Research Statement

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Introduction

The recent development of efficient algorithms for optimization on the cone of positive semidefinite matrices has created a fundamental shift in how research is conducted. Specifically, semidefinite programming has been widely adopted in control theory and has led to a greater understanding of linear finite dimensional centralized systems. Unfortunately, this same understanding has not yet been extended to nonlinear, infinite dimensional or decentralized systems. In fact, many critical questions in analysis and control of these systems have been shown to be NP-hard. My research interest lies in finding ways to address these types of problems. One approach is to expand the definition of solution. Instead of giving a single necessary and sufficient condition, expressible as a semidefinite program, one can construct a nested sequence of sufficient conditions, of increasing accuracy, which converge to a necessary and sufficient condition and are expressible as semidefinite programs. Examples of this technique as well as others can be found below, in a brief description of my research.

Internet Congestion Control

The application which has motivated much of my research is stability analysis of proposed protocols for Internet congestion control. The analysis of congestion control protocols has received much attention recently. This work has been motivated by concern about the ability of current protocols to ensure stability and performance as the number of users and amount of bandwidth continues to increase. Although the protocols that have been used in the past have performed well as the Internet has increased in size, as capacities and delays increase instability will become a problem. In my work, I have considered global stability with delay of congestion control protocols which attempt to solve a distributed network optimization problem. These systems are described by differential equations with delay and contain non-static nonlinearity. Because of the nonlinearity and delays, proving convergence is difficult. By combining frequency and time domain techniques using a generalized passivity framework, I have shown for certain protocols that global stability with delay holds under the same conditions as local stability. These tight bounds, verified by experimental evidence, allow one to accurately predict when congestion control will fail. This knowledge and experience serves as an aid to the development of the next generation of congestion control protocols.

An Approach to Analysis and Synthesis

My research at Stanford has produced a number of new results and tools concerning the analysis of systems with delay, nonlinearity and decentralized structure. Specifically, I have proposed two new types of refutation which can be used to construct the positive quadratic Lyapunov-Krasovskii functionals necessary for stability of linear time-delay systems. Furthermore, I have shown how these refutations can be parameterized using the space of positive semidefinite matrices. This has resulted in a nested sequence of sufficient conditions, of increasing accuracy and expressible as semidefinite programs, which prove stability of linear time-delay systems. Thus for any desired level of accuracy, these results give a condition, expressible as a semidefinite program, which will test stability to that level of accuracy. In addition, because of the structure of the refutations, I have also been able to generalize these results to time-delay systems with nonlinearities and parametric uncertainty.

Significantly, these results provide an opportunity for further research. In particular, the question has arisen as to whether one can generalize other aspects of finite dimensional linear systems theory to linear time-delay systems. In recent work, I have been able to develop an algorithm to construct the inverses of the linear operators defining the quadratic functionals discussed above. By using these inverses, one can construct full-state feedback controllers for linear time-delay systems. This is the first step toward further results on algorithms for optimal control and minimum error estimation of this type of system. In addition, the Lyapunov-Krasovskii functionals discussed above can also be used to analyze other types of systems, such as those with distributed nonlinear dynamics or modeled by particular kinds of partial differential equations.

Research Interests and Direction

The analysis of nonlinear systems is a subject of much recent interest. Although linearization is a well-established approach to analysis, results obtained in this manner are always, at best, local. When considering changes in critical systems such as the Internet, the guarantee of convergence associated with a global stability result carries significant weight. Furthermore, some dynamical systems, such as are found in biology or high performance aircraft, are dominated by nonlinear behavior. In such cases, the practical value of linear analysis is limited. Apart from nonlinearity, the study of decentralized systems and systems with delay is also the topic of much active research. Communications systems, an issue of significant current interest, are often decentralized and, by virtue of their geographic reach, inevitably contain delay. Earth based telescopes are currently being built which consist of thousands of mirror segments; each individually actuated but required to move in a coordinated manner. Such systems are difficult to model using a state space of reasonable dimension.

In addressing these topics, my previous research and experience provide an immediate opportunity. Additionally, some specific future research goals which I believe to be of particular importance are, in no particular order; generalization of the KYP lemma to irrational polynomials, model reduction of nonlinear polynomial systems to nonlinear polynomial systems of lower dimension, modeling of distributed dynamics using partial differential equations, nonlinear estimation and nonlinear model validation.