Spacecraft Dynamics and Control

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Lecture 14: Interplanetary Mission Planning

Introduction

In this Lecture, you will learn:

Sphere of Influence

- Definition

Escape and Re-insertion

- The light and dark of the Oberth Effect

Patched Conics

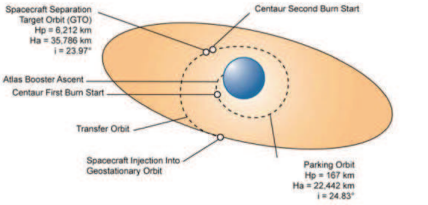
- Heliocentric Hohmann

Planetary Flyby

- The Gravity Assist

Consider a Simple Earth-Moon Trajectory.

1. Launch
2. Establish Parking Orbit
3. Escape Trajectory
4. Arrive at Destination
5. Circularize or Depart Destination



The big difference is that now there are 3 bodies.

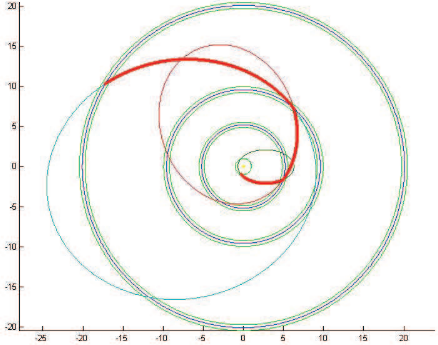
- We only know how to solve the 2-body problem.
- Solving the 3-body problem is beyond us.

Patched Conics

For interplanetary travel, the problem is even more complicated.

Consider the Figure

- The motion is elliptic about the sun.
- The motion is affected by the planets
  - Interference only occurs in the green bands.
  - Motion about planets is hyperbolic.



The solution is to break the mission into segments.

- During each segment we use *two-body motion*.
- The third body is a disturbance.

Sphere of influence

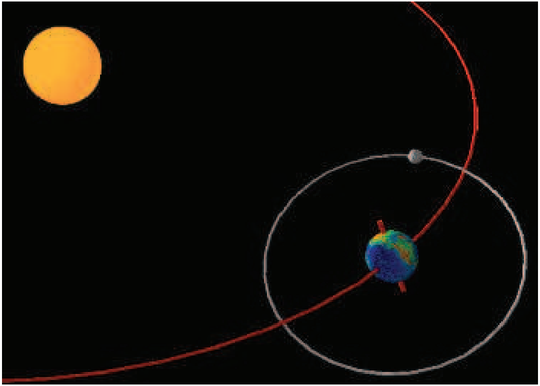
The Wrong Definition

Question: Who is in charge??

- The Sphere of Influence of A stops when A is no longer the dominant force.
- What do we mean by dominant?

Wrong Definition:
The Sphere of Influence of A is the region A exerts the largest gravitational force.

This would imply the moon is not in earth's Sphere of Influence!!!



Sphere of influence

The Sun's Perspective

Sun Perspective: Lets group the forces as central and disturbing. Consider motion of a spacecraft relative to the sun:

$$\ddot{\vec{r}}_{sv} + Gm_s \frac{\vec{r}_{sv}}{\|\vec{r}_{sv}\|^3} = -Gm_p \left[\frac{\vec{r}_{pv}}{\|\vec{r}_{pv}\|^3} + \frac{\vec{r}_{sp}}{\|\vec{r}_{sp}\|^3} \right]$$

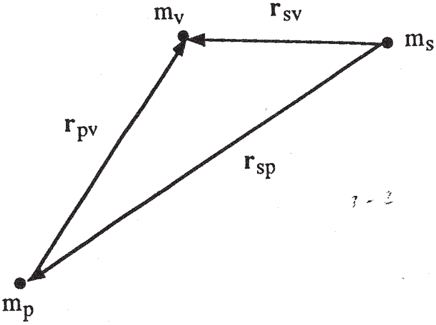
where p denotes planet, v denotes vehicles and s denotes sun.

The Central "Force" is

$$\ddot{\vec{r}}_{central,s} = -Gm_s \frac{\vec{r}_{sv}}{\|\vec{r}_{sv}\|^3}$$

The Disturbing "Force" is

$$\ddot{\vec{r}}_{dist,s} = -Gm_p \left[\frac{\vec{r}_{pv}}{\|\vec{r}_{pv}\|^3} + \frac{\vec{r}_{sp}}{\|\vec{r}_{sp}\|^3} \right]$$



Planet Perspective: The motion of the spacecraft relative to the planet is

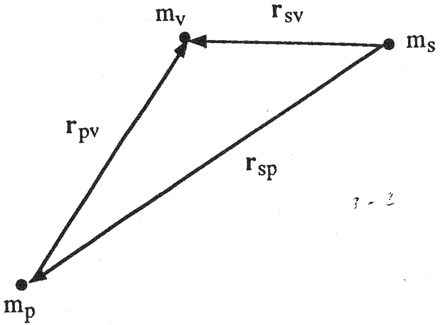
$$\ddot{\vec{r}}_{pv} + Gm_p \frac{\vec{r}_{pv}}{\|\vec{r}_{pv}\|^3} = -Gm_s \left[\frac{\vec{r}_{sv}}{\|\vec{r}_{sv}\|^3} - \frac{\vec{r}_{sp}}{\|\vec{r}_{sp}\|^3} \right]$$

The Central "Force" for the planet is

$$\ddot{\vec{r}}_{central,p} = -Gm_p \frac{\vec{r}_{pv}}{\|\vec{r}_{pv}\|^3}$$

The Disturbing "Force" for the planet is

$$\ddot{\vec{r}}_{dist,p} = -Gm_s \left[\frac{\vec{r}_{sv}}{\|\vec{r}_{sv}\|^3} - \frac{\vec{r}_{sp}}{\|\vec{r}_{sp}\|^3} \right]$$



Definition 1.

An object is in the **Sphere of Influence**(SOI) of body 1 if

$$\frac{\|\ddot{\vec{r}}_{dist,1}\|}{\|\ddot{\vec{r}}_{central,1}\|} < \frac{\|\ddot{\vec{r}}_{dist,2}\|}{\|\ddot{\vec{r}}_{central,2}\|}$$

for any other body 2.

That is, the ratio of disturbing "force" to central "force" determines which planet is in control.

For planets, an approximation for determining the SOI of a planet of mass m_p at distance d_p from the sun is

$$R_{SOI} \cong \left(\frac{m_p}{m_s}\right)^{2/5} d_p$$

Sphere of influence

Table 7.1 Sphere of Influence Radii

Celestial Body	Equatorial Radius (*km*)	SOI Radius (*km*)	SOI Radius (*body radii*)
Mercury	2487	1.13×10^5	45
Venus	6187	6.17×10^5	100
Earth	6378	9.24×10^5	145
Mars	3380	5.74×10^5	170
Jupiter	71370	4.83×10^7	677
Neptune	22320	8.67×10^7	3886
Moon	1738	6.61×10^4	38

Example: Lunar Lander

Problem: Suppose we want to plan a lunar-lander mission. Determine the spheres of influence to consider for a patched-conic approach.

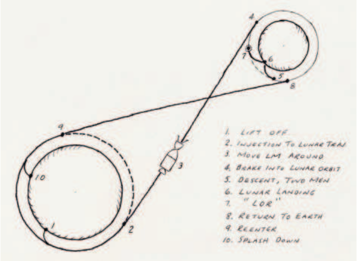
- The Sphere of influence of the earth is of radius 924,000km.
- The sphere of influence of the moon is of radius 66,100km.

Solution: The moon orbits at a distance of 384,000km. The spacecraft will transition to the lunar sphere at distance

$$r = 384,000 - 66,100 = 317,900km$$

We will probably also need a plane change. A reasonable mission design is

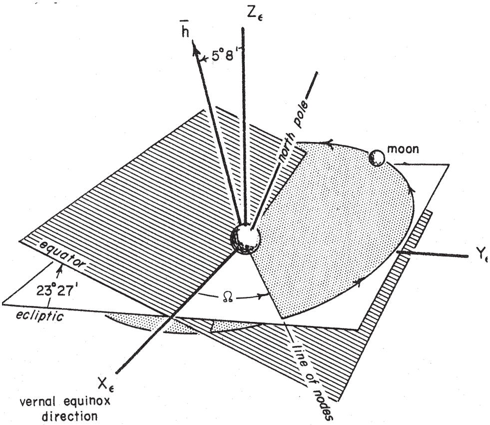
1. Depart earth on a Hohmann transfer to radius $317,900$ km.
2. Perform inclination change near apogee.
3. Enter sphere of influence of the moon.
4. Establish parking orbit.



Example: Lunar Lander

Why is a Plane Change is needed.

- Note that the lunar orbit is inclined at about 5.8° to the ecliptic plane.
- The inclination of the lunar orbit is almost fixed with respect to the ecliptic.
- Not fixed relative to the equatorial plane (Saros cycle - Solar and $J2$).
- Inclination to equator varies by $21.3^\circ \pm 5.8^\circ$ every 18 years.



5 Stage Lunar Intercept Mission

First Stage Lunar Tug Assist

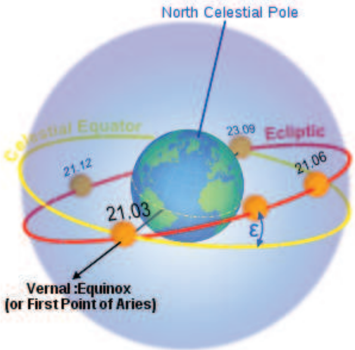
Let's start by going through an example.

- Can serve as a template.
- Will illustrate the issues

Problem: Design an Earth-Venus rendez-vous. Final Venus orbit should be posigrade of altitude 500km.

First Step: Maneuver into a suitable parking orbit.

- Orbital plane aligned with ecliptic plane
  - $i \cong 23^\circ$
- Circular orbit.
  - Radius $r \cong 6578km$



Interplanetary Hohmann Transfer

Design a Hohmann transfer from Earth to Venus.

The perigee and apogee velocities of the transfer ellipse are

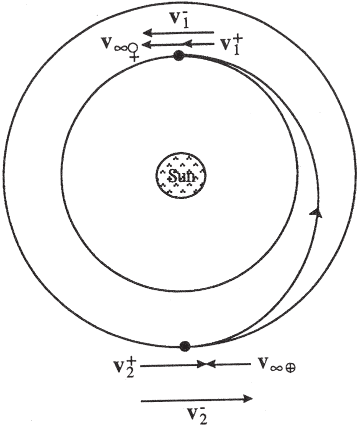
$$v_1^+ = v_p = \sqrt{2\mu_{sun}\frac{r_e}{r_v(r_e + r_v)}} = 37.73km/s$$

$$v_2^+ = v_a = \sqrt{2\mu_{sun}\frac{r_v}{r_e(r_e + r_v)}} = 27.29km/s$$

Where

- r_e is dist. from sun to earth
- r_v is dist. from sun to venus

Because Venus is an inner planet, apogee velocity occurs at Earth



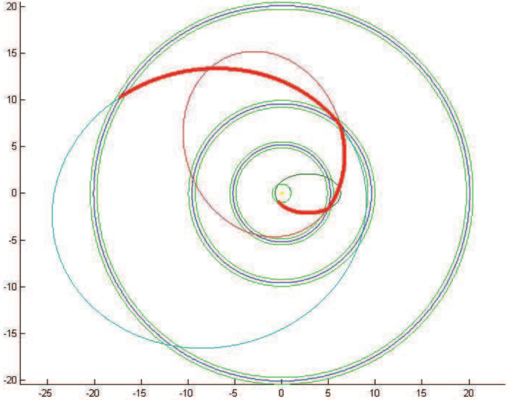
The Hohmann transfer is defined using the Sphere of Influence of the *Sun*

- Velocities are in the **Heliocentric Frame**.

Interplanetary Hohmann Transfer

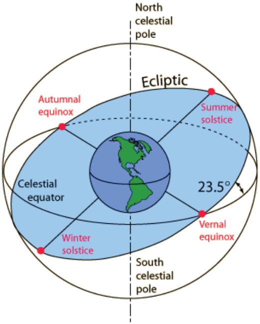
We can use the Hohmann transfer (2-body, Elliptic orbits) because the voyage will take place almost exclusively in the sun's sphere of influence.

- The earth orbits at radius $1au = 1.5 \cdot 10^8 km = 23,518ER$.
- The SOI of the earth is only $145ER$, or $.5\%$.



All planets in the solar system orbit the sun in the ecliptic plane.

- We need to transition to this plane.
- Transition must occur when the orbital plane and ecliptic planes intersect.

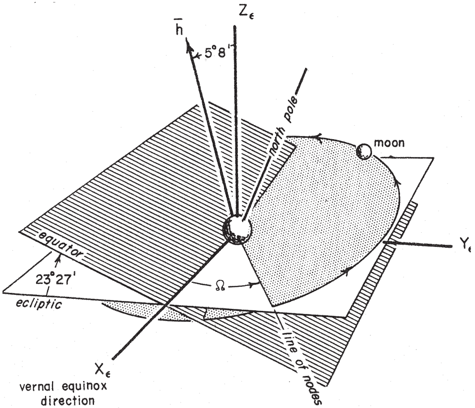


Any orbit about the earth passes through the ecliptic twice per orbit.

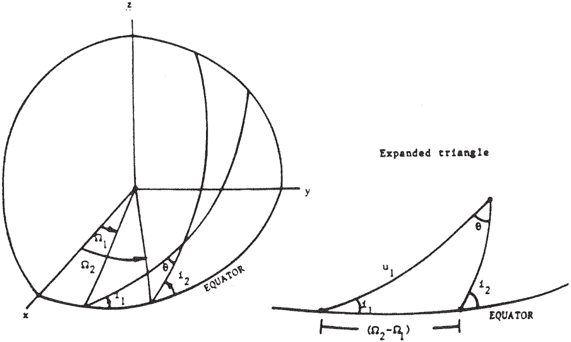
- But not at the ascending node (w/r to the equatorial plane).
- But not at the correct time.

To change to the ecliptic plane:

- Burn at ascending node w/r to the ecliptic plane.
- Execute a plane change.



We have already covered these kinds of maneuvers!

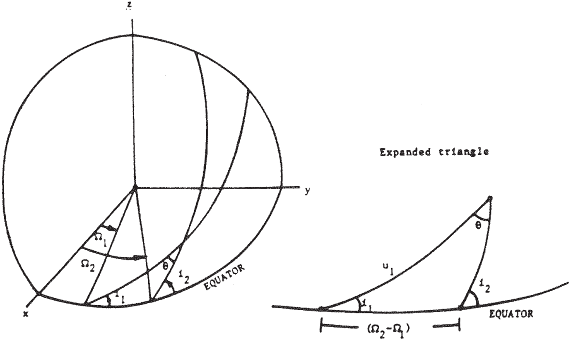


Our desired orbit has

- $i_2 = \epsilon = 23.5°$ - Inclination to the ecliptic
- $\Omega_2 = 0°$ - by definition: Ω is measured from FPOA (intersection of equatorial and ecliptic planes).

If our initial orbit has inclination i_1 and RAAN Ω_1, then the angle change is

$$\cos\theta = \cos i_1 \cos i_2 + \sin i_1 \sin i_2 \cos(\Omega_2 - \Omega_1)$$



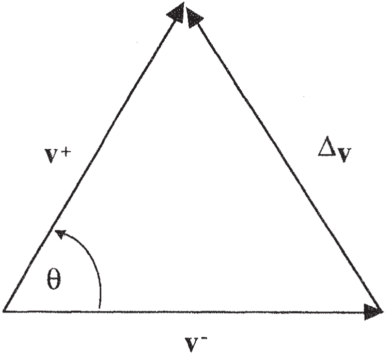
The position in the orbit is given by

$$\cos(\omega + f) = \frac{\cos i_1 \cos\theta - \cos i_2}{\sin i_1 \sin\theta}$$

Where recall

- $i_2 = \epsilon = 23.5°$

The Plane Change



The Δv required required for the plane change is then

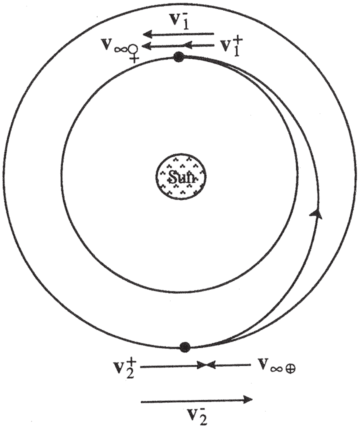
$$\Delta v = 2v \sin\frac{\theta}{2}$$

Problem: How much Δv do we need to achieve v_a?

- v_a is in the heliocentric frame.
- We start in the earth-frame
  - The earth frame is moving with velocity

$$v_e = \sqrt{\frac{\mu_s}{\|\vec{r}_{se}\|}} = 29.78km/s$$

- We must find $v_{\infty,e}$ - velocity with respect to the earth.



We have

$$v_2+ = v_a = v_e^- + v_{\infty,e}$$

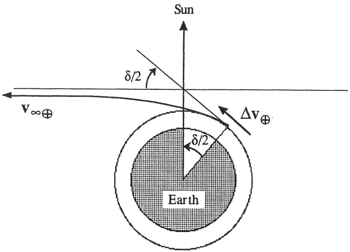
Thus our desired velocity with respect to the earth is

$$v_{\infty,e} = v_a - v_e^- = 27.29 - 29.78 = -2.49km/s$$

Problem: How to achieve the initial $v_{\infty,e} = -2.49km/s$?

- We need to escape earth orbit.
- Must have leftover velocity (excess velocity) of $2.49km/s$.
  - Implies the total energy after burn is

$$E_+ = \frac{1}{2} v_{\infty,e}^2 = 2.067$$

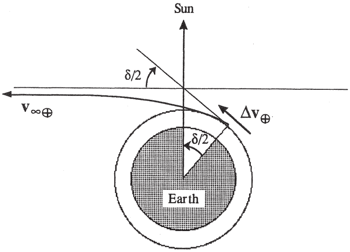


Interplanetary Hohmann Transfer

Suppose the spacecraft starts in a circular parking orbit of radius $r_{park} = 6578$.

- The velocity before the burn will be

$$v_{park} = \sqrt{\frac{\mu_e}{r_{park}}} = 7.7843 km/s$$



- The velocity after burn $v_{b,e}$ can be found by solving the energy equation.

$$E = \frac{1}{2}v_{b,e}^2 - \frac{\mu_e}{r_{park}} = +2.067$$

  Solving for $v_{b,e}$, we get

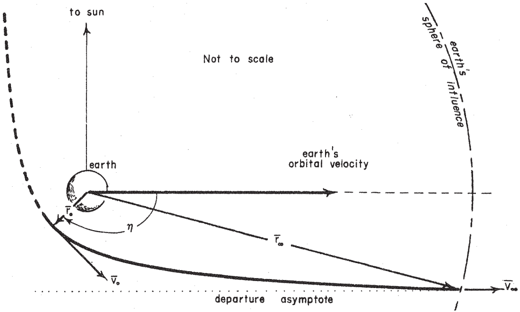
$$v_{b,e} = \sqrt{2E + 2\frac{\mu_e}{r_{park}}} = \sqrt{v_{\infty,e}^2 + 2\frac{\mu_e}{r_{park}}} = 11.195 km/s$$

- This yields a Δv_e of

$$\Delta v_e = v_{b,e} - v_{park} = 3.4106 km/s$$

Light Side / Dark Side:

- The earth rotates counterclockwise about the sun.
- Vehicles typically orbit counterclockwise about the earth.

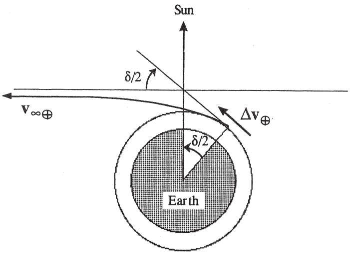


In this configuration, the burn occurs

- On the dark side for $\Delta v_{\infty,e} > 0$
  - Missions to outer planets.
- On the light side for $\Delta v_{\infty,e} < 0$
  - Missions to inner planets

Timing: The Δv should occur at $\delta/2$ before midnight/noon, where the turning angle

$$\delta = 2\sin^{-1}\frac{1}{e}$$



Eccentricity (e) can be found as:

- Energy: $E = \frac{1}{2}v_{\infty,e}^2 = 2.067 = -\frac{\mu}{2a}$ yields

$$a = -\frac{\mu}{v_{\infty,e}^2} = -\frac{\mu}{2E} = -96,420km$$

- Perigee: $r_{p,e} = r_c = a(1-e) = 6578km$ yields

$$e = 1 - \frac{r_{p,e}}{a} = 1.0682$$

This yields a turning angle of

$$\delta = 2.423rad = 138.83^\circ$$

Thus the spacecraft should depart at $\delta/2 = 69.4^\circ$ before noon/midnight.

Arrival at Venus

At arrival, our excess velocity w/r to Venus ($v_{\infty,v}$) will be

$$v_{\infty,v} = v_p - v_v = v_1^- - v_1^+ = 37.81km/s - 35.09km/s = 2.71km/s$$

where

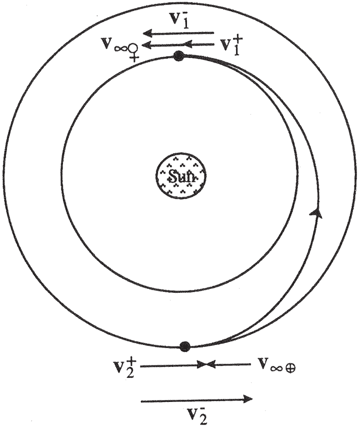
- $v_1^+ = v_v$ is the velocity of venus

$$v_1^+ = v_v = \sqrt{\frac{\mu_s}{r_v}}$$

- v_p is the periapse velocity of the Hohmann transfer

Because $v_{\infty,v} > 0$, the spacecraft will approach Venus from behind.

- Spacecraft is catching up to planet (not vice-versa)
- The back door



Arrival at Venus

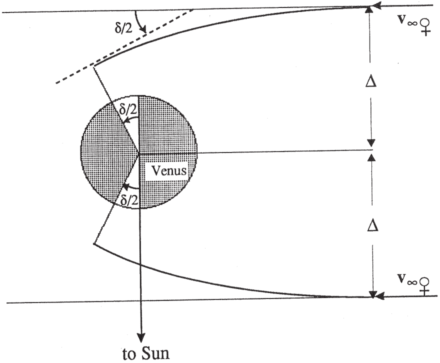
Venus Data:

$$R_v = 6187km, \quad \mu_v = 324859, \quad a_{venus} = 1.08 \cdot 10^8$$

Desired Orbit: Circular, posigrade grade (counterclockwise) with

$$r_c = 6187 + 500 = 6687km$$

For a counterclockwise orbit, we want to approach Venus on the **Dark Side**



Arrival at Venus

For orbital insertion, we want to perform a retrograde burn at periapse of the incoming hyperbola.

To achieve a circular orbit of radius $r_c = 6687km$, we need the periapse of our incoming hyperbola to occur at

$$r_{p,v} = a(1-e) = 6687km$$

The energy of the incoming hyperbola is given by the excess velocity as

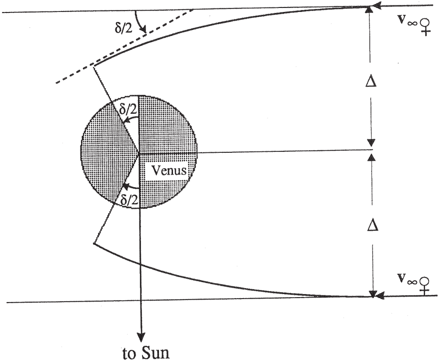
$$E = \frac{1}{2}v_{\infty,v}^2 = 3.67$$

This fixes the semimajor axis at

$$a = -\frac{\mu_v}{v_{inf,v}^2} = -44,232km$$

Thus to achieve $r_p = a(1-e)$, we need

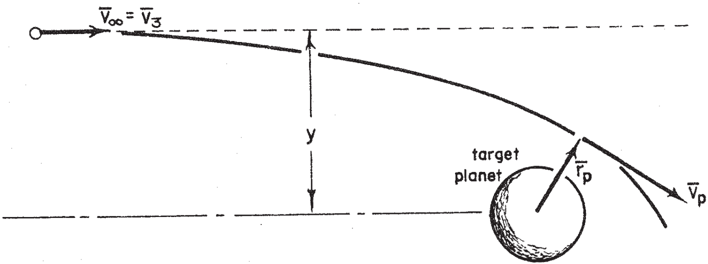
$$e = 1 - \frac{r_p}{a} = 1.15$$



Arrival at Venus

To achieve the desired $e = 1.15$, we control the conditions at the *Patch Point*.

- We do this through the angular momentum, h.



We can control the **Target Radius**, Δ through small adjustments far from the planet. Angular momentum can be exactly controlled through target radius, Δ.

$$h_v = v_{\infty,v}\Delta$$

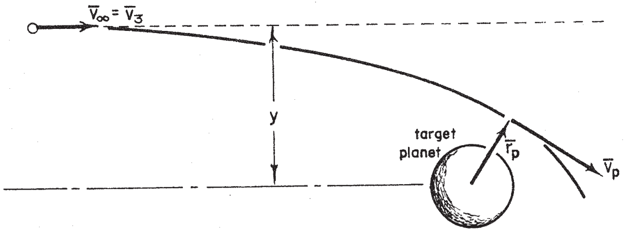
Arrival at Venus

Solution: For a given a, e is determined by $p = a(1-e^2)$.

- But p is defined by angular momentum (and thus target radius).

$$p = \frac{h^2}{\mu_v} = \frac{\Delta^2 v_{\infty,v}^2}{\mu_v}$$

- For $a = -44,232km$ and $e = 1.15$, we get $p = 14,265km$.



Given a desired p we solve for target radius, Δ,

$$\Delta = \sqrt{\frac{p\mu_v}{v_{\infty,v}^2}} = \sqrt{\frac{a(1-e^2)\mu_v}{v_{\infty,v}^2}} = 25,120km$$

Injection into Circular Orbit

Finally, we need to slow down to achieve circular orbit.

- The velocity at periapse (6687km) is given by the vis-viva equation.

$$v = \sqrt{\frac{2\mu_v}{r_{p,v}} - \frac{\mu_v}{a}} = 10.223 km/s$$

- The velocity of a circular orbit is

$$v_c = \sqrt{\mu_v} r_{p,v} = 6.97 km/s$$

Thus the Δv required to circularize the orbit is

$$\Delta v = 6.97 - 10.223 = -3.253 km/s$$

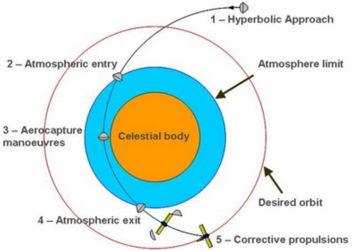


Figure: Aerobraking can also assist with Δv

Messenger Probe to Mercury

Gravity Assist Trajectories

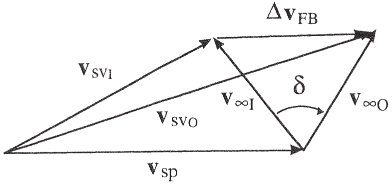
Concept: Planets rotate the relative velocity vector.

- The relative motion changes as

$$\vec{v}_f - \vec{v}_{planet} = R_1(\delta)\,(\vec{v}_i - \vec{v}_{planet})$$

- In the inertial frame (2 dimensions) this means

$$\vec{v}_f = \begin{bmatrix} \cos\delta & -\sin\delta \\ \sin\delta & \cos\delta \end{bmatrix} (\vec{v}_i - \vec{v}_{planet}) + \vec{v}_{planet}$$



Example: If $\delta = 180°$ and $\vec{v}_i = \begin{bmatrix} -20 \\ 0 \end{bmatrix} km/s$ and $\vec{v}_p = \begin{bmatrix} 20 \\ 0 \end{bmatrix} km/s$, then

$$v_f = R(180°) \begin{bmatrix} -40 \\ 0 \end{bmatrix} km/s + \begin{bmatrix} 20 \\ 0 \end{bmatrix} km/s = \begin{bmatrix} 40 \\ 0 \end{bmatrix} km/s + \begin{bmatrix} 20 \\ 0 \end{bmatrix} km/s = \begin{bmatrix} 60 \\ 0 \end{bmatrix} km/s$$

Thus a probe can potentially *triple* its velocity!

Gravity Assist Trajectories

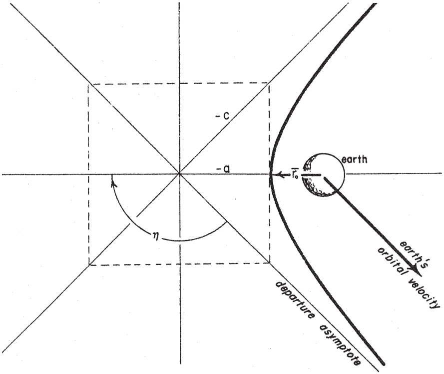
To achieve the desired turning angle, we must control the geometry

The turning angle δ is given by

$$\delta = 2\sin^{-1}\frac{1}{e}$$

Recall that energy of the orbit is fixed.
Thus we can solve for

$$a = -\mu_{planet}/\|\vec{v}_i - \vec{v}_{planet}\|^2$$



Then the eccentricity can be fixed by the target radius as

$$\Delta = \sqrt{\frac{a(1-e^2)\mu_{planet}}{\|\vec{v}_i - \vec{v}_{planet}\|^2}} = 25,120km$$

In 3 dimensions, the calculations are more complex.

Gravity Assist Trajectories

Example: Jupiter flyby

Problem: Suppose we perform a Hohman transfer from Earth to Jupiter. What is the best-case gravity assist we can expect?

Solution: The velocity of arrival at apogee (Jupiter) in the Heliocentric frame is:

$$v_a = \sqrt{2\mu_{sun}\frac{r_e}{r_j(r_j + r_e)}} = 7.414km/s$$

The velocity of jupiter itself is

$$v_j = \sqrt{\frac{\mu_s}{d_j}} = 13.0573km/s$$

Since this is an outer planet, we approach from the front door. In the jupiter $R - T - N$ frame we have

$$\vec{v}_s = \begin{bmatrix} 7.414 \\ 0 \end{bmatrix}, \qquad \vec{v}_j = \begin{bmatrix} 13.0573 \\ 0 \end{bmatrix}$$

The velocity of the spacecraft relative to jupiter is

$$\vec{v}_s - \vec{v}_j = \begin{bmatrix} -5.6429 \\ 0 \end{bmatrix} km/s$$

Example: Jupiter flyby

Jupiter Data: Radius $r_j = 11.209ER$; Distance $d_j = 5.2028AU$; $\mu_j = 317.938\mu_e$.

The velocity of the spacecraft relative to jupiter is

$$\vec{v}_s - \vec{v}_j = \begin{bmatrix} -5.6429 \\ 0 \end{bmatrix} km/s$$

Thus we can calculate the energy of the hyperbolic approach as

$$a = -\frac{\mu_s}{\|\vec{v}_s - \vec{v}_j\|^2} = -3.98E6km$$

The closest we can approach jupiter is its radius. If we use this for periapse, we get

$$e = 1 - \frac{r_j}{a} = 1.018$$

The eccentricity yields the maximum turning angle as

$$\delta = 2\sin^{-1}\left(\frac{1}{e}\right) = 158.44^\circ$$

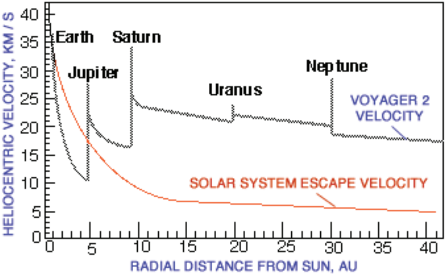
Example: Jupiter flyby

Applying this rotation (light-side approach), we get

$$\vec{v}_f = \begin{bmatrix} \cos\delta & -\sin\delta \\ \sin\delta & \cos\delta \end{bmatrix} (\vec{v}_i - v_{planet}) + \vec{v}_{planet} = \begin{bmatrix} 18.305 \\ 2.076 \end{bmatrix}$$

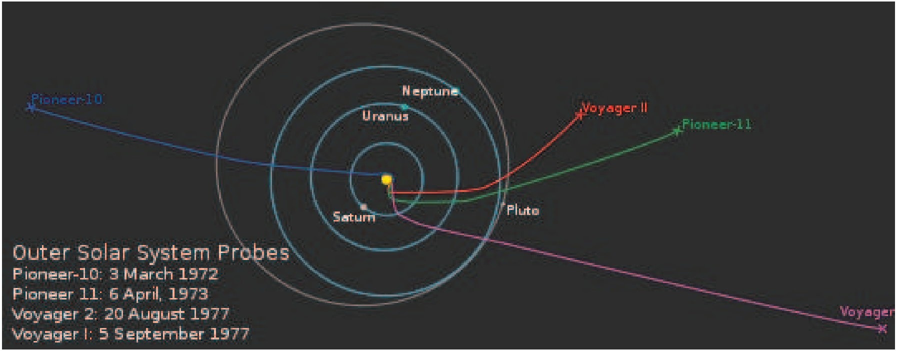
The magnitude of the Δv from this flyby is 11.01km/s. A factor of 2.5.

Note that if we could have reversed our direction of flight (clockwise approach), we could achieve a $\Delta v = 20.05 km/s$.



Trajectories for Voyager 1 and Voyager 2 Spacecraft

Trajectories for Voyager 1, Voyager 2, and Pioneer Spacecraft



Summary

This Lecture you have learned:

Sphere of Influence

- Definition

Escape and Re-insertion

- The light and dark of the Oberth Effect

Patched Conics

- Heliocentric Hohmann

Planetary Flyby

- The Gravity Assist