Chp-2 Boolean Algebra and logic Gates :

Boolean Algebra -

BA is a mathematics that provides various operations and rules to perform arithmetic and algebraic operations on binary variables and numbers.

- It is based on binary number system.

Based on three fundamental logical operations namely, AND, OR, NOT.

⇒ Laws of BA -

AND Laws -

① $A \cdot 0 = 0$ ⓘⓘ $A \cdot 1 = A$ ⓘⓘⓘ $A \cdot A = A$ ⓘⓥ $A \cdot A' = 0$

OR law -

① $A + 0 = A$ ⓘⓘ $A + 1 = 1$ ⓘⓘⓘ $A + A = A$ ⓘⓥ $A + A' = 1$

Complementation Laws -

ⓘ $0' = 1$ ⓘⓘ $1' = 0$ ⓘⓘⓘ $A = 0$ then $A' = 1$ ⓘⓥ $A = 1$ then $A' = 0$ ⓥ $(A')' = A$

Commutative Laws -

① $A + B = B + A$ ⓘⓘ $A \cdot B = B \cdot A$

Associative Laws -

ⓘ $(A + B) + C = A + (B + C)$ ⓘⓘ $(A \cdot B) \cdot C = A \cdot (B \cdot C)$

Distributive Law

① $A \cdot (B + C) = AB + AC$ ② $A + BC = (A + B)(A + C)$

Redundant Literal Rule -

① $A + A'B = A + B$ ② $A(A' + B) = AB$

Idempotence laws.

① $A \cdot A = A$ ② $A + A = A$

Absorption laws.

(i) $A + A \cdot B = A$ (ii) $A(A+B) = A$

DeMorgan's Th^m

(i) $\overline{A+B} = \bar{A} \cdot \bar{B}$

(ii) $\overline{AB} = \bar{A} + \bar{B}$

De Morgan's first th^m:

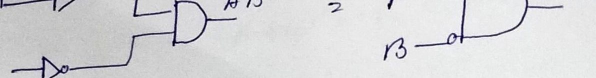
States that the complement of a sum of variable is equal to the product of their individual complements.

In other words, the complement of two or more ORed variables is equivalent to the AND of the complements of each of the individual variables i.e.

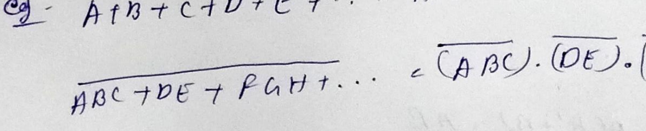
$$\overline{A+B} = \bar{A} \cdot \bar{B}$$

$$(A+B)' = A' \cdot B'$$

LHS



RHS-



eg- $\overline{A+B+C+D+E+\ldots} = \bar{A}\,\bar{B}\,\bar{C}\,\bar{D}\,\bar{E} \ldots$

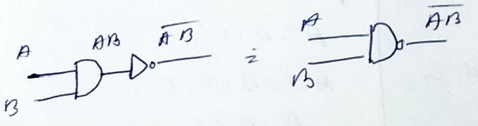
$\overline{ABC + DE + FGH + \ldots} = (\overline{ABC}) \cdot (\overline{DE}) \cdot (\overline{FGH}) \ldots$

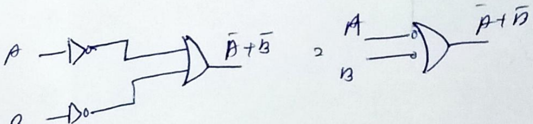
De Morgan's 2nd th^m

The Complement of the product of variables is equivalent to the Sum of their individual components.

$$\overline{AB} = \bar{A} + \bar{B}.$$

$$(AB)' = A' + B'$$





eg- $\overline{ABCDE} = \bar{A} + \bar{B} + \bar{C} + \bar{D} + \bar{E}$

$$\overline{(ABC)(DE)(FG)} = \overline{ABC} + \overline{DE} + \overline{FG}.$$

① Apply De Morgan's th^m.

$$F = \overline{AB(\overline{C+D})EF}$$

$$= \overline{AB} + (C+D) + \overline{EF}$$

② $F = \overline{AB + \overline{CD}}$

$$= \overline{AB} \cdot CD$$

Canonical and Standards forms

(i) Sum of Product (SOP) form:

It is formed by adding the product terms. These product terms are also called 'min-terms'.

Representated by 'm'.

SOP is sum of minterm as is represented as.

F in $SOP = \Sigma_m(0,3)$

Here F is sum of minterm 0 and 3.

$A=0, B=0, C=0$

$A' . B' . C'$

$A=1, B=0, C=1$

$A . B' . C$

SOP eg:

A	B	C	X
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

$X = \Sigma_m(1,3,6)$

$= A'B'C + AB'C' + ABC'$

(ii) Product of Sum –

- formed by multiplying the sum terms. These sum terms are called 'max terms'. Represented by M.

F in $POS = \pi_m(1,2)$

ex ~~A0~~ $A=0, B=1, C=0$; Maxterm is $A+B'+C$

$A=1, B=1, C=1$, $A'+B'+C$

A B C	X
0 0 0	0
0 0 1	1
0 1 0	0
0 1 1	1
1 0 0	0
1 0 1	0
1 1 0	1
1 1 1	0

$X(POS) = \Sigma_m (0, 2, 4, 5, 7)$

$= (A+B+C) \cdot (A+B'+C) \cdot (A'+B+C) \cdot (A'+B+C') \cdot (A'+B'+C')$.

* Conversion of a Boolean Exp in POS form to Standard POS form.

1. Write down all sum terms of given Boolean exp.

2. If one or more variables are missing in any sum term, then add the products of each of the missing variables and its complement to that term.

3. Expand the terms according to the rules of BA.

4. Finally drop out the redundant terms from the Expression.

eg:- Convert 3-variable BE in POS form to S POS.

$f(A, B, C) = (A+\bar{B}) \cdot (\bar{B}+C) \cdot (A+\bar{C})$

$\Rightarrow (A+\bar{B}+C\bar{C}) \cdot (A\bar{A}+\bar{B}+C) \cdot (A+B\bar{B}+\bar{C})$

$= (A+\bar{B}+C)(A+\bar{B}+\bar{C})(A+\bar{B}+C)(\bar{A}+\bar{B}+C)(A+B+\bar{C})(A+\bar{B}+\bar{C})$

$= (A+\bar{B}+C)(\bar{A}+\bar{B}+C)(A+B+\bar{C})(A+\bar{B}+\bar{C})$