Spacecraft Dynamics and Control

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Lecture 12: Orbital Perturbations

Introduction

In this Lecture, you will learn:

Perturbation Basics

- The Satellite-Normal Coordinate System
- Equations for
  - $\dot{a}$, $\dot{i}$, $\dot{\Omega}$, $\dot{\omega}$, $\dot{e}$

Drag Perturbations

- Models of the atmosphere.
- Orbit Decay
- Δv budgeting.
- Effect on eccentricity.

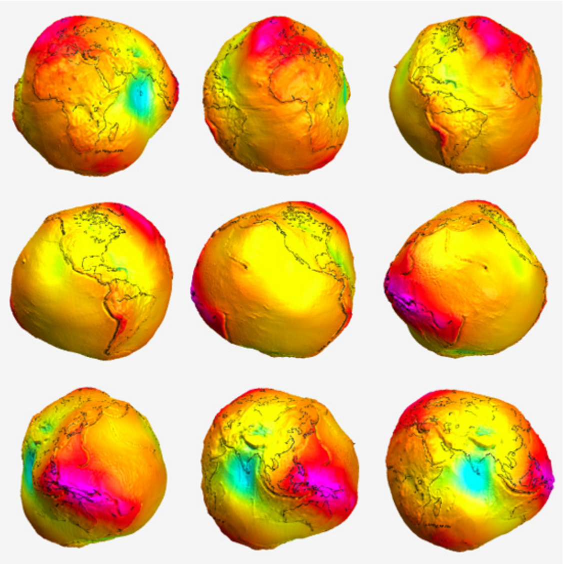
Introduction to Perturbations

So far, we have only discussed idealized orbits.

- Solutions to the 2-body problem.
- All orbital elements are fixed (except f).

In reality, there are many other forces at work:

- Drag
- Non-spherical Earth
- Lunar Gravity
- Solar Radiation
- Tidal Effects



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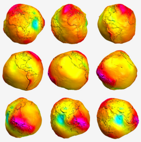


- Perturbations can be good or bad.
- Perturbations allow us to break free of the Δv budget.
- There is not much flexibility in the restricted two-body problem. All maneuvering is accomplished using Δv budget (Gravity assist being an exception)
- Perturbations allow us to identify new forces which, if used correctly, can reduce our dependency on Δv budget.

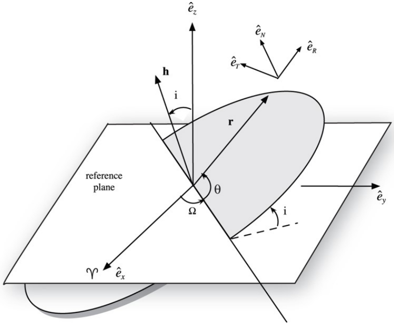
By definition, perturbations don't point to the center of mass

- Where do they point?
- Need a new coordinate system.

$$\vec{F} = R\hat{e}_R + N\hat{e}_N + T\hat{e}_T$$

Satellite-Normal CS (R-T-N):

- $\hat{e}_R$ points along the earth $\rightarrow$ satellite vector.
- $\hat{e}_N$ points in the direction of $\vec{h}$
- $\hat{e}_T$ is defined by the RHR
  - $\hat{e}_T \cdot v > 0$.



Generalized Perturbation Analysis

Satellite-Normal Coordinate System

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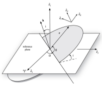


In Frenet coordinates, $\hat{e}_N$ is the same, $\hat{e}_T$ is tangential to motion, and $\hat{e}_R = \hat{e}_T \times \hat{e}_N$.

Generalized Perturbation Analysis

Now suppose we have an expression for the disturbing force:

$$\vec{F} = R\hat{e}_R + N\hat{e}_N + T\hat{e}_T$$

How does this affect $\dot{a}$, $\dot{i}$, $\dot{\Omega}$, $\dot{\omega}$, $\dot{e}$?

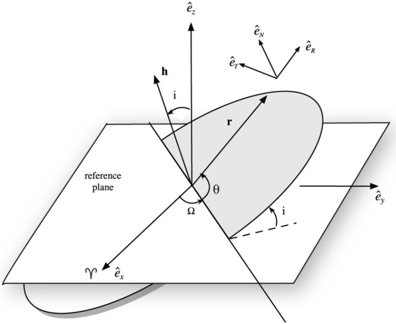
Most elements depend on $\vec{h}$ **and** E**:**

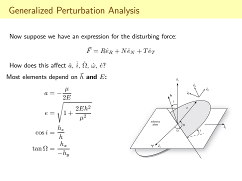
$$a = -\frac{\mu}{2E}$$

$$e = \sqrt{1 + \frac{2Eh^2}{\mu^2}}$$

$$\cos i = \frac{h_z}{h}$$

$$\tan \Omega = \frac{h_x}{-h_y}$$





- Here we see the direct relationship between physical parameters h, E and orbital parameters a, e.
- In the presence of perturbations, angular momentum and energy of the satellite are not conserved.
- Hence, in the presence of perturbations, the orbit is no longer truly elliptic. Hence the orbital elements are not perfect parameters of motion. However, deviations from the ellipse occur over long time-horizons and so we assume a quasi-stationary elliptic motion and include adjustments to the ellipse in the form of orbit-averaged versions of $\dot{a}$, $\dot{i}$, $\dot{\Omega}$, $\dot{\omega}$, $\dot{e}$. Also, we don't have anything better.

Energy and Momentum Perturbation

We have the orbital elements in terms of $\vec{h}$ and E.

1. Find expressions for $\dot{\vec{h}}$ and $\dot{E}$.
2. Translate into expressions for $\dot{a}$, $\dot{e}$, etc.

Example 1: Semimajor axis.

$$a = -\frac{\mu}{2E}$$

Chain Rule:

$$\dot{a} = \frac{da}{dE}\frac{dE}{dt} = \frac{\mu}{2E^2}\dot{E}$$

Example 2: Eccentricity.

$$e = \sqrt{1 + \frac{2Eh^2}{\mu^2}}$$

Chain Rule:

$$\dot{e} = \frac{de}{dh}\frac{dh}{dt} + \frac{de}{dE}\frac{dE}{dt} = \frac{1}{2e}(e^2 - 1)\left[2\frac{\dot{h}}{h} - \frac{\dot{E}}{E}\right]$$

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$$p = \frac{h^2}{\mu} = a(1-e^2) = -\frac{\mu}{2E}(1-e^2), \qquad \text{So} \qquad (1-e^2) = -\frac{2Eh^2}{\mu^2}$$

So

$$e = \sqrt{1 + \frac{2Eh^2}{\mu^2}} = \left(1 + \frac{2Eh^2}{\mu^2}\right)^{\frac{1}{2}}$$

So

$$\frac{de}{dh} = \frac{1}{2}\left(1 + \frac{2Eh^2}{\mu^2}\right)^{-\frac{1}{2}}\frac{4Eh}{\mu^2} = \frac{2Eh}{\mu^2 e} = -\frac{h^2}{\mu a h e} = -\frac{p}{ahe} = \frac{(e^2-1)}{he}$$

and likewise

$$\frac{de}{dE} = \frac{1}{2}\left(1 + \frac{2Eh^2}{\mu^2}\right)^{-\frac{1}{2}}\frac{2h^2}{\mu^2} = \frac{h^2}{\mu^2 e} = \frac{p}{\mu e} = \frac{2a(1-e^2)}{2\mu e} = \frac{e^2-1}{2Ee}.$$

Energy and Momentum Perturbation

So now the key is to find expressions for $\dot{h}$ and $\dot{E}$. Let $\vec{F}$ be the disturbing force per unit mass (watch those units!) in RTN coordinates:

$$\vec{F} = \begin{bmatrix} R \\ T \\ N \end{bmatrix}$$

Energy: Energy is Force times distance.

$$dE = \vec{F} \cdot d\vec{r}$$

So in RTN coordinates,

$$\begin{aligned} \dot{E} &= \vec{F} \cdot \vec{v} \\ &= \vec{F} \cdot \left(\dot{r}\hat{e}_R + r\dot{\theta}\hat{e}_T \right) \\ &= \dot{r}R + r\dot{\theta}T \end{aligned}$$

Momentum: Newton's Second Law:

$$\begin{aligned} \dot{\vec{h}} &= \vec{r} \times \vec{F} \\ &= rT\hat{e}_N - rN\hat{e}_T \end{aligned}$$

With magnitude $\dot{h} = d/dt\sqrt{\vec{h} \cdot \vec{h}}$

$$\begin{aligned} \dot{h} &= \frac{\vec{h} \cdot \dot{\vec{h}}}{h} = \frac{(h\vec{e}_N) \cdot (rT\hat{e}_N - rN\hat{e}_T)}{h} \\ &= rT \end{aligned}$$

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- Energy is NOT conserved. Some disturbances can sap energy (e.g. drag). Some can increase energy (e. g. solar wind)
- We have assumed quasi-elliptic motion, so...
- Recall $\vec{v} = \dot{r}\hat{e}_R + r\dot{\theta}\hat{e}_T$ is the velocity in RTN - recall Lecture 2!
- Recall $\vec{r}$ is always in the orbital plane! So $\hat{e}_N \cdot \vec{r} = 0$.
- Also recall $\vec{h} = h\vec{e}_N$.

Semi-Major Axis Perturbation

Using $r = \dfrac{h^2/\mu}{1 + e\cos f}$ and the approximation $\dot{\theta} = \frac{d}{dt}(\omega + f) \cong \dot{f} = h/r^2$, we combine

$$\dot{a} = \frac{\mu}{2E^2}\dot{E}$$

with

$$\dot{E} = \dot{r}R + r\dot{\theta}T$$

where $E = -\frac{\mu}{2a}$ to get:

Semi-major Axis

$$\dot{a} = 2\frac{a^2}{\mu}\left[R\frac{\mu e \sin f}{h} + T\frac{h}{r}\right]$$

or, in terms of a, e, and f,

$$\dot{a} = 2\sqrt{\frac{a^3}{\mu(1-e^2)}}\left[eR\sin f + T(1 + e\cos f)\right]$$

Semi-Major Axis Perturbation

Using $r = \dfrac{h^2/\mu}{1 + e\cos f}$ and the approximation $\dot\theta = \frac{d}{dt}(\omega + f) \cong \dot f = h/r^2$, we combine

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$$\dot a = 2\sqrt{\frac{a^3}{\mu(1-e^2)}}\left[eR\sin f + T(1 + e\cos f)\right]$$

Recall by definition $h = r \cdot v_\perp = r \cdot (r\dot f) = r^2 \dot f$.

Since $r = \dfrac{h^2/\mu}{1 + e\cos f}$, we have used the chain rule to get

$$\dot r = \frac{h^2/\mu}{(1+e\cos f)^2} e\sin f \dot f = \frac{r^2}{h^2/\mu} e \sin f \dot f = \frac{e\sin f}{h^2/\mu} r^2 \dot f = \frac{\mu e \sin f}{h}$$

Eccentricity Perturbation

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$$\dot{e} = \frac{1}{2e}(e^2 - 1)\left[2\frac{\dot{h}}{h} - \frac{\dot{E}}{E}\right]$$

with

$$\dot{E} = \dot{r}R + r\dot{\theta}T \qquad \text{and} \qquad \dot{h} = rT$$

where $E = -\frac{\mu}{2a}$ to get

Eccentricity:

$$\dot{e} = \sqrt{\frac{a(1-e^2)}{\mu}}\left[R\sin f + T(\cos f + \cos E_{ecc})\right]$$

where E_{ecc} is eccentric anomaly,

$$\tan\frac{E_{ecc}}{2} = \sqrt{\frac{1-e}{1+e}}\tan\frac{f}{2}$$

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In the last equation, we used the expression for r

$$r = a(1 - e\cos E_{ecc})$$

Energy and Momentum Perturbation

Inclination and RAAN

Inclination: From

$$\cos i = \frac{h_z}{h}$$

we have from the chain rule

$$\frac{d}{dt} i = \frac{1}{-\sin i} \frac{h\dot{h}_z - \dot{h} h_z}{h^2}$$

from which we can get

$$\frac{d}{dt} i = \sqrt{\frac{a(1-e^2)}{\mu}} \frac{N \cos(\omega + f)}{1 + e \cos f}$$

RAAN: From

$$\tan \Omega = \frac{h_x}{-h_y}$$

we have from the chain rule

$$\dot{\Omega} = \cos^2 \Omega \frac{h_x \dot{h}_y - \dot{h}_x h_y}{h_y^2}$$

from which we can get

$$\dot{\Omega} = \sqrt{\frac{a(1-e^2)}{\mu}} \frac{N \sin(\omega + f)}{\sin i (1 + e \cos f)}$$

Although complicated, we can also find $\dot{\omega}$.

$$\dot{\omega} = -\dot{\Omega} \cos i + \sqrt{\frac{a(1-e^2)}{e^2 \mu}} \left(-R \cos f + T \frac{(2 + e \cos f) \sin f}{1 + e \cos f} \right)$$

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Use Rotation matrices to convert:

$$\begin{aligned}
\dot{\vec{h}} &= \vec{r} \times \vec{F} \\
&= rT\hat{e}_N - rN\hat{e}_T = \begin{bmatrix} 0 \\ -rN \\ rT \end{bmatrix}_{RNT} \\
&= \begin{bmatrix} \dot{h}_x \\ \dot{h}_y \\ rT\cos i - rN \cos\theta \sin i \end{bmatrix}_{ECI}
\end{aligned}$$

Where in the last step, we used the rotation matrix $R_{RTN \to ECI} = R_3(\Omega) R_1(i) R_3(\theta)$ from Lecture 7. However, the expression for $\dot{h}_x$, $\dot{h}_y$ is too complicated for these slides.

Levitated Orbit Example

Problem: Suppose a satellite of 100kg in circular polar orbit of 42,164km experiences a continuous solar pressure of $.1$ Newton in $\hat{e}_N$ direction. How do the orbital elements vary with time?

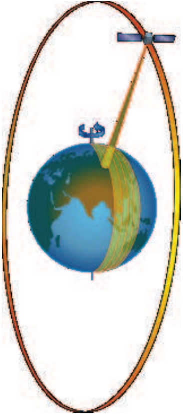
Solution: The Force per unit mass is
$N = F/m = .001 m/s^2 = 1E-6 km/s^2$.
Since $T = R = e = 0$, and $f \cong E_{ecc} \cong M = nt$

$$\dot{a} = 2\sqrt{\frac{a^3}{\mu(1-e^2)}}\left[eR\sin f + T(1+e\cos f)\right] = 0$$

$$\dot{e} = \sqrt{\frac{a(1-e^2)}{\mu}}\left[R\sin f + T(\cos f + \cos E_{ecc})\right] = 0$$

For inclination, we have

$$\frac{d}{dt}i = N\sqrt{\frac{a(1-e^2)}{\mu}}\frac{\cos(\omega+f)}{1+e\cos f} = N\sqrt{\frac{a}{\mu}}\cos nt$$



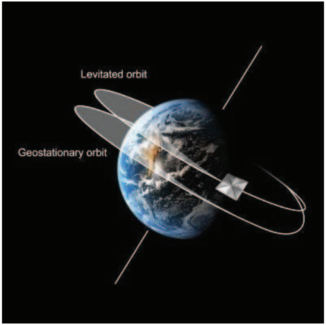
Levitated Orbit Example

The formula for inclination integrates out to

$$\Delta i(t) = N\sqrt{\frac{a}{\mu}}\frac{1}{n}\sin nt = .00446 \sin nt \; radians$$

Similarly, since $i \cong 90^\circ$

$$\dot{\Omega} = N\sqrt{\frac{a(1-e^2)}{\mu}}\frac{\sin(\omega+f)}{\sin i(1+e\cos f)} = N\sqrt{\frac{a}{\mu}}\sin nt$$



We have

$$\Delta\Omega(t) = -N\sqrt{\frac{a}{\mu}}\frac{1}{n}\cos nt = -.00446 \cos nt \; radians$$

The effect is a "Displaced" orbit. The size of the displacement is .0045rad * 42164 km = 188km. See "Light Levitated Geostationary Cylindrical Orbits are Feasible" by S. Baig and C. R. McInnes.

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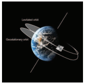

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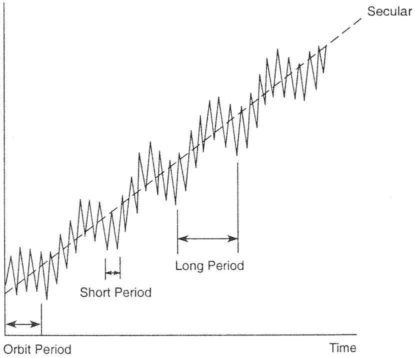
- At ascending node, pulled forward $(+\hat{e}_N)$ by 188km due to $+\Delta\Omega$, no Δi
- At descending node, pulled forward $(+\hat{e}_N)$ by 188km due to $-\Delta\Omega$, no Δi
- At north pole, pulled forward $(+\hat{e}_N)$ by 188km due to $-\Delta i$, no $\Delta\Omega$
- At south pole, pulled forward $(+\hat{e}_N)$ by 188km due to $+\Delta i$, no $\Delta\Omega$

Periodic and Secular Variation

The preceding example illustrated the effect of periodic variation.

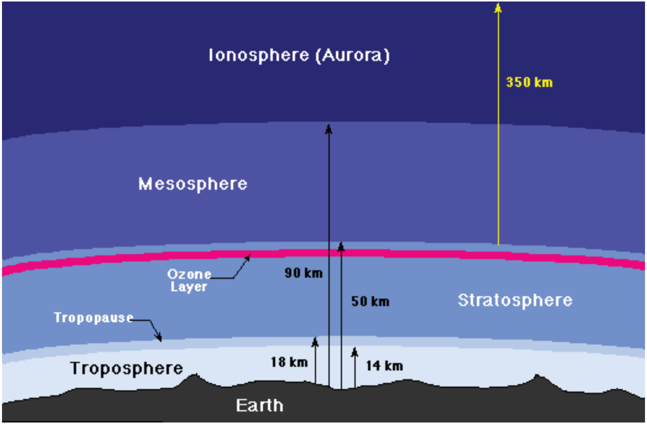


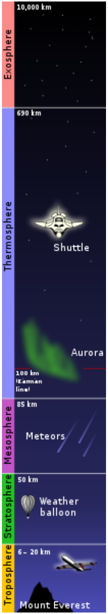
There are three types of disturbances

- **Short Periodic** - Cycles every orbital period.
- **Long Periodic** - Cycles last longer than one orbital period.
- **Secular** - Does not cycle. Disturbances mount over time.

Secular Disturbances must be corrected.

Earth's atmosphere extends into space.





The ionosphere extends well past 350km.

- ISS orbit lies between 330 and 400km.

2023-04-11

Lecture 12
└─Spacecraft Dynamics
└─Atmospheric Drag

Atmospheric Drag

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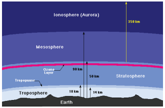

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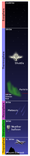


- Its called the ionosphere because all the atmospheric gasses have lost their electrons.

The Ionosphere

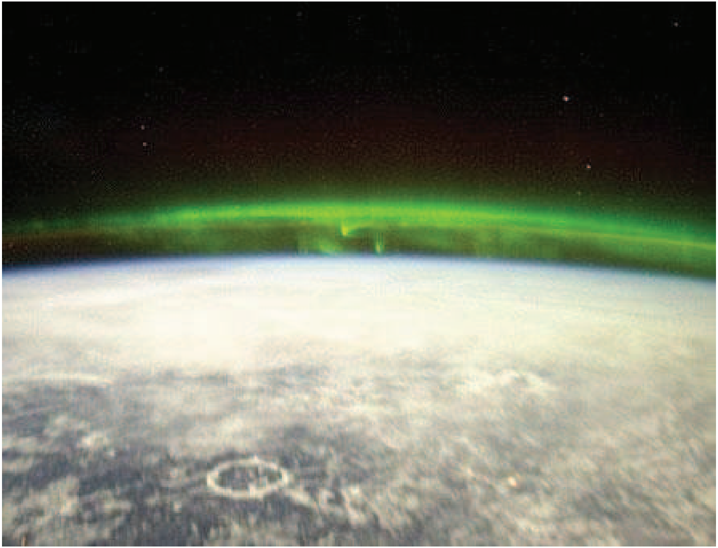


Figure: The Aurora Borealis Shows the Ionosphere Extending Well into Orbital Range

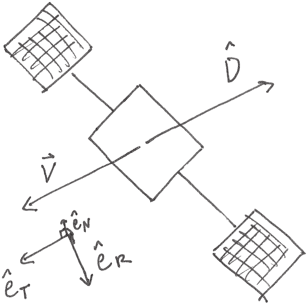
The Drag Perturbation

Drag force for satellites is the same as for aircraft

$$F_D = C_D Q A = \frac{1}{2}\rho v^2 C_D A$$

By definition, drag is opposite to the velocity vector.

- Since by definition, $\vec{v} \perp \vec{h}$, $N = 0$
- For now, ignore the rotation of the earth (adds $\Delta v = \omega_e r \cong .5km/s$).
- For now, assume circular orbit, so $\vec{v} = v\hat{e}_T$.



Ballistic Coefficient:

$$B = \frac{m}{C_D A}$$

Then as first approximation,

$$N = R = 0, \qquad T = -\frac{1}{2}\frac{\rho}{m} C_D A v^2 = -\frac{1}{2}\frac{\rho v^2}{B}$$

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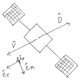

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Then as first approximation,

$$N = R = 0, \qquad T = -\frac{1}{2}\frac{\rho}{m} C_D A v^2 = -\frac{1}{2}\frac{\rho v^2}{B}$$

- Q is dynamic pressure.
- $\vec{v}$ is in the orbital plane and $\vec{h}$ is perpendicular to the orbital plane.
- A is the area of the spacecraft projected onto the $\hat{e}_N - \hat{e}_R$ plane.
- C_D measures how aerodynamic the spacecraft is.
- Drag can also generate lift (C_L)! A component in the $\hat{e}_R$ direction (or even the $\hat{e}_N$ direction)

The Drag Effect on Orbital Elements

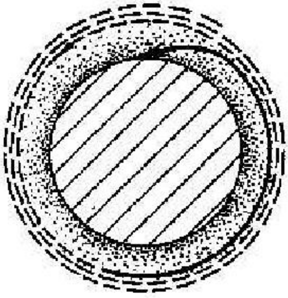
Circular Orbits, Constant Density

First note that since $N = 0$, the orbital plane does not change

- $\dot{\Omega} = 0$.
- $\frac{d}{dt} i = 0$.

Semi-Major Axis: Since $e = 0$, the dominant effect is on a.

$$\dot{a} = 2\sqrt{\frac{a^3}{\mu(1-e^2)}}\left[eR\sin f + T(1+e\cos f)\right]$$
$$= -\sqrt{\frac{a^3}{\mu}}\frac{\rho}{m}C_D A v^2 = -\sqrt{\frac{a^3}{\mu}\frac{\mu^2}{a^2}}\frac{\rho}{B}$$
$$= -\sqrt{a\mu}\frac{\rho}{B}$$



Integrating with respect to time (assuming constant ρ) yields

$$a(t) = \left(\sqrt{a(0)} - 2\sqrt{\mu}\frac{\rho}{B}t\right)^2$$

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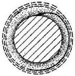


Integrating with respect to time (assuming constant ρ) yields

$$a(t) = \left(\sqrt{a(0)} - 2\sqrt{\mu}\frac{\rho}{B}t\right)^2$$

- $v = \sqrt{\mu/r}$ for circular orbits.
- Unfortunately, $\rho(t)$ is NOT constant.

Example: International Space Station

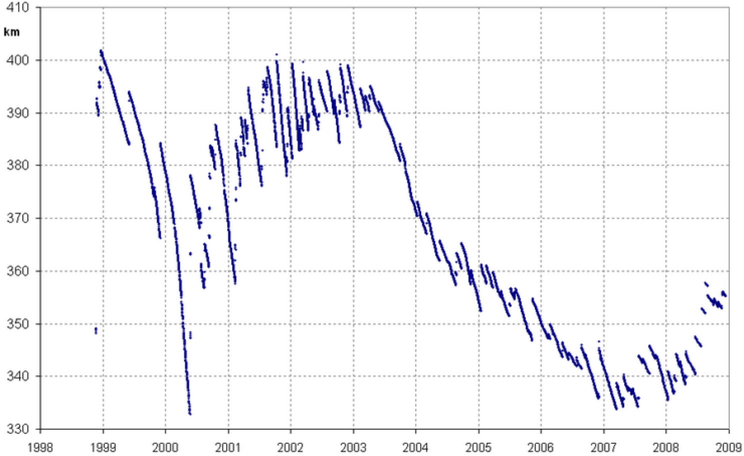


Figure: Orbit Decay of the International Space Station

Density Variation

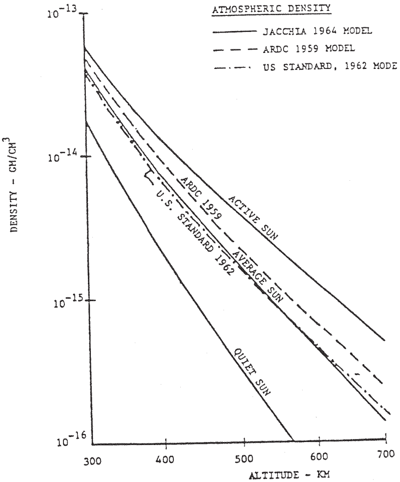
The atmospheric density is not even remotely constant

Exponential Growth:

- Extends to $1.225 * 10^{-3} g/cm^3$ at sea level.
- Orbits below Kármán Line (100km) will not survive a single orbit.
  - Suborbital flight.

Solar Activity: We have different models of the atmosphere depending on solar activity level.

- Unlike aircraft applications
- Variation mainly occurs in ionosphere
- Solar wind changes earth's EM field



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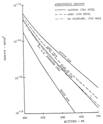
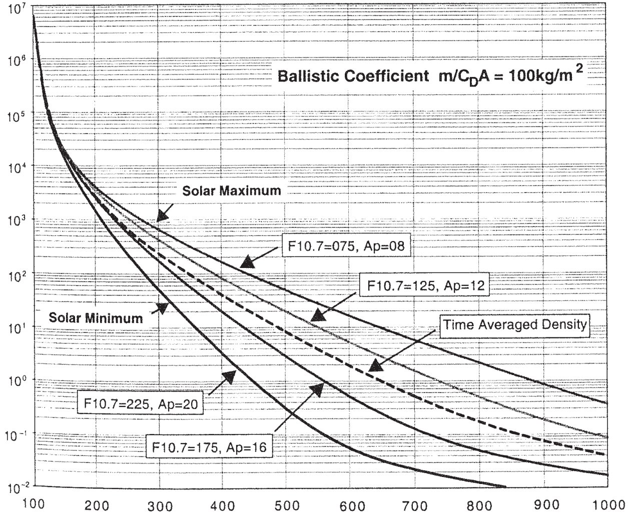


- Most density models of the atmosphere start to fail at the ionosphere.
- Kármán Line is named after Theodore van Kármán (1881–1963)
- A nominal aircraft at the Kármán Line would have to travel at orbital velocity to generate more lift than weight.
- Usually differentiates the fields of aeronautics and astronautics

Stationkeeping

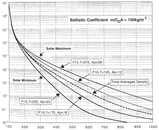
All Satellites must budget Δv (m/s/yr) to compensate for atmospheric drag.



The problem with budgeting is predicting solar activity.

Stationkeeping

All Satellites must budget Δv (m/s/yr) to compensate for atmospheric drag.



The problem with budgeting is predicting solar activity.

This data is scaled to Ballistic Coefficient.

- So if your ballistic coefficient is 10 times lower, you need 10 times the Δv!

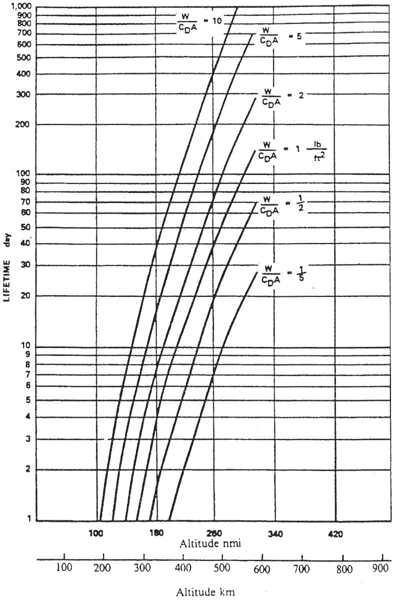
Spacecraft Lifetime

Without stationkeeping, orbits will decay quickly.

Definition 1.

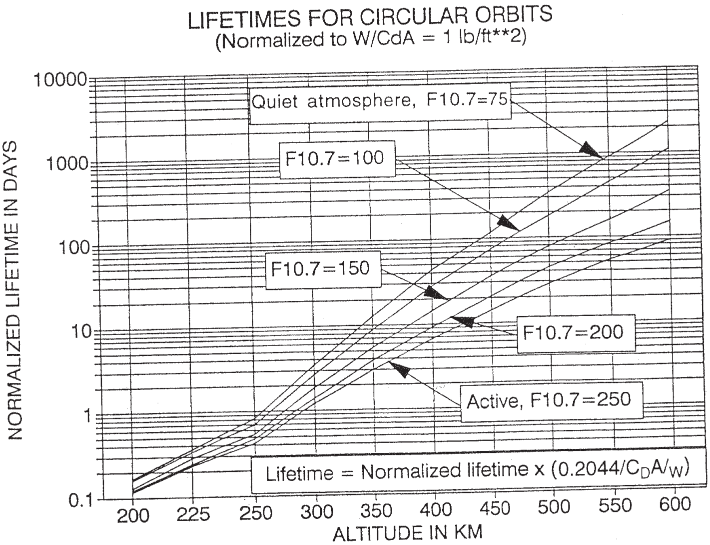
The **Lifetime** of a spacecraft is the time it takes to reach the 100km Kármán Line.

- The Figure shows mean value of lifetime.
- Actual values will depend on solar activity.

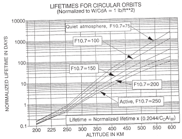


Spacecraft Lifetime

Solar Activity Effect

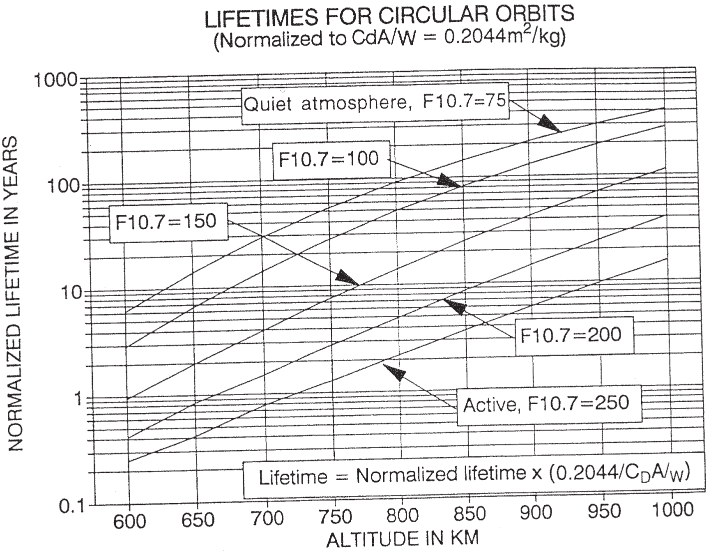


Spacecraft Lifetime
Solar Activity Effect



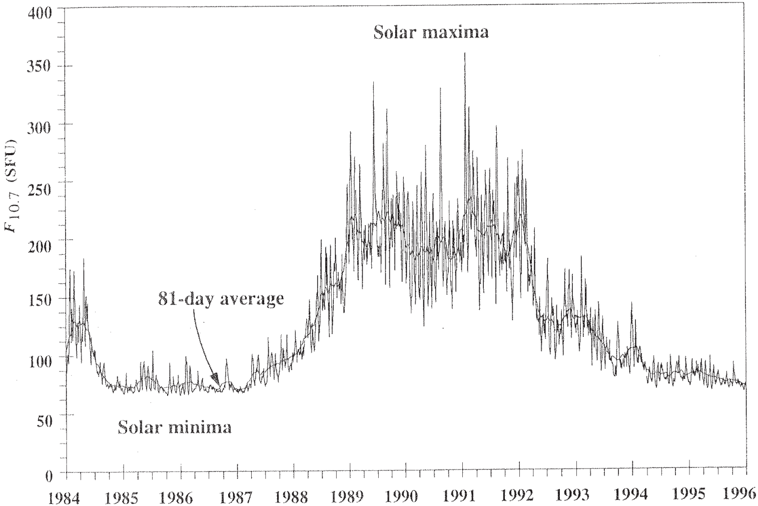
Plot is normalized for a ballistic coefficient and US customary units.

- To get actual lifetime, multiply number from plot by $.2044\frac{W}{C_D A}$ in metric units.


LIFETIMES FOR CIRCULAR ORBITS
(Normalized to CdA/W = 0.2044m²/kg)
Quiet atmosphere, F10.7=75
F10.7=100
F10.7=150
F10.7=200
Active, F10.7=250
Lifetime = Normalized lifetime x (0.2044/CDA/W)
NORMALIZED LIFETIME IN YEARS
ALTITUDE IN KM
1000
100
10
1
0.1
600
650
700
750
800
850
900
950
1000

Solar Activity

Solar Activity varies substantially with time. $F_{10.7}$ measures normalized solar power flux at EM wavelength 10.7cm.



Solar Activity is Hard to Predict

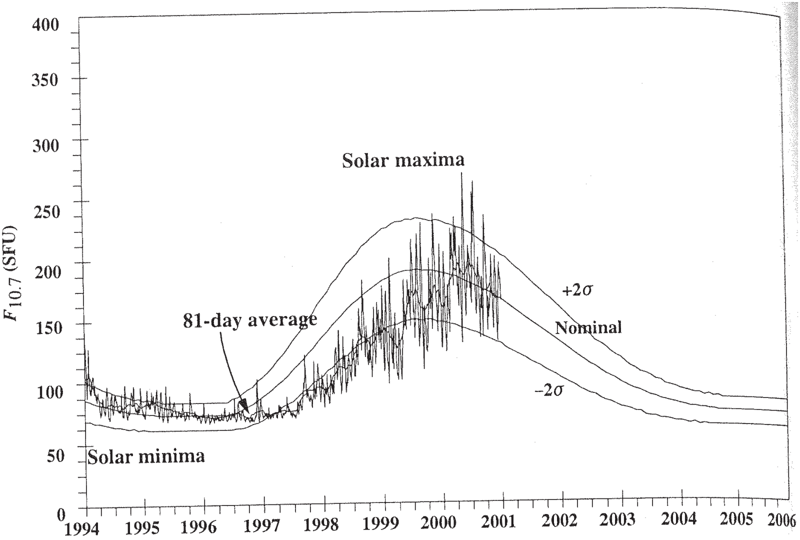
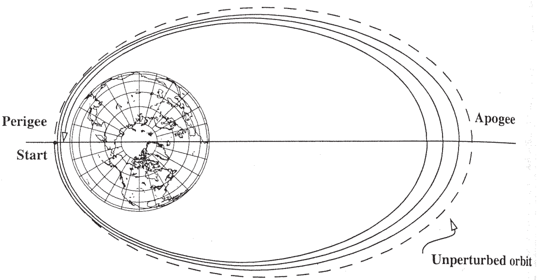


Figure: Shatten Prediction Model with Actual Data

Drag Effects on Eccentric Orbits

Eccentric orbits are particularly prone to drag.



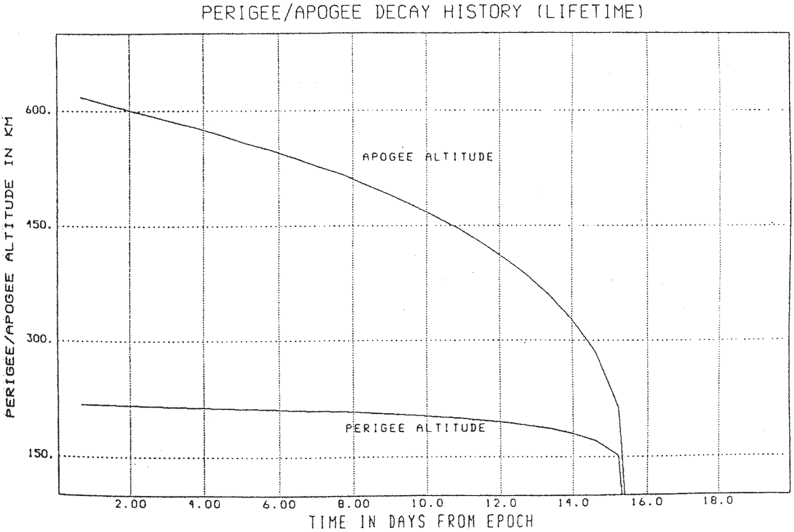
- Even if a is large, drag at perigee is high.
- Very difficult to integrate, due to changing density
- Using Exponential Density model,

$$\Delta e_{rev} = -2\pi \frac{C_D A}{m} a \rho_{perigee} e^{-ae/H} [I_1 + e(I_0 + I_2)/2]$$

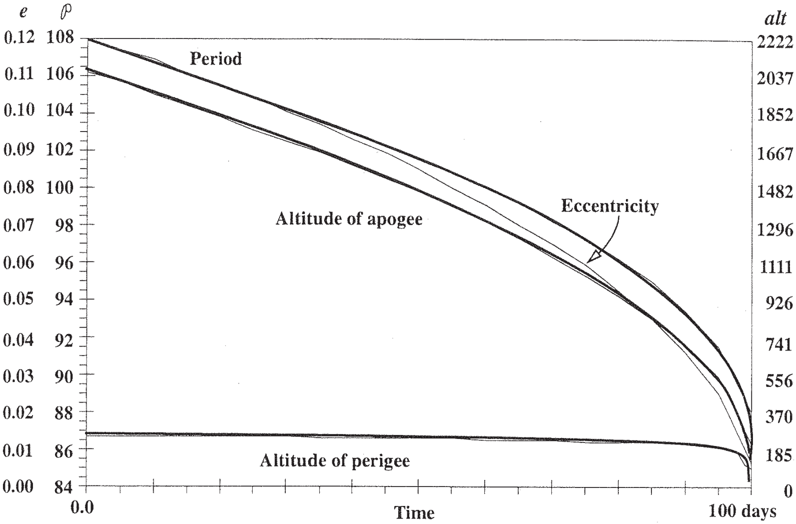
  - ρ_p is density at perigee. H is a height constant. I_i are Bessel functions
- Δa is also complicated.

Decay of Eccentricity

Although drag occurs at perigee, apogee is lowered.



Drag Effects on Eccentric Orbits



Hayabusa Re-entry

Summary

This Lecture you have learned:

Perturbation Basics

- The Satellite-Normal Coordinate System
- Equations for
  - $\dot{a}$, $\dot{i}$, $\dot{\Omega}$, $\dot{\omega}$, $\dot{e}$

Drag Perturbations

- Models of the atmosphere.
- Orbit Decay
- Δv budgeting.
- Effect on eccentricity.

Next Lecture: Earth's Shape and Sun-synchronous Orbits.

Equations

$$\dot{a} = 2\sqrt{\frac{a^3}{\mu(1-e^2)}}\left[eR\sin f + T(1+e\cos f)\right]$$

$$\dot{e} = \sqrt{\frac{a(1-e^2)}{\mu}}\left[R\sin f + T(\cos f + \cos E_{ecc})\right]$$

$$\frac{d}{dt}i = \sqrt{\frac{a(1-e^2)}{\mu}}\frac{N\cos(\omega+f)}{1+e\cos f}$$

$$\dot{\Omega} = \sqrt{\frac{a(1-e^2)}{\mu}}\frac{N\sin(\omega+f)}{\sin i(1+e\cos f)}$$

$$\dot{\omega} = -\dot{\Omega}\cos i + \sqrt{\frac{a(1-e^2)}{e^2\mu}}\left(-R\cos f + T\frac{(2+e\cos f)\sin f}{1+e\cos f}\right)$$

Drag (circular orbit):

$$N = R = 0, \qquad T = -\frac{1}{2}B\rho v^2 = -\frac{1}{2}B\rho\frac{\mu}{a}.$$