Research Statement

Matthew M. Peet

Introduction Consider the following general convex optimization problem.

$$\begin{aligned} \min \quad & ax \\ \text{subject to:} \quad & Bx \in C \end{aligned}$$

I often make the claim that most problems in controls can be expressed in this form for some convex function a, affine transformation B, and suitably chosen convex cone C. Generally, the process of casting a problem as convex optimization involves lifting the problem into a higher dimensional space. Thus in only a few cases will the resulting optimization problem actually be solvable by an efficient numerical algorithm. In particular, some necessary conditions for solvability are:

- An efficient algorithm for testing membership in the cone C
- That x is finite dimensional

To date, the fastest algorithms can only solve problems on the the cone of positive matrices. However, many important problems are of this form. In the following description of my research, I describe new techniques I have developed for solving optimization problems which are not expressible in the standard positive semidefinite form.

Communication Networks The need for efficient exchange of information has led to the creation of a wide variety of telecommunication networks. Familiar examples of networks with a wide geographic reach and a large number of users include the internet, telephone, postal and power distribution services. In each of these cases, the goal is to transmit some relevant equivalent of information, be it in the form of packets, bandwidth, letters or voltage, without the aid of a strong centralized control mechanism. In each of the cases listed above, certain ad hoc decentralized transmission controls have been adopted. These ad hoc protocols generally work well as short term solutions, but as the systems expand they almost inevitably lead to widespread system failure, as has occurred on occasion for the internet and power networks.

In my work at Stanford, I examined certain classes of decentralized control mechanisms which have been proposed in the context of internet congestion control. These protocols had previously been shown to maintain control even in the presence of delay if disruptions or changes to the system were very small. In my own work, I was able to prove that the same proposed system is able to maintain control regardless of the size of the perturbations. These results, which have been confirmed by experimental evidence, not only prove effectiveness of the proposed protocol, but also allow one to predict under what conditions certain other mechanisms are likely to fail.

At a fundamental level, the design of decentralized control systems in general and internet congestion control protocols in particular are attempts to solve optimization problems using distributed resources. Specifically, we consider decentralized optimization problems of the following form.

$$\begin{aligned} \max \quad & \sum_{i=1}^{k} f_i(x_i) \\ \text{subject to:} \quad & Bx \geq c \end{aligned}$$

In large systems such as the internet, $k \to \infty$ and so centralized computation becomes infeasible. Furthermore, even if one could design a centralized controller, the presence of information constraints would make implementation impossible. Therefore, we develop local control mechanisms which, when combined, create an algorithm for massively parallel computation on a semi-infinite array of nodes. My research at Stanford, then, was to prove convergence of these algorithms in the absence of a centralized clock. In my papers, I showed that when one applies certain generalized coordinate transformations to individual subsystems, each resource is passive in a certain sense. In this way, it can be shown that arbitrary interconnections of such systems converge to the optimal distribution of resources.

Due to a number of factors, there is no universally agreed upon approach to solving even some of the simplest problems in decentralized control. Therefore, analysis of decentralized control mechanisms is likely to be an active and fruitful area of research for some time to come. Particular areas of study which are of interest to me include the analysis of other promising congestion control protocols, almost none of which have been shown to converge in the asynchronous case. Another topic of interest in this area includes application of similar transformations to systems of power distribution. These systems, while having a similar information structure, differ in significant ways such as the absence of a positive queue, a poorly understood feature of current internet congestion control.

Positive Forms and Numerical Algorithms The field of control theory has recently been significantly altered by the development of fast numerical algorithms for solving general convex optimization problems using barrier methods. These advances, combined with the codification of computational complexity, have contributed to what is by now a fairly sophisticated understanding of what algorithms are capable of. The results are often surprising. For example, while things as seemingly trivial as how to color a map or plan a world tour have been shown to be impossibly complex, seemingly complicated chores like designing the optimal linear controller for a large chemical refinery or determining the optimal data routing for the internet can be relatively easy. An understanding of the fundamental differences between these kinds of problems is critical when approaching the challenges in design and control of engineering systems.

My research on interior-point algorithms addresses the problem of whether communication networks will destabilize when signals are delayed. The problem is that the use of Linear Matrix Inequalities(LMIs) for the analysis of delayed systems is not well understood. In addition, complexity results show that analysis of delayed systems may be fundamentally hard(NP-hard). In one of my main contributions to the literature, I proved that delayed systems can be analyzed in the sense of Lyapunov by considering certain "semi-decidable" LMI conditions. It is interesting to note that while my results do not contradict the statement on computational complexity of delayed systems, neither do they support the hypothesis that delayed systems are impossibly difficult to understand. In fact, these results form the basis of an approach which also allows us to consider systems with nonlinearity and uncertainty. This work offers the possibility of defining a large class of problems that, while not easy, are also not impossibly complex.

To better present my work in this area, it is helpful to return to a special type of convex optimization problem. Specifically, consider the convex cone of positive linear operators. This cone defines the set of positive quadratic functions on a given space. It is important to be able to optimize over the cone of positive quadratic forms, since it frequently arises in applications such as Lyapunov stability and machine learning. For example, the cone of positive matrices is used to analyze stability of ordinary differential equations on $\mathbb{R}^n$. Now consider the example of positive integral operators on L_2. Every such operator is defined by a kernel function, N, and defines a quadratic form as

$$V(x) = \int\int x(s)N(s,t)x(t)dsdt.$$

Note that in this case, positivity of N does not guarantee a positive quadratic form. In a recent paper, I gave necessary and sufficient LMI conditions for N to define a positive operator given certain conditions on N. Examples of my work in this area also include positive multiplier operators on subspaces of L_2 and extensions to non-quadratic forms. These types of result can be used to synthesize Lyapunov functions for most systems which are described by semigroups on L_2. Particular examples of systems which I have analyzed using these methods include systems with delay and classes of partial differential equations.

Ongoing research topics include extensions to full state feedback controller synthesis. Recent results have shown progress in computing these controllers. Additionally, an intriguing subject of considerable recent interest is the field of machine learning. In these types of systems, the learning algorithm is given by a positive integral operator defined by a kernel function. Typically these kernels are constructed in such a way that the algorithm has finite learning capacity. By using kernels that have certain properties in common with the Gaussian, however, I may be able to create systems with increased learning capacity. Another topic of work which has generated considerable excitement is a generalization of the KYP lemma to irrational functions and multiple variables. Progress on such a result is important to the continued relevance of my work and a topic of considerable weight in my current studies. Important applications of my work include mathematical biology and and certain types of fluid dynamics.

Research Directions and Motivation Specific aspects of my past research and future plans can be found in the previous two sections of this statement. My topics of interest tend to be those which are generally considered to be "hard". These include analysis and control of nonlinear, infinite-dimensional and decentralized systems. In general I aggressively seek out new opportunities for research. As part of exploring new possibilities, I actively look for and participate in regional, national and international collaborative efforts with colleagues. Examples of my recent work which has been based on such collaboration includes that on numerical algorithms to study stability of flows described by linear partial differential equations using a frequency-domain approach; on time-domain approaches to stability of nonlinear partial differential equations; on estimating H_∞ norms for delayed systems; and a recent result generalizing the Weierstrass theorem to sets of continuous functions subject to linear constraints. Please refer to my papers and curriculum vitae for details on the subjects presented in this statement and on other work not mentioned here.