Cubic bezier Curve Eqⁿ.

Unit 3 CGM

Curves.

A curve is a set of point that are connected to form a smooth line.

Curve Representation:

① **Explicit**: It is type of mathematical curve representation where y is expressed as a direct function of x.

$$\boxed{y = f(x).}$$

ex:- Straight line,

$$y = mx + c$$

② **Implicit**: Define the set of points on a curve by employing a procedure that can test to see if a point is on the curve..

$$\boxed{f(x, y) = 0}$$

ex- circle

③ **Parametric**: Curves having parametric form are called parametric curves.

A parametric curve is represented using a parameter t, where both x & y are expressed as functions of t.

$$x = f(t), \quad y = g(t)$$

where t varies over a given range

Surface and design:

Surfaces in computer graphics are the 3D equivalents of curves and are represented using mathematical functions. Common surfaces types include.

- Parametric surface: Defined using two parameters (u, v)

$$x = f(u, v) \qquad y = g(u, v)$$

- Implicit surface: Defined as $f(x, y, z) = 0$ - like spheres.

Designs: Involve the creation of smooth and complex surfaces using techniques like Bezier and B-spline surfaces.

Bezier Curve:

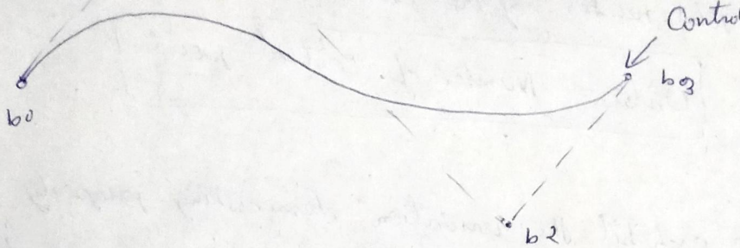
It is a parametric curve defined by a set of control points to be approximated and their relative position determine shape of curve.

The no. of control points determine the degree of Bezier polynomial.

→ Two points are ends of curve. Other points determine shape of curve.

→ Bezier curve generally follows the shape of the defining polygon.

→ No straight line intersects a bezein curve more times than it intersects it's control polygon.

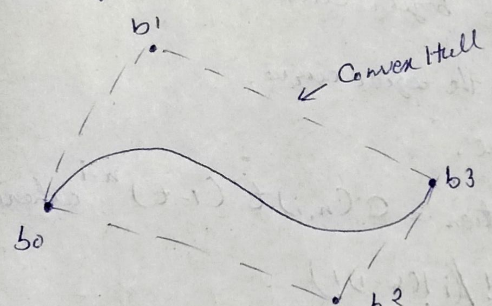


Here,

- The bezier curve is defined by a set of control points b_0, b_1, b_2 and b_3.
- Points ~~b~~ b_0 and b_3 are ends of curve.
- Points b_1 and b_2 determine the shape of the curve.

Properties of Bezier curve :

① Bezier curve is always contained within a polygon called as convex hull of its control points.



- Bezier curve with convex hull.

② Bezier curve generally follows the shape of its defining polygon.

- The first and last points of the curve are coincident with the first and last point of defining polygon.

③ The degree of ~~polygon~~ polynomial defining the curve segment is one less than the total number of control points.

Degree = no. of control points − 1.

(4) The Order of the polynomial defining the curve segment is equal to the total number of the control points.

$\boxed{\text{Order} = \text{Number of control points}}$

(5) Bezier curve exhibit the variation deminishing property.

- It means the curve do not oscillate about any straight line more often than the defining polygon.

BC Eqn:

$$\boxed{P(t) = \sum_{i=0}^{n} B_i J_{n,i}(t)}$$

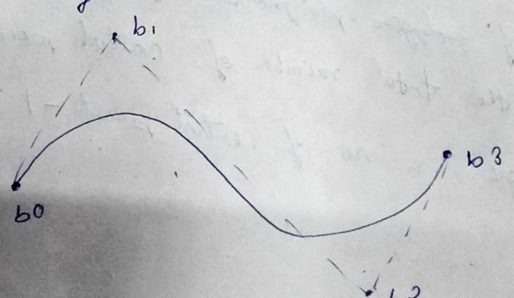
- t is any parameter where $0 <= t <= 1$
- $P(t)$ = Any pt lying on bezier curve.
- B_i = i^{th} control point of the bezier curve.
- n = degree of the curve.
- $J_{n,i}(t)$ = Blending function = $C(n,i) t^i (1-t)^{n-i}$ where

$$C(n,i) = n! / (i!(n-i)!)$$

Cubic Bezier Curve:

Cubic bezier curve is a curve with degree 3.

- The total number of control pts in a cubic bezier curve is 4.



Cubic bezier Curve.

Cubic bezier curve eq^n.

$\Rightarrow$ $P(t) = \sum_{i=0}^{n} B_i J_{n,i}(t)$

$J_{n,i}(t) = C(n,i)\, t^i (1-t)^{n-i}$

$$\boxed{J_{n,i}(t) = \frac{n!}{i!(n-i)!} t^i (1-t)^{n-i}}$$

For cubic bezier curve:

$n = 3$.

$\Rightarrow$ $P(t) = \sum_{i=0}^{3} B_i J_{3,i}(t)$

Expanding the above eq^n.

$P(t) = B_0 J_{3,0}(t) + B_1 J_{3,1}(t) + B_2 J_{3,2}(t) + B_3 J_{3,3}(t)$ .... ①

Now,

$$J_{3,0}(t) = \frac{3!}{0!(3-0)!} t^0 (1-t)^{3-0} = (1-t)^3 \text{ --- } ②$$

$$J_{3,1}(t) = \frac{3!}{1!(3-1)!} t^1 (1-t)^{3-1} = 3t(1-t)^2 \text{ --- } ③$$

$$J_{3,2}(t) = \frac{3!}{2!(3-2)!} t^2 (1-t)^{3-2} = 3t^2(1-t)^2 \text{ --- } ④$$

$$J_{3,3}(t) = \frac{3!}{3!(3-3)!} t^3 (1-t)^{3-3} = t^3 \text{ --- } ⑤$$

Using 2, 3, 4, 5 in 1 we get.

$$P(t) = B_0 (1-t)^3 + B_1\, 3t(1-t)^2 + B_2\, 3t(1-t)^3 + B_3 t^3$$

This is req parametric eq^n for cubic bezier curve.