Spacecraft Dynamics and Control

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Lecture 11: Intro to Rocketry

Introduction

In this Lecture, you will learn:

Introduction to Rocketry

- Mass Consumption
- Specific Impulse and Rocket Types
- Δv limitations
- Staging

Numerical Problem: Suppose our mission requires a dry weight of 30kg. How much propellant is required to achieve a circular orbit of altitude $200km$?

Questions about Propulsion

We have talked a bit about Δv.

- How is Δv created?
- How expensive is it?
- Is it really instantaneous?

Δv budget

A Typical mission uses a lot of Δv.

- How much propellant will we need?
- What is the maximum Δv budget?

Propulsion Function	Typical Requirement
Orbit transfer to GEO (orbit insertion)	
• Perigee burn	2,400 m/s
• Apogee burn	1,500 (low inclination) to 1,800 m/s (high inclination)
Initial spinup	1 to 60 rpm
LEO to higher orbit raising ΔV	60 to 1,500 m/s
• Drag-makeup ΔV	60 to 500 m/s
• Controlled-reentry ΔV	120 to 150 m/s
Acceleration to escape velocity from LEO parking orbit	3,600 to 4,000 m/s into planetary trajectory
On-orbit operations (orbit maintenance)	
• Despin	60 to 0 rpm
• Spin control	±1 to ±5 rpm
• Orbit correction ΔV	15 to 75 m/s per year
• East-West stationkeeping ΔV	3 to 6 m/s per year
• North-South stationkeeping ΔV	45 to 55 m/s per year
• Survivability or evasive maneuvers (highly variable) ΔV	150 to 4,600 m/s
Attitude control	3–10% of total propellant mass
• Acquisition of Sun, Earth, Star	Low total impulse, typically <5,000 N•s, 1 K to 10 K pulses, 0.01 to 5.0 sec pulse width
• On-orbit normal mode control with 3-axis stabilization, limit cycle	100 K to 200 K pulses, minimum impulse bit of 0.01 N•s, 0.01 to 0.25 sec pulse width
• Precession control (spinners only)	Low total impulse, typically <7,000 N•s, 1 K to 10 K pulses, 0.02 to 0.20 sec pulse width
• Momentum management (wheel unloading)	5 to 10 pulse trains every few days, 0.02 to 0.10 sec pulse width
• 3-axis control during ΔV	On/off pulsing, 10 K to 100 K pulses, 0.05 to 0.20 sec pulse width

Some Definitions

In a staged Launch system, the mass varies with time.

- Dry weight is the weight without propellant.
    - This is the final weight.
    - Craft plus payload
- There are several variations of dry weight.

Weight Parameters	Comments
1. *Spacecraft Dry Weight*	Weight of all spacecraft subsystems and sensors, including weight growth allowance of 15–25% at concept definition
plus Propellant	Weight of propellant required by the spacecraft to perform its mission when injected into its mission orbit
Yields	
2. *Loaded Spacecraft Weight*	Mission-capable spacecraft weight (wet weight)
plus Upper Stage Vehicle Weight	Weight of any apogee or perigee kick motors and stages added to the launch system
Yields	
3. *Injected Weight*	Total weight achieving orbit
plus Booster Adapter Weight	May also include airborne support equipment on the Space Shuttle
Yields	
4. *Boosted Weight*	Total weight that must be lifted by the launch vehicle
plus Performance Margin	The amount of performance retained in reserve (for the booster) to allow for all other uncertainties.
Yields	
5. *Payload Performance Capability*	This is the payload weight contractors say their launch systems can lift

How to create Thrust: Newton's Second Law

Approximation: Consider the expulsion of a piece of propellant, Δm.

Initial State:

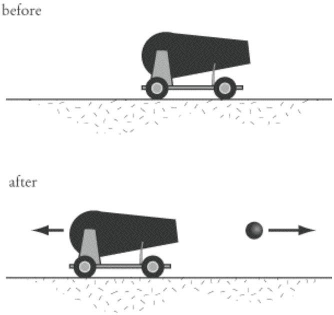
- Propellant and Rocket move together.
- Total Momentum:

$$h_i = (m_r + \Delta m)v$$

Final State:

- Propellant and Rocket move separately.
- Rocket has velocity $v + \Delta v$
- Propellant has velocity $v - c$.
  - c is the exhaust velocity
- Total Momentum:

$$h_f = m_r(v + \Delta v) + \Delta m(v - c)$$



Conservation of Momentum:

- Setting $h_i = h_f$, we obtain:

$$(m_r + \Delta m)v = m_r(v + \Delta v) + \Delta m(v - c)$$

- Solving for Δv, we obtain

$$\Delta v = \frac{\Delta m}{m_r} c$$

Continuous Thrust

For a single piece of propellant, we have

$$\Delta v = \frac{\Delta m}{m_r} c$$

Dividing by Δt and taking the limit, we get

$$\dot{v}(t) = \frac{\dot{m}_r(t)}{m_r(t)} c$$

where we often assume constant mass flow rate $\dot{m}_r(t)$.

Returning to the differential form, we can directly integrate

$$dv = \frac{c}{m_r} dm_r$$

to obtain

$$\Delta v = v - v_0 = c \ln \left[\frac{m(t_0)}{m(t_f)} \right]$$

Which is quite different from the approximation $\Delta v = \frac{\Delta m}{m_r} c$!

Sizing the Propellant

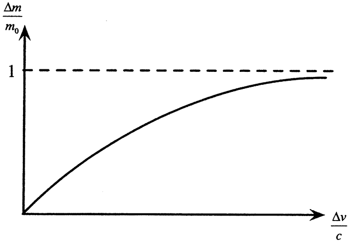
Now we have an expression for thrust as a function of weight.

$$\Delta v = v - v_0 = c \ln \left[\frac{m(t_0)}{m(t_f)} \right]$$

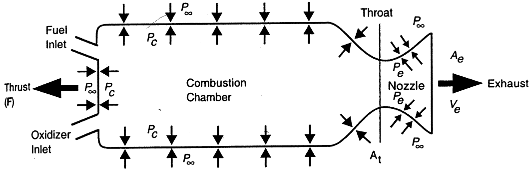
where recall $m_0 = m(t_0)$ is the mass before thrust and $m(t_f)$ is the mass after.

- Δv is a function of the ratio of wet weight to dry weight
- For a given maneuver, we can calculate the required propellant

$$\frac{\Delta m}{m_0} = 1 - e^{-\frac{\Delta v}{c}}$$



Clearly the efficiency of the propulsion depends on the relative velocity of the propellant, c.



The *effective* velocity of propellant is determined by configuration of the rocket:

$$c = V_e + \frac{A_e}{\dot{m}}[P_e - P_\infty]$$

However: To maximize V_e, we want $P_e = P_\infty$.

- In space, this implies we want $\frac{A_e}{A_t}$ as large as possible.
- On the ground, there is an optimal A_e.

Another reason for staging.

Pressure Changes affect Efficiency on Saturn V

Definition 1.

The **Specific Impulse** is the ratio of the momentum imparted to the weight (on earth) of the propellant.

$$I_{sp} = \frac{\Delta mc}{\Delta mg} = \frac{c}{g}$$

Since $\Delta v = c \ln \left[\frac{m_0}{m_f}\right]$, specific impulse gives a measure of how efficient the propellant is.

Propulsion Technology	Orbit Insertion		Orbit Maintenance and Maneuvering	Attitude Control	Typical Steady State I_{sp} (s)
	Perigee	Apogee			
Cold Gas			✔	✔	30–70
Solid	✔	✔			280–300
Liquid					
Monopropellant			✔	✔	220–240
Bipropellant	✔	✔	✔	✔	305–310
Dual mode	✔	✔	✔	✔	313–322
Hybrid	✔	✔	✔		250–340
Electric		✔	✔		300–3,000

Example

Problem: Suppose our mission requires a dry weight of $m_L = 30$kg. Using an I_{sp} of $300s$, how much propellant is required to achieve a circular orbit of altitude $200km$?

Solution: A Circular orbit at 200km requires a total velocity of

$$v = \sqrt{\frac{\mu}{r}} = \sqrt{\frac{398600}{6578}} = 7.78km/s$$

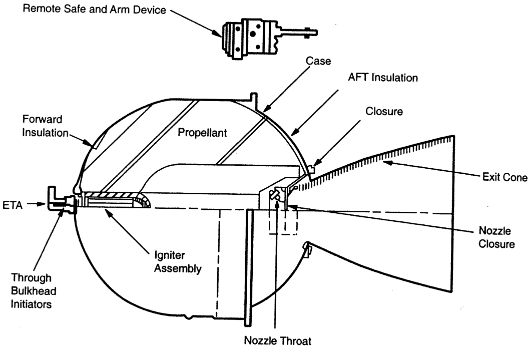
Add $1.72km/s$ to account for gravity and drag. This totals $9.5km/s = 9500m/s$. The $I_{sp} = 300s$, which means $c = 3000m/s$. Thus we have

$$\frac{m_p}{m_0} = 1 - e^{-\frac{\Delta v}{c}} = .9579$$

Since $m_0 = m_L + m_p$, $m_p = m_L \left(\frac{.9579}{1-.9579}\right) = 682$kg.

- Which is sort of a lot!
- What about structural mass and Δv for orbital maneuvers?
- We'll return to this problem later

Solid Rocket Motors



Advantages:

- Simple
- Reliable
- Low Cost

Disadvantages:

- Limited Performance
- Not Adjustable (Safety)
- High Thrust

Solid Rocket Motors

Motor	Total Impulse (N·s)	Loaded Weight (kg)	Propellant Mass Fraction	Avg. Thrust (lb_f)	Avg. Thrust (N)	Max. Thrust (N)	Effective I_{sp} (sec)	Status
IUS SRM-1 (ORBUS-21)	2.81×10^7	10,374	0.94	44,610	198,435	260,488	295.5	Flown
LEASAT PKM	9.26×10^6	3,658	0.91	35,375	157,356	193,200	285.4	Flown
STAR 48A	6.78×10^6	2,559	0.95	17,900	79,623	100,085	283.9	Flown
STAR 48B(S)	5.67×10^6	2,135	0.95	14,845	66,034	70,504	286.2	Qualified
STAR 48B(L)	5.79×10^6	2,141	0.95	15,160	67,435	72,017	292.2	Qualified
STAR 62	7.12×10^6	2,459					293.5	In develop.
STAR 75	2.13×10^7	8,066	0.93	44,608	198,426	242,846	288.0	In develop.
IUS SRM-2 (ORBUS-6)	8.11×10^6	2,995	0.91	18,020	80,157	111,072	303.8	Flown
STAR 13B	1.16×10^5	47	0.88	1,577	7,015	9,608	285.7	Flown
STAR 30BP	1.46×10^6	543	0.94	5,960	26,511	32,027	292.0	Flown
STAR 30C	1.65×10^6	626	0.95	7,140	31,760	37,031	284.6	Flown
STAR 30E	1.78×10^6	667	0.94	7,910	35,185	40,990	289.2	Flown
STAR 37F	3.02×10^6	1,149	0.94	9,911	44,086	49,153	291.0	Flown

Figure: Thiokol (ATK Launch Systems) = STAR, LEASAT; United Technologies = IUS

Liquid Monopropellants

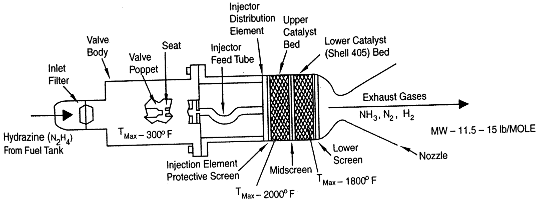


Figure: Typical Hydrazine Monopropellant

Advantages:

- Simple
- Reliable
- Low Cost

Disadvantages:

- Lower Performance than bipropellant

Liquid Bipropellants

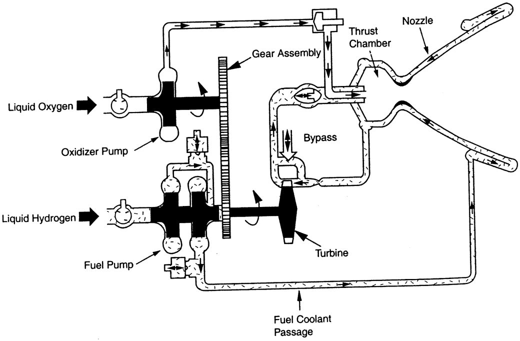


Figure: Centaur O_2-H_2 upper stage.

Advantages:

- High Performance
- Adjustable

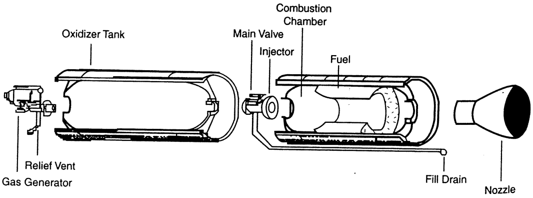
Disadvantages:

- Complicated
- Dangerous
- Sometimes Toxic

Liquid Bipropellants

Type	Propellant	Energy	Vacuum I_{sp} (sec)	Thrust Range (N)	Thrust Range (lb_f)	Avg Bulk Density (g/cm^3)
Cold Gas	N_2, NH_3, Freon, helium	High pressure	50–75	0.05–200	0.01–50	0.28*, 0.60, 0.96*
Solid Motor	†	Chemical	280–300	$50–5 \times 10^6$	$10–10^6$	1.80
Liquid:						
Monopropellant	H_2O_2, N_2H_4	Exothermic decom-position	150–225	0.05–0.5	0.01–0.1	1.44, 1.0
Bipropellant	O_2 and RP-1	Chemical	350	$5–5 \times 10^6$	$1–10^6$	1.14 and 0.80
	O_2 and H_2	Chemical	450	$5–5 \times 10^6$	$1–10^6$	1.14 and 0.07
	N_2O_4 and MMH (N_2H_4, UDMH)	Chemical	300–340	$5–5 \times 10^6$	$1–10^6$	1.43 and 0.86 (1.0, 0.79)
	F_2 and N_2H_4	Chemical	425	$5–5 \times 10^6$	$1–10^6$	1.5 and 1.0
	OF_2 and B_2H_6	Chemical	430	$5–5 \times 10^6$	$1–10^6$	1.5 and 0.44
	ClF_5 and N_2H_4	Chemical	350	$5–5 \times 10^6$	$1–10^6$	1.9 and 1.0
Dual Mode	N_2O_4/N_2H_4	Chemical	330	3–200	—	1.9 and 1.0
Water Electrolysis	$H_2O \rightarrow H_2 + O_2$	Electric / chemical	340–380	50–500	10–100	1.0
Hybrid	O_2 and rubber	Chemical	225	$225–3.5 \times 10^5$	50–75,000	1.14 and 1.5
Electrothermal:						
Resistojet	N_2, NH_3, N_2H_4, H_2	Resistive heating	150–700	0.005–0.5	0.001–0.1	0.28*, 0.60, 1.0, 0.019*
Arcjet	NH_3, N_2H_4, H_2	Electric arc heating	450–1,500	0.05–5	0.01–1	0.60, 1.0, 0.019*
Electrostatic:						
Ion	Hg/A/Xe/Cs	Electrostatic	2,000–6,000	$5 \times 10^{-6}–0.5$	$10^{-6}–0.1$	13.5/0.44*/2.73*/1.87
Colloid	Glycerine	Electrostatic	1,200	$5 \times 10^{-6}–0.05$	$10^{-6}–0.01$	1.26
Hall Effect Thruster	Xenon	Electrostatic	1,500–2,500	$5 \times 10^{-6}–0.1$	$10^{-6}–0.02$	0.22
Electromagnetic:						
MPD‡	Argon	Magnetic	2,000	25–200	5–50	0.44*
Pulsed Plasma	Teflon	Magnetic	1,500	$5 \times 10^{-6}–0.005$	$10^{-6}–0.001$	2.2
Pulsed Inductive	Argon N_2H_4	Magnetic Magnetic	4,000 2,500	2–200 2–200	0.5–50 0.5–50	0.44 1.0

Hybrid Rockets



Advantages:

- Throttled
- Non-Explosive

Disadvantages:

- Requires Oxidizer
- Bulky

Flown on SpaceShipOne (Developed by SpaceDev, N02, $I_{sp} = 250s$, Max Thrust 74kN)

Hybrid Rockets

Motor	Average Thrust (lb_f)	Average Thrust (kN)	Burn Duration (sec)	Fuel	Oxidizer	Comments
American Rocket Company						
H-500	75,000	333	70	HTPB	LOx	Qualified for flight
H-250	32,000	142		HTPB	LOx	In development
H-50	10,000	44		HTPB	LOx	In development
U-50	6,500	29		HTPB	LOx	In development
U-1	100	0.44		HTPB	LOx	In development
United Technologies						
	40,000	178	300	HTPB	IRFNA	Flown on Firebolt air-launched target drone, 1968
StarsTruck						
	40,000	178		CTBN	LOx	Flown on Dolphin water-launched sounding rocket, 1984
USAF Academy						
H-1	55	0.25	2.3	HTPB	GOx	Flown on 4-ft tall rocket for student project, 1991

Figure: American Rocket Company = SpaceDev

Electric Propulsion

Electrothermal:

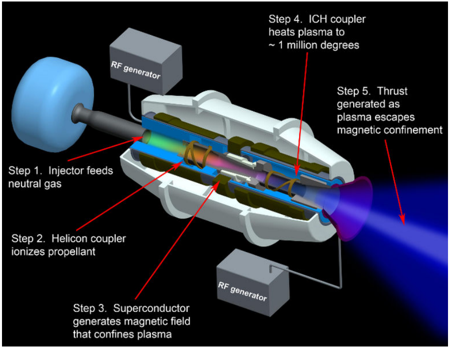
- Ohmic Heating

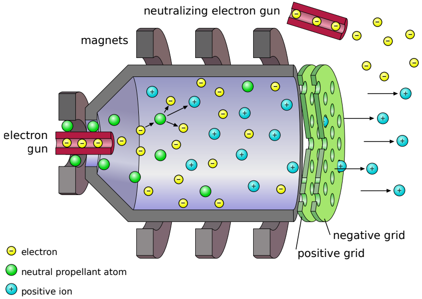
Electrostatic:

- Repulsion/Attraction

Electromagnetic:

- Ions accelerated by EM waves





Advantages:

- Very High Performance

Disadvantages:

- Low Thrust
- Requires Electricity

Electric Propulsion

The choice of Electrothermal/Electrostatic/Electromagnetic depends on available power.

Electrothermal	Electrostatic	Electromagnetic
• Gas heated via resistance element or arc and expanded through nozzle • Resistojets • Arcjets	• Ions electrostatically accelerated • Hall effect (HET) • Ion • Field emission	• Plasma accelerated via interaction of current and magnetic field • Pulsed plasma (PPTs) • Magnetoplasmadynamic (MPD) • Pulsed inductive (PIT)
Power Range; 0.4–2 kW	1–50 kW	50 kW–1 MW
Specific Impulse, I_{sp}; 300–800 sec	1,000–3,000 sec	2,000–5,000 sec

Electric Propulsion

Concept	Characteristics					
	Specific Impulse, (sec)	Input Power, (kW)	Thrust/ Power, (mN/kW)	Specific Mass, (kg/kW)	Propellant	Supplier
Resistojet	296	0.5	743	1.6	N_2H_4	Primex
	299	0.9	905	1	N_2H_4	Primex, TRW
Arcjet	480	0.85	135	3.5	NH_3	IRS/ITT
	502	1.8	138	3.1	N_2H_4	Primex
	>580	2.17	113	2.5	N_2H_4	Primex
	800	26*	—	—	NH_3	TRW, Primex, CTA
Pulsed Plasma Thruster (PPT)	847	< 0.03†	20.8	195	Teflon	JHU/APL
	1,200	< 0.02†	16.1	85	Teflon	Primex, TSNIIMASH, NASA
Hall Effect Thruster (HET)	1,600	1.5	55	7	Xenon	IST, Loral, Fakel
	1,638	1.4*	—	—	Xenon	TSNIIMASH, NASA
	2,042	4.5	54.3	6	Xenon	SPI, KeRC
Ion Thruster (IT)	2,585	0.5	35.6	23.6	Xenon	HAC
	2,906	0.74	37.3	22	Xenon	MELCO, Toshiba
	3,250	0.6	30	25	Xenon	MMS
	3,280	2.5	41	9.1	Xenon	HAC, NASA
	3,400	0.6	25.6	23.7	Xenon	DASA

Staging

Previously, we assumed the rocket only consisted of payload and propellant: $m_0 = m_L + m_p$.

$$\frac{m_p}{m_0} = 1 - e^{-\frac{\Delta v}{c}} \qquad \Delta v = c \ln\left[\frac{m(t_0)}{m(t_f)}\right] = c \ln\left[\frac{m_L + m_p}{m_L}\right]$$

Which would mean the only way to increase Δv is to decrease payload or increase the size of the rocket.

However: Payload is not the only part of the rocket.

- Rocket engines and storage tanks are heavy.
- Typically, *structure* accounts for $\cong 1/7$ of the propellant weight

$$m_0 = m_L + m_s + m_p = (m_L + 1/7 m_p) + m_p$$

- While $1/7$ may not seem a lot, without staging, it limits the total Δv to

$$\Delta v = c \ln\left(\frac{m_p}{m_p/7}\right) = c \ln 7 \cong 2c \cong 6 km/s \qquad \text{(assuming } m_L = 0\text{)}$$

But $\Delta v = 8$km/s is needed for low earth orbit (LEO) - not accounting for drag or gravity losses (2km/s)!

OMG: Space flight is IMPOSSIBLE!

- It's a conspiracy.

Structural Mass on Saturn V

Structural Coefficient

Definition 2.

The ratio of structure to propellant is called the structural coefficient, ϵ:

$$\epsilon = \frac{m_s}{m_s + m_p}$$

In the ideal case, structural weight would be discarded as soon as it is no longer required.

- Continuous Staging

In this ideal scenario, we would have

$$\Delta v = (1 - \epsilon) c \ln \left[\frac{m_0}{m_L} \right]$$

- The structure simply decreases the efficiency of the fuel!

In **Staging**, we discard structure at discrete points in time.

- Staging can never be better than $\Delta v = (1 - \epsilon) c \ln \left[\frac{m_0}{m_L} \right]$.

3-stage Scenario

Suppose we divide the structural and propulsive weight into three components

1. First Stage: m_{s1}, m_{p1}
2. Second Stage: m_{s2}, m_{p2}
3. Third Stage: m_L, m_{p3}

Then Δv is the combined Δv of all three stages.

$$\begin{aligned}
\Delta v_T &= \Delta v_1 + \Delta v_2 + \Delta v_3 \\
&= c \ln \left[\frac{m_{p1} + m_{p2} + m_{p3} + m_{s1} + m_{s2} + m_L}{m_{p2} + m_{p3} + m_{s1} + m_{s2} + m_L} \right] \\
&+ c \ln \left[\frac{m_{p2} + m_{p3} + m_{s2} + m_L}{m_{p3} + m_{s2} + m_L} \right] + c \ln \left[\frac{m_{p3} + m_L}{m_L} \right]
\end{aligned}$$

Optimal choice of m_{p1}, m_{p2} and m_{p3} is difficult.
For a fixed total mass, m_0, we can maximize

- Payload weight
- Total Δv

A good rule is $m_{p1} = 3m_{p2} = 9m_{p3}$.

1,2 and 3stage Scenarios (Fixed Total Mass)

TABLE 6.3 ONE-, TWO-, AND THREE-STAGE ROCKETS STAGES (EQUAL ε AND λ) $c = 3048$ m/sec ($I_{sp} = 311$ sec)

	1 stage	2 stage	3 stage	
			specified m_L	specified Δv
m_{01}	15,000	15,000	15,000	15,000
m_{02}	–	3,873	6,082	4,926
m_{03}	–	–	2,466	1,618
m_{s1}	2,000	1,590	1,274	1,393
m_{s2}	–	410	517	457
m_{s3}	–	–	209	150
m_{p1}	12,000	9,537	7,644	8,681
m_{p2}	–	2,463	3,099	2,851
m_{p3}	–	–	1,257	936
m_L	1,000	1,000	1,000	531
ε	0.143	0.143	0.143	0.138
λ	0.0714	0.348	0.682	0.489
Δv (m/sec)	4,906	6,157	6,515	7,905
$m_{s_{TOT}}$	2,000	2,000	2,000	2,000
$m_{p_{TOT}}$	12,000	12,000	12,000	12,469
Z	5	2.75	2.039	2.374

Staging on Minuteman ICBM

Summary

This Lecture you have learned:

Introduction to Rocketry

- Mass Consumption
- Specific Impulse and Rocket Types
- Δv limitations
- Staging