L1

Topic Function, Domain and Range

What is function?

A function in mathematic, is an expression or rule or law that defines a relationship between two variable, one is independent variable another ~~one~~ is dependent variable.

input (independent variable)

output (dependent variable)

input → $\boxed{\text{Rule}}$ → output

Relation

$A = \pi r^2$ $A \rightarrow$ dependent variable

A depends on r

A is a function of r

A function is a relation but all relations are not function.

Formal defination of a function

~~is~~ if a variable y depends on x variable. x in such a way that each variable of x determines exactly one value of y, Then we say y is a function of x.

This relationship is commonly symblies as

y = function

$y = g(x)$, $y = ~~g~~h(x)$;

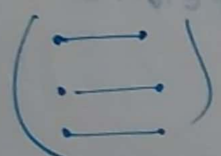
one input $\longrightarrow$ $\boxed{R}$ $\longrightarrow$ one output

This relation is a function.

Some function that one function

$y = 2x+3$

x	y
-1	1
0	3
10	23



One to one

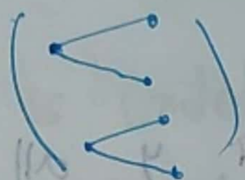
one input one output that is it's a function

$y = f(x)$

~~$y = x^2 + 1$~~

$y^2 = x$

x	y
1	1, −1
16	4, −4
9	3, −3



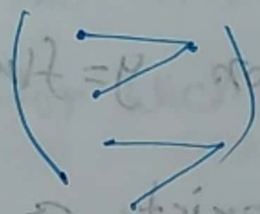
one to many

That is not a function

$$\boxed{\begin{aligned} y^2 &= 9 \\ y &= \pm\sqrt{9} \end{aligned}}$$

$y = x^2$

x	y
1	1
−1	
0	0
2	4
−2	

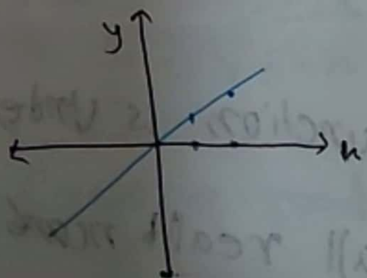


many to one

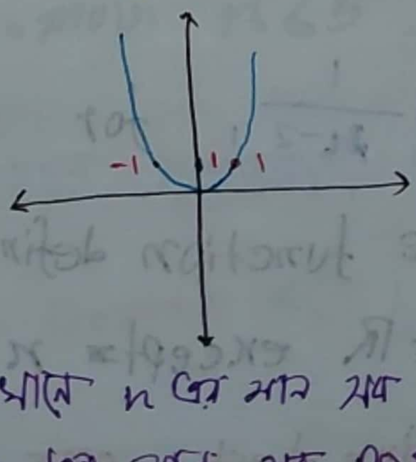
That is Function

graph

$y = x$

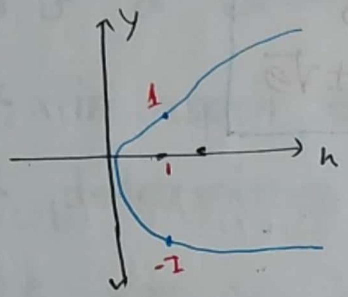


$y = x^2$



এখানে x এর মান সব real numbers এবং
y এর মান সব positive real numbers

$y^2 = x$ $y = \pm\sqrt{x}$ x Always positive



এখানে y all real number আর x positive real number কারণ ($\sqrt{x}$) negative হলে undefined হয়

When a function is defined?

A function a relation $y = f(x)$ is defined means for each x there exist a y each input has it's output.

$y = \frac{1}{x}$, $x = 0$, $y = \frac{1}{0}$ (undefined)

$y = \sqrt{x}$, $x = -2$, $y = \sqrt{-2}$ (($\sqrt{-ve}$) undefined)

$f(x) = \frac{1}{x-2}$, for $x = 2$, function is undefined or the function defined for all real number of $x \in \mathbb{R}$ except $x = 2$;

vertical line test one point এক হবে function

L2

\# $f(x) = \sqrt{x+1}$; $x \geq -1$

The function is undefined for $x < -1$
or The function is defined for $x \geq -1$

Domain and Range

Domain:

Let $y = f(x)$, The domain of $f(x)$ is the set of values of x, for which the function is defined.

$y = 2x+1 \rightarrow$ Domain $x \in \mathbb{R}$ (set of all real number)

\# $y = \frac{5}{x+2} \rightarrow$ Domain $x \in \mathbb{R}$, $x \neq -2$

-2 দিলে নিচে 0 আসবে যা undefined তাই

\# $y = \sqrt{5-x} \longrightarrow$ Domain $x \leq 5$

Square root এর ভিতরে negative হতে পারবে না,

x এর সকল মানকেই Domain বলে।

Range:

if $y = f(x)$, Then the range of $f(x)$ is the set of values of y obtained from corresponding domain (that is the set of x values)

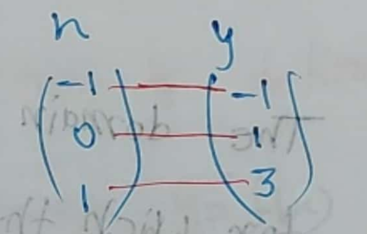
$y = 2x+1$

Domain of $x = \{-1, 0, 1\}$ → Restrict

$x = 1 \quad y = 3$

$x = 0 \quad y = 1$

$x = -1 \quad y = -1$

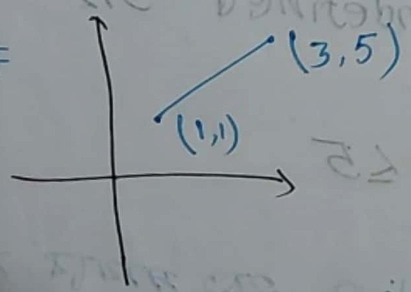


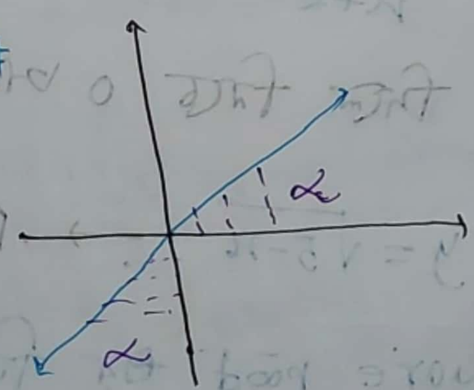
Domain Range

y এর মানকেই Range বলে

Graph for domain and Range

#





Domain: $1 \leq x \leq 3$

Range: $1 \leq y \leq 5$

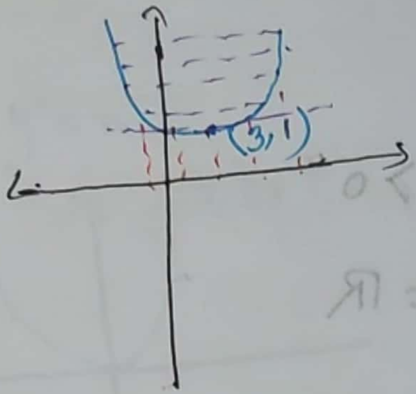
Restricted হচ্ছে
min and max value হয় জন্যে।

Domain: $-\infty < x < \infty$; $x \in R$

Range: $-\infty < y < \infty$; $y \in R$

natural

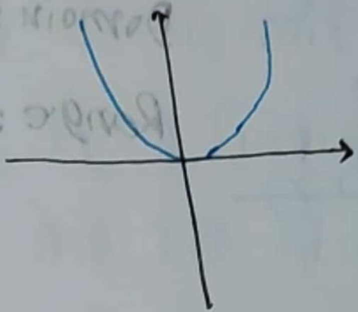
#



Domain : $-\infty < x < \infty$

Range : $1 \leq y$

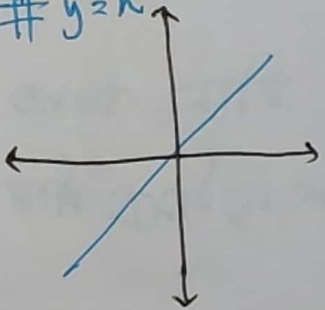
$y = x^2$



Domain : $x \in \mathbb{R}$

Range : $y \geq 0$

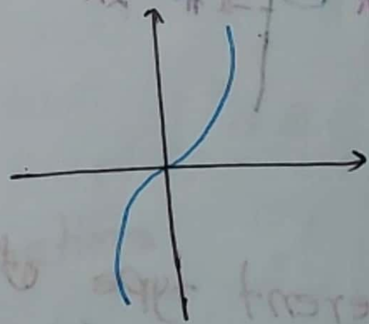
$y = x$



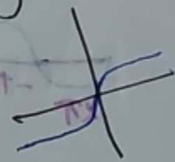
Domain : $x \in \mathbb{R}$

Range : $y \in \mathbb{R}$

$y = x^3$



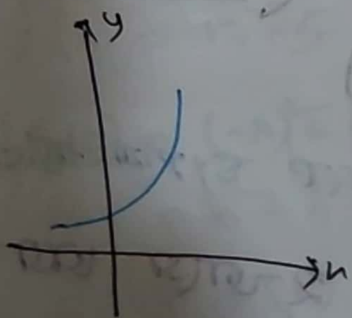
$y = \sqrt[3]{x}$



Domain : $x \in \mathbb{R}$

Range : ~~x~~ $y \in \mathbb{R}$

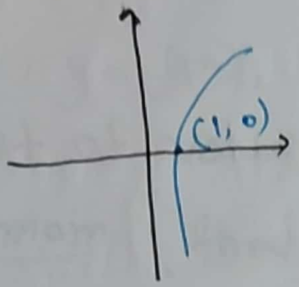
$y = e^x$



Domain : $x \in \mathbb{R}$

Range : $y > 0$

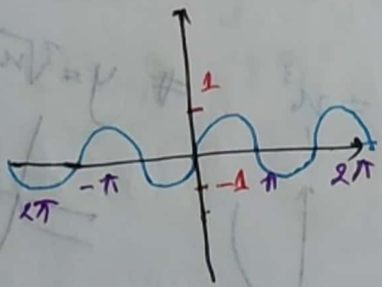
$y = \ln x$



Domain: $x > 0$

Range: $y \in \mathbb{R}$

$y = \sin x$ or $y = \cos x$



Domain: $x \in \mathbb{R}$

Range: $-1 \le y \le 1$

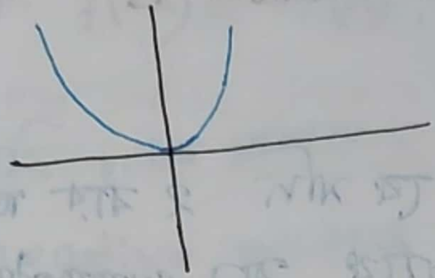
Different type of function:

Even function

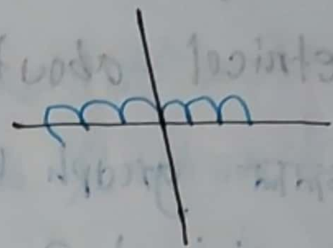
An even function is a function which is symmetrical about y ~~axi~~ axis (y axis divides the graph into two equal part)

যাকে সমান ২ ভাগে ভাগ করা যায় তাকে symmetrical বলে। যেই function এর graph কে সমান ২ ভাগে ভাগ করা যায় ~~তাকে~~ সেই function কে ~~symm~~ even function বলে।

$y = x^2$



$y = f(x)$



☐ Mathmetically even নাকি odd তা বোঝাতে হবে x এর পরিবর্তে $(-x)$ ~~বস~~ বসাইলে যদি আগের-টা পাওয়া যায় তবে even

$f(x) = x^2$

$f(-x) = (-x)^2$
$= x^2$
$= f(x)$

→ even function

$\cos(-x) = \cos x$

$f(x) = x^2 + 5$

$f(-x) = (-x)^2 + 5$
$= x^2 + 5$
$= f(x)$

☐ $f(x) = x^3 - x + 1$

$f(-x) = (-x)^3 - (-x) + 1$
$= -x^3 + x + 1$

$\therefore f(x) \neq f(-x)$

↓

not even

origina	(x, y)
even	$(-x, y)$

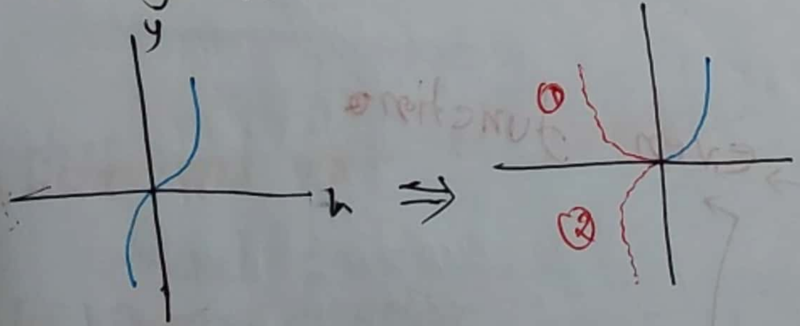
Odd function

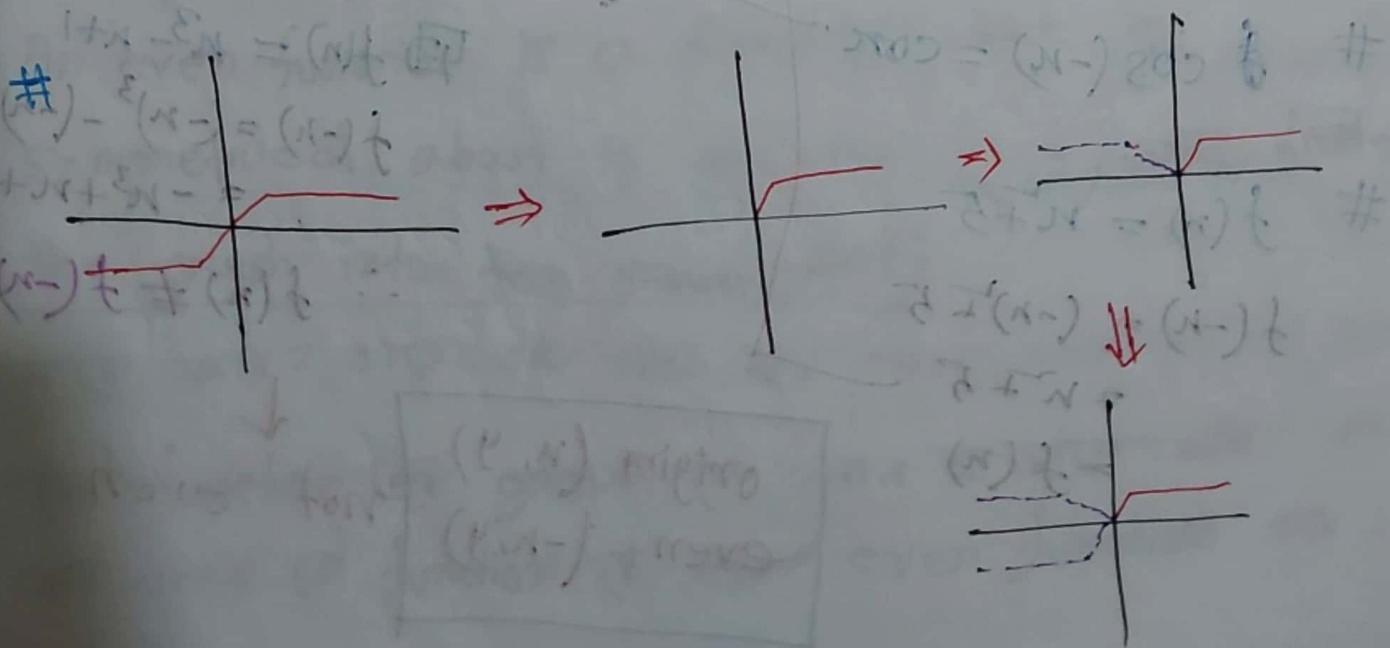
An odd function is a function which is symmetrical about origin

অরিজিনাল graph এর ~~অরি~~ অরিজিন কে যদি 2 বার reflect করলে original টা পাওয়া যায় তাকে বলে symmetrical origin.

যে function এর graph কে থেকে তা পাওয়া যায় সেই function কে odd function বলে।

$y = x^3$





⊞ Mathmatically

If $f(-x) = -f(x)$; then, $f(x)$ is an odd function

যদি x এর বদলে $(-x)$ বসাইলে ফাংশনটাই পাওয়া যায় কিন্তু negative আকারে তাহলে ও odd function

$f(x) = x^3$

$$f(-x) = (-x)^3$$
$$= -x^3$$
$$= -f(x)$$

$f(x) = \sin x$

$$f(-x) = \sin(-x)$$
$$= -\sin x$$
$$= -f(x)$$

Therefore, They are odd function.

original = (x, y)
odd = $(-x, -y)$
even = $(-x, y)$

L3

Start for ch-4

Inverse function:

যদি x এর কোনো value এর জন্য প্রথম ~~fungt~~ function কে যা পাবো তা ব্যবহার করে দ্বিতীয় function solve করে ফিরে x এর input ফেরত পাওয়া যায় তাহলে ~~একে~~ তাদেরকে একে অপরের Inverse বলে।

$f(x) = 3x - 2$

$x = 5$, $f(x) = 13$

$g(x) = \frac{x+2}{3}$

$= \frac{13+2}{3} = 5$

input $x=5$ → $f(x)$ → output (13)

x ← Inverse ??

$f(x)$ and $g(x)$ inverse to each other.

inverse হতে হলে one to one function must হতে হবে।

মানে 1 input, ~~∞~~ 1 output

Lets $y = f(x)$ be an one to one function (bijective function)

ex: $f(x) = 2x+1$

$x = 2$, $f(x) = 5$

input output

$f^{-1}(x) = \frac{x-1}{2}$

$= \frac{5-1}{2}$

$= 2$

$f(x) = 2x+1 \longrightarrow$ flow diagram

$x \to$ multiplied by $2 \to 2x \to$ add $1 = 2x+1 = y$

$\frac{y-1}{2}$ —— divided by 2 $\longleftarrow y-1 \leftarrow$ subtract 1

important step

$\therefore f^{-1}(x) = \frac{x-1}{2}$

$f(x) = 2x+1$

$\therefore f^{-1}(x) = \frac{x-1}{2}$

linear $y = f(x) = mx+c$

$f(x) = 3x-2$

$f^{-1}(x) = \frac{\cancel{3}x+2}{3}$

$f(x) = \cancel{3x}\ \frac{x}{3}+1$

$f^{-1}(x) = (x-1)\times 3$

How to find the inverse of a function

~~$y = f(x)$~~

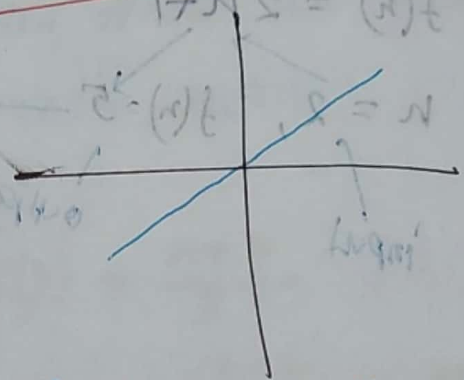
step 1: $y = f(x)$

step 2: Inter change the variables (x & y)

step 3: subject or separate y

step: $f^{-1}(x) = y$

$\boxed{\text{only one - one function then it's inverse}}$



linear equation is always one-one function. also had it's inverse

Ex: $f(x) = \frac{3x-1}{2}$

$y = \frac{3x-1}{2}$

interchange the variable

$x = \frac{3y-1}{2}$

$\Rightarrow 2x = 3y-1$

$\Rightarrow 3y = 2x+1$

$\Rightarrow y = \frac{2x+1}{3}$

$\therefore f^{-1}(x) = \frac{2x+1}{3}$

$\begin{bmatrix} x=5, & f(x)=7 \\ x=7 & f^{-1}(x)=5 \end{bmatrix}$

~~প্র~~ The inverse function of a quadratic equation

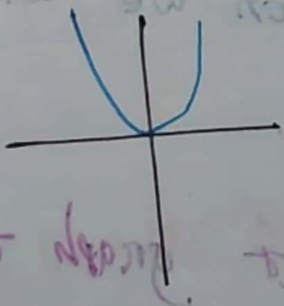
Any equation of the form $y = ax^2 + bx + c$, $a \neq 0$ is known as quadratic function or $y = x^2$

এই equation এর পাওয়ার সর্বোচ্চ (2) তাকে quadratic equation বলে।

quadratic হলে inverse বের করার সময় আগের coefficient ~~be~~ suare form বের করতে হবে তার।

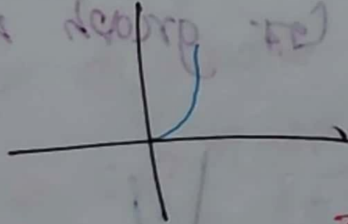
ex:
$$y = 3x^2 - 4x + 1$$
$$x = t^2 + 5$$
$$y = x^2 + 3x$$
$$p = 3 - q^2$$
all are quadratic equation

$y = x^2$



~~no inver~~
has not inverse
কারণ one-to-one function না

$y = x^2$ $[x \geq 0]$



$x = y^2$

interchange x & y

$y^2 = x$

$y = \sqrt{x}$

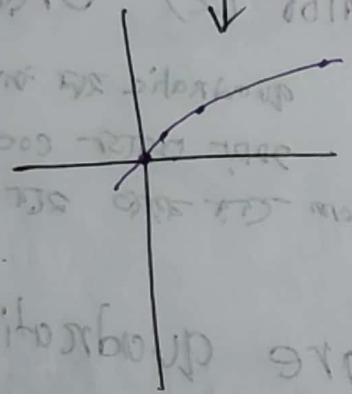
$f^{-1}(x) = \sqrt{x}$

has inverse
কারণ এখন এটা one-to-one function
কারণ input 1 টা output ও একটা

The graph of an inverse function:

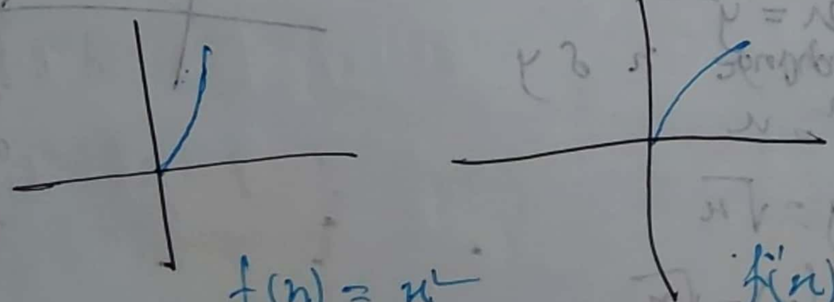
\# $f(x) = x^2$; $x \geq 0$ $\longrightarrow$

$\therefore f^{-1}(x) = \sqrt{x}$



→ if we reflect the graph of $f(x)$ along the line $y=x$; Then we will get the graph of $f^{-1}(x)$

অবিচ্ছিন্ন function এর graph কে reflect করলেই inverse এর graph পাওয়া যায়



$f(x) = x^2$ $\qquad$ $f^{-1}(x) = \sqrt{x}$

(4,5)

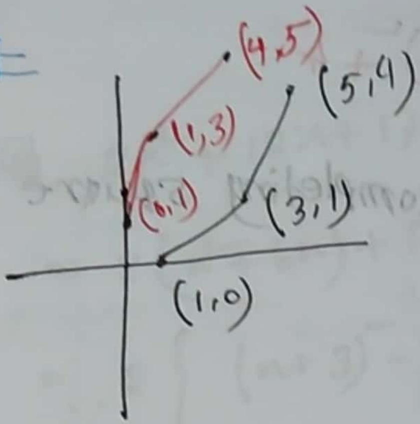
(5,4)

(1,3)

(0,1)

(3,1)

(1,0)



for inverse

(0,1),

(1,3)

(4,5)

find the inverse of y

$y = x^2 - 3$ $x \geq 0$

interchange x and y

~~$x = y$~~

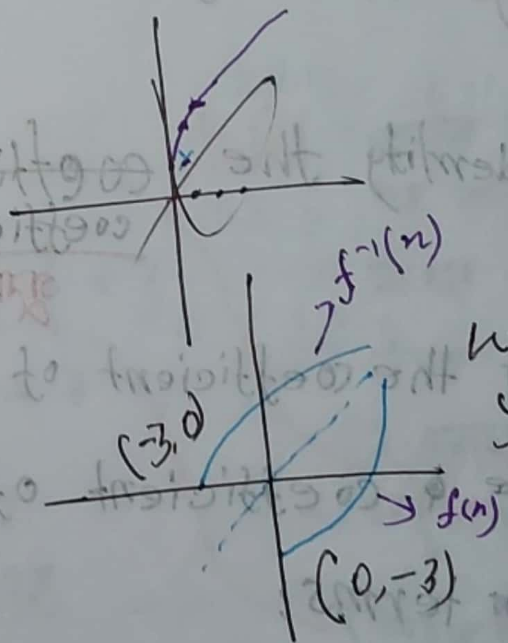
$f(x) = x^2 - 3$

$x = y^2 - 3$

$\Rightarrow y^2 = x + 3$

$\Rightarrow y = \sqrt{x+3}$

$\Rightarrow f^{-1}(x) = \sqrt{x+3}$



$x = 0,$

$y = 0^2 - 3$

$= -3$

Find the inverse of $y = x^2 - 4x + 5$??

how to convert a quadratic equation $(ax^2 + bx + c)$ into a completing square form

$a(x+b)^2 + c$

~~converting~~ $ax^2+bx+c = a(x+h)^2+k$

completing square form

$$x^2+4x+5 = (x+2)^2+1$$
$$= x^2+2x2+2^2+1$$
$$= x^2+4x+5$$

things to noted

coefficient এর পর নিয়ম

(i) Identify the ~~coeffiaint~~ coefficient of $x^2 = 1$ থাকলে সহজ

গুণাঙ্ক

(ii) If the coefficient of x^2 is not 1 Then take the coefficient of x^2 out from the first two ~~for~~ terms:

like: $2x^2+12x+17$

$$2(x^2+6x)+17$$

1 না থাকলে প্রথম দুটো term থেকে common নিতে হবে

(iii) Half of the coefficient of x

$\boxed{\times}$ $2x^2+12x+17$

$\Rightarrow 2(x^2+6x)+17$

$= 2\{(x+3)^2-3^2\}+17$

$= 2(x+3)^2-18+17$

$= 2(x+3)^2-1$

→ verify

$2(x^2+6x+9)-1$

$= 2x^2+12x+18-1$

$= 2x^2+12x+17$

$\boxed{\times}$ $-2x^2+6x+7$

$\Rightarrow -2(x^2-3x)+7$

$\Rightarrow -2\left\{\left(x-\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2\right\}+7$

$= -2\left(x-\frac{3}{2}\right)^2+2\times\frac{9}{4}+7$

$= -2\left(x-\frac{3}{2}\right)^2+\frac{18}{4}+7$

$= -2\left(x-\frac{3}{2}\right)^2+\frac{18+28}{4}$

$= -2\left(x-\frac{3}{2}\right)^2+\frac{46}{4}$

N.B। প্রথমে দেখতে হবে x^2 এর সহগ 1 কিনা। ~~1 হলে~~ না হলে x^2 এর; ~~x~~ এর সাথের term থেকে কমন নিয়ে 1 বানাতে হবে। তারপর x^2 কে (x + x এর সহগের অর্ধেক) − (অর্ধেক এর)^2 ✓

Domain and range of a inverse function:

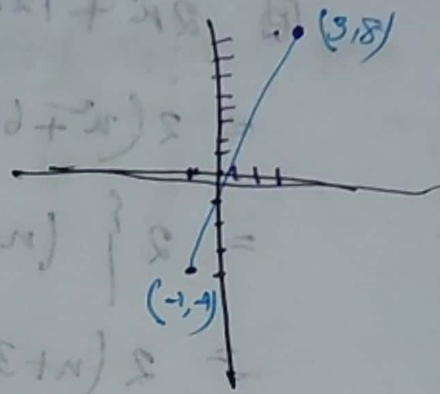
$f(x) = 3x - 1$; $-1 \le x \le 3$

$y = 3x - 1$ $\quad$ $x = -1,\ y = -4$

$x = 3 \quad y = 8$

$(-1, -4)$ start

$(3, 8)$ end



$f(x)$

According to graph

Domain: $-1 \le \cancel{8}x \le 3$

Range : $-4 \le y \le 8$

now for inverse

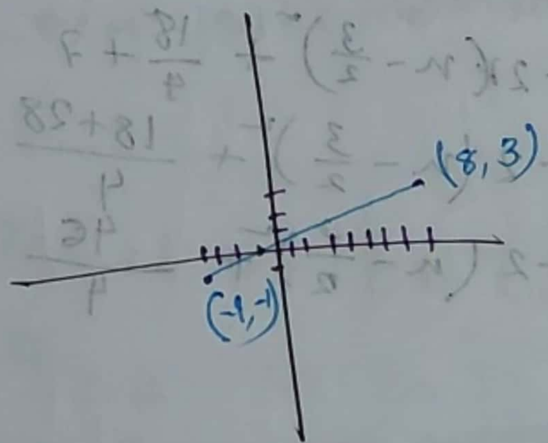
$\therefore$ $y = 3x - 1$

interchange

~~x =~~

$x = 3y - 1$

$\Rightarrow y = \frac{x+1}{3}$



for inverse $(-4, -1)$ start

$(\cancel{8}, \overset{3}{\cancel{8}})$ end

$f^{-1}(x)$

Domain: $-4 \le x \le 8$

Range : $-1 \le y \le 3$

1→4

L4

$f(x)$	Domain	Range
$f^{-1}(x)$	Range	Domain

$f(x) = 1+\sqrt{1-3x}$

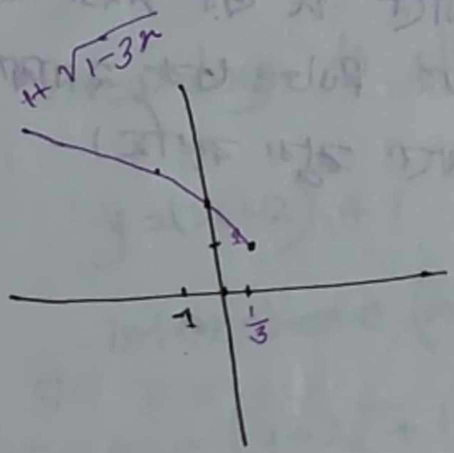
Domain: $x \leq \frac{1}{3}$

$1-3x \geq 0$

$1 \geq 3x$

$3x \leq 1$

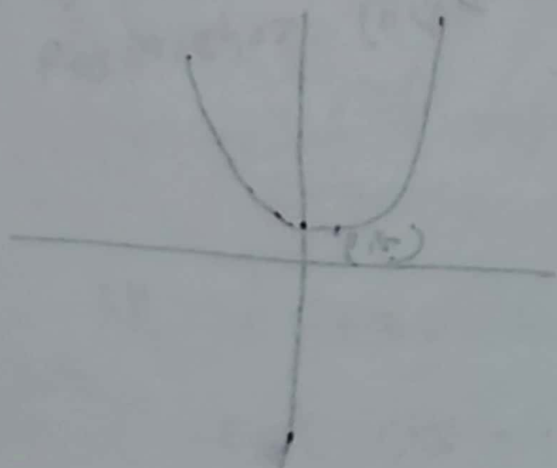
$x \leq \frac{1}{3}$

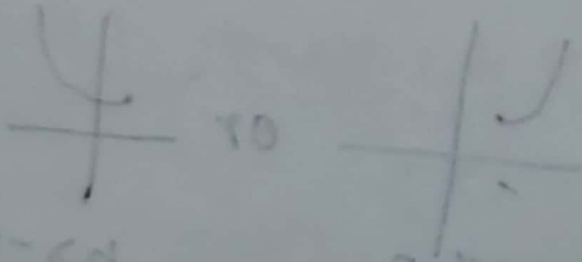


$x=\frac{1}{3}$, $y=1$

$x=0$, $y=2$

$x=-1$, $y=3$





☐ How to find the inverse of a quadratic function?

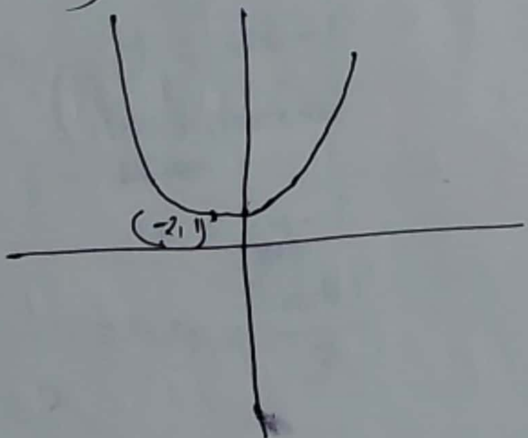
→ To find the inverse at first we have to convert the given quadratic equation into a completing square form.

ওখানে x এর সহজ । তাই Rule এর ২ নাম্বার থেকে শুরু করছি।

$$f(x) = x^2 + 4x + 5$$
$$= (x+2)^2 - 2^2 + 5$$
$$= (x+2)^2 + 1$$

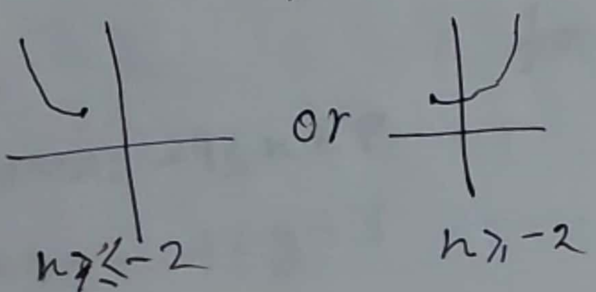
Domain: $x \in \mathbb{R}$

Range: $1 \leq y$



quadratic equation এ সাধারণত inverse থাকে না।

inverse করার জন্য Domain এ Restriction আনতে লাগে।



question

What will be the domain for the inverse of $f(x)$?

$\rightarrow x \geq -2$ or $x \leq -2$

problem

$\rightarrow$ Given $f(x) = (x+2)^2 + 1$; $x \geq -2$

find the $f^{-1}(x)$?

According to inverse Rules

$$y = (x+2)^2 + 1$$

interchange (x & y)

$$x = (y+2)^2 + 1$$

$$(x-1) = (y+2)^2$$

$$\Rightarrow y+2 = \sqrt{x-1}$$

($\pm$) হয় নাই কারন আগেই Restriction দেওয়া - আছে

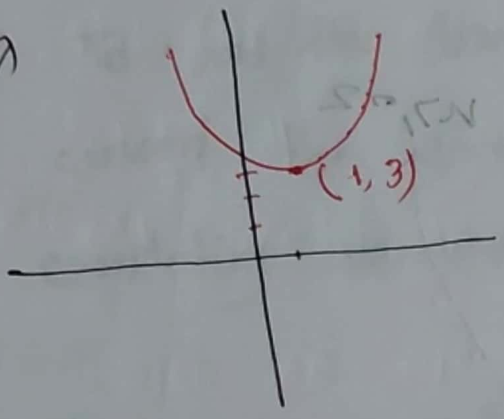
$$\Rightarrow y = \sqrt{(x-1)} - 2$$

$$\therefore f^{-1}(x) = \sqrt{x-1} - 2$$

Domain: $[1, \infty)$

⊗ problem

$2(x-1)^2+3 \quad x=1,\ y=3$



note

$2(x^2-2x+1)+3$

$=2x^2-4x+5$

$x=1;\quad y=1$

question:

(1) inverse find

if $f(x)=2(x-1)^2+3$, $x\geq 1$

domain

নিজের মতো নিতে পারলে $x\geq$ something দিয়ে কাজ করবো।

$2(x-1)^2+3$

$y=2(x-1)^2+3$

interchange (x, y)

$x=2(y-1)^2+3$

$\Rightarrow x-3=2(y-1)^2$

$\Rightarrow (y-1)^2=\dfrac{x-3}{2}$

$\Rightarrow y=\sqrt{\dfrac{x-3}{2}}+1$

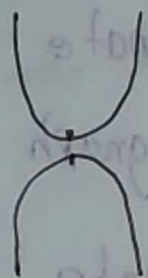
± দেইনাই তাই domain Restricted

ধ $y = a(x-h)^2 + k \rightarrow (h, k)$ → $\boxed{\text{Domain}}$

$y = a(x+h)^2 + k \rightarrow (-h, k)$ — Turning point/ vertex

if $a > 0$ (+ve) → valley shape

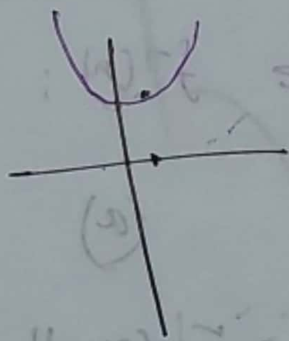
if $a < 0$ (−ve) → hill shape



parabola

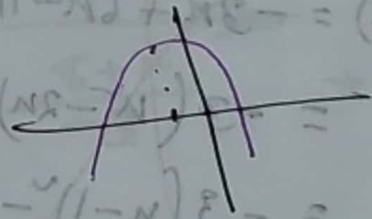
Ex:

$y = \frac{2}{a}(x-1)^2 + 3$



(1, 3)

$y = -\frac{1}{a}(x+1)^2 + 3$



(−1, 3)

like $y = mx + c$

যদি m এর মান possitive হয় তাহলে valley shape

" m " " negative " " hill shape

$\boxed{k \rightarrow \text{Range}}$

min: $a > 0$ max: $a < 0$

problem

Given that $f(x) = -3x^2 + 6x - 11$

a) write $f(x)$ in the form $a(x-h)^2 + k$

(b) State the domain and Range of $f(x)$

(c) state the co-ordinate of the vertex of $f(x)$ and skatch the graph of $f(x)$

(d) stated an appropiate domain of $f(x)$ to have the inverse

(e) find the $f^{-1}(x)$; stating it's domain and Range.

(a)

$$f(x) = -3x^2 + 6x - 11$$
$$= -3(x^2 - 2x) - 11$$
$$= -3(x-1)^2 - 11$$

$$f(x) = -3(x^2 - 2x) - 11$$
$$= -3\left((x-1)^2 - 1^2\right) - 11$$
$$= -3(x-1)^2 + 3 - 11$$
$$= -3(x-1)^2 - 8$$

hill shape

(b)

$f(x) \Rightarrow$ Domain : $x \in \mathbb{R}$

Range: $y \leq -8$

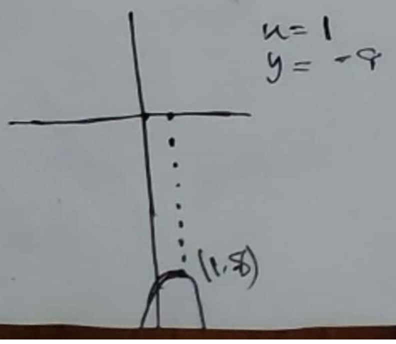
$a < 0$

k

$max = -8$

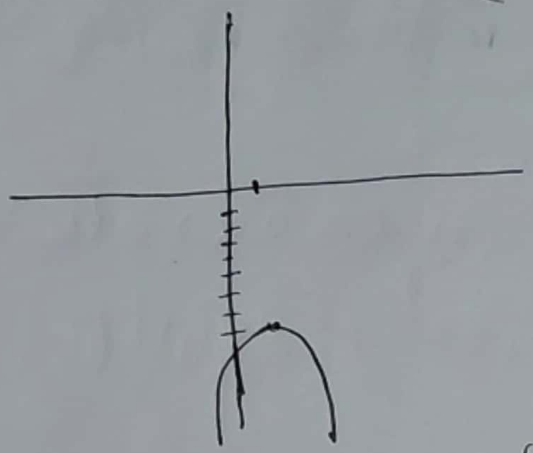
quadratic equation of Domain

Always real number



~~Exercise of inverse function~~

(c)



$x = 1$

$y = -8$

$m = -3$ hill shape

(d)

if $x \geq 1$ or $x \leq 1$

(e)

$y = -3(x-1)^2 - 8$

interchange x and y

$x = -3(y-1)^2 - 8$

$\cancel{\Rightarrow x + 8 = -3y + 3}$

$\cancel{\Rightarrow 3y = 3 - x + 8}$

$\cancel{\Rightarrow y = \frac{8 - 5x}{3}}$

$\Rightarrow x + 8 = -3(y-1)^2$

$\Rightarrow (y-1)^2 = -\frac{8+x}{3}$

$y = \sqrt{\frac{8+x}{-3}} + 1$

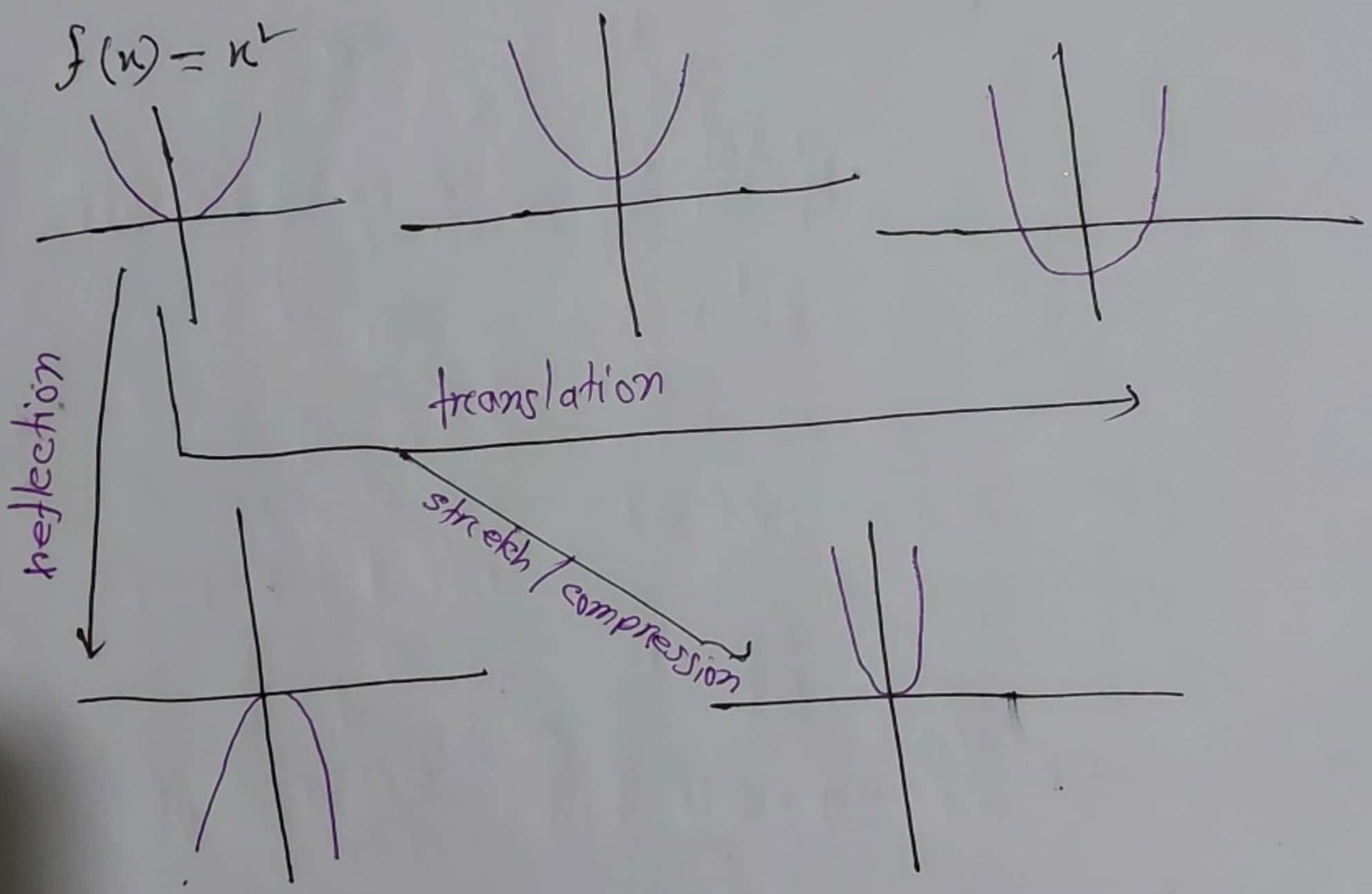
Domain: $x \leq -8$

Range: $\cancel{y \in \mathbb{R}}$ $y \geq 1$

Transformation of function

① Translation: The translation of a function shifts the graph from one position to another position. (no change in shape)

$f(x) = x^2$



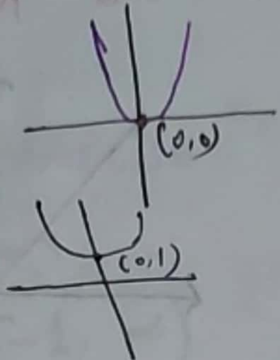
② ~~Transformation~~

~~In Translation is are kind of~~ transformation.

~~Translator moves~~

Exercise এর পরে আবার শুরু করেছি

(4) $f(x) = x^2$



$f(x)+1 \longrightarrow$

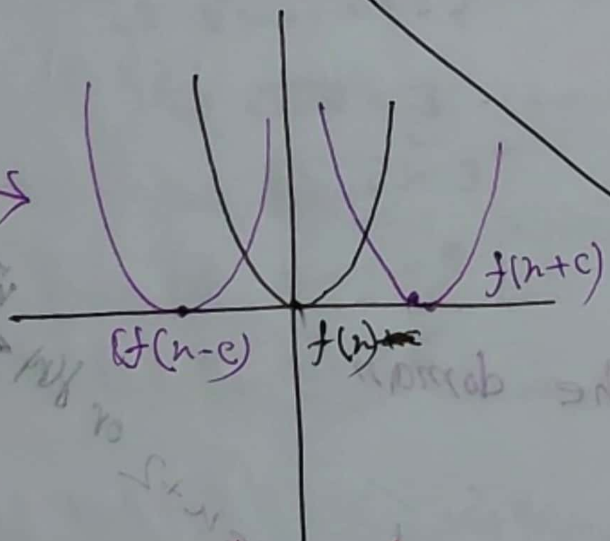
① $f(x)+c$ moves the graph $\boxed{c \text{ unit}}$ upword

② $f(x)-c$ moves the graph $\boxed{c \text{ unit}}$ downword

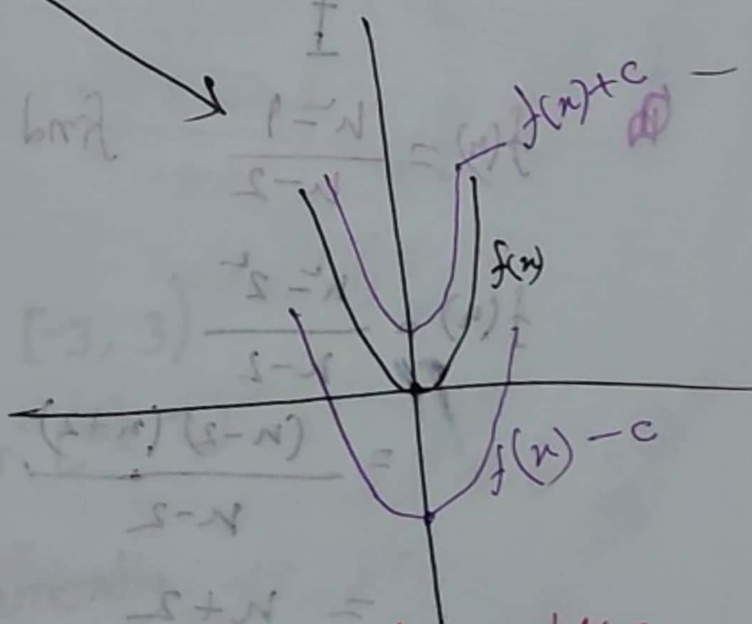
③ $f(x+c)$ moves the graph ~~Right~~ Left by c unit

④ $f(x-c)$ " " ~~Left~~ Right by c unit

Horigontal



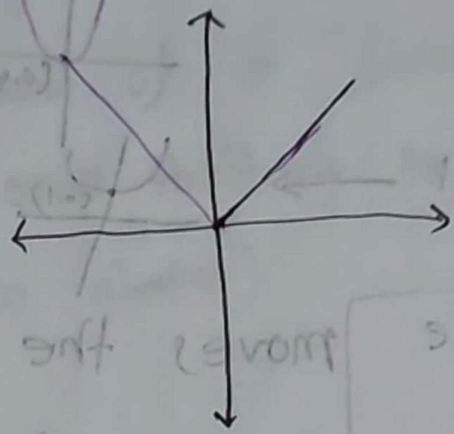
Horigontal translation



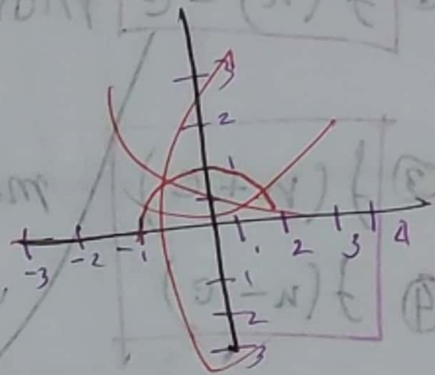
vertcn translation

1-5 problem solve for Domain and Range

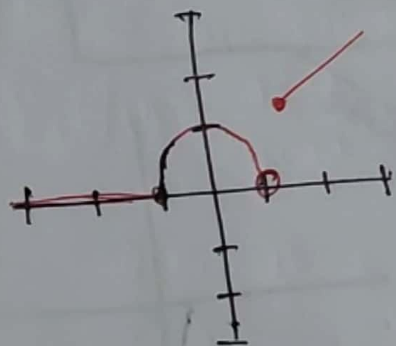
Rules of Absolute value



$$|x| = \begin{cases} x, & x \geq 0 \\ -x; & x < 0 \end{cases}$$



$$f(x) = \begin{cases} 0 & ; x \leq -1 \\ \sqrt{1-x^2} & ; -1 < x < 1 \\ x & ; x \geq 1 \end{cases}$$



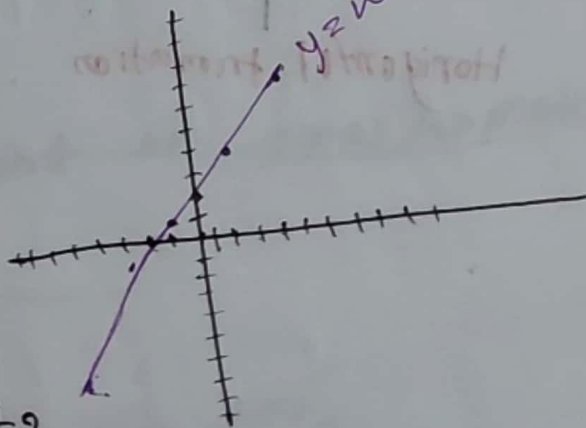
$f(x) = \dfrac{x^2-9}{x-2}$ find the domain

$$f(x) = \frac{x^2-2^2}{x-2}$$

$$= \frac{(x-2)(x+2)}{x-2}$$

$$= x+2$$

$y = x+2$ or $f(x) = \dfrac{x^2-9}{x-2}$



Domain: $x \in \mathbb{R}$ except $x \neq 2$

Rang : ~~$y < 0$ or $y \geq 2$~~ $\mathbb{R}$ $f(x) \in \mathbb{R} \neq 0$

~~$f(x)$~~ =

Quick check Exercise

1) $f(x) = \sqrt{x+1} + 4$

a) Domain: $[-1, \infty)$

c) $f(t^2-1) = \cancel{\sqrt{(t^2-1)^2+1}+4} = \sqrt{t^2-1+1} + 4$

$= \cancel{\sqrt{t^4-2t^2+1}} + 4 = \sqrt{t^2} + 4$

$= t + 4$

=

d) $f(x) = 7$ if $x = ?$

e) The range: $[4, \infty)$

$f(x) = \sqrt{x+1} + 4$

$\therefore \sqrt{x+1} + 4 = 7$

$\Rightarrow \sqrt{x+1} = 3$

$\Rightarrow (\sqrt{x+1})^2 = 3^2$

$\Rightarrow x + 1 = 9$

$\Rightarrow x = 8.$

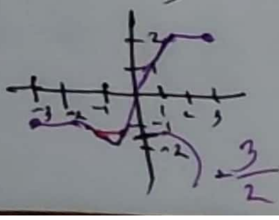
2

$y = f(x)$

a) Domain: $-3 \le x < 3$ or $[-3, 3)$

b) Range: $-2 \le y \le 2$ or $[-2, 2]$

d) $f(\frac{1}{2}) = \approx 1 \rightarrow$ Apparently

e) 

$y = -\frac{3}{2}$

$x \approx -0.999$ and $x \approx -1.0999$

Exercise:

~~$y=1, \quad x \approx -2, \quad x \approx -3$~~

ⓐ $y=1$, ~~approx~~ approximatly $-2 < x < -3$, $2 < x < 3$

ⓒ $x=3$ y $0 \leq y \leq 1$

~~ⓔ $y < -3$ $3 > y > -3$,~~

ⓔ $2 < y < 3$, $-2 < x < -3$ and $-2 < y < -3$, $-2 < x < -3$

ⓖ

ⓐ $f(x) = \dfrac{x^2+x}{x+1}$; Domain: $x \in \mathbb{R}$; $x \neq -1$

$g(x) = x$ Domain; $x \in \mathbb{R}$

ⓑ $f(x) = \dfrac{x\sqrt{x}+\sqrt{x}}{x+1}$ Domain: $[0, \alpha)$ or $0 \leq x$

$g(x) = \sqrt{x}$ Domain: $[0, \alpha)$ or $0 \leq x$

⑦ b $f(x) = \begin{cases} \frac{1}{x} & x > 3 \\ 2x & x \leq 3 \end{cases}$

$* \; f(0) = 2x$
$= 0$

$* \; f(-2) = 2x$
$= -4$

~~$* \; f(3t)$~~

$$f(3t) = \begin{cases} \frac{1}{3t} & ; 3t > 3 \\ 2 \cdot 3t & ; 3t \le 3 \end{cases}$$

$$= \begin{cases} \frac{1}{3t} & ; t > 1 \\ 6t & ; t \le 1 \end{cases}$$

19

ⓐ $f(x) = \frac{1}{x-3}$

Domain: $x \in \mathbb{R};\ x \neq 3$

Range: $y \in \mathbb{R};\ y \neq 0$

ⓑ $f(x) = \frac{x}{|x|}$

Domain: $x \in \mathbb{R};\ x \neq 0$

Range: $\{-1, 1\}$

ⓒ $g(x) = \sqrt{x^2 - 3}$

Domain: $[\sqrt{3}, \alpha) \cup [-\sqrt{3}, -\alpha)$

Range: $0 \le y$ or $[0, \alpha)$

$x^2 - 3 \geq 0$

$x^2 \geq 3$

$x \geq \pm\sqrt{3}$

10

a) $f(x) = \sqrt{3-x}$

$\therefore 3 - x \geq 0$

$\Rightarrow 3 \geq x$

$\therefore$ Domain: $[3, -\alpha)$

Range: $[0, \alpha)$

b) $F(x) = \sqrt{4-x^2}$

$\therefore 4 - x^2 \geq 0$

$\Rightarrow 4 \geq x^2$

$\Rightarrow x \leq \pm\sqrt{4}$

$\Rightarrow x \leq \pm 2$

Domain: $[2, -2]$

Range: $[0, 2]$

c) $g(x) = 3 + \sqrt{x}$

Domain: $[0, \alpha)$

Range: $[3, \alpha)$

d) $G(x) = x^3 - 2$

Domain: $x \in \mathbb{R}$

Range: $y \in \mathbb{R}$

e) $h(x) = 3\sin x$

Domain: $(-\alpha, \alpha)$

Range: $[-3, 3]$

f) $H(x) = (\sin\sqrt{x})^{\frac{1}{2}}$

Domain: $[0, \alpha)$

Range: $[0, \alpha)$

15 $y = \sqrt{25 - x^2}$

16 $y = -\sqrt{25 - x^2}$

17 $y = \begin{cases} \sqrt{25-x^2} & ; -5 \leq x \leq 0 \\ -\sqrt{25-x^2} & ; \cancel{5 \leq x < 0} \; 0 < x \leq 5 \end{cases}$

9

d) $y = \sqrt{x^2-2x+5}$

$x^2-2x+5 \geq 0$

$(x-1)^2+4 \geq 0$

$y = \sqrt{x^2-2x+5}$

$y^2 = (x-1)^2+4$

$\Rightarrow y^2-(x-1)^2 = 2^2$

কোনো কিছুর √ এর ভিতর negative আসে না তাই

Domain : $x \in \mathbb{R}$

Range : $y \geq 2$ ~~[illegible]~~

e) $H(x) = \dfrac{1}{1-\sin x}$

$\therefore$ Domain $-1 \leq \sin x \leq 1$

Range : $y \geq \frac{1}{2}$

$\dfrac{1}{1-\sin x} \geq 0$

$\Rightarrow 1-\sin x \leq 0$

$-1 \leq \sin x \leq 1$

$1-\sin x \neq 0$

$\sin x \neq 1$

$x \neq (4n+1)\frac{\pi}{2}$

CamScanner

(e) f) $H(x) = \sqrt{\dfrac{x^2-4}{x-2}}$

$= \sqrt{\dfrac{x^2-2^2}{x-2}}$

$= \sqrt{\dfrac{(\cancel{x-2})(x+2)}{(\cancel{x-2})}}$

$= \sqrt{x+2}$

এখানে $x-2$ কাটতে হলে ভেবে কাটতে লাগবে কারণ $x-2$ নিজেই (0) হতে পারে আর 0 দিয়ে কোনো কিছু কাটা যায় না তাই কাটি $x \neq 2$

$\therefore x+2 \geq 0$

$\Rightarrow x \geq -2$

$\therefore$ Domain: $[-2, \infty)$ exept 2

or $[-2, 2) \cup (2, \infty)$

or $-2 \leq x < 2$; $2 < x < \infty$

Range: $[0, 2) \cup (2, \infty)$

~~Freaks~~ Composite of function

একটি function এর value কে অন্য একটি function দ্বারা replace করা।

Given that, $f(x)$ and $g(x)$ are two function the composition ~~defined~~ ~~b~~ of $f(x)$ and $g(x)$ created a new function $fg(x)$ and $gf(x)$

Ex: $f(x) = 3x-1$; $g(x) = 2x^2+5$

$f\circ g(x) = 3(2x^2+5)-1$

$g\circ f(x) = 2x^2+5$

$= 2(3x-1)^2+5$

Ex: ~~$f\circ g(x)$~~ $f(x) = x^2$, $g(x) = x+3$

$f\circ g(x) = f(g(x))$

$= f(x+3)$

$= (x+3)^2$

Domain g

↓

Range g / Dom f

↓

Range f

Ex: $f(x) = x^2+3$, $g(x) = \sqrt{x}$

a) $f \circ g(x) = f(g(x))$

$= f(\sqrt{x})$

$= (\sqrt{x})^2 + 3$

$= x+3$

b) $g \circ f(x) = \cancel{x^2+3}$

$= g(f(x))$

$= g(x^2+3)$

$\cancel{= (\sqrt{x})^2+3}$

$\cancel{= x+3}$

$= \sqrt{x^2+3}$

Ex: $(f \circ g \circ h)(x) = f(g(h(x))) \implies$ $f(x) = \sqrt{x}$, $g(x) = \frac{1}{x}$, $h(x) = x^3$

$= f(g(x^3))$

$= f\left(\frac{1}{x^3}\right)$

$= \sqrt{\frac{1}{x^3}}$

4 Transformation of function

The function of a function ~~shel~~ shifts the graph from one to another position. (No change in shape)

① $f(x-a)$ $\boxed{\text{right}}$ shift by $\boxed{\text{a Unit}}$ (x axis)

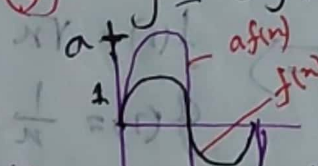
② $f(x+a)$ $\boxed{\text{Left}}$ shift by a unit (x axis)

(3) $f(x)+a$ $\boxed{\text{upword}}$ shift by a unit (y axis)

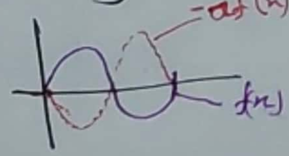
(4) $f(x)-a$ $\boxed{\text{downword}}$ " by a unit (y axis)

~~(5) y = af(x) upword or~~

(5) $y = af(x)$ (x axis) স্থির রাইখা y axis বরাবর বাড়বে (ফুলবে বা চিকন হবে)

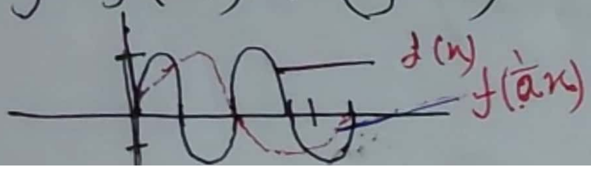


(6) $y = -af(x)$ (x axis) স্থির রাইখা -y axis বরাবর বাড়বে।



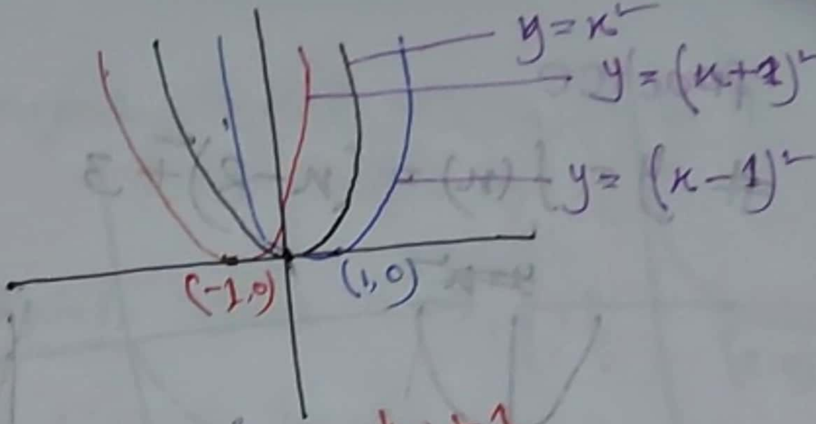
⑦ $y = f(2x) \rightarrow$ (y axis) স্থির রাইখা (x axis) বরাবর বাড়বে

⑧ $y = f(-2x) \rightarrow$ (y axis) " " (-x axis) " "

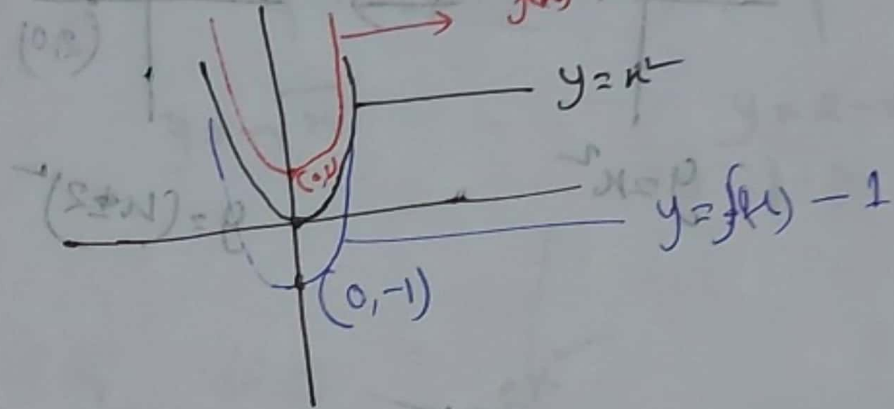


Ex: for 1 to 1

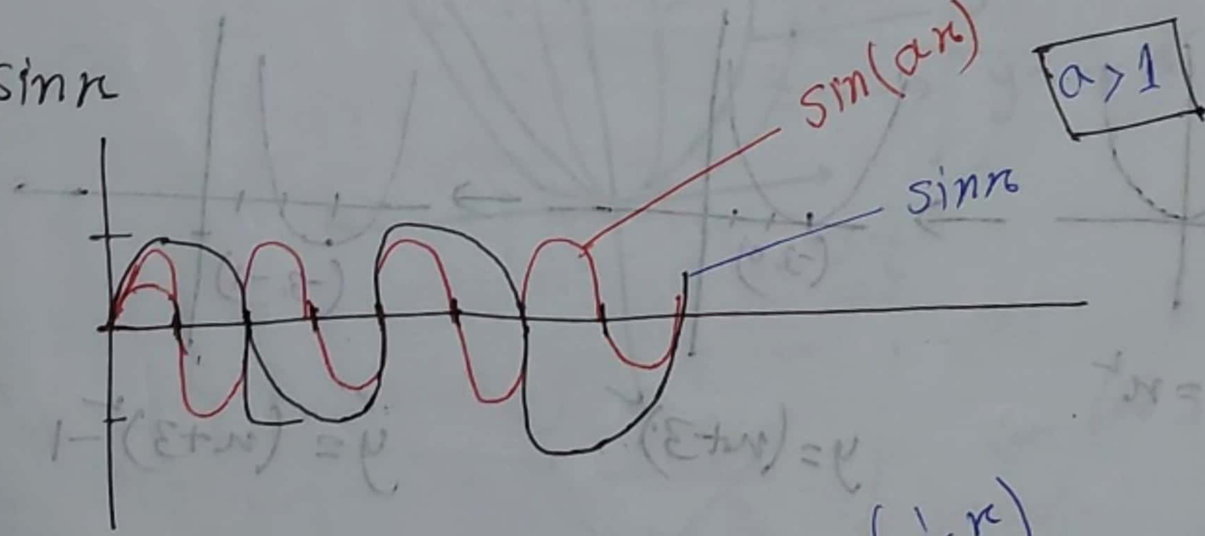
$f(x) = x^2$



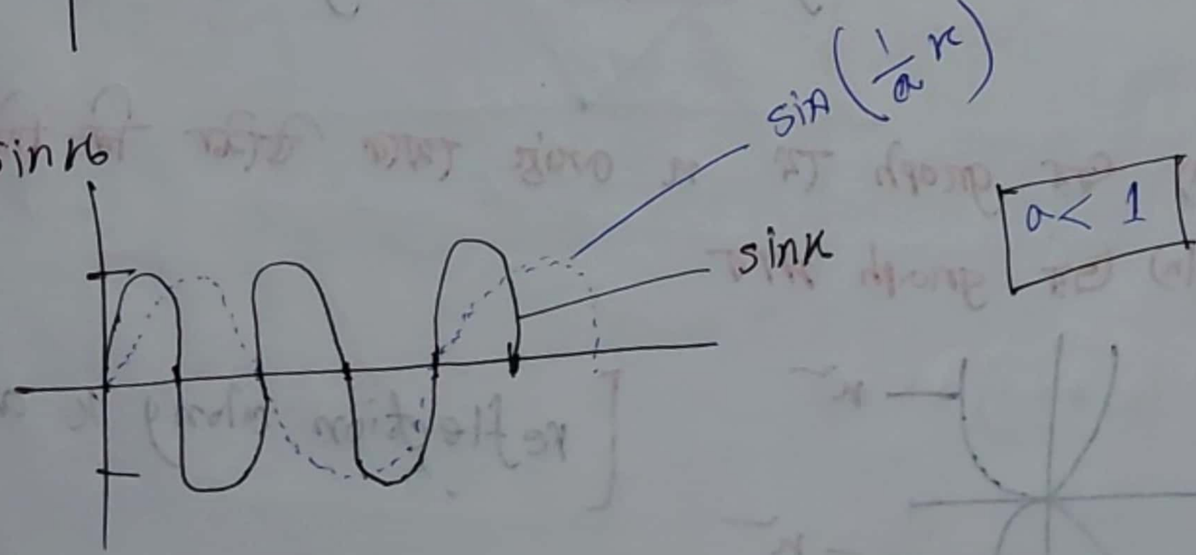
$f(x) = x^2$



sin x

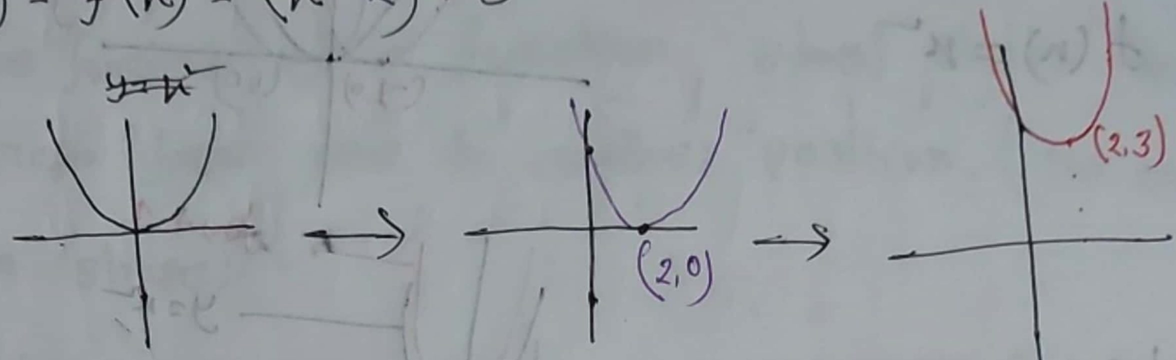


sin x



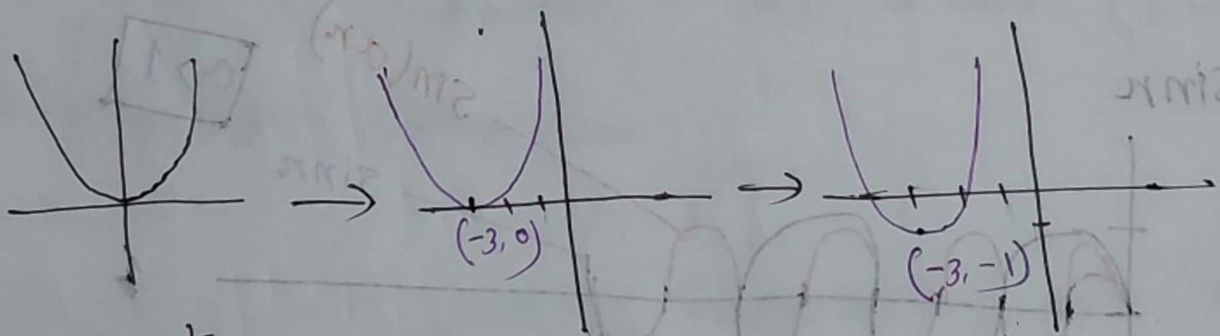
practice

$y = f(x) = (x-2)^2 + 3$



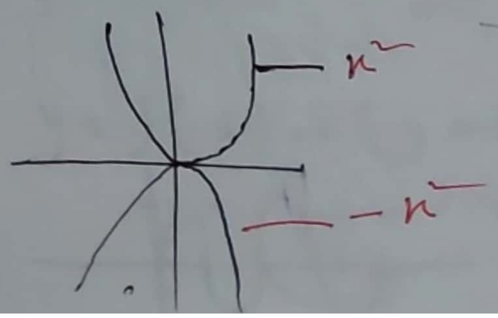
$y = x^2$ $\quad$ $y = (x-2)^2$ $\quad$ $y = (x-2)^2 + 3$

$y = f(x) = (x+3)^2 - 1$



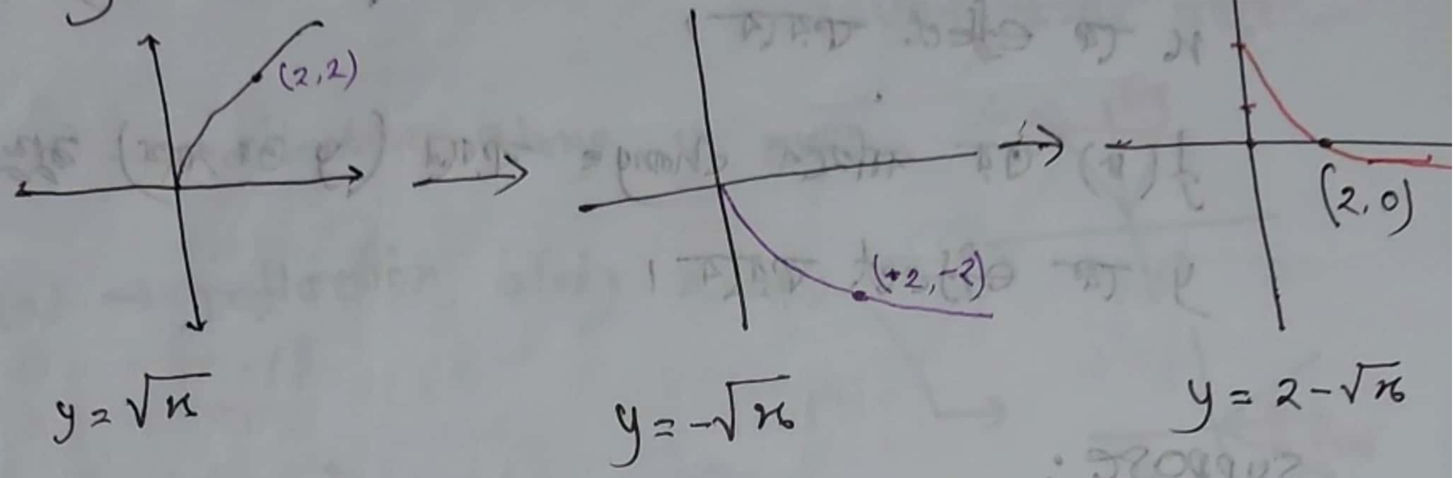
$y = x^2$ $\quad$ $y = (x+3)^2$ $\quad$ $y = (x+3)^2 - 1$

F(x) এর graph কে x axis থেকে উল্টো দিক দিলেই
-F(x) এর graph পাব



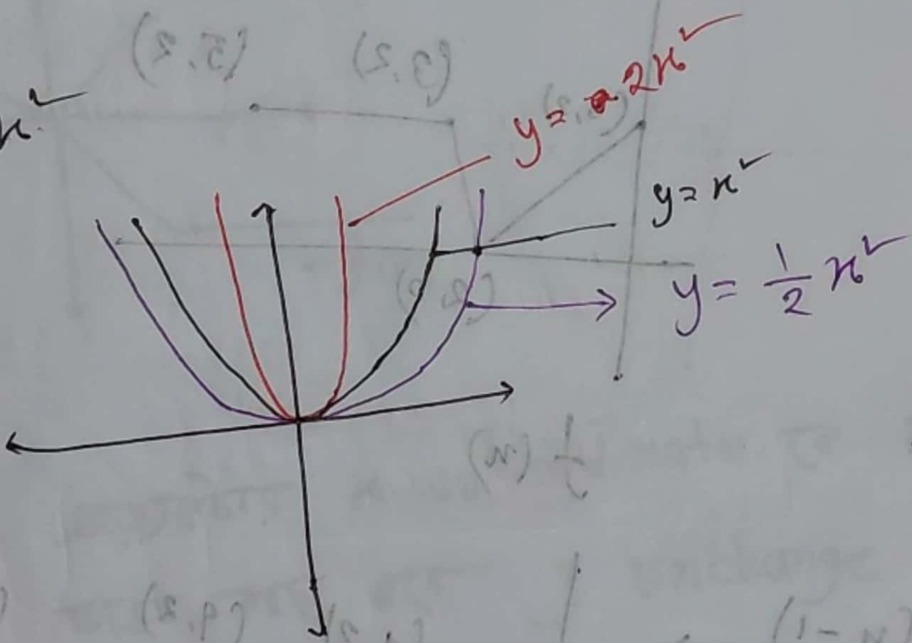
[reflection along x axis]

$y = 2 - \sqrt{x}$



4

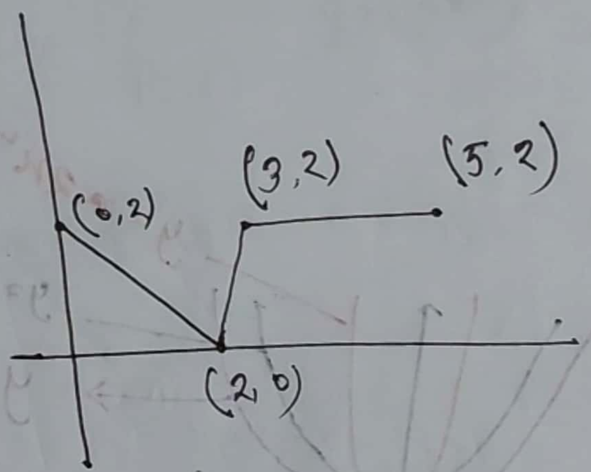
$f(x) = x^2$



N.B| $f(x)$ এর ভিতরে change করলে (x এর সাথে) শুধু x কে effect করবে।

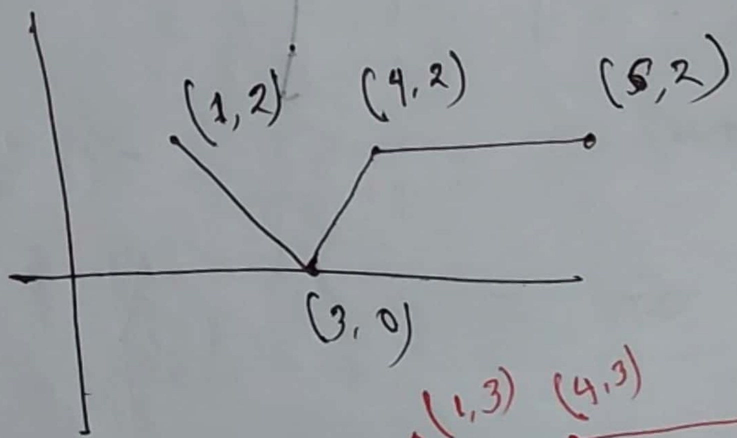
$f(x)$ এর বাহিরে change করলে (y এর সাথে) শুধু y কে effect করবে।

suppose:

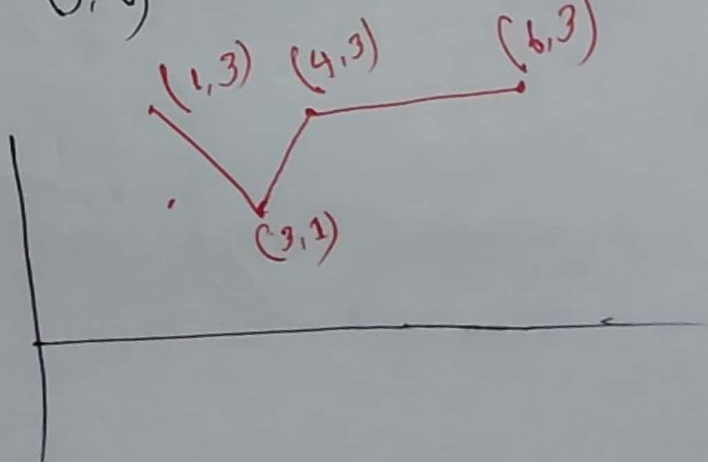


$f(x)$

$f(x-1) \Rightarrow$



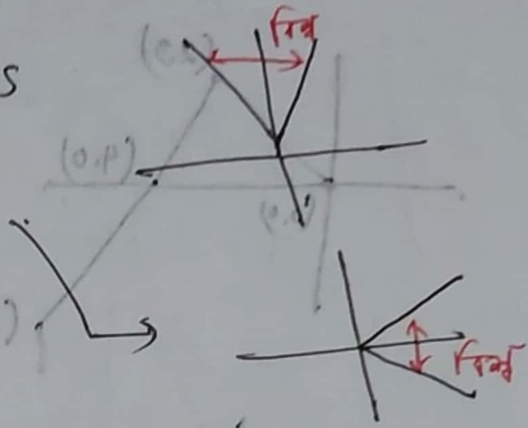
$f(x-1)+1 \Rightarrow$

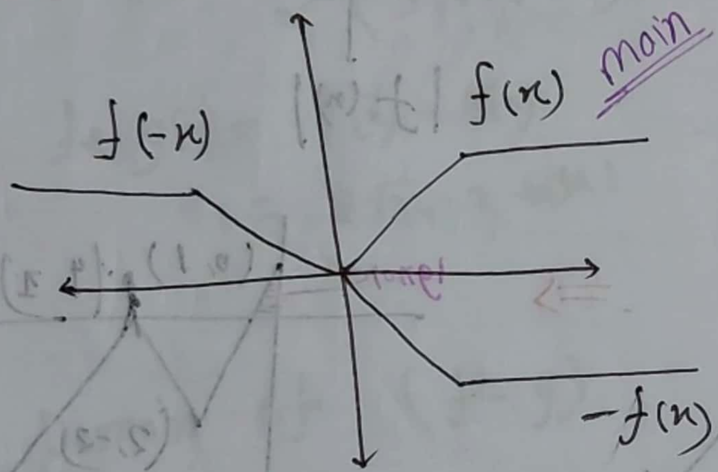


N.B| # $y = f(x-2)+1$; 2 unit ডানে যাবে
1 unit উপরে উঠবে

$f(-x) \rightarrow$ reflection along y axis

$-f(x) \rightarrow$ reflection along x axis





$f(2x) \rightarrow$ প্রত্যেকটার x co-ordinate কে 2 দ্বারা ভাগ করতে হবে y unchange

$(4,3) \rightarrow (2,3)$

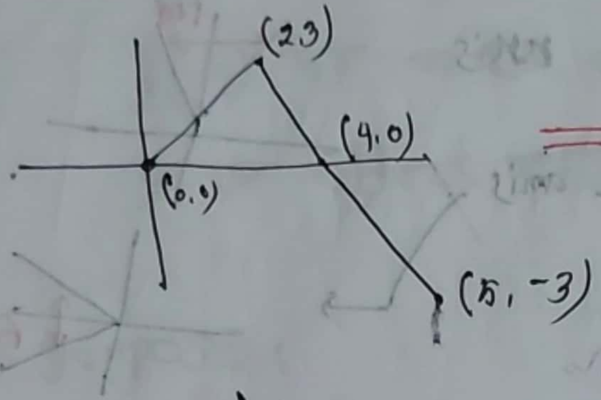
$f\left(\frac{1}{3}x\right) \rightarrow$ প্রত্যেকটার x co-ordinate কে 3 দ্বারা গুণ করতে হবে y unchange

$(1,3) \rightarrow (3,3)$

$2f(x)$

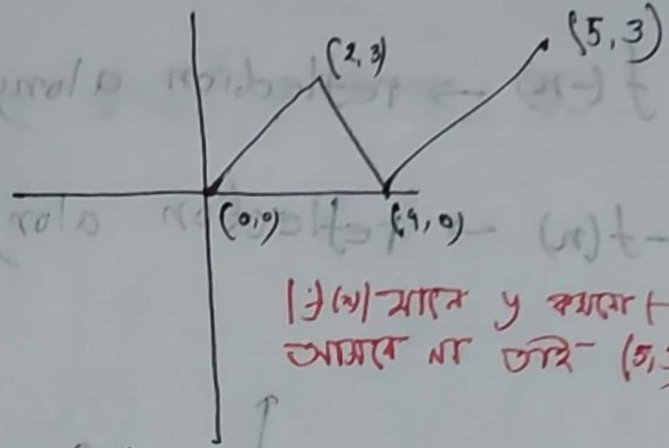
$(1,3) \rightarrow (1,6)$

$y = f(x)$, Draw graph of $1 - |F(x)|$



$f(x)$

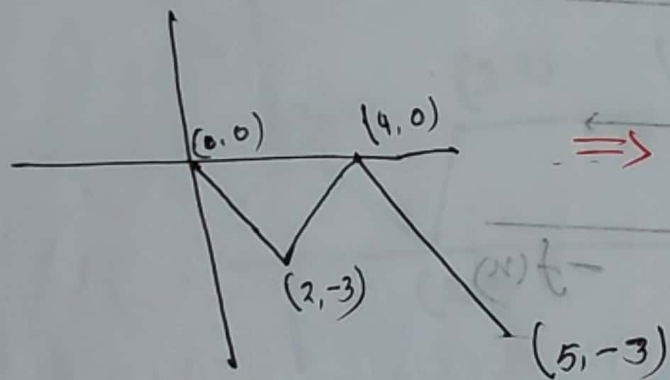
$\Rightarrow$



|f(x)| মানে y কখনো (−) আসবে না তাই (5,3)

$|f(x)|$

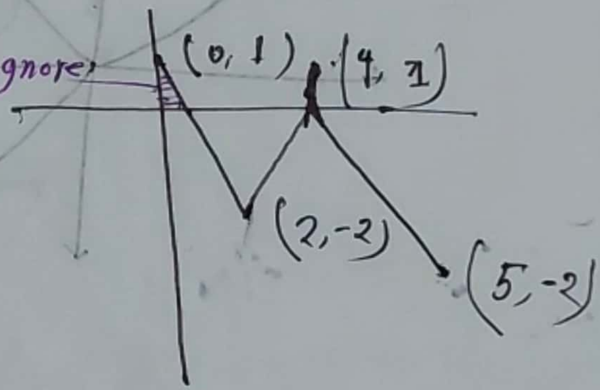
$\Rightarrow$



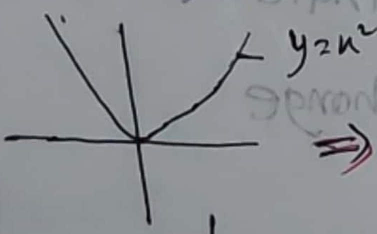
মানে y এর সব মান negative

$-|f(x)|$

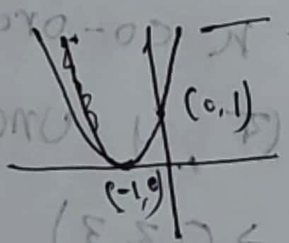
$\Rightarrow$



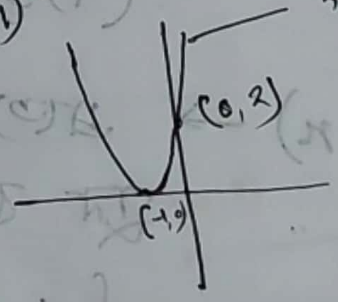
Ex: $-2(x+1)^2 - 3$



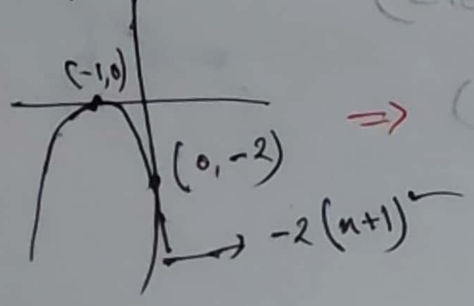
$\Rightarrow$



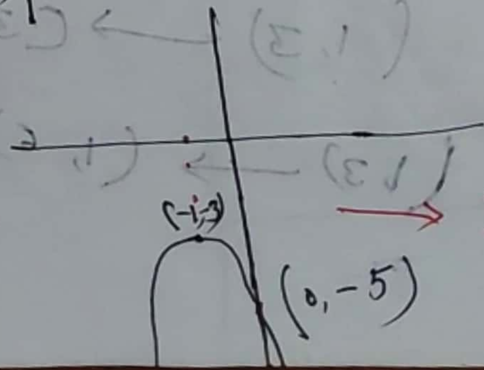
$\Rightarrow$



$\Rightarrow$



$\Rightarrow$



$\longrightarrow y = -2(x+1)^2 - 3$

Some exercise for component function

① $f(x) = 3\sqrt{x} - 2, \quad g(x) = |x|$ Domain of $(f+g)$

$\therefore$ $\cancel{f \circ g(x) = f(g(x))}$

$\cancel{= f(|x|)}$

$\cancel{= f(3|\sqrt{x}| - 2)}$

$f + g(x) = f(x) + g(x)$

$= 3\sqrt{x} - 2 + |x|$

Domain: $0 \leq x$; $[0, \infty)$

② Domain of $(f - g)$

$f - g(x) = f(x) - g(x)$

$= 3\sqrt{x} - 2 - x$

Domain: $0 \leq x$

③ Domain of (fg)

$fg(x) = f(x) \times g(x)$

$= (3\sqrt{x} - 2)x \quad ; x \geq 0$

$= 3\sqrt{x} \cdot x - 2x$

$= 3\sqrt{x} \cdot \sqrt{x} \cdot \sqrt{x} - 2x$

$= 3 \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} - 2x$

$= 3x^{\frac{3}{2}} - 2x$

Domain: $0 \leq x$

~~কারণ $g(x) = \sqrt{x}$ মানে~~

যেহেতু $\sqrt{x}$ আছে (negative দেওয়া যাবে না।)

$y = f(x)$

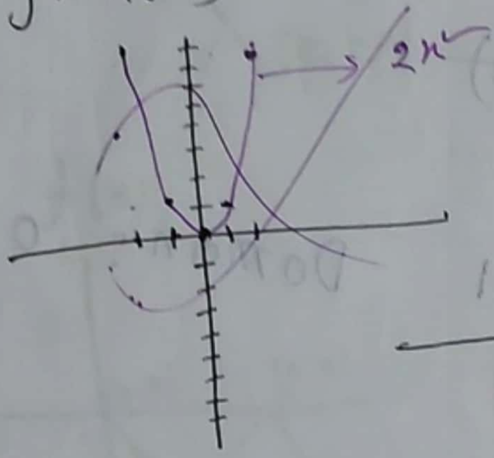
$y = a f(x) \quad a > 1$

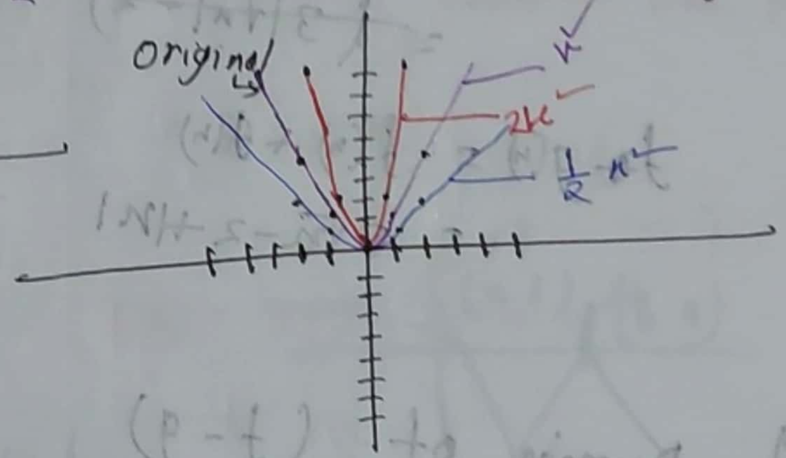
ও কোণের বস্তু করে নিলে ছোট হবে।

$y = f(x) = x^2$

$y = 2f(x) = 2x^2$

x	-2	-1	0	1	2
$y = x^2$	4	1	0	1	4
$y = 2x^2$	8	2	0	2	8
$y = \frac{1}{2}x^2$	2	0.5	0	.5	2





N.B| যদি x এর ভিতরে change আসে তাহলে x অক্ষেই পরিবর্তন করে কিন্তু উল্টো

$f(x+3) \rightarrow$ বামে যাবে ২ ঘর

$f(2x) \rightarrow$ x এর সাথে ২ ভাগ হবে

$f(\frac{1}{2}x) \rightarrow$ x " " ২ গুণ হবে

যদি x এর বাহিরে change আসে তাহলে y অক্ষেই পরিবর্তন করে যেভাবে আছে সেভাবেই

$f(x)+2 \rightarrow$ উপরে ২ ঘর

$f(x)-2 \rightarrow$ নিচে ২ ঘর

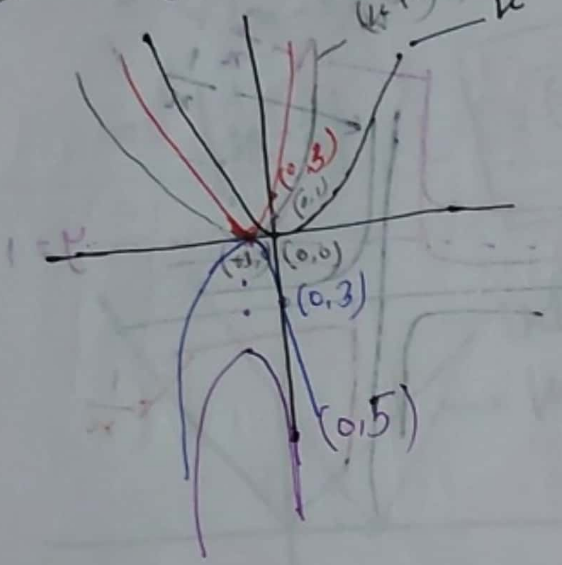
$2f(x) \rightarrow$ ২ গুণ হবে

$\frac{1}{2}f(x) \rightarrow$ ২ দ্বারা ভাগ হবে।

0.2

Book exercise

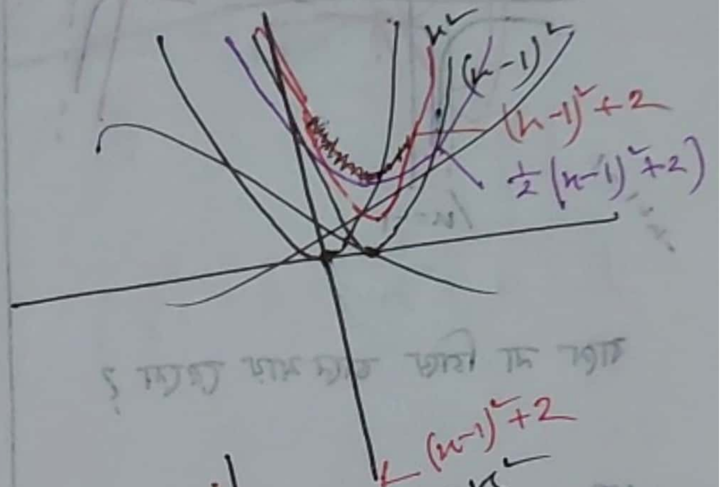
5 $y = -2(x+1)^2 - 3$

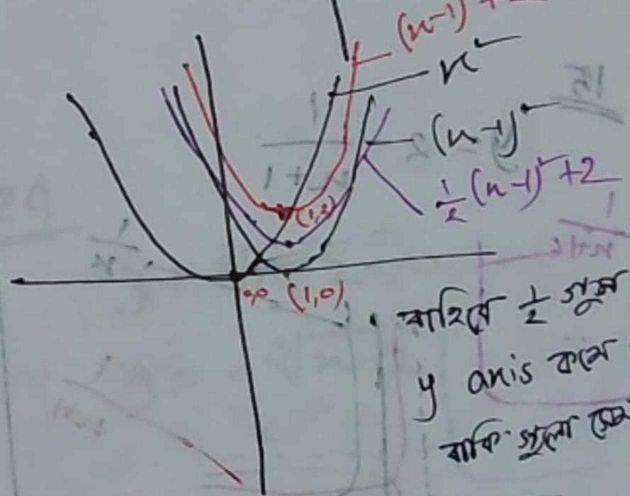


8 $y = \frac{1}{2}(x^2 - 2x + 3)$

$= \frac{1}{2}(x^2 - 2 \cdot x \cdot 1 + 1^2 - 1^2 + 3)$

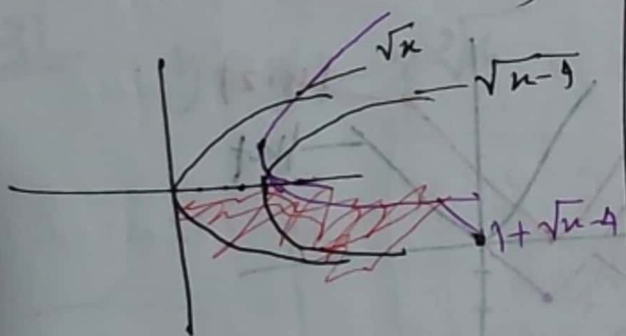
$= \frac{1}{2}((x-1)^2 + 2)$



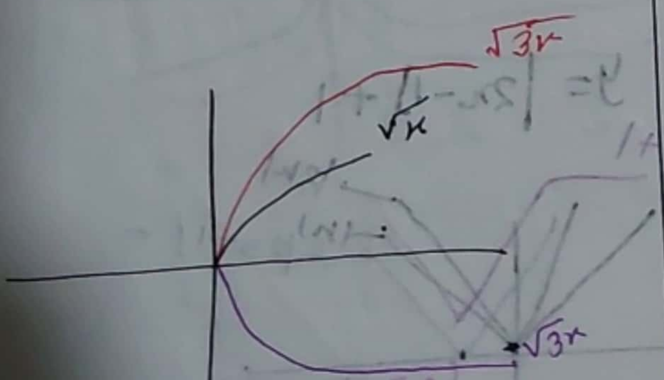


বাহিরে $\frac{1}{2}$ গুণ দেখে
y axis কমে গেছে
বাকি গুলো সেম

10 $y = 1 + \sqrt{x-4}$

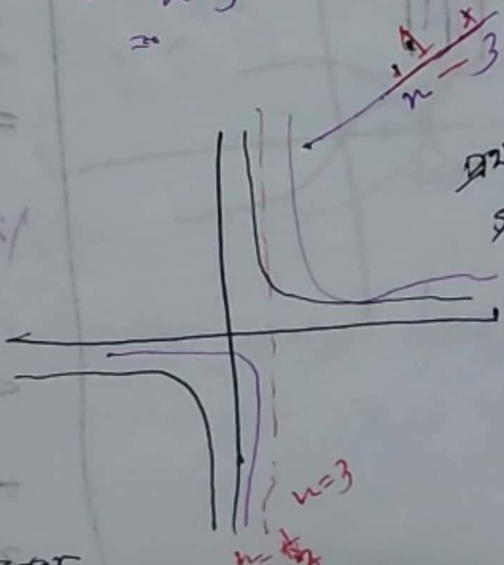


12 $y = -\sqrt{3x}$



এখানে $\sqrt{x}$ এর ভিতরে ৩ আছে
আর কিছু কাজ করছে ৩ সাধারণ গুনের মতো

13 $y = \frac{1}{x-3} = \frac{1}{x} - \frac{1}{3}$



প্রথমে $\frac{1}{x}$ এর
গ্রাফ করে -3
x এর সাপেক্ষে
বাড়বো

$x =$ কতো হলে হর 0 হবে

$y = \dfrac{\text{লবে } x \text{ এর সহগ}}{\text{হরে } x \text{ এর সহগ}}$

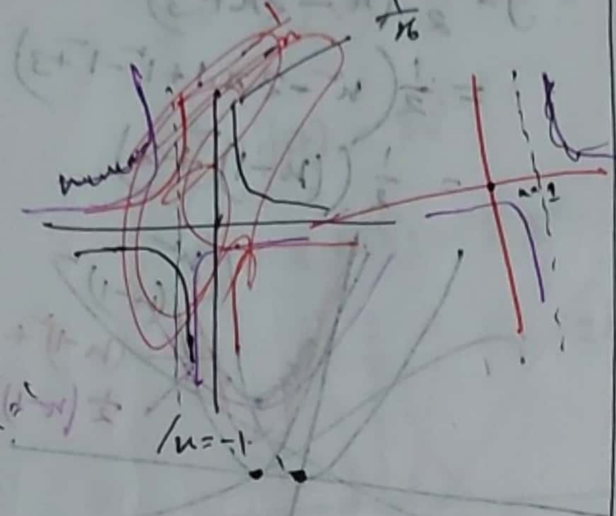
14

$$y = \frac{1}{-1+x}$$

$x = 1$

$y = \frac{0}{-1} = 0$

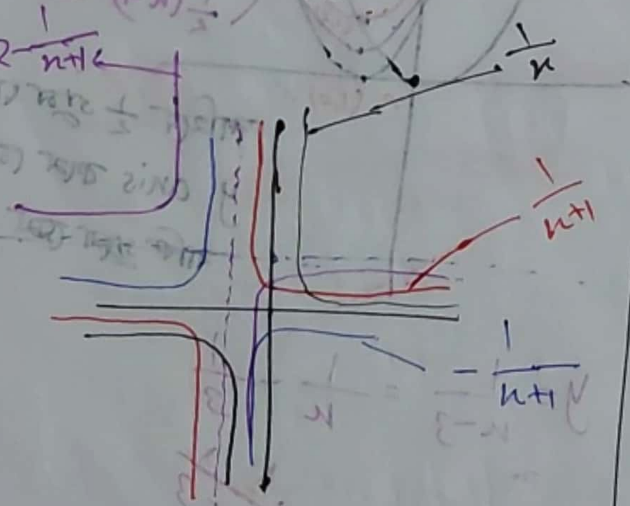
linear



বামে না গিয়ে ডানে যায় কেনো ?

15

$$y = 2 - \frac{1}{x+1}$$

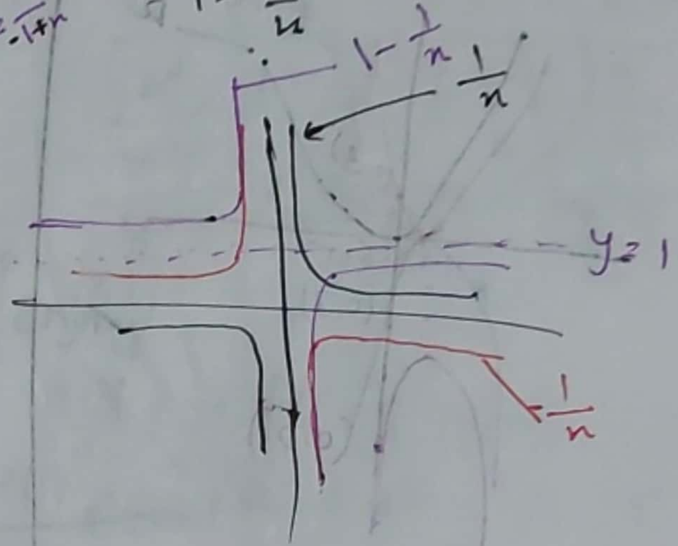


16

$$y = \frac{x-1}{x}$$

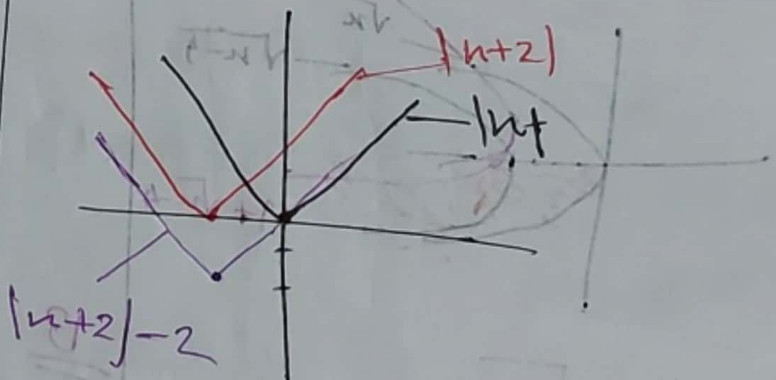
$$= \frac{x}{x} - \frac{1}{x}$$

$$= 1 - \frac{1}{x}$$



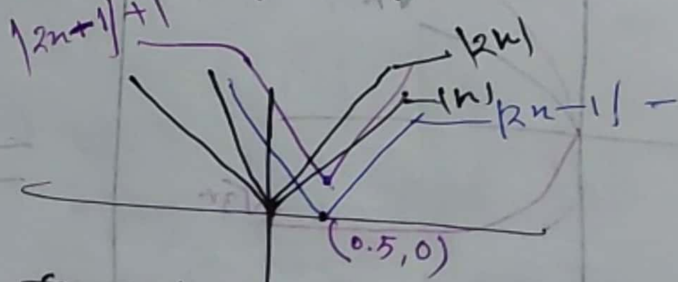
17

$$y = |x+2| - 2$$



19

$$y = |2x-1| + 1$$



এখানে $|2x-1|$ করার সময় আগে x এর সাথে 2 গুণ আছে তা খেয়াল রাখতে হবে।

20

$y = \sqrt{x^2 - 4x + 4}$

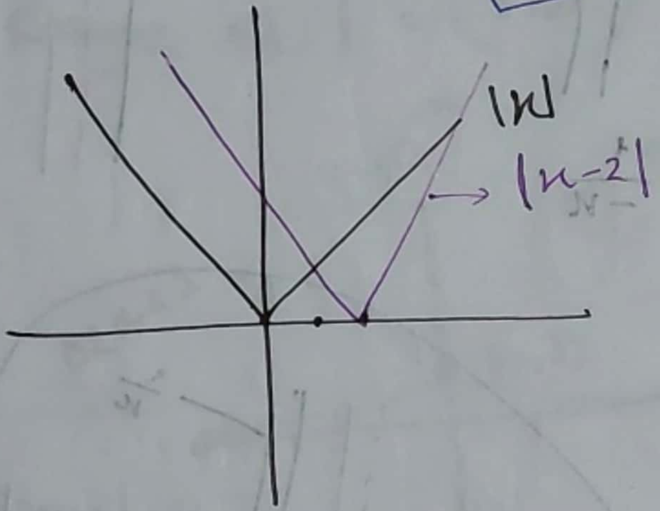
$= \sqrt{(x^2 - 2.x.2 + 2^2)}$

$= \sqrt{(x-2)^2}$

$= |x-2|$

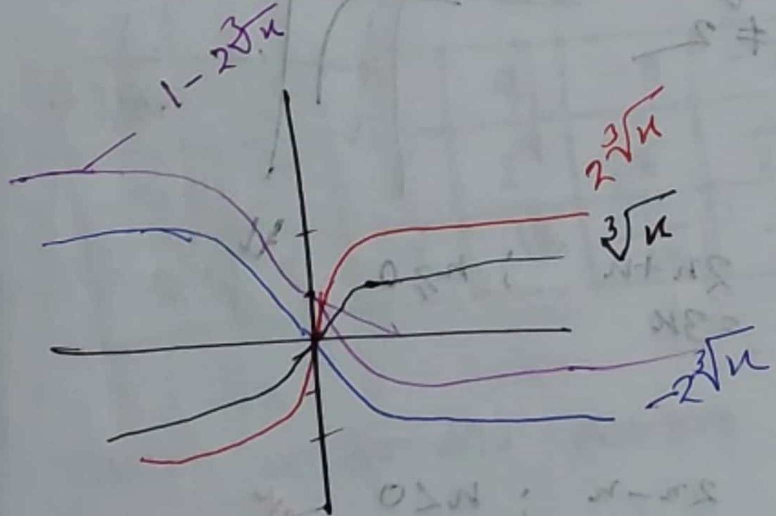
$$\boxed{\sqrt{(\text{equation})^2} = |\text{equation}|}$$

Basic



21

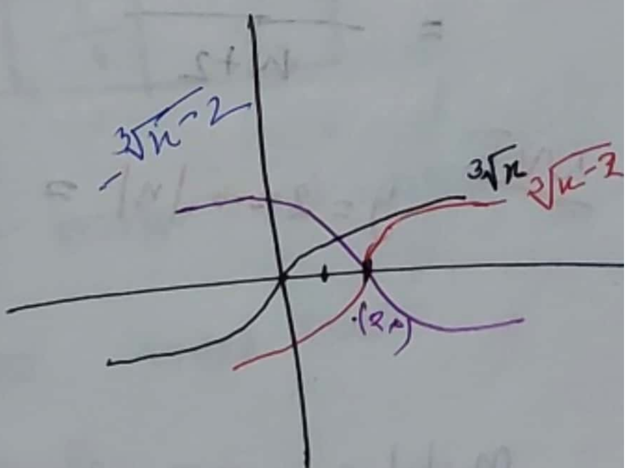
$y = 1 - 2\sqrt[3]{x}$



24

$y + \sqrt[3]{x-2} = 0$

$y = -\sqrt[3]{x-2}$



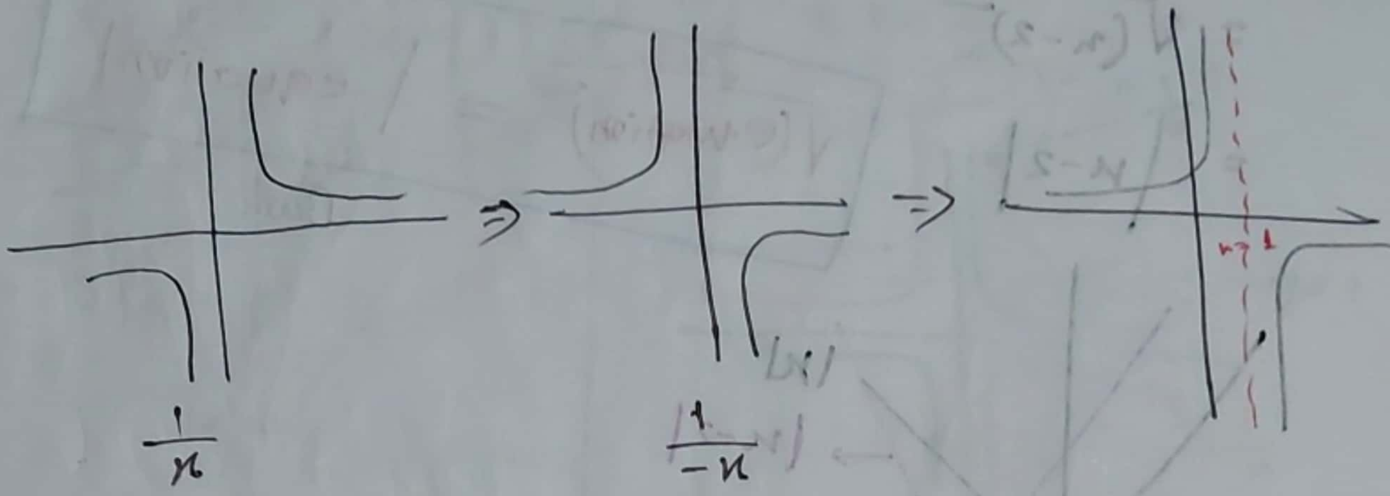
14

$$y = \frac{1}{1-x}$$

$1 - x = 0$

$-x = -1$

$x = 1$



V.V.I

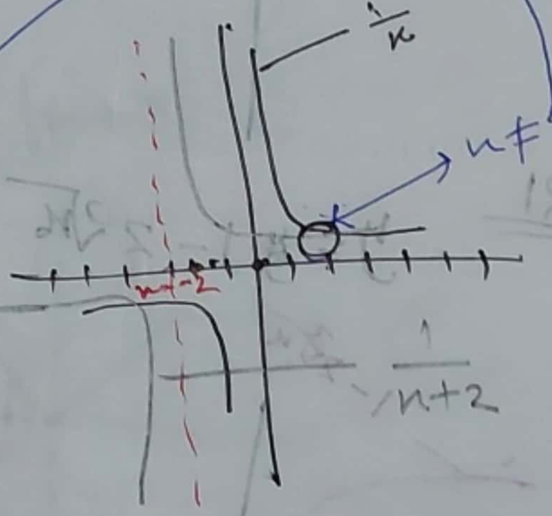
$$y = \frac{x-2}{x^2-4}$$

$$= \frac{x-2}{x^2-2^2}$$

$$= \frac{(x-2)}{(x+2)(x-2)}$$

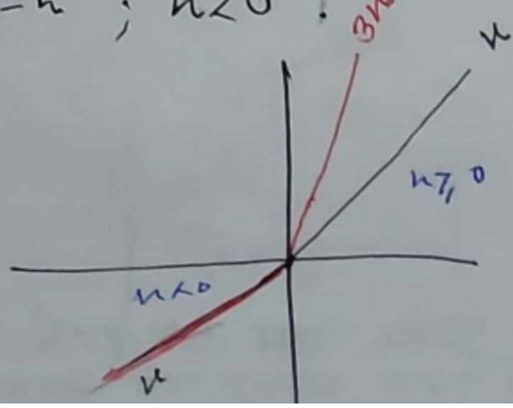
$$= \frac{1}{x+2}$$

$x \neq 2$



V.V.I

$$y = 2x + |x| = \begin{cases} 2x + x = 3x & ; x \geq 0 \\ 2x - x = x & ; x < 0 \end{cases}$$



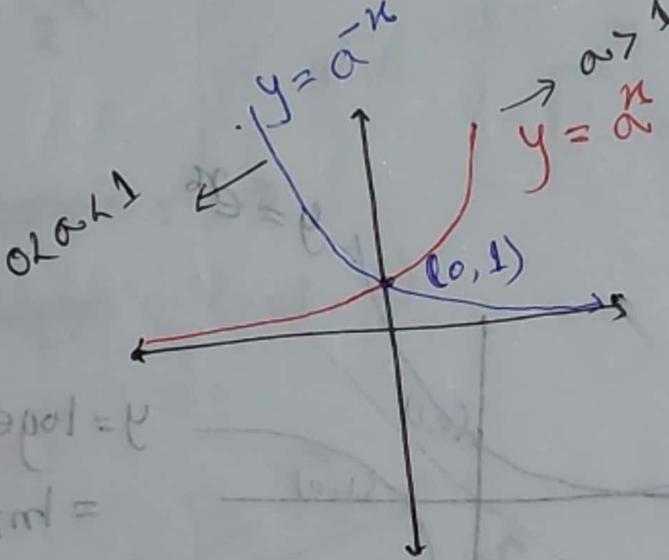
Modulus থাকলে piecewise করতে হবে।

Ct-2

ক) Exponential Function

$f(x) = a^x \qquad a > 0$

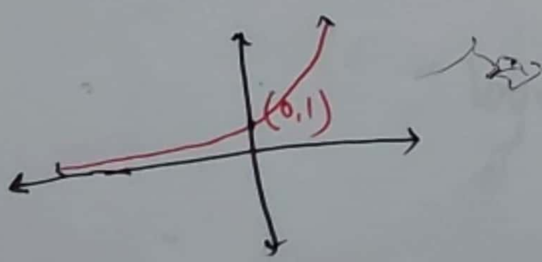
Domain of $f = \mathbb{R}$
Range of $f = (0, +\infty)$
} ~~Also~~ Always for exponential function.



$y = a^x$

x	0	1	4	−4	−3	−2	−1
a=2	2^0	2^1	2^4	$\frac{1}{2^4}$	$\frac{1}{2^3}$	$\frac{1}{2^2}$	$\frac{1}{2^1}$
y	1	2	16	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$

এখানে x এর মান positive বা negative যেই-অনুযায়ী—
y অনেক ছোট পাওয়া যায় তাই graph এমন—



$a^x = m \longrightarrow \log_a m = x$

$y = a^x = f(x)$

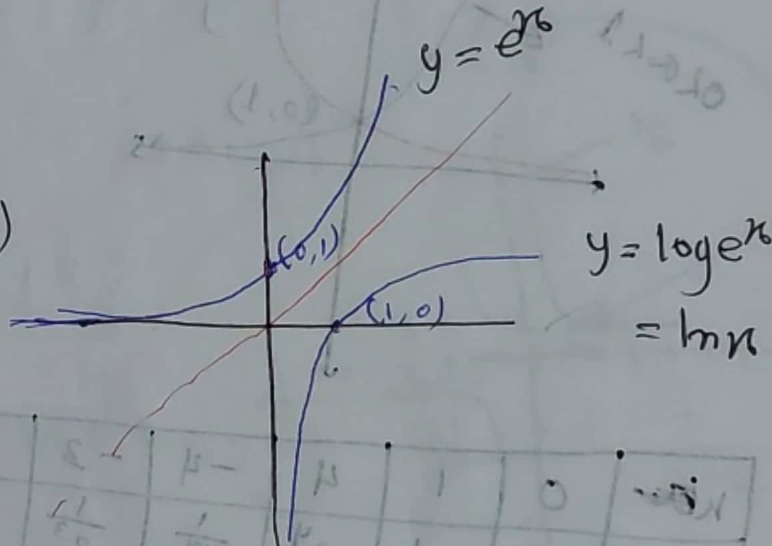
$\log_a y = x$

$\Rightarrow f^{-1}(y) = \log_a y$

$\Rightarrow f^{-1}(x) = \log_a x$

Domain $f^{-1} = (0, \infty)$

Range $f^{-1} = \mathbb{R}$



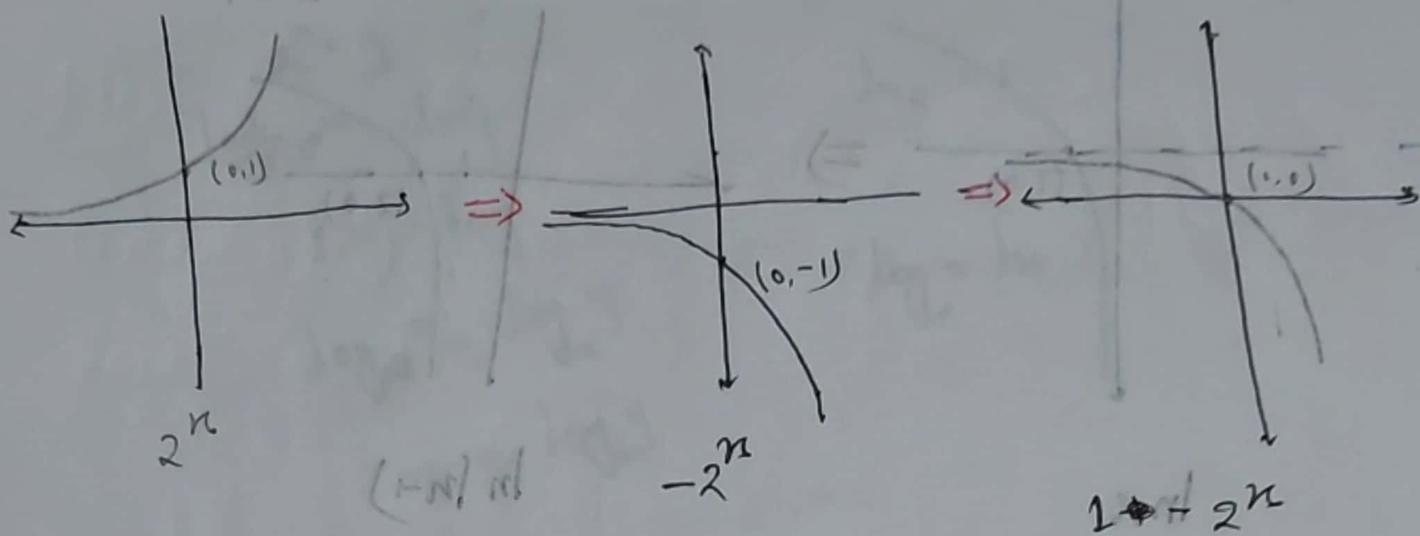
lnx :

(loge^x) lnx Always $y = e^x$ এর inverse হয়

$f(x) = 1 - 2^x$

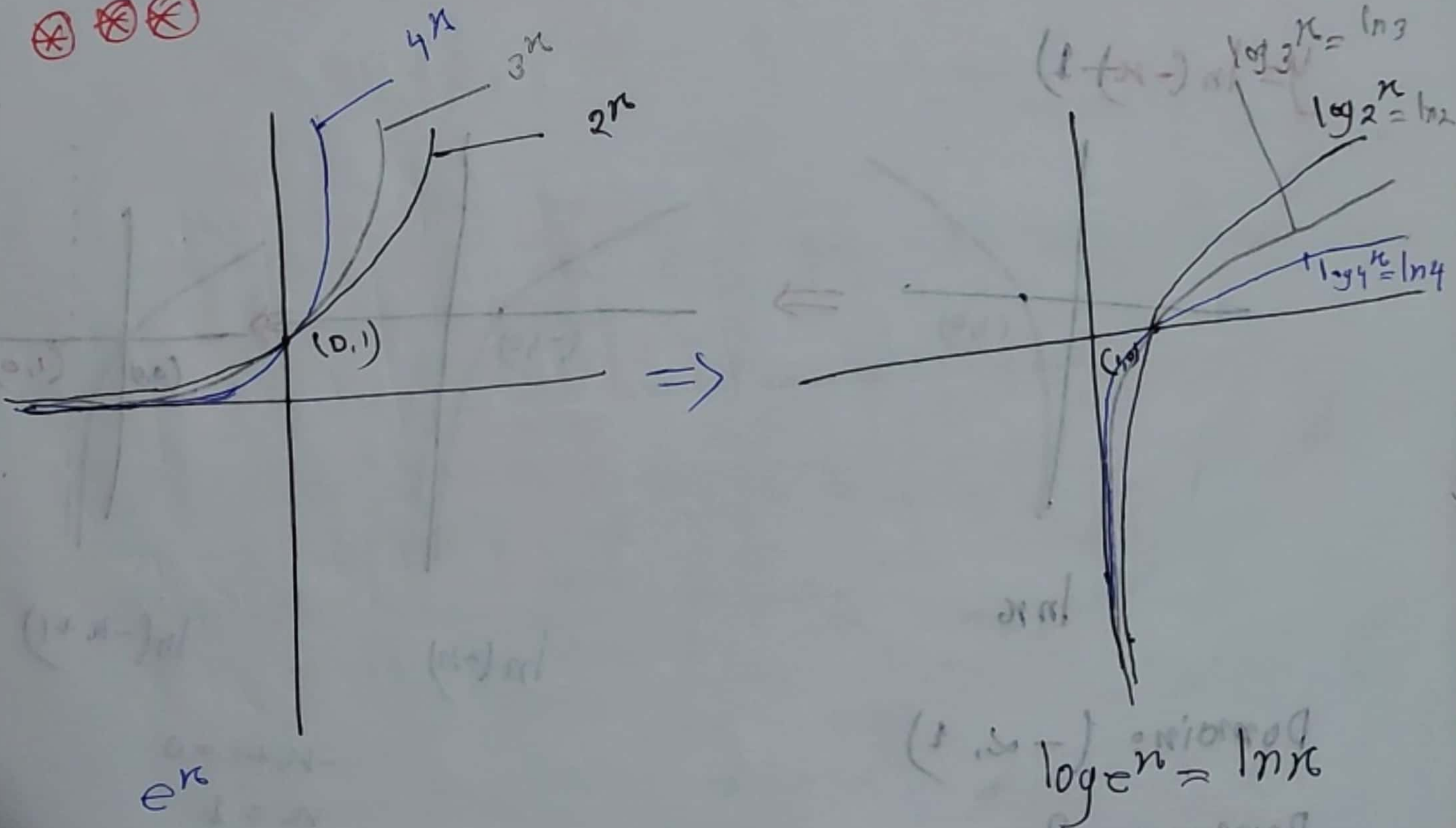
(2^{-x}) Reflect along y axis

(-2^x) Reflect along x axis



Domain : $\mathbb{R}$

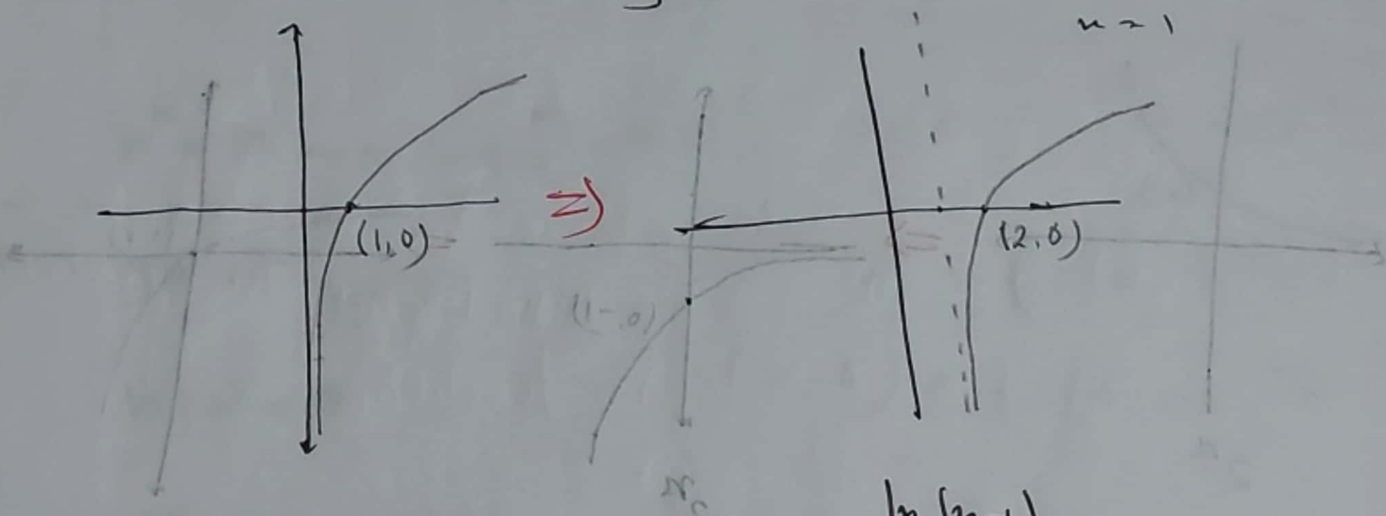
Range : $(-\infty, 1)$



practice

$y = \ln(x-1)$ $\quad e^x = y$

$x - 1 = 0,$
$x = 1$

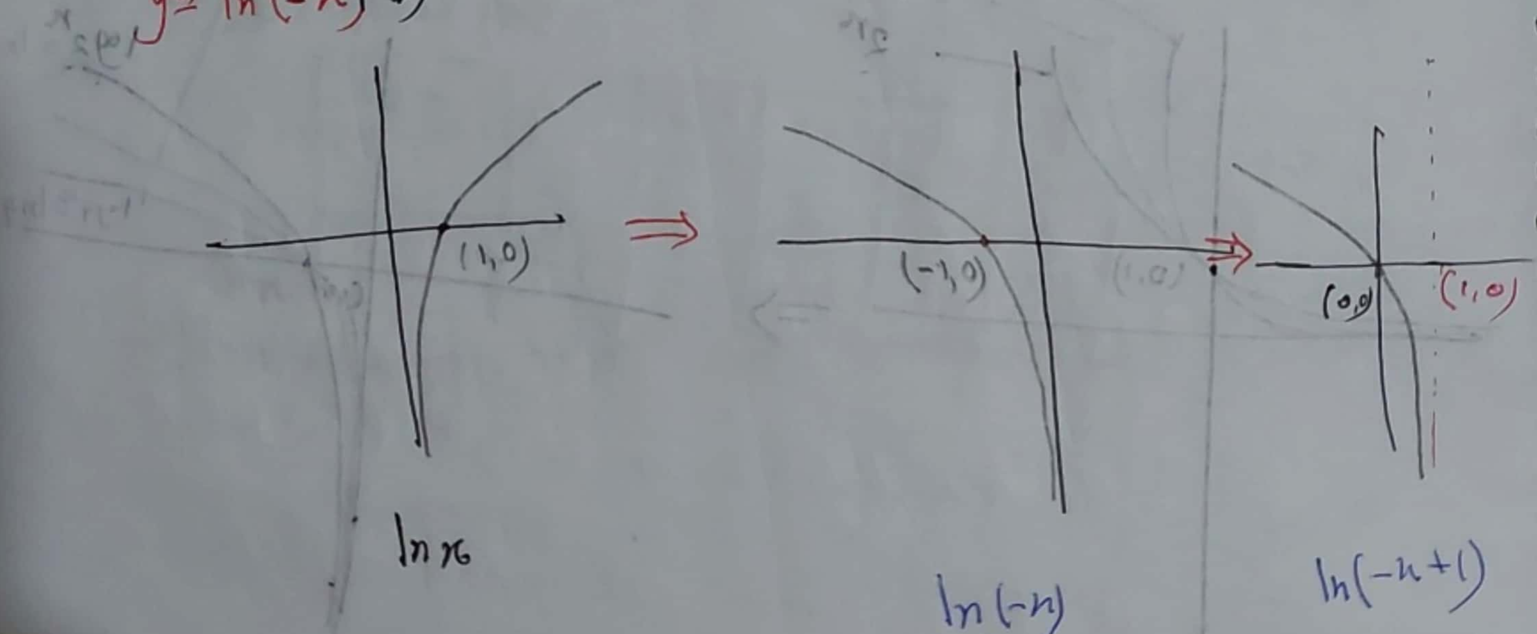


$\ln x$ $\quad \ln(x-1)$

Domain: $(1, \infty)$

Range = $\mathbb{R}$

$y = \ln(-x+1)$



$\ln x$ $\quad \ln(-x)$ $\quad \ln(-x+1)$

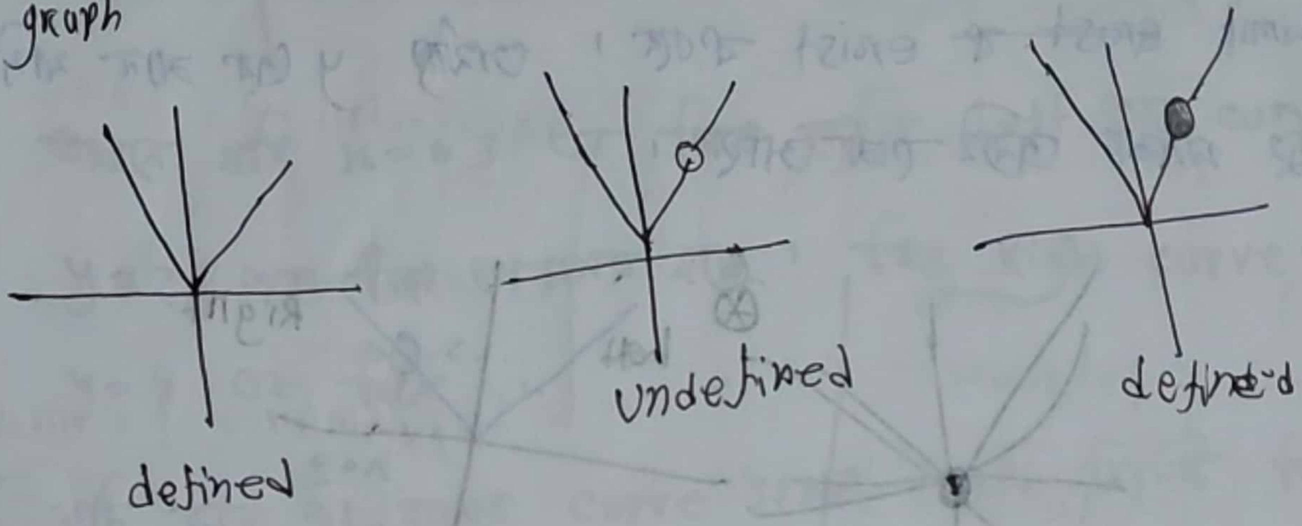
Domain: $(-\infty, 1)$

Range: $\mathbb{R}$

$-x + 1 = 0$
$x = 1$

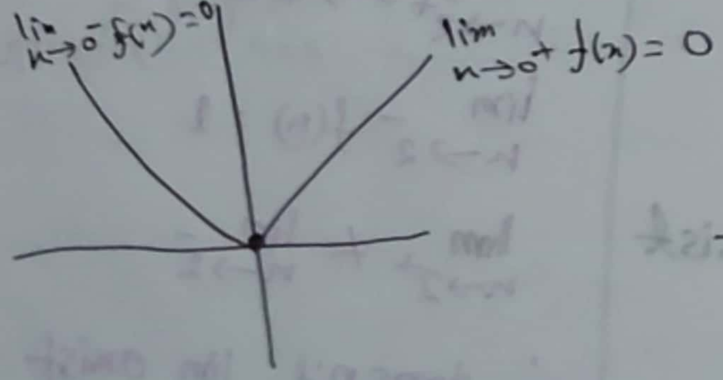
Limit and continuity

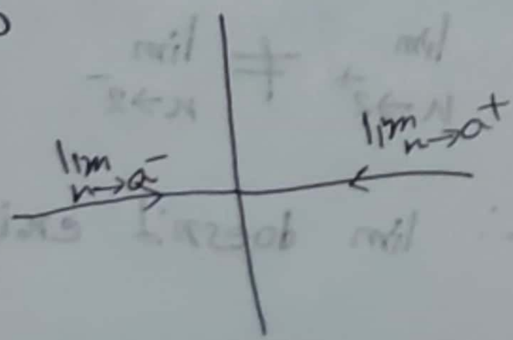
যেই graph এ ফোটা (0) থাকবে না or defined graph



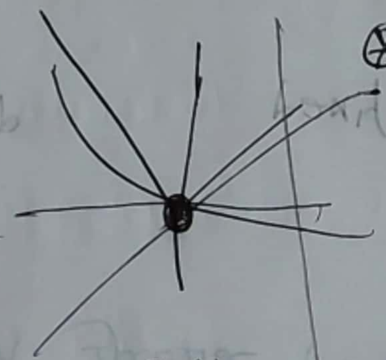
Limit মানে (approximate value) কাছাকাছি value exact value না।

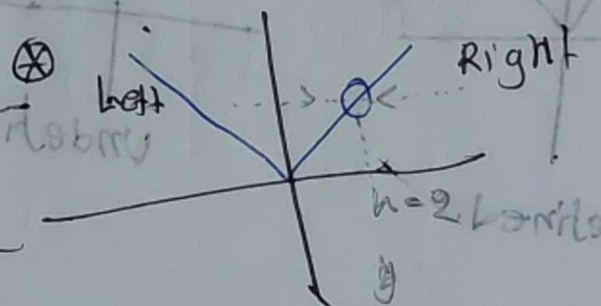
limit বের করার নিয়ম হচ্ছে যদি (●) থাকে সেখানের limit চাইছে সেখানের তাহলে (●) বাম পাশের ও ডান পাশের মান y এর কোথায় এসে meet করতেছে তা





x এর value এর জন্য Right and Left side এর limit এর meet point যদি same হয় তাহলে Limit ~~exist~~ exist করবে। অর্থাৎ y এর মান যদি দুই পাশের একই সেম আসে।

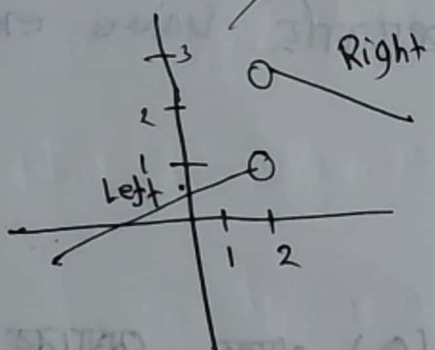




$$\lim_{x\to 2^+} f(x) = 2$$

$$\lim_{x\to 2^-} f(x) = 2$$

$\therefore$ lim exist

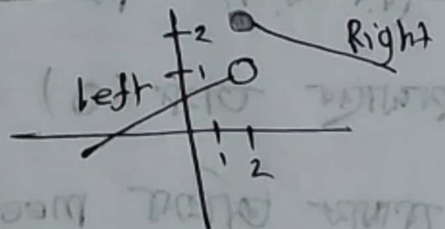


$$\lim_{x\to 2^+} f(x) = 3$$

$$\lim_{x\to 2^-} f(x) = 1$$

$$\lim_{x\to 2^+} \neq \lim_{x\to 2^-}$$

$\therefore$ lim doesn't exist

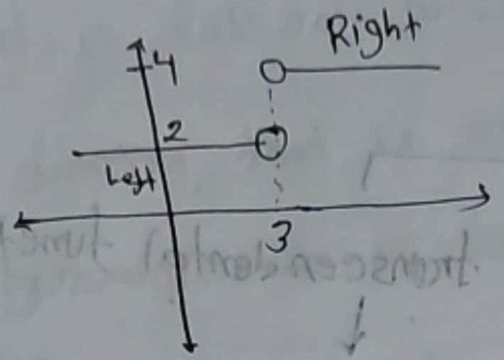


$$\lim_{x\to 2^+} f(x) = 2$$

$$\lim_{x\to 2^-} f(x) = 1$$

$$\lim_{x\to 2^+} \neq \lim_{x\to 2^-}$$

$\therefore$ doesn't lim exist



এখানে যদি $x = 3$ এর দিকে দেখি Left এর curve টি $y = 2$ এর দিকে অগ্রসর হচ্ছে। কিন্তু Right curve টি $y = 4$ এর দিকে " "

[যদি ডান এবং বামের curve সমূহ একটি নির্দিষ্ট বিন্দুর দিকে অগ্রসর না হয় তবে limit exist করবে না।]

∴ এই graph এর $x = 3$ তে limit নাই।

$\lim_{x \to a^+} f(x) \Rightarrow$ [Right hand limit]

$\lim_{x \to a^-} f(x) \longrightarrow$ [Left hand limit]

Function

Algebiac Function

transcendental function

$f(x) = 2x^2 + 1$

$g(x) = \frac{x}{x^2+3}$

- polynomial
- Rational function
- others

exponential $y = e^{2x-1}$

logarithmic $y = \ln x$

trigonometric $y = \sin x$

What is polynomial function?

x का power non-negative and not fractional

$f(x) = x^3 + \frac{1}{x} + 7 \longrightarrow$ not polynomial x^{-1}

$f(x) = x^7 + 2x^3 + 8 \longrightarrow$ polynomial of degree 7

$f(x) = x^{\frac{3}{2}} + x^7 + 9 \longrightarrow$ not polynomial

What is Rational function ?

A function that is expressible as a ratio of two polynomials ~~is~~ is called a rational function

ex: $\frac{x}{2x+5}$, $\frac{x^2+x+1}{2x^2+2x+3}$, $\frac{3}{x^3+1}$

Limit of a function

Let y be a function of x, $y=f(x)$

x is independent, y is dependent variable

A limit is a number that a function approaches (closing to) as the independent variable of the function approaches to a given number

$f(x) = 3x+2 \longrightarrow$ the limit of $f(x) = 8$

as x approaches to 2

$f(n) = 3 + \frac{1}{n}$

$n = 10, \ f(n) = 3.1$

$n = 100, \ f(n) = 3.01$

$n = 1000, \ f(n) = 3.001$

$n = \infty \quad f(n) \to (3 \ldots)$

if n approaches to ∞, $f(n) = ??$

$$\lim_{n \to \infty} f(n) = 3$$

এখানে n হলো মানের y হলো 3 এর কাছাকাছি যাবে কিন্তু 3 হবে না

প্র $\lim_{n \to a} f(n) = L$

The symbol $n \to a$ tells us that we are letting n approaches to a (n tends to a)

$L \longrightarrow$ tells us the value of $f(n)$ when $n \to a$

প্র $\lim f(n) = n^2 + 1$, find the limiting value of $f(n)$ when $n = -2$

$$\therefore \lim_{n \to -2} f(n) = (-2)^2 + 1$$
$$= 5$$

polynomial এর ~~value~~ limiting value সরাসরি বসালে আসে

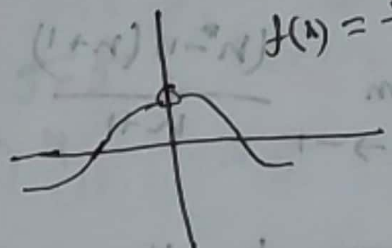
ex: $\lim_{x\to 2} f(x) = 3x^2+1$

$= 3(-2)^2+1$

$= 13$

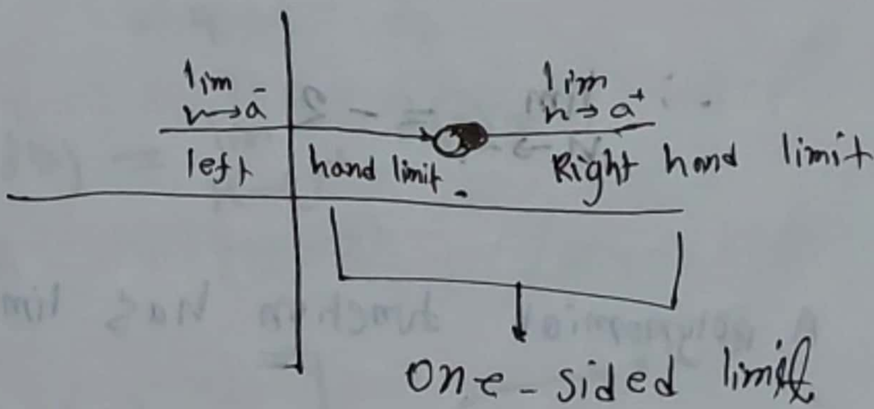
$\lim_{x\to 0} \frac{\sin x}{x} = 1$

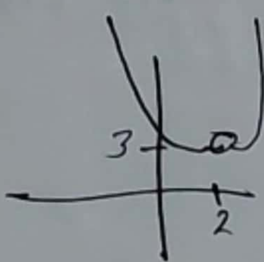
$f(x) = \frac{\sin x}{x}$



polynomial always continuous

$\lim_{x\to a} f(x) = L$





→ lim f(x) → does exist ⇒ yes

→ if exist, what is the limit → 3 = y

Q: $\lim_{x \to 2} (x^2 + x - 1)$

$= 2^2 + 2 - 1$

$= 5$

Q: find $\lim_{x \to -1} \frac{x^2 - 1}{x + 1}$

Rational হলে তাকে polynomial বানিতে হবে

$= \lim_{x \to -1} \frac{(x-1)(x+1)}{x+1}$

$= \lim_{x \to -1} (x - 1)$

$= -1 - 1$

$= -2$

$\therefore \lim_{x \to -1} = -2$

A polynomial function has limit at any point

Q. $\boxed{4}$ $f(x) = \begin{cases} x^2 - 5 ; & x < 3 \\ \sqrt{x+13} ; & x \geq 3 \end{cases}$

Does $\lim\limits_{x \to 3} f(x)$ exist ?

$$L.H.L = \lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} (x^2 - 5)$$

$$= 3^2 - 5$$

$$= 4$$

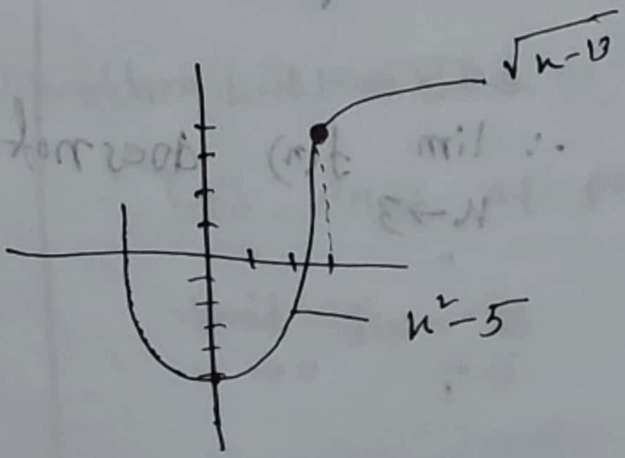
$$R.H.L = \lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (\sqrt{x+13})$$

$$= \sqrt{3+13}$$

$$= 4$$

Sim since $\lim\limits_{x \to 3^-} f(x) = \lim\limits_{x \to 3^+} f(x) = 4$

$\therefore$ limit exist



4)
$$f(x) = \begin{cases} 2x+3 & ; \; x > 3 \\ 7 & ; \; x = 3 \\ x-2 & ; \; x < 3 \end{cases}$$

Does $\lim f(x)$ exist ?

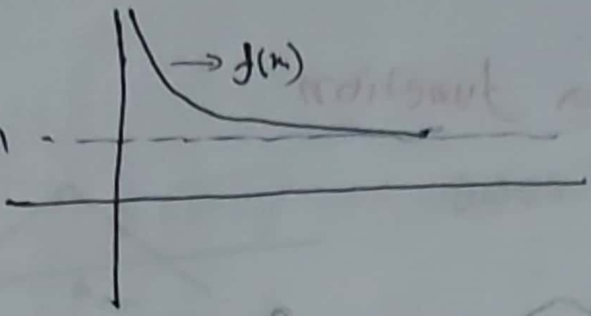
$\cancel{\lim_{x \to 3}}$

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} (x-2)$$
$$= 3-2$$
$$= 1$$

$$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (2x+3)$$
$$= 2\times 3 + 3$$
$$= 9$$

since L.H.L $\neq$ R.H.L

$\therefore \lim_{x \to 3} f(x)$ does not exist

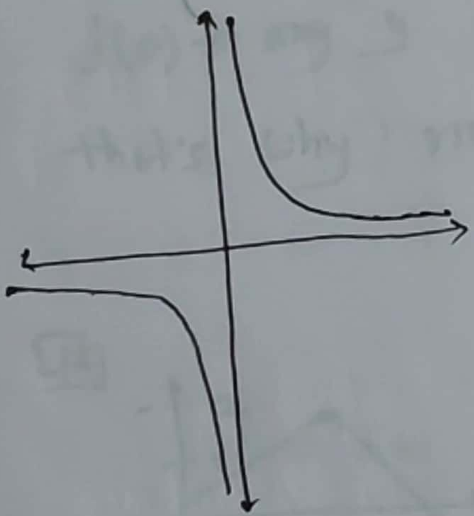


$$\lim_{x \to \infty} f(x) = ?$$

এখানে x এর মান যত বাড়বে y এর মান তত 1 এর কাছাকাছি যাবে।

Infinite limit

$y = f(x) = \frac{1}{x}$



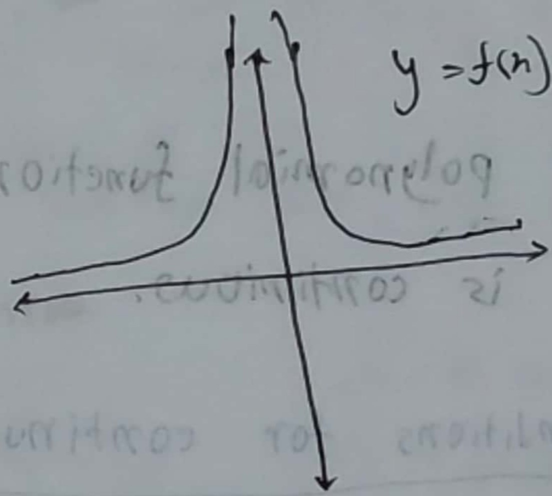
$$\lim_{x \to 0^-} f(x) = -\infty$$

$$\lim_{x \to 0^+} f(x) = +\infty$$

L.H.L $\neq$ R.H.L

$\therefore$ limit Doesn't exist

$y = f(x) = \frac{1}{x^2}$



$$\lim_{x \to 0^+} f(x) = +\infty$$

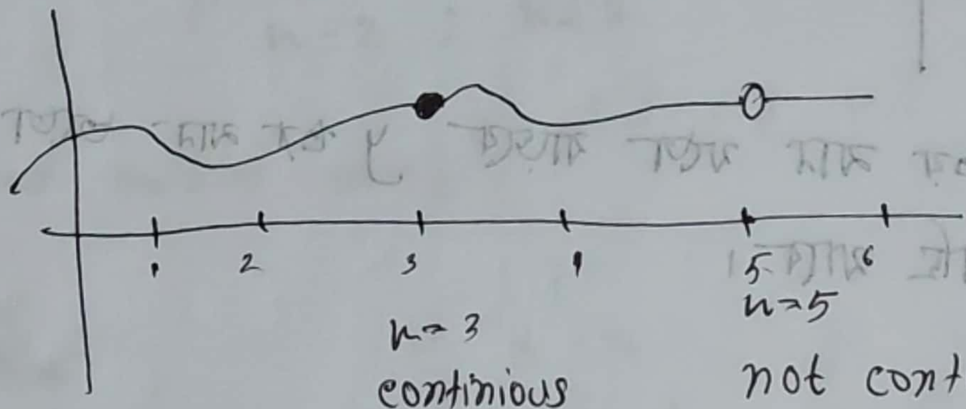
$$\lim_{x \to 0^-} f(x) = +\infty$$

L.H.L = R.H.L

$\therefore$ limit exist at point $x=0$

$$\lim_{x \to 0} f(x) = \infty$$

Continuity of a function



N.B polynomial function always and every where is continious.

Conditions for continuity

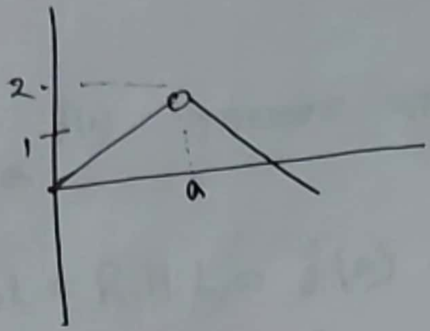
(i) $f(a)$ is defined

(ii) $\lim_{x \to a} f(x)$ exists

(iii) if the function value is equals to limiting value at $x = a$

$\therefore \lim_{x \to a} f(x) = f(a)$

3



Does limit exist or continuous?

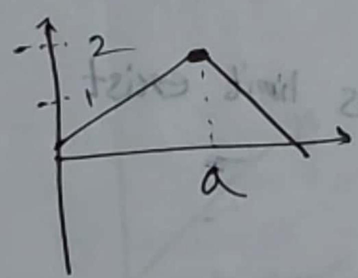
$\lim_{x \to a^-} f(x) = 2$ since $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$

$\lim_{x \to a^+} f(x) = 2$ ∴ limit exist

$f(a) \neq$ any y

that's why not continious

4



Does limit exist and continious?

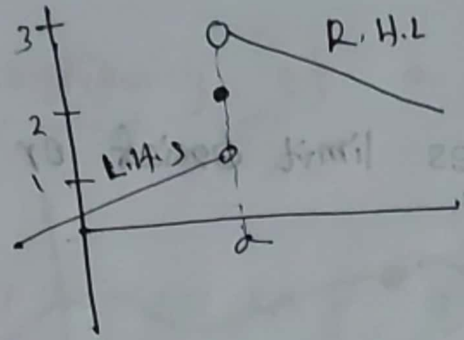
$\lim_{x \to a^-} f(x) = 2$ Hence, $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$

$\lim_{x \to a^+} f(x) = 2$ ∴ limit exist

$f(a) = L$

or $f(a) = \lim_{x \to a^+} f(x) = \lim_{x \to a^-}$ ∴ limit continuo

$\boxed{4}$



$\lim_{x\to a^+} f(x) = 3$, $\lim_{x\to a^-} f(x) = 1$

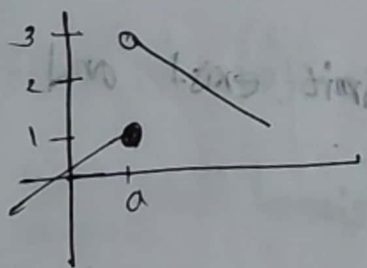
$\therefore \lim_{x\to a^+} f(x) \neq \lim_{x\to a^-}$

Therefore limit does not exist

When $x = a$,

$f(x) = 2$

$\boxed{5}$



Does limit exist.

$\lim_{x\to a^+} f(x) = 3$ $\lim_{x\to a^-} f(x) = 1$

$\lim_{x\to a^+} f(x) \neq \lim_{x\to a^-} f(x)$ $\therefore$ limit Does not exist

N.B|

→ $\lim_{x \to a} f(x)$ means যদি limit থাকে তাহলে তা হবে ?

→ L.H.L = R.H.L = $f(a)$ → continuous

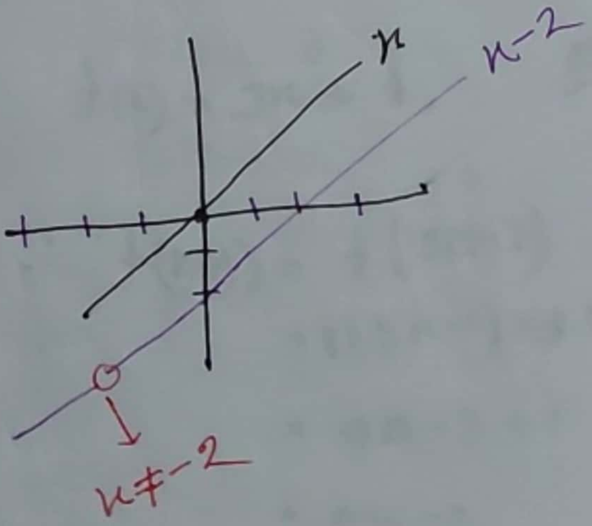
L.H.L = R.H.L $\neq f(a)$ → does not continuous

L.H.L $\neq$ R.H.L = $f(a)$ → " "

$$f(x) = \frac{x^2-4}{x+2}$$ does limit exist ? How ? Continuous?

$$= \frac{(x+2)(x-2)}{x+2} \quad x \neq -2$$

$$= x-2$$



Exercise for inverse function

যদি fog এবং gof এর মান একই আসে তাহলে একে অপরের inverse

1) a) $f(x) = 4x$, $g(x) = \frac{1}{4}x$

$\therefore f(g(x)) = f\left(\frac{1}{4}x\right)$

$= 4\left(\frac{1}{4}x\right)$

$= x$

and $g(f(x)) = g(4x)$

$= \frac{1}{4} \times 4x$

$= x$

Since $f(g(x)) = g(f(x))$

$\therefore$ inverse function of each other

b) $f(x) = 3x + 1$, $g(x) = 3x - 1$

$\therefore f(g(x)) = f(3x-1)$

$= 3(3x-1) + 1$

$= 9x - 3 + 1$

$= 9x - 2$

$g \circ f = g(f(x))$

$= g(3x+1)$

$= 3(3x+1) - 1$

$= 9x + 3 - 1$

$= 9x + 2$

Since $g \circ f \neq f \circ g$

$\therefore$ not inverse

c) $f(x) = \sqrt[3]{x-2}$ $\quad g(x) = x^3 + 2$

$$f \circ g = f(g(x))$$
$$= f(x^3+2)$$
$$= \sqrt[3]{x^3+2-2}$$
$$= x$$

$$g \circ f = g(f(x))$$
$$= g(\sqrt[3]{x-2})$$
$$= (\sqrt[3]{x-2})^3 + 2$$
$$= x-2+2$$
$$= x$$

Since $f \circ g = g \circ f$

$\therefore$ inverse function each other

d) $f(x) = x^4$, $g(x) = \sqrt[4]{x}$

$$f \circ g = f(g(x))$$
$$= f(\sqrt[4]{x})$$
$$= (\sqrt[4]{x})^4$$
$$= x$$

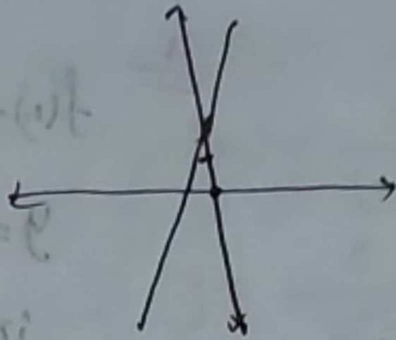
$$g \circ f = g(f(x))$$
$$= g(x^4)$$
$$= (\sqrt[4]{x^4})$$
$$= x$$

Since $f \circ g = g \circ f$

$\therefore$ inverse function each other

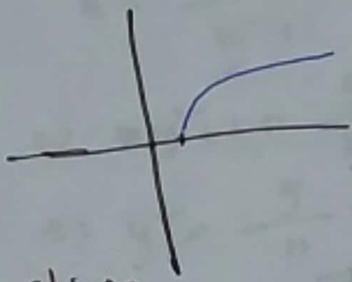
3

(a) $f(x) = 3x+2$



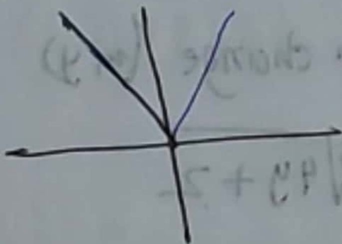
yes one to one function

b) $f(x) = \sqrt{x-1}$



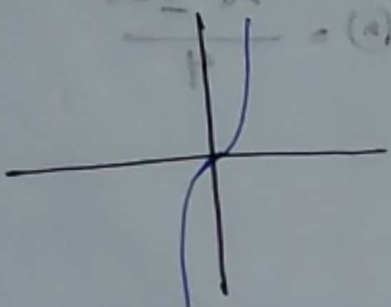
yes one to one function

c) $f(x) = |x|$



not one to one

d) $f(x) = x^3$

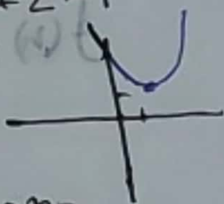


one to one function

e) $f(x) = x^2 - 2x + 2$

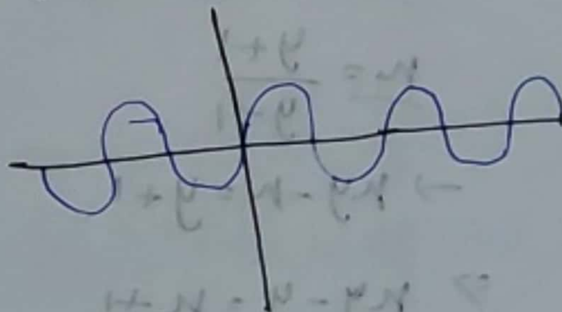
$= x^2 - 2 \cdot x + 1^2 + 2 - 1^2$

$= (x-1)^2 + 1$



not one to one

f) $f(x) = \sin x$



not one to one

9

$f(x) = 7x - 6$

$y = 7x - 6$

inter change (x, y)

$x = 7y \pm 6$

$7y = x + 6$

$y = \frac{x+6}{7}$

$f^{-1}(x) = \frac{x+6}{7}$

11

$f(x) = 3x^3 - 5$

$y = 3x^3 - 5$

inter change (x, y)

$x = 3y^3 - 5$

$3y^3 = x + 5$

$y^3 = \frac{x+5}{3}$

$y = \sqrt[3]{\frac{x+5}{3}}$

$\therefore f^{-1}(x) = \sqrt[3]{\frac{x+5}{3}}$

10

$f(x) = \frac{x+1}{x-1}$

$y = \frac{x+1}{x-1}$

inter chang (x, y)

$x = \frac{y+1}{y-1}$

$\rightarrow xy - x = y + 1$

$\Rightarrow xy - y = x + 1$

$\Rightarrow y(x-1) = x + 1$

$\Rightarrow y = \frac{x+1}{x-1}$

$f^{-1}(x) = \frac{x+1}{x-1}$

12

$f(x) = \sqrt[5]{4x+2}$

$y = \sqrt[5]{4x+2}$

inter change (x, y)

$x = \sqrt[5]{4y+2}$

$\Rightarrow 4y + 2 = x^5$

$\Rightarrow y = \frac{x^5 - 2}{4}$

$\therefore \cancel{y}\ f^{-1}(x) = \frac{x^5 \pm 2}{4}$

13

13 $f(x) = 3/x^2$; $x < 0$

$y = \frac{3}{x^2}$

interchang (x, y)

$x = \frac{3}{y^2}$

$y^2 = \frac{3}{x}$

$y = \pm\sqrt{\frac{3}{x}}$

$f^{-1}(x) = -\sqrt{\frac{3}{x}}$

for condition

~~13~~ 14

$f(x) = \frac{5}{(x^2+1)}$; $x \geq 0$

$y = \frac{5}{x^2+1}$

interchang (x, y)

$x = \frac{5}{y^2+1}$

$\Rightarrow y^2+1 = \frac{5}{x}$

$\Rightarrow y^2 = \frac{5}{x} - 1$

$\Rightarrow y = +\sqrt{\frac{5}{x} - 1}$

for condition

17

$f(x) = (x+2)^4 \quad ; \quad x \geq 0$

$y = (x+2)^4$ — Domain $[0, \alpha)$, Rang $[16, \alpha)$

interchange (x, y)

$x = (y+2)^4$

$y = \sqrt[4]{x} - 2$ $\quad \therefore$ Domain: $f^{-1}(x) = [16, \alpha)$

$f^{-1}(x) = \sqrt[4]{x} - 2$ $\quad$ Range: $f^{-1}(x) = [0, \alpha)$

18

$f(x) = \sqrt{x+3}$ $\quad$ Dom $f(x) = [-3, \alpha)$

$y = \sqrt{x+3}$ $\quad$ ~~Dom~~ Range $f(x) = [0, \alpha)$

inter change

~~y~~ $x = \sqrt{y+3}$ $\quad$ Dom $f^{-1}(x) = [0, \alpha)$

$y = x^2 - 3$ $\quad$ Rang $f^{-1}(x) = [-3, \alpha)$

~~f~~ $f^{-1}(x) = x^2 - 3$

?

~~Domain~~:

CamScanner

19 $f(x) = -\sqrt{3-2x}$

$y = -\sqrt{3-2x}$

inter change (x, y)

$x = -\sqrt{3-2y}$

$\Rightarrow 3 - 2y = x^2$

$\Rightarrow 2y = 3 - x^2$

$\Rightarrow y = \frac{3-x^2}{2}$

$\Rightarrow f^{-1}(x) = \frac{3-x^2}{2}$

Domain $f(x)$ ~~$[\frac{1}{2}, -\alpha, \frac{3}{2}]$~~ $(-\alpha, \frac{3}{2}]$

Range $f(x)$ ~~$[-\alpha, 0)$~~ $(-\alpha, 0]$

Domain $f^{-1}(x) =$ ~~$(-\alpha, 0]$~~ $[-\alpha, 0]$

Rang $f^{-1}(x) = [-\alpha, \frac{3}{2}]$

?

30 $f(x) = x - 5x^2; \quad x \geq 1$

$y = x - 5x^2$

inter change (x, y)

$x = y - 5y^2$

$\Rightarrow y(1-5y) = x$

$\Rightarrow y = \frac{x}{1-5y}$

$f^{-1}(x) = \frac{x}{1-5x}$

Domain $f(x)$: $[1, \alpha)$

Range $f(x)$: $[-\alpha, -4]$

Domain $f^{-1}(x)$: $[-\alpha, -4]$

Range $f^{-1}(x)$: $[1, \alpha)$

Exercise limit and continuo continuity

11) $f(x) = 5x^4 - 3x + 7 \rightarrow$ polynomial function always continuous

Domain: $(-\alpha, \alpha)$

12) $f(x) = \sqrt[3]{x-8} \rightarrow$ continuous $\sqrt[3]{x}$ এর real number কাজে যাবে

Domain $(-\alpha, \alpha)$

~~$x - 8 \neq 0$~~

13) $f(x) = \dfrac{x+2}{x^2+4} \rightarrow$ Square থাকায় এর value positive আসবে তাই continuous

~~$x^2 + 4 \neq 0$~~

Domain: $(\alpha, -\alpha)$

14) $f(x) = \dfrac{x+2}{x^2-4}$

$= \dfrac{x+2}{x^2-2^2}$

~~$= x^2 - 2^2 \neq 0$~~

$= \dfrac{\cancel{x+2}}{(\cancel{x+2})(x-2)}$ $x \neq -2, 2$

$= \dfrac{1}{x-2}$ discontinuous at $x = 2, -2$

$x - 2 \neq 0$

$x \neq 2$

Domain: $\mathbb{R} \rightarrow \{-2, 2\}$

15

$$f(x) = \frac{x}{2x^2+x} \qquad x \neq 0, -\frac{1}{2}$$

$$= \frac{x}{x(2x+1)}$$

$$= \frac{1}{2x+1}$$

$\therefore f(x)$ is discontinuous at $x = 0, -\frac{1}{2}$

$$2x+1 \neq 0$$

$$x = -\frac{1}{2}$$

Domain: $\mathbb{R} \to \left\{0, -\frac{1}{2}\right\}$

N.B

আগে Domain বের করতে হবে যদি Restriction থাকে তাহলে Discontinuous

16 ?

17 $f(x) = \frac{3}{x} + \frac{x-1}{x^2-1}$; $x \neq 0, \pm 1$

$x^2 - 1 \neq 0$

$x \neq 1$ Discontinuous at $x = 0, \pm 1$

$x = \pm\sqrt{1}$

Domain $\mathbb{R} \to \{0, -1, 1\}$

18 $f(x) = \frac{5}{x} + \frac{2x}{x+4}$; $x \neq 0, -4$

$x + 4 \neq 0$

$x \neq -4$ Discontinuous at, $x = 0, -4$,

Domain $\mathbb{R} \to \{0, -4\}$

19 $f(x) = \frac{x^2 + 6x + 9}{|x| + 3}$

Domain: $\mathbb{R}$ $\therefore$ continuous

20/ $f(x) = \left| 4 - \frac{8}{x^4 + x} \right|$ $x \neq 0, -1$

$\therefore$ Discontinuous at $x = 0, -1$

$x^4 + x \neq 0$

$x(x^3 + 1) \neq 0$

Domain $\mathbb{R} \to \{0, -1\}$

২। N.B| Piecewise function এর ক্ষেত্রে condition এর common value ~~টা~~ function এ বসালে যদি একই value আসে তাহলে continuous.

$$f(x) = \begin{cases} 2x+3; & x \leq 4 \\ 7+\frac{16}{x}; & x>4 \end{cases}$$

$f(x) = 2x+3$
$= 2\times 4+3$
$= 11$

$f(x) = 7+\frac{16}{x}$
$= 7+\frac{16}{4}$
$= 11$

or Domain - $\mathbb{R}$
$\therefore$ continuous

$\therefore$ continuous

Ex:

$$f(x) = \begin{cases} 3x+8; & x \leq 8 \\ 3+\frac{11}{x} & x>8 \end{cases}$$

?

$f(x) = 3x+8$
$= 3\times 8+8$
$= 32$

$f(x) = 3+\frac{11}{x}$
$= 3+\frac{11}{8}$
$= \frac{32+11}{8}$
$= \frac{39}{8}$

Discontinuous

Ex:

$$f(x) = \begin{cases} x+1; & 0 < x < 3 \\ x-1; & x > 4 \end{cases}$$

Dis continuous

কারণ 3 এবং 4 এর মধ্যে কোনো x এর value দেওয়া হয় নাই।

1.3 Assymptote line

⊞ Horizontal Assymptote

→ $\lim_{x \to -\infty} f(x) =$ যা হবে তাই A Horizontal Assymptote line

→ $\lim_{x \to +\infty} f(x) =$ যা হবে তাই Horizontal Assymptote line [আলাদা আলাদা মান হইলে Assymptote আলাদা আলাদা হবে।]

→ $\boxed{\text{Except } [\infty, \infty]}$

⊞ vertical Assymptote

→ প্রথমে Domain বের করতে হবে। Domain এ যা Restrict থাকবে তা দিয়ে Left and Right hand limit বের করলে তা যদি $\pm\infty$ আসে তাহলে ঐ Restrict point টাই vertical Assymptote line

⊛ $f(x) = \frac{1}{x}$

→ Horizontal

$\lim_{x \to \infty^-} f(x) = 0$

$\lim_{x \to \infty^+} f(x) = 0$

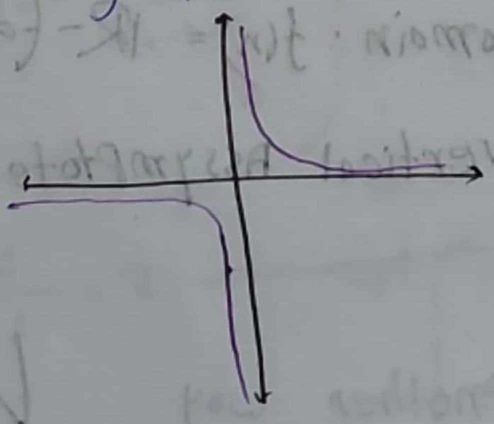
Horizontal Assymptote: $y = 0$

Vertical

Domain ↔ $f(x) \Rightarrow R \rightarrow \{0\}$

∴ vertical Assymptote: $x = 0$

Another way to find



Horizontal $y = 0$

vertical $x = 0$

(c) $y = f(x) = 2x + 7$

Horizontal

$$\lim_{x \to \infty^+} f(x) = \infty$$

$$\lim_{x \to \infty^-} f(x) = -\infty$$

$\therefore$ Horizontal Assympmote নাই

vertical

Domain: $\mathbb{R} \to (-\infty, \infty)$

vertical Assymptote নাই

Somple question 29

a) $y = f(x) = x + 7$

Horizontal

$$\lim_{x \to \infty^+} f(x) = \infty$$

$$\lim_{x \to \infty^-} f(x) = -\infty$$

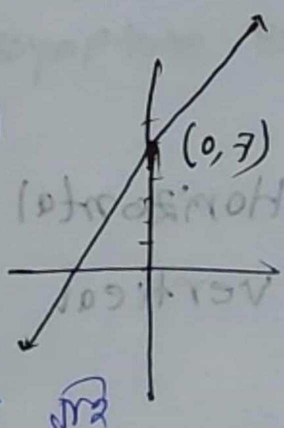
Horizontal Assympatote নাই

vertical

Domain: $f(x) = \mathbb{R} - (-\infty, \infty)$

vertical Assymptote নাই

Another way



Assymptote নাই

b) $f(x) = \frac{2}{x-3}$

$\left[\begin{array}{l} x = \text{x হর যার জন্য 0 হবে} \\ y = \dfrac{\text{~~হ~~ লবের x এর সহগ}}{\text{~~লবের~~ হরের x এর সহগ}} \end{array}\right.$

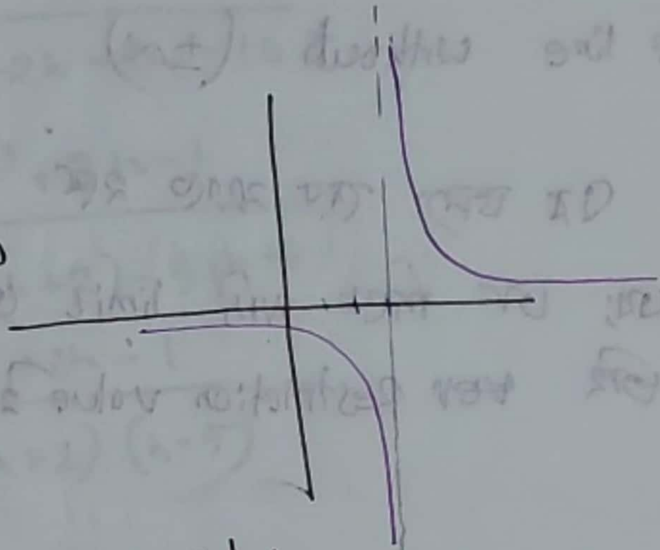
~~Horizontal~~

$\cancel{\lim_{x \to \alpha} x}$

$x = 3$

$y = \frac{0}{\cancel{-}1} = 0$

$\to \cancel{0}$



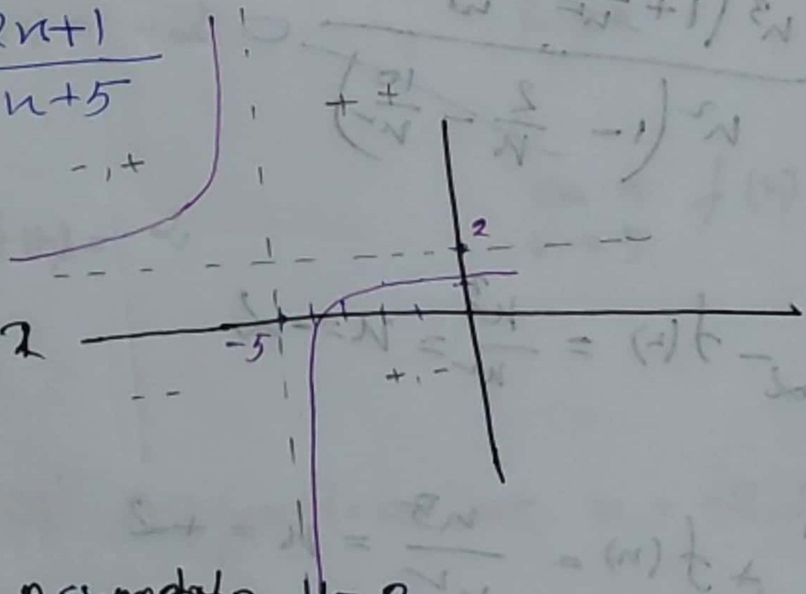
Horizontal Assymptote $y = 0$

vertical Assymptote $x = 3$

c) $f(x) = \frac{2x+1}{x+5}$

$x = -5$

$y = \frac{2}{1} = 2$



Horizontal Assymptote $y = 2$

vertical Assymptote $x = -5$

d) → Horizonta Assymptote বের করার জন্য x এর মান অসীম ধরে নিতে হবে

→ $\lim\limits_{x \to \mp\alpha} f(x)$ এর যা value আসবে তাই Horizontal Assymptote line without $(\pm\alpha)$

→ vertical এর জন্য বের করতে হবে Domain এর Restriction এর; তা দিয়ে যদি limit এর value $\neq \pm\alpha$) আসে তাহলে তাই ~~ver~~ Restriction value ই vertical Assymptote

for Horizontal

$$y = \frac{x^3+2x-1}{x^2-2x-10}$$

$$= \frac{x^3\left(1+\cancel{\frac{2}{x^2}}-\cancel{\frac{1}{x^3}}\right)}{x^2\left(1-\cancel{\frac{2}{x}}-\cancel{\frac{10}{x^2}}\right)}$$

$$\lim_{x \to \alpha^-} f(x) = \frac{x^3}{x^2} = x = -\alpha$$

$$\lim_{x \to \alpha^+} f(x) = \frac{x^3}{x^2} = x = +\alpha$$

$\therefore$ Horizontal Assymptote line নাই

⊗ for vertical Assymptote

$$y = \frac{x^3+2x-1}{x^2-3x-10}$$

$$= \frac{x^3+2x-1}{x^2+2x-5x-10}$$

$$= \frac{x^3+2x-1}{x(x+2)-5(x+2)}$$

$$= \frac{x^3+2x-1}{(x+2)(x-5)}$$

$\therefore x \neq -2, 5$

$\lim\limits_{x \to -2^-} f(x) = -\alpha$

$\lim\limits_{x \to -2^+} f(x) = \alpha$

$\lim\limits_{x \to 5^-} f(x) = -\alpha$

$\lim\limits_{x \to 5^+} f(x) = \alpha$

$\therefore$ vertical Assymptote line $x = -2, 5$

United International University
School of Science and Engineering
CT–01, Fall 2023
Course Code : MATH 1151, Sec: L
Course Title: Fundamental Calculus

Answer all questions

1. Use the given graph (Figure 1) of $y = f(x)$ to sketch the following functions:

   (a) $y = |f(x)| - 1$; $\quad y = \frac{1}{2}f(x+1)$; $\quad y = f(2x) - 1$. [6]

   (b) The given graph (Figure 1) of the function is defined for $x \geq 0$. Complete the graph for $x < 0$ to make it as (a) an odd function, and (b) an even function. [2]

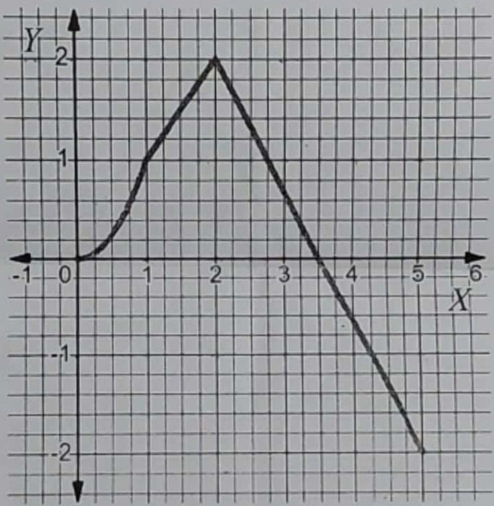


Figure 1:

2. Find the domain and range of the following function. [2]

$$y = \sqrt{x^2 - 4x - 5}$$

Best of Luck

United International University

School of Science and Engineering

Mid Term Examination   Trimester: Fall-2023

Course Title: Fundamental Calculus

Course Code: Math 1151   Marks: 30   Time: 1 Hour 45 Mins

Answer all the questions. Answer all parts of a question together.

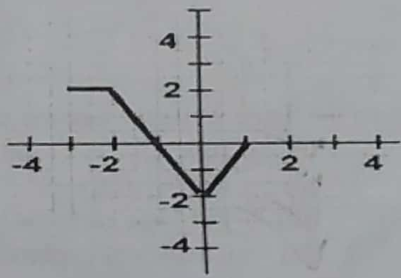
1\. (a) **Draw** the graph of the following functions and **find** their domain and range. [5]

(i) $y = 2 - 2x - x^2$ (ii) $y = 1 - \sin 2x$

(b) The graph of $f(x)$ is given. Use it to **sketch** the graph of the following functions. [5]

(i) $2 + f(1 - x)$

(ii) $1 - 2\left|f\left(\frac{x}{2}\right)\right|$ $-h+2=0$ $h=-1$



2\. (a) Evaluate $f(-5)$, $f(-1)$, and $f(3)$ for the piecewise defined function. Then **sketch** the graph of the function. [3]

$$f(x) = \begin{cases} 2; & x < -3 \\ \sqrt{9 - x^2}; & -3 \le x < 3 \\ x - 2; & x \ge 3 \end{cases}$$

(b) Use the table to evaluate the following expressions: [3]

x	-3	-1	2	5
$f(x)$	7	2	-1	-1
$g(x)$	9	3	-8	2

(i) $(f \circ g)(-1)$ (ii) $(g \circ g)(5)$ (iii) $(g \circ f)(2)$ -3

(c) The graph of $f(x)$ is given. [4]

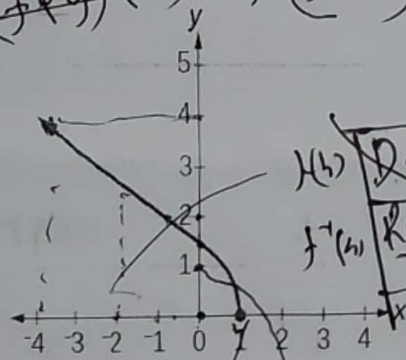
(i) **Determine** whether $f(x)$ is one to one function, or not.

(ii) **Complete** the following table.

x	0	2	4
$f^{-1}(x)$	3/2	5/2	-4

(iii) **Sketch** the graph of $f^{-1}(x)$ along with $f(x)$.

(iv) **What** is the domain and range of $f^{-1}(x)$?



$f(g(-1))$

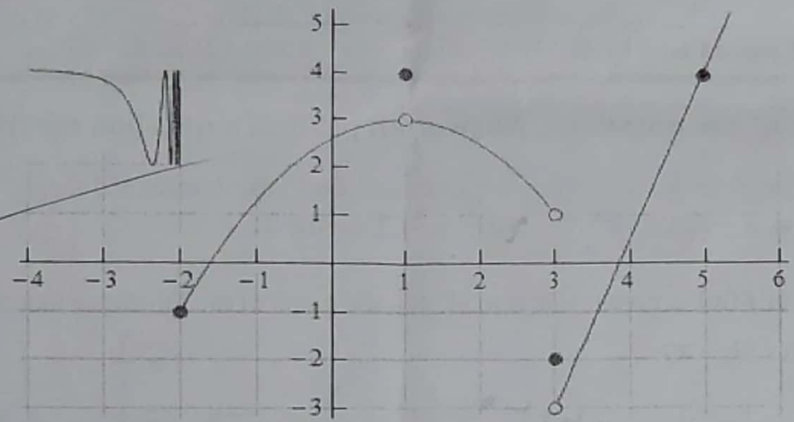
Please Turn Over

$u = b^y$

$y = b^x$

3. (a) **Find** the inverse of $f(x) = 2 + 3^{-x}$, **draw** the graph of $f(x)$ and **its inverse** in the same diagram. Also, **state** the domain and range of the inverse function. [3]

(b) The graph of the function $y = f(x)$ is given. [5]



From the figure **write** the answers of the following questions:

(i) $\lim_{x\to-2^-} f(x)$ and $\lim_{x\to3^+} f(x)$.

(ii) $\lim_{x\to1} f(x)$.

(iii) $f(1)$ and **state** the horizontal asymptote(s) of $f(x)$.

(iv) **Check** and **explain** the continuity of $f(x)$ at $x = -2$ and 5.

(c) **Find** value of the constant k, if possible, that will make the function $f(x)$ continuous everywhere. [2]

$$f(x) = \begin{cases} x^3 + 2k; & x > -1 \\ x + 5; & x \le -1 \end{cases}$$

$x^3 + 5$

$\lim_{x\to-1^-} x+5 = 4$

$\lim_{x\to-1^+} = 1+2k$

$1 + 2k = 4$

$2k = 3$

BEST OF LUCK!!!

a) $2^n - 1$
b) 2^{n-1}
c) -2^n
d) 2^{-n}

United International University
School of Science and Engineering
CT–02, Fall 2023
Course Code : MATH 1151, Sec: **L**
Course Title: Fundamental Calculus

Answer all questions

1. Starting with the graph of $y = 2^x$, write the equation of the graph that results from
   (a) shifting 1 units downward. $2^n - 1$ [1]
   (b) shifting 1 units to the right. [1]
   (c) reflecting about the x-axis. [1]
   (d) reflecting about the y-axis. [1]

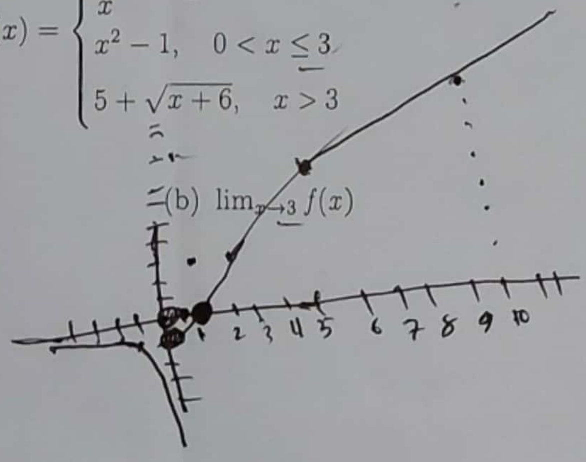
2. Find the inverse of the function. Draw the graph of the function and its inverse in the same diagram. Also, state the domain and range of the inverse. [4]

   $f(x) = 2 + e^{-x}$

3. Consider the following function and find [2]

$$f(x) = \begin{cases} \dfrac{1}{x}, & x \le 0 \\ x^2 - 1, & 0 < x \le 3 \\ 5 + \sqrt{x+6}, & x > 3 \end{cases}$$

   (a) $\lim_{x \to 0} f(x)$ (b) $\lim_{x \to 3} f(x)$



Best of Luck