Reliability Engineering Comprehensive Reference

Fundamental Concepts

Reliability Definition

- Reliability (R(t)) is the probability that a system will perform its intended function under stated conditions for a specified period of time
- Unreliability (F(t)) $= 1 - R(t)$
- Mean Time Between Failures (MTBF) $= \frac{\text{Total Operating Time}}{\text{Number of Failures}}$
- Mean Time To Failure (MTTF) - Used for non-repairable systems
- Mean Time To Repair (MTTR) - Average time required to repair a failed component

Availability Metrics

- Availability (A) $= \frac{\text{MTTF}}{\text{MTTF+MTTR}}$
- Inherent Availability (Ai) $= \frac{\text{MTTF}}{\text{MTTF+MTTR}}$
- Operational Availability (Ao) $= \frac{\text{Operating Time}}{\text{Operating Time+Downtime}}$
- Achieved Availability (Aa) $= \frac{\text{MTTF}}{\text{MTTF}+\bar{M}}$ where $\bar{M}$ is mean active maintenance time

Probability Distributions in Reliability

Exponential Distribution

- Most commonly used for constant failure rate systems
- Probability Density Function: f(t) $= \lambda e^{-\lambda t}$
- Reliability Function: R(t) $= e^{-\lambda t}$
- Failure Rate: $\lambda(t) = \lambda$ (constant)
- MTTF $= 1/\lambda$

Weibull Distribution

- Most versatile distribution in reliability engineering
- Three-Parameter Form:
  - f(t) $= (\beta/\eta)((t-\gamma)/\eta)^{(\beta-1)} exp(-((t-\gamma)/\eta)^{\beta})$
- Parameters:
  - β (Beta): Shape parameter
  - η (Eta): Scale parameter
  - γ (Gamma): Location parameter
- Reliability Function: R(t) $= exp(-((t-\gamma)/\eta)^{\beta})$
- Characteristics:
  - $\beta < 1$: Decreasing failure rate
  - $\beta = 1$: Constant failure rate (reduces to exponential)
  - $\beta > 1$: Increasing failure rate

Normal Distribution

- Used for wear-out failures
- Probability Density Function: f(t) = $(1/(\sigma\sqrt{2\pi}))e^{-(t-\mu)^2/(2\sigma^2)}$
- Parameters:
  - μ: Mean life
  - σ: Standard deviation

Lognormal Distribution

- Used for repair times and maintenance actions
- PDF: f(t) = $(1/(t\sigma\sqrt{2\pi}))e^{-(\ln(t)-\mu)^2/(2\sigma^2)}$
- Where μ and σ are the mean and standard deviation of ln(t)

System Reliability Analysis

Series Systems

- Overall Reliability: $R_s = R_1 \times R_2 \times ... \times R_n$
- System fails if any component fails
- MTTF(system) = $1/(\lambda_1 + \lambda_2 + ... + \lambda_n)$

Parallel Systems

- Overall Reliability: $R_p = 1 - [(1 - R_1) \times (1 - R_2) \times ... \times (1 - R_n)]$
- System functions if at least one component works
- Improves system reliability

k-out-of-n Systems

- System works if at least k components out of n work
- Reliability: $R(k, n) = \sum_{i=k}^{n} \binom{n}{i} R^i (1 - R)^{n-i}$

Failure Analysis

Failure Modes and Effects Analysis (FMEA)

- Risk Priority Number (RPN) = Severity × Occurrence × Detection
- Scale typically 1-10 for each factor
- Higher RPN indicates higher risk

Fault Tree Analysis (FTA)

Basic Events Symbols:

- Circle: Basic event
- Diamond: Undeveloped event
- Rectangle: Intermediate event
- House: External event

Gate Symbols:

- AND gate: Output occurs if all inputs occur
- OR gate: Output occurs if any input occurs
- Exclusive OR gate: Output occurs if exactly one input occurs

Common Cause Failures (CCF)

- Beta Factor Model: CCF $= \beta \times \lambda$ where β is the fraction of failures that are common cause

Maintenance Strategies

Preventive Maintenance

- Time-Based Maintenance (TBM)
- Usage-Based Maintenance (UBM)
- Optimal Maintenance Interval: $T^* = \sqrt{\frac{2 \times C_p}{C_f \times \lambda}}$ where:
- C_p = Preventive maintenance cost
- C_f = Failure repair cost
- λ = Failure rate

Condition-Based Maintenance

- P-F Interval: Time between potential failure detection and functional failure
- Key Parameters:
  - Inspection interval $<$ P-F interval
  - Cost of monitoring $<$ Cost of failure $\times$ Probability of failure

Reliability-Centered Maintenance (RCM)

Seven Questions: 1. Functions and performance standards 2. Functional failures 3. Failure modes 4. Failure effects 5. Failure consequences 6. Proactive tasks 7. Default actions

Life Data Analysis

Life Testing

Types:

- Complete data
- Right censored
- Left censored
- Interval censored

Acceleration Factors

Arrhenius Model:

- $AF = exp[\frac{E_a}{k}(\frac{1}{T_1} - \frac{1}{T_2})]$

where:

- E_a = Activation energy
- k = Boltzmann's constant
- T_1, T_2 = Temperatures in Kelvin

Standards and Specifications

Military Standards

- MIL-STD-785: Reliability Program Requirements
- MIL-HDBK-217: Reliability Prediction
- MIL-STD-2173: Reliability-Centered Maintenance

Commercial Standards

- ISO 9001: Quality Management Systems
- IEC 61508: Functional Safety
- SAE JA1011/1012: RCM Implementation

Key Performance Indicators (KPIs)

Reliability Metrics

- Reliability Growth Rate
- Failure Rate Trend
- Mean Time Between Critical Failures (MTBCF)
- System Availability
- First Time Fix Rate (FTFR)

Maintenance Metrics

- Planned Maintenance Percentage (PMP)
- Schedule Compliance
- Backlog Trend
- Mean Time to Repair (MTTR)
- Overall Equipment Effectiveness (OEE)

Statistical Testing and Analysis

Hypothesis Testing

- Null Hypothesis (H_0)
- Alternative Hypothesis (H_1)

- Type I Error (α)
- Type II Error (β)
- Power = $1 - \beta$

Confidence Intervals

For exponential distribution:

- Lower bound = $\frac{2T}{\chi^2(\alpha/2)}$
- Upper bound = $\frac{2T}{\chi^2(1-\alpha/2)}$ where T is total test time

Goodness of Fit Tests

- Kolmogorov-Smirnov Test
- Anderson-Darling Test
- Chi-Square Test

Cost Analysis

Life Cycle Cost (LCC)

Components: 1. Acquisition Cost 2. Operating Cost 3. Maintenance Cost 4. Disposal Cost

Cost of Poor Reliability

Factors: - Warranty Claims - Lost Production - Repair Costs - Customer Dissatisfaction - Brand Damage

Safety and Risk Assessment

Risk Assessment Matrix

Severity Levels: 1. Catastrophic 2. Critical 3. Marginal 4. Negligible

Probability Levels: 1. Frequent 2. Probable 3. Occasional 4. Remote 5. Improbable

Safety Integrity Levels (SIL)

- SIL 1: 10^{-1} to 10^{-2} failures per hour
- SIL 2: 10^{-2} to 10^{-3} failures per hour
- SIL 3: 10^{-3} to 10^{-4} failures per hour
- SIL 4: 10^{-4} to 10^{-5} failures per hour

Testing and Confidence

Sample Size Determination

- Zero-failure testing: $n = \frac{\ln(1-C)}{\ln(R)}$ where C = confidence level, R = required reliability
- For binomial success/failure: $n = \frac{\ln(1-C)}{\ln(1-p)}$ where p = probability of failure

Confidence Calculations

- Two-sided confidence bounds for exponential MTTF:
  - Lower: $\frac{2T}{\chi^2_{1-\alpha/2,2f}}$
  - Upper: $\frac{2T}{\chi^2_{\alpha/2,2f}}$
- One-sided bounds use $\chi^2_{1-\alpha,2f}$

Environmental Stress Screening

ESS vs Burn-in

- ESS: Dynamic stressing to precipitate latent defects
- Burn-in: Static conditions to age products
- Key differences:
  - ESS uses multiple stresses
  - ESS targets manufacturing defects
  - Burn-in targets infant mortality

ESS Program Development

- Stress Selection Criteria:
  1. Related to failure mechanisms
  2. Not exceeding design limits
  3. Measurable and controllable
- Common Stresses:
  - Temperature cycling
  - Vibration
  - Power cycling
  - Combined environments

Human Reliability Analysis

Performance Shaping Factors

- Task complexity
- Time pressure
- Environmental conditions
- Training and experience
- Procedures and documentation

- Supervision and teamwork
- Fatigue and stress

Error Prevention Strategies

1. Design for human factors
2. Clear procedures and instructions
3. Training and certification
4. Error-proofing (Poka-Yoke)
5. Regular feedback and improvement

Design of Experiments

Taguchi Methods

- Signal-to-noise ratios:
  - Larger is better: $S/N = -10\log(\frac{1}{n}\sum\frac{1}{y_i^2})$
  - Nominal is best: $S/N = 10\log(\frac{\bar{y}^2}{s^2})$
  - Smaller is better: $S/N = -10\log(\frac{1}{n}\sum y_i^2)$

Loss Functions

- Quality loss: $L(y) = k(y-T)^2$ where T = target value, k = cost coefficient
- Process capability indices:
  - $C_p = \frac{USL-LSL}{6\sigma}$
  - $C_{pk} = min(\frac{USL-\mu}{3\sigma}, \frac{\mu-LSL}{3\sigma})$

Statistical Life Measures

B-Life Analysis

- B-life: Time at which X% of units have failed
- For Normal Distribution:
  - $B_x = \mu + z_p\sigma$ where z_p is standard normal value at $(x/100)$ probability
- For Weibull Distribution:
  - $B_x = \eta[-\ln(1-x/100)]^{1/\beta}$

Population Parameters

- Sample Variance Confidence Interval:
  - $\frac{(n-1)s^2}{\chi^2_{\alpha/2}} \leq \sigma^2 \leq \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}$
- Population Mean Confidence Interval:
  - $\bar{x} \pm t_{\alpha/2,n-1}\frac{s}{\sqrt{n}}$

Acceleration Testing

Common Models

1. Arrhenius (Temperature):
    - $AF = exp[\frac{E_a}{k}(\frac{1}{T_1} - \frac{1}{T_2})]$
2. Coffin-Manson (Mechanical Stress):
    - $AF = (\frac{\Delta\varepsilon_1}{\Delta\varepsilon_2})^m$
3. Inverse Power Law (Stress):
    - $AF = (\frac{S_1}{S_2})^n$
4. Eyring (Multiple Stresses):
    - $AF = (\frac{T_1}{T_2})exp[\frac{B}{k}(\frac{1}{T_1} - \frac{1}{T_2}) + C(V_1 - V_2)]$

Reliability Growth Models

AMSAA-Duane Model

- Cumulative Failures: $N(t) = \lambda t^\beta$
- Instantaneous Failure Rate: $r(t) = \lambda\beta t^{\beta-1}$
- Cumulative Failure Rate: $r_c(t) = \lambda t^{\beta-1}$
- Cumulative MTBF: $M_c(t) = \frac{1}{\lambda}t^{1-\beta}$

Spare Parts Analysis

Poisson Process Spares

- Probability of x spares needed: $P(X = x) = \frac{(\lambda t)^x e^{-\lambda t}}{x!}$
- Probability of more than n spares: $P(X > n) = 1 - \sum_{x=0}^{n} \frac{(\lambda t)^x e^{-\lambda t}}{x!}$

System with Spares

- Reliability with n spares: $R_s(t) = e^{-\lambda t} \sum_{i=0}^{n} \frac{(\lambda t)^i}{i!}$
- Mean Time To System Failure: $MTSF = \frac{1}{\lambda} \sum_{i=0}^{n+1} i$

Fail-Safe Design

Principles

1. System remains safe when component fails
2. Failure detection and indication
3. Redundancy in critical functions
4. Graceful degradation

Implementation Methods

- Structural redundancy
- Functional redundancy

- Analytical redundancy
- Safe-state default
- Monitoring and diagnostics