Spacecraft Dynamics and Control

Matthew M. Peet
Arizona State University

Lecture 14: Interplanetary Mission Planning

Introduction

In this Lecture, you will learn:

Sphere of Influence

- Definition

Escape and Re-insertion

- The light and dark of the Oberth Effect

Patched Conics

- Heliocentric Hohmann

Planetary Flyby

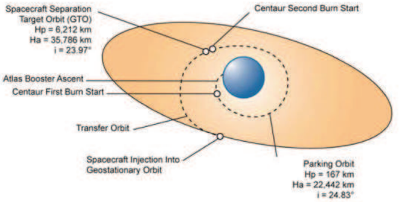
- The Gravity Assist

The Sphere of Influence Model

Three-Body Motion

Consider a Simple Earth-Moon Trajectory.

1. Launch
2. Establish Parking Orbit
3. Escape Trajectory
4. Arrive at Destination
5. Circularize or Depart Destination



The big difference is that now there are 3 bodies.

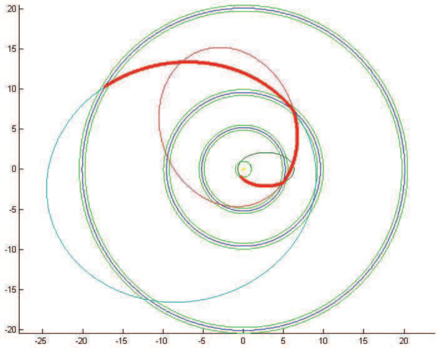
- We only know how to solve the 2-body problem.
- Solving the 3-body problem is beyond us.

Patched Conics

For interplanetary travel, the problem is even more complicated.

Consider the Figure

- The motion is elliptic about the sun.
- The motion is affected by the planets
  - Interference only occurs in the green bands.
  - Motion about planets is hyperbolic.
  - Direction and Magnitude of $\vec{v}$ changes.



The solution is to break the mission into segments.

- During each segment we use *two-body motion*.
- The third body is a disturbance.

Sphere of Influence (SOI)

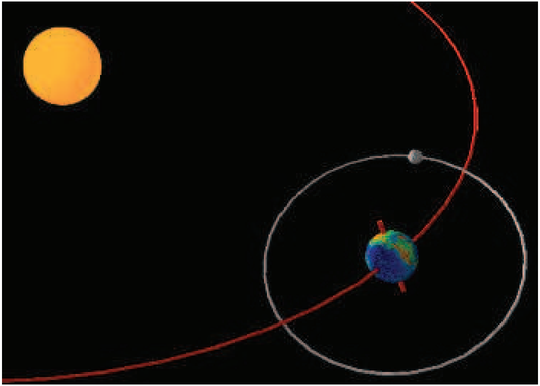
The **WRONG** Definition

Question: Who is in charge??

- The Sphere of Influence of A stops when A is no longer the dominant force.
- What do we mean by dominant?

Wrong Definition:
The Sphere of Influence of A is the region wherein A exerts the largest gravitational force.

Why Wrong?
This would imply the moon is not in earth's Sphere of Influence!!!



Sun Perspective: Lets group the forces as central and disturbing.
Consider motion of a spacecraft relative to the sun:

$$\ddot{\vec{r}}_{sv} + \underbrace{Gm_s \frac{\vec{r}_{sv}}{\|\vec{r}_{sv}\|^3}}_{\text{Effect of sun on object}} = -Gm_p \left[\underbrace{\frac{\vec{r}_{pv}}{\|\vec{r}_{pv}\|^3}}_{\text{Effect of planet on object}} + \underbrace{\frac{\vec{r}_{sp}}{\|\vec{r}_{sp}\|^3}}_{\text{Effect of planet on sun}} \right]$$

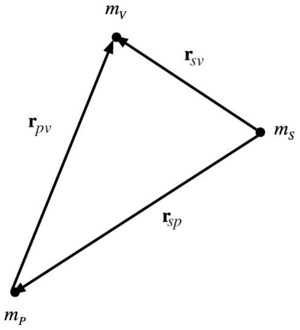
where p denotes planet, v denotes vehicles and s denotes sun.

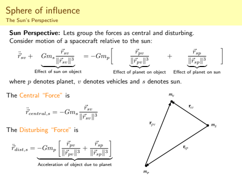
The Central "Force" is

$$\ddot{\vec{r}}_{central,s} = -Gm_s \frac{\vec{r}_{sv}}{\|\vec{r}_{sv}\|^3}$$

The Disturbing "Force" is

$$\ddot{\vec{r}}_{dist,s} = \underbrace{-Gm_p \left[\frac{\vec{r}_{pv}}{\|\vec{r}_{pv}\|^3} + \frac{\vec{r}_{sp}}{\|\vec{r}_{sp}\|^3} \right]}_{\text{Acceleration of object due to planet}}$$





- For the sun-moon system, e.g., the vectors

$$\frac{\vec{r}_{pv}}{\|\vec{r}_{pv}\|^3} >> \frac{\vec{r}_{sp}}{\|\vec{r}_{sp}\|^3} \cong 0$$

so

$$\frac{\ddot{\vec{r}}_{dist,s}}{\ddot{\vec{r}}_{central,s}} \cong \frac{m_p}{m_s} \frac{\|\vec{r}_{sv}\|^2}{\|\vec{r}_{pv}\|^2}$$

- So if $\|\vec{r}_{pv}\|$ is small and $\|\vec{r}_{sv}\|$ is big, the disturbing force dominates.

Planet Perspective: The relative motion of the spacecraft with respect to the planet is

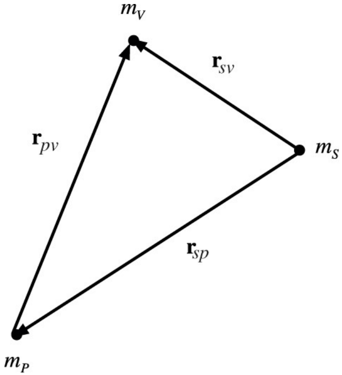
$$\ddot{\vec{r}}_{pv} + \underbrace{Gm_p \frac{\vec{r}_{pv}}{\|\vec{r}_{pv}\|^3}}_{\text{Effect of planet on object}} = -Gm_s \left[\underbrace{\frac{\vec{r}_{sv}}{\|\vec{r}_{sv}\|^3}}_{\text{Effect of sun on object}} - \underbrace{\frac{\vec{r}_{sp}}{\|\vec{r}_{sp}\|^3}}_{\text{Effect of sun on planet}} \right]$$

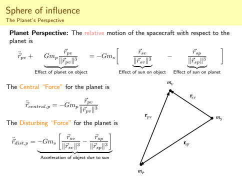
The Central "Force" for the planet is

$$\ddot{\vec{r}}_{central,p} = -Gm_p \frac{\vec{r}_{pv}}{\|\vec{r}_{pv}\|^3}$$

The Disturbing "Force" for the planet is

$$\ddot{\vec{r}}_{dist,p} = \underbrace{-Gm_s \left[\frac{\vec{r}_{sv}}{\|\vec{r}_{sv}\|^3} - \frac{\vec{r}_{sp}}{\|\vec{r}_{sp}\|^3} \right]}_{\text{Acceleration of object due to sun}}$$





- When the vehicle is near the planet, $\vec{r}_{sp} \cong \vec{r}_{sv}$ and hence

$$\frac{\vec{r}_{sp}}{\|\vec{r}_{sp}\|^3} \cong \frac{\vec{r}_{sv}}{\|\vec{r}_{sv}\|^3}$$

  so $\ddot{\vec{r}}_{dist,p} \cong 0$ and

$$\frac{\ddot{\vec{r}}_{dist,p}}{\ddot{\vec{r}}_{central,p}} \cong \frac{m_s}{m_p} \cdot 0 \cong 0$$

  and hence the relative size of the disturbance is small.
- Sphere of influence is based on the relative distance.

Definition 1.

An object is in the **Sphere of Influence**(SOI) of body 1 if

$$\frac{\|\ddot{\vec{r}}_{dist,1}\|}{\|\ddot{\vec{r}}_{central,1}\|} < \frac{\|\ddot{\vec{r}}_{dist,2}\|}{\|\ddot{\vec{r}}_{central,2}\|}$$

for any other body 2.

That is, the ratio of disturbing “force” to central “force” determines which planet is in control.

For planets, an approximation for determining the SOI of a planet of mass m_p at distance d_p from the sun is

$$R_{SOI} \cong \left(\frac{m_p}{m_s}\right)^{2/5} d_p$$

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$$\frac{m_p}{m_s}\frac{\|\vec{r}_{sv}\|^2}{\|\vec{r}_{pv}\|^2} > \frac{m_s\left[\frac{\vec{r}_{sv}}{\|\vec{r}_{sv}\|^3} - \frac{\vec{r}_{sp}}{\|\vec{r}_{sp}\|^3}\right]}{m_p\frac{\vec{r}_{pv}}{\|\vec{r}_{pv}\|^3}} \cong \frac{m_s\left[\vec{r}_{sv} - \vec{r}_{sp}\right]}{m_p\frac{\vec{r}_{pv}\|\vec{r}_{sv}\|^3}{\|\vec{r}_{pv}\|^3}}$$

$$\frac{m_p^2}{m_s^2}\frac{\|\vec{r}_{sv}\|^5}{\|\vec{r}_{pv}\|^5} > \frac{\left[\vec{r}_{sv} - \vec{r}_{sp}\right]}{\vec{r}_{pv}} \cong 1$$

$$\frac{m_p^2}{m_s^2}\|\vec{r}_{sv}\|^5 > \|\vec{r}_{pv}\|^5$$

$$\|\vec{r}_{pv}\| < \left(\frac{m_p}{m_s}\right)^{2/5}\|\vec{r}_{sv}\|$$

Sphere of influence

Table 7.1 Sphere of Influence Radii

Celestial Body	Equatorial Radius (km)	SOI Radius (km)	SOI Radius ($body\ radii$)
Mercury	2487	1.13×10^5	45
Venus	6187	6.17×10^5	100
Earth	6378	9.24×10^5	145
Mars	3380	5.74×10^5	170
Jupiter	71370	4.83×10^7	677
Neptune	22320	8.67×10^7	3886
Moon	1738	6.61×10^4	38

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- The sphere of influence of a planet is defined w/r another mass.
- Distance from earth to the moon is 385,000km
- e.g. Note that sphere of influence of the Moon (w/r to the earth) is inside the sphere of influence of the Earth (w/r to the sun)!
- The SOI of the earth w/r to the moon is different that the SOI w/r to the sun!

Example: Lunar Lander

Problem: Suppose we want to plan a lunar-lander mission. Determine the spheres of influence to consider for a patched-conic approach.

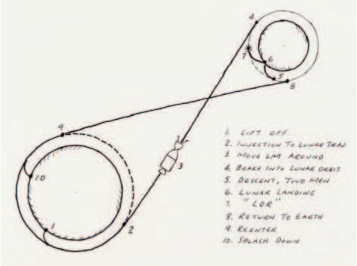
- The SOI of the earth is of radius 924,000km.
- The SOI of the moon is of radius 66,100km.

Solution: The moon orbits at a distance of 385,000km. The spacecraft will transition to the lunar sphere at distance

$$r = 385,000 - 66,100 = 318,900km$$

We will probably also need a plane change. A reasonable mission design is

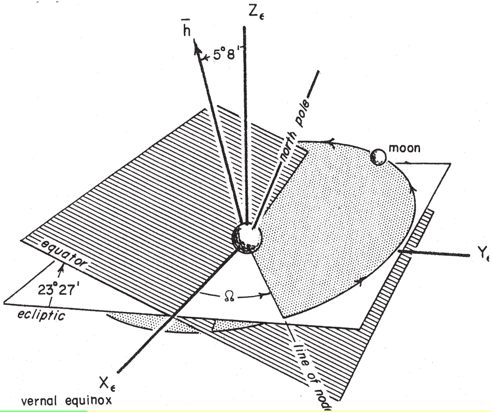
1. Depart earth on a Hohmann transfer to radius $317,900$ km.
2. Perform inclination change near apogee.
3. Enter sphere of influence of the moon.
4. Establish parking orbit.

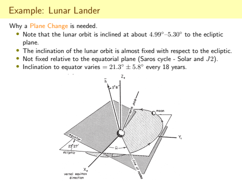


Example: Lunar Lander

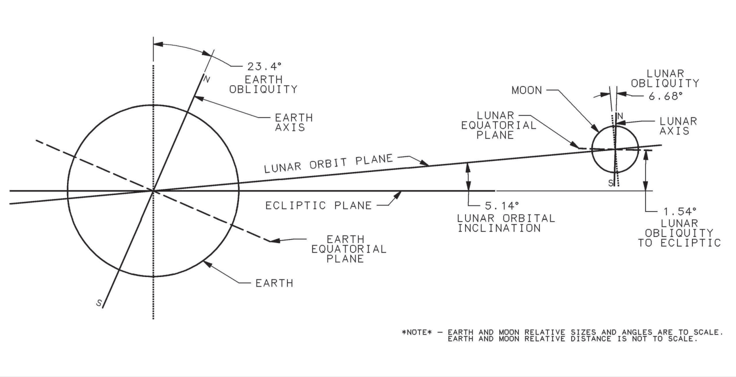
Why a Plane Change is needed.

- Note that the lunar orbit is inclined at about $4.99°–5.30°$ to the ecliptic plane.
- The inclination of the lunar orbit is almost fixed with respect to the ecliptic.
- Not fixed relative to the equatorial plane (Saros cycle - Solar and $J2$).
- Inclination to equator varies $= 21.3° \pm 5.8°$ every 18 years.





- The orbit of the moon is significantly perturbed by the sun.
- Somewhat similar to J2 perturbation, but centered on ecliptic.
- RAAN of lunar orbit processes with period of 18 years.



More Illustrations of the Lunar Orbit

5 Stage Lunar Intercept Mission

First Stage Lunar Tug Assist

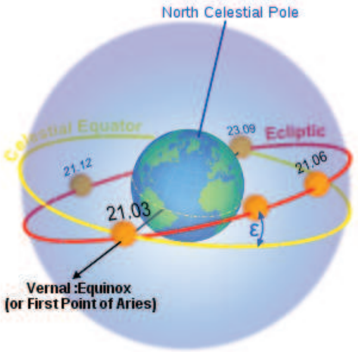
Stages of Interplanetary Mission Planning

1. Establish Orbit in Ecliptic Plane (Low Earth Orbit) with counter-clockwise rotation
2. Burn to escape with excess velocity v_∞
3. Establishes Velocity in Solar Frame
   3.1 $v_p = v_e + v_\infty$ for dark-side burn (Outer planets)
   3.2 $v_a = v_e - v_\infty$ for light-side burn (Inner planets)
4. Propagate Hohman (or Lambert) to destination
   4.1 Find v_a for outer planets
   4.2 Find v_p for inner planets
5. Compute relative velocity (v_r) in planet (Venus) frame $v_r = \|v_p - v_v\|$
   5.1 For flyby, use targeting radius to find turning angle.
   5.2 For insertion, use targeting radius to find r_p.
6. Compute post-flyby relative velocity and convert to Heliocentric frame.

Problem: Design an Earth-Venus rendez-vous. Final orbit around Venus should be posigrade and have altitude 500km.

First Step: Align parking orbit with ecliptic plane.

- All planets move in the ecliptic plane
  - $i \cong 23.4^\circ$
- Circular orbit.
  - Radius $r \cong 6578km$



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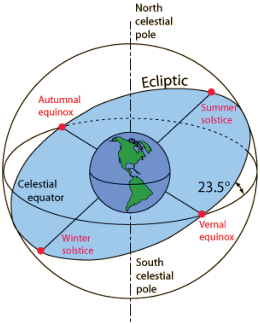
Moving to the Ecliptic Plane

All planets in the solar system orbit the sun in the ecliptic plane.

- Transition must occur when the orbital plane and ecliptic planes intersect.



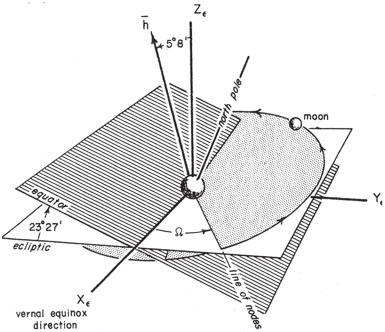
Any earth-centered orbit passes through the ecliptic twice per orbit.

- But not at the ascending node (w/r to the equatorial plane).
- But not at the correct time (f??).

Transition to the ecliptic

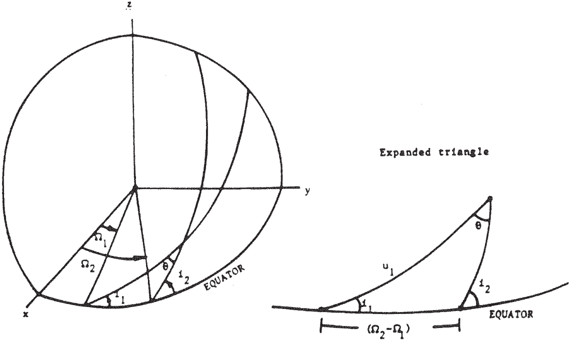
To change to the ecliptic plane:

- Burn at ascending node w/r to the ecliptic plane.
- Execute a plane change.



Requires a change in both Ω and i

- New $\Omega = 0$
- New $i = 23.27°$



Our desired orbit has

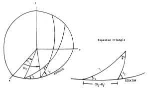
- $i_2 = \epsilon = 23.5°$ - Inclination to the ecliptic
- $\Omega_2 = 0°$ - by definition: Ω is measured from FPOA (intersection of equatorial and ecliptic planes).

If our initial orbit has inclination i_1 and RAAN Ω_1, then the angle change is

$$\cos\theta = \cos i_1 \cos i_2 + \sin i_1 \sin i_2 \cos(\Omega_2 - \Omega_1)$$

Interplanetary Hohmann Transfer

Transition to the ecliptic



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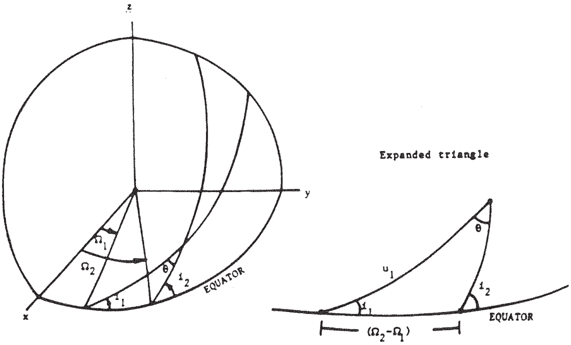
$$\cos\theta = \cos i_1 \cos i_2 + \sin i_1 \sin i_2 \cos(\Omega_2 - \Omega_1)$$

- It is not possible to launch directly into the ecliptic from the U.S. (Recall for Kennedy $\phi_{gc} = 28.5^\circ$)
- However, we may choose launch time θ_{LST} in order to select RAAN Ω_1
- For the ecliptic, $i_2 = 23.5^\circ$.
- For Kennedy, $i_1 = 28.5^\circ$
- For the ecliptic plane, $\Omega_2 = 0^\circ$.
- To minimize Δv, we want to minimize θ. To do this, we may select $\Omega_1 = 0^\circ$, which yields

$$\theta = \cos^{-1}\left(\cos(28.5^\circ)\cos(23.5^\circ) + \sin(28.5^\circ)\sin(23.5^\circ) * \cos(0^\circ)\right) = 5^\circ$$

- If combined with a burn to escape, the Δv for a 5° plane change is almost negligible!



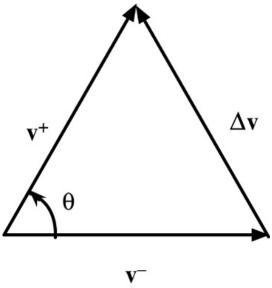
The position in the orbit is given by

$$\cos(\omega + f) = \frac{\cos i_1 \cos\theta - \cos i_2}{\sin i_1 \sin\theta}$$

Where recall

- $i_2 = \epsilon = 23.5°$

The Plane Change



The Δv required required for the plane change is then

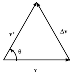
$$\Delta v = 2v\sin\frac{\theta}{2}$$

or

$$\Delta v^2 = v(t_k^-)^2 + v(t_k^+)^2 - 2v(t_k^-)v(t_k^+)\cos\Delta\theta$$

if combined with a velocity change ($v(t_k^-)$ to $v(t_k^+)$).

The Plane Change



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if combined with a velocity change ($v(t_k^-)$ to $v(t_k^+)$).

In truth, we try and avoid large plane changes. Typically, it is better to launch directly into the ecliptic plane. This is normally possible if the launch site is below 23.5° latitude and the launch time is carefully chosen.

We need to know the magnitude and direction of velocity in the Heliocentric Frame.

The perigee and apogee velocities of the Heliocentric transfer ellipse are

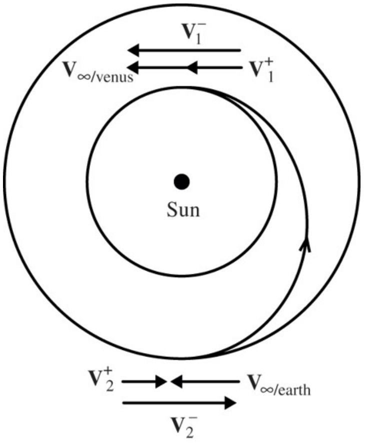
$$v_1^+ = v_p = \sqrt{2\mu_{sun}\frac{r_e}{r_v(r_e+r_v)}} = 37.73km/s$$

$$v_2^+ = v_a = \sqrt{2\mu_{sun}\frac{r_v}{r_e(r_e+r_v)}} = 27.29km/s$$

Where

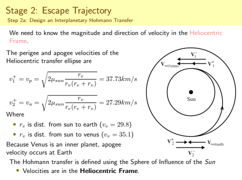
- r_e is dist. from sun to earth ($v_e = 29.8$)
- r_v is dist. from sun to venus ($v_v = 35.1$)

Because Venus is an inner planet, apogee velocity occurs at Earth



The Hohmann transfer is defined using the Sphere of Influence of the *Sun*

- Velocities are in the **Heliocentric Frame**.



Stages of Interplanetary Mission:

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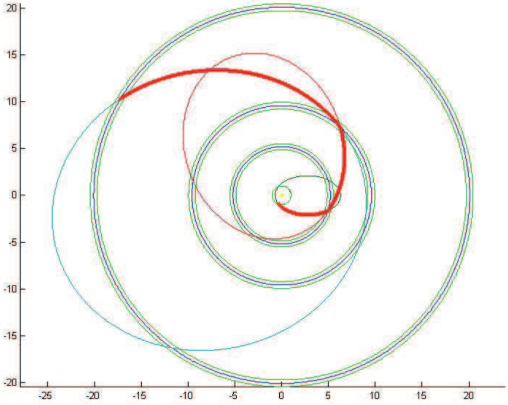
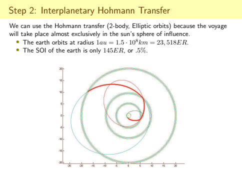
Step 2: Interplanetary Hohmann Transfer

We can use the Hohmann transfer (2-body, Elliptic orbits) because the voyage will take place almost exclusively in the sun's sphere of influence.

- The earth orbits at radius $1au = 1.5 \cdot 10^8 km = 23,518ER$.
- The SOI of the earth is only $145ER$, or $.5\%$.



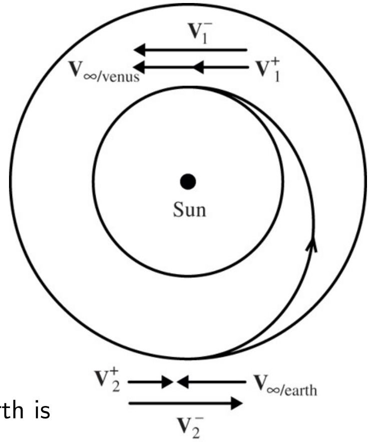


- None of the trajectories in this diagram are Hohmann transfers (although the first is nearly so)
- The phasing must be perfect for a Hohman transfer, and so these are only possible for single-planet routes, with no gravity assist.
- The Δv at planet 2 to intersect planet 3 is chosen by solving **Lambert's Problem**.

Interplanetary Hohmann Transfer

Injection (v_a)

Problem: We need to know the Δv magnitude relative to earth's motion.



- $v_a = v_2^+$ is w/r to inertial frame.
- Earth is moving in the inertial frame.
  - The earth frame is moving with velocity

$$v_2^- = v_e = \sqrt{\frac{\mu_s}{\|\vec{r}_{se}\|}} = 29.78 km/s$$

- What is this v_a velocity relative to earth?

We have

$$v_2^+ = v_a = v_2^- + v_{\infty,e}$$

Thus our desired velocity with respect to the earth is

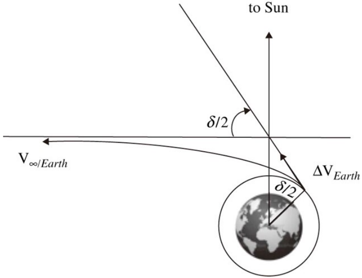
$$\Delta v_e = v_{\infty,e} = v_a - v_e^- = 27.29 - 29.78 = -2.49 km/s$$

- The magnitude of Δv_e is determined by *excess velocity*
- The direction of Δv_e is determined by timing

Problem: How to achieve the initial $v_{\infty,e} = -2.49km/s$?

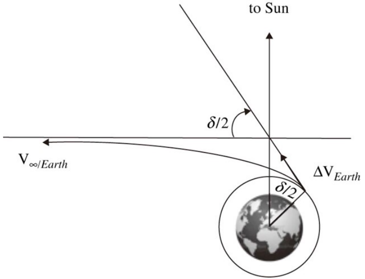
- We need to escape earth orbit.
- Must have leftover velocity (excess velocity) of $2.49km/s$.
    - Implies the total energy (w/r to the earth) after burn is

$$E_+ = \frac{1}{2} v_{\infty,e}^2 = 3.1223$$



Interplanetary Hohmann Transfer

Suppose the spacecraft is in a circular parking orbit of radius $r_{park} = 6578$km.



- The velocity before the burn will be

$$v_{\text{park}} = \sqrt{\frac{\mu_e}{r_{\text{park}}}} = 7.7843 km/s$$

- The velocity after burn (v_{after}) can be found by solving the energy equation.

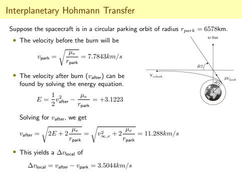
$$E = \frac{1}{2}v_{\text{after}}^2 - \frac{\mu_e}{r_{\text{park}}} = +3.1223$$

Solving for v_{after}, we get

$$v_{\text{after}} = \sqrt{2E + 2\frac{\mu_e}{r_{\text{park}}}} = \sqrt{v_{\infty,e}^2 + 2\frac{\mu_e}{r_{\text{park}}}} = 11.288 km/s$$

- This yields a Δv_{local} of

$$\Delta v_{\text{local}} = v_{\text{after}} - v_{\text{park}} = 3.5044 km/s$$



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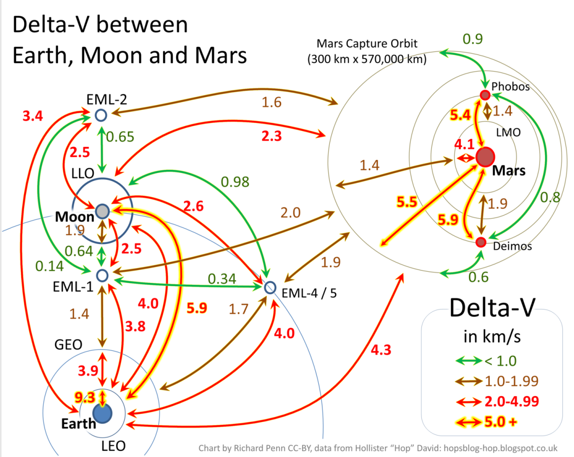
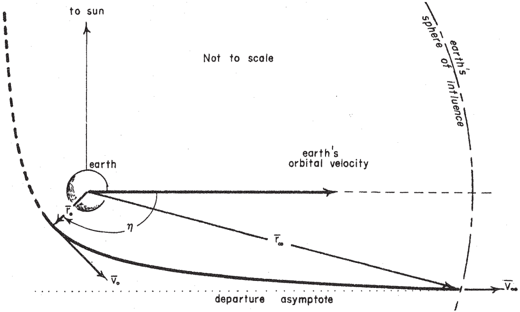
Note that $\Delta v = 3.4$ km/s is less than the Δv to reach GEO.



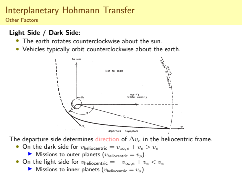
Light Side / Dark Side:

- The earth rotates counterclockwise about the sun.
- Vehicles typically orbit counterclockwise about the earth.



The departure side determines direction of Δv_e in the heliocentric frame.

- On the dark side for $v_{\text{heliocentric}} = v_{\infty,e} + v_e > v_e$
  - Missions to outer planets ($v_{\text{heliocentric}} = v_p$).
- On the light side for $v_{\text{heliocentric}} = -v_{\infty,e} + v_e < v_e$
  - Missions to inner planets ($v_{\text{heliocentric}} = v_a$).



Stages of Interplanetary Mission:

1. Establish Orbit in Ecliptic Plane (Low Earth Orbit) with counter-clockwise rotation
2. Burn to escape with excess velocity v_x
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   3.1 $v_p = v_e + v_x$ for dark-side burn (Outer planets)
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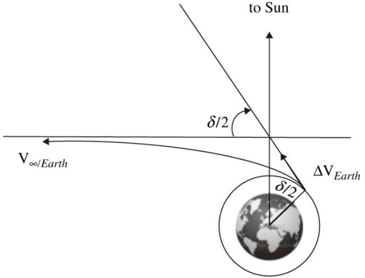
6. Compute post-flyby relative velocity and convert to Heliocentric frame.

Interplanetary Hohmann Transfer

Other Factors

Timing: The Δv should occur at $\delta/2$ before midnight/noon, where δ is the turning angle

$$\delta = 2\sin^{-1}\frac{1}{e}$$



Eccentricity (e) can be found as:

- Energy: $E = \frac{1}{2}v_{\infty,e}^2 = 2.067 = -\frac{\mu}{2a}$ yields

$$a = -\frac{\mu}{v_{\infty,e}^2} = -\frac{\mu}{2E} = -96,420km$$

- Perigee: $r_{p,e} = r_c = a(1-e) = 6578km$ yields

$$e = 1 - \frac{r_{p,e}}{a} = 1.0682$$

This yields a turning angle of

$$\delta = 2.423rad = 138.83^\circ$$

Thus the spacecraft should depart at $\delta/2 = 69.4^\circ$ before noon/midnight.

Arrival at Venus

At arrival, our excess velocity w/r to Venus ($v_{\infty,v}$) will be

$$v_{\infty,v} = v_p - v_v = v_1^- - v_1^+ = 37.81km/s - 35.09km/s = 2.71km/s$$

where

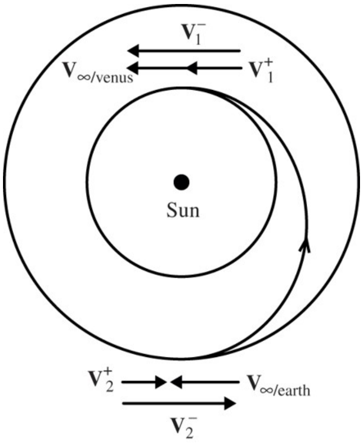
- $v_1^+ = v_v$ is the velocity of venus

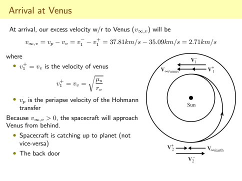
$$v_1^+ = v_v = \sqrt{\frac{\mu_s}{r_v}}$$

- v_p is the periapse velocity of the Hohmann transfer

Because $v_{\infty,v} > 0$, the spacecraft will approach Venus from behind.

- Spacecraft is catching up to planet (not vice-versa)
- The back door





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Arrival at Venus

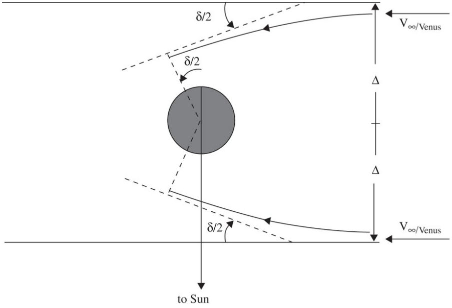
Venus Data:

$$R_v = 6187km, \quad \mu_v = 324859, \quad a_{\text{venus}} = 1.08 \cdot 10^8$$

Desired Orbit: Circular, posigrade (counterclockwise) with

$$r_c = 6187 + 500 = 6687km$$

For a counterclockwise orbit, we want to approach Venus on the **Dark Side**



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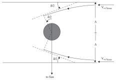


- If we were travelling to an outer planet, we would approach on the **Light Side** to achieve a counterclockwise orbit
- This is because for outer planets, we are moving slower than the planet
- Hence the planet is approaching us.
- We would enter the SOI from the left.

Arrival at Venus

For orbital insertion, we want to perform a retrograde burn at periapse of the incoming hyperbola.

To achieve a circular orbit of radius $r_c = 6687km$, we need the periapse of our incoming hyperbola to occur at

$$r_{p,v} = a(1-e) = 6687km.$$

The energy of the incoming hyperbola is given by the excess velocity as

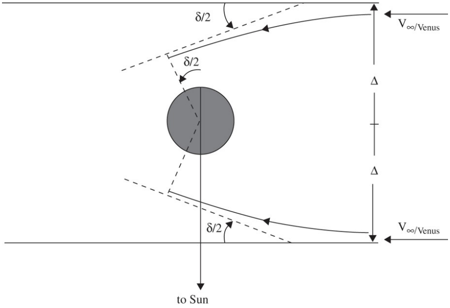
$$E = \frac{1}{2} v_{\infty,v}^2 = 3.67.$$

This fixes the semimajor axis at

$$a = -\frac{\mu_v}{v_{inf,v}^2} = -44,232km.$$

Thus to achieve $r_p = a(1-e)$, we need

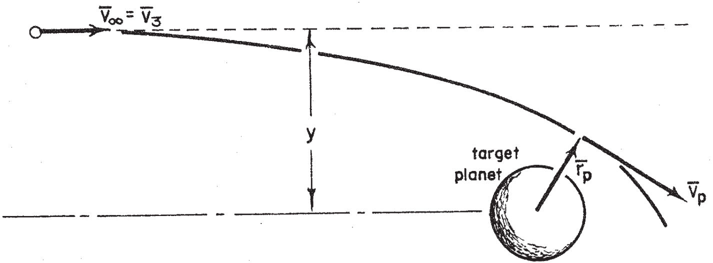
$$e = 1 - \frac{r_p}{a} = 1.15.$$



Arrival at Venus

To achieve the desired $e = 1.15$, we control the conditions at the *Patch Point*.

- We do this through the angular momentum, h.



We can control the **Target Radius**, Δ through small adjustments far from the planet. Angular momentum can be exactly controlled through target radius, Δ.

$$h_v = v_{\infty,v}\Delta$$

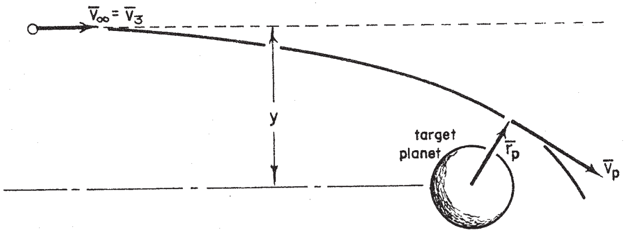
Arrival at Venus

Solution: For a given a, e is determined by $p = a(1-e^2)$.

- But p is defined by angular momentum (and thus target radius).

$$p = \frac{h^2}{\mu_v} = \frac{\Delta^2 v_{\infty,v}^2}{\mu_v}$$

- For $a = -44,232km$ and $e = 1.15$, we get $p = 14,265km$.



Given a desired p we solve for target radius, Δ,

$$\Delta = \sqrt{\frac{p\mu_v}{v_{\infty,v}^2}} = \sqrt{\frac{a(1-e^2)\mu_v}{v_{\infty,v}^2}} = 25,120km$$

Injection into Circular Orbit

Finally, we need to slow down to achieve circular orbit.

- The velocity at periapse (6687km) is given by the vis-viva equation.

$$v = \sqrt{\frac{2\mu_v}{r_{p,v}} - \frac{\mu_v}{a}} = 10.223km/s$$

- The velocity of a circular orbit is

$$v_c = \sqrt{\mu_v} r_{p,v} = 6.97km/s$$

Thus the Δv required to circularize the orbit is

$$\Delta v = 6.97 - 10.223 = -3.253km/s$$

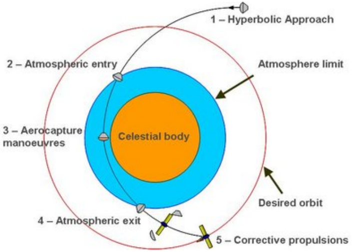


Figure: Aerobraking can also assist with Δv

Escape Velocity at 6687: $v_{esc} = \sqrt{\frac{2\mu_v}{r_{p,v}}} = 9.8577$

Min Δv for Injection: $\Delta v_{min} = v - v_{esc} = .3653$

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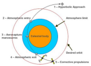

Figure: Aerobraking can also assist with Δv

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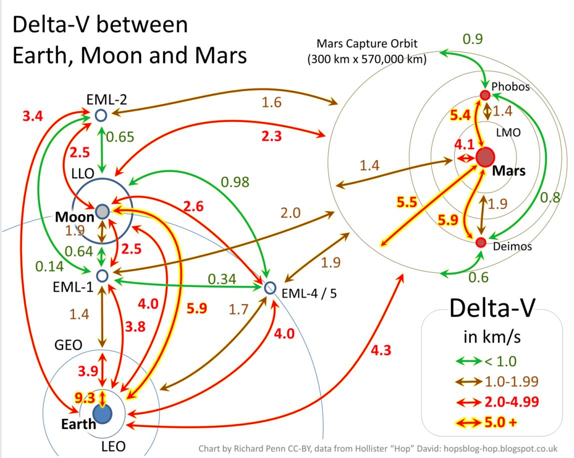
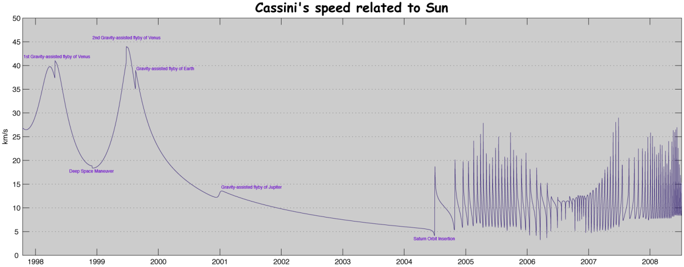
Min Δv for Injection: $\Delta v_{min} = v - v_{esc} = .3653$

- Aerocapture is used to reduce a hyperbolic orbit to an elliptic orbit.
- Aerocapture has never been used except in Kerbel Space Program and 2010.
- Aerobraking is used to reduce the apogee of an elliptic orbit over many rotations.
- Requires a very detailed model of the atmosphere to be safe.
- Many aerobraking maneuvers are performed using Earth's atmosphere!

Messenger Probe to Mercury


Delta-V between
Earth, Moon and Mars
Mars Capture Orbit
(300 km x 570,000 km)
0.9
Phobos
5.4
1.4
LMO
4.1
Mars
1.4
5.5
5.9
1.9
0.8
Deimos
0.6
EML-2
3.4
1.6
0.65
2.5
2.3
LLO
0.98
Moon
2.6
2.0
1.9
0.64
2.5
0.14
EML-1
0.34
1.9
EML-4 / 5
1.4
4.0
5.9
1.7
3.8
4.0
GEO
4.3
3.9
9.3
Earth
LEO
Delta-V
in km/s
< 1.0
1.0-1.99
2.0-4.99
5.0 +
Chart by Richard Penn CC-BY, data from Hollister "Hop" David: hopsblog-hop.blogspot.co.uk


Cassini's speed related to Sun
km/s
1st Gravity-assisted flyby of Venus
2nd Gravity-assisted flyby of Venus
Gravity-assisted flyby of Earth
Deep Space Maneuver
Gravity-assisted flyby of Jupiter
Saturn Orbit Insertion
0
5
10
15
20
25
30
35
40
45
50
1998
1999
2000
2001
2002
2003
2004
2005
2006
2007
2008

Gravity Assist Trajectories

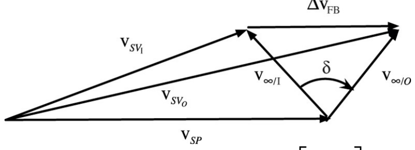
Concept: Planets rotate the relative velocity vector.

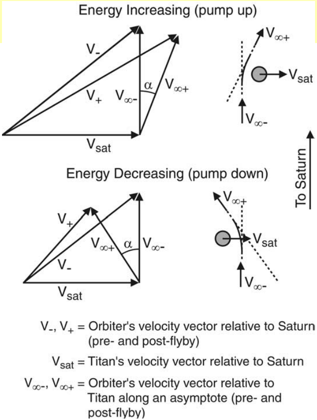
- The relative motion changes as

$$\vec{v}_f - \vec{v}_{planet} = R_1(\delta)\,(\vec{v}_i - \vec{v}_{planet})$$

- In the inertial frame (2 dimensions) this means

$$\vec{v}_f = \begin{bmatrix} \cos\delta & -\sin\delta \\ \sin\delta & \cos\delta \end{bmatrix} (\vec{v}_i - \vec{v}_{planet}) + \vec{v}_{planet}$$



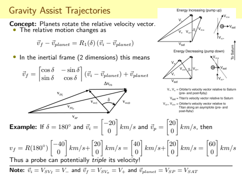


Example: If $\delta = 180°$ and $\vec{v}_i = \begin{bmatrix} -20 \\ 0 \end{bmatrix} km/s$ and $\vec{v}_p = \begin{bmatrix} 20 \\ 0 \end{bmatrix} km/s$, then

$$v_f = R(180°)\begin{bmatrix} -40 \\ 0 \end{bmatrix} km/s + \begin{bmatrix} 20 \\ 0 \end{bmatrix} km/s = \begin{bmatrix} 40 \\ 0 \end{bmatrix} km/s + \begin{bmatrix} 20 \\ 0 \end{bmatrix} km/s = \begin{bmatrix} 60 \\ 0 \end{bmatrix} km/s$$

Thus a probe can potentially *triple* its velocity!

Note: $\vec{v}_i = V_{SV_I} = V_-$ and $\vec{v}_f = V_{SV_o} = V_+$ and $\vec{v}_{planet} = V_{SP} = V_{SAT}$



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Gravity Assist Trajectories

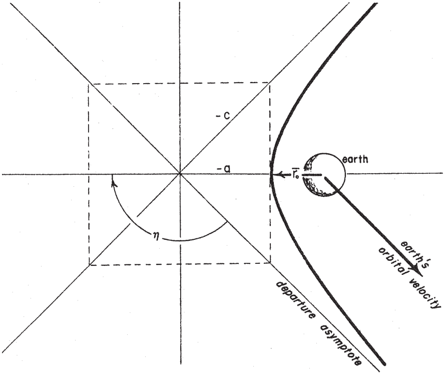
To achieve the desired turning angle, we must control the geometry

The turning angle δ is given by

$$\delta = 2\sin^{-1}\frac{1}{e}$$

Recall that energy of the orbit is fixed.
Thus we can solve for

$$a = -\mu_{planet}/\|\vec{v}_i - \vec{v}_{planet}\|^2$$



Then the eccentricity can be fixed by the target radius as

$$\Delta = \sqrt{\frac{a(1-e^2)\mu_{planet}}{\|\vec{v}_i - \vec{v}_{planet}\|^2}}$$

In 3 dimensions, the calculations are more complex.

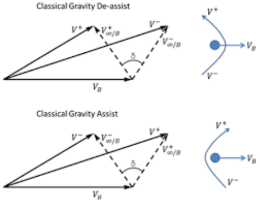
Problem: Suppose we perform a Hohman transfer from Earth to Jupiter. What is the best-case gravity assist we can expect?

Solution: The velocity of arrival at apogee (Jupiter) in the Heliocentric frame is:

$$\vec{v}_i = v_a = \sqrt{2\mu_{sun}\frac{r_e}{r_j(r_j+r_e)}} = 7.414 km/s$$

The velocity of Jupiter itself is

$$\vec{v}_{planet} = v_j = \sqrt{\frac{\mu_s}{d_j}} = 13.0573 km/s$$



Since this is an outer planet, we approach from the front door. In the Jupiter $R-T-N$ frame we have

$$\vec{v}_i = \begin{bmatrix} 7.414 \\ 0 \end{bmatrix}, \qquad \vec{v}_{planet} = \begin{bmatrix} 13.0573 \\ 0 \end{bmatrix}$$

The velocity of the spacecraft relative to Jupiter is $\vec{v}_\infty = \vec{v}_i - \vec{v}_p = \begin{bmatrix} -5.6429 \\ 0 \end{bmatrix}$

Example: Jupiter flyby

Jupiter Data: Radius $r_j = 11.209ER$; Distance $d_j = 5.2028AU$; $\mu_j = 317.938\mu_e$.

The velocity of the spacecraft relative to jupiter is

$$\vec{v}_\infty = \vec{v}_i - \vec{v}_p = \begin{bmatrix} -5.6429 \\ 0 \end{bmatrix} km/s$$

Thus we can calculate the energy of the hyperbolic approach as

$$a = -\frac{\mu_j}{\|\vec{v}_i - \vec{v}_p\|^2} = -3.98E6km$$

The closest we can approach jupiter is its radius. If we use this for periapse, we get

$$e = 1 - \frac{r_j}{a} = 1.018$$

The eccentricity yields the maximum turning angle as

$$\delta = 2\sin^{-1}\left(\frac{1}{e}\right) = 158.44^\circ$$

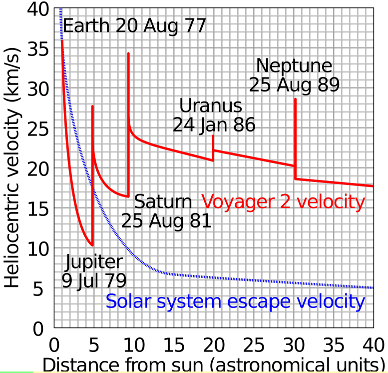
Example: Jupiter flyby

Applying this rotation (light-side approach), we get

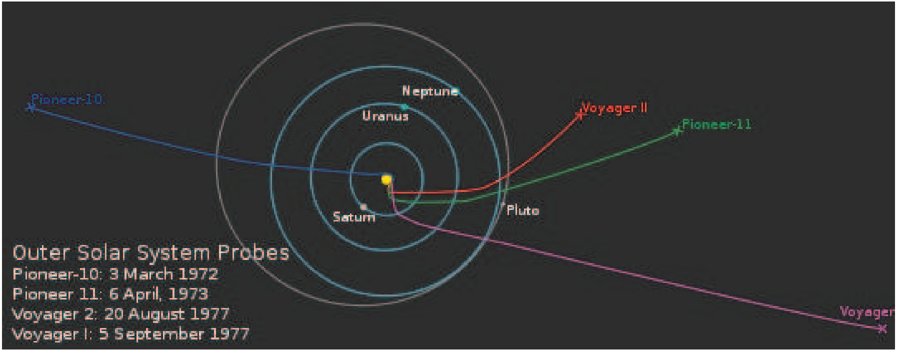
$$\vec{v}_f = \begin{bmatrix} \cos\delta & -\sin\delta \\ \sin\delta & \cos\delta \end{bmatrix} (\vec{v}_i - v_{planet}) + \vec{v}_{planet} = \begin{bmatrix} 18.305 \\ 2.076 \end{bmatrix}$$

The magnitude of the Δv from this flyby is 11.01km/s. A factor of 2.5.

Note that if we could have reversed our direction of flight (clockwise approach), we could achieve a $\Delta v = 20.05 km/s$.



Trajectories for Voyager 1, Voyager 2, and Pioneer Spacecraft



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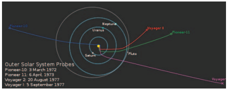

Image credit (previous page): By Cmglee

Image credit: By Phoenix7777

Summary

This Lecture you have learned:

Sphere of Influence

- Definition

Escape and Re-insertion

- The light and dark of the Oberth Effect

Patched Conics

- Heliocentric Hohmann

Planetary Flyby

- The Gravity Assist