

# **Spacecraft Dynamics and Control**

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Lecture 11: Intro to Rocketry

# Introduction

In this Lecture, you will learn:

## Introduction to Rocketry

- Mass Consumption
- Specific Impulse and Rocket Types
- $\Delta v$  limitations
- Staging

**Numerical Problem:** Suppose our mission requires a dry weight of 30kg. How much propellant is required to achieve a circular orbit of altitude 200km?

# Questions about Propulsion

We have talked a bit about  $\Delta v$ .

- How is  $\Delta v$  created?
- How expensive is it?
- Is it really instantaneous?

# $\Delta v$ budget

A Typical mission uses a lot of  $\Delta v$ .

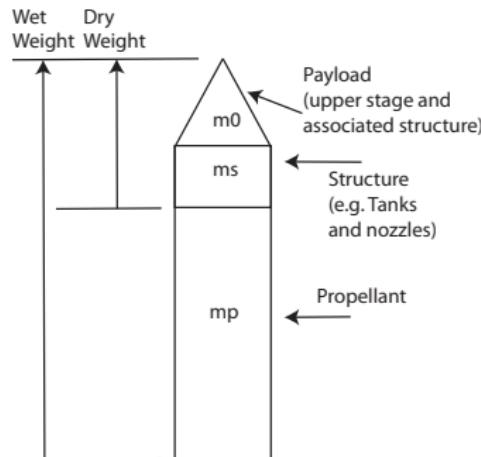
- How much propellant will we need?
- What is the maximum  $\Delta v$  budget?

Propulsion Function	Typical Requirement
<i>Orbit transfer to GEO (orbit insertion)</i> <ul style="list-style-type: none"><li>• Perigee burn</li><li>• Apogee burn</li></ul>	2,400 m/s 1,500 (low inclination) to 1,800 m/s (high inclination)
<i>Initial spinup</i>	1 to 60 rpm
<i>LEO to higher orbit raising <math>\Delta V</math></i> <ul style="list-style-type: none"><li>• Drag-makeup <math>\Delta V</math></li><li>• Controlled-reentry <math>\Delta V</math></li></ul>	60 to 1,500 m/s 60 to 500 m/s 120 to 150 m/s
<i>Acceleration to escape velocity from LEO parking orbit</i>	3,600 to 4,000 m/s into planetary trajectory
<i>On-orbit operations (orbit maintenance)</i> <ul style="list-style-type: none"><li>• Despin</li><li>• Spin control</li><li>• Orbit correction <math>\Delta V</math></li><li>• East-West stationkeeping <math>\Delta V</math></li><li>• North-South stationkeeping <math>\Delta V</math></li><li>• Survivability or evasive maneuvers (highly variable) <math>\Delta V</math></li></ul>	60 to 0 rpm $\pm 1$ to $\pm 5$ rpm 15 to 75 m/s per year 3 to 6 m/s per year 45 to 55 m/s per year 150 to 4,600 m/s
<i>Attitude control</i> <ul style="list-style-type: none"><li>• Acquisition of Sun, Earth, Star</li><li>• On-orbit normal mode control with 3-axis stabilization, limit cycle</li><li>• Precession control (spinners only)</li><li>• Momentum management (wheel unloading)</li><li>• 3-axis control during <math>\Delta V</math></li></ul>	3–10% of total propellant mass Low total impulse, typically <5,000 N·s, 1 K to 10 K pulses, 0.01 to 5.0 sec pulse width 100 K to 200 K pulses, minimum impulse bit of 0.01 N·s, 0.01 to 0.25 sec pulse width Low total impulse, typically <7,000 N·s, 1 K to 10 K pulses, 0.02 to 0.20 sec pulse width 5 to 10 pulse trains every few days, 0.02 to 0.10 sec pulse width On/off pulsing, 10 K to 100 K pulses, 0.05 to 0.20 sec pulse width

# Some Definitions

In a staged Launch system, the mass varies with time.

- Dry weight is the weight without propellant.
  - ▶ This is the final weight.
  - ▶ Craft plus payload
- There are several variations of dry weight.



Weight Parameters	Comments
1. <i>Spacecraft Dry Weight</i> plus Propellant Yields	Weight of all spacecraft subsystems and sensors, including weight growth allowance of 15–25% at concept definition
2. <i>Loaded Spacecraft Weight</i> plus Upper Stage Vehicle Weight Yields	Weight of propellant required by the spacecraft to perform its mission when injected into its mission orbit
3. <i>Injected Weight</i> plus Booster Adapter Weight Yields	Mission-capable spacecraft weight (wet weight) Weight of any apogee or perigee kick motors and stages added to the launch system
4. <i>Boosted Weight</i> plus Performance Margin Yields	Total weight achieving orbit May also include airborne support equipment on the Space Shuttle
5. <i>Payload Performance Capability</i>	Total weight that must be lifted by the launch vehicle The amount of performance retained in reserve (for the booster) to allow for all other uncertainties.
	This is the payload weight contractors say their launch systems can lift

# Lecture 11

## Spacecraft Dynamics

### Some Definitions

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- ▶ This is the final weight.
- Craft plus payload
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Definitions	Comments
Dry weight	Weight of hardware assembled for flight and ready for launching.
Propellant	Weight of propellant required for the mission.
Launch Mass	Weight of hardware assembled for flight and ready for launching.
Structural Mass	Weight of hardware required for the mission.
Payload	Weight of payload required for the mission.
Total Mass	The sum of all masses required for the mission.
Final Weight	The weight of the hardware when it reaches the end of its mission.
Propellant plus Payload	The weight of the hardware when it reaches the end of its mission.
Structural plus Payload	The weight of the hardware when it reaches the end of its mission.
Structural plus Propellant	The weight of the hardware when it reaches the end of its mission.
Structural plus Payload plus Propellant	The weight of the hardware when it reaches the end of its mission.
Structural plus Payload plus Propellant plus Dry weight	The total gross weight condition of the craft.

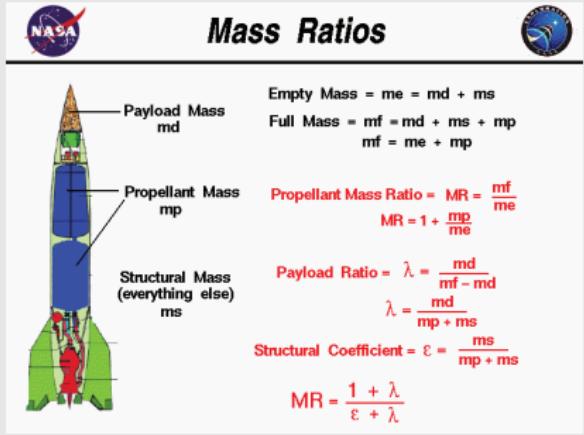


figure from NASA

# How to create Thrust: Newton's Second Law

**Approximation:** Consider the expulsion of a piece of propellant,  $\Delta m$ .

## Initial State:

- Propellant and Rocket move together.
- Total Momentum:

$$h_i = (m_r + \Delta m)v$$

## Final State:

- Propellant and Rocket move separately.
- Rocket has velocity  $v + \Delta v$
- Propellant has velocity  $v - c$ .
  - ▶  $c$  is the exhaust velocity
- Total Momentum:

$$h_f = m_r(v + \Delta v) + \Delta m(v - c)$$

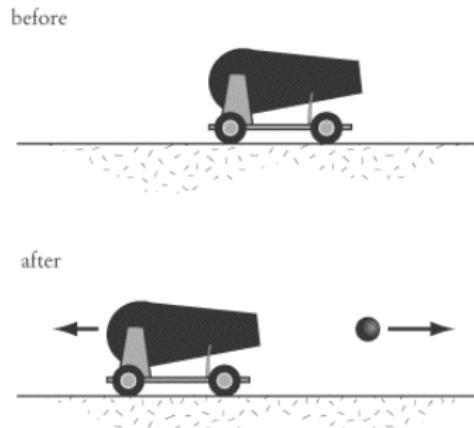
## Conservation of Momentum:

- Setting  $h_i = h_f$ , we obtain:

$$(m_r + \Delta m)v = m_r(v + \Delta v) + \Delta m(v - c)$$

- Solving for  $\Delta v$ , we obtain

$$\Delta v = \frac{\Delta m}{m_r}c$$



# Lecture 11

## Spacecraft Dynamics

### └ How to create Thrust: Newton's Second Law

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- Solving for  $\Delta v$ , we obtain:  $\Delta v = \frac{\Delta m}{m_r}v$

- In this slide,  $h_i$  is the initial linear momentum of rocket and propellant mass
- $v$  is the initial velocity of the spacecraft
- $m_r$  is the mass of the rocket
- $\Delta m$  is the mass of the propellant.
- $h_f$  is the final linear momentum of the rocket combined with the propellant. By conservation of momentum,  $h_i = h_f$
- $\Delta v$  is the change in velocity of the rocket.
- Note:** This is for a single particle of propellant -  $\Delta m$  and can not be used to calculate  $\Delta v$  directly. We will integrate this equation of many particles of propellant to get the true  $\Delta v$ .

# Continuous Thrust

For a single particle of propellant, we have

$$\Delta v = \frac{\Delta m}{m_r} c$$

Dividing by  $\Delta t$  and taking the limit as  $\Delta t \rightarrow 0$ , we get

$$\dot{v}(t) = \frac{\dot{m}_r(t)}{m_r(t)} c$$

where we often assume constant mass flow rate  $\dot{m}_r(t)$ .



Returning to the differential form, we can directly integrate

$$dv = \frac{c}{m_r} dm_r$$

to obtain

$$\Delta v = v(t_f) - v(t_0) = c \ln \left[ \frac{m(t_0)}{m(t_f)} \right]$$

Which is quite different from the approximation  $\Delta v = \frac{\Delta m}{m_r} c$ !

# Lecture 11

## Spacecraft Dynamics

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Which is quite different from the approximation  $\Delta v = \frac{\Delta m}{m_0} c$

- $\dot{m}_r = \lim_{\Delta t \rightarrow 0} \frac{\Delta m}{\Delta t}$  is the rate at which we are using up propellant. Note this doesn't affect  $\Delta v$ !
- Although liquid and hybrid rockets can control this rate, in practice, we want to make it as large as possible so that the  $\Delta v$  happens quickly.
- $m(t_0)$  is the mass before the burn.  $m(t_f)$  is the mass after the burn.
- $v(t_f)$  is the velocity after the burn.  $v(t_0)$  is the velocity before the burn.

# Sizing the Propellant

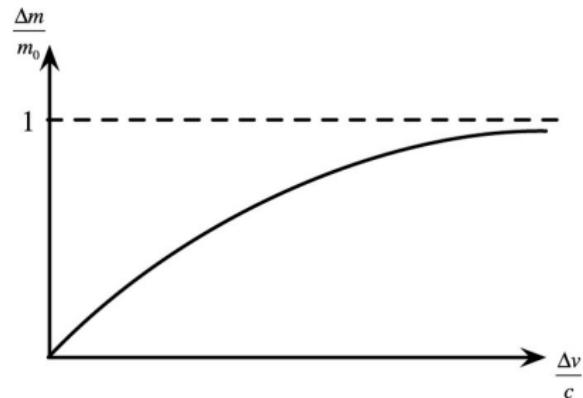
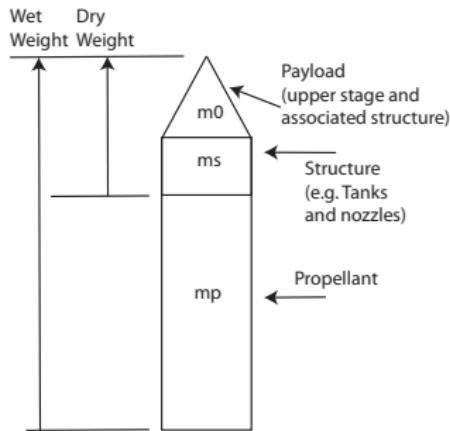
Now we have an expression for  $\Delta v$  as a function of wet and dry weights.

$$\Delta v = v(t_f) - v(t_0) = c \ln \left[ \frac{m(t_0)}{m(t_f)} \right]$$

where recall  $m_0 = m(t_0)$  is the mass before thrust and  $m(t_f)$  is the mass after.

- $\Delta v$  is a function of the ratio of wet weight to dry weight
- For a given maneuver, we can calculate the required propellant

$$\frac{\Delta m}{m_0} = 1 - e^{-\frac{\Delta v}{c}}$$



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## Spacecraft Dynamics

### Sizing the Propellant

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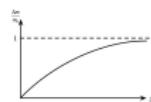
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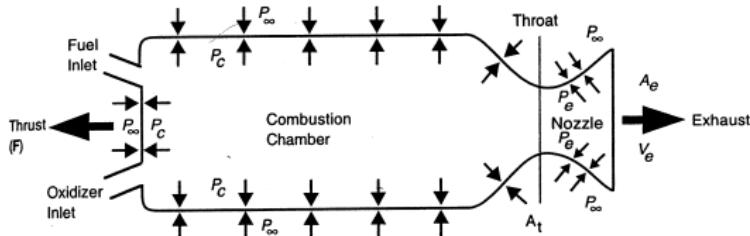


- $m(t_0)$  is the wet weight (with propellant).
- $m(t_f)$  is the dry weight (after all propellant has been used up)
- First equation is called the rocket equation (May 10, 1897), derived by Konstantin E. Tsiolkovsky (1857-1935). Recluse who lived in a log cabin outside Moscow. First person to conceive of space elevator (inspired by Eiffel tower).

# Effective Exhaust Velocity

The efficiency of the rocket depends on the relative velocity of the propellant,  $c$ .

- However, there is also a force due to pressure,  $F = A_e(P_e - P_\infty)$ .



The effective velocity,  $c$ , of propellant is determined by configuration of the rocket:

$$c = V_e + \frac{A_e}{\dot{m}} [P_e - P_\infty]$$

**Note:**  $P_e$  gives a boost to thrust, but at the cost of a *lower*  $V_e$

- As  $V_e$  increases,  $P_e$  drops (particles accelerate out of high-P regions)
- It is always best to maximize  $V_e$  (we want  $P_e = P_\infty$ ).
- In space, this implies we want  $\frac{A_e}{A_t}$  as large as possible.
- Propellant is usually rated by  $c$  and not  $V_e$ !

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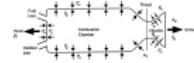
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- It is always best to maximize  $V_e$  (we want  $P_e = P_\infty$ )
- In space, this implies we want  $\frac{dP}{dr}$  as large as possible.
- Propellant is usually rated by  $c$  and not  $V_e$ !

- In previous slides, we have been using  $c$  instead of  $V_e$  for exhaust velocity so as not to confuse you later. However, these formulae should be used with **effective** exhaust velocity,  $c$ .
- $P_\infty$  is the atmospheric pressure.
- $P_e$  is the pressure at exit from the nozzle.
- $A_t$  is the area of the throat.
- $A_e$  is the area of the nozzle exit.
- The effective exhaust velocity of  $H_2-O_2$  propellant in space is 4,440 m/s
- On the ground, there is an optimal  $A_e$  corresponding to  $P_e = P_\infty$ .
- Expansion ratio ( $A_e/A_t$ ) of 117:1 for Merlin 1D Falcon Heavy upper stage.

# Pressure Changes affect Efficiency on Saturn V

# Specific Impulse

## Definition 1.

The **Specific Impulse** is the ratio of the momentum imparted to the weight (on earth) of the propellant.

$$I_{sp} = \frac{\Delta mc}{\Delta mg} = \frac{c}{g}$$

Since  $\Delta v = c \ln \left[ \frac{m_0}{m_f} \right]$ , specific impulse gives a measure of how efficient the propellant is.

Propulsion Technology	Orbit Insertion		Orbit Maintenance and Maneuvering	Attitude Control	Typical Steady State $I_{sp}$ (s)
	Perigee	Apogee			
Cold Gas			✓	✓	30–70
Solid	✓	✓			280–300
Liquid					
Monopropellant			✓	✓	220–240
Bipropellant	✓	✓	✓	✓	305–310
Dual mode	✓	✓	✓	✓	313–322
Hybrid	✓	✓	✓		250–340
Electric			✓		300–3,000

## Lecture 11

## └ Spacecraft Dynamics

## └ Specific Impulse

## Specific Impulse

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Since  $\Delta v = c \ln \left( \frac{m_0}{m_f} \right)$ , specific impulse gives a measure of how efficient the propellant is.

Propellant	Hydrogen	Hydrogen	Hydrogen	Hydrogen	Fuel
Octane	x	x	x	x	20.00
Gasoline	x	x	x	x	20.00
Alcohol	x	x	x	x	20.00
Ammonium	x	x	x	x	20.00
Nitrogen	x	x	x	x	20.00
Hydrogen	x	x	x	x	20.00
Water	x	x	x	x	20.00

## Example

**Problem:** Suppose our mission requires a dry weight of  $m_L = 30\text{kg}$ . Using an  $I_{sp}$  of  $300\text{s}$ , how much propellant is required to achieve a circular orbit of altitude  $200\text{km}$ ?

**Solution:** A Circular orbit at  $200\text{km}$  requires a total velocity of

$$v = \sqrt{\frac{\mu}{r}} = \sqrt{\frac{398600}{6578}} = 7.78\text{km/s}$$

Add  $1.72\text{km/s}$  to account for gravity and drag. This totals  $9.5\text{km/s} = 9500\text{m/s}$ . The  $I_{sp} = 300\text{s}$ , which means  $c = 3000\text{m/s}$ . Thus we have

$$\frac{m_p}{m_0} = 1 - e^{-\frac{\Delta v}{c}} = .9579$$

Since  $m_0 = m_L + m_p$ ,  $m_p = m_L \left( \frac{.9579}{1-.9579} \right) = 682\text{kg}$ .

- Which is sort of a lot!
- What about structural mass and  $\Delta v$  for orbital maneuvers?
- We'll return to this problem later

# Lecture 11

## Spacecraft Dynamics

### Example

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**Problem:** Suppose our mission requires a dry weight of  $m_L = 30\text{kg}$ . Using an  $I_{sp}$  of 300s, how much propellant is required to achieve a circular orbit of altitude 200km?

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$$v = \sqrt{\frac{\mu}{r}} = \sqrt{\frac{398600}{6378}} = 7.78\text{km/s}$$

Add 1.72km/s to account for gravity and drag. This totals 9.5km/s = 9500m/s. The  $I_{sp} = 300\text{s}$ , which means  $c = 3000\text{m/s}$ . Thus we have

$$\frac{m_p}{m_0} = 1 - e^{-\frac{c}{I_{sp}}} = .3579$$

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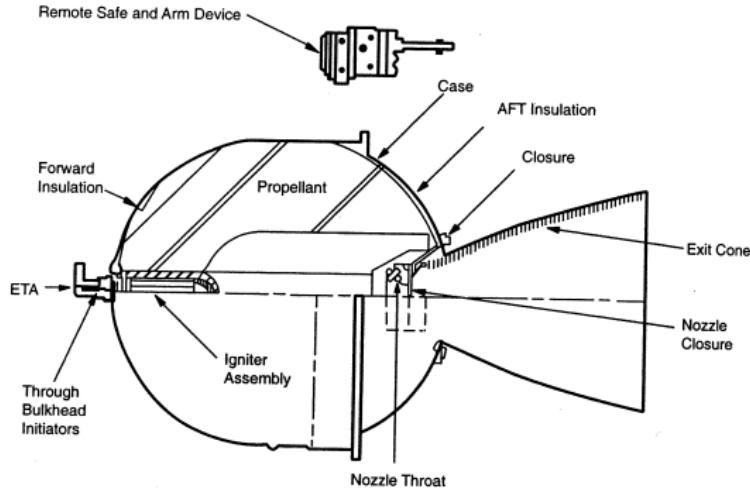
- Which is sort of a lot!

- What about structural mass and  $\Delta v$  for orbital maneuvers?

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- $m_0$  is wet mass, dry mass plus propellant.
- $\Delta m = m_p$

# Solid Rocket Motors



## Advantages:

- Simple
- Reliable
- Low Cost

## Disadvantages:

- Limited Performance
- Not Adjustable (Safety)
- Toxic Byproducts

# Solid Rocket Motors

Motor	Total Impulse (N·s)	Loaded Weight (kg)	Propellant Mass Fraction	Avg. Thrust (lbf)	Avg. Thrust (N)	Max. Thrust (N)	Effective $I_{sp}$ (sec)	Status
IUS SRM-1 (ORBUS-21)	$2.81 \times 10^7$	10,374	0.94	44,610	198,435	260,488	295.5	Flown
LEASAT PKM	$9.26 \times 10^6$	3,658	0.91	35,375	157,356	193,200	285.4	Flown
STAR 48A	$6.78 \times 10^6$	2,559	0.95	17,900	79,623	100,085	283.9	Flown
STAR 48B(S)	$5.67 \times 10^6$	2,135	0.95	14,845	66,034	70,504	286.2	Qualified
STAR 48B(L)	$5.79 \times 10^6$	2,141	0.95	15,160	67,435	72,017	292.2	Qualified
STAR 62	$7.12 \times 10^6$	2,459					293.5	In develop.
STAR 75	$2.13 \times 10^7$	8,066	0.93	44,608	198,426	242,846	288.0	In develop.
IUS SRM-2 (ORBUS-6)	$8.11 \times 10^6$	2,995	0.91	18,020	80,157	111,072	303.8	Flown
STAR 13B	$1.16 \times 10^5$	47	0.88	1,577	7,015	9,608	285.7	Flown
STAR 30BP	$1.46 \times 10^6$	543	0.94	5,960	26,511	32,027	292.0	Flown
STAR 30C	$1.65 \times 10^6$	626	0.95	7,140	31,760	37,031	284.6	Flown
STAR 30E	$1.78 \times 10^6$	667	0.94	7,910	35,185	40,990	289.2	Flown
STAR 37F	$3.02 \times 10^6$	1,149	0.94	9,911	44,086	49,153	291.0	Flown

Figure: Thiokol (ATK Launch Systems) = STAR, LEASAT; United Technologies = IUS

# Liquid Monopropellants

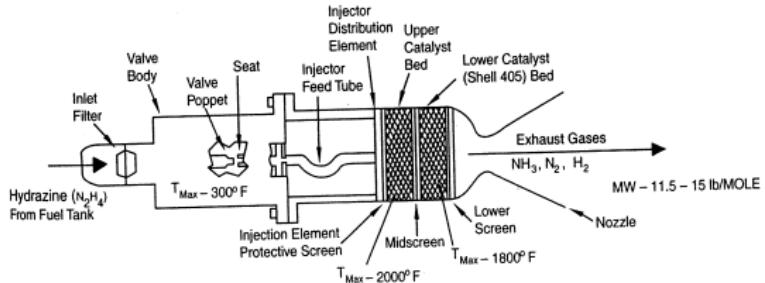


Figure: Typical Hydrazine Monopropellant

## Advantages:

- Simple
- Reliable
- Low Cost

## Disadvantages:

- Lower Performance than bipropellant

# Liquid Bipropellants

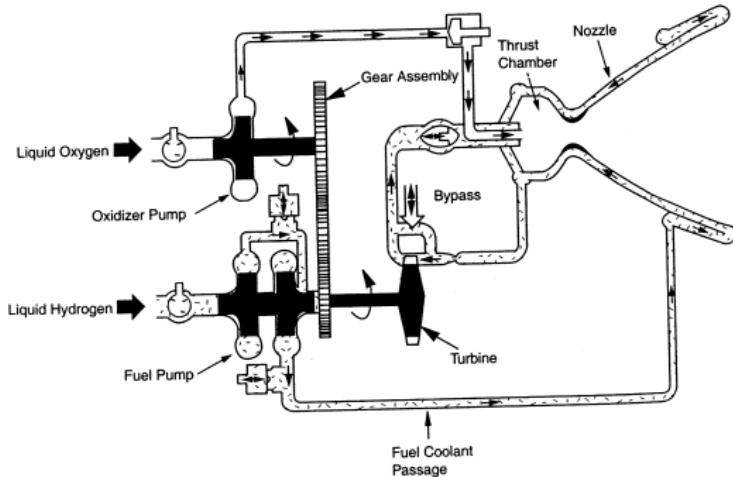


Figure: Centaur O<sub>2</sub>-H<sub>2</sub> upper stage.

## Advantages:

- High Performance
- Adjustable

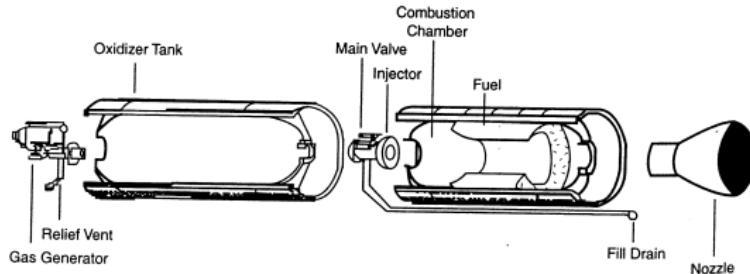
## Disadvantages:

- Complicated
- Dangerous
- Sometimes Toxic

# Liquid Bipropellants

Type	Propellant	Energy	Vacuum $I_{sp}$ (sec)	Thrust Range (N)	Thrust Range (lb <sub>f</sub> )	Avg Bulk Density (g/cm <sup>3</sup> )
Cold Gas	N <sub>2</sub> , NH <sub>3</sub> , Freon, helium	High pressure	50–75	0.05–200	0.01–50	0.28*, 0.60, 0.96*
Solid Motor	†	Chemical	280–300	50–5 × 10 <sup>6</sup>	10–10 <sup>6</sup>	1.80
Liquid:						
Monopropellant	H <sub>2</sub> O <sub>2</sub> , N <sub>2</sub> H <sub>4</sub>	Exothermic decomposition	150–225	0.05–0.5	0.01–0.1	1.44, 1.0
Bipropellant	O <sub>2</sub> and RP-1	Chemical	350	5–5 × 10 <sup>6</sup>	1–10 <sup>6</sup>	1.14 and 0.80
	O <sub>2</sub> and H <sub>2</sub>	Chemical	450	5–5 × 10 <sup>6</sup>	1–10 <sup>6</sup>	1.14 and 0.07
	N <sub>2</sub> O <sub>4</sub> and MMH (N <sub>2</sub> H <sub>4</sub> , UDMH)	Chemical	300–340	5–5 × 10 <sup>6</sup>	1–10 <sup>6</sup>	1.43 and 0.86 (1.0, 0.79)
	F <sub>2</sub> and N <sub>2</sub> H <sub>4</sub>	Chemical	425	5–5 × 10 <sup>6</sup>	1–10 <sup>6</sup>	1.5 and 1.0
	OF <sub>2</sub> and B <sub>2</sub> H <sub>6</sub>	Chemical	430	5–5 × 10 <sup>6</sup>	1–10 <sup>6</sup>	1.5 and 0.44
	ClF <sub>5</sub> and N <sub>2</sub> H <sub>4</sub>	Chemical	350	5–5 × 10 <sup>6</sup>	1–10 <sup>6</sup>	1.9 and 1.0
Dual Mode	N <sub>2</sub> O <sub>4</sub> /N <sub>2</sub> H <sub>4</sub>	Chemical	330	3–200	—	1.9 and 1.0
Water Electrolysis	H <sub>2</sub> O → H <sub>2</sub> + O <sub>2</sub>	Electric / chemical	340–380	50–500	10–100	1.0
Hybrid	O <sub>2</sub> and rubber	Chemical	225	225–3.5 × 10 <sup>5</sup>	50–75,000	1.14 and 1.5
Electrothermal:						
Resistojet	N <sub>2</sub> , NH <sub>3</sub> , N <sub>2</sub> H <sub>4</sub> , H <sub>2</sub>	Resistive heating	150–700	0.005–0.5	0.001–0.1	0.28*, 0.60, 1.0, 0.019*
Arcjet	NH <sub>3</sub> , N <sub>2</sub> H <sub>4</sub> , H <sub>2</sub>	Electric arc heating	450–1,500	0.05–5	0.01–1	0.60, 1.0, 0.019*
Electrostatic:						
Ion	Hg/A/Xe/Cs	Electrostatic	2,000–6,000	5 × 10 <sup>-6</sup> –0.5	10 <sup>-6</sup> –0.1	13.5/0.44*/2.73*/1.87
Colloid	Glycerine	Electrostatic	1,200	5 × 10 <sup>-6</sup> –0.05	10 <sup>-6</sup> –0.01	1.26
Hall Effect Thruster	Xenon	Electrostatic	1,500–2,500	5 × 10 <sup>-6</sup> –0.1	10 <sup>-6</sup> –0.02	0.22
Electromagnetic:						
MPD‡	Argon	Magnetic	2,000	25–200	5–50	0.44*
Pulsed Plasma	Teflon	Magnetic	1,500	5 × 10 <sup>-6</sup> –0.005	10 <sup>-6</sup> –0.001	2.2
Pulsed Inductive	Argon N <sub>2</sub> H <sub>4</sub>	Magnetic Magnetic	4,000 2,500	2–200 2–200	0.5–50 0.5–50	0.44 1.0

# Hybrid Rockets



## Advantages:

- Throttled
- Non-Explosive

Flown on SpaceShipOne (Developed by SpaceDev, Oxidizer - N<sub>2</sub>O,  $I_{sp} = 250s$ , Max Thrust 74kN)

## Disadvantages:

- Requires Oxidizer
- Bulky

# Hybrid Rockets

Motor	Average Thrust (lb <sub>f</sub> )	Average Thrust (kN)	Burn Duration (sec)	Fuel	Oxidizer	Comments
<i>American Rocket Company</i>						
H-500	75,000	333	70	HTPB	LOx	Qualified for flight
H-250	32,000	142		HTPB	LOx	In development
H-50	10,000	44		HTPB	LOx	In development
U-50	6,500	29		HTPB	LOx	In development
U-1	100	0.44		HTPB	LOx	In development
<i>United Technologies</i>						
	40,000	178	300	HTPB	IRFNA	Flown on Firebolt air-launched target drone, 1968
<i>StarsTruck</i>						
	40,000	178		CTBN	LOx	Flown on Dolphin water-launched sounding rocket, 1984
<i>USAF Academy</i>						
H-1	55	0.25	2.3	HTPB	GOx	Flown on 4-ft tall rocket for student project, 1991

Figure: American Rocket Company = SpaceDev

# Lecture 11

## Spacecraft Dynamics

### Hybrid Rockets

Hybrid Rockets

Model	Length mm	Diameter mm	Average Burn Rate mm/s	Burn Duration (sec)	Fuel	Grander	Comments
<i>American Rocket Company</i>							
H-30	75,000	300	76	HTPB	12x	Qualified for flight	
H-20	50,000	140	107	HTPB	12x	In development	
H-10	30,000	100	137	HTPB	12x	In development	
8.0	8,000	58	100	HTPB	12x	In development	
8.1	950	0.44	100	HTPB	12x	In development	
<i>Other Technologies</i>							
	40,000	178	300	HTPB	10xNA	Flame on Fleisch missed large share	
<i>Genesys</i>							
	40,000	178	—	CBP	12x	Flame on Cognac-water launched sounding	
<i>CBP Assembly</i>							
8.1	55	0.05	2.2	HTPB	10x	Flame on a 4.4g model for student project 1980	

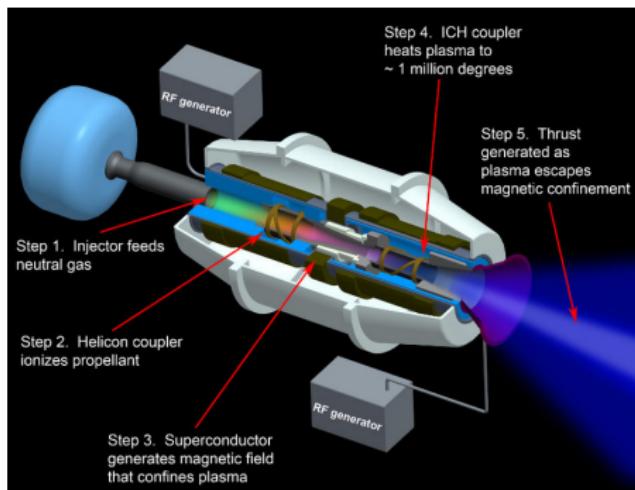
Figure: American Rocket Company :: SpaceDev

- HTPB is a common plastic/rubber hybrid. Also used occasionally in solid rockets
- Commercial sources include paraffin and spandex.

# Electric Propulsion

## Electrothermal:

- Ohmic Heating

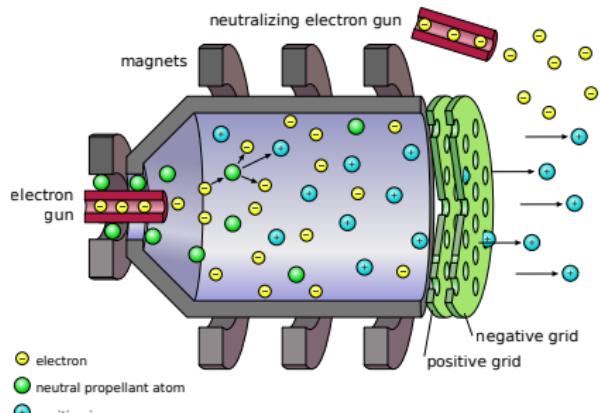


## Electrostatic:

- Repulsion/Attraction

## Electromagnetic:

- Ions accelerated by EM waves



## Advantages:

- Very High Performance

## Disadvantages:

- Low Thrust
- Requires Electricity

# Lecture 11

## Spacecraft Dynamics

### Electric Propulsion

#### Electric Propulsion

##### Electrothermal:

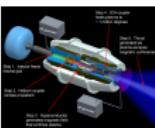
- Omic Heating

##### Electrostatic:

- Repulsion/Attraction

##### Electromagnetic:

- Ions accelerated by EM waves



##### Advantages:

- Very High Performance

##### Disadvantages:

- Low Thrust
- Requires Electricity

On left is a magnetoplasmadynamic thruster (MPD)

- Requires MW power for radio heating and magnetic confinement
- Requires a nuclear reactor for power.
- Only experimental MPDs have flown to date.

On right is a gridded ion thruster

- The US in the 1960s focused on GITs, while the Soviet union focused on Hall Effect Thrusters (HET)
- Common for stationkeeping (in GEO)
- Isp in the range 3k-10k (maybe 21k)

# Electric Propulsion

The choice of Electrothermal/Electrostatic/Electromagnetic depends on available power.

Electrothermal	Electrostatic	Electromagnetic
<ul style="list-style-type: none"><li>Gas heated via resistance element or arc and expanded through nozzle</li><li>Resistojets</li><li>Arcjets</li></ul>	<ul style="list-style-type: none"><li>Ions electrostatically accelerated</li><li>Hall effect (HET)</li><li>Ion</li><li>Field emission</li></ul>	<ul style="list-style-type: none"><li>Plasma accelerated via interaction of current and magnetic field</li><li>Pulsed plasma (PPTs)</li><li>Magnetoplasmadynamic (MPD)</li><li>Pulsed inductive (PIT)</li></ul>
Power Range; 0.4–2 kW	1–50 kW	50 kW–1 MW
Specific Impulse, $I_{sp}$ ; 300–800 sec	1,000–3,000 sec	2,000–5,000 sec

# Lecture 11

## Spacecraft Dynamics

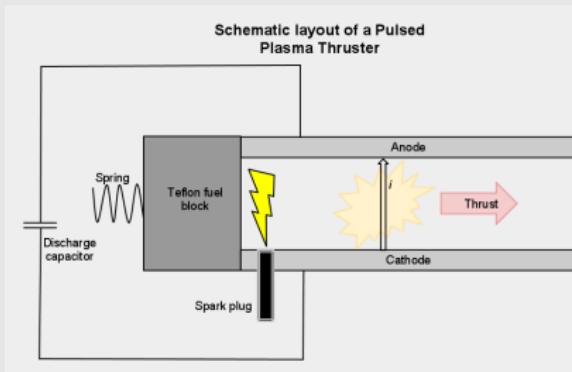
### Electric Propulsion

The choice of Electrothermal/Electrostatic/Electromagnetic depends on available power.

Electrothermal	Electrostatic	Electromagnetic
<ul style="list-style-type: none"> <li>Gas heated via resistance element &amp; arc and expand through nozzle</li> <li>Resistojet</li> <li>Arcjets</li> </ul>	<ul style="list-style-type: none"> <li>Ions electrostatically accelerated in electric field of current and magnetic field</li> <li>Inertial electrostatic (PPT)</li> <li>Magnetoplasmodynamic (MHD)</li> <li>Polar-inductive (PII)</li> </ul>	<ul style="list-style-type: none"> <li>Plasma accelerated via interaction of current and magnetic field</li> <li>Electron cyclotron resonance heating (ECRH)</li> <li>Magnetic confinement fusion (MCF)</li> <li>Polar-inductive (PII)</li> </ul>
Power Range 0.0–9 kW	1–60 kW	50 kW–1 MW
Specific impulse, $\Delta v$ 300–600 sec	1,000–3,000 sec	2,000–5,000 sec

- SRP average generation is 3.2MW.

PPT's have flown since 1964. In 2000, NASA's research PPT generated  $c=13,700 \text{ m/s}$  ( $I_{sp}=1,370$ ), with thrust of 860 micro-N, requiring power of 70 W.



# Electric Propulsion

Concept	Characteristics					
	Specific Impulse, (sec)	Input Power, (kW)	Thrust/Power, (mN/kW)	Specific Mass, (kg/kW)	Propellant	Supplier
<i>Resistojet</i>	296	0.5	743	1.6	N <sub>2</sub> H <sub>4</sub>	Primex
	299	0.9	905	1	N <sub>2</sub> H <sub>4</sub>	Primex, TRW
<i>Arcjet</i>	480	0.85	135	3.5	NH <sub>3</sub>	IRS/ITT
	502	1.8	138	3.1	N <sub>2</sub> H <sub>4</sub>	Primex
	>580	2.17	113	2.5	N <sub>2</sub> H <sub>4</sub>	Primex
	800	26*	—	—	NH <sub>3</sub>	TRW, Primex, CTA
<i>Pulsed Plasma Thruster (PPT)</i>	847	< 0.03†	20.8	195	Teflon	JHU/APL
	1,200	< 0.02†	16.1	85	Teflon	Primex, TSNIIMASH, NASA
<i>Hall Effect Thruster (HET)</i>	1,600	1.5	55	7	Xenon	IST, Loral, Fakel
	1,638	1.4*	—	—	Xenon	TSNIIMASH, NASA
	2,042	4.5	54.3	6	Xenon	SPI, KeRC
<i>Ion Thruster (IT)</i>	2,585	0.5	35.6	23.6	Xenon	HAC
	2,906	0.74	37.3	22	Xenon	MELCO, Toshiba
	3,250	0.6	30	25	Xenon	MMS
	3,280	2.5	41	9.1	Xenon	HAC, NASA
	3,400	0.6	25.6	23.7	Xenon	DASA

# Lecture 11

## Spacecraft Dynamics

### Electric Propulsion

Characteristic						
	Specific Impulse (s)	Input Power (kW)	Thrust/ Power (N/kW)	Specific Mass (kg/kg)	Propellant	Supplier
Reaction	206	0.8	743	1.6	NH <sub>3</sub>	Praxair
	206	0.8	908	1	NH <sub>3</sub>	Praxair, TRW
Arcjet	200	0.8	605	3.5	NH <sub>3</sub>	TRW
	102	1.8	108	3.1	NH <sub>3</sub>	Praxair
Ion	3000	2.17	113	3.5	NH <sub>3</sub>	Praxair
	906	2.4	106	4.1	NH <sub>3</sub>	TRW, Praxair, CTA
Tether/Magnetic Thruster (PTT)	1.000	< 0.0007	28.8	165	Tether	Praxair, TDRSS/MOHIC, NASA
	1.200	< 0.0007	14.1	65	Tether	Praxair, TDRSS/MOHIC, NASA
Solid Electrode Thruster (NET)	1.000	1.5	55	7	Karen	Han, Liwei, Peilei
	1.000	1.4*	—	—	Karen	TSURIBAISHI, NASA
Ion Thruster (IT)	2.040	4.5	54.3	—	Karen	SPL, KARI
	2.000	0.8	58	23.8	Karen	MLDOO, Toshiba
Hall Thruster (HT)	2.000	0.74	37.3	22	Karen	MLDOO
	2.050	0.6	30	25	Karen	IMB
Multi-Stage Thruster (MST)	2.000	0.8	41	8.1	Karen	MLDOO, NASA
	2.000	0.6	25.6	23.7	Karen	DASA

Teflon melts at 327° C (260° C for cooking). Boils at 400° C

# Staging

Previously, we assumed the rocket only consisted of payload and propellant:

$$m_0 = m_L + m_p.$$

$$\frac{m_p}{m_0} = 1 - e^{-\frac{\Delta v}{c}} \quad \Delta v = c \ln \left[ \frac{m(t_0)}{m(t_f)} \right] = c \ln \left[ \frac{m_L + m_p}{m_L} \right]$$

Which would mean the only way to increase  $\Delta v$  is to decrease payload or increase the size of the rocket.

**However:** Payload is not the only part of the rocket.

- Rocket engines and storage tanks are heavy.
- Typically, *structure* accounts for  $\cong 1/7$  of the propellant weight

$$m_0 = m_L + m_s + m_p = (m_L + 1/7m_p) + m_p$$

- While  $1/7$  may not seem a lot, without staging, it limits the total  $\Delta v$  to

$$\Delta v = c \ln \left( \frac{m_p}{m_p/7} \right) = c \ln 7 \cong 2c \cong 6 \text{ km/s} \quad (\text{assuming } m_L = 0)$$

But  $\Delta v = 8 \text{ km/s}$  is needed for low earth orbit (LEO) - not accounting for drag or gravity losses ( $2 \text{ km/s}$ )!

**OMG:** Space flight is IMPOSSIBLE!

- It's a conspiracy.

# Lecture 11

## Spacecraft Dynamics

### Staging

#### Staging

Previously, we assumed the rocket only consisted of payload and propellant:

$$m_0 = m_L + m_p$$

$$\frac{m}{m_0} = 1 - e^{-\frac{4\pi}{c}}$$

$$\Delta v = c \ln \left[ \frac{m(t_0)}{m(t_f)} \right] = c \ln \left[ \frac{m_L + m_p}{m_L} \right]$$

Which would mean the only way to increase  $\Delta v$  is to decrease payload or increase the size of the rocket.

However: Payload is not the only part of the rocket.

- \* Rocket engines and storage tanks are heavy.
- \* Typically, structure accounts for  $\approx 1/7$  of the propellant weight

$$m_0 = m_L + m_s + m_p = (m_L + 1/7m_L) + m_p$$

- \* While  $1/7$  may not seem a lot, without staging, it limits the total  $\Delta v$  to

$$\Delta v = c \ln \left( \frac{m_0}{m_0} \right) = c \ln 7 \approx 2c \approx 6km/s \quad (\text{assuming } m_L = 0)$$

But  $\Delta v = 8km/s$  is needed for low earth orbit (LEO) - not accounting for drag or gravity losses ( $2km/s$ )!

OMG: Space flight is IMPOSSIBLE!

- \* It's a conspiracy.

This is why Hohmann was so excited about relatively small changes in  $c$

$$\Delta v_{\max} = c \ln 7 \cong 2c \cong 6km/s \quad (\text{assuming } m_L = 0)$$

# Structural Mass on Saturn V

# Structural Mass on Saturn V

**[From ESA]** - Cameras mounted on the Soyuz Fregat upper stage that sent Sentinel-1A into space on 3 April 2014. It shows liftoff, the stages in the rocket's ascent and the Sentinel-1A satellite being released from the Fregat upper stage to start its life in orbit around Earth.

The 2.3 tonne satellite lifted off on a Soyuz rocket from Europe's Spaceport in Kourou, French Guiana at 21:02 GMT (23:02 CEST). The first stage separated 118 sec later, followed by the fairing (209 sec), stage 2 (287 sec) and the upper assembly (526 sec). After a 617 sec burn, the Fregat upper stage delivered Sentinel into a Sun-synchronous orbit at 693 km altitude. The satellite separated from the upper stage 23 min 24 sec after liftoff.

# Structural Coefficient

## Definition 2.

The ratio of structure to total mass is called the structural coefficient,  $\epsilon$ :

$$\epsilon = \frac{m_s}{m_s + m_p}$$

In the ideal case, structural weight would be discarded as soon as it is no longer required.

- Continuous Staging

In this ideal scenario, we would have

$$\Delta v = (1 - \epsilon)c \ln \left[ \frac{m_0}{m_L} \right]$$

- The structure simply decreases the efficiency of the fuel!

In **Staging**, we discard structure at discrete points in time.

- Staging can never be better than  $\Delta v = (1 - \epsilon)c \ln \left[ \frac{m_0}{m_L} \right]$ .

## Lecture 11

## └ Spacecraft Dynamics

## └ Structural Coefficient

## Structural Coefficient

## Definition 2.

The ratio of structure to total mass is called the structural coefficient,  $\epsilon$ :

$$\epsilon = \frac{m_s}{m_p + m_s}$$

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- Continuous Staging

In this ideal scenario, we would have

$$\Delta v = (1 - \epsilon) c \ln \left[ \frac{m_0}{m_f} \right]$$

\* The structure simply decreases the efficiency of the fuel

In Staging, we discard structure at discrete points in time.

\* Staging can never be better than  $\Delta v = (1 - \epsilon) c \ln \left[ \frac{m_0}{m_f} \right]$ .

- Prussing uses structural coefficient  $\epsilon$
- Prussing uses the term “mass ratio” to refer to the full weight of a stage over the empty weight (wet weight over dry weight).

$$Z = \frac{m_p + m_s + m_L}{m_s + m_L} = \frac{1 + \lambda}{\epsilon + \lambda}$$

- Prussing uses the term “payload ratio” to indicate the ratio of payload to structural mass plus propellant mass.

$$\lambda = \frac{m_L}{m_p + m_s}$$

- I use the term “structural mass fraction” to indicate the mass of structure needed for every mass of fuel

$$\frac{m_s}{m_p} = \frac{\epsilon}{1 - \epsilon}$$

# $\Delta v$ for staging

Total  $\Delta v$  is the sum of the  $\Delta v$ 's from each stage:

$$\Delta v = \Delta v_1 + \Delta v_2 + \Delta v_3$$

So the  $\Delta v$  of each stage is

$$\Delta v_i = c \ln \frac{m_{0,i}}{m_{f,i}} = c \ln \frac{m_{0,i}}{m_{L,i} + m_{s,i}} = c \ln Z_i = c \ln \left( \frac{1 + \lambda_i}{\lambda_i + \epsilon_i} \right)$$

The total mass of each state is the payload, propellant and structural mass

$$m_{0,i} = m_{p,i} + m_{s,i} + m_{L,i}$$

The payload mass for each stage is the total mass of the following stages

$$m_{L,i} = m_{0,i+1} = m_{p,i+1} + m_{s,i+1} + m_{L,i+1}$$

**When designing a multi-stage rocket:** the only thing you are allowed to choose is  $m_{p,i}$ . These are the variables. Everything else is fixed by these choices.

$$m_{s,i} = \frac{\epsilon_i}{1 - \epsilon_i} m_{p,i}$$

## 3-stage Scenario

Suppose we divide the structural and propulsive weight into three components

1. First Stage:  $m_{s1}$ ,  $m_{p1}$
2. Second Stage:  $m_{s2}$ ,  $m_{p2}$
3. Third Stage:  $m_L$ ,  $m_{p3}$

Then  $\Delta v$  is the combined  $\Delta v$  of all three stages.

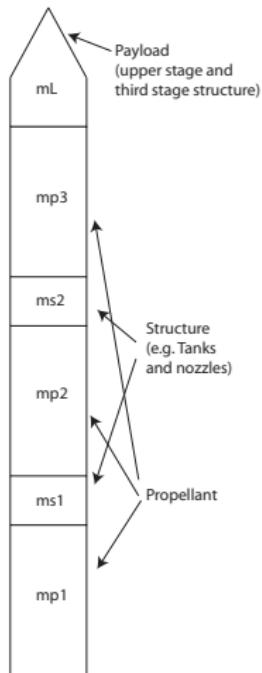
$$\begin{aligned}\Delta v_T &= \Delta v_1 + \Delta v_2 + \Delta v_3 \quad (m_L \text{ here includes } m_{s3}) \\ &= c \ln \left[ \frac{m_{p1} + m_{p2} + m_{p3} + m_{s1} + m_{s2} + m_L}{m_{p2} + m_{p3} + m_{s1} + m_{s2} + m_L} \right] \\ &\quad + c \ln \left[ \frac{m_{p2} + m_{p3} + m_{s2} + m_L}{m_{p3} + m_{s2} + m_L} \right] + c \ln \left[ \frac{m_{p3} + m_L}{m_L} \right]\end{aligned}$$

Optimal choice of  $m_{p1}$ ,  $m_{p2}$  and  $m_{p3}$  is difficult.

For a fixed total mass,  $m_0$ , we can maximize

- Payload weight
- Total  $\Delta v$

A good rule of thumb is  $m_{p1} = 3m_{p2} = 9m_{p3}$ .

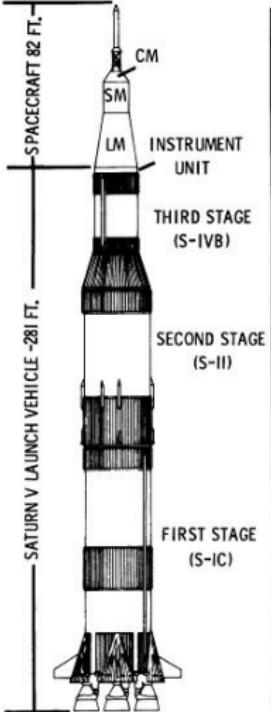


# Lecture 11

## Spacecraft Dynamics

### 3-stage Scenario

## SATURN V LAUNCH VEHICLE



FIRST STAGE (S-IC)	
DIAMETER	33 FEET
HEIGHT	138 FEET
WEIGHT	5,031,023 LBS. FUELED 294,200 LBS.DRY
ENGINES	FIVE F-1
PROPELLANTS	LIQUID OXYGEN (3,258,280 LBS.) (KEROSENE) - (1,417,334 LBS.)
THRUST	7,680,982 LBS.
SECOND STAGE (S-II)	
DIAMETER	33 FEET
HEIGHT	81.5 FEET
WEIGHT	1,074,590 LBS. FUELED 84,367 LBS. DRY
ENGINES	FIVE J-2
PROPELLANTS	LIQUID OXYGEN (829,114 LBS.) LIQUID HYDROGEN (158,231 LBS.)
THRUST	1,163,854 LBS.
INTERSTAGE	8,890 LBS.
THIRD STAGE (S-IVB)	
DIAMETER	21.7 FEET
HEIGHT	58.3 FEET.
WEIGHT	261,834 LBS. FUELED 25,750 LBS. DRY
ENGINES	ONE J-2
PROPELLANTS	LIQUID OXYGEN (190,785 LBS.) LIQUID HYDROGEN (43,452 LBS.)
THRUST	203,615 LBS.
INTERSTAGE	8,081 LBS.
INSTRUMENT UNIT	
DIAMETER	21.7 FEET
HEIGHT	3 FEET
WEIGHT	4,254 LBS.

NOTE: WEIGHTS AND MEASURES GIVEN ABOVE ARE FOR THE NOMINAL VEHICLE CONFIGURATION FOR APOLLO 10. THE FIGURES MAY VARY SLIGHTLY DUE TO CHANGES BEFORE LAUNCH TO MEET CHANGING CONDITIONS.

### 3-stage Scenario

Suppose we divide the structural and propulsive weight into three components

1. First Stage:  $m_{s1}, m_{p1}$

2. Second Stage:  $m_{s2}, m_{p2}$

3. Third Stage:  $m_{s3}, m_{p3}$

Then  $\Delta v$  is the combined  $\Delta v$  of all three stages.

$$\Delta v = \Delta v_1 + \Delta v_2 + \Delta v_3 (m_0 \text{ here includes } m_{s3}) \\ = c \ln \left[ \frac{(m_{s1} + m_{p1} + m_{s2} + m_{p2} + m_{s3} + m_{p3})}{(m_{s1} + m_{p1} + m_{s2} + m_{p2} + m_{s3})} \right] \\ + c \ln \left[ \frac{(m_{s2} + m_{p2} + m_{s3} + m_{p3})}{(m_{s2} + m_{p2} + m_{s3})} \right] + c \ln \left[ \frac{(m_{s3} + m_{p3})}{m_{s3}} \right]$$

Optimal choice of  $m_{s1}, m_{p1}$  and  $m_{s2}, m_{p2}$  is difficult.

For a fixed total mass,  $m_0$ , we can maximize

▪ Payload weight

▪ Total  $\Delta v$

A good rule of thumb is  $m_{p1} = 3m_{s2} = 9m_{s3}$ .



Structural coefficient by stage  
(assuming interstage is  
structural mass on previous  
stage:)

$$\epsilon_1 = .0602$$

$$\epsilon_2 = .0868$$

$$\epsilon_3 = .0985$$

figure from National Air and Space Museum

Comparison: Ariane IV (1988):

$$\epsilon_1 = .0696$$

$$\epsilon_2 = .0957$$

$$\epsilon_3 = .01008$$

$$\text{mass} = 500,000 - 1,000,000 lb$$

Comparison: BFR

(Starship+Superheavy)

$$\epsilon_1 = .0651 \text{ (est.)}$$

$$\epsilon_2 = .0909$$

$$\text{mass} = 11,000,000 lb$$

# 1,2 and 3stage Scenarios (Fixed Total Mass)

TABLE 6.3 ONE-, TWO-, AND THREE-STAGE ROCKETS STAGES (EQUAL  $\epsilon$  AND  $\lambda$ )  $c = 3048$  m/sec ( $I_{sp} = 311$  sec)

	1 stage	2 stage	3 stage	
			specified $m_L$	specified $\Delta v$
$m_{01}$	15,000	15,000	15,000	15,000
$m_{02}$	—	3,873	6,082	4,926
$m_{03}$	—	—	2,466	1,618
$m_{s1}$	2,000	1,590	1,274	1,393
$m_{s2}$	—	410	517	457
$m_{s3}$	—	—	209	150
$m_{p1}$	12,000	9,537	7,644	8,681
$m_{p2}$	—	2,463	3,099	2,851
$m_{p3}$	—	—	1,257	936
$m_L$	1,000	1,000	1,000	531
$\epsilon$	0.143	0.143	0.143	0.138
$\lambda$	0.0714	0.348	0.682	0.489
$\Delta v$ (m/sec)	4,906	6,157	6,515	7,905
$m_{sTOT}$	2,000	2,000	2,000	2,000
$m_{pTOT}$	12,000	12,000	12,000	12,469
Z	5	2.75	2.039	2.374

# Staging on Minuteman ICBM

# Summary

This Lecture you have learned:

## Introduction to Rocketry

- Mass Consumption
- Specific Impulse and Rocket Types
- $\Delta v$  limitations
- Staging