Name:	ID:

CSE 2213: Discrete Math

Class Test 04

Section H

Time: 25 Minutes

Total Marks: 20

1. a. How many ways can you form a sequence of 4 character, consisting of any of uppercase letters, lowercase letters or hexadecimal numbers such that no two characters are repeated? (2.5)

total = 26 + 26 + 16 = 68

∴ total ways = 68 × 67 × 66 × 65 × 64

= 1250895360 (Ans.)

b. There are n1 computer science courses and n2 computer engineering courses available at a certain university. A student has to select r1 courses from computer science courses and r2 courses from computer engineering courses. If the order of the courses taken are important, then how many ways can a student complete the courses? **[r1 = last digit of our id, n1 = 2*r1, r2 = 2nd last digit of your id, n2 = 2*r2]** (2.5)

let, $n_1 = 6$, $r_1 = 3$, $n_2 = 2$, $r_1 = 1$

Total ways = (6 × 5 × 4) × (2)

= 240 (Ans.)

2. Five cards are dealt off of a standard 52-card deck and lined up in a row. (5)
   a. How many such line-ups are there that have at least one black card?
   b. How many such line-ups are there in which the cards are either all red or all spades?

a) All possible lineups − num of lineups that don't have any black card (all red)

= (52×51×50×49×48) − (26×25×24×23×22) = 303981600 (Ans)

b) all ~~black~~ red + all ~~hearts~~ spades = 7893600 + (13×12×11×10×9)

= 8048040 (Ans.)

3. a) In Discrete Mathematics course 10 students got A, 8 students got B, 7 students got C and 9 students got D. A student is selected at random without knowing. (A, B, C, D total four grades)
   a. How many students must be selected to be sure of having at least two students of the same grade? b. How many students must be selected to be sure of having at least four C grades? (5)

a) $\lceil \frac{N}{k} \rceil = m$

$\Rightarrow \lceil \frac{N}{4} \rceil = 2$

$\Rightarrow 1 < \frac{N}{4} \leq 2$

$\Rightarrow 4 < N \leq 8$

$\therefore N = 5$

(Ans.)

b) not a pigeonhole principle.

other grades = ~~10~~ 8 + 7 + 10 students = 25

After 25 students if another 4 students are picked up, the criteria is fulfilled

∴ Total = 25 + 4 = 29 students (Ans.)

4. Use induction to establish the following identity for any integer $n \geq 1$: (5)

$1-3+9-\dots+(-3)^n = \frac{1-(-3)^{n+1}}{4}$

5. Use induction to show that, $1+\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\dots+\frac{1}{3^n}=\frac{3}{2}(1-\frac{1}{3^{n+1}})$ for all positive integers n. (Bonus Marks: 3)

4. Basis case: $n=0$

$L.H.S = (-3)^0 = 1$

$R.H.S = \frac{1-(-3)^{0+1}}{4}$

$= \frac{1-(-3)}{4}$

$= 1$

$\therefore L.H.S = R.H.S$

Induction case: let, $n=k$

$1-3+9-\dots+(-3)^k = \frac{1-(-3)^{k+1}}{4}$ — (I)

for $n=k+1$,

$1-3+9-\dots+(-3)^{k+1} = \frac{1-(-3)^{k+2}}{4}$ — (II)

adding $(-3)^{k+1}$ on both sides of (I),

$1-3+9-\dots+(-3)^k+(-3)^{k+1} = \frac{1-(-3)^{k+1}}{4} + (-3)^{k+1}$

$= \frac{1}{4} - \frac{(-3)^{k+1}}{4} + (-3)^{k+1}$

$= \frac{1}{4} - (-3)^{k+1}\left[\frac{1}{4}-1\right]$

$= \frac{1}{4} - (-3)^{k+1}\left(-\frac{3}{4}\right)$

$= \frac{1}{4} - \frac{(-3)^{k+1+1}}{4}$

$= \frac{1-(-3)^{k+2}}{4}$

which is eqn (II)

[Proved)

5. Basis case; $n=0$, $L.H.S = \frac{1}{3^0} = 1$

$R.H.S = \frac{3}{2}\left(1-\frac{1}{3^{0+1}}\right) = \frac{3}{2}\times\frac{3-1}{3} = \frac{\cancel{3}}{\cancel{2}}\times\frac{\cancel{2}}{\cancel{3}} =$

$\therefore L.H.S = R.H.S$

Induction case: let, $n=k$,

$1+\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\dots+\frac{1}{3^k} = \frac{3}{2}\left(1-\frac{1}{3^{k+1}}\right)$ — (I)

for $n=k+1$,

$1+\frac{1}{3}+\frac{1}{3^2}+\dots+\frac{1}{3^{k+1}} = \frac{3}{2}\left(1-\frac{1}{3^{k+2}}\right)$ — (II)

adding $\frac{1}{3^{k+1}}$ on both sides of (I),

$1+\frac{1}{3}+\frac{1}{3^2}+\dots+\frac{1}{3^k}+\frac{1}{3^{k+1}} = \frac{3}{2}\left(1-\frac{1}{3^{k+1}}\right)+\frac{1}{3^{k+1}}$

$= \frac{3}{2} - \frac{3}{2.3^{k+1}} + \frac{1}{3^{k+1}}$

$= \frac{3}{2} - \frac{1}{2.3^{k+1-1}} + \frac{1}{3^{k+1}}$

$= \frac{3}{2} - \frac{1}{2.3^k} + \frac{1}{3^k.3}$

$= \frac{3}{2} - \frac{1}{2.3^k}\left(\frac{\cancel{2}}{\cancel{2}}1 - \frac{2}{3}\right)$

$= \frac{3}{2} - \frac{1}{2.3^k}\left(\frac{3-2}{3}\right)$

$= \frac{3}{2} - \frac{1}{2.3^k}\cdot\frac{1}{3}$

$= \frac{3}{2} - \frac{1}{2.3^{k+1}}$

$= \frac{3}{2} - \frac{3}{2.3^{k+1+1}}$

$= \frac{3}{2}\left(1-\frac{1}{3^{k+2}}\right)$ = eqn (II)

[Proved)