Spacecraft Dynamics and Control

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Lecture 15: Attitude Dynamics and Control

Attitude Dynamics

In this Lecture we will cover:

- Mission Requirements
- Forms of Attitude Control

The Problem of Attitude Stabilization

- Actuators

Newton's Laws

- $\sum \vec{M}_i = \frac{d}{dt}\vec{H}$
- $\sum \vec{F}_i = m\frac{d}{dt}\vec{v}$

Rotating Frames of Reference

- Equations of Motion in Body-Fixed Frame
- Often Confusing

Orientation of a spacecraft

Let's begin with some examples

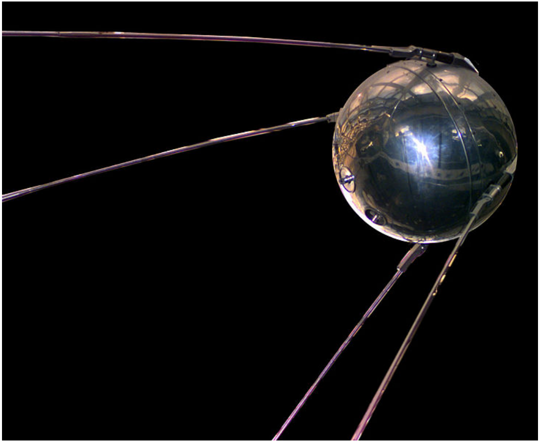


Figure: Sputnik I Satellite

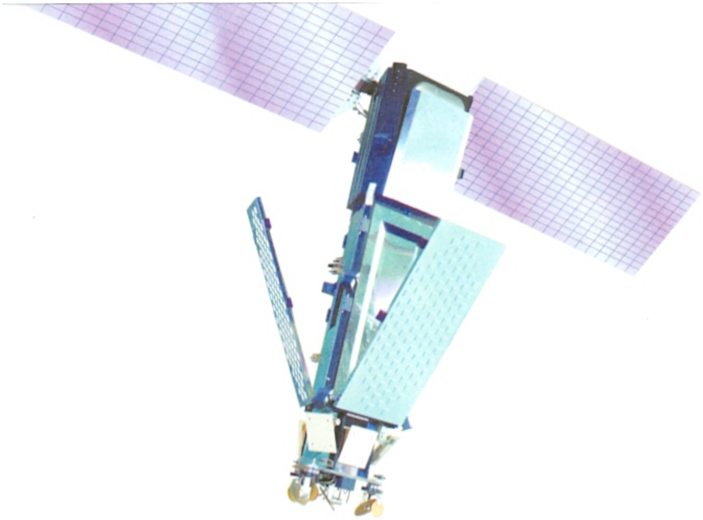
Orbit: 947 x 228, 65° inclination. Spin Stabilized.

NASA Tracking and Data Relay Satellite (TDRS)

There have been 11 TDRS missions.

- Enable continuous real-time communications between space and ground.
- GEO, TDRS-K is currently at $\Omega = 150°$ (Final $\Omega = 171°$).

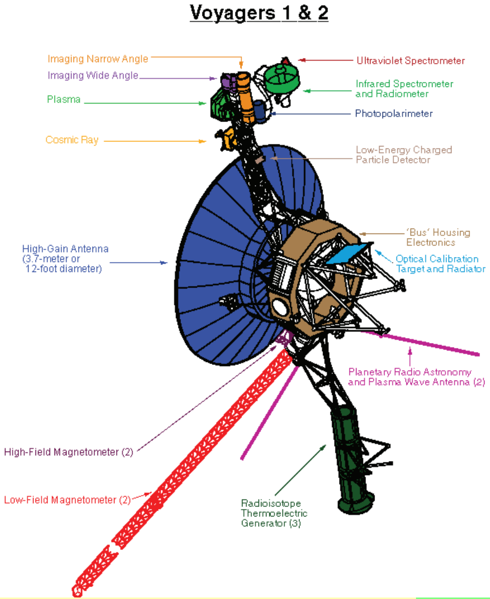
Iridium Satellite Constellation. Launched 1992-1999



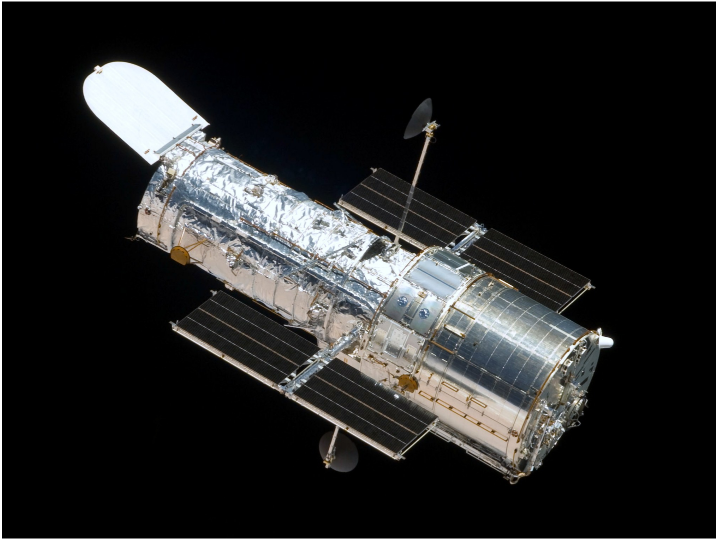
Telephone. Originally commercial, now pseudo-military. 66 active satellites.
Orbit: 780km, 75° inclination, 6 orbital planes.

Voyager I and II (1977)

Interplanetary Exploration



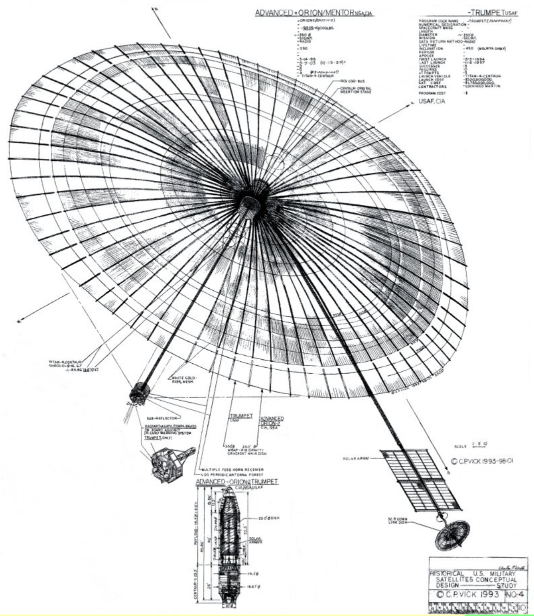
Hubble Space Telescope (HST) - 1990



Orbit: 613 x 620, 28.5° inclination

Advanced Orion/MENTOR

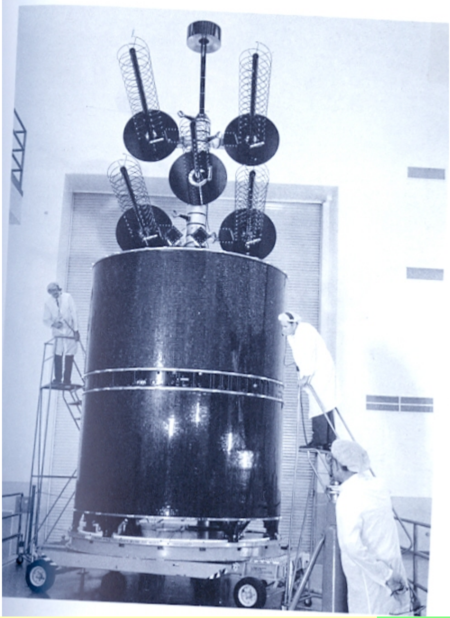
- Signals Intelligence
- Duration 1994-???
- L-32 is the largest satellite ever launched with a dish size of $\cong 100$m



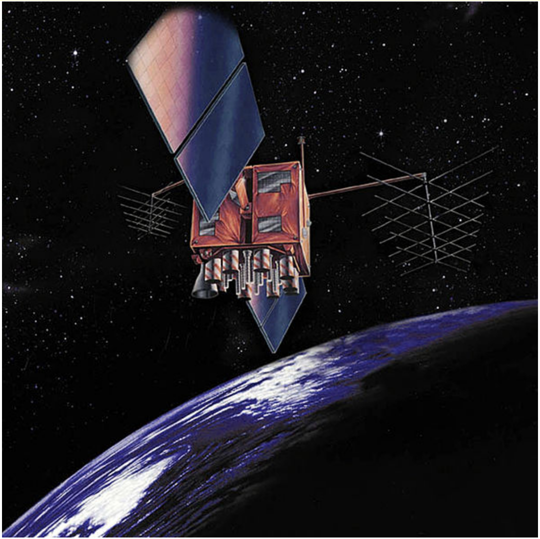
TACSAT I (1969-1972)

Military Tactical Communications Satellite.

Orbit: GEO, $\Omega = 107°$



GPS Block IIR (2005-2009)



Orbit: 20,182km, 55° inclination.
GPS constellation has 24 satellites in 6 planes.

GPS Constellation

Figure: GPS Constellation is designed to have at least 4 satellites visible at any given time anywhere in the world.

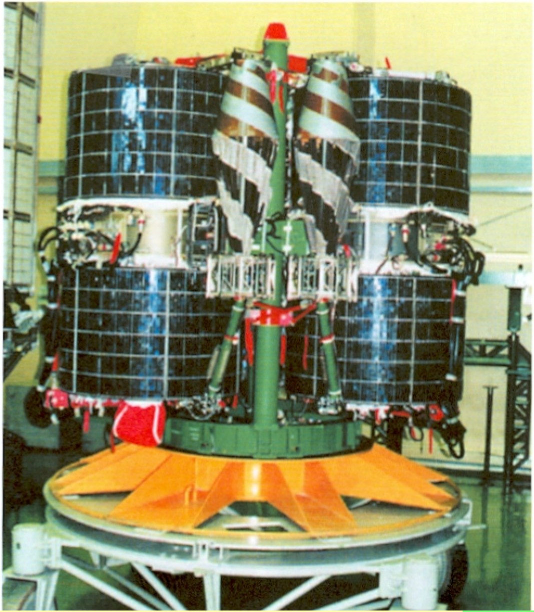
Strela-3 Satellite Constellation (1985)

Russian Military Communications

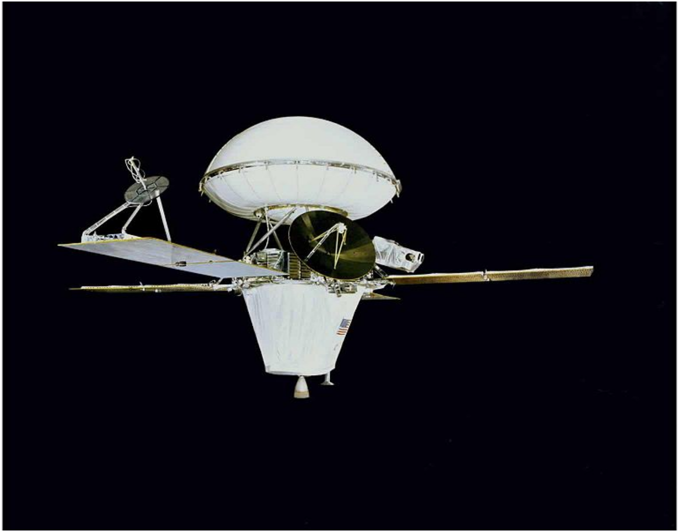
- 12 Satellites
- Gravity-Gradient Stabilization

Note: 2009 Satellite Collision between Strela 2M and Irridium 33

Orbit: 1440 x 1450, $82.5°$ inclination.



Viking I



Interplanetary Mission to Mars.
Orbit: 320 x 56,000km, $39.3°$ inclination. $e = .8822$.

Attitude Stability

What can go wrong?

Figure: Tumbling Satellites are sometimes visible in the Night Sky

Attitude Stability and Control

Attitude stability is required for almost all satellite applications.

- Communication
- Reconnaissance
- Navigation
- Exception
  - LAGEOS
  - ECHO I, II

Even Sputnik was spin-stabilized

Problem: Unlike aircraft, spacecraft cannot rely on aerodynamic forces to provide stability.

- If a spacecraft is not attitude stabilized, small disturbances will cause it to tumble.

Question: How to stabilize a satellite

There are many varieties and methods for attitude control of spacecraft.

- 3-axis Stabilization
  - Thrusters $(.1 - .5^\circ)$
  - Control-Moment Gyros (CMGs) $(.001 - 1^\circ)$
  - Momentum wheels $(.001 - 1^\circ)$
- Spin Stabilization $(.1 - 1^\circ)$
  - Also good for pre-insertion.
- Gravity-Gradient Stabilization (5°)
- Magnetic Torquers (5°)

Lets go through a few of these.

No Control (LAGEOS, ECHO)

Figure: LAGEOS Geodesy Satellite

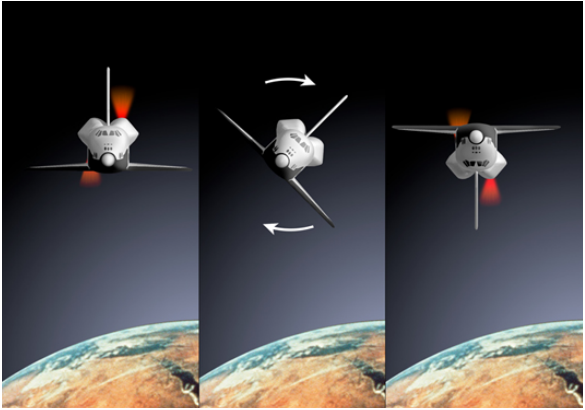
Figure: ECHO II communication satellite

Mechanisms for Attitude Control

Thrusters

Thrusters are grouped in pairs in order to provide pure moment

- no change in orbit.

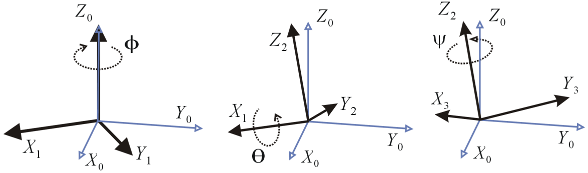


- Thrusters are typically bang-bang
- Provide discrete units of angular momentum (spin up).

$$\Delta h = F \Delta x \Delta t$$

Thrusters may alter orientation and angular velocity.

- e.g. Through rotation matrices



Only two sets of thrusters are needed to achieve any orientation (Euler Angles).

1. Rotate about $\hat{b}_3$ until $\hat{b}_1$ lines in $\hat{a}_2 - \hat{a}_1$ plane.
2. Rotate about $\hat{b}_1$ until $\hat{b}_2$ lines in $\hat{a}_2 - \hat{a}_1$ plane.
3. Rotate about $\hat{b}_3$ until $\hat{b}_1 = \hat{a}_1$ and $\hat{b}_2 = \hat{a}_2$.

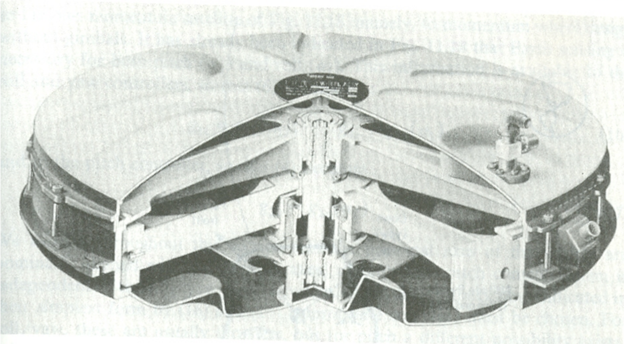
Usually better to have 3 sets of thrusters to minimize fuel

- Otherwise small changes can lead to big rotations.

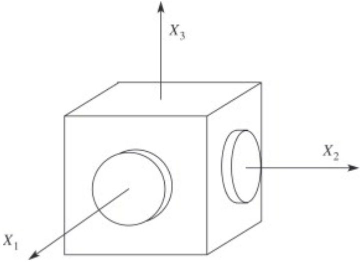
Mechanisms for Control

Reaction wheels

Because thrusters are not extremely accurate, they are rarely used to control the finer motions of attitude. Maintenance maneuvers are usually left to other mechanisms.



Reaction wheels: A momentum wheel rotates about a given axis aligned with the body-axis of the spacecraft.



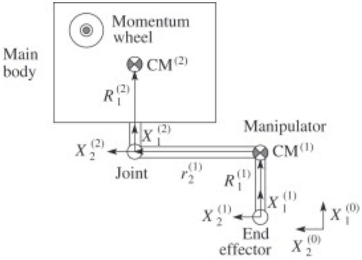
Dynamics: Consider rotation about the x-axis.

- Let J_x be the moment of inertia of the Spacecraft.
- Let I_x be the moment of inertia of the flywheel.
- By conservation of angular momentum:

$$I_x(\omega_f + \omega_s) + J_x\omega_s = 0$$

  - ω_s is the angular velocity of the the craft in inertial space.
  - ω_f is the angular velocity of flywheel w/r to the craft.

Figure: Spacecraft rotation via 3-1-3 Euler Angles

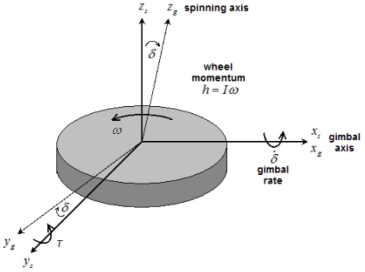


So if the craft has some velocity ω_s in the $\hat{b}_1$-direction and the reaction wheel is aligned with this axis, we can null out the velocity by spinning up to

$$\omega_f = -\frac{J_x\omega_s + I_x\omega_s}{I_x} = -\frac{J_x + I_x}{I_x}\omega_S.$$

If we have reaction wheels in the $\hat{b}_2$ and $\hat{b}_3$ directions, we can create any angular velocity vector.

- Flywheels can be use to correct for small deviations (Telescopes).
  - However, accumulated momentum may cause the flywheels to spin too fast.
  - Will need to eventually find a way to dump momentum.



Control Moment Gyros are different from reaction wheels in that they have a fixed rate of rotation (ω_{CMG})

- Thus the magnitude of the angular momentum vector, $\|\vec{h}\|$ will be fixed.
- The direction of the angular momentum vector will vary, however.

Single Gimbal Control is achieved by rotation of the gyroscope through an angle δ.

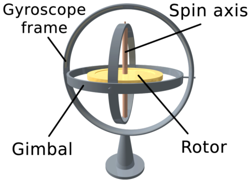
- This can only be used for 2-axis stabilization

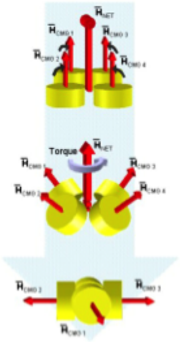
Single-Gimbal Control Moment Gyro

Figure: Effect of Single Control Moment Gyro

Dual-Gimbal Control Moment Gyro

Alternatively, a dual-gimbal CMG may be used





Suppose

- The initial angular momentum vector of the CMG is $\bar{h}$
- The desired angular momentum vector of the spacecraft is $\bar{h}_d$
- The final position vector of the CMG is $R_3(\theta_3)R_1(\theta_2)R_3(\theta_1)\bar{h}$

By conservation of angular momentum

$$\bar{h} = \bar{h}_d + R_3(\theta_3)R_1(\theta_2)R_3(\theta_1)\bar{h}$$

Dual-Gimbal Control Moment Gyro

Definition 1.

Given $\bar{h}$, the **Momentum Envelope** is set of solutions of

$$\bar{h}_d = (I - R_3(\theta_3)R_1(\theta_2)R_3(\theta_1))\bar{h}$$

for some set of Euler rotations, $\theta_1, \theta_2, \theta_3$.

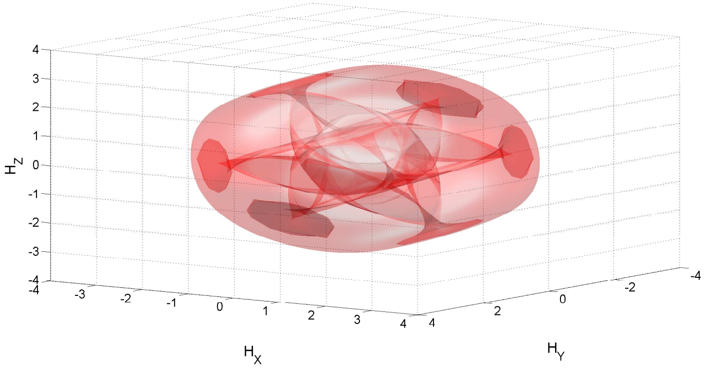
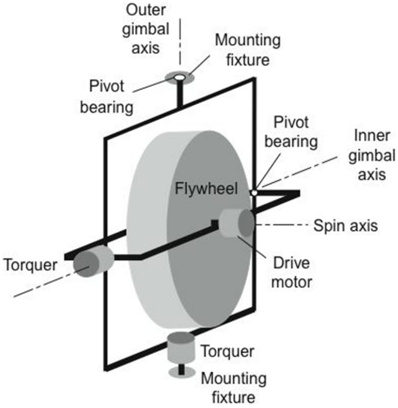


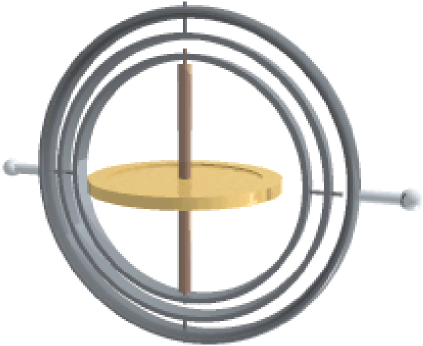
Figure: Momentum Envelope for a pyramidal 4-CMG array. Note the singularities.

Dual-Gimbal Control Moment Gyro

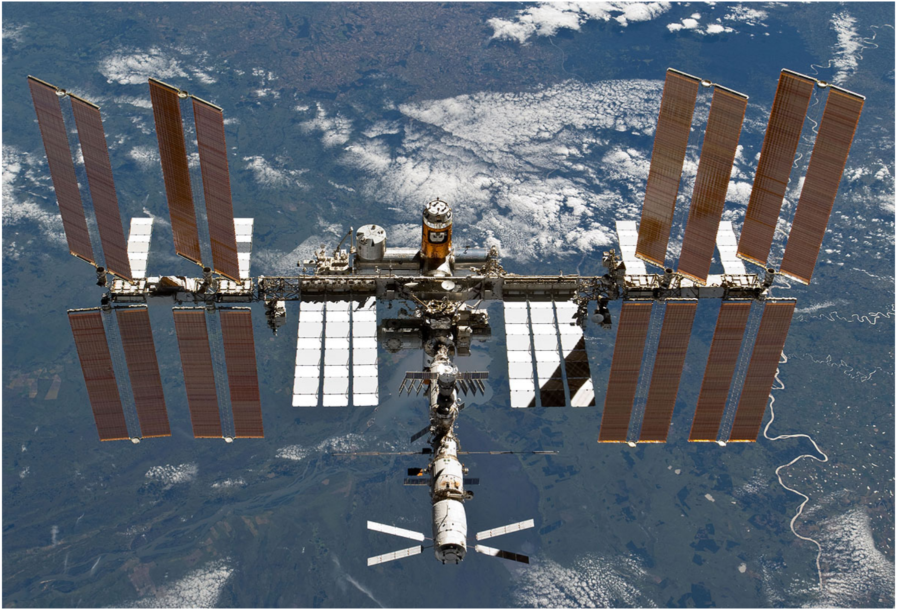
The effect of the singularities is most easily understood as **Gimbal Lock**

- Two axes of the gyroscope align.
- Rotation about that axis requires no torque.
  - Freely spinning.





Control Moment Gyros on the International Space Station



Control Moment Gyros on the International Space Station

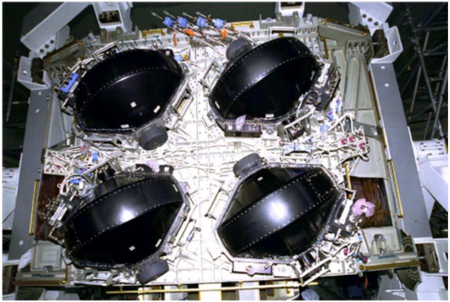


Figure: ISS Z1 truss with 4-CMG array

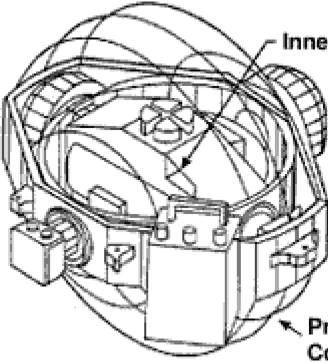
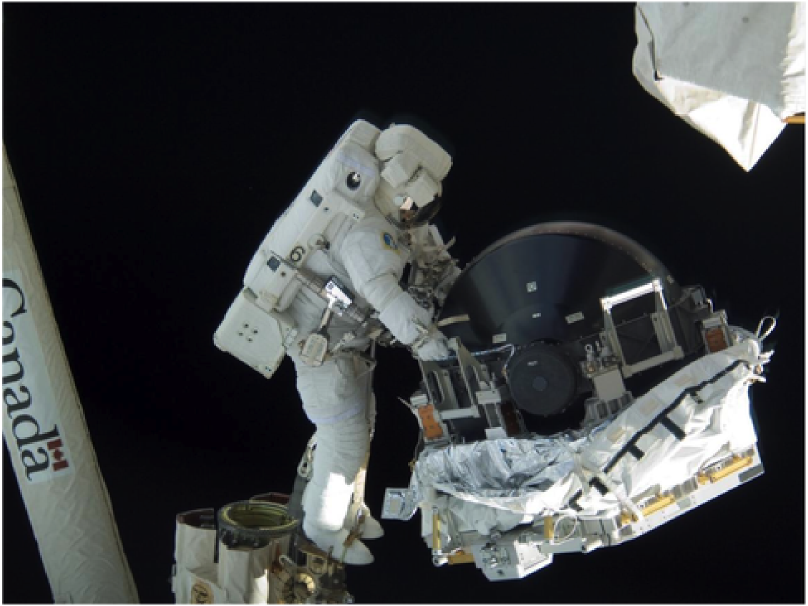


Figure: Cutaway of ISS CMG

The CMGs are double-gimbal, so hardware orientation is not important.

Replacement of CMG on the International Space Station



Control Moment Gyros on the International Space Station

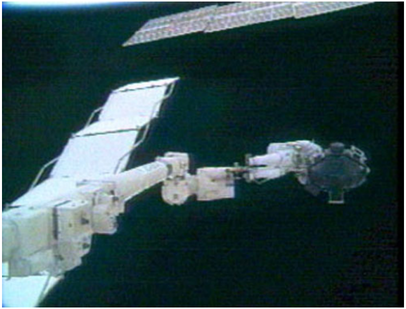


Figure: Replacement of CMG on ISS in 2005 (STS-114)

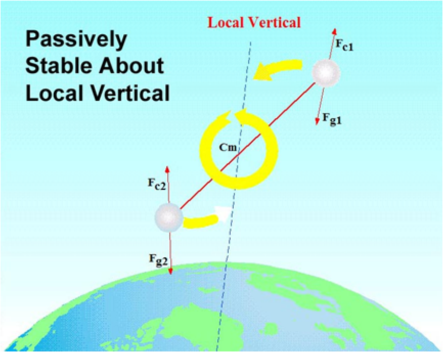
Mechanisms for Control

Gravity Gradient Stabilization

Gravitational attraction varies as

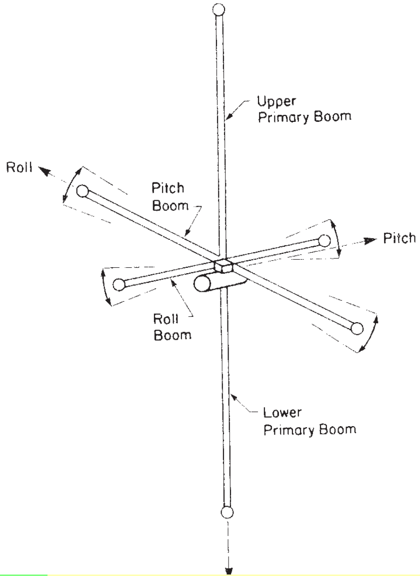
$$\|F\| = \frac{\mu m}{r^2}$$

For very long spacecraft, lower section will feel additional gravitational attraction.



Mechanisms for Control

Gravity Gradient Stabilization



Gravity Gradient Stabilization

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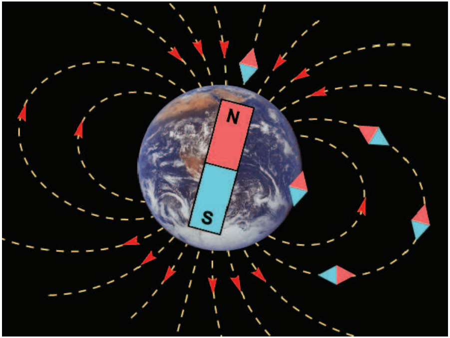
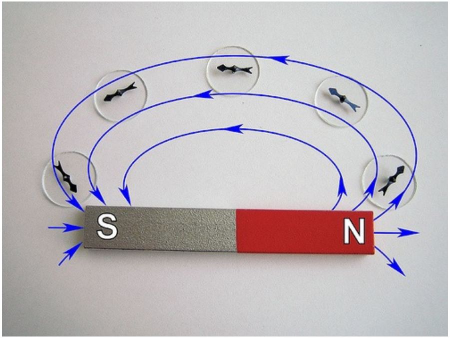
Magnetic Torquers

In addition to gravity, the **Magnetic Field** of the earth can be used to provide attitude control.

Idea: The earth has a magnetic field, $\vec{\mathbf{B}}_e(x, y, z)$.

- the interaction of two magnets produces force





What if we put a magnet on the spacecraft?

- Turn it into a giant flying compass.

Magnetic Torquers

Instead of using fixed magnets, we use electromagnets to create an arbitrary magnetic dipole moment, $\vec{M}$ for the spacecraft.

Maxwell's Equations lead us to

$$\vec{T} = \vec{M} \times \vec{\mathbf{B}}_e$$

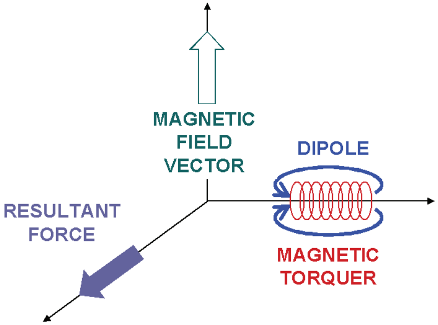
Thus

$$\begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} = \begin{bmatrix} 0 & B_z & -B_y \\ -B_z & 0 & B_x \\ B_y & -B_x & 0 \end{bmatrix} \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix}$$

Unfortunately,

$$\begin{bmatrix} 0 & B_z & -B_y \\ -B_z & 0 & B_x \\ B_y & -B_x & 0 \end{bmatrix}$$

is not invertible.



- Magnetic fields cannot rotate the spacecraft about a field-line.
- Pitch or Yaw forces - No Roll.

Magnetic Torquers

The magnetic dipole moment is created by torque-rods.

Unfortunately the magnitude of the torque is limited by:

- The magnitude of earth's magnetic field is inversely proportional to radius.

$$\|\mathbf{B}_e\| \cong \frac{7.96 \cdot 10^{15} Wb - m}{r^3}$$

- The magnetic dipole of the torque rod ($\|M\| \cong 10Am^2 - 100Am^2$).
- The angle to the field line (α).

$$T = \|M\|\|B\| \sin \alpha$$

Magnetic Torquers

In a 400km orbit with a $100A - m^2$ dipole at $30°$ field orientation, we can obtain a torque of

$$T = \frac{7.96 \cdot 10^{15}}{6778000^3} \cdot 100 \cdot \sin(30°) = 1.28 \cdot 10^{-3} N - m$$

Magnetic Torque is not typically used for active attitude control.

- Used to dump angular momentum over time from
  - Reaction Wheels
  - CMGs
- Combined with momentum wheel for roll-control.

Solar Sail Stabilization

Multi-Functional Transport Satellite (MTSAT)

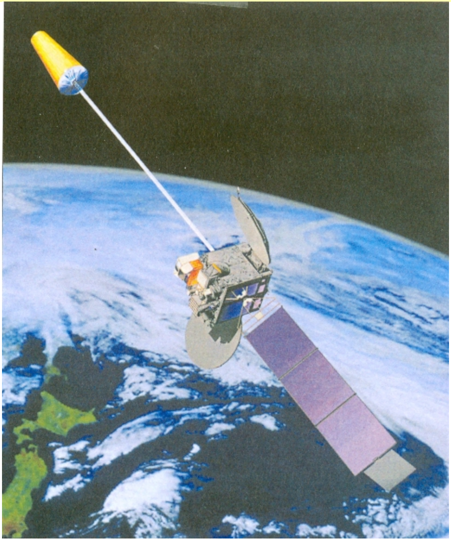


Figure: Japanese Air-traffic Control/Navigation/Meteorology Satellite MTSAT

Mechanisms for Control

Spin Stabilization

Historically, the most common form of stabilization have been spin stabilization.

Idea: Give the craft an angular momentum vector which is fixed in the body-fixed axis.

- Think of rifles vs. muskets

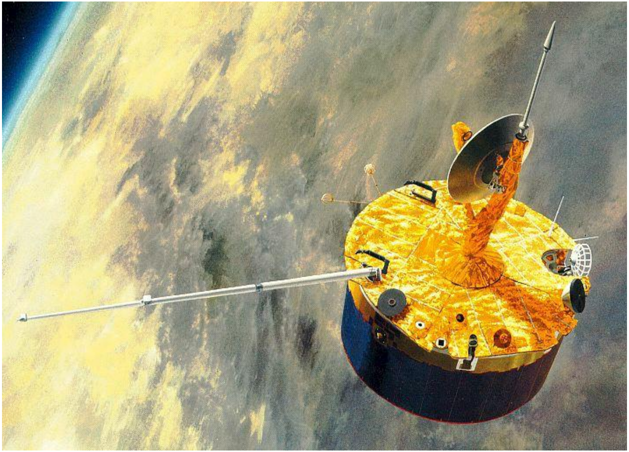
Positives:

- By Newton's second Law: A large angular momentum vector requires large torques to change.
- Very little active maintenance required.

Negatives:

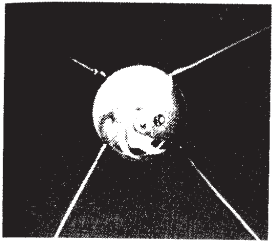
- Spin motion complicates communication, navigation, etc.
- Changing attitude after spin-up is very difficult.
- Angular momentum vector is not fixed in the body-axes!
  - We will study this issue in more detail.

Pioneer Venus Orbiter (1978-1992)

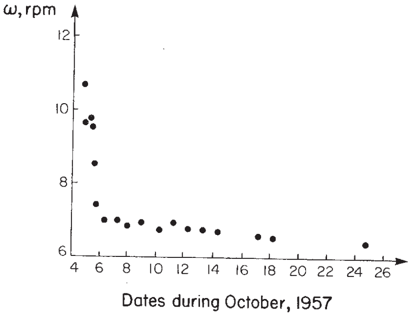


Mapping/Communication. Dual-Spin Stabilized.

Spin stabilization can decay.



(a) Sputnik 1



(b) Spin Rate Data, based on [Beletskii]

Figure: Decay in Spin Control of Sputnik I Satellite

Conclusion

In this lecture we have covered:

- Mission Requirements
- Forms of Attitude Control

Next Lecture:

Equations of Motion

- How to differentiate Vectors in Rotating Frames
- Derivation of the Nonlinear 6DOF Equations of Motion

Euler Angles

- Definition of Euler Angles
- Using Rotation Matrices to transform vectors
- Derivatives of the Euler angles
  - Relationship to p-q-r in Body-Fixed Frame