AGI from the perspectives of Categorical Logic and Algebraic Geometry

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Abstract. To "situate" AGI in the context of current mathematics, so that mathematics-minded professionals can more easily see whether current mathematical ideas can be fruitfully applied to AGI.

Keywords: AGI · categorical logic · homotopy type theory · Curry-Howard isomorphism · algebraic geometry.

1 Goal of this paper

The bottleneck of AGI development is the speed of learning algorithms. The cost of training GPT-4 was rumored to be $100M (by Sam Altman). To speed up learning, one needs to introduce inductive biases, according to the No Free Lunch theorem. One principled way to introduce inductive bias is by imposing the structure of logic. One reasons that, if humans have discovered the structure of logic in this world, an intelligent program may re-discover the same structure. So our question is: what is the mathematical structure of logic?

Traditionally this line of research is called algebraic logic, starting from Leibniz and Boole, up to more recent times Tarski's cylindrical algebra and Paul Halmos' work.

2 Our conclusions so far

2.1 Basic categorical logic

2.2 Modal logic

2.3 Homotopy type theory

2.4 Algebraic geometry

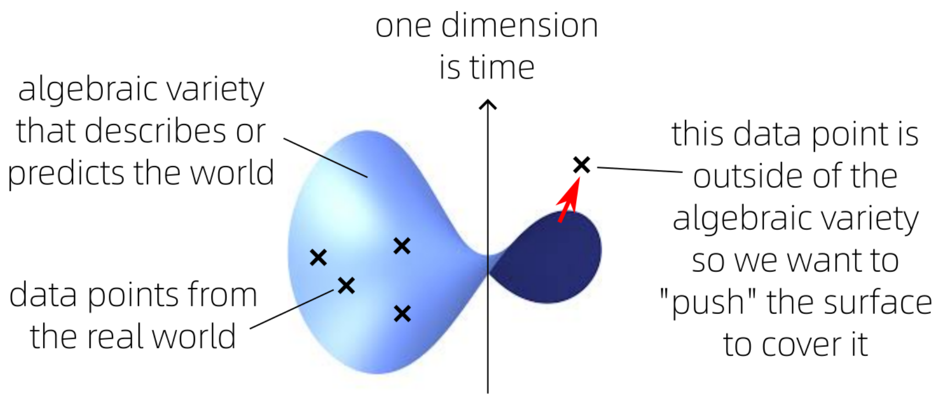
The fundamental duality in algebraic geometry is:

$$\left\{\begin{array}{c}\text{spaces, or}\\ \text{varieties}\end{array}\right\} \longleftrightarrow \left\{\begin{array}{c}\text{commutative}\\ \mathbf{k}\text{-algebras}\end{array}\right\} \tag{1}$$

within this correspondence, we have:

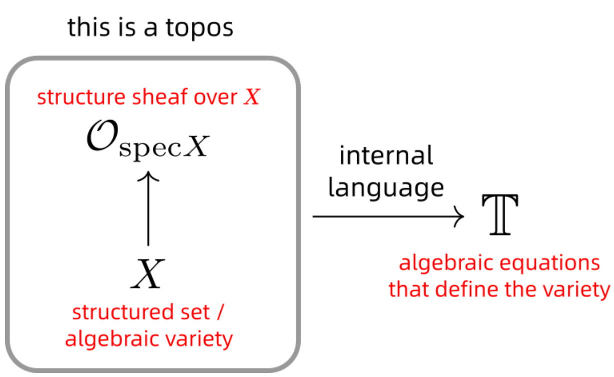
$$\left\{\begin{array}{c}\text{points in}\\ \text{geometry}\end{array}\right\} \longleftrightarrow \left\{\text{prime ideals}\right\} \tag{2}$$

My visualization of the "Grothendieck picture" of AGI is like this:



One of the interesting discoveries in categorical logic is that every topos admits an **internal language**. This is a simple consequence of Curry-Howard: since a type-space corresponds to a logic proposition, and categorical logic interprets a type-space as an object in the category, thus every category (satisfying extra conditions) can be interpreted as having an "internal" logic. The converse of this correspondence is the **classifying topos** of a theory:

$$\underset{\text{classifying topos}}{\mathcal{E}_{\mathbb{T}}} \overset{\text{internal language}}{\rightleftarrows} \underset{\text{theory}}{\mathbb{T}} \tag{3}$$



(4)

An approach suggested by Yuri Manin is turn logic into an algebra, such as the Boolean ring. But this can only handle propositional logic. Varieties defined by such Boolean polynomials [1] live in the space $\mathbb{Z}_2^n$, the discrete hypercube.

Theorem 1. *This is a sample theorem. The run-in heading is set in bold, while the following text appears in italics. Definitions, lemmas, propositions, and corollaries are styled the same way.*

Acknowledgements Please place your acknowledgments at the end of the paper, preceded by an unnumbered run-in heading (i.e. 3rd-level heading).

Table 1. Table captions should be placed above the tables.

Heading level	Example	Font size and style
Title (centered)	**Lecture Notes**	14 point, bold
1st-level heading	**1 Introduction**	12 point, bold
2nd-level heading	**2.1 Printing Area**	10 point, bold
3rd-level heading	**Run-in Heading in Bold.** Text follows	10 point, bold
4th-level heading	*Lowest Level Heading.* Text follows	10 point, italic

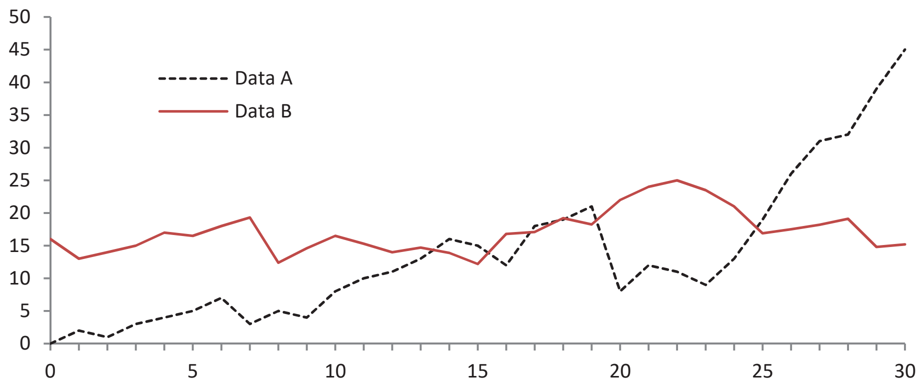


Fig. 1. A figure caption is always placed below the illustration. Please note that short captions are centered, while long ones are justified by the macro package automatically.

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