from p"
* "q when p"
* "q unless $\neg p$"

** যার আগে If বসে সে sufficient
যার আগে only if বসে সে necessary

sufficient implies necessary

symbol of Implies

Sufficient condition

If you will pass ~~only if~~ you study

p q

যেহেতু q এর আগে only if আছে তাই p এর আগে if বসে নিলাম

$p \rightarrow q$

পাস করাটা sufficient, কিন্তু পড়াশোনা করাটা দরকার

Necessary condition

you will pass only if you study

p q

$p \rightarrow q$

p	q	$p \rightarrow q$
T	F	T
T	F	F
F	T	T
F	F	T

Bioconditional statement ($\rightleftarrows = \leftrightarrow$)

* (If and only if) থাকলেই Bio conditional

* you can take the flight if and only if you buy a ticket — necessary and sufficient

p = you can take the flight, q = buy a ticket

$\Rightarrow p \leftrightarrow q$

* p is necessary and sufficient for q
* "p iff q"
* if p then q, and conversely

দুটোই value থাকলে T True হবে

p	q	$p \Leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

$1+1=2$ only if $2+2=4$ = T

$1+2=4$ only if $2+2=4$ = F

$1+2=3$ only if $2+2=5$ = F

$1+1=3$ only if $2+2=5$ = T

converse, contrapositive, Inverse

* converse $q \rightarrow p$

$\underbrace{\text{If it rains}}_{p}$, $\underbrace{\text{I will stay home}}_{q}$ $\quad p \rightarrow q$

If $\underbrace{\text{I will stay home}}_{q}$, $\underbrace{\text{it is raining}}_{p}$ $\quad q \rightarrow p$

* contrapositive. $\neg q \rightarrow \neg p$

If $\underbrace{\text{I donot stay home}}_{\neg q}$, $\underbrace{\text{it is not raining}}_{\neg p}$

$\Rightarrow \neg q \rightarrow \neg p$

*** original এবং contrapositive এর meaning same

Inverse $\Rightarrow \neg p \rightarrow \neg q$

$\underbrace{\text{If it does not rain}}_{\neg p}$, I $\underbrace{\text{will not stay home}}_{\neg q}$

$\Rightarrow \neg p \rightarrow \neg q$

Problem

Exercise

$p \equiv$ You get an A on the final exam.

$q \equiv$ You do every exercise.

$r \equiv$ You get an A in this class.

(i) $p \wedge \neg q$

(ii) $p \wedge q \wedge r$

(iii) $r \rightarrow p$

proposition to English sentence.

Let p and q be the propositions

"swimming at the new Jersy is ~~called~~ allowed"

and "sharks have been spotted near the shore.

a/ $p \rightarrow q$ = If swimming at the new Jersy is allowed, then, sharks have been spotted near the shore

b/ $p \leftrightarrow q$ = swimming at the new Jersy is allowed if and only if sharks have been spotted near the shore.

3rd cls
04-2-23

P →

Logical Equibalance.

$p \to q \equiv \neg p \vee q$

p	q	¬p	p → q	¬p ∨ q
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

সমান

$(p \wedge q) \to r \equiv (p \to r) \wedge (q \to r)$

p	q	r	p ∧ q	(p ∧ q) → r	p → r	q → r	(p → r) ∧ (q → r)
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	F	T	T	T	T
T	F	F	F	T	F	T	F
F	T	T	F	T	T	T	T
F	T	F	F	T	T	F	F
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

not same.

Law

Identity

$p \wedge T \equiv p$

$p \vee F \equiv p$

Domination

$p \vee T \equiv T$

$p \wedge F \equiv F$

Idempotent

$p \wedge p \equiv p$

$p \vee p \equiv p$

Double negation

$\neg(\neg p) \equiv p$

Commutative

$p \vee q \equiv q \vee p$

$p \wedge q \equiv q \wedge p$

Associative

$(p \vee q) \vee r \equiv p \vee (q \vee r)$

$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

Distributive.

$(p \wedge q) \vee (q \wedge s) \equiv (p \vee r) \wedge (p \vee s) \wedge (q \vee r) \wedge (q \vee s)$

Demorgan's Law

$\neg(p \wedge q) \equiv \neg p \vee \neg q$

$\neg(p \cancel{\wedge} \vee q) \equiv \neg p \wedge \neg q$

▣ Negation

$p \vee \neg p \equiv T$

$p \wedge \neg p \equiv F$

▣ Absorption

$p \vee (p \wedge q) \equiv p$

$p \wedge (p \vee q) \equiv p$

▣ p only if q

if p then q

$p \rightarrow q$

▣ p if q

if q then p

$q \rightarrow p$

Golden Law

▣ conditional

$p \rightarrow q \equiv \neg p \vee q$

▣ contrapositive

$p \rightarrow q \equiv \cancel{\neg p \rightarrow} \neg q \rightarrow \neg p$

▣ Biconditional

$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

*** প্র proved that is $(p \wedge q) \to (p \vee q)$ is a tautology

p	q	$p \wedge q$	$p \vee q$	$(p \wedge q) \to (p \vee q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T is a tautology

প্র tautology means is every element is true

$\neg(p \to q) \equiv p \wedge \neg q$

$\equiv$ L.H.S $= \neg(p \to q)$

$\equiv \neg(\neg p \vee q)$

$\equiv \neg(\neg p) \wedge \neg q$

$\equiv p \wedge \neg q$

$\equiv$ R.H.S

prove that

$(p \to q) \wedge (q \to p) \longrightarrow (p \to r)$ is a tautology

$\equiv \neg((p \to q) \wedge (q \to r)) \vee (p \to r)$

$\equiv \neg(p \to q) \vee \neg(q \to r) \vee (p \to r)$

$\equiv \neg(\neg p \vee q) \vee \neg(\neg q \vee r) \vee (\neg p \vee r)$

$\equiv (\neg(\neg p) \wedge \neg q) \vee (\neg(\neg q) \wedge \neg r) \vee \neg p \vee r$

$\equiv (p \wedge \neg q) \vee (q \wedge \neg r) \vee \neg p \vee r$

$\equiv (p \wedge \neg q) \vee \neg p \vee (q \wedge \neg r) \vee r$

$\equiv ((p \vee \neg p) \wedge (\neg q \vee \neg p)) \vee ((q \vee r) \wedge (\neg r \vee r))$

$\equiv (T \wedge (\neg q \vee \neg p)) \vee ((q \vee r) \wedge T)$

$\equiv (\neg q \vee \neg p) \vee (q \vee r)$

$\equiv \neg q \vee \neg p \vee q \vee r$

$\equiv \neg q \vee q \; \neg p \vee r$

$\equiv T \vee p \vee r$

$\equiv T$

$\therefore$ it's a tautology

*** $(p \wedge q) \to r \equiv (p \to r) \wedge (q \to r)$

$\equiv \neg(p \wedge q) \vee r$

$\equiv (\neg p \vee \neg q) \vee r$

$\equiv \neg p \vee \neg q \vee r \vee r$

Logically not possible possibly by the Truth Table.

now

$(p \to r) \wedge (q \to r)$

$\equiv (\neg p \vee r) \wedge (\neg q \vee r)$

$\equiv ((\neg p \wedge \neg q) \vee (\neg p \wedge r)) \vee ((r \wedge \neg q) \vee (r \wedge r))$

$\equiv (\neg(p \vee q) \vee (\neg p \wedge r)) \vee ((\neg q \wedge r) \vee r)$

$\equiv \neg(p \vee q) \vee$

$\equiv \neg p \vee r \vee \neg q \vee r$

$\equiv (p \to r) \vee (q \to r)$

$\equiv$

$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

L.H.S

$\equiv \neg(p \leftrightarrow q)$

$\equiv \neg((p \rightarrow q) \wedge (q \rightarrow p))$

$\equiv \neg(p \rightarrow q) \vee \neg(q \rightarrow p)$

$\equiv \neg(\neg p \vee q) \vee \neg(\neg q \vee p)$

$\equiv (\neg(\neg p) \wedge \neg q) \vee (\neg(\neg q) \wedge \neg p)$

$\equiv (p \wedge \neg q) \vee (q \wedge \neg p)$

$\equiv (p \vee q) \wedge (p \vee \neg p) \wedge (\neg q \vee q) \wedge (\neg q \vee \neg p)$

$\equiv (p \vee q) \wedge T \wedge T \wedge (\neg q \vee \neg p)$

$\equiv (p \vee q) \wedge (\neg q \vee \neg p)$

now

$p \leftrightarrow \neg q$

$\equiv (p \rightarrow \neg q) \wedge (\neg q \rightarrow p)$

$\equiv (\neg p \vee \neg q) \wedge (\neg(\neg q) \vee p)$

$\equiv (\neg q \vee \neg p) \wedge (q \vee p)$

$\equiv (q \vee p) \wedge (\neg q \vee \neg p)$ Ans