Karnaugh Maps (K-Map)

- For simplifying Boolean expressions, the K-map does not require the knowledge of theorems of BA.
- K-Map involves less numbers of steps in simplification process of logical expression as compared to other simplification techniques.

(i) Two variable maps:

x \ y	0	1
0	m_0 $x'y'$	m_1 $x'y$
1	m_2 xy'	m_3 xy

(ii) Three variable map

x \ yz	00	01	11	10
0	$x'y'z'$ m_0	$x'y'z$ m_1	$x'yz$ m_3	$x'yz'$ m_2
1	$xy'z'$ m_4	$xy'z$ m_5	xyz m_7	xyz' m_6

Simplify -

$F(x,y,z) = \Sigma(2,3,4,5)$

x \ yz	00	01	11	10
0			1	1
1	1	1		

$F = x'y + xy'$

$F(x,y,z) = \Sigma(0,2,4,5,6)$

x \ yz	00	01	11	10
0	1			1
1	1	1	1	1

$F = x'z' + yz' + xy' + xz$

$F(x,y,z) = \Sigma(0,2,4,5,6)$

x \ yz	00	01	11	10
0	1			1
1	1	1		1

$F = xy' + z'$

$F = A'C + A'B + AB'C + BC$ (a) Express it in Sum of minterms

(b) Find the minimal sum of product exp.

Sol:-

A \ BC	00	01	11	10
0		1	1	1
1		1	1	

(b) $F = C + A'B$

(a) $F = \Sigma(1, 2, 3, 5, 7)$

Four Variable MAP

	00	01	11	10
00	m_0	m_1	m_3	m_2
01	m_4	m_5	m_7	m_6
11	m_{12}	m_{13}	m_{15}	m_{14}
10	m_8	m_9	m_{11}	m_{10}

Simplify -

$F(w,x,y,z) = \Sigma(0,1,2,4,5,6,8,9,12,13,14)$

wx \ yz	00	01	11	10
00	1	1		1
01	1	1		1
11	1	1		1
10	1	1		

$F = y' + w'z' + xz'$

$F = A'B'C' + B'CD' + A'BCD' + AB'C'$

AB \ CD	00	01	11	10
00	1	1		1
01				1
11				
10	1	1		1

$F = A'B'C' + A'CD' + AB'C' + B'D'$

$= B'C' + A'CD' + B'D'$

Five Variable MAP

	A = 0				A = 1			
BC \ DE	00	01	11	10	00	01	11	10
00	0	1	3	2	16	17	19	18
01	4	5	7	6	20	21	23	22
11	12	13	15	14	28	29	31	30
10	8	9	11	10	24	25	27	26

$F(ABCDE) = (0, 2, 4, 6, 9, 13, 21, 23, 25, 29, 31)$

A = 0

BC \ DE	00	01	11	10
00	1			1
01	1		9	1
11		1		
10		1		

A = 1

BC \ DE	00	01	11	10
00				
01		1	1	1
11		1	1	
10		1		

$F = A'B'E' + A'BD'E + ACE + ABD'E$

$= A'B'E' + BD'E + ACE$

Don't Care Condition in K-MAP

$f(A, B, C, D) = \Sigma m(0, 1, 4, 5, 6, 10, 11, 13) + d(2, 3)$

Sol.

AB \ CD	00	01	11	10
00	1	1	x	x
01	1			1
11		1		
10			1	1

$F = A'C' + BC'D + A'D' + B'C$

$f(A, B, C, D) = \pi M(1, 5, 6, 12, 13, 14) + d(2, 4)$

⇒

AB \ CD	00	01	11	10
00		0		X
01	X	0	X	0
11	0	0		0
10				

~~$F = B'C' + AC'D + ACD' + BD'$~~

$F = (B' \cdot C) + (A'C'D) + (A'C D') + (BD')$

$= (B + \bar{C})(A + C + \bar{D})\cancel{(A + \bar{C} + D)}(\bar{B} + D)$

$= (B + \bar{C})(A + C + \bar{D})(\bar{B} + D)$

Quine-McCluskey Tabular Method

$$F(A,B,C,D) = \Sigma_m(0, 1, 3, 7, 8, 9, 11, 15)$$

Step 1. Represent Given number in binary.

N	A B C D
0	0 0 0 0 ✓
1	0 0 0 1 ✓
3	0 0 1 1 ✓
7	0 1 1 1 ✓
8	1 0 0 0 ✓
9	1 0 0 1 ✓
11	1 0 1 1
15	1 1 1 1

Step 2 - Form a group based on Number

group	Term	A B C D
group-0	(0)	0 0 0 0 ✓
group-1	(1)	0 0 0 1 ✓
	(8)	1 0 0 0 ✓
group-2	(3)	0 0 1 1 ✓
	(9)	1 0 0 1 ✓
group-3	(7)	0 1 1 1 ✓
	(11)	1 0 1 1 ✓
group 4	(15)	1 1 1 1 ✓

Step 3. Find Matched pair with one bit difference & mark location.

group	Pair	A B C D
group 0	(0-1)	0 0 0 – ✓
	(0-8)	– 0 0 0 ✓
group 1	(1-3)	0 0 – 1 ✓
	(1-9)	– 0 0 1 ✓
	(8-9)	1 0 0 – ✓
group 2	(3-7)	0 – 1 1
	(3-11)	– 0 1 1 ✓
	(9-11)	1 0 – 1 ✓
group 3	(7-15)	– 1 1 1 ✓
	(11-15)	1 – 1 1 ✓

Step 4 - Repeat step 3 till end.

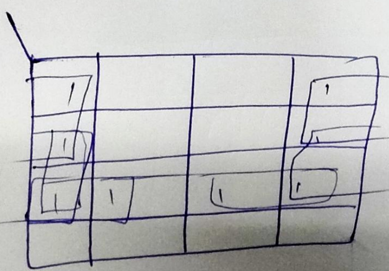
group	Pair	A B C D
group 0	(0-1-8-9)	- 0 0 -
$\bar{B}\bar{C}$	(0-8-1-9)	- 0 0 -
group 1	(1-3-9-11)	- 0 - 1
$\bar{B}D$	(1-9-3-11)	- 0 - 1
	(	
group 2	(3-7-11-15)	- - 1 1
CD	(3-11-7-15)	- - 1 1

Step 5 - Implicant Table

PI	Minterms	0	1	3	7	8	9	11	15
$\bar{B}\bar{C}$	0+8-9	x	x			x	x		
$\bar{B}D$	1-3-9-11		x	x			x	x	
CD	3-7-11-15			x	x			x	x

$$\boxed{Y = \bar{B}\bar{C} + CD}$$

Compare with K MAP



$$\boxed{Y = \bar{B}\bar{C} + CD}$$