

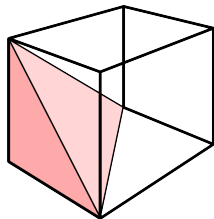
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# Measuring the “size” of hypothesis spaces over $\mathbb{R}$ from the perspective of No Free Lunch

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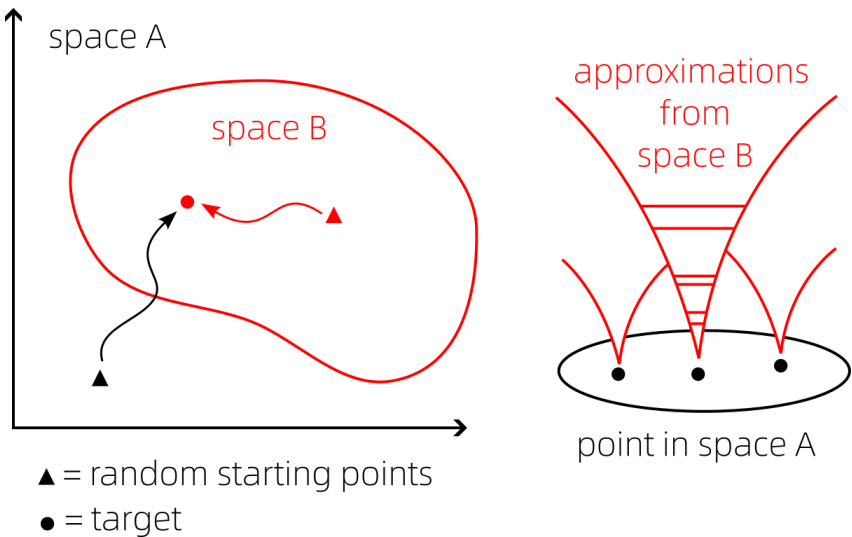
## Motivating example



(1)

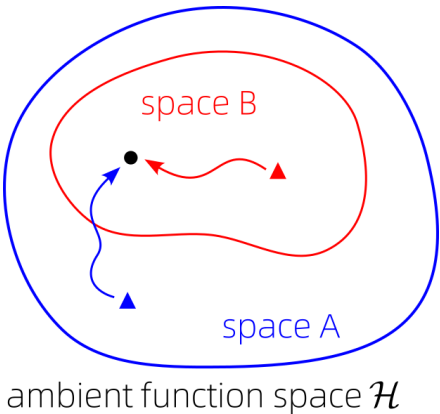
The **input space** is  $X = \mathbb{R}^n = n$ -dimensional hypercube. Permutation symmetry implies that only one corner of the hypercube need be considered, this is the **fundamental domain**. The **volume** of this domain over the entire hypercube shrinks exponentially as  $n$  grows, which appears to suggest that this symmetry is very significant for efficiency consideration. We want to quantify the notion of this efficiency.

We want to compare the **hypothesis spaces**  $A$  and  $B$ :  $A$  = unconstrained, fully-connected neural network with  $L$  layers and  $N$  total weights. The parameter space  $\Theta_A = \mathbb{R}^N$ .  $B$  = symmetric neural network with a special structure but its parameter space is also  $\Theta_B = \mathbb{R}^M$ .



(2)

It’s easier to consider  $A$  and  $B$  both embedding in an ambient function space  $\mathcal{H}$ :



(3)

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