AGI via Combining Logic with Deep Learning

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Abstract. An integration of deep learning and logic is proposed, based on the Curry-Howard isomorphism. Under this interpretation, it turns out that Google's BERT, which many currently state-of-the-art language models are derived from, can be regarded as a special form of logic. Moreover, this structure can be incorporated under a reinforcement-learning framework to form a minimal AGI architecture. This paper also surveys some category theory of logic and topos theory, which may be helpful to practitioners of AGI.

Keywords: deep learning, symbolic logic, logic-based AI, neural-symbolic integration, Curry-Howard isomorphism, category theory, topos theory, fuzzy logic

0 Introduction: the Curry-Howard Isomorphism

As the risk of sounding too elementary, we would go over some basic background knowledge, that may help those readers who are unfamiliar with this area of mathematics.

The Curry-Howard isomorphism expresses a connection between logic **syntax** and its underlying **proof** mechanism. Consider the mathematical declaration of a **function** f with its domain and co-domain:

$$f : A \to B. \tag{1}$$

This notation comes from type theory, where A and B are **types** (which we can think of as sets or general spaces) and the function f is an **element** in the function space $A \to B$, which is also a type.

What the Curry-Howard isomorphism says is that we can regard $A \to B$ as a **logic** formula $A \Rightarrow B$ (an implication), and the function f as a **proof** process that maps a proof of A to a proof of B.

The following may give a clearer picture:

$$\begin{array}{cc} \boxed{\text{logic}} & A \Longrightarrow B \\ & \text{- - - - - - - - - -} \\ \boxed{\text{program}} & \blacksquare \overset{f}{\longmapsto} \blacksquare \quad . \end{array} \tag{2}$$

What we see here is a logic formula "on the surface", with an underlying proof mechanism which is a **function**. Here the $\blacksquare$'s represent proof objects or witnesses. The logic propositions A and B coincide with the **domains** (or **types**) specified by type theory. Hence the great educator Philip Wadler calls it "propositions as types". [1] Other textbooks on the Curry-Howard isomorphism include: []

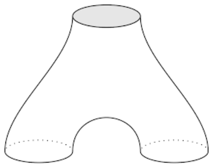
The gist of our theory is that Deep Learning provides us with neural networks (ie. non-linear functions) that serve as the proof mechanism of logic via the Curry-Howard isomorphism. With this interpretation, we can impose the mathematical structure of logic (such as symmetries) onto neural networks. Such constraints serve as **inductive bias** that can accelerate learning, according to the celebrated "No Free Lunch" theory.

In particular, logic propositions in a conjunction (such as $A \wedge B$) are commutative, ie. invariant under permutations, which is a "symmetry" of logic. This symmetry essentially decomposes a logic "state" into a set of propositions, and seems to be a fundamental feature of most logics known to humans. Imposing this symmetry on neural networks gives rise to symmetric neural networks, which can be easily implemented thanks to separate research on the latter topic. This is discussed in §**??**.

As an aside, the Curry-Howard isomorphism also establishes connections to diverse disciplines. Whenever there is a space of elements and some operations over them, there is a chance that it has an underlying "logic" to it. For example, in quantum mechanics, Hilbert space and Hermitian operators. Another example: in String Theory, strings

[1] See his introductory video: https://www.youtube.com/watch?v=IOiZatlZtGU .

and cobordisms between them (The following is the famous “pair of pants” cobordism, representing a process in time that splits one string into two):



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1 Prior Research

1.1 Neuro-Symbolic Integration

There has been a long history of attempts to integrate symbolic logic with neural processing, with pioneers such as Ron Sun, Dov Gabbay, among others. We consider 2 example approaches below.

Pascal Hitzler (*et. al.*)’s “Core Method” [**Hitzler2011**] is model-based (as in model theory in logic). An interpretation $\mathcal{I}$ is a function that assigns truth values to the set of all possible ground atoms. To a logic program P is associated a semantic operator $\mathcal{T}_P : \mathcal{I}_P \to \mathcal{I}_P$, where $\mathcal{I}_P$ is an interpretation endowed with the topology of the Cantor set. Then P is approximated by a neural network $f : \mathcal{I}_P \to \mathbb{R}$.

Pedro Domingos’ ∂-ILP. [**Domingos**]

Geoffrey Hinton’s GLOM theory. [**Hinton**]

1.2 Cognitive Architectures and Reinforcement Learning

Reinforcement Learning (RL). In the 1980’s, Richard Sutton [13] introduced reinforcement learning as an AI paradigm, drawing inspiration from Control Theory and Dynamic Programming. In retrospect, RL already has sufficient generality to be considered an AGI theory, or at least as a top-level framework for describing AGI architectures.

Relation to AIXI. AIXI is an abstract AGI model introduced by Marcus Hutter in 2000 [**Hutter2000**]. AIXI’s environmental setting is the external “world” as observed by some sensors. The agent’s internal model is a universal Turing machine (UTM), and the optimal action is chosen by maximizing potential rewards over all programs of the UTM. In our (minimal) model, the UTM is constrained to be a neural network, where the NN’s **state** is analogous to the UTM’s **tape**, and the optimal weights (program) are found via Bellman optimality.

Relation to Quantum mechanics and Path Integrals. At the core of RL is the Bellman equation, which governs the update of the utility function to reach its optimal value. This equation (in discrete time) is equivalent to the Hamilton-Jacobi equation in differential form. Nowadays they are unified as the Hamilton-Jacobi-Bellman equation, under the name “optimal control theory” [10]. In turn, the Hamilton-Jacobi equation is closely related to the Schrödinger equation in quantum mechanics:

$$\boxed{\text{Bellman eqn.}} \;---\; \boxed{\text{Hamilton-Jacobi eqn.}} \;---\; \boxed{\text{Schrödinger eqn.}} \tag{4}$$

but the second link is merely “heuristical”; it is the well-studied “quantization” process whose meaning remains mysterious to this day. Nevertheless, the path integral method introduced by Richard Feynmann can be applied to RL algorithms, eg. [**Kappen**].

The Hamilton-Jacobi equation gives the RL problem a “symplectic” structure; Such problems are best solved by so-called symplectic integrators. Surprisingly, in the RL / AI literature, which has witnessed tremendous growth in recent years, there is scarcely any mention of the Hamilton-Jacobi connection, while the most efficient heuristics (such as policy gradient, Actor-Critic, etc.) seem to exploit other structural characteristics of the “world”.

2 The Mathematical Structure of Logic

Currently, the most mathematically advanced and satisfactory description of logic seems to base on category theory, known as categorial logic and topos theory. This direction was pioneered by William Lawvere in the 1950-60’s. The body of work in this field is quite vast, but we shall briefly survey the main points that are relevant to AGI.

2.1 Dependent Type Theory

The Curry-Howard isomorphism identifies *propositional* intuitionistic logic with type theory. As such, the arrow → in type theory is "used up" (it corresponds to the implication arrow ⇒ in intuitionistic logic). However, predicates are also a kind of functions (arrows), so how could we accomodate predicates in type theory such that Curry-Howard continues to hold? This is the idea behind Martin Löf's Dependent Type Theory.

$$\begin{tikzcd} \overbrace{\text{Human (Socrates)}}^{A} \arrow[d] & \Rightarrow & \overbrace{\text{Mortal (Socrates)}}^{B} \arrow[d] \\ \Omega & & \Omega \end{tikzcd} \tag{5}$$

The type of B in the dependent sum $\sum\limits_{A} B$ depends on A. The sum of the entire family of A (indexed by B) is similar to the product $A \times B$.

The type of B in the dependent product $\prod\limits_{A} B$ depends on A. The product of the entire family of A is similar to the exponentiation B^A.

2.2 Topos Theory

The most important commutative diagram in Topos theory is this:

$$\begin{tikzcd} X \arrow[r, "!"] \arrow[d, tail, "m"'] & 1 \arrow[d, "\text{true}"] \\ Y \arrow[r, "\chi_m"'] & \Omega \end{tikzcd} \tag{6}$$

The logic of sheaves is intuitionistic.

2.3 Fuzzy logic and fuzzy topos

Every set is a pullback of the true map. Every fuzzy set should be the pullback of the fuzzy true map?

3 Permutation Symmetry and Symmetric Neural Networks

One basic characteristic of (classical) logic is that the conjuction ∧ is **commutative**:

$$\mathsf{P} \wedge \mathsf{Q} \quad \Leftrightarrow \quad \mathsf{Q} \wedge \mathsf{P}. \tag{7}$$

3.1 BERT as a Logic

4 "No Free Lunch" Theory

5 Experiment

A simple test of the symmetric neural network, under reinforcement learning (policy gradient), has been applied to the Tic-Tac-Toe game. [2]

[2] Code is on GitHub:

The state of the game is represented as a set of 9 propositions, where all the propositions are initialized as "null" propositions. During each step of the game, a new proposition is added to the set (ie. over-writing the null propositions). Each proposition encodes who the player is, and which square (i, j) she has chosen. In other words, it is a predicate of the form: `move(player,i,j)`. The neural network takes all 9 propositions as input, and outputs a new proposition; Thus it is a permutation-invariant function.

In comparison, the game state of traditional RL algorithms (eg. AlphaGo []) usually is represented as a vector of dimension same as the chessboard (eg. 3×3 in Tic-Tac-Toe and 8×8 in Chess). This state vector remains the same constant length even if there are very few pieces on the chessboard. Our logic-based representation may offer some advantages over the board-vector representation, and likely induces a different way of "reasoning" about the game.

In our Tic-Tac-Toe experiment, convergence of learning is observed, but the algorithm fell short of achieving the highest score (19 instead of 20), and the score displayed unstable oscillating behavior after it got near the optimal value. Further investigation is required, but it seems to be a promising start.

6 Conclusion and Future Directions

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(Diagrams in this paper can be re-organized in the traditional format, please contact the author if this is desired.)

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7 Reinforcement-learning architecture

The comparison of RL with neuroscience helps to crack the brain code:

- What constitute the brain's **state**?
- What is the **state transition function**?
- What enables **learning** (in the state transtion function)?

We propose an AGI architecture:

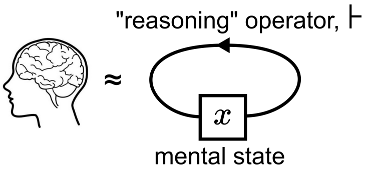
1. with **reinforcement learning** (RL) as top-level framework
   - State space = mental space
2. **Logic** structure is imposed on the **knowledge representation** (KR)
   - State transitions are given by logic rules = actions in RL
   - The logic state $\boldsymbol{x}$ is decomposable into **propositions** (§**??**)
3. The set of logic rules is approximated by a deep neural network
   - Just the most basic kind of feed-forward neural network (FFNN) is required
   - Logic conjunctions are **commutative**, so working-memory elements can be presented in any order (§**??**)
   - **Stochastic** actions are represented by **Gaussian kernels** (radial basis functions) (§**??**), thus partly avoiding the curse of dimensionality

The rest of this paper will explain these design features in detail.

The **metaphor** in the title of this paper is that of RL controlling an autonomous agent to navigate the maze of "thoughts space", seeking the optimal path:

(8)

The main idea is to regard "thinking" as a **dynamical system** operating on **mental states**:



(9)

A mental state is a **set of propositions**, for example:

- I am in my room, writing a paper for AGI-2019.
- I am in the midst of writing the sentence, "I am in my room, ..."
- I am about to write a gerund phrase "writing a paper..."

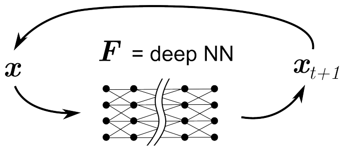
Thinking is the process of **transitioning** from one mental state to another. As I am writing now, I use my mental states to keep track of where I am at within the sentence's syntax, so that I can construct my sentence grammatically.

7.1 Actions = cognitive state-transitions = "thinking"

Our system consists of two main algorithms:

1. Learning the transition function $\vdash$ or $\boldsymbol{F} : \boldsymbol{x} \mapsto \boldsymbol{x}'$. $\boldsymbol{F}$ represents the **knowledge** that constrains thinking. In other words, the learning of $\boldsymbol{F}$ is the learning of "static" knowledge.
2. Transitioning from $\boldsymbol{x}$ to $\boldsymbol{x}'$. This corresponds to "thinking" under the guidance of the static knowledge $\boldsymbol{F}$.

In our architecture, $\boldsymbol{F}$ can be implemented as a simple feed-forward neural network (where "deep" simply means "many layers"):



(10)

Since a recurrent NN is Turing-complete, this can be viewed as a minimalist AGI. But its learning may be too slow without further **inductive bias** (*cf* the "no free lunch" theorem [17]) — so we will further modify $\boldsymbol{F}$ by imposing the logic structure of reasoning on it (§**??** and §**??**).

In principle, every state is potentially **reachable** from every other state, if a logic rule exists between them. Now we use a deep FFNN to represent the set of all logic rules. This is a key efficiency-boosting step, because deep neural networks allows to use a polynomial number of parameters to represent an exponential number of mappings.

Note that parts of the state $\boldsymbol{x}$ would be reserved and directly connect to the **input** and **output** of the AGI system.

8 Logic structure

8.1 Logic is needed as an inductive bias

We know that the transition function $\boldsymbol{F}$ is analogous to $\vdash$, the logic consequence or entailment operator. So we want to impose this logic structure on $\boldsymbol{F}$.

By logic structure we mean that $\boldsymbol{F}$ would act like a **knowledge base** KB containing a large number of logic **rules**, as in the setting of classical logic-based AI.

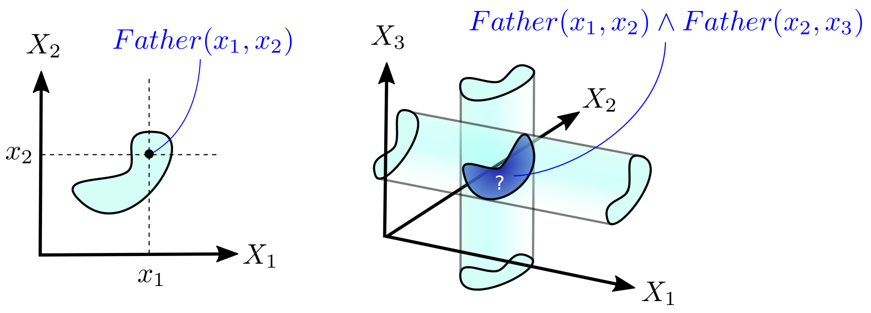
8.2 Geometry induced by logic rules

A logic rule is a conditional formula with variables. For example:

$$\forall X\ \forall Y\ \forall Z.\quad \text{father}(X,Y) \wedge \text{father}(Y,Z) \Rightarrow \text{grandfather}(X,Z) \tag{11}$$

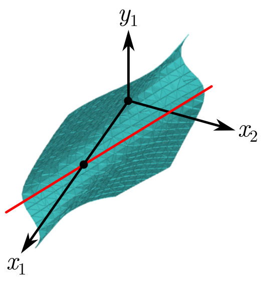
where the red lines show what I call "linkages" between different appearances of the same variables.

Quantification of logic variables, with their linkages, result in **cylindrical** and **diagonal** structures when the logic is interpreted *geometrically*. This is the reason why Tarski discovered the **cylindric algebra** structure of first-order predicate logic [14] [15] [1] [5] [6]. Cylindrical shapes can arise from quantification as illustrated below:



(12)

And "linkages" cause the graph of the $\vdash$ map to *pass through* diagonal lines such as follows:



(13)

We are trying to use neural networks to approximate such functions (*ie*, these geometric shapes). One can visualize, as the shape of neural decision-boundaries approximate such diagonals, the matching of first-order objects gradually go from partial to fully-quantified ∀ and ∃. This may be even better than if we fix the logic to have exact quantifications, as quantified rules can be learned gradually. There is also *empirical* evidence that NNs can well-approximate logical maps, because the *symbolic* matching and substitution of logic variables is very similar to what occurs in *machine translation* between natural languages; In recent years, deep learning is fairly successful at the latter task.

8.3 Form of a logic rule

So what exactly is the logic structure? Recall that inside our RL model:

- state $\boldsymbol{x} \in \mathbb{X}$ = mental state = set of logic propositions $\mathsf{P}_i \in \mathbb{P}$
- environment = state space $\mathbb{X}$ = mental space
- actions $\boldsymbol{a} \in \mathbb{A}$ = logic rules

For our current prototype system, an action = a logic **rule** is of the form:

$$\underbrace{\overbrace{\mathsf{C}^1_1\,\mathsf{C}^1_2\,\mathsf{C}^1_3 \;\wedge\; \underbrace{\mathsf{C}^2_1\,\mathsf{C}^2_2\,\mathsf{C}^2_3}_{} \;\wedge\; .... \;\wedge\; \mathsf{C}^k_1\,\mathsf{C}^k_2\,\mathsf{C}^k_3}^{\text{conjunction of } k \text{ literal propositions}}}_{\text{each literal made of } m \text{ atomic concepts, } m = 3 \text{ here}} \quad\Rightarrow\quad \overbrace{\mathsf{C}^0_1\,\mathsf{C}^0_2\,\mathsf{C}^0_3}^{\text{conclusion}} \tag{14}$$

where a **concept** can be roughly understood as a **word vector** as in Word2Vec [19]. Each $\mathsf{C} \in \mathbb{R}^d$ where d is the dimension needed to represent a single word vector or concept.

We use a "free" neural network (*ie*, standard feed-forward NN) to approximate the set of *all* rules. The **input** of the NN would be the state vector $\boldsymbol{x}$:

$$\mathsf{C}^1_1\,\mathsf{C}^1_2\,\mathsf{C}^1_3 \;\wedge\; \mathsf{C}^2_1\,\mathsf{C}^2_2\,\mathsf{C}^2_3 \;\wedge\; .... \;\wedge\; \mathsf{C}^k_1\,\mathsf{C}^k_2\,\mathsf{C}^k_3. \tag{15}$$

We fix the number of conjunctions to be k, with the assumption that conjunctions of length $< k$ could be filled with "dummy" (always-true) propositions.

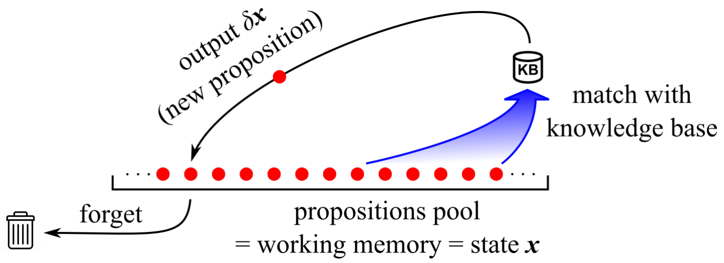
The **output** of the NN would be the conditional **probability** of an action:

$$P(\text{action} \mid \text{state}) := \pi(\,\mathsf{C}_1\,\mathsf{C}_2\,\mathsf{C}_3 \mid \boldsymbol{x}). \tag{16}$$

Note that we don't just want the action itself, we need the **probabilities** of firing these actions. The **Bellman update** of reinforcement learning should update the conditional probabilities over such actions (§**??**).

8.4 Structure of a logic-based AI system

Besides the intrinsic structure of a logic, the AI system has a structure in the sense that it must perform the following operations iteratively, in an endless loop:



(17)

- **Matching** — the KB of rules is matched against the current state $\boldsymbol{x}$, resulting in a (stochastically selected, *eg* based on ϵ-greedy) rule:

$$\begin{aligned} \boxed{\text{Match}} \quad (\boldsymbol{x} \overset{?}{=} \text{KB}) \;&: \mathbb{X} \to (\mathbb{X} \to \mathbb{P}) \\ \boldsymbol{x} &\mapsto \boldsymbol{r} \end{aligned} \tag{18}$$

  — In categorical logic, matching is seen as finding the **co-equalizer** of 2 terms which returns a **substitution** [4] [8] [9] [7]. The substitution is implicit in our formulation and would be *absorbed* into the neural network in our architecture.
  — Matching should be performed over the entire **working memory** = the state $\boldsymbol{x}$ which contains k literals. This is combinatorially time-consuming. The celebrated ***Rete*** algorithm [18] turns the set of rules into a tree-like structure which is efficient for solving (**??**).

- **Rule application** — the rule is applied to the current state $\boldsymbol{x}$ to produce a new literal proposition $\delta\boldsymbol{x}$:

$$\boxed{\text{Apply}} \quad \begin{aligned} \boldsymbol{r} : \mathbb{X} &\to \mathbb{P} \\ \boldsymbol{x} &\mapsto \boldsymbol{r}(\boldsymbol{x}) = \delta\boldsymbol{x} \end{aligned} \tag{19}$$

- **State update** — the state $\boldsymbol{x}$ is *destructively* updated where one literal $\mathsf{P}_j \in \boldsymbol{x}$ at the j-th position is **forgotten** (based on some measure of attention / interestingness) and over-written with $\delta\boldsymbol{x}$:

$$\boxed{\text{Update}} \quad \boldsymbol{x} = (\mathsf{P}_1, \mathsf{P}_2, ..., \mathsf{P}_j, ..., \mathsf{P}_k) \mapsto (\mathsf{P}_1, \mathsf{P}_2, ..., \delta\boldsymbol{x}, ..., \mathsf{P}_k) \tag{20}$$

All these operations are represented by functions parametrized by some variables Θ and they must be made *differentiable* for gradient descent.

8.5 Commutativity of logic conjunctions

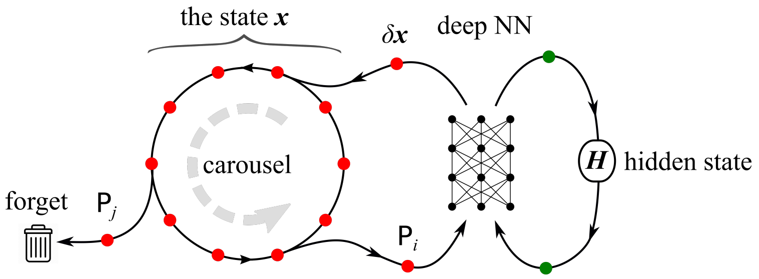
One basic characteristic of (classical) logic is that the conjuction $\wedge$ is **commutative**:

$$\mathsf{P} \wedge \mathsf{Q} \quad \Leftrightarrow \quad \mathsf{Q} \wedge \mathsf{P}. \tag{21}$$

This restriction may significantly reduce the size of the search space. If we use a neural network to model the deduction operator $\vdash: \mathbb{P}^k \to \mathbb{P}$, where $\mathbb{P}$ is the space of literal propositions, then this function should be **symmetric** in its input arguments.

I have considered a few solutions to this problem, including an algebraic trick to build "symmetric" neural networks (but it suffers from combinatorial inefficiency), and using Fourier transform to get a "spectral" representation of the state, which remained rather vague and did not materialize.

As of this writing [3] I have settled on the "carousel" solution: All the propositions in working memory will enter into a loop, and the reasoning operator acts on a hidden state $\boldsymbol{H} =$ ● and one proposition $\mathsf{P}_i =$ ● at a time:



(22)

Notice that the working memory $\boldsymbol{x}$ is itself a hidden state, so $\boldsymbol{H}$ can be regarded as a *second-order* hidden state.

This architecture has the advantage of being simple and may be biologically plausible (the human brain's working memory).

I believe in the maxim: *Whatever can be done in time can be done in space.* The diagram (**??**), when unfolded in time, can be expressed in this functional form:

$$\boldsymbol{F}_{\text{sym}}(\mathsf{P}_0, ..., \mathsf{P}_k) = \boldsymbol{f}(\mathsf{P}_k, \boldsymbol{f}(\mathsf{P}_{k-1}, \boldsymbol{f}(..., \boldsymbol{f}(\mathsf{P}_0, \vec{\emptyset})))). \tag{23}$$

$\boldsymbol{F}_{\text{sym}}$ means that the function is invariant under the action of the symmetric group $\mathfrak{S}_k$ over propositions. Such symmetric NNs have been proposed in [3] [2] [**Ravanbakhsh2016**] [12] [**Qi2016**] [11] [20].

8.6 Logic actions

The output of a logic rule is a proposition $\delta\boldsymbol{x} \in \mathbb{P}$, the space of propositions. This is a continuous space (due to the use of Word2Vec embeddings).

[3] Convolutional NNs are only *translation*-invariant. The Transformer [16] architecture is *equivariant* under permutations (meaning permuted inputs give permuted outputs), but it implicitly contains a recurrence similar to ours.

From the perspective of reinforcement learning, we are performing an action $\boldsymbol{a}$ (the logic rule) from the current state $\boldsymbol{x}$. The neural network in (**??**) is a parametrized function $\boldsymbol{F}_\Theta$ that accepts $\boldsymbol{x}$ and outputs $\delta\boldsymbol{x}$. We want to **gradient-descent** on Θ to get the optimal set of actions, ie, a **policy**.

To solve the RL problem, 2 main options are: **value-iteration** (*eg* Q-learning) and **policy-iteration**.

In **Q-learning**, we try to learn the action-value function $Q(\boldsymbol{a}|\boldsymbol{x}) \to \mathbb{R}$. The policy is obtained by choosing $\arg\max\limits_{\boldsymbol{a}} Q(\boldsymbol{a}|\boldsymbol{x})$ at each step. In our logic setting, the action space $\mathbb{A} \ni \boldsymbol{a}$ is continuous. As is well known, if we use an NN to represent $Q(\boldsymbol{a}|\boldsymbol{x})$, the evaluation of $\arg\max\limits_{\boldsymbol{a}}$ would be rather awkward, which is why Q-learning is widely seen as ill-suited for continuous actions.

It is much easier for the NN to directly output (a fixed number of) stochastic actions, thus avoiding the *curse of dimensionality.* Such a function is a **stochastic policy** $\pi(\boldsymbol{a}|\boldsymbol{x})$.

In other words, we can use a fixed number of Gaussian kernels (radial basis functions) to approximate the conditional probability distribution of π over $\mathbb{A}$. For each state $\boldsymbol{x}$, our NN outputs a probabilistic *choice* of c actions. So we only need to maintain c "peaks" given by Gaussian kernels. Each peak is determined by its mean $\delta\boldsymbol{x}_i$ and variance σ. Both parameters are to be learned.

The size of the FFNN in (**??**) seems well within the capacity of current hardware.

8.7 Forgetting uninteresting propositions

In (**??**) and (**??**), some propositions need to be forgotten based on some measure of **interestingness**. One way to measure interestingness is through the value function of a state, $V(\boldsymbol{x})$, where $\boldsymbol{x}$ consists of propositions $\boldsymbol{p}_i$. Suppose that $V(\boldsymbol{x})$ is learned by a neural network, then it may be possible to extract (backwards) the weight by which a proposition $\boldsymbol{p}_i$ contributed to the value V. For this to work, the function $V(\boldsymbol{x})$ should be deliberately **regularized** so that it would *generalize broadly.* Notice that $V(\boldsymbol{x})$ would also be *symmetric* in the $\boldsymbol{p}_i$'s so it would have the architecture of (**??**). $V(\boldsymbol{x})$ is very similar to $Q(\boldsymbol{a}|\boldsymbol{x})$ but should be learned separately because they serve different purposes.

9 Remaining work

- Replace the recurrent NN architecture with symmetric NNs
- In this minimal architecture there is no **episodic memory** or **meta-reasoning** ability, but these can be added to the architecture and are not bottleneck problems. For example, meta-reasoning can be added via turning the input to introspection.
- Implementation of the system is currently under way.