KERMIT: logicalization of BERT

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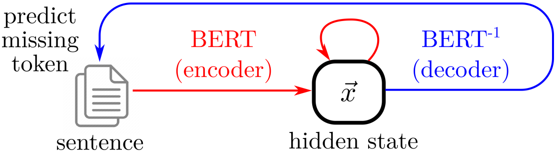
BERT's ground-breaking significance: closed-loop training

- BERT uses ordinary text corpuses to induce knowledge, forming representations that have universality:



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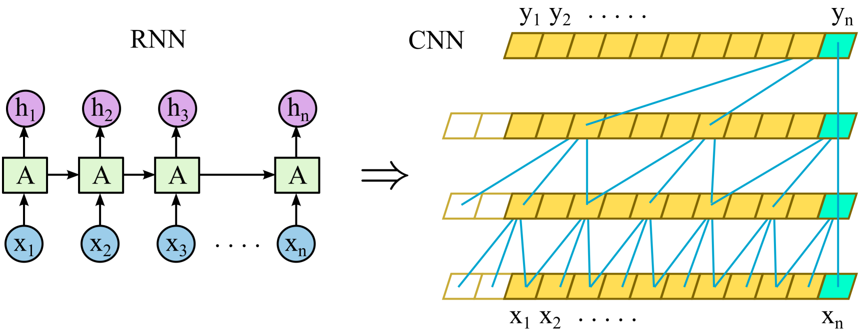
  In other words, the hidden state compresses the meaning of sentences, that can be used in other scenarios
- This implies that human-level AI can be *induced* from existing corpora, without the need to retrace human infant development
- This training technique came from an earlier paper, unrelated to BERT's internal architecture

BERT's internal architecture

BERT results from combining several ideas:

- BERT is basically a seq-to-seq transformation
- Seq-to-seq was originally solved by RNNs
- But RNNs are slow, researchers proposed to replace them with CNNs



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- CNN with attention mechanism gives rise to Transformer
- My idea is to incorporate symmetric NN into BERT while following this line of thinking

Symmetry in logic

- Words form sentences, analogous to concepts forming propositions in logic
- From an abstract point of view, logic can be seen as an algebra with 2 operations: a non-commutative multiplication ($\cdot$, for composition of concepts) and a commutative addition ($\wedge$, for conjunction of propositions)
- For example:

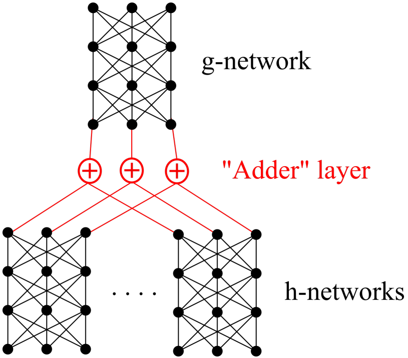
$$\begin{aligned} A \wedge B &\equiv B \wedge A \\ \text{it's raining} \wedge \text{lovesick} &\equiv \text{lovesick} \wedge \text{it's raining} \end{aligned} \tag{3}$$

- Word2Vec was also ground-breaking, but it was easy to go from Word2Vec to Sentence2Vec: just concatenate the vectors
  Sentences correspond to propositional logic
- A set of propositions requires symmetric NN to process, as elements of the set are permutation invariant

Symmetric neural network

- The symmetric NN problem has been solved by 2 papers:
  [PointNet 2017] and [DeepSets 2017]
- Any symmetric function can be represented by the following form (a special case of the Kolmogorov-Arnold representation of functions):

$$f(x, y, ...) = g(h(x) + h(y) + ...) \tag{4}$$



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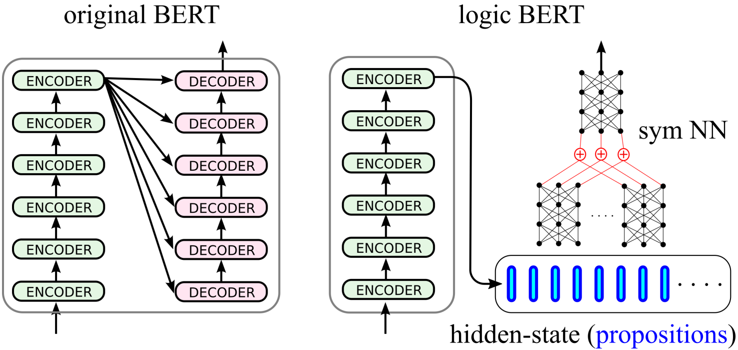
Logicalization of BERT

- We can convert BERT's hidden state into a set of propositions, by replacing the original decoder with a sym NN:



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- Permutation invariance of the ▭'s is automatically satisfied by the architecture of the new decoder
- The original encoder can be retained, as it is a universal seq-2-seq mapping
- The decoder imposes symmetry on the hidden state; Error propagation is expected to cause its representation to change
- Of course, this remains to be proven by experiment ☺

Connection between AI and logic

- If AI is based on logic, there must exist a precise connection between them
- BERT seems to be performing some kind of transformations between sentences, such sentences are simply compositions of word-embedding vectors:

$$\text{Socrates} \cdot \text{is} \cdot \text{human} \xrightarrow{BERT} \text{Socrates} \cdot \text{is} \cdot \text{mortal} \tag{7}$$

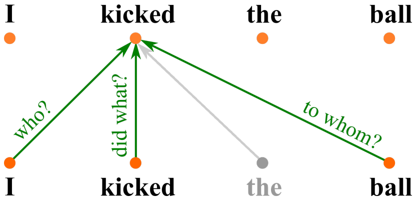
  While this may seem crude, it is effectively the same as a logic formula:

$$\forall x.\ \text{Human}(x) \rightarrow \text{Mortal}(x) \tag{8}$$

  Surprisingly, by the Curry-Howard correspondence, this formula corresponds to the mapping (7) above!
- In another article we shall explore this connection. One could say the mathematical structure of logic is "eternal"; It will provide guidance for the long-term development of AI

What is attention?

- Attention originated with Seq2seq, then BERT introduced self-attention
- The essence of attention is weighing
- For each input, attention weighs the relevance of every other input and draws information from them accordingly to produce the output
- In BERT, attention is a relation among words in a sentence:



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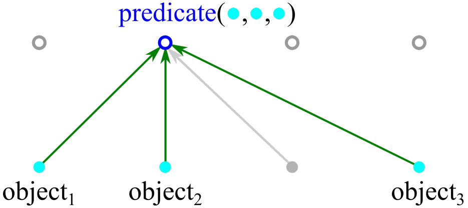
- From a logic point of view, words $\neq$ propositions
- In logic, the distinction between sub-propositional and propositional levels is of crucial importance!

Predicates and propositions

- The word “predicate” comes from Latin “to declare”
- In logic, a predicate is a declaration without a subject or object; In other words, it is a proposition with “holes”
- Proposition = predicate + objects
- Eg: Human(John), Loves(John, Mary)
- From the logic point of view, the output of attention is the fusion of a predicate with its objects:



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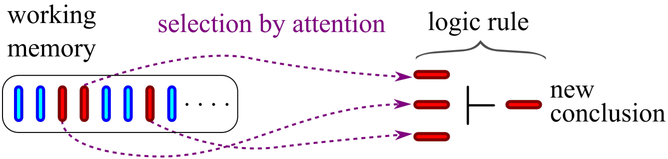
- Or figuratively:

$$\begin{aligned} \text{predicate} + \text{objects} &= \text{proposition} \\ \circ + \bullet\,\bullet\,\bullet \ldots. &= \boxed{} \end{aligned} \tag{11}$$

Attention at the propositional level

- Analogously, attention on higher levels may process relations among propositions, but here this mechanism seems rather inadequate
- When forming sentences from words, there are relatively few syntactic patterns, such as:

  subject · verb · object
- But there are no *a priori* patterns for forming deductions from propositions; Logically related things need not be similar
- Eg: need to pee $\wedge$ not in toilet $\Rightarrow$ hold it
  But "pee" and "in toilet" is learned *a posteriori*
- We wish for attention to select propositions that are relevant for deduction:



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- But to choose N propositions from a set of N, there would be $\binom{M}{N}$ subsets, an exponential number

“Attention is all you need” ?

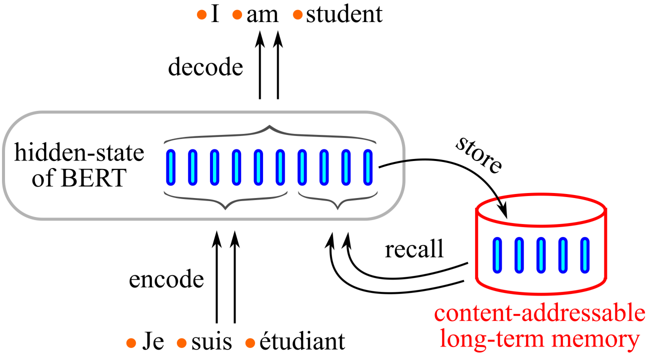
- At the propositional level, attention uses a weight matrix to memorize the relatedness among propositions, such a method seems inefficient
- Suppose q is the proposition under focus, $p_1, p_2, ...$ are some possibly relevant propositions, $\bowtie$ denotes matching by attention, we want to output their relatedness:

$$q \bowtie \quad p_1, p_2, ... \mapsto \text{related?} \tag{13}$$

  but each case takes one row of matrix for storage
- This is a “flat” representation of cases
- If we try to use hierarchical techniques to handle this, it becomes more and more like deep learning, we might as well just use a neural network!
- In other words, use a deep symmetric NN, let it learn internally how to select relevant propositions

Content-addressable long-term memory

- The original BERT's hidden state lacked a logical structure; It was not clear what it contains exactly. With logicalization, propositions inside BERT can be stored into a long-term memory:



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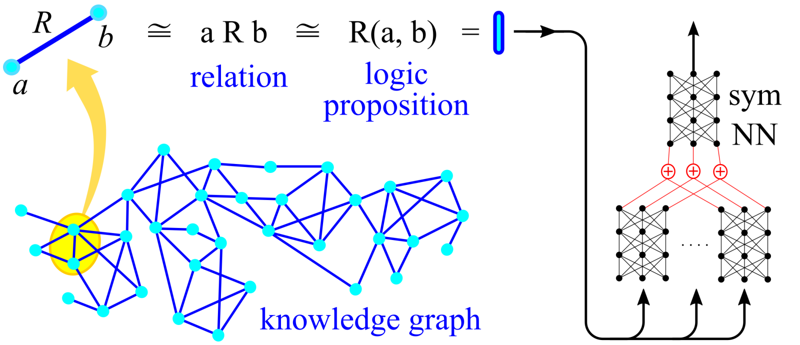
Eg. "The sun is hot", "Water flows downhill" are facts that stay constant

- Name: Knowledge-Enhanced Reasoning with Memorized Items
- This is getting very close to strong AI, and depends crucially on logicalization
- The content-addressable memory idea came from Alex Graves *et al*'s Neural Turing Machine [2014]

Knowledge graphs

- One cannot feed a knowledge graph directly into a neural network, as the input must be a vector. A solution is to break the graph into edges, where each edge is equivalent to a relation or proposition. One could say that graphs are isomorphic to logic



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- As edges are invariant under permutations, it seems that we must use symmetric NNs to process them

Doubts about logic

- Many people question: Do our brains really use symbolic logic to think?
- To say the least, all our languages are essentially in logical form
- Our impression is that the brain constructs "mental models" of the world and "reads off" conclusions from such models
- Consider a description: "Wife cheats on husband, stubs him with knife"

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- What is she wearing? What color is her dress? Such details are imagined and unwarranted
- So what kind of details can our model have? The answer is: it cannot have ANY detail, except those entailed or constrainted by logic
- Models may be constructed from abstract logic propositions; Models with a lot of sensory details are implausible
- Perhaps the brain is much closer to formal logic than we'd thought

References

Questions, comments welcome ☺️

[1] Alex Graves, Greg Wayne, and Ivo Danihelka. "Neural Turing Machines". In: *CoRR* abs/1410.5401 (2014). arXiv: 1410.5401. URL: http://arxiv.org/abs/1410.5401.

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[3] Zaheer et al. "Deep sets". In: *Advances in Neural Information Processing Systems* 30 (2017), pp. 3391–3401.