Unit II CGM

2D Translation:

Process of moving an object from one position to another in two dimensional plane.

$$x' = x + T_x$$
$$y' = y + T_y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \end{bmatrix}$$

$$\begin{bmatrix} x' & y' \end{bmatrix} = \begin{bmatrix} x & y \end{bmatrix} + \begin{bmatrix} T_x & T_y \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & T_x \\ 0 & 1 & T_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

2D Rotation:

In Anticlockwise:

$$x' = r\cos(\phi + \theta)$$
$$= x\cos\theta - y\sin\theta$$

$$y' = r\sin(\phi + \theta)$$
$$= r\sin\phi\cos\theta + r\cos\phi\sin\theta$$
$$= x\sin\theta + y\cos\theta$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x\cos\theta - y\sin\theta \\ x\sin\theta + y\cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Row major.

$$\begin{bmatrix} x' & y' \end{bmatrix} = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

Clockwise rotation -

$x' = r\cos(\phi - \theta)$

$= r\cos\phi\cos\theta + r\sin\phi\sin\theta$

$[x' = x\cos\theta + y\sin\theta]$

$y' = r\sin(\phi - \theta)$

$= r\sin\phi\cos\theta - r\cos\phi\sin\theta$

$[y' = y\cos\theta - x\sin\theta]$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} x' & y' \end{bmatrix} \quad \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Scaling

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

~~$[x' \; y'] = [x \; y]$ [~~

Reflection :

X axis

$x' = x$

$y' = -y$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Y axis

$x' = -x$

$y' = y$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Shearing :

x Axis

$x' = x + S_x y$

$y' = y$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & S_x \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ S_x & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

y Axis

$x' = x$

$y' = y + S_y x$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ S_y & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ S_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Euclidean VS Homogeneous

Euclidean coordinates represent points in a standard Cartesian space.

Homogeneous coordinates add extra dimension.

In EC a point in 2D space is represented by (x, y) while HC, it would be represented by (x, y, ω) where 'ω' is the additional 'weight' value.

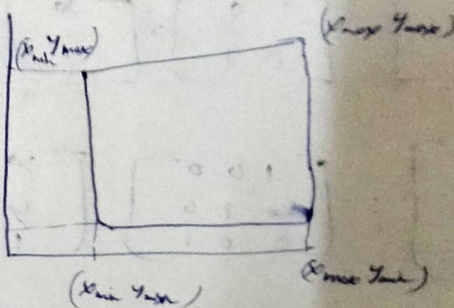
Point Clipping:

* Clipping in computer graphics is the process of removing parts of image, object, or signal that are outside of a specified viewing area.

⇒

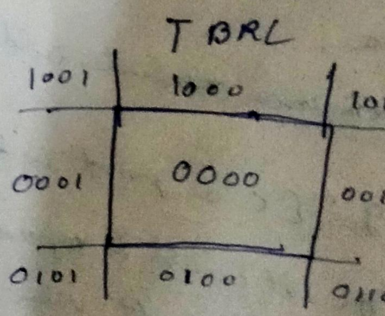
$X_{min} \le X \le X_{max}$

$Y_{min} \le X \le Y_{max}$.



Line Clipping:

Cohen Sutherland Algo:



① Line Inside.

A & B

0000

② Line Completely outside

A 0001
B 0001
———
0001

Logical And

$!= 0000$

⇓

Completely outside

③ Partially inside.

A 0001
B 0000
———
0000 → partially inside.

Find intersection pt.

Left = $x = x_{min}$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{y - y_1}{x_m - x_1}$$

$$y - y_1 = m(x_{min} - x_1)$$

$$y = y_1 + m(x_{min} - x_1)$$

Right

$$y = y_1 + m(x_{max} - x_1)$$

Top = $y = y_{max}$

$$m = \frac{y - y_1}{x - x_1} = \frac{y_{max} - y_1}{x - x_1}$$

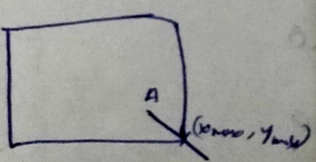
$$x = x_1 + (y_{max} - y_1)/m$$

Bottom

$y = y_{min}$

$$x = x_1 + (y_{min} - y_1)/m$$

Corner



$A(x, y)$

$A'(x_{max}, y_{min})$

~~Liang Barsky Line Clipping Algo~~

3D Translation-

$x' = x + T_x$

$y' = y + T_y$

$z' = z + T_z$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

3D Rotation-

X Axis Rotation-

$x' = x$

$y' = y \cos\theta - z \sin\theta$

$z' = y \sin\theta + z \cos\theta$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Y Axis Rotation

$x' = z \sin\theta + x \cos\theta$

$y' = y.$

$z = y \cos\theta - x \sin\theta$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Z Axis

$x' = x\cos\theta - y\sin\theta$

$y' = x\sin\theta + y\cos\theta$

$z' = z$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Scaling

$x' = x\, S_x$

$y' = y\, S_y$

$z' = z\, S_z$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Reflection

xy

$x' = x$

$y' = y$

$z' = -z$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

yz

$x' = -x$

$y' = y$

$z' = z$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

xz

$x' = x, \; y' = -y, \; z' = z$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

3D viewing pipeline

Modeling Coordinates → Modeling Transformation → World Coordinates → Viewing Transformation → Viewing Coordinates → Projection Coordinates Transf → Projection Coordinates → Workstation Transformation → Device Coordinates

Projection

Parallel Projection

Here the projection rays (lines from object to the projection plane) are parallel to each other

- Orthographic → Projection lines are perpendicular to projection plane.
- Oblique → Projection lines are at an angle to the projection plane.
  - Cavalier → Preserves original length
  - Cabinet → Reduces depth scale.

Perspective Projection.

. Here the projection rays converge at a single pt called projection center.

Axonometric projection is a type of // projection where the object is rotated along one or more of its axes to reveal multiple sides of it in a single view.

Types

1. Isometric Projection

- All three axes (x, y, z) are equally inclined to the projection plane.

2. Dimetric projection.

- Two of three axes ~~are~~ have equal foreshortening, while the third axis has a different foreshortening ratio.

3. Trimetric.

All three axes have different foreshortening ratios.