Proof Techniques ($p \to q$)

Hypothesis and conclution

my clam:- If it rains, I will stay home

How to prove it?

we cannot prove the statement untill it's ~~raix~~ ~~ra~~ raining hypothesis is true.

▣ prove that, If an integer n is even, then $3n+2$ is also even.

hypothesis :- n is even

conclution :- $3n+2$ is even

Let n be even.

Let $n = 2k$, where k is an integer

$\therefore 3n+2$

$= 3(2k)+2$

$= 2(3k+1)$

$\therefore 3n+2$ is even

since k is an integer

$3k+1$ must also be an integer

$\therefore 2(3k+1)$ must be even

$\therefore 3n+2$ is even

prove that, If n is an ~~intg~~ integer, and $3n+2$ is even, then n is also even.

Hypothesis :- $3n+2$ is even

conclution :- n is even

Let, $3n+2$ be even

Let, $3n+2 = 2k$, where k is an ~~ing~~ integer

$\therefore 3n+2 = 2k$

$\Rightarrow 3n = \cancel{2+} 2k-2$

$\Rightarrow n = \frac{2k-2}{3}$

Let $3n+2$ be even, but n is odd

Let $n = 2k+1$, where k is an integer

$\therefore 3n+2 = 3(2k+1)+2$

$= 6k+3+2$

$= 6k+5$

$= 6k+4+1$

$= 2(3k+2)+1$

since k is an integer, $(3k+2)$ must be integer

$\therefore 2(3k+2)+1$ must be odd

$\therefore 3n+2$ is odd

this contradicts with our assumption than $3n+2$ is even.

$\therefore n$ can not be ~~odd~~ odd

$\therefore n$ is even

proof by contrapositive $(\neg q \to \neg p)$

Hypothesis : $3n+2$ is even

conclution : n is even

It is sufficient to proof that,

$\underbrace{\text{if } n \text{ is odd}}_{H}$, the $\underbrace{3n+2 \text{ is odd}}_{C}$

Let n be odd

Let $n = 2k+1$, where k is an integer.

$\therefore 3n+2 = 3(2k+1)+2$

$= 6k+3+2$

$= 2(3k+2)+1$

since k is an integer,

$3k+2$ must be an integer

$\therefore 2(3k+2)+1$ must be odd

$\therefore 3n+2$ is odd.

(proved)

Indirect proof

prove by contradiction
(condition না থাকলেও হবে)

prove by contraposition
(conditions থাকবে)

contradiction $p \to \neg q$

prove by contradiction that $\sqrt{2}$ is an irrational number

Let $\sqrt{2}$ be a rational number

Let $\sqrt{2} = \frac{p}{q}$, when p, q are ~~ing~~ integer, $q \neq 0$
and p, q has no common factors othen then 1

$\therefore \frac{p^2}{q^2} = 2$

$\Rightarrow p^2 = 2q^2$

Since q^2 is an integer, $2q^2$ must be even

$\therefore p^2$ is even and p is even

Let $p = 2k$, where, k is an integer

$\therefore (2k)^2 = 2q^2$

$\Rightarrow 4k^2 = 2q^2$

$\Rightarrow q^2 = 2k^2$

Since k^2 is an integer, $2k^2$ must be even.

$\therefore q^2$ is even, and thus q is even

$\therefore 2$ is a common factor of p and q

This, contradicts with our assumption, that p, q has no common factors other than 1

$\therefore \sqrt{2}$ cannot be rational

$\therefore \sqrt{2}$ is irrational.

(proved)

ডাইরেক্ট proof

prove that, the product of two rational number is rational.

Hypothesis :- r_1 and r_2 are two rational numbers.

conclusion :- $r_1 r_2$ is rational.

Let $R_1 = \frac{p_1}{q_1}$, $R_1 = \frac{p_2}{q_2}$ here $p_1 q_1$ $p_2 q_2$ are ~~rat~~ integer and $q_1, q_2 \neq 0$

$\therefore R_1 R_2 = \frac{p_1}{q_1} \times \frac{p_2}{q_2} = \frac{p_1 p_2}{q_1 q_2}$,

Since p_1, p_2 are integer $p_1 p_2$ is also integers

Since q_1, q_2 are non zero integers, $q_1 q_2$ is also & non zero integer

$\therefore \frac{p_1 q_2}{q_1 q_2}$ is rational

$\therefore R_1 R_2$ is rational

(proved)

[note; irrational proof a indirect proof হবে]

prove that, if m, n, p are integers, and $m+n$ and $n+p$ are even then $m+p$ is also even.

Hypothesis: $m+n$ and $n+p$ are even

conclusion: $m+p$ is even

Let $m+n = 2k_1$ ~~is even~~ $n+p = 2k_2$

where k_1, k_2 are integer

$\therefore m = 2k_1 - n$ $\quad p = 2k_2 - n$

$\therefore m+p = 2k_1 - n + 2k_2 - n$

$= 2(k_1 + k_2 - n)$

since k_1, k_2, n are integers

$(k_1 + k_2 - n)$ must be an integers

$\therefore 2(k_1 + k_2 - n)$ must be even

$\therefore m+p$ even (proved)

by contrapositive

prove that, If n is irrational, then $\frac{1}{n}$ is also irrational

Hypothesis :- n is irrational

conclution :- $\frac{1}{n}$ is irrational

it is sufficient that,

If $\frac{1}{n}$ is ~~ir~~rational, then n is also rational.

Let $\frac{1}{n} = \frac{p}{q}$, where p, q are integer and $q \neq 0$

and p, q has no common factor other

then 1

Here p cannot be zero in that case,

$\frac{1}{n} = \frac{0}{q} = q = 0$

$\therefore n = \frac{1}{\frac{p}{q}} = \frac{q}{p}$

Since q, p are integer and $p \neq 0$

$\therefore \frac{q}{p}$ must be rational

$\therefore n$ is rational.

by contradiction.

prove that, the sum of an irrational and a rational number is irrational.

Hypothesis :- x is an irrational number and y is a rational number.

Conclusion :- $(x+y)$ is irrational

Let $y = \frac{p}{q}$ where, are integers and $\cancel{p}\ q \neq 0$

Let $x+y$ be rational and $x+y = \frac{p_1}{q_1}$ where p', q' are integer $q' \neq 0$

$$\therefore x + \frac{p}{q} = \frac{p'}{q'}$$

$$\Rightarrow x = \frac{p'}{q'} - \frac{p}{q}$$

$$= \frac{p'q - pq'}{qq'}$$

since p, q, p', q' are integers $p'q - pq'$ is an integers

since q, q' are non zero integer qq' is a non zero integers.

$\therefore \frac{p'q - pq'}{qq'} = x$ is rational.

this contradict to the hypothesis that x is irrational

$\therefore$ $x+y$ cannot be rational

$\therefore$ $x+y$ is irrational

Vaccuous proof

$P(0)$, where $P(n) \equiv (n > 1) \rightarrow (n^2 \geq n)$

$F \rightarrow \; ? \; F \Rightarrow T$

Trivial proof

$P(0)$, where $P(n) \equiv \forall a \forall b \, ((a \geq b) \rightarrow (a^n \geq b^n))$

$? \quad T \quad \Rightarrow T$