Lab 1: Errors and Error Propagation Nick Jacquet

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ERROR PROPAGATION EXERCISES

Determine the calculated value using the given values in the given equations. Be sure to **include the units in your answer**. Using the error propagation method described above, calculate the percent error in the calculated value. For this exercise, answers should be with the proper number of significant figures and **your percent error is to be given to two significant figures.**

Hand in this answer sheet. Work the problems neatly on scratch paper and staple your work to this sheet.

1. $A=xy$, $x=3.0cm\pm0.1cm$, $y=4.0cm\pm0.1cm$ — $12\,cm^2$ ± 4.2 %
2. $f=x+y$, for x and y given in problem # 1 — $7.0\,cm$ ± 2.0 %
3. $f=x-y$, for x and y given in problem # 1 — $-1.0\,cm$ ± 14 %
4. $z=3x+2y$, for x and y given in problem # 1 — 17 ± 2.1 %
5. $g=\frac{2h}{t^2}$ for $h=2.00m\pm3\%$, $t=0.630s\pm4\%$ — $10.1\,m/s$ ± 8.5 %
6. $T=2\pi\sqrt{\frac{M}{k}}$, $M=2.5Kg\pm6\%$, $k=\frac{100N}{m}\pm2\%$ — $0.99\,\frac{kg\cdot m}{N}$ ± 3.2 %
7. $d=\left(\frac{5.00}{cm^2g}\right)(Mt^3)$, $M=30.0g\pm2\%$, $L=(20.3\pm0.2)cm$ — $1.25\times10^6\,cm$ ± 3.6 %
8. $z=x^2+y^2$, $x=3.0cm\pm2\%$, $y=4.0cm\pm2\%$ — $25\,cm^2$ ± 2.9 %
9. $z=\frac{5a^3-(2cm)b^2}{C}$, $a=2.0cm\pm1\%$, $b=3.0cm\pm1\%$, $C=11.0cm\pm2\%$ — $2.0\,cm^2$ ± 5.9 %
10. $h=d\sin\theta$, $d=1.00m\pm0.05m$, $\theta=10^\circ\pm1^\circ$ — $0.17m$ ± 11 %

Hint: Convert 1° to radians

1) $A=xy$, $x=3.0cm\pm0.1cm$, $y=4.0cm\pm0.1cm$

$$\frac{\delta A}{A}=\sqrt{\frac{\left(\frac{\delta A}{\delta x}\delta x\right)^2}{(A)^2}+\frac{\left(\frac{\delta A}{\delta y}\delta y\right)^2}{(A)^2}}$$

$$\frac{\delta A}{\delta x}=\frac{dA}{dx}\Big|_{y\,constant}=\frac{d}{dx}xy=y$$

$$\frac{\delta A}{\delta y}=\frac{dA}{dy}\Big|_{x\,constant}=\frac{d}{dy}xy=x$$

$$\frac{\delta A}{A}=\sqrt{\left(\frac{y\,\delta x}{xy}\right)^2+\left(\frac{x\,\delta y}{xy}\right)^2}=\sqrt{\left(\frac{\delta x}{x}\right)^2+\left(\frac{\delta y}{y}\right)^2}$$

$$=\sqrt{\left(\frac{0.1}{3}\right)^2+\left(\frac{0.1}{4}\right)^2}=\sqrt{.0011+.000625}=0.041\overline{6}653$$

$$=0.041\overline{66}\times100\%=\boxed{4.2\%}$$

2) $f=x+y$, $x=3.0cm\pm0.1cm$

$y=4.0cm\pm0.1cm$

$y = 4.0\,cm \pm 0.1\,cm$

$$\frac{\delta f}{f} = \sqrt{\frac{\left(\frac{\delta f}{\delta x}\delta x\right)^2}{(f)^2} + \frac{\left(\frac{\delta f}{dy}+y\right)^2}{(f)^2}}$$

$$\frac{\delta f}{\delta x} = \frac{df}{dx}\Big|_{y\,const.} = \frac{d}{dx}x+y=1$$

$$\frac{\delta f}{dy} = \frac{df}{dy}\Big|_{x\,const.} = \frac{d}{dy}x+y=1$$

$$\frac{\delta f}{f} = \sqrt{\left(\frac{\delta x}{x+y}\right)^2 + \left(\frac{\delta y}{x+y}\right)^2} = \sqrt{\left(\frac{0.1}{3.0+4.0}\right)^2 + \left(\frac{0.1}{3.0+4.0}\right)^2}$$

$$= \sqrt{0.000408} = 0.020199 = \boxed{2.0\%}$$

3) $f = x - y$

$x = 3.0\,cm \pm 0.1\,cm$

$y = 4.0\,cm \pm 0.1\,cm$

$$\frac{\delta f}{f} = \sqrt{\left(\frac{\frac{\delta f}{\delta x}\delta x}{f}\right)^2 + \left(\frac{\frac{\delta f}{\delta y}\delta y}{f}\right)^2}$$

$$\frac{\delta f}{\delta x} = \frac{df}{dx}\Big|_{y\,const.} = \frac{d}{dx}x-y=1 \quad (y\ const.)$$

$$x_{const} = \frac{\delta f}{\delta y} = \frac{df}{dy}\Big|_{x\,const.} = \frac{d}{dy}x-y=-1$$

$x_{const} = \frac{\delta x}{\delta y} = \frac{dx}{dy}\Big|_{x_{const.}} = \frac{d}{dy}\, x - y = -1$

$$\frac{\delta f}{f} = \sqrt{\left(\frac{\delta x}{x-y}\right)^2 + \left(\frac{-\delta y}{x-y}\right)^2}$$

$$= \sqrt{\left(\frac{0.1}{3-4}\right)^2 + \left(\frac{-0.1}{3-4}\right)^2}$$

$$= \sqrt{0.01+0.01} = 0.14142 = \boxed{14\%}$$

4) $z = 3x + 2y$

$x = 3.0\,cm \pm 0.1\,cm$

$y = 4.0\,cm \pm 0.1\,cm$

$$\frac{\delta z}{z} = \sqrt{\left(\frac{\frac{\delta z}{\delta x}\,\delta x}{z}\right)^2 + \left(\frac{\frac{\delta z}{\delta y}\,\delta y}{z}\right)^2}$$

$y_{const} = \frac{\delta z}{\delta x} = \frac{dz}{dx}\Big|_{y_{const}} = \frac{d}{dx}(3x+2y) = 3$

$$x_{const} = \frac{\delta z}{\delta y} = \frac{dz}{dy}\Big|_{x_{const}} = \frac{d}{dy}(3x+2y) = 2$$

$$= \frac{\delta z}{z} = \sqrt{\left(\frac{3\delta x}{3x+2y}\right)^2 + \left(\frac{2\delta y}{3x+2y}\right)^2}$$

$$= \sqrt{0.000311419 + 0.000138408} =$$

$$= \sqrt{0.000449827} = 0,021209 = \boxed{2.1\%}$$

5) $g = \frac{2h}{t^2}$

$h = 2,00m \pm 3\%$

$t = 0,630s \pm 4\%$

$g = \frac{2h}{t^2} \Rightarrow t^{-2}2h$

$h = 2,00m \pm 0,06m$

$t = 0,630 \pm 0,0252$

$$1 - 0.6\omega - 0.\overline{20}$$

$$\frac{\delta g}{g} = \sqrt{\left(\frac{\frac{\delta g}{\delta h}\delta h}{g}\right)^2 + \left(\frac{\frac{\delta g}{\delta t}\delta t}{g}\right)^2}$$

$$t_{const} = \frac{\delta g}{\delta h} = \frac{dg}{dh}\bigg|_{t\,const} = \frac{d}{dh} t^{-2} 2h = \frac{2}{t^2}$$

$$h_{const} = \frac{\delta g}{\delta t} = \frac{dg}{dt}\bigg|_{h\,const} = \frac{d}{dt} t^{-2} 2h = \frac{-4h}{t^3}$$

$$\frac{dg}{g} = \sqrt{\left(\frac{\frac{2}{t^2}dh}{\frac{2h}{t^2}}\right)^2 + \left(\frac{\frac{-4h}{t^3}dt}{\frac{2h}{t^2}}\right)^2}$$

$$= \sqrt{\left(\frac{\frac{2(0.06)}{(0.630)^2}}{\frac{2(2)}{(0.630)^2}}\right)^2 + \left(\frac{\frac{-4(2)(0.02)}{(0.630)^2}}{\frac{(2)(2)}{(0.630)^2}}\right)^2}$$

$$= \sqrt{\left(\frac{0.12}{4}\right)^2 + \left(\frac{-0.2016}{2.52}\right)^2} = \sqrt{0.0004 + 0.0064}$$

$$= 0.08544 = \boxed{8.5\%}$$