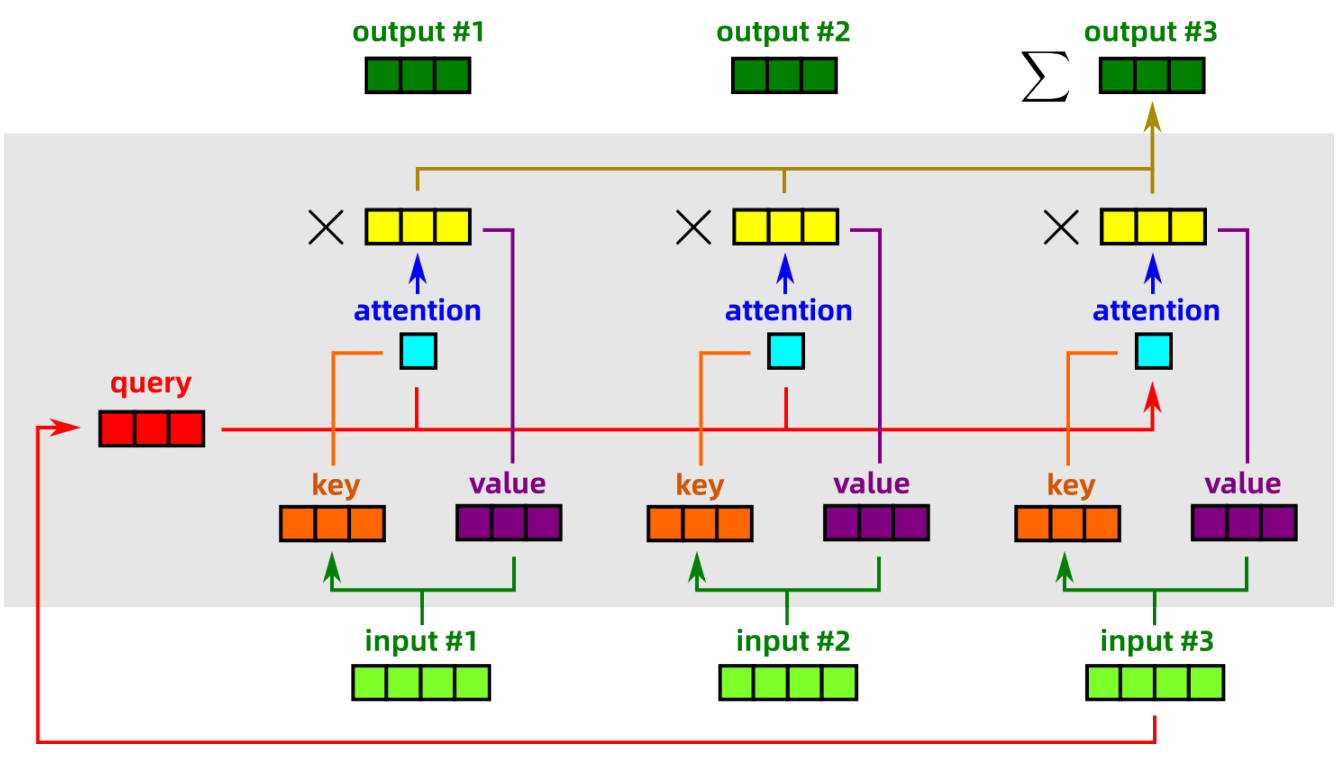
Transformer as Logic-Base

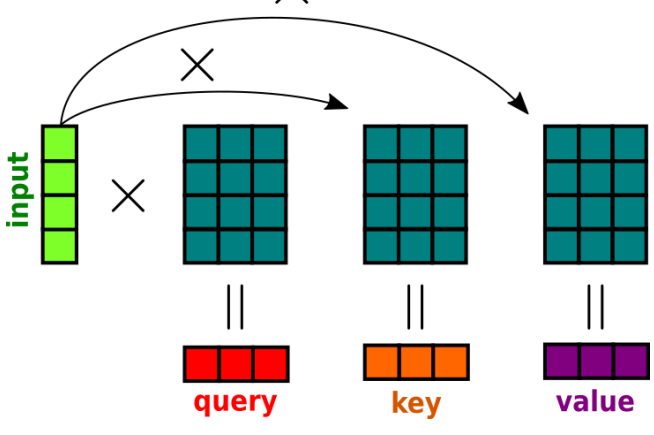
In this infographic I'd explain a major finding that is the culmination of many years of my research: the Transformer is a symbolic-logic machine.

For your convenience let's refresh on the Transformer's Self-Attention mechanism:



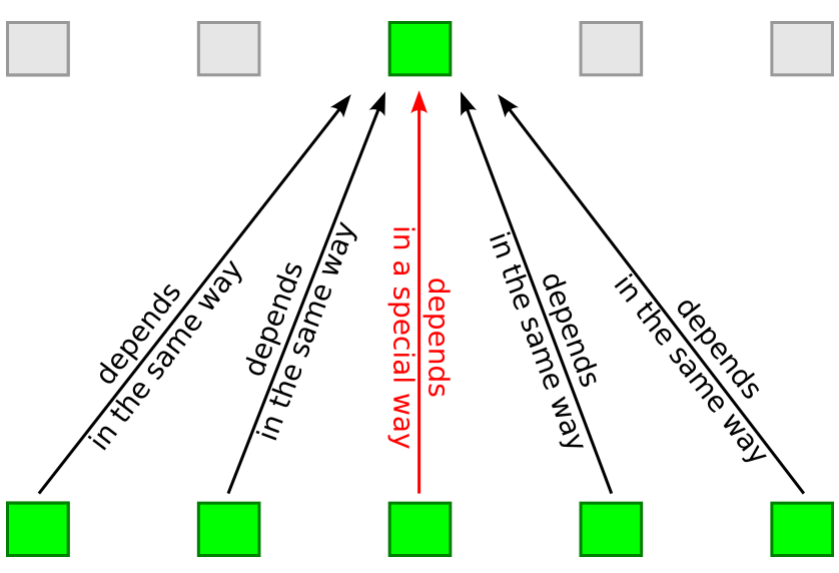
(1)

"Input" tokens are translated to Q, K, V (query, key, value)'s via matrix multiplication, which can be regarded as a kind of table look-up, or memory store:



(2)

From an abstract point of view, the Transformer has the following structure, which gives rise to its equivariance property (if input elements are swapped in a certain order, the output elements changes the same way):



(3)

The equivariance property corresponds to the exchangeability of logic propositions:

$$A \wedge B \quad \Leftrightarrow \quad B \wedge A \tag{4}$$

For example:

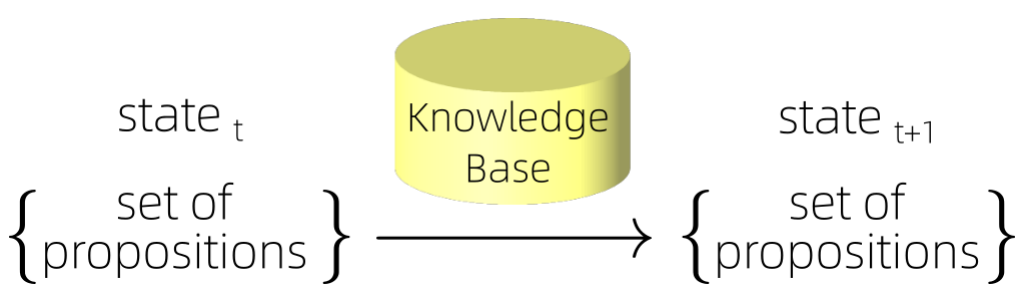
$$\text{it's raining} \wedge \text{I'm heart-broken} \quad \Leftrightarrow \quad \text{I'm heart-broken} \wedge \text{it's raining} \tag{5}$$

Propositions are made up of atomic concepts, but here, at the sub-propositional level, atoms cannot be permuted freely, eg:

$$\text{I} \cdot \text{love} \cdot \text{her} \quad \neq \quad \text{she} \cdot \text{loves} \cdot \text{me} \tag{6}$$

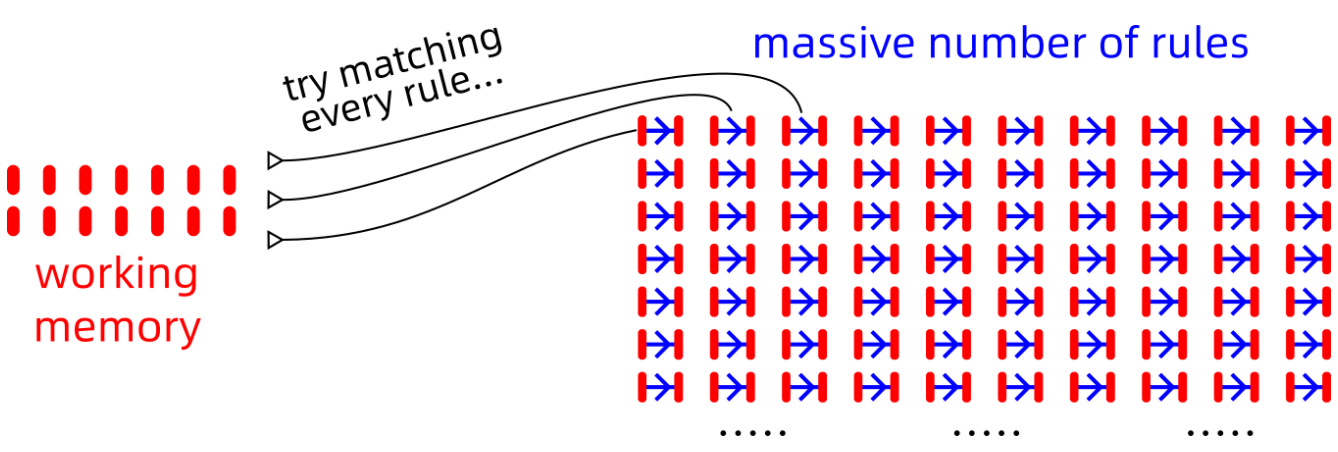
otherwise there would be no such things as heart-breaks.

Now let's refresh a bit on classical logic-based AI. This is its basic architecture:



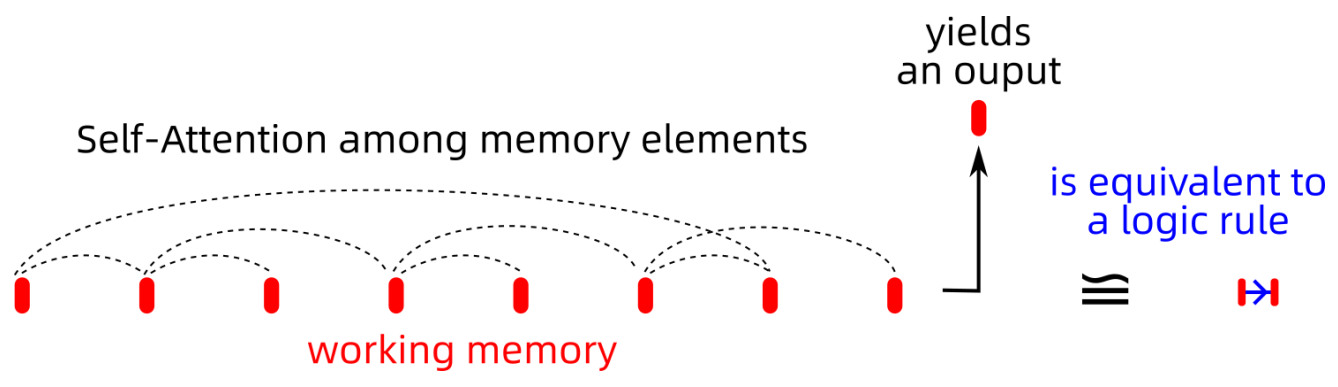
(7)

There would be a huge number of rules in the Knowledge Base, and the system needs to match these rules one by one against propositions in the system's state (= working memory):



(8)

For the Transformer, it is a kind of memory stored between input elements (stored as the Q, K, V matrices), and it implicitly plays the role of a logic rule-base:

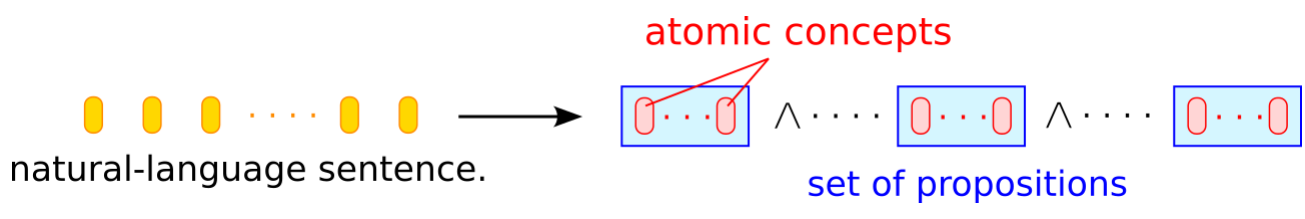


(9)

A crucial insight is that the Self-Attention mechanism has its origin in NTMs (Neural Turing Machines) proposed by Graves *et al* 2014. The Turing machine needs to have a "memory tape" but in the neural setting this memory must be *differentiable*. If the memory is addressed by an index $i \in \mathbb{N}$, then it won't be differentiable. So they came up with a content-addressable memory mechanism where a memory matrix is looked up using the "query-key-value" method. A nice explantion of NTMs can be found in the book *Fundamentals of Deep Learning* [Buduma, Locascio 2017].

Now consider LLMs (Large Language Models) such as BERT and GPT.

Given a natural-language sentence, we'd like to convert or decompose it into a bunch of logic propositions:

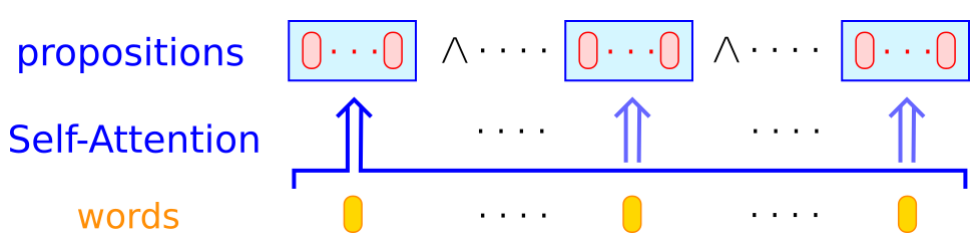


(10)

The structure on the right of (10) is a mental state of a logical AI system. It is composed of (exchangeable) propositions, which are in turn made up of atomic concepts. This 2-level structure is characteristic of all logical systems.

Surprisingly, I found out that the Transformer completely satisfies this 2-level logic structure.

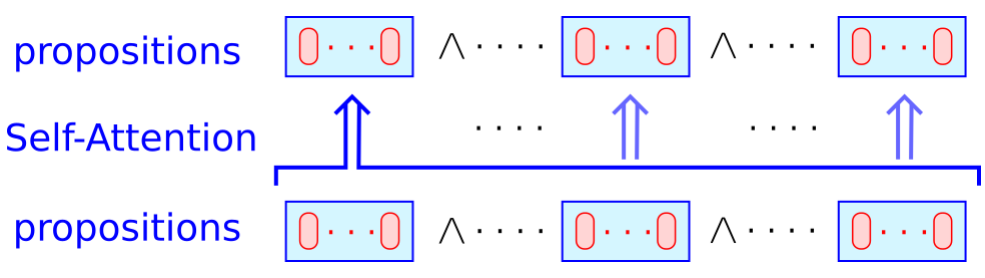
On the first layer, a Transformer transforms each input word token into one proposition:



(11)

The crucial point here is that the propositions are composed of atoms (⬭), this is achieved in the Transformer by adding vectors (that represent atomic concepts), ie, by superposition. Note also that the Transformer is equivariant, so we must add "positional encoding" to each word, to indicate their ordering.

At higher layers, there is no need for positional encoding, and logic propositions can be freely exchanged, exactly as what happens in Transformers:



(12)

Note that in the above, every ⇑ arrow uses the same (Q, K, V) matrices as "rule-base", that may limit the number of rules that can be represented. To circumvent this, Multi-Head Attention allows to use different (Q, K, V) matrices on the same layer.