Spacecraft and Aircraft Dynamics

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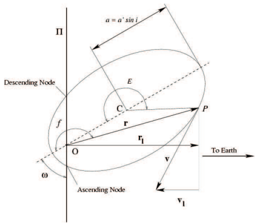
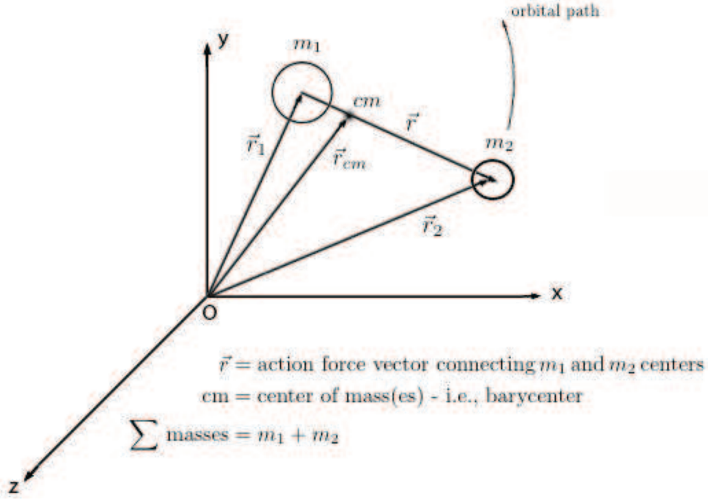
Lecture 1: Spacecraft Dynamics

- Determining Orbital Elements
  - Know Kepler's Laws of motion, Frames of Reference (ECI, ECEF, etc.)
  - Given position and velocity, determine orbital elements.
  - Given orbital elements and time, determine position + velocity.
- Plan Earth-Orbit Transfers
  - Identify Required Orbit.
  - Find Optimal Transfer.
  - Determine Thrust and Timing.
- Plan Interplanetary Transfers
  - Design Gravity-Assist Maneuvers.
  - Use Patched-Conics.



Things we probably won't cover but would like to.

- Satellite Dynamics
  - Pointing, Tracking Problems
  - Vibration Damping
  - 6 DOF motion
- Controllers
  - Control Moment Gyros
  - Spin Stabilization
  - Gravity-Gradient Stabilization
  - Attitude Thrusters
- Propulsion
  - Chemical Rockets
  - Nuclear Rockets
  - Solar Sails
  - Ion engines
  - Gravity Assist
- Linear Orbit Theory (Perturbations)
  - Earth-Oblateness
  - Drag
  - Solar Wind
- 3-body orbits
- Ground Tracking
- Orbit Estimation

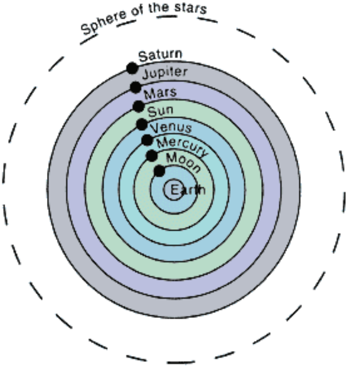


Modeling the system is 90% of the problem.

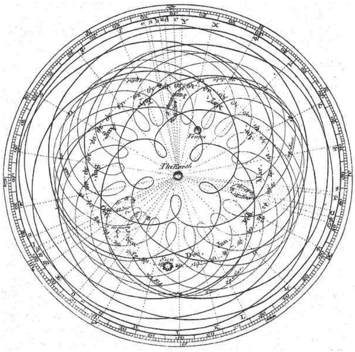
Developing the model

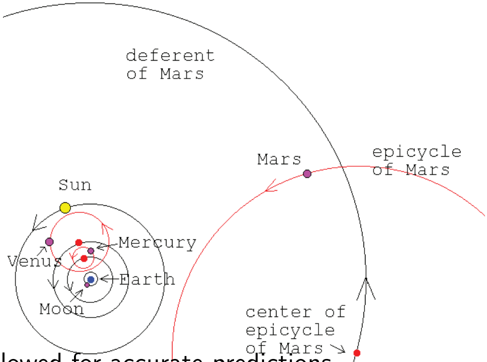
Ptolemy (ca. 100-178 AD)

Ptolemy observed that the moon and sun move in a circular motion about the spherical earth (daily and yearly).



- Wrote the Almagest
- Hypothesizes that all planets move in a similar manner.
- Beat out Sun-centered, rotating-earth model of Aristarchus.





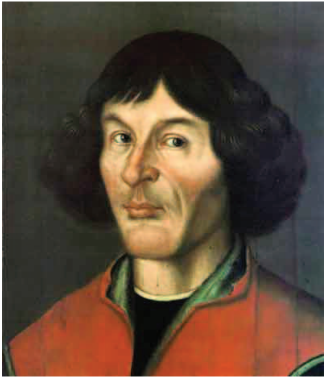
Concentric Circles:

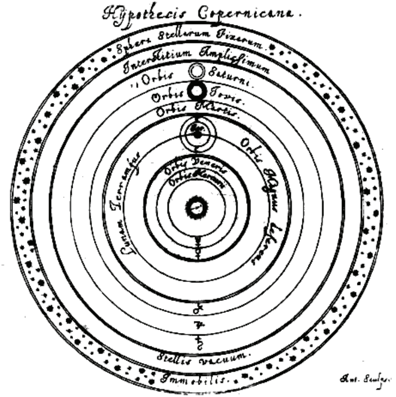
- Daily Motion
- Yearly Motion
- Other Epicycles

Allowed for accurate predictions

- Equinoxes
- Eclipses
- Latitude

Increasing accuracy of observation made model of Ptolemy obsolete

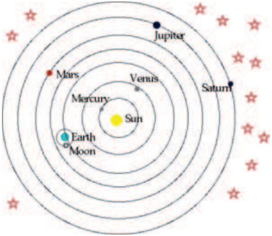




1. Swap earth/sun. 2. Earth is spinning. 3. Moon still orbits earth
Question: Would Ptolemy have won without the moon?

Developing the model

Copernican Model



Positives

- Aesthetically Appealing
  - Epicycles are much smaller
  - Less movement/rotation
- Intuitively appealing

Negatives

- No physical Explanation
  - Relies on Metaphysics, not physics
- No proof
- No empirical validation
- Still assumes circular orbits at constant speed
  - Still requires Epicycles, albeit smaller ones

Developing the model

Galileo Galilei (1564-1642)

The Ptolemy Model was **"Disproven"** by the observations of Galileo

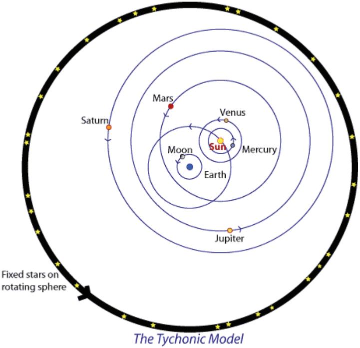
- Built the first decent telescope
- Observed the moons of Jupiter.
  - Showed that planets could orbit other planets
- Observed the phases of venus.
  - Death blow for Ptolemy's model
- An incorrect theory of tides.
- Imprisoned by church.

Developing the model

Tycho Brahe (1546-1601)

Tycho Brahe was a man with

- A fake nose (silver-gold alloy)
- A colorful personality
- A very bad model



However, he also had very good *Equipment* and *Methodology*.

- Most accurate pre-telescope equipment available
- Would catalogue the all relevant stars every night.
- Refused to share data.
  - Data was stolen by Kepler post-mortem.

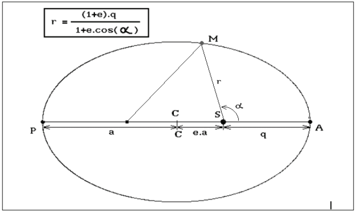
Developing the model

Johannes Kepler (1571-1630)

- Contemporary with Sir Francis Bacon (1561-1626), father of empirical science.
- Became an assistant to Tycho in order to get access to data.
  - Rudophine Tables.
- Primarily observed the motion of Mars
- Formulated experimentally the three laws of planetary motion
  - No derivation.
- Postulated that earth exhibits a central force.
- A correct theory of tides.
- Ignored by Galileo, Descartes

Law 1: Planets move in elliptic orbits with one focus at the planet they orbit.

- The ellipse is a well-understood mathematical concept from geometry
- There are several well-studied parameters of the ellipse
    - a - semi-major axis
    - b - semi-minor axis
    - e - eccentricity



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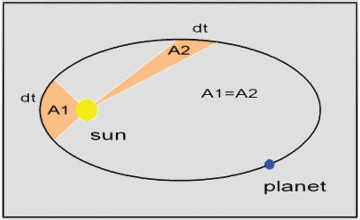
Law 2: Planets sweep out equal areas of the ellipse in equal time.

- First model to posit that planets slow down and speed up.
- $\dot{S}$ is constant for each planet
- Allows for quantitative predictions of locations and time.
  - Allowed him to formulate Rudophine tables.



Developing the model

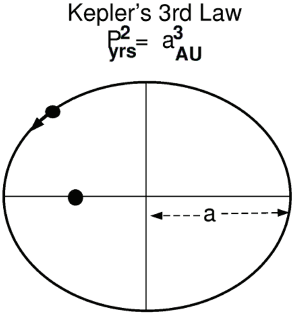
Third Law of Planetary Motion

Law 3: The square of the period of the orbit is proportional to the cube of the semi-major axis.

- A simple corollary of the second law?
  - Second law applies to each orbit
  - Area of ellipse:

$$Area_{ellipse} = a^2\sqrt{1-e^2}$$

- Third law implies the rate of sweep changes from orbit to orbit

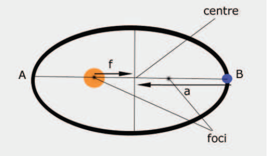


Developing the model

More on Kepler's model

Got almost everything right

- Postulated a central-force hypothesis
  - First theory based on physics.
  - Remember, to three laws of motion, so no inertia.
- Created a correct explanation for tidal motion
- Made the most accurate predictions



Kepler's model was not initially accepted

- ignored by big names (Galileo, Descartes, etc.)
  - Galileo had his own tides model
- Used but not believed

Still no physical explanation. Must wait almost 60 years for an explanation.

Developing the model

Isaac Newton (1643-1727)

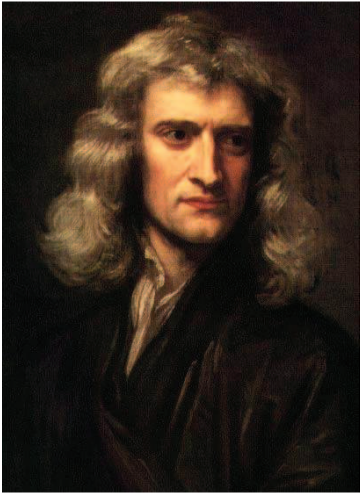
- Most influential person in history?

A quantitative model for motion

- Discovered differential equations
- Discovered modeling nature using equations
  - Now we can use differential equations to model Everything.

Newton used differential equations to model

- Force
- Velocity
- Acceleration
- Inertia
- Gravity



Developing the model

PhilosophiæNaturalis Principia Mathematica

Three Laws of Motion:

Law 1 Every body remains at rest or in uniform motion unless acted upon by an unbalanced external force.

Law 2 A body of mass m, subject to force F, undergoes an acceleration a where

$$F = ma$$

Law 3 The mutual forces of action and reaction between two bodies are equal, opposite and collinear. (In popular culture: Every action creates an equal and opposite reaction.)

Law of Universal Gravitation:

- All bodies exert a force on all others
  - Proportional to mass
  - Inversely proportional to the square of the distance

$$F = G\frac{m_1 m_2}{r^2}$$

- From this, Newton derived Kepler's laws of planetary motion.
  - Or perhaps derived universal gravitation from Kepler's laws.

Developing the model

Some other contributions

Also discovered

- Refracting Telescope
- Integral calculus
- Infinite sequences and series
- Model for wave motion
  - Theory of Color
  - Speed of Sound
- Algorithms for solving nonlinear equations
  - Newton's method is still the most common optimization algorithm.
  - We will use it in this class.

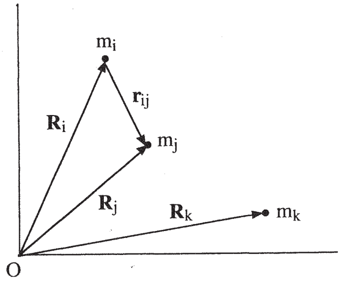
Recall the force on mass 1 due to mass 2 is

$$m_1\ddot{\vec{r}}_1 = \vec{F}_1 = G\frac{m_1 m_2}{\|\vec{r}_{12}\|^3}\vec{r}_{12}$$

where we denote $\vec{r}_{12} = \vec{r}_2 - \vec{r}_1$. Clearly $\vec{r}_{12} = -\vec{r}_{21}$. The motion of mass 2 due to mass 1 is

$$m_2\ddot{\vec{r}}_2 = \vec{F}_2 = G\frac{m_2 m_1}{\|\vec{r}_{21}\|^3}\vec{r}_{21}$$

The problem is nonlinear coupled ODE with 6 degrees of freedom.



Solution: Consider relative motion (only $\vec{r}_{12}$)

$$\ddot{\vec{r}}_{12} = -\frac{G(m_1+m_2)}{\|\vec{r}_{12}\|^3}\vec{r}_{12}$$

This is our model.

Orbital Mechanics

In this class, we study

Definition 1.

Orbital Mechanics is the study of motion about a center of mass.

More generally, the field is

Definition 2.

Celestial Mechanics is the study of heavenly bodies

- Also includes
  - Black holes
  - Dark matter
  - Big Bang Theory
  - Relativistic mechanics

We will stick to orbits.

Conclusion

In this Lecture, you learned:

History

- Orbital mechanics is old
- Much of science was developed as part of the foundation of this discipline
  - Physical Modeling
  - Models using differential equations
  - Empirical Science

The Model

- Universal Gravitation
- Two-Body Problem
  - 3DOF equations of relative motion

Next Lecture: The two-body problem

Universal Invariants

- Angular Momentum
- Linear Momentum

N-body Problem

- Introduction
- Invariants

Derivation of Kepler's Laws

- Kepler's First Law