

# AGI via Combining Logic and Deep Learning

AGI 2021 Conference

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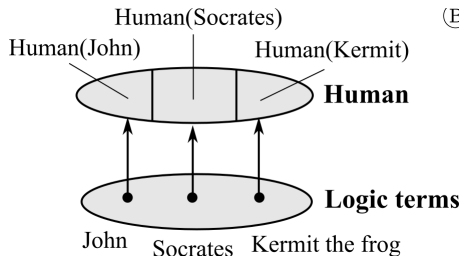
# Part I

## Categorical Logic

- Quantifier as adjunctions

# Fibrations

- Fibration

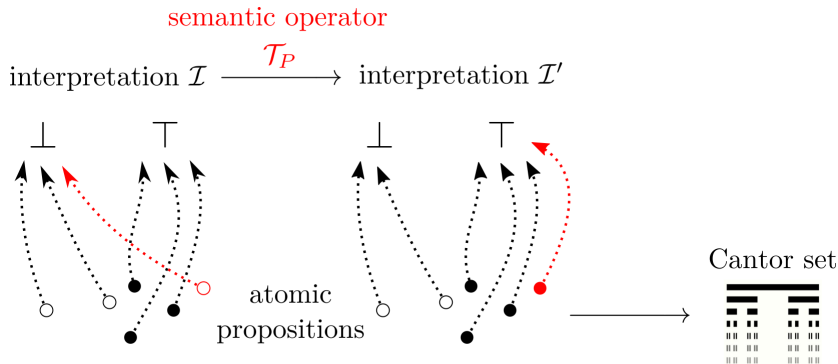


## Part II

### Model-based vs Syntax-based

# Hitzler's Core Method

- The **semantic operator**  $\mathcal{T}_P$  updates an interpretation to another interpretation:



# Hitzler's Core Method

- The **level mapping**  $\iota$  takes an interpretation to a real number:

$$\begin{array}{ccc} \mathcal{I} & \xrightarrow{\mathcal{T}_P} & \mathcal{I} \\ \downarrow \iota & & \downarrow \iota \\ \mathcal{K} & \xrightarrow{f_P} & \mathcal{K} \end{array} \quad \iota(\mathcal{I}) = \sum_{\text{atom} \in \mathcal{I}} b^{-\ell(\text{atom})} \in \mathbb{R}$$

approximated by  
neural network



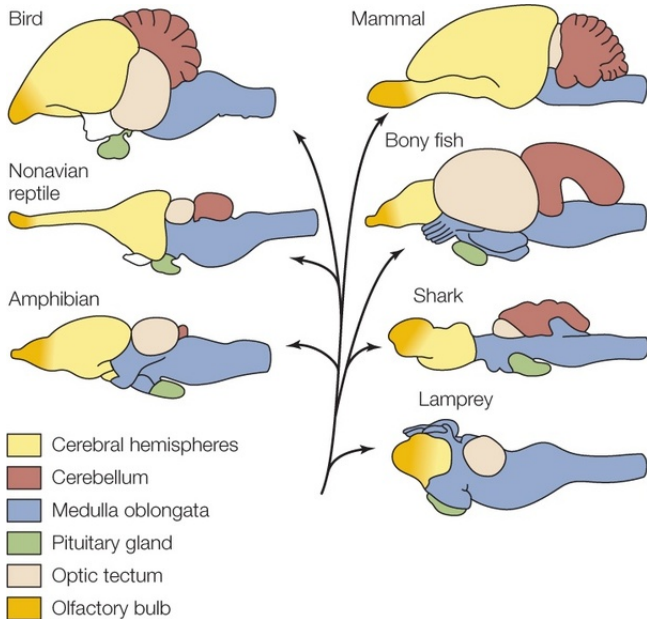
## Part III

# Reinforcement Learning

This is a lamprey



# Evolution of the Neocortex



# Neocortex vs Reinforcement Learning

Neocortical Learning



+1

+100

+10



food



sex



money

associations

action<sub>1</sub>

action<sub>2</sub>

action<sub>3</sub>

Reinforcement Learning



reward  $R_x$

$$Q \leftarrow Q + \eta \left[ R_x + \gamma \max_{a'} Q' - Q \right]$$

(Q-learning)

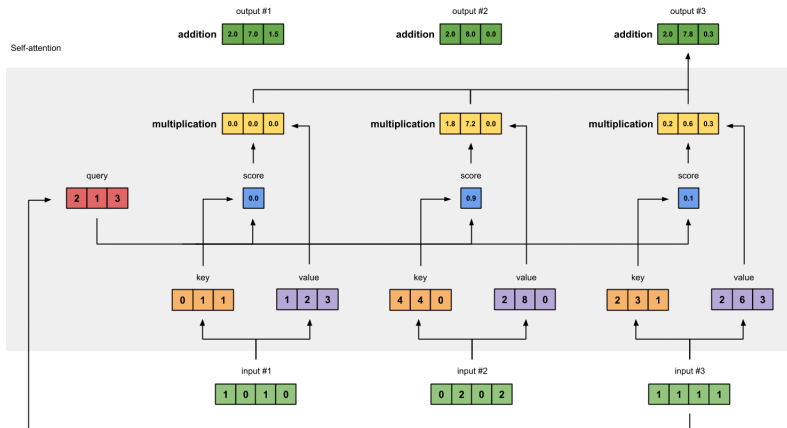
action  $a$

## Part IV

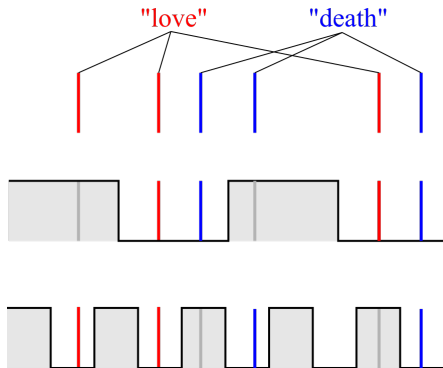
BERT / GPT

# Equivariance of the Transformer

- Many people already know this:  
the Transformer module is **equivariant** in its inputs and outputs:



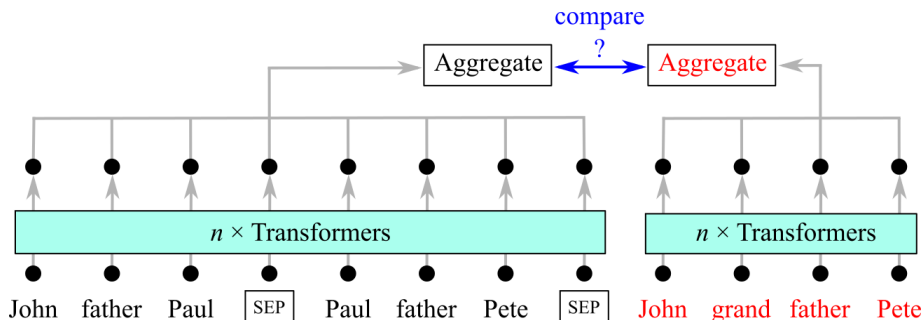
# Additivity of Word Embeddings



- This suggests that word embeddings can be added together to give **composite** meanings

# BERT's Hidden Representation

- This may be a close description of how BERT performs question-answering or logic deduction:





## Part V

# BERT + Knowledge Graphs

# Memory Formation

- Memory

# Memory Recall

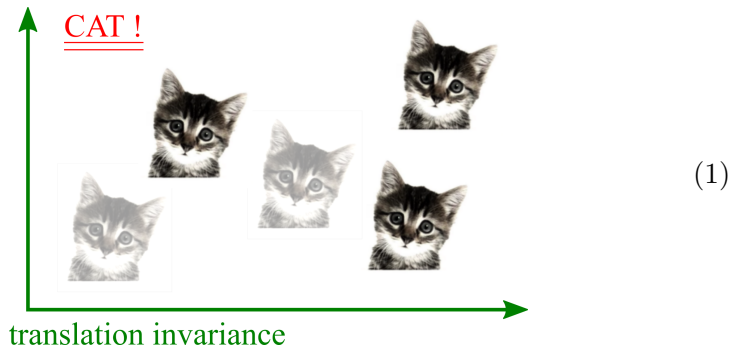
- Memory retrieval

## Part VI

### Old Stuffs

# The success of CNN in computer vision

- In geometry, vision is said to possess the property of **translation invariance**:



- **Convolution** is an operation invariant under translation:

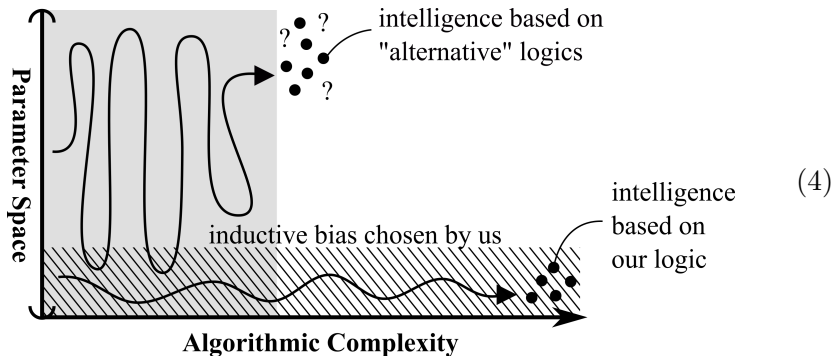
$$(T_x \circ f) * g = T_x \circ (f * g) \quad (2)$$

- Yann LeCun *et al* exploited the symmetry of CNNs to accelerate learning, successfully solved the visual recognition problem



# Symmetry and inductive bias

- In mathematics, symmetry often simplifies computation, which is why mathematicians love to study symmetries
- In machine learning, one introduces **inductive bias** to narrow down the search space:



- Oftentimes, if inductive bias is chosen correctly, solution is found quickly, otherwise problem becomes **intractable**

## Richard Sutton's view

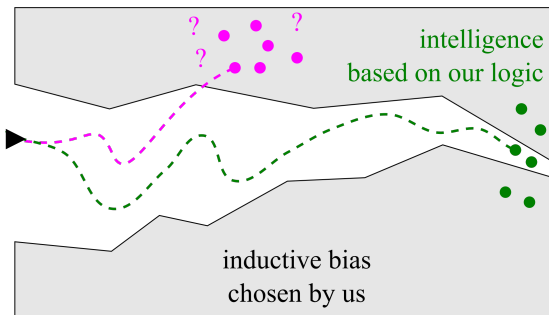
- In contrast, Sutton expressed the view that AI can be solved merely by **increasing computing power**, under the reinforcement learning framework





- Our choice is just one out of many possible **forms** of logic:

intelligence based on "alternative" logics



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- This is not only a theoretical issue;  
Indeed, AI labs around the world had begun the search for AGI with various strategies!

# Doubts about logicism

- Admittedly, the human brain does **not** think like symbolic logic
- By symbolic logic, we mean any “linear” (sequential) representation
- In contrast, **pictures** and **music** are *multi*-dimensional
- The brain may use representations that are not sequential or symbolic
- However, logic is a very efficient representation (from a computer perspective) even though it deviates from the biological brain
- Neuroscience is too difficult to crack, thus not a practical route to AGI
- Human cognition may be much closer to logic than we’ve thought. If we’re given a description: “woman finds new lover, murders husband”:

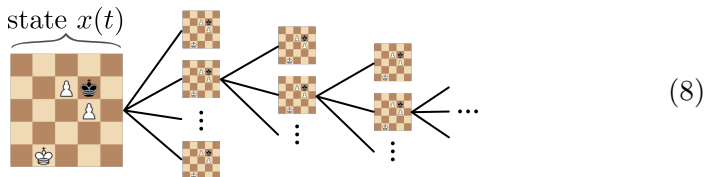


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We may not know: Is the woman blonde? What is she wearing? In other words, our “mental model” is devoid of details and it may not possess any more information than just a sequence of symbols.

# Reinforcement learning

- Think of the “**state**” in reinforcement learning like a **board position** in a chess game:



- Reinforcement learning seeks to maximize the total **rewards** accrued over a (possibly infinite) **time horizon**:

$$\text{maximize } S = \int_0^{\infty} L \, dt \quad (9)$$

- Such maximization gives the AI **intelligence** because it is often beneficial to **delay** rewards, eg: to plot a clever chess move
- But reinforcement learning is a **brute force** approach; we need to give the model some additional **inductive bias**, eg, in the form of **logical structure**

# Structure of logic

- The idea is: introduce symmetries of **logic** into deep learning to solve the AGI problem
- Because human cognition has logical structure, this inductive bias may help us find a solution to AGI faster
- Logic is a complicated structure, but its simplest symmetry is the **commutativity** (or permutation invariance) of **propositions**:

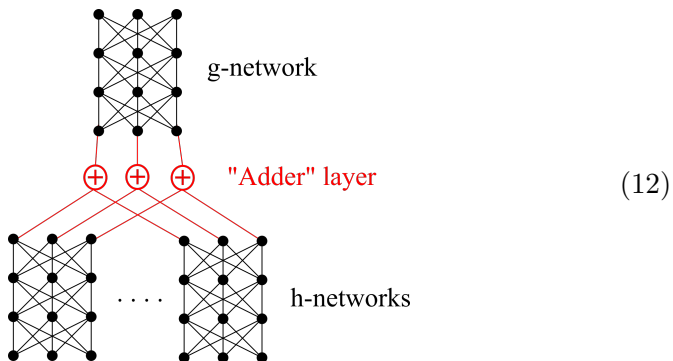
$$\begin{array}{ccc} A \wedge B & \equiv & B \wedge A \\ \text{it's raining} \wedge \text{lovesick} & \equiv & \text{lovesick} \wedge \text{it's raining} \end{array} \quad (10)$$

- Its importance may be analogous to translation invariance in vision
- The significance of commutativity is: it **decomposes** the AI system's **mental state** into individual **propositions**

# Symmetric neural networks

- Permutation invariance can be handled by **symmetric** neural networks
- I wasted 2 years trying to solve this problem, and then found out it had been solved 3 years before: [PointNet 2017] and [DeepSets 2017] and their mastery of mathematics is significantly above me!
- Any symmetric function can be represented by the following form (a special case of the Kolmogorov-Arnold representation of functions):

$$f(x, y, \dots) = g(h(x) + h(y) + \dots) \quad (11)$$



- Sym NN gives a powerful boost in efficiency  $\propto n!$  where  $n = \#inputs$
- The code for Sym NN is just a few lines of Tensorflow:

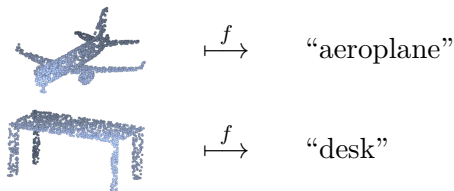
```
h = Dense(3, activation='tanh')
ys = []
for i in range(9):
    ys.append( h(xs[i]) )
y = Keras.stack(ys, axis=1)
Adder = Lambda(lambda x: Keras.sum(x, axis=1))
y = Adder(y)
g = Dense(3)
output = g(y)
```

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- Very easy to adopt this to existing models such as BERT and reinforcement learning
- I have successfully tested it on the game of TicTacToe:  
<https://github.com/Cybernetic1/policy-gradient>

# For example: symmetric NN for object recognition

- Imagine objects represented as **point clouds**:

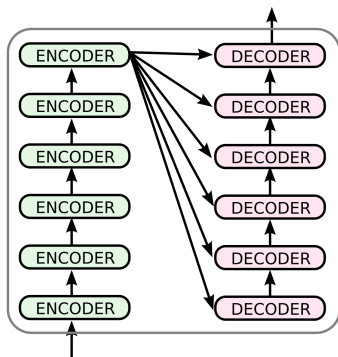


- It does not matter **in what order** the points are in a sequence; the function  $f(x_1, \dots, x_n)$  is symmetric in its arguments (the points)
- Permutation invariance is **essential** for this to work

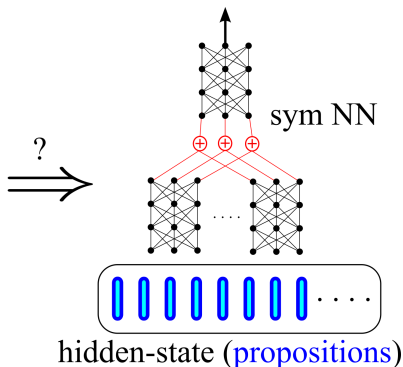
# Logicalization of BERT

- Similarly, we can convert BERT's hidden state into a set of propositions, by replacing the original **decoder** with a sym NN:

original BERT / Transformer



logic BERT ?



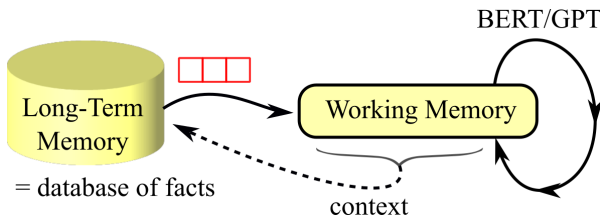
(14)

- The original **encoder** can be retained. As the **decoder** imposes symmetry on the hidden state, error propagation is expected to cause its representation to change
- Of course, this remains to be proven by experiment 😊



# Advantages of logical AI (1)

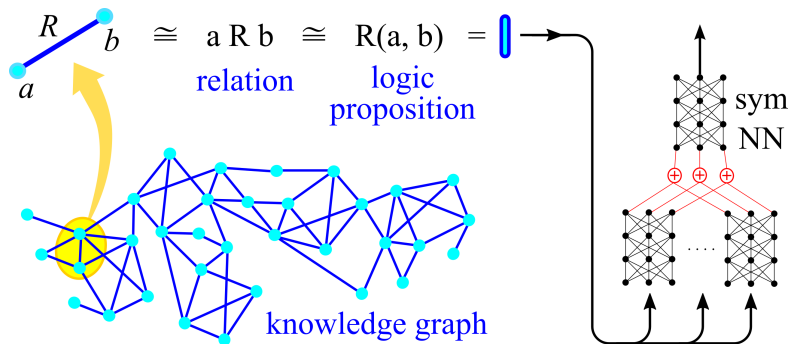
- With logic, it becomes easy to design cognitive architectures, eg: long-term memory module



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## Advantages of logical AI (2)

- Integrate seamlessly with **knowledge graphs**
- graphs are made up of edges,  
edges = relations between nodes = propositions:



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# References

Thanks for watching 😊

## **Illustration credits:**

- Translation invariance, from Udacity Course 730, Deep Learning (L3 Convolutional Neural Networks ▷ Convolutional Networks)