Induction Machines

Pinjia Zhang

Induction Motors

- Induction Motor Construction
- Basic Induction Motor Concepts
  - Induced Torque
  - Slip
  - Electrical Frequency on the Rotor
- Equivalent Circuit of an Induction Motor
- Induction Motor Equivalent Circuit Model Parameters
  - DC Test
  - No-Load Test
  - Locked-Rotor Test

Induction Motors

- Power and Torque in Induction Motor
- Torque-Speed Characteristics
- Torque-Speed Curve Variation
- Starting Induction Motor
- Speed Control of Induction Motor
- Solid-State Induction Motor Drives
- Induction Generator

The AC machine operation – rotor structure

- Three-phase windings installed on ***stator*** to generate ***rotating magnetic field*** (magnetomotive force) in ***air-gap*** between stator and rotor.
- ***Synchronous Machine***
  - If the rotor is a ***magnetic material*** and has a magnetic field, the rotor magnetic will follow the air-gap magnetic – ***the synchronous machine operation theory***
- ***Induction Machine***
  - If the rotor is only a ***conductor*** and has not any own magnetic field, the voltage will be induced on rotor conductor by the moving air-gap flux crossing stationary rotor conductor.
  - Finally, the magnetic field induced by rotor will also follow the air-gap magnetic field – ***the induction machine operation theory***

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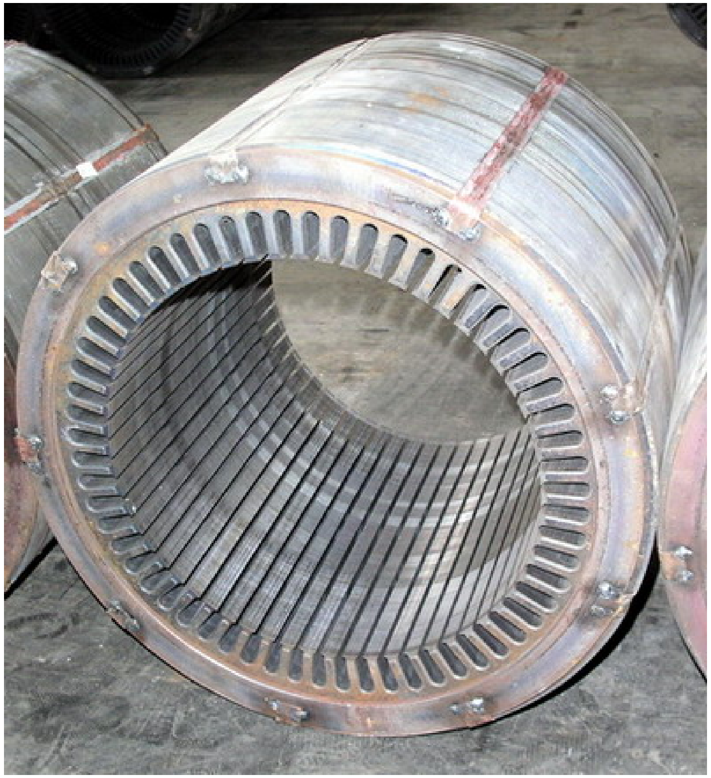
Induction motor construction

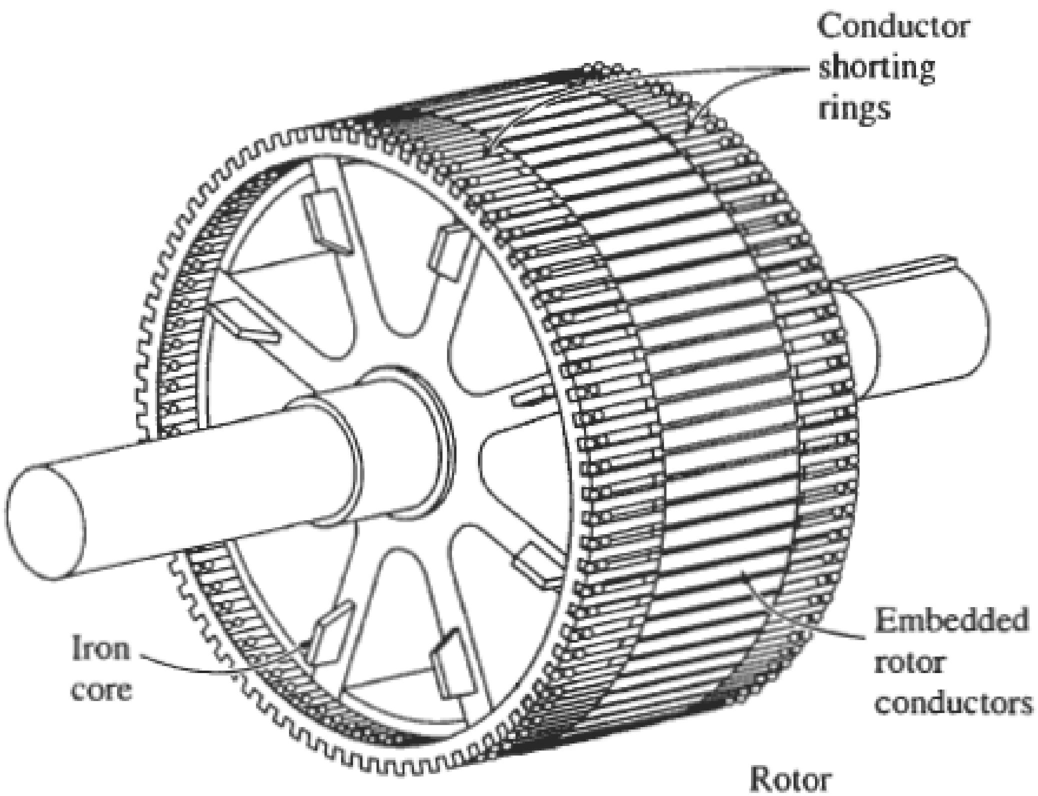
- The stator construction is the same as that of synchronous – all AC machine has the same stator construction
- There are two types of rotor conductor constructions in induction machine
  - Squirrel-cage rotor or cage rotor
    - The rotor is constructed by only **conductor bar**
  - Wound rotor
    - The rotor is constructed by **fully three phase windings**

Typical induction motor stator construction – three phase winding



Three phase currents on three phase windings to produce rotating magnetic field in air gap

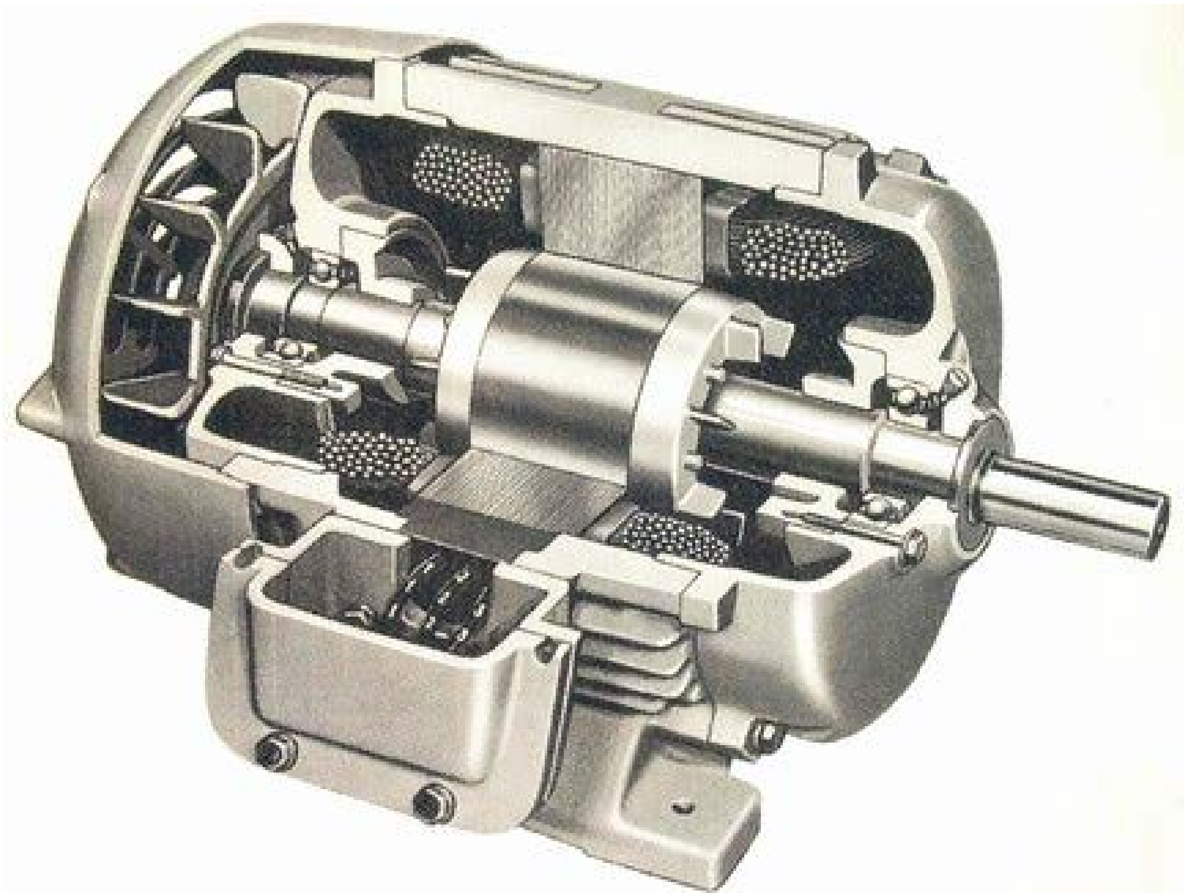
Squirrel-cage rotor construction



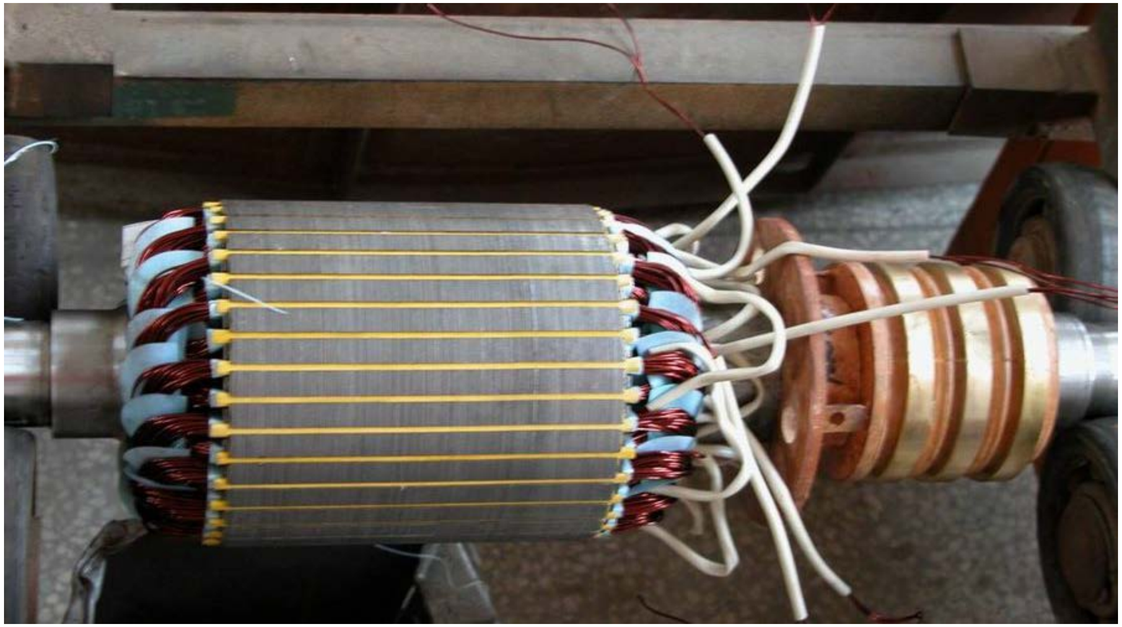
Conductor bars are short circuit by conductor shorting rings to flow the induced current

Squirrel-cage rotor construction

Cutaway diagram of Squirrel-cage induction motor construction



Typical wound rotors for induction motors



Three phase windings on rotor are usually Y-connected, and the ends are tied to stator via the slip rings on rotor's shaft

The three phase windings are short circuit by connection three-phase windings on stator

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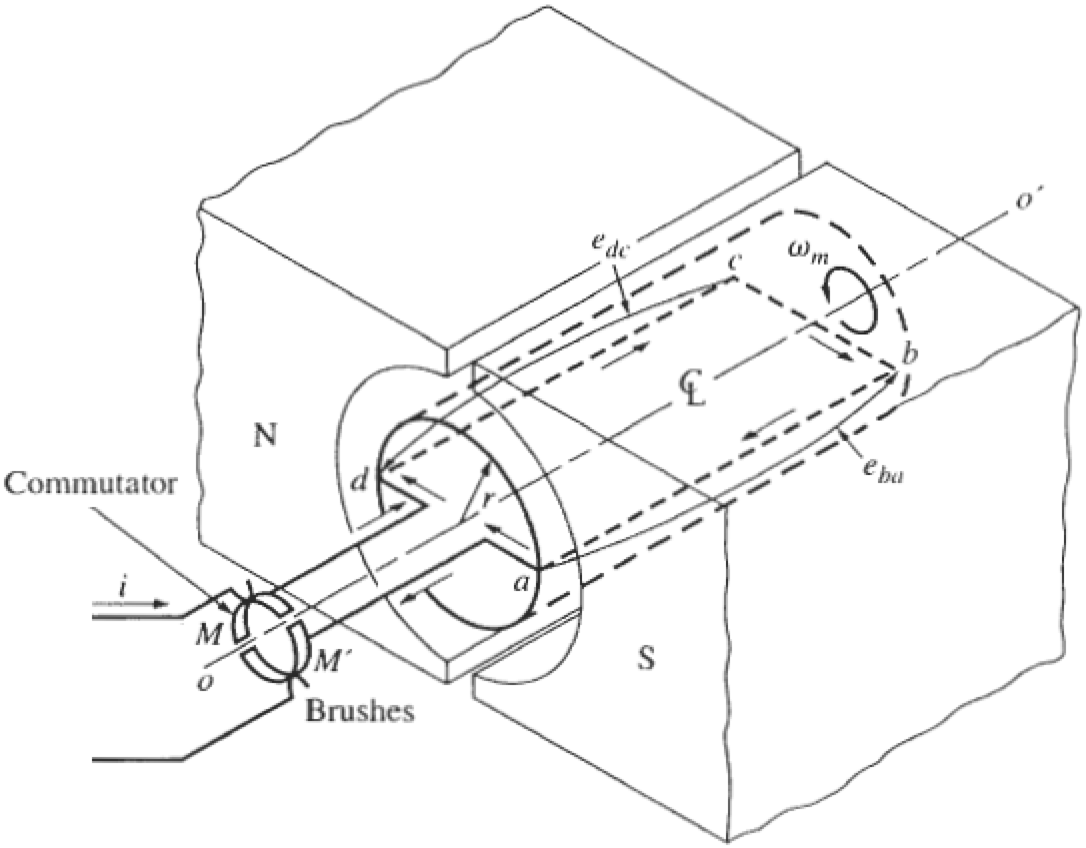
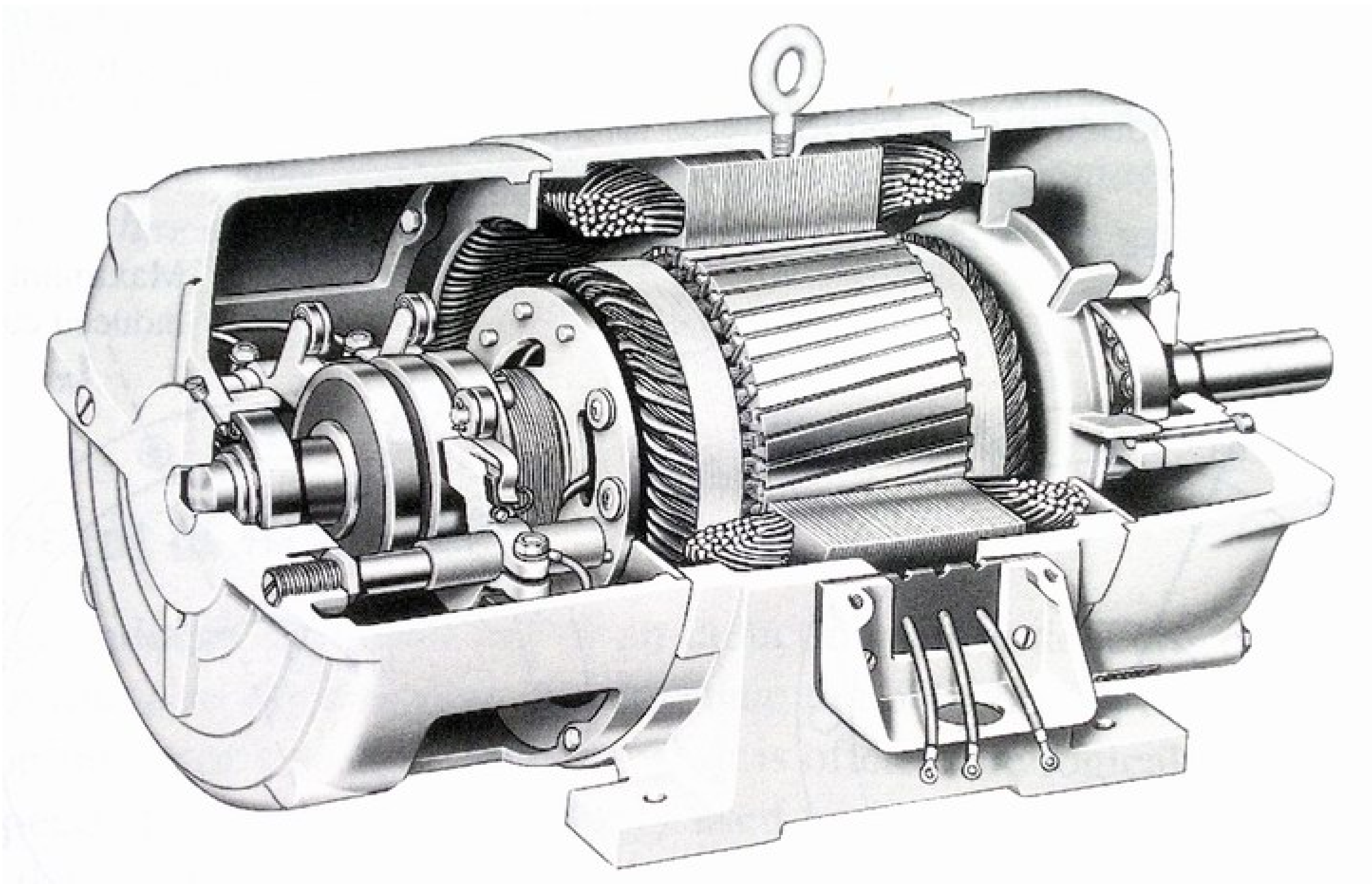
Slip ring commutator



Cutaway diagram of a wound-rotor induction motor



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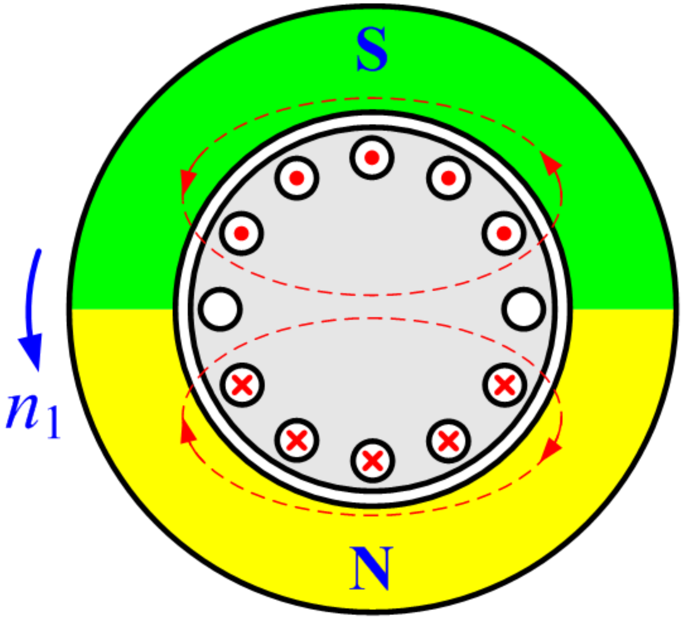
Comparison between cage and wound rotor

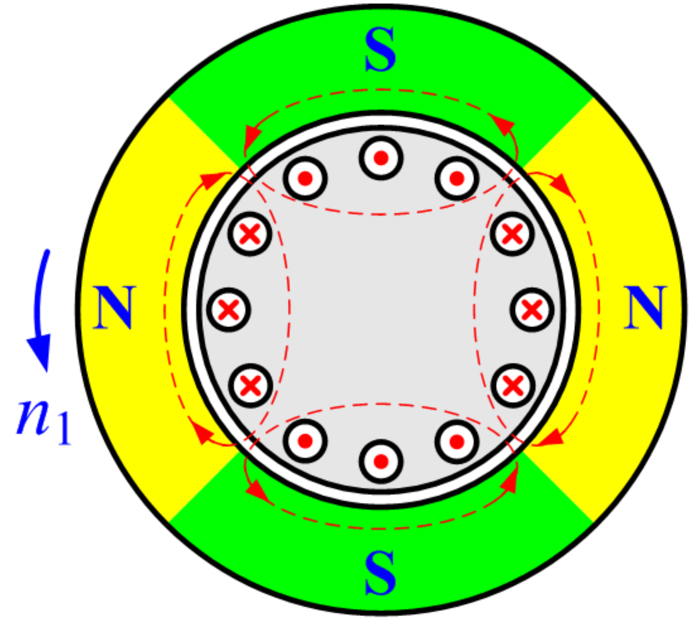
- The cage type induction motors are more smaller, harder and easier maintenance .
- The wound type induction motors are easier control
  - The rotor resistance can vary the torque-speed curve
- Due to the maintenance of brushes and slip rings, the wound-rotor induction motors are rarely used in practical applications.
- However, the wound-motors are usually used in lab for research.
  - Slip power recovery drive

Introduction of induction motor operation

Squirrel cage rotor

- The number of poles depend on the stator





- It forms a multi-phase system

$$m_2 = Q_2 \qquad N_2 = \frac{1}{2}, \quad k_{dp2} = 1$$

Basic induction motor concepts – induced torque on cage rotor bar

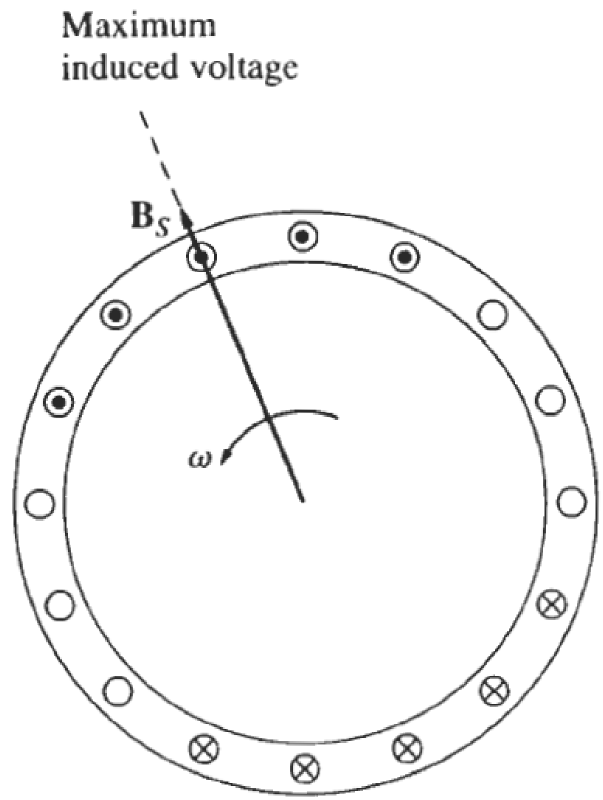
- The rotation magnetic field produced by stator windings

$$\boxed{n_{sync} = \frac{120 f_e}{P}}$$

- The magnetic field pass over the rotor bar will induce voltage

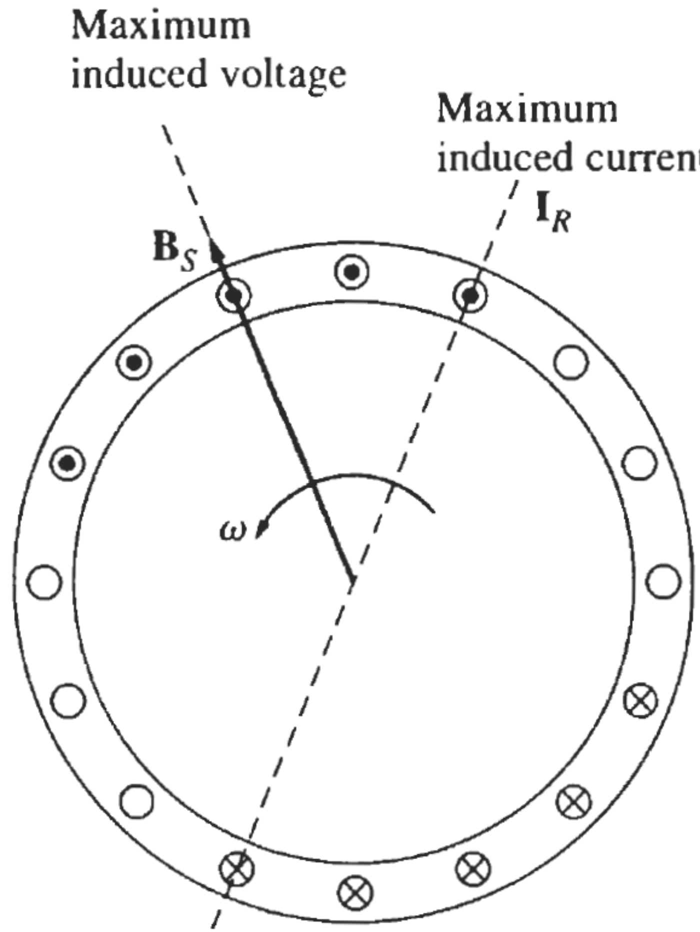
$$e_{\text{ind}} = (\mathbf{v} \times \mathbf{B}) \bullet \mathbf{l}$$

  - Notably, v is the velocity of the bar **relative** to the **magnetic field**



Induced current due to inductive rotor loop

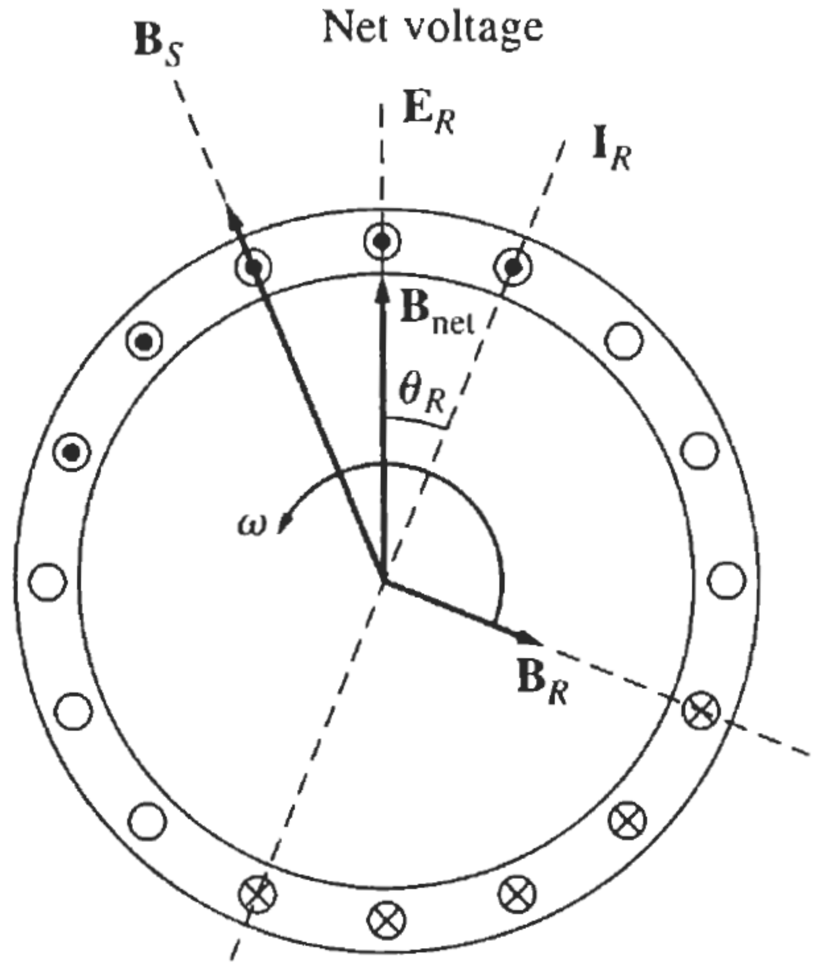
- The induced current in rotor loop is lag behind induced voltage, due to inductive assembly of rotor.
- The rotor current I_R will further produce rotor flux density B_R.



Induced torque by rotor and stator flux density – Finite upper speed limitation

- The induced torque in the machine is given by

  $\tau_{\text{ind}} = k\mathbf{B}_R \times \mathbf{B}_S$
- The induced torque is counterclockwise (CCW)
- ***Finite upper limit of rotor speed***
  - Can the rotor speed ω_R equals to the speed of B_S (synchronous speed) ?
  - The rotor speed ω_R must be less than the speed of B_S (Or $E_R = I_R = B_R = 0$)



Speed of Induction motor

- The rotor speed ω_R must be less than the synchronous speed to induce voltage on it.
- Actually, since the I_R is proportional to the induced torque (or load torque), the speed difference (slip speed) between B_S and ω_R will become larger when larger load.

- ***Notably, both the speed of B_S and B_R are synchronous speed !!***

Concept of rotor slip

- The voltage induced in a rotor bar of an IM depends on the speed of the rotor ***relative to the magnetic field***.
- Two terms used to define the relative motion of the rotor and the magnetic fields
  - ***Slip speed***

$$\boxed{n_{\text{slip}} = n_{\text{sync}} - n_m}$$

n_{slip} = slip speed of the machine
n_{sync} = speed of the magnetic fields
n_m = mechanical shaft speed of motor

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Slip and mechanical speed expression

- ***Slip*** – percentage slip speed
  - ***s = 0, the rotor is at synchronous speed***
  - ***s = 1, the rotor is at stationary***

$$s = \frac{n_{\text{slip}}}{n_{\text{sync}}} (\times\ 100\%)$$

$$\boxed{s = \frac{n_{\text{sync}} - n_m}{n_{\text{sync}}} (\times\ 100\%)}$$

$$\boxed{s = \frac{\omega_{\text{sync}} - \omega_m}{\omega_{\text{sync}}} (\times\ 100\%)}$$

- The mechanical speed also can be expressed via slip and synchronous speed

$$\boxed{n_m = (1 - s) n_{\text{sync}}}$$

$$\boxed{\omega_m = (1 - s) \omega_{\text{sync}}}$$

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Electrical frequency on rotor circuits

- Why B_R and B_S are both at synchronous speed ?
- The answer can be found via rotor frequency
- The induction machine is sometimes called the ***rotating transformer***
  - However, the frequency at rotor circuits **is not equal to** the frequency at stator
- ***Rotor frequency***
  - If the rotor is locked, the IM is just the transformer, $\omega_s = \omega_r$
  - If the rotor is at synchronous speed, $\omega_r = 0$
  - The frequency on rotor circuit is

$$\boxed{f_r = sf_e}$$

$$f_r = \frac{n_{\text{sync}} - n_m}{n_{\text{sync}}} f_e$$

$$f_r = (n_{\text{sync}} - n_m) \frac{P}{120 f_e} f_e$$

$$\boxed{f_r = \frac{P}{120}(n_{\text{sync}} - n_m)}$$

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Example 7-1

Example 7–1. A 208-V, 10-hp, four-pole, 60-Hz, Y-connected induction motor has a full-load slip of 5 percent.

(a) What is the synchronous speed of this motor?
(b) What is the rotor speed of this motor at the rated load?
(c) What is the rotor frequency of this motor at the rated load?
(d) What is the shaft torque of this motor at the rated load?

Solution

(a) The synchronous speed of this motor is

$$n_{\text{sync}} = \frac{120 f_e}{P}$$

$$= \frac{120(60\text{ Hz})}{4\text{ poles}} = 1800\text{ r/min}$$

(b) The rotor speed of the motor is given by

$$n_m = (1 - s)n_{\text{sync}} \tag{7–6}$$

$$= (1 - 0.95)(1800\text{ r/min}) = 1710\text{ r/min}$$

(c) The rotor frequency of this motor is given by

$$f_r = sf_e = (0.05)(60\text{ Hz}) = 3\text{ Hz} \tag{7–8}$$

Alternatively, the frequency can be found from Equation (7–9):

$$f_r = \frac{P}{120}(n_{\text{sync}} - n_{\text{m}}) \tag{7–9}$$

$$= \frac{4}{120}(1800\text{ r/min} - 1710\text{ r/min}) = 3\text{ Hz}$$

(*d*) The shaft load torque is given by

$$\tau_{\text{load}} = \frac{P_{\text{out}}}{\omega_m}$$

$$= \frac{(10 \text{ hp})(746 \text{ W/hp})}{(1710 \text{ r/min})(2\pi \text{ rad/r})(1 \text{ min/60 s})} = 41.7 \text{ N} \bullet \text{m}$$

The shaft load torque in English units is given by Equation (1–17):

$$\tau_{\text{load}} = \frac{5252P}{n}$$

where τ is in pound-feet, P is in horsepower, and n_m is in revolutions per minute. Therefore,

$$\tau_{\text{load}} = \frac{5252(10 \text{ hp})}{1710 \text{ r/min}} = 30.7 \text{ lb} \bullet \text{ft}$$

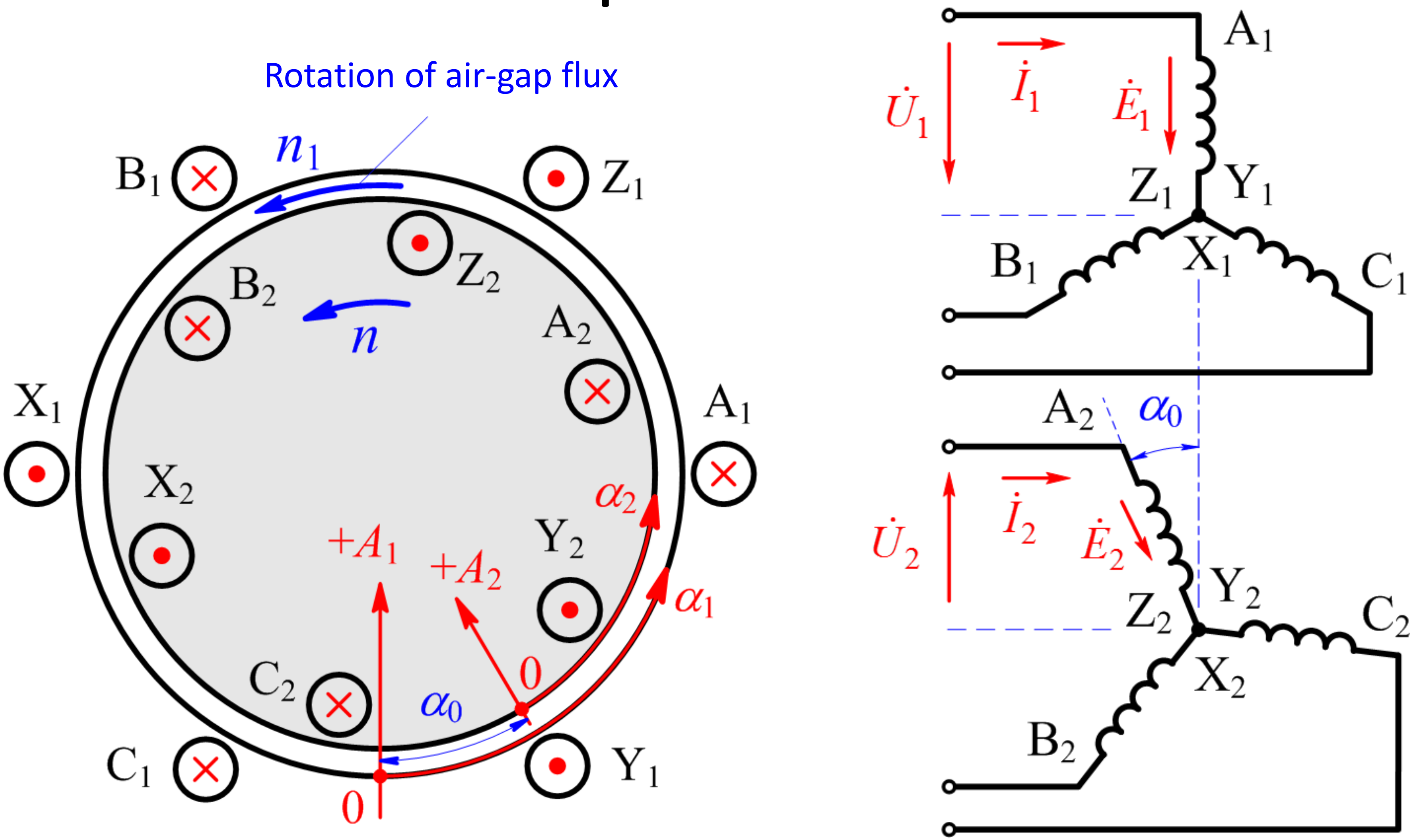
Frequency in induction machine

- B_S is at synchronous speed
- Rotor is at the speed less than synchronous speed
- Rotor circuit is at the slip speed (frequency)
- B_R = rotor rotation speed ω_r + electrical speed (frequency) in rotor circuit = synchronous speed

Fundamentals of induction motor operation

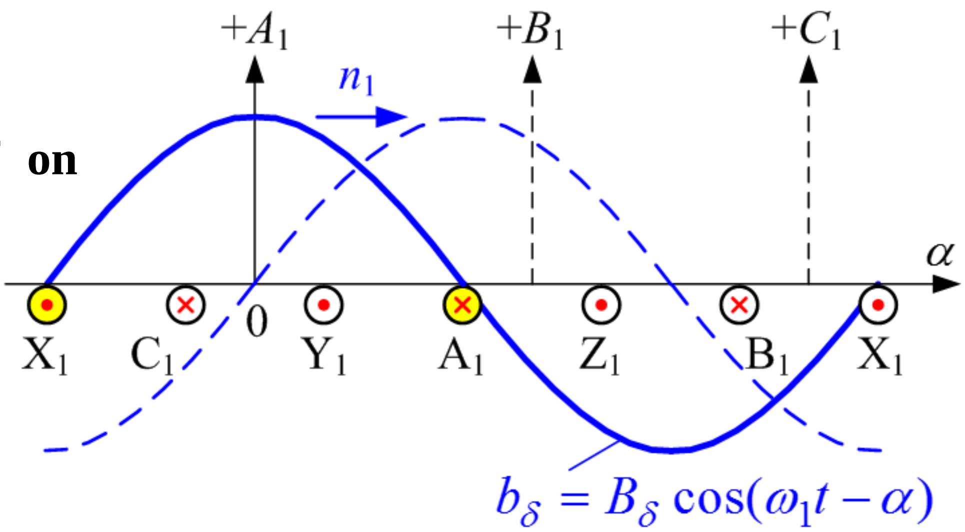
Locked-rotor, rotor open circuit operation

Locked-rotor, rotor open circuit operation



Locked-rotor, rotor open circuit operation

- **Rotating magnetic flux induces emf on both stator and rotor:**



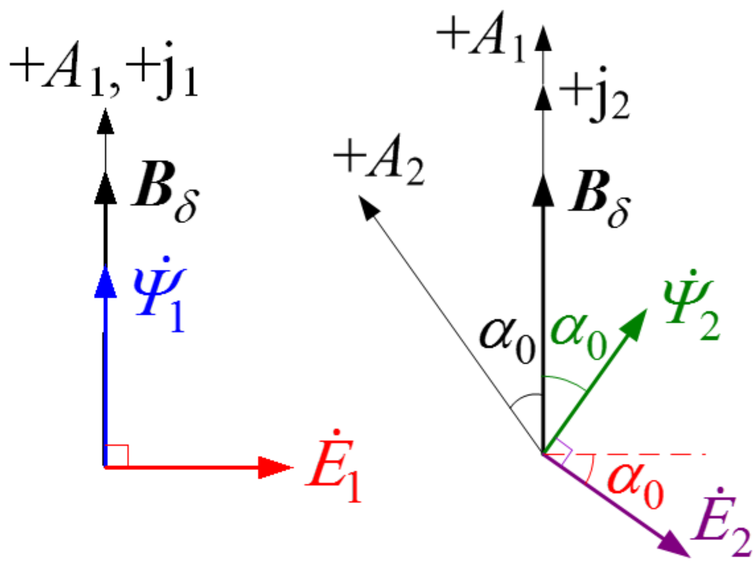
$$E_{1m} = \omega_1 \Psi_{m1} = \omega_1 N_1 k_{dp1} \Phi_m$$

$$E_{2m} = \omega_1 \Psi_{m2} = \omega_1 N_2 k_{dp2} \Phi_m$$

$$\boxed{k_e = \frac{E_1}{E_2} = \frac{N_1 k_{dp1}}{N_2 k_{dp2}}}$$

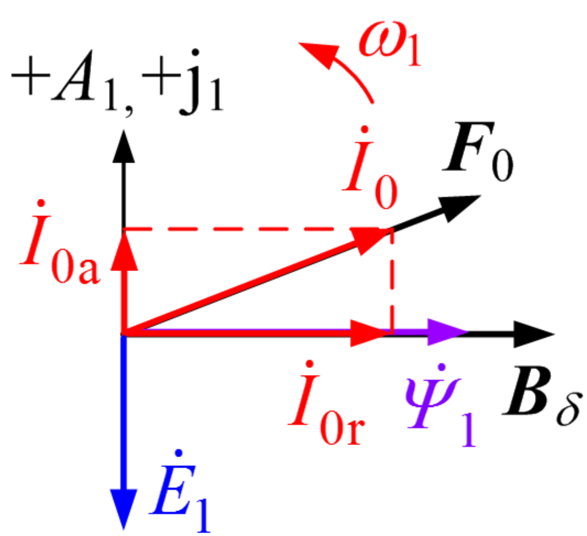
Locked-rotor, rotor open circuit operation

- Due to space location, stator and rotor emf have a phase shift



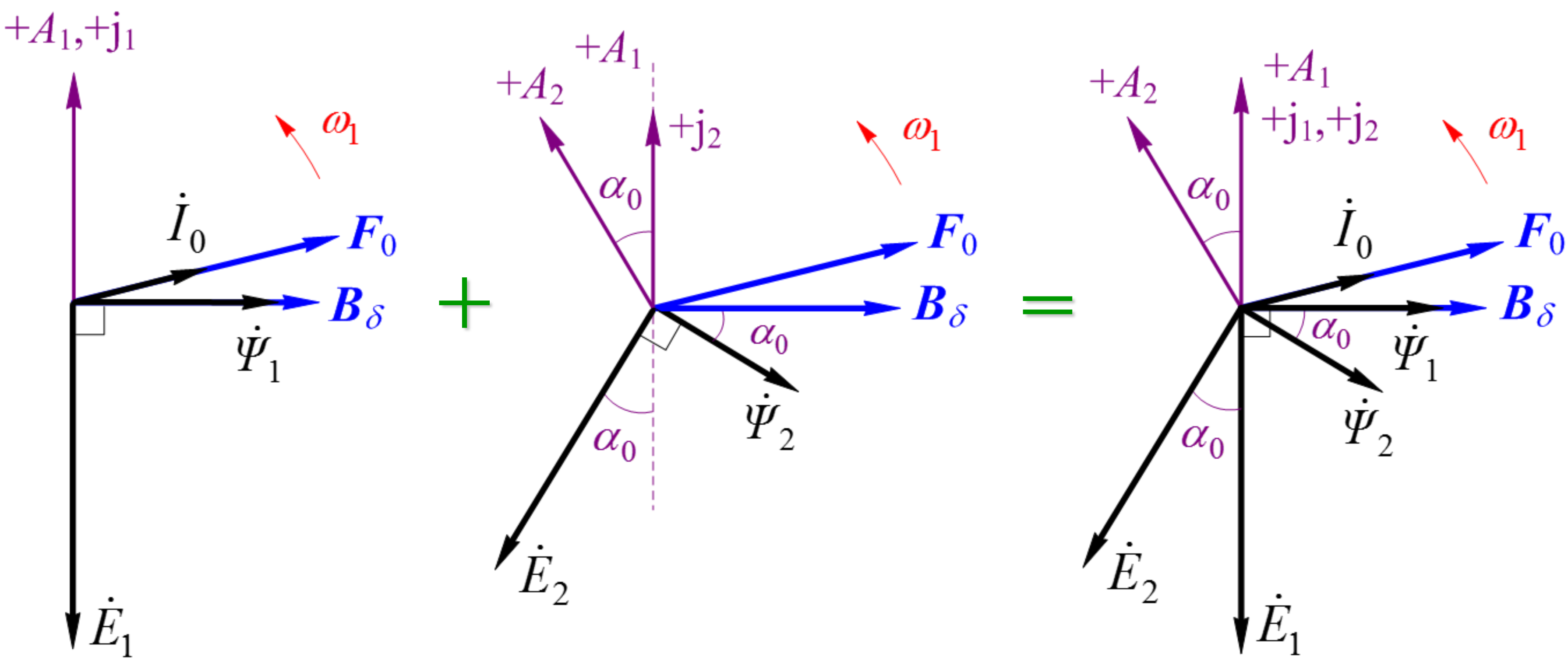
Locked-rotor, rotor open circuit operation

- Magnetizing current
  - Due to iron losses, mmf leads flux by a small angle

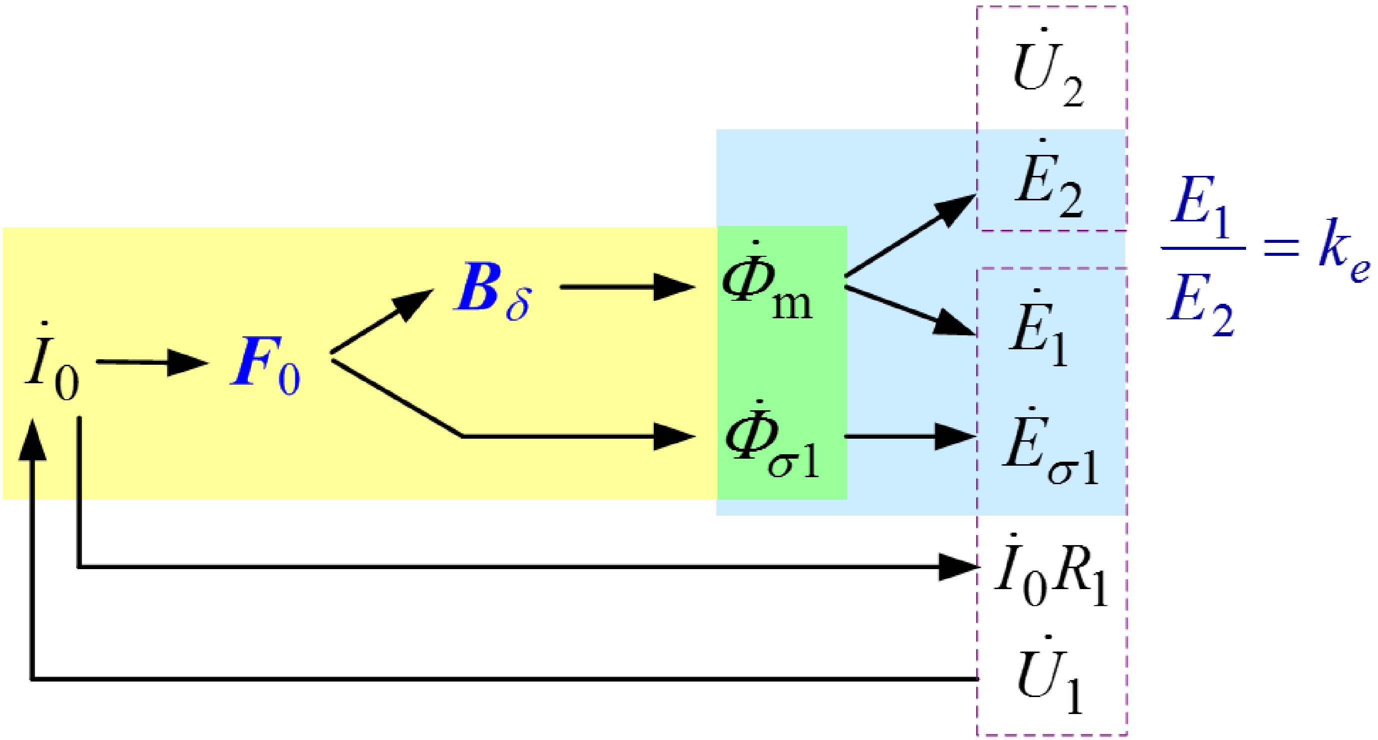


Phasor diagram

stator **rotor** **overall**



Locked-rotor, rotor open circuit operation



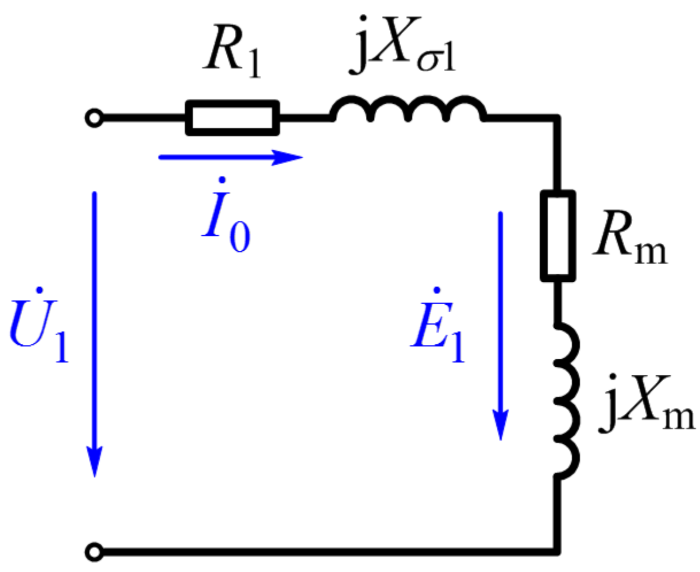
Like a transformer!

Locked-rotor, rotor open circuit operation

- Equivalent circuit

$$\dot{U}_1 = \dot{I}_0 Z_m + \dot{I}_0 Z_1 = \dot{I}_0 (Z_1 + Z_m)$$

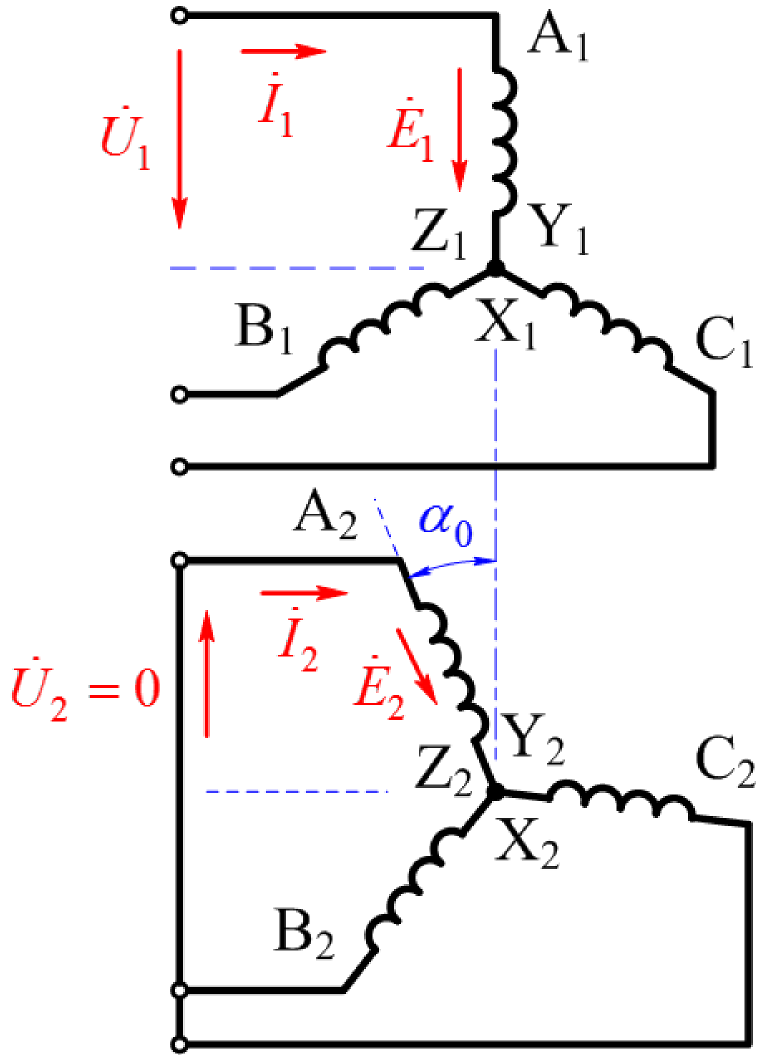
$Z_1 = R_1 + jX_{\sigma 1}$ $\qquad$ $Z_m = R_m + jX_m$



$$\dot{U}_2 = \dot{E}_2$$

Locked-rotor, rotor short circuit operation

Locked-rotor, rotor short circuit operation



Like a transformer!

Locked-rotor, rotor short circuit operation

Stator & rotor mmf (rotate at the same speed)

$$F_1 = \frac{m_1}{2}\frac{4}{\pi}\frac{\sqrt{2}}{2}\frac{N_1 I_1}{p}k_{\mathrm{dp1}} \qquad F_2 = \frac{m_2}{2}\frac{4}{\pi}\frac{\sqrt{2}}{2}\frac{N_2 I_2}{p}k_{\mathrm{dp2}}$$

$$\boldsymbol{F}_0 = \boldsymbol{F}_1 + \boldsymbol{F}_2$$

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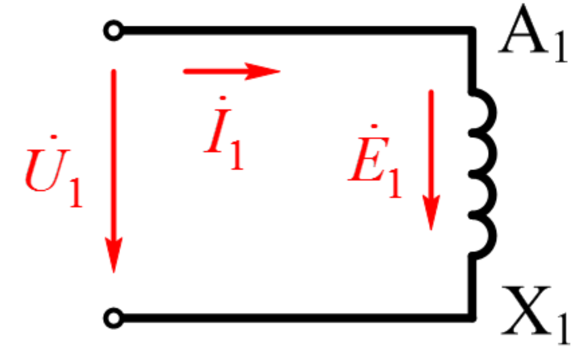
Locked-rotor, rotor short circuit operation

- Stator

$$\dot{U}_1 = -\dot{E}_1 - \dot{E}_{\sigma 1} + \dot{I}_1 R_1$$

$$\dot{E}_{\sigma 1} = -\mathrm{j}\dot{I}_1 X_{\sigma 1}$$

$$\dot{U}_1 = -\dot{E}_1 + \dot{I}_1(R_1 + \mathrm{j}X_{\sigma 1}) = -\dot{E}_1 + \dot{I}_1 Z_1$$

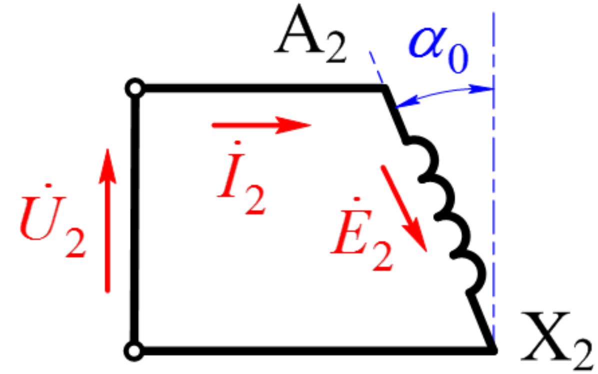


- Rotor

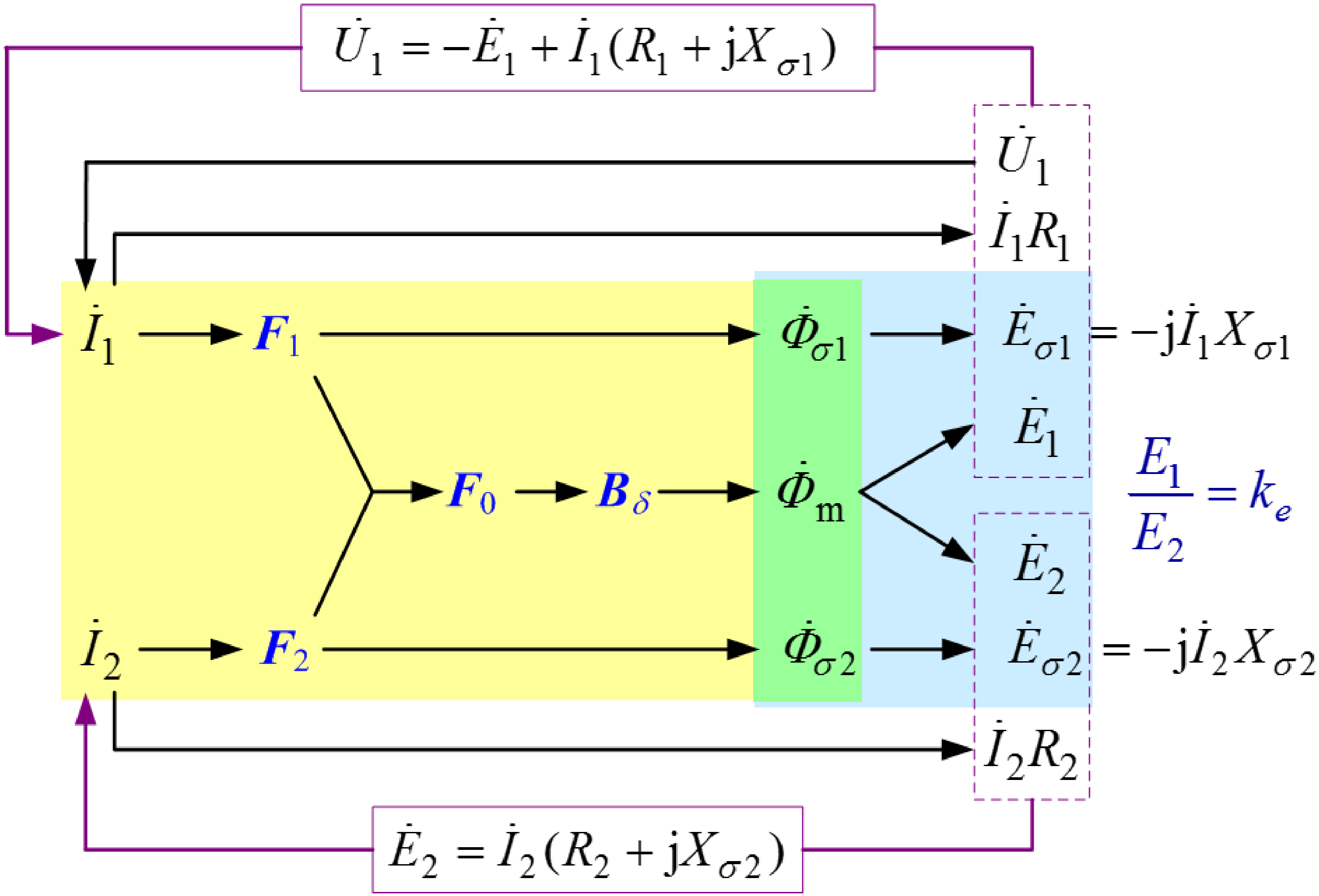
$$\dot{E}_2 + \dot{E}_{\sigma 2} = \dot{I}_2 R_2$$

$$\dot{E}_{\sigma 2} = -\mathrm{j}\dot{I}_2 X_{\sigma 2}$$

$$\dot{E}_2 = \dot{I}_2(R_2 + \mathrm{j}X_{\sigma 2}) = \dot{I}_2 Z_2$$



Locked-rotor, rotor short circuit operation



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Locked-rotor, rotor short circuit operation

Parameter conversion

$$k_e = \frac{E_1}{E_2} = \frac{N_1 k_{\mathrm{dp1}}}{N_2 k_{\mathrm{dp2}}}$$

$$\dot{E}_2' = \frac{N_1 k_{\mathrm{dp1}}}{N_2 k_{\mathrm{dp2}}} \dot{E}_2 = k_e \dot{E}_2$$

$$\dot{I}_2' = \frac{m_2 N_2 k_{\mathrm{dp2}}}{m_1 N_1 k_{\mathrm{dp1}}} \dot{I}_2 = \frac{1}{k_i} \dot{I}_2$$

$$k_i = \frac{m_1 N_1 k_{\mathrm{dp1}}}{m_2 N_2 k_{\mathrm{dp2}}} = \frac{m_1}{m_2} k_e$$

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Locked-rotor, rotor short circuit operation

Parameter conversion

$$Z_2' = \frac{\dot{E}_2'}{\dot{I}_2'} = \frac{k_e \dot{E}_2}{\frac{1}{k_i}\dot{I}_2} = k_e k_i \frac{\dot{E}_2}{\dot{I}_2} = k_e k_i Z_2$$

$$R_2' = k_e k_i R_2 \quad , \quad X'_{\sigma 2} = k_e k_i X_{\sigma 2}$$

$$F_1 = K I_1 \text{，} F_0 = K I_0 \text{，} F_2 = K I_2'$$

$$\left(K = \frac{m_1}{2} \frac{4}{\pi} \frac{\sqrt{2}}{2} \frac{N_1 k_{\mathrm{dp1}}}{p} \right)$$

$$\dot{I}_1 = \dot{I}_0 + (-\dot{I}_2')$$

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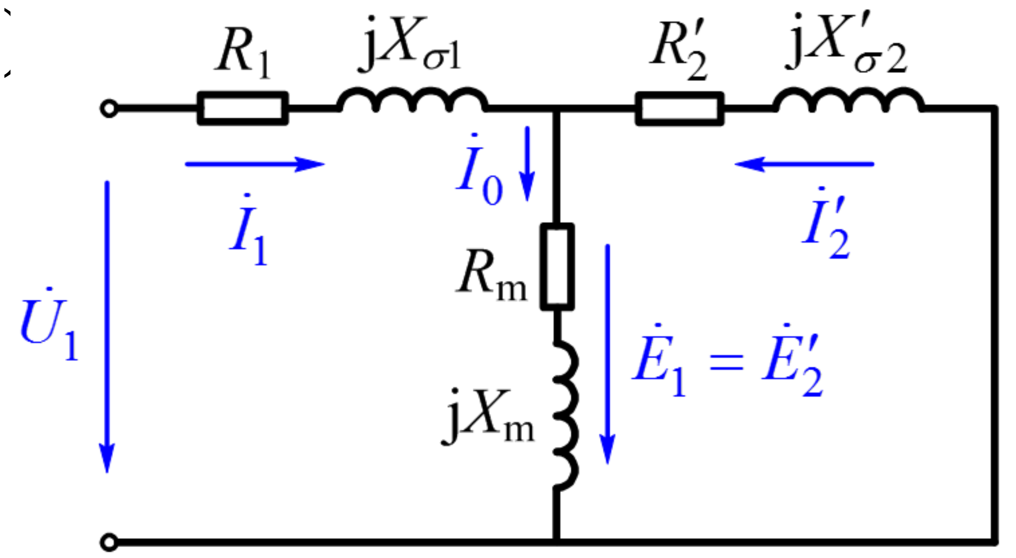
Locked-rotor, rotor short circuit operation

- Equivalent circuit

$$
\begin{cases}
\dot{U}_1 = -\dot{E}_1 + \dot{I}_1(R_1 + \mathrm{j}X_{\sigma 1}) \\
\dot{E}'_2 = \dot{I}'_2(R'_2 + \mathrm{j}X'_{\sigma 2}) \\
\dot{E}_1 = \dot{E}'_2 \\
\dot{E}_1 = -\dot{I}_0(R_\mathrm{m} + \mathrm{j}X_\mathrm{m}) \\
\dot{I}_1 + \dot{I}'_2 = \dot{I}_0
\end{cases}
$$



The same as a transformer!

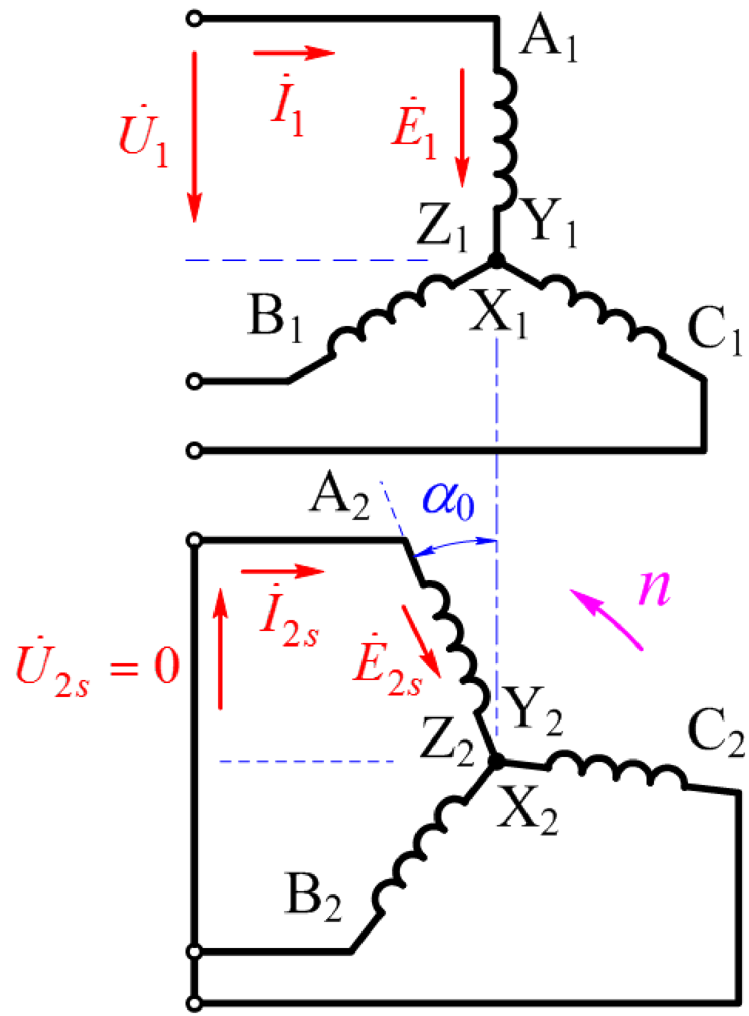
Rotating rotor, rotor short circuit operation

Rotating rotor, rotor short circuit operation

- slip

$$s = \frac{n_1 - n}{n_1}$$

$$F_1 = \frac{m_1}{2} \frac{4}{\pi} \frac{\sqrt{2}}{2} \frac{N_1 I_1}{p} k_{\mathrm{dp1}}$$



Rotating rotor, rotor short circuit operation

Rotor sees a rotation speed of

$$f_2 = \frac{p(n_1 - n)}{60} = \frac{pn_1}{60}\frac{n_1 - n}{n_1} = sf_1$$

$$F_2 = \frac{m_2}{2}\frac{4}{\pi}\frac{\sqrt{2}}{2}\frac{N_2 \boldsymbol{I}_{2s}}{p}k_{\mathrm{dp}2}$$

Rotate at n_2 with respect to the rotor

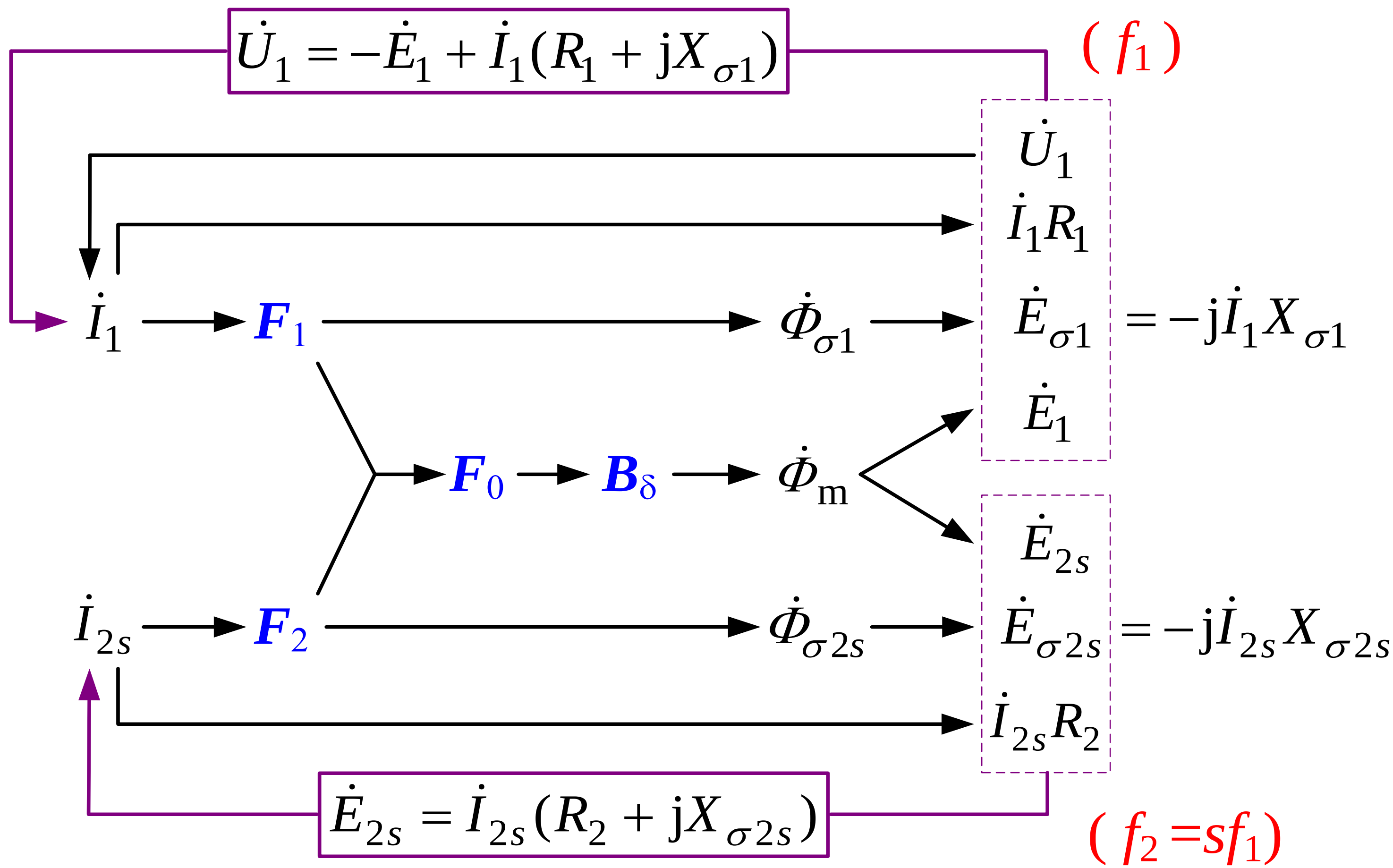
$$n_2 = \frac{60\,f_2}{p} = s\frac{60\,f_1}{p} = sn_1$$

$$n_2 + n = sn_1 + (1 - s)n_1 = n_1$$

Rotating at the same speed as F_1

$$\boldsymbol{F}_0 = \boldsymbol{F}_1 + \boldsymbol{F}_2$$

Rotating rotor, rotor short circuit operation



Rotating rotor, rotor short circuit operation

- Rotor electrical circuit

rotor emf

$$\dot{E}_{2s} + \dot{E}_{\sigma 2s} = \dot{I}_{2s} R_2 \longrightarrow \boxed{\dot{E}_{2s} = \dot{I}_{2s}(R_2 + \mathrm{j}X_{\sigma 2s})}$$

$$E_{2s} = 4.44 f_2 N_2 k_{\mathrm{dp2}} \Phi_{\mathrm{m}} = 4.44 s f_1 N_2 k_{\mathrm{dp2}} \Phi_{\mathrm{m}}$$

$$\boxed{E_{2s} = sE_2}$$

rotor leakage reactance

$$X_{\sigma 2s} = 2\pi f_2 L_{\sigma 2} = 2\pi s f_1 L_{\sigma 2} \longrightarrow \boxed{X_{\sigma 2s} = sX_{\sigma 2}}$$

Rotating rotor, rotor short circuit operation

- Rotor electrical circuit

$$\dot{E}_{2s} = \dot{I}_{2s}(R_2 + \mathrm{j}X_{\sigma 2s})$$

$$\dot{E}_2 = \dot{I}_2(\frac{R_2}{s} + \mathrm{j}X_{\sigma 2})$$

$$I_2 = I_{2s}$$

$$\varphi_2 = \arctan\frac{X_{\sigma 2}}{R_2 / s} = \arctan\frac{X_{\sigma 2s}}{R_2}$$

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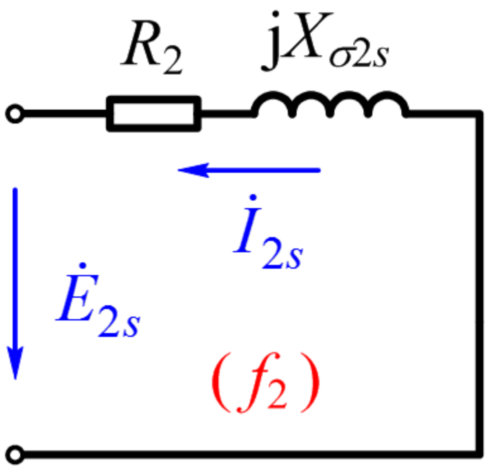
Rotating rotor, rotor short circuit operation

$$\dot{E}_{2s} = \dot{I}_{2s}(R_2 + \mathrm{j}X_{\sigma 2s})$$

$$\dot{E}_2 = \dot{I}_2(\frac{R_2}{s} + \mathrm{j}X_{\sigma 2})$$



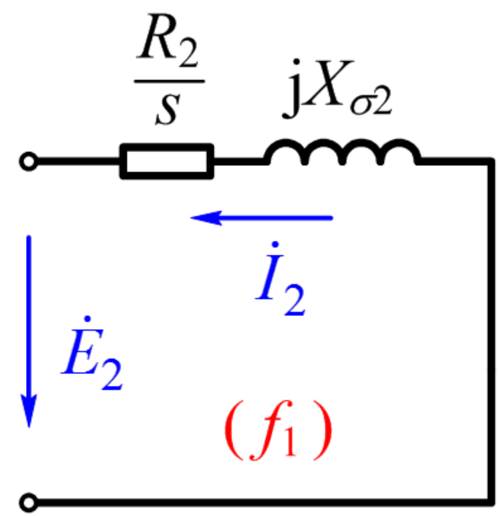
rotating

$$\dot{E}_{2s} \xrightarrow{E_2 = \frac{E_{2s}}{s}} \dot{E}_2$$

$$R_2 \longrightarrow \frac{R_2}{s}$$

$$X_{\sigma 2s} \xrightarrow{X_{\sigma 2} = \frac{X_{\sigma 2s}}{s}} X_{\sigma 2}$$

$$\dot{I}_{2s} \xrightarrow{I_2 = I_{2s}} \dot{I}_2$$



Locked-rotor

Rotating rotor, rotor short circuit operation

- Equivalent circuit

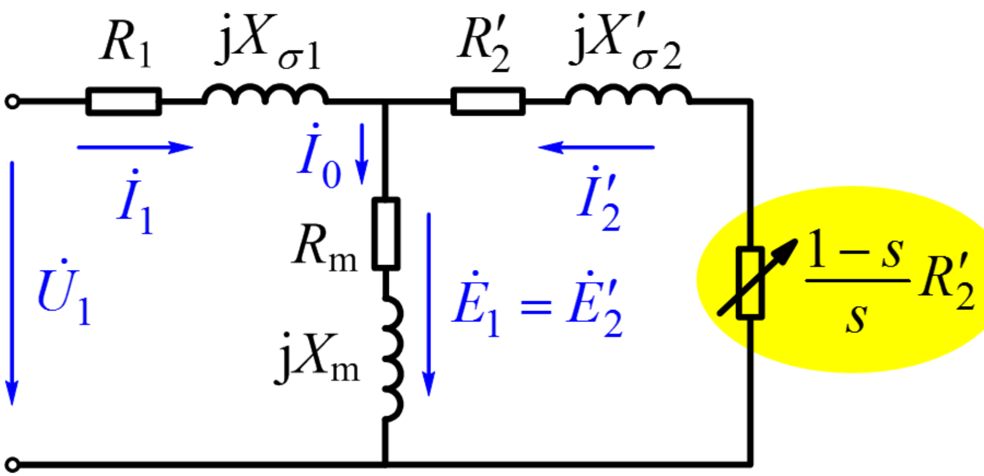
$$\dot{U}_1 = -\dot{E}_1 + \dot{I}_1(R_1 + \mathrm{j}X_{\sigma 1})$$

$$\dot{E}_2' = \dot{I}_2'(\frac{R_2'}{s} + \mathrm{j}X'_{\sigma 2})$$

$$\dot{E}_1 = \dot{E}_2'$$

$$\dot{E}_1 = -\dot{I}_0(R_\mathrm{m} + \mathrm{j}X_\mathrm{m})$$

$$\dot{I}_1 + \dot{I}_2' = \dot{I}_0$$



Rotating rotor, rotor short circuit operation

Conversion table

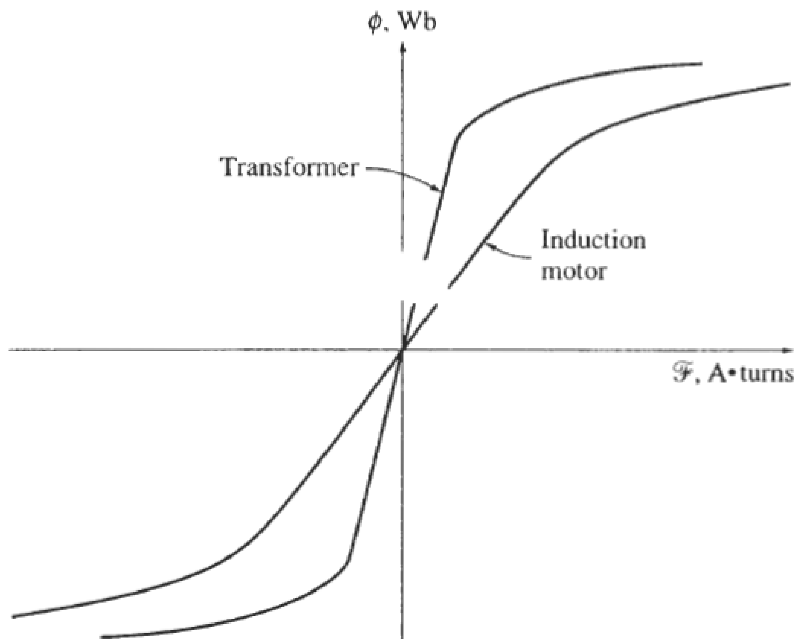
Actual value	Frequency conversion		Winding conversion	
	Conversion ratio	Converted value	Conversion ratio	Converted value
f_2	f_2/s	f_1	$\times 1$	f_1
$\dot{E}_{2s}$	E_{2s}/s	$\dot{E}_2$	$\times k_e$	$\dot{E}_2'$
R_2	R_2/s	R_2/s	$\times k_e k_i$	R_2'/s
$X_{\sigma 2s}=\omega_2 L_{\sigma 2}$	$X_{\sigma 2s}/s$	$X_{\sigma 2}$	$\times k_e k_i$	$X_{\sigma 2}'$
$\dot{I}_{2s}$	$I_{2s}=I_2$	$\dot{I}_2$	$\times 1/k_i$	$\dot{I}_2'$

$$k_e = \frac{E_1}{E_2} = \frac{N_1 k_{\mathrm{dp1}}}{N_2 k_{\mathrm{dp2}}}$$

$$k_i = \frac{m_1 N_1 k_{\mathrm{dp1}}}{m_2 N_2 k_{\mathrm{dp2}}} = \frac{m_1}{m_2} k_e$$

Magnetizing curve of induction machine

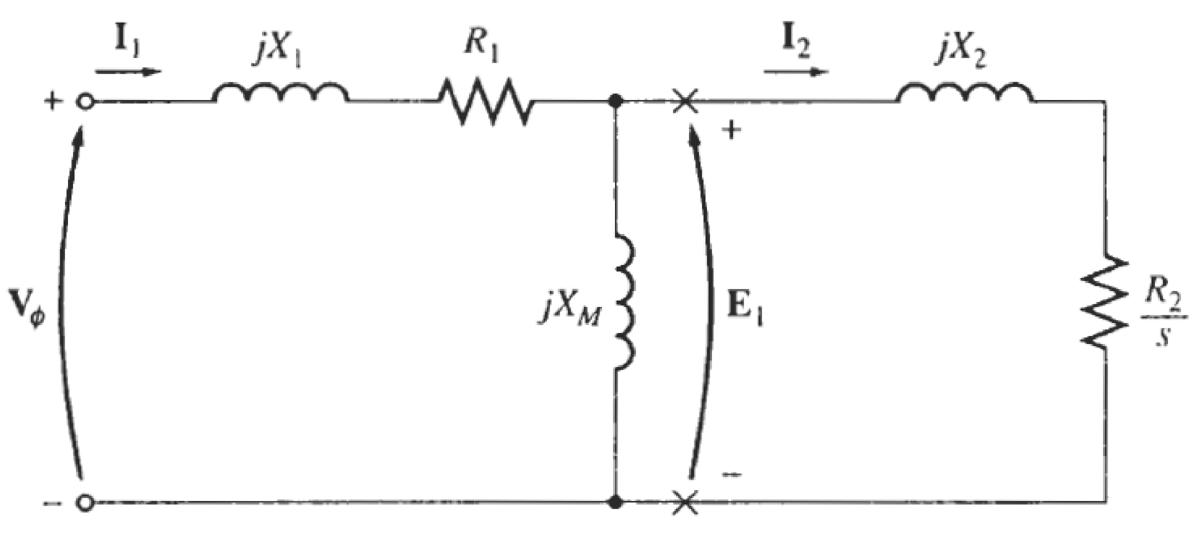
- The slop in magnetizing curve is smaller than that of transformer, due to the air-gap between stator and rotor
- Thus, much higher magnetizing current is necessary



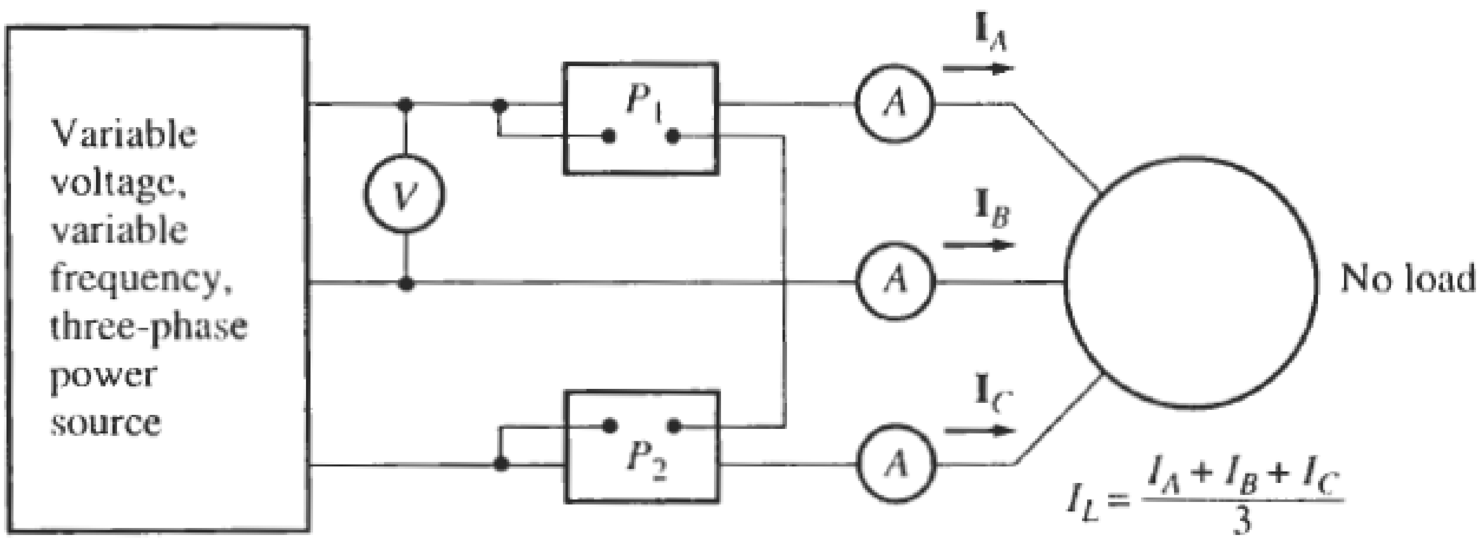
Determining circuit model parameters

Determining circuit model parameters

- No-load test – open circuit test for Xfmr
- Dc test for stator resistance
- Locked-rotor test– short circuit test for Xfmr

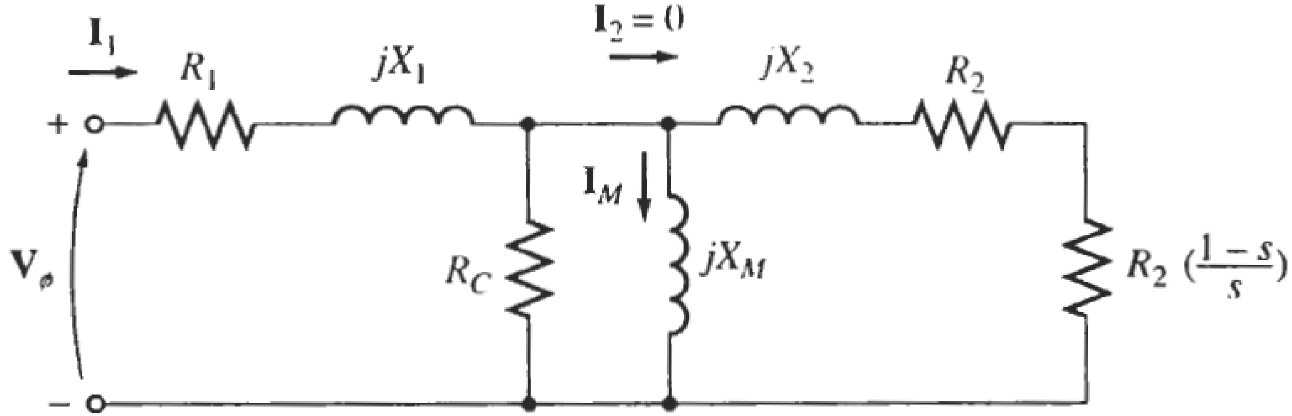


No-Load Test

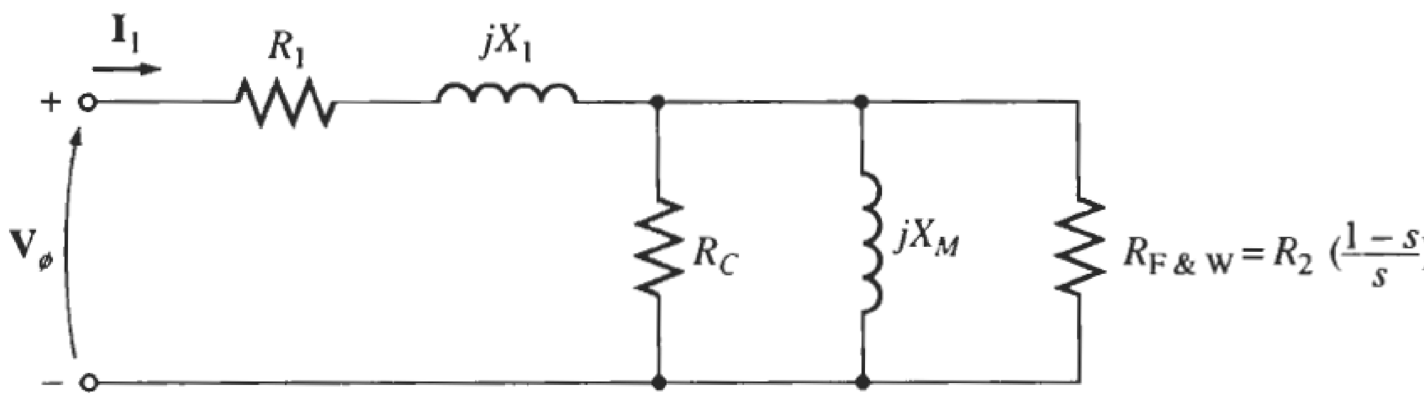


Variable voltage, variable frequency, three-phase power source
V
P_1
P_2
A
A
A
$\mathbf{I}_A$
$\mathbf{I}_B$
$\mathbf{I}_C$
No load
$I_L = \dfrac{I_A + I_B + I_C}{3}$

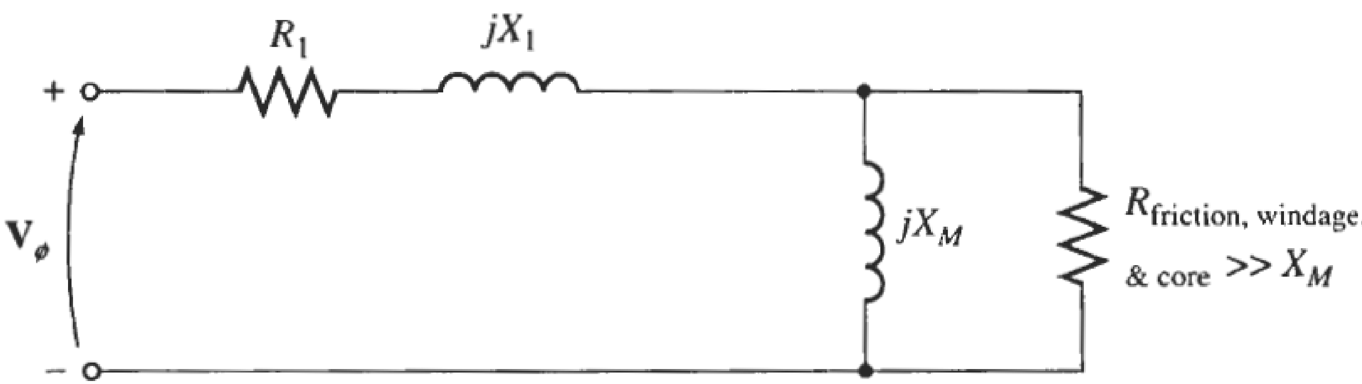
Initial equivalent circuit:



Since $R_2\ (\frac{1-s}{s}) >> R_2$ and $R_2\ (\frac{1-s}{s}) >> X_2$, this circuit reduces to:



Combining $R_{F\,\&\,W}$ and R_C yields:



No-Load Test

- The rotor copper losses are negligible because the current I_2 is extremely small
- The input power P_{in} is

$$
\begin{aligned}
P_{\text{in}} &= P_{\text{SCL}} + P_{\text{core}} + P_{\text{F\&W}} + P_{\text{misc}} \\
&= 3I_1^2 R_1 + P_{\text{rot}}
\end{aligned}
$$

$$\boxed{P_{\text{SCL}} = 3I_1^2 R_1}$$

$$P_{\text{rot}} = P_{\text{core}} + P_{\text{F\&W}} + P_{\text{misc}}$$

$$\boxed{\left|Z_{\text{eq}}\right| = \frac{V_\phi}{I_{1,\text{nl}}} \approx X_1 + X_M}$$

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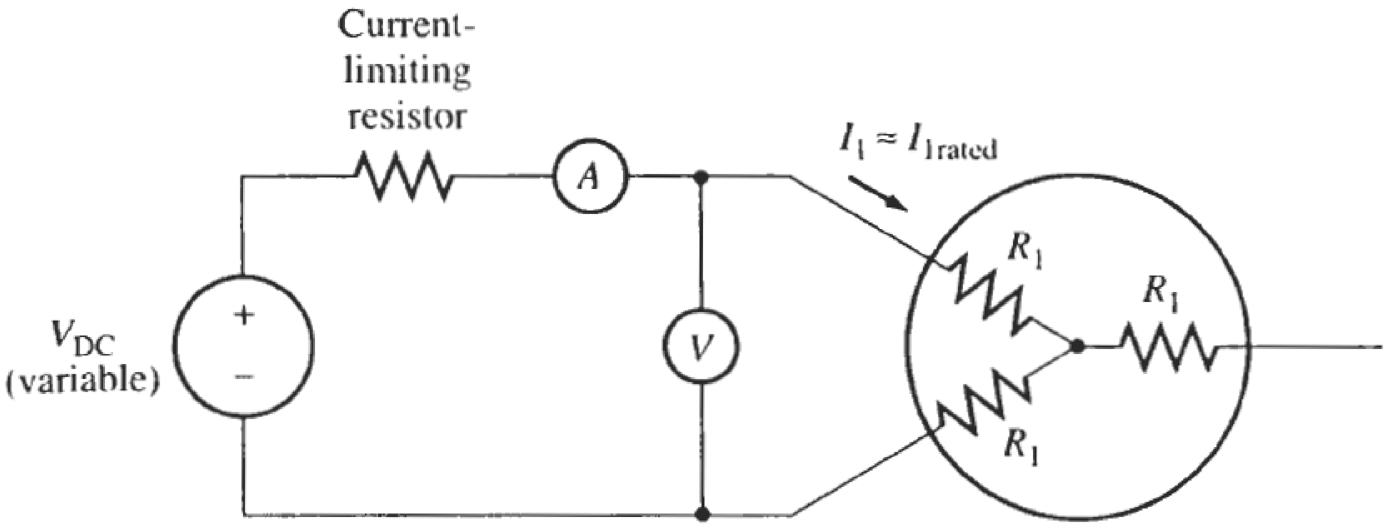
DC Test for Stator Resistance



$$2R_1 = \frac{V_{DC}}{I_{DC}}$$

$$\boxed{R_1 = \frac{V_{DC}}{2I_{DC}}}$$

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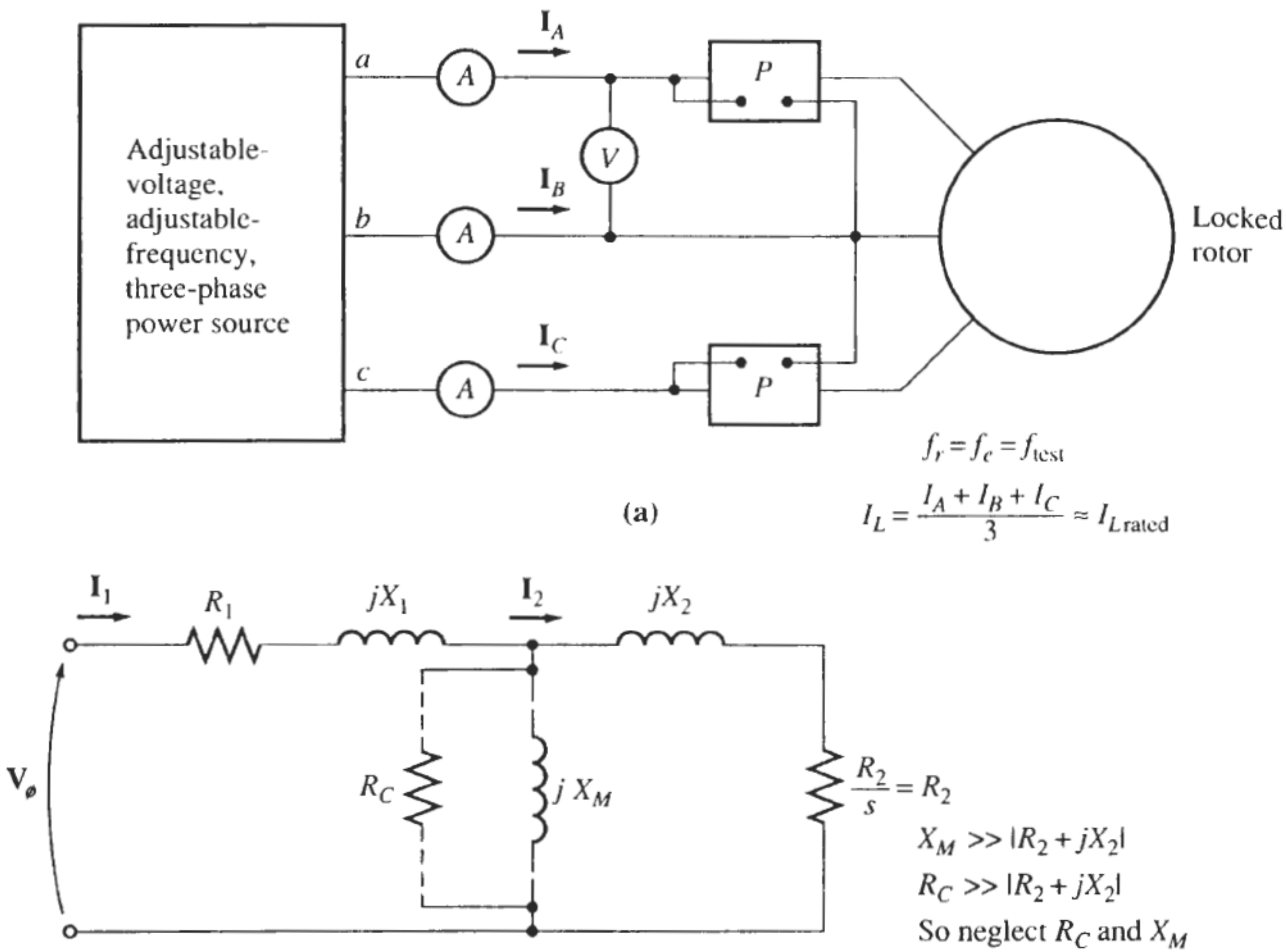
Locked-Rotor Test



Locked-Rotor Test

- Since the rotor frequency is much smaller than the 60Hz (only 2 to 4 percent), lower input frequency is necessary for more accurate solutions. (to reduce the skin effect of resistance)
- Th $P = \sqrt{3} V_T I_L \cos\theta$ ance r $|Z_{LR}| = \dfrac{V_\phi}{I_1} = \dfrac{V_T}{\sqrt{3} I_L}$ ied by frequency

$$PF = \cos\theta = \frac{P_{in}}{\sqrt{3} V_T I_L}$$

$$\begin{aligned} Z_{LR} &= R_{LR} + jX'_{LR} \\ &= |Z_{LR}|\cos\theta + j|Z_{LR}|\sin\theta \end{aligned}$$

Locked-Rotor Test

- The locked-rotor resistance

$$R_{LR} = R_1 + R_2$$

- The locked-rotor reactance

$$X'_{LR} = X'_1 + X'_2$$

- The rotor resistance R_2 is

$$R_2 = R_{LR} - R_1$$

- The rotor reactance X_2 is

$$X_{LR} = \frac{f_{rated}}{f_{test}} X'_{LR} = X_1 + X_2$$

The way to determine X_1 and X_2

	X_1 and X_2 as functions of X_{LR}	
Rotor Design	X_1	X_2
Wound rotor	$0.5\ X_{LR}$	$0.5\ X_{LR}$
Design A	$0.5\ X_{LR}$	$0.5\ X_{LR}$
Design B	$0.4\ X_{LR}$	$0.6\ X_{LR}$
Design C	$0.3\ X_{LR}$	$0.7\ X_{LR}$
Design D	$0.5\ X_{LR}$	$0.5\ X_{LR}$

Example 7-8

Example 7–8. The following test data were taken on a 7.5-hp, four-pole, 208-V, 60-Hz, design A, Y-connected induction motor having a rated current of 28 A.

DC test:

$V_{DC} = 13.6\ \text{V}$ $\qquad I_{DC} = 28.0\ \text{A}$

No-load test:

$V_T = 208\ \text{V}$ $\qquad f = 60\ \text{Hz}$

$I_A = 8.12\ \text{A}$ $\qquad P_{in} = 420\ \text{W}$

$I_B = 8.20\ \text{A}$

$I_C = 8.18\ \text{A}$

Locked-rotor test:

$V_T = 25\ \text{V}$ $\qquad f = 15\ \text{Hz}$

$I_A = 28.1\ \text{A}$ $\qquad P_{in} = 920\ \text{W}$

$I_B = 28.0\ \text{A}$

$I_C = 27.6\ \text{A}$

(a) Sketch the per-phase equivalent circuit for this motor.

Solution

(a) From the dc test,

$$R_1 = \frac{V_{DC}}{2I_{DC}} = \frac{13.6\ \text{V}}{2(28.0\ \text{A})} = 0.243\ \Omega$$

From the no-load test,

$$I_{L,av} = \frac{8.12\ \text{A} + 8.20\ \text{A} + 8.18\ \text{A}}{3} = 8.17\ \text{A}$$

$$V_{\phi,nl} = \frac{208\ \text{V}}{\sqrt{3}} = 120\ \text{V}$$

Therefore,

$$|Z_{nl}| = \frac{120\ \text{V}}{8.17\ \text{A}} = 14.7\ \Omega = X_1 + X_M$$

When X_1 is known, X_M can be found. The stator copper losses are

$$P_{SCL} = 3I_1^2 R_1 = 3(8.17\ \text{A})^2(0.243\ \Omega) = 48.7\ \text{W}$$

Therefore, the no-load rotational losses are

$$\begin{aligned} P_{rot} &= P_{in,nl} - P_{SCL,nl} \\ &= 420\ \text{W} - 48.7\ \text{W} = 371.3\ \text{W} \end{aligned}$$

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From the locked-rotor test,

$$I_{\mathrm{L,av}} = \frac{28.1\ \mathrm{A} + 28.0\ \mathrm{A} + 27.6\ \mathrm{A}}{3} = 27.9\ \mathrm{A}$$

The locked-rotor impedance is

$$|Z_{\mathrm{LR}}| = \frac{V_\phi}{I_A} = \frac{V_T}{\sqrt{3}\,I_A} = \frac{25\ \mathrm{V}}{\sqrt{3}(27.9\ \mathrm{A})} = 0.517\ \Omega$$

and the impedance angle θ is

$$\begin{aligned}\theta &= \cos^{-1}\frac{P_{\mathrm{in}}}{\sqrt{3}V_T I_L} \\ &= \cos^{-1}\frac{920\ \mathrm{W}}{\sqrt{3}(25\ \mathrm{V})(27.9\ \mathrm{A})} \\ &= \cos^{-1} 0.762 = 40.4^\circ\end{aligned}$$

Therefore, $R_{\mathrm{LR}} = 0.517 \cos 40.4^\circ = 0.394\ \Omega = R_1 + R_2$. Since $R_1 = 0.243\ \Omega$, R_2 must be $0.151\ \Omega$. The reactance at 15 Hz is

$$X'_{\mathrm{LR}} = 0.517 \sin 40.4^\circ = 0.335\ \Omega$$

The equivalent reactance at 60 Hz is

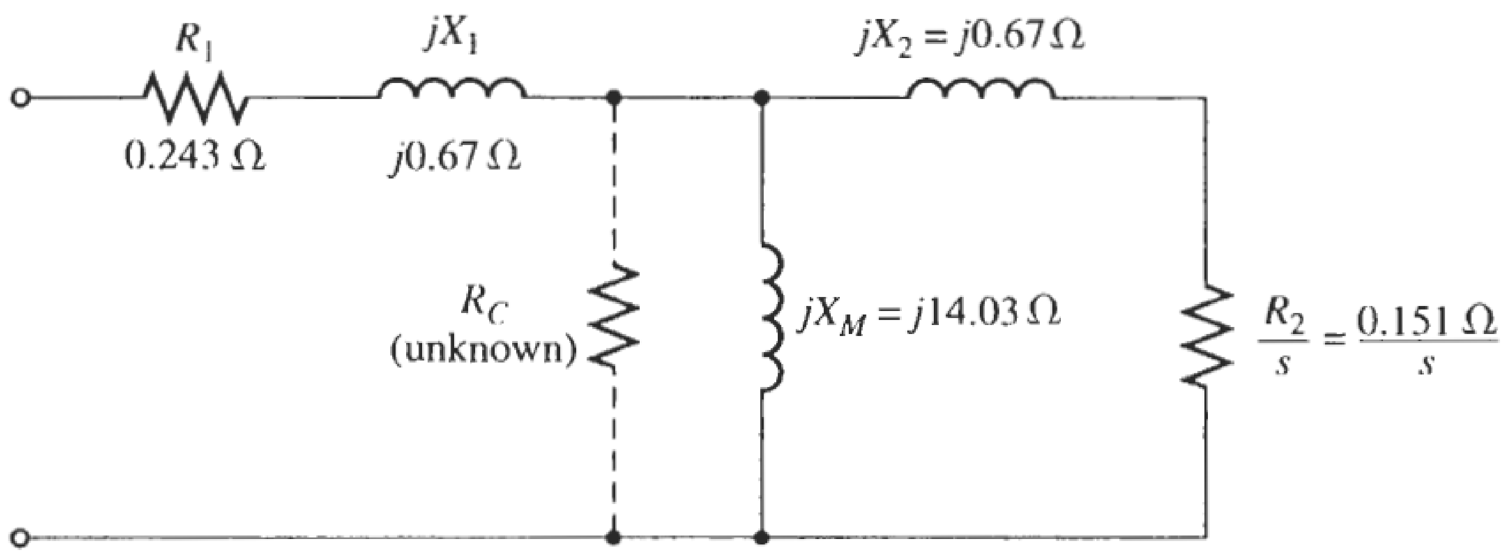
$$X_{\mathrm{LR}} = \frac{f_{\mathrm{rated}}}{f_{\mathrm{test}}} X'_{\mathrm{LR}} = \left(\frac{60\ \mathrm{Hz}}{15\ \mathrm{Hz}}\right) 0.335\ \Omega = 1.34\ \Omega$$

For design class A induction motors, this reactance is assumed to be divid equally between the rotor and stator, so

$$X_1 = X_2 = 0.67\ \Omega$$

$$X_M = |Z_{nl}| - X_1 = 14.7\ \Omega - 0.67\ \Omega = 14.03\ \Omega$$

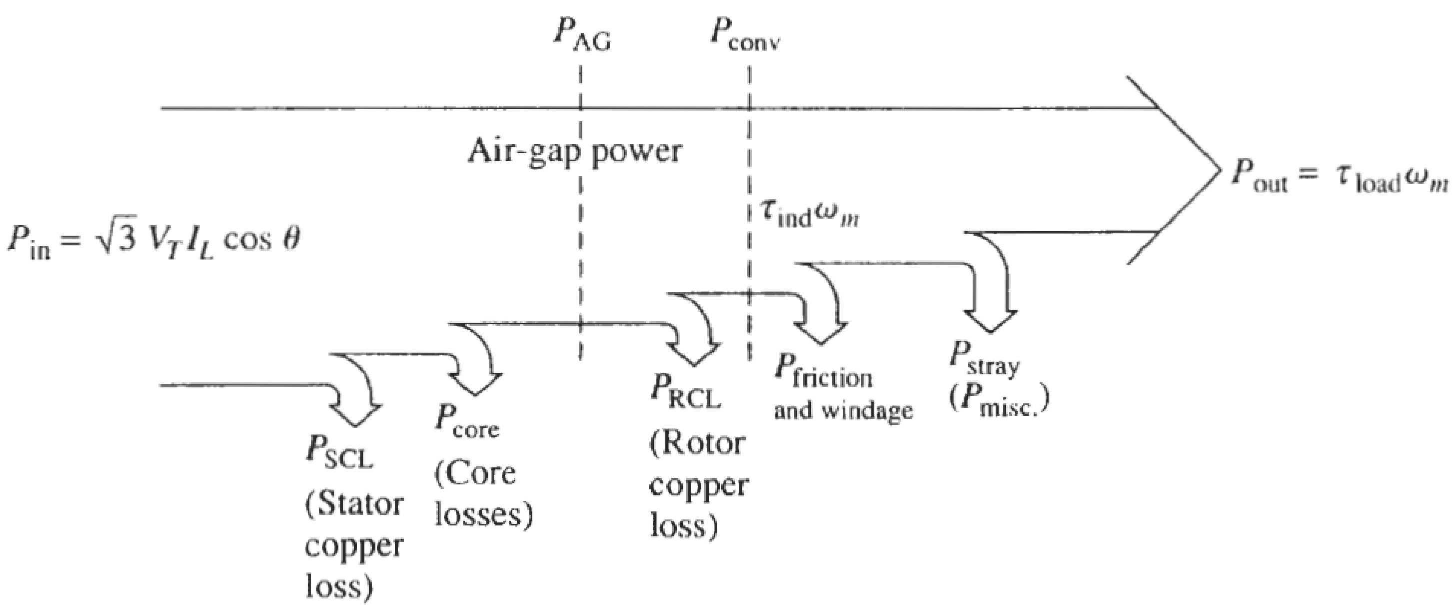
The final per-phase equivalent circuit is shown in Figure 7–56.



Power and torque in induction motors – Loss analysis

Power and torque in induction motors – Loss analysis

- Since the core loss is proportional to the operation frequency, the rotor core loss is much smaller than that of the stator.
- P_{conv} = mechanical power = P_{out} + rotational loss (mechanical loss)
- $P_{AG} = P_{conv}$ + rotor loss



Example 7-2

Example 7–2. A 480-V, 60-Hz, 50-hp, three-phase induction motor is drawing 60 A at 0.85 PF lagging. The stator copper losses are 2 kW, and the rotor copper losses are 700 W. The friction and windage losses are 600 W, the core losses are 1800 W, and the stray losses are negligible. Find the following quantities:

(*a*) The air-gap power P_{AG}
(*b*) The power converted P_{conv}
(*c*) The output power P_{out}
(*d*) The efficiency of the motor

(a) The air-gap power is just the input power minus the stator I^2R losses. The input power is given by

$$P_{in} = \sqrt{3} V_T I_L \cos\theta$$
$$= \sqrt{3}(480\ \text{V})(60\ \text{A})(0.85) = 42.4\ \text{kW}$$

From the power-flow diagram, the air-gap power is given by

$$P_{AG} = P_{in} - P_{SCL} - P_{core}$$
$$= 42.4\ \text{kW} - 2\ \text{kW} - 1.8\ \text{kW} = 38.6\ \text{kW}$$

(b) From the power-flow diagram, the power converted from electrical to mechanical form is

$$P_{conv} = P_{AG} - P_{RCL}$$
$$= 38.6\ \text{kW} - 700\ \text{W} = 37.9\ \text{kW}$$

(c) From the power-flow diagram, the output power is given by

$$P_{out} = P_{conv} - P_{F\&W} - P_{misc}$$
$$= 37.9\ \text{kW} - 600\ \text{W} - 0\ \text{W} = 37.3\ \text{kW}$$

or, in horsepower,

$$P_{out} = (37.3\ \text{kW})\frac{1\ \text{hp}}{0.746\ \text{kW}} = 50\ \text{hp}$$

(d) Therefore, the induction motor's efficiency is

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\%$$

$$= \frac{37.3\ \text{kW}}{42.4\ \text{kW}} \times 100\% = 88\%$$

Power and torque in an induction motor

- From input electric power to output mechanical power
- The input current to a phase circuit

$$\mathbf{I}_1 = \frac{\mathbf{V}_\phi}{Z_{\text{eq}}} \qquad Z_{\text{eq}} = R_1 + jX_1 + \frac{1}{G_C - jB_M + \dfrac{1}{V_2/s + jX_2}}$$

- The air-gap power is obtained from input power

$$P_{\text{AG}} = P_{\text{in}} - P_{\text{SCL}} - P_{\text{core}} \qquad P_{\text{SCL}} = 3I_1^2 R_1 \qquad P_{\text{core}} = 3E_1^2 G_C$$

- Due to energy balance, the air-gap power can also be obtained from output power

$$P_{\text{AG}} = 3I_2^2 \frac{R_2}{s}$$

Power and torque in an induction motor

- The $P_{AG} = P_{conv}$ + rotor copper loss
- The actual rotor copper loss is

$$P_{RCL} = 3I_2^2 R_2$$

- The power converted (mechanical power) is

$$P_{conv} = P_{AG} - P_{RCL}$$

$$= 3I_2^2 \frac{R_2}{s} - 3I_2^2 R_2$$

$$= 3I_2^2 R_2\left(\frac{1}{s} - 1\right)$$

$$\boxed{P_{conv} = 3I_2^2 R_2\left(\frac{1-s}{s}\right)}$$

- Alternative expression of rotor copper is

$$P_{RCL} = sP_{AG}$$

Power and torque in an induction motor

- The mechanical power from air-gap power

$$P_{\text{conv}} = P_{\text{AG}} - P_{\text{RCL}}$$
$$= P_{\text{AG}} - sP_{\text{AG}}$$

$$\boxed{P_{\text{conv}} = (1 - s)P_{\text{AG}}}$$

- The finally output power

$$\boxed{P_{\text{out}} = P_{\text{conv}} - P_{\text{F\&W}} - P_{\text{misc}}}$$

- The induced torque (induced from electric-mechanical power conversion)

$$\tau_{\text{ind}} = \frac{P_{\text{conv}}}{\omega_m}$$

$$\tau_{\text{ind}} = \frac{(1 - s)P_{\text{AG}}}{(1 - s)\omega_{\text{sync}}}$$

$$\boxed{\tau_{\text{ind}} = \frac{P_{\text{AG}}}{\omega_{\text{sync}}}}$$

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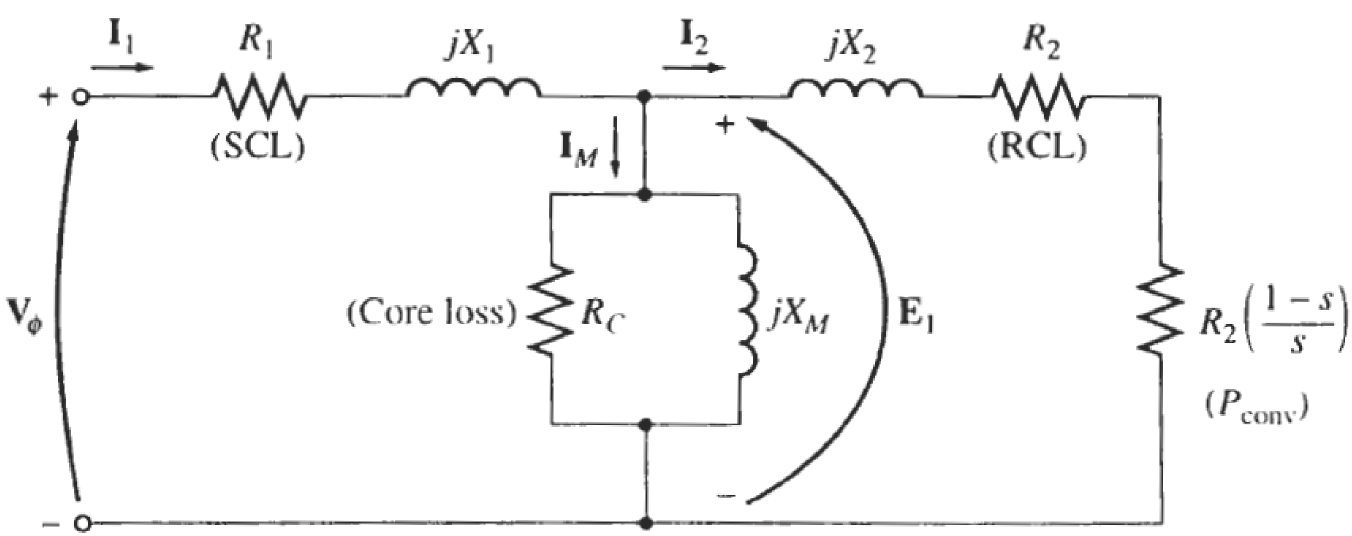
Separate the rotor copper loss and power converted in equivalent circuit

- The rotor resistance can be separated into two parts – the real rotor copper part and power converted part

$$R_{\text{conv}} = \frac{R_2}{s} - R_2 = R_2\left(\frac{1}{s} - 1\right)$$

$$\boxed{R_{\text{conv}} = R_2\left(\frac{1-s}{s}\right)}$$



Example 7-3

Example 7–3. A 460-V, 25-hp, 60-Hz, four-pole, Y-connected induction motor has the following impedances in ohms per phase referred to the stator circuit:

$$R_1 = 0.641\ \Omega \qquad R_2 = 0.332\ \Omega$$
$$X_1 = 1.106\ \Omega \qquad X_2 = 0.464\ \Omega \qquad X_M = 26.3\ \Omega$$

The total rotational losses are 1100 W and are assumed to be constant. The core loss is lumped in with the rotational losses. For a rotor slip of 2.2 percent at the rated voltage and rated frequency, find the motor's

(a) Speed

(b) Stator current

(c) Power factor

(d) P_{conv} and P_{out}

(e) τ_{ind} and τ_{load}

(f) Efficiency

The per-phase equivalent circuit of this motor is shown in Figure 7–12, and the power-flow diagram is shown in Figure 7–13. Since the core losses are lumped together with the friction and windage losses and the stray losses, they will be treated like the mechanical losses and be subtracted after P_{conv} in the power-flow diagram.

(a) The synchronous speed is

$$n_{\text{sync}} = \frac{120 f_e}{P} = \frac{120(60\text{ Hz})}{4\text{ poles}} = 1800\text{ r/min}$$

or $$\omega_{\text{sync}} = (1800\text{ r/min})\left(\frac{2\pi\text{ rad}}{1\text{ r}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 188.5\text{ rad/s}$$

The rotor's mechanical shaft speed is

$$\begin{aligned} n_m &= (1 - s)n_{\text{sync}} \\ &= (1 - 0.022)(1800\text{ r/min}) = 1760\text{ r/min} \end{aligned}$$

or $$\begin{aligned} \omega_m &= (1 - s)\omega_{\text{sync}} \\ &= (1 - 0.022)(188.5\text{ rad/s}) = 184.4\text{ rad/s} \end{aligned}$$

(b) To find the stator current, get the equivalent impedance of the circuit. The first step is to combine the referred rotor impedance in parallel with the magnetization branch, and then to add the stator impedance to that combination in series. The referred rotor impedance is

$$
\begin{aligned}
Z_2 &= \frac{R_2}{s} + jX_2 \\
&= \frac{0.332}{0.022} + j0.464 \\
&= 15.09 + j0.464\ \Omega = 15.10\angle 1.76^\circ\ \Omega
\end{aligned}
$$

The combined magnetization plus rotor impedance is given by

$$
\begin{aligned}
Z_f &= \frac{1}{1/jX_M + 1/Z_2} \\
&= \frac{1}{-j0.038 + 0.0662\angle -1.76^\circ} \\
&= \frac{1}{0.0773\angle -31.1^\circ} = 12.94\angle 31.1^\circ\ \Omega
\end{aligned}
$$

Therefore, the total impedance is

$$
\begin{aligned}
Z_{\text{tot}} &= Z_{\text{stat}} + Z_f \\
&= 0.641 + j1.106 + 12.94\angle 31.1^\circ\ \Omega \\
&= 11.72 + j7.79 = 14.07\angle 33.6^\circ\ \Omega
\end{aligned}
$$

The resulting stator current is

$$\mathbf{I}_1 = \frac{\mathbf{V}_\phi}{Z_{\text{tot}}}$$

$$= \frac{266\angle 0^\circ \text{ V}}{14.07\angle 33.6^\circ\ \Omega} = 18.88\angle -33.6^\circ \text{ A}$$

(*c*) The power motor power factor is

$$\text{PF} = \cos 33.6^\circ = 0.833 \qquad \text{lagging}$$

(*d*) The input power to this motor is

$$P_{\text{in}} = \sqrt{3}V_T I_L \cos\theta$$

$$= \sqrt{3}(460 \text{ V})(18.88 \text{ A})(0.833) = 12{,}530 \text{ W}$$

The stator copper losses in this machine are

$$P_{\text{SCL}} = 3I_1^2 R_1 \qquad (7\text{–}25)$$

$$= 3(18.88 \text{ A})^2(0.641\ \Omega) = 685 \text{ W}$$

The air-gap power is given by

$$P_{\text{AG}} = P_{\text{in}} - P_{\text{SCL}} = 12{,}530 \text{ W} - 685 \text{ W} = 11{,}845 \text{ W}$$

Therefore, the power converted is

$$P_{\text{conv}} = (1 - s)P_{\text{AG}} = (1 - 0.022)(11{,}845\ \text{W}) = 11{,}585\ \text{W}$$

The power P_{out} is given by

$$P_{\text{out}} = P_{\text{conv}} - P_{\text{rot}} = 11{,}585\ \text{W} - 1100\ \text{W} = 10{,}485\ \text{W}$$

$$= 10{,}485\ \text{W}\left(\frac{1\ \text{hp}}{746\ \text{W}}\right) = 14.1\ \text{hp}$$

(e) The induced torque is given by

$$\tau_{\text{ind}} = \frac{P_{\text{AG}}}{\omega_{\text{sync}}}$$

$$= \frac{11{,}845\ \text{W}}{188.5\ \text{rad/s}} = 62.8\ \text{N} \bullet \text{m}$$

and the output torque is given by

$$\tau_{\text{load}} = \frac{P_{\text{out}}}{\omega_m}$$

$$= \frac{10{,}485\ \text{W}}{184.4\ \text{rad/s}} = 56.9\ \text{N} \bullet \text{m}$$

(*f*) The motor's efficiency at this operating condition is

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\%$$

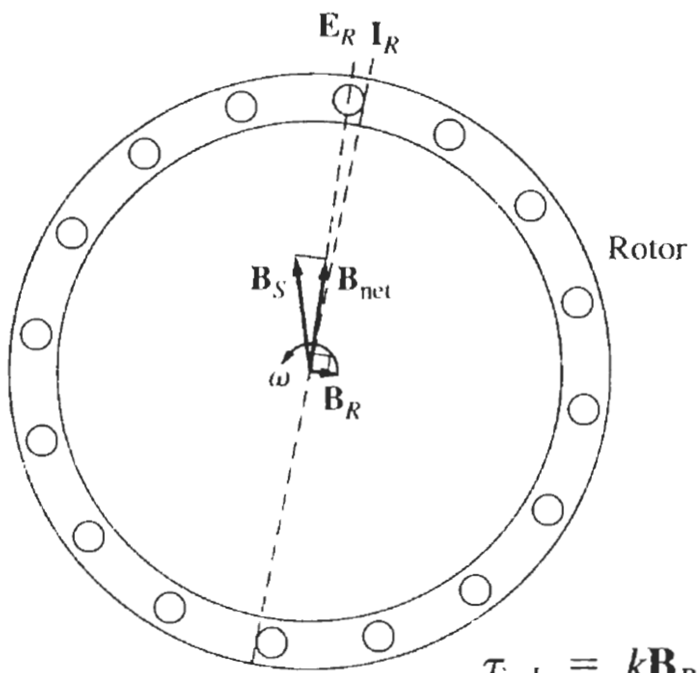
$$= \frac{10{,}485 \text{ W}}{12{,}530 \text{ W}} \times 100\% = 83.7\%$$

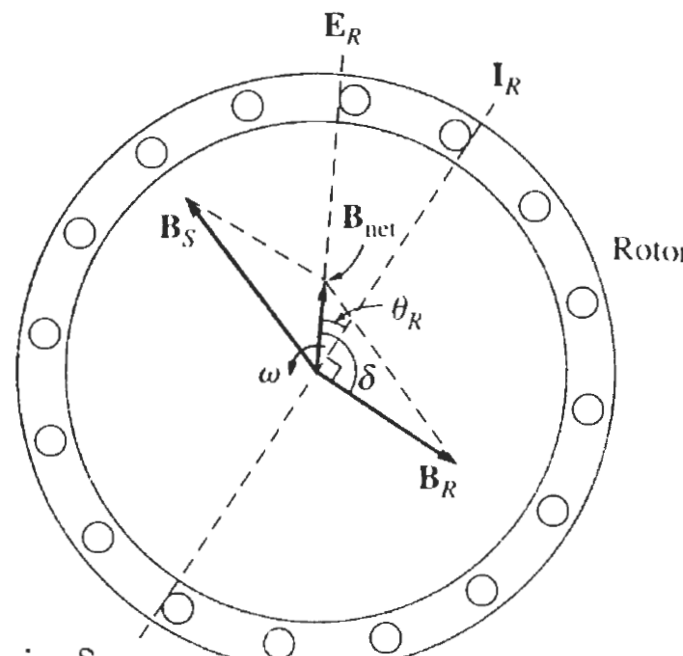
Torque-speed characteristics

- How does the torque of an induction motor change as the load changes?
- How much torque can an induction motor suppply at starting conditions?
- How much does the speed of an induction motor drop as its shaft load increases?
- The relationship among the motor’s torque, speed and power.
- First, the relation is developed via the physical standpoint.
- Then, the quantitative equation for torque as a function of slip will be obtained.

Induced torque from a physical standpoint

- The magnetic field in an induction motor under light loads and heavy loads
- At light load, the relative motion (slip) is small, the E_R, I_R, and B_R decrease
- At heavy load, the relative motion (slip) is large, the E_R, I_R, and B_R increase.
- While slip increases, sf_e and rotor reactance also increase. Thus θ_R increases
- While δ increases, B_R increases. However, $\sin\delta$ is max at δ=90 degrees





$$\tau_{\text{ind}} = k\mathbf{B}_R\mathbf{B}_{\text{net}}\sin\delta$$

The effect of individual terms on slip (speed) change

- The torque equation

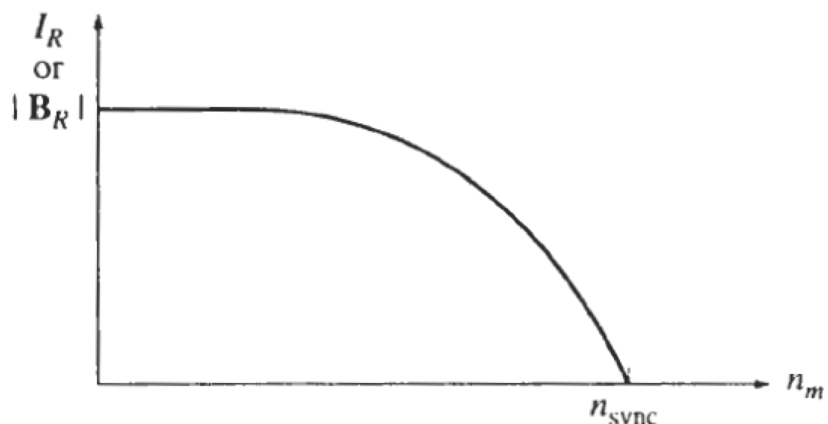
$$\tau_{\text{ind}} = k\mathbf{B}_R\mathbf{B}_{\text{net}} \sin \delta$$

- Three individual terms B_R, B_{net} and $\sin\delta$ are considered individually

B_R

- If no saturation, rotor magnetic field is proportional to current I_R (load)

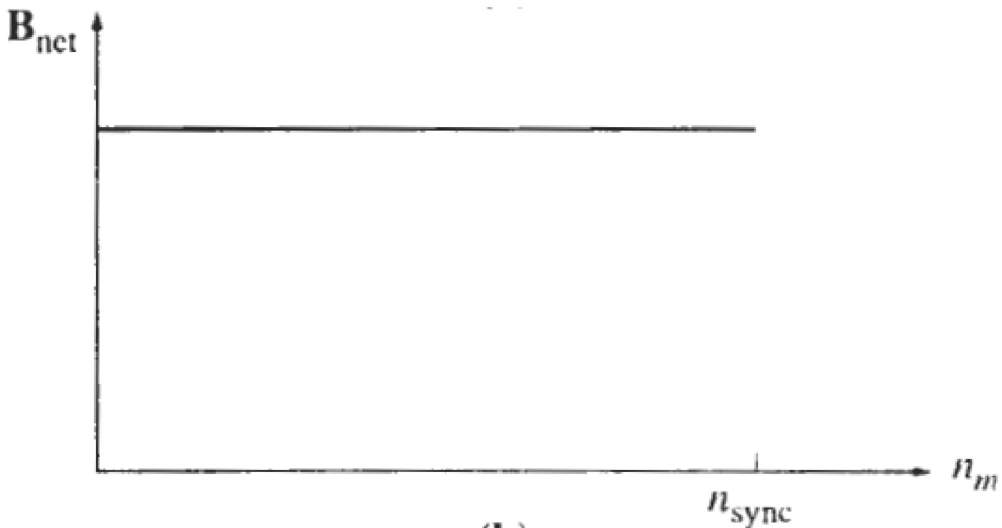
$$\mathbf{I}_R = \frac{\mathbf{E}_{R0}}{R_R/s + jX_{R0}}$$



(a)

B_{net}

- For constant stator voltage, neglect stator resistance and leakage reactance,



(b)

sinδ

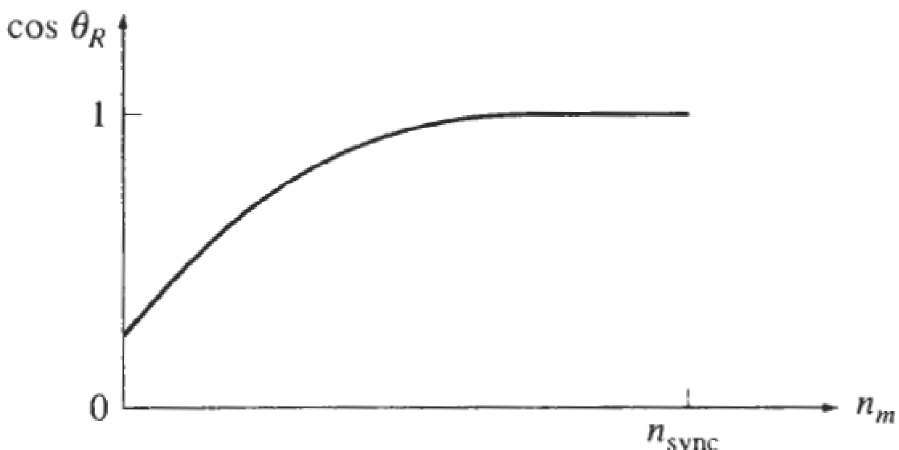
- δ is the rotor power factor angle plus 90 degrees

$$\delta = \theta_R + 90°$$

$$\theta_R = \tan^{-1}\frac{X_R}{R_R} = \tan^{-1}\frac{sX_{R0}}{R_R}$$

$$PF_R = \cos\theta_R$$

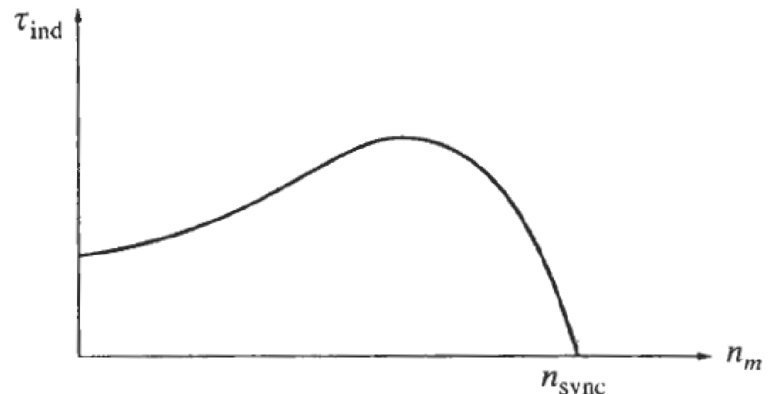
$$\boxed{PF_R = \cos\left(\tan^{-1}\frac{sX_{R0}}{R_R}\right)}$$



(c)

The resulting torque-speed curve

- Combine the above three curves
- The resulting curve can be divided into three regions
  - ***Low slip region***
    - Linear region, the IM normal operation region
    - Rotor reactance is small and PF=1
  - ***Moderate slip region***
    - Pull out torque region
  - ***High slip region***
    - Starting torque region (normally 150% full load)
    - IM can full load starting



(d)

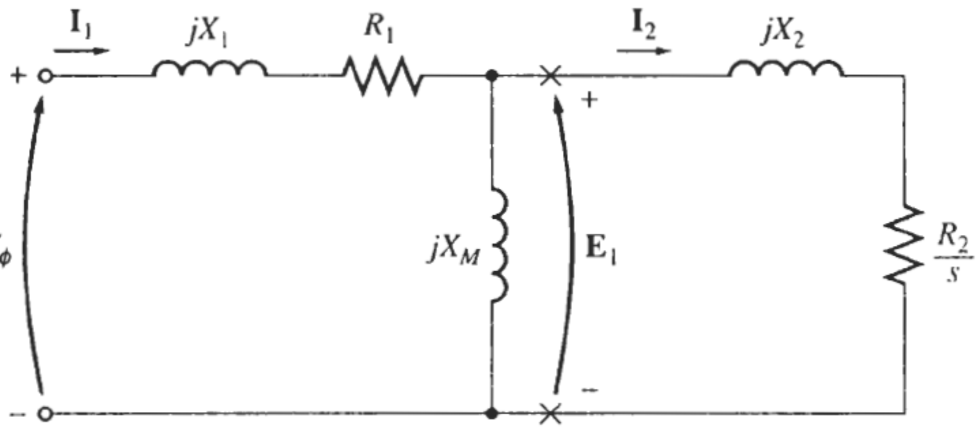
Quantitative equation expression of torque-speed relation

- Derive the induce torque equation from equivalent circuit power analysis
- The second equation is preferred, since ω_{sync} is easier to be obtained
- P_{AG} is the power absorbed by R_2/s

$$\tau_{ind} = \frac{P_{conv}}{\omega_m}$$

$$\tau_{ind} = \frac{P_{AG}}{\omega_{sync}}$$

$$P_{AG} = 3I_2^2 \frac{R_2}{s}$$

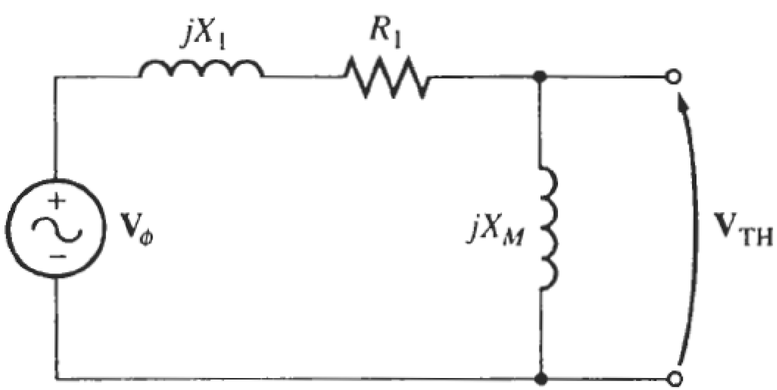


Thevenin equivalent circuit analysis for I_2

- The Thevenin equivalent circuit analysis is used to calculate current I_2
- The Thevenin voltage
- The magnitude of Thevenin voltage

$$\mathbf{V}_{TH} = \mathbf{V}_\phi \frac{Z_M}{Z_M + Z_1} = \mathbf{V}_\phi \frac{jX_M}{R_1 + jX_1 + jX_M}$$

$$V_{TH} = V_\phi \frac{X_M}{\sqrt{R_1^2 + (X_1 + X_M)^2}}$$

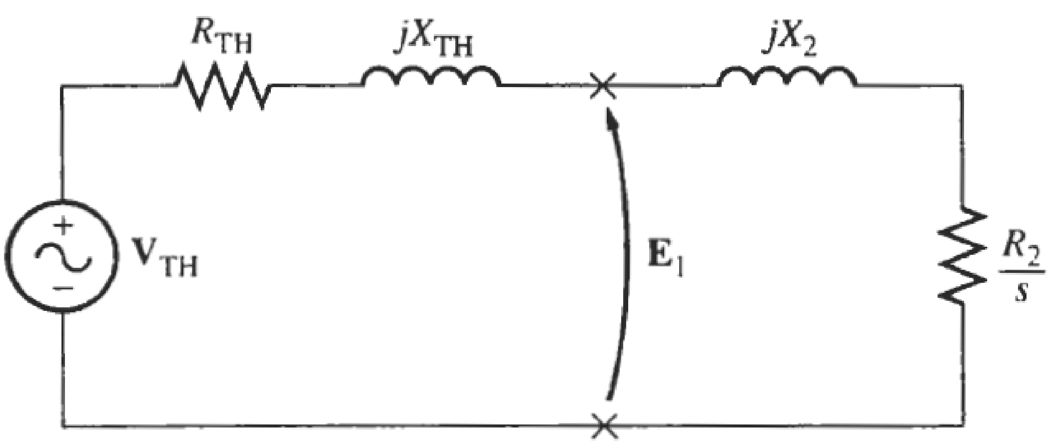


$$\mathbf{V}_{TH} = \frac{jX_M}{R_1 + jX_1 + jX_M}\mathbf{V}_\phi$$

$$V_{TH} = \frac{X_M}{\sqrt{R_1{}^2 + (X_1 + X_M)^2}} V_\phi$$

Simplified Thevenin equivalent circuit

- The simplifier equivalent circuit can be obtained by considering $X_M >> X_1$ and $X_M >> R_1$
- Thus, the Thevenin voltage V_{TH} and impedance Z_{TH} are



(c)

$$V_{TH} \approx V_\phi \frac{X_M}{X_1 + X_M}$$

$$Z_{TH} = \frac{Z_1 Z_M}{Z_1 + Z_M}$$

Thevenin impedance

- The Thevenin impedance can be approximate as

- The Thevenin resistance and reactance are

$$Z_{\mathrm{TH}} = R_{\mathrm{TH}} + jX_{\mathrm{TH}} = \frac{jX_M(R_1 + jX_1)}{R_1 + j(X_1 + X_M)}$$

$$\boxed{R_{\mathrm{TH}} \approx R_1\left(\frac{X_M}{X_1 + X_M}\right)^2}$$

$$\boxed{X_{\mathrm{TH}} \approx X_1}$$

Induce torque calculation from P_{AG}

- The I_2 current can be obtained from simplified equivalent circuit

$$\mathbf{I}_2 = \frac{\mathbf{V}_{TH}}{Z_{TH} + Z_2}$$

$$= \frac{\mathbf{V}_{TH}}{R_{TH} + R_2/s + jX_{TH} + jX_2}$$

- The magnitude of current is

$$I_2 = \frac{V_{TH}}{\sqrt{(R_{TH} + R_2/s)^2 + (X_{TH} + X_2)^2}}$$

- The air-gap power P_{AG} is

$$P_{AG} = 3I_2^2 \frac{R_2}{s}$$

$$= \frac{3V_{TH}^2 R_2/s}{(R_{TH} + R_2/s)^2 + (X_{TH} + X_2)^2}$$

- Finally, the induce torque is

$$\tau_{ind} = \frac{P_{AG}}{\omega_{sync}}$$

$$\boxed{\tau_{ind} = \frac{3V_{TH}^2 R_2/s}{\omega_{sync}[(R_{TH} + R_2/s)^2 + (X_{TH} + X_2)^2]}}$$

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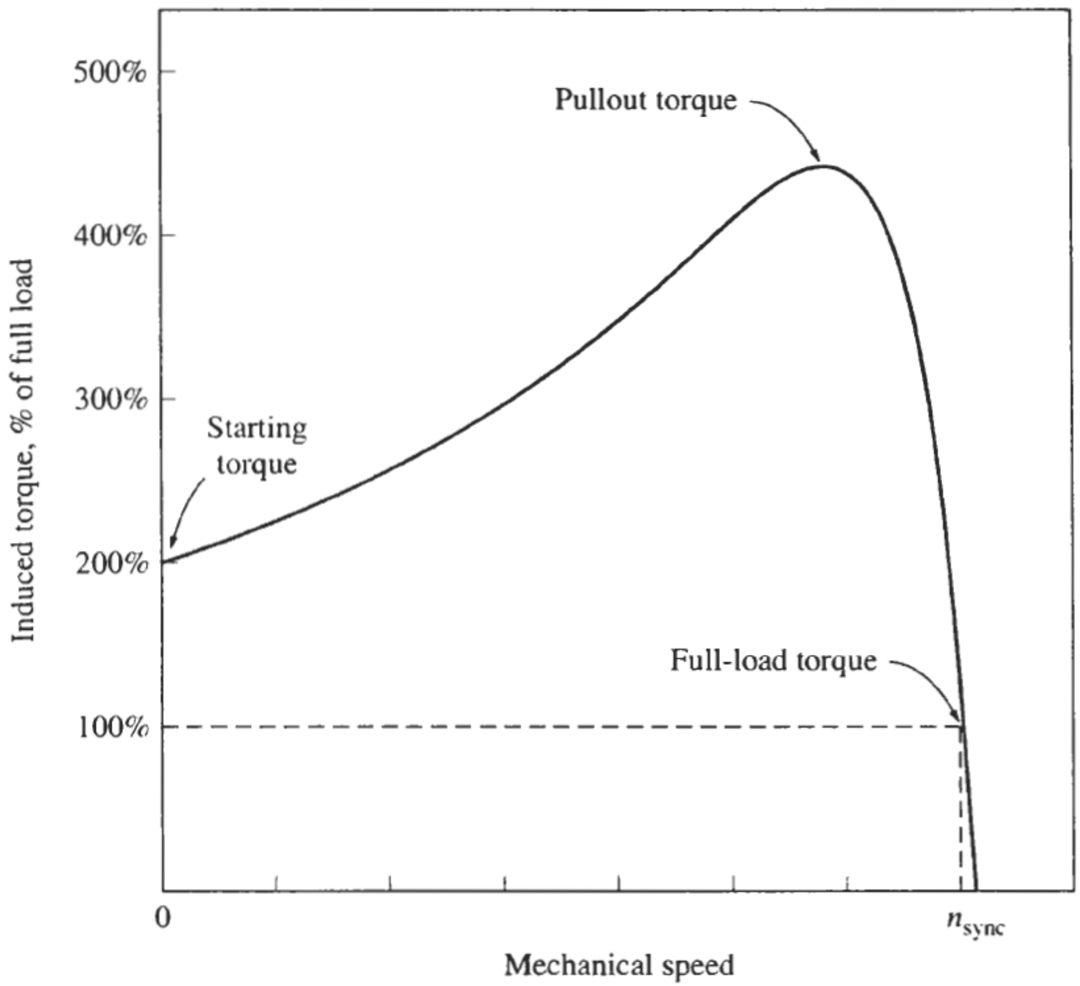
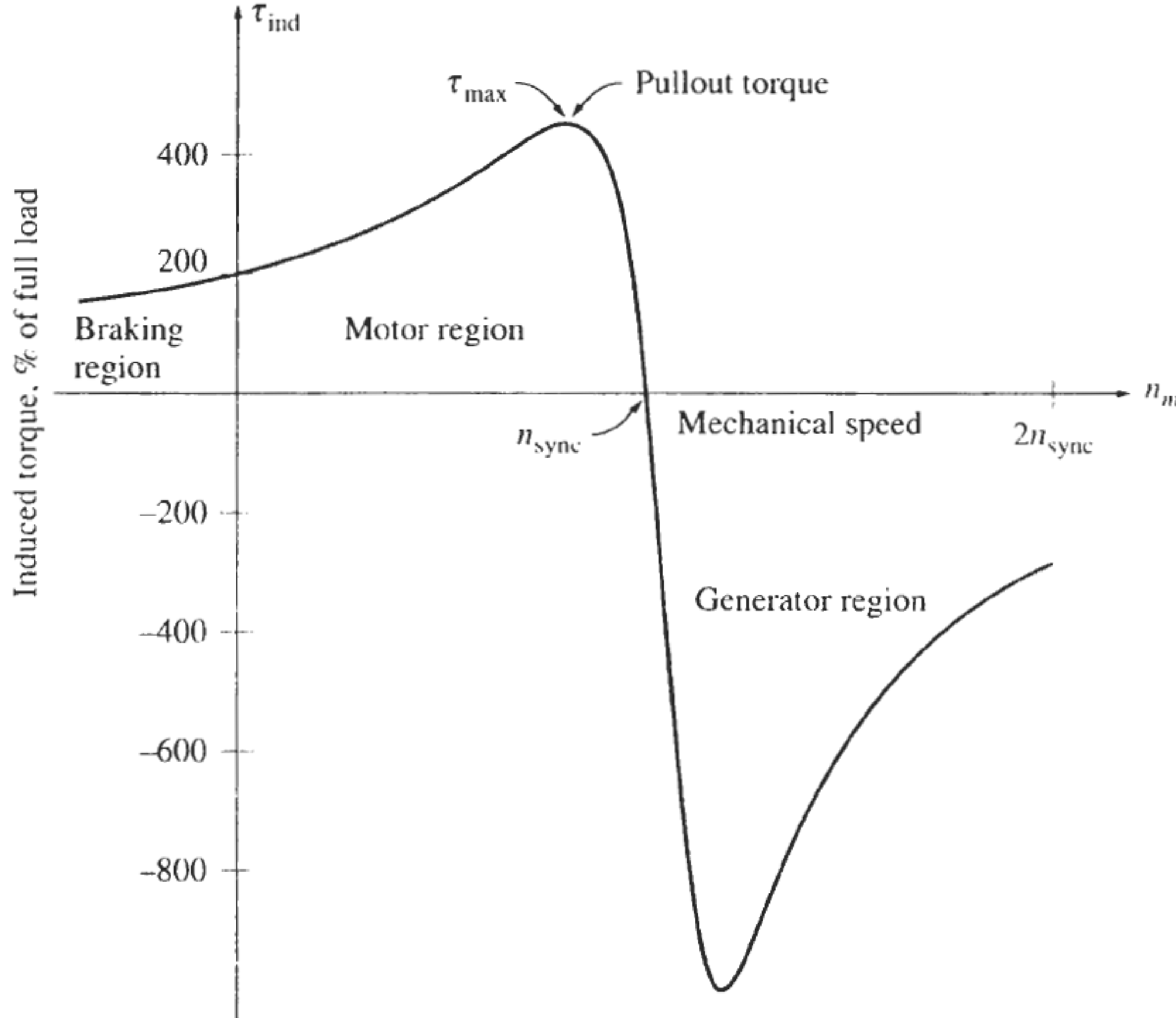
Torque speed curve
- speed between 0~n_{sync}





Speed beyond $0 \sim n_{sync}$

Summarize of torque-speed curve

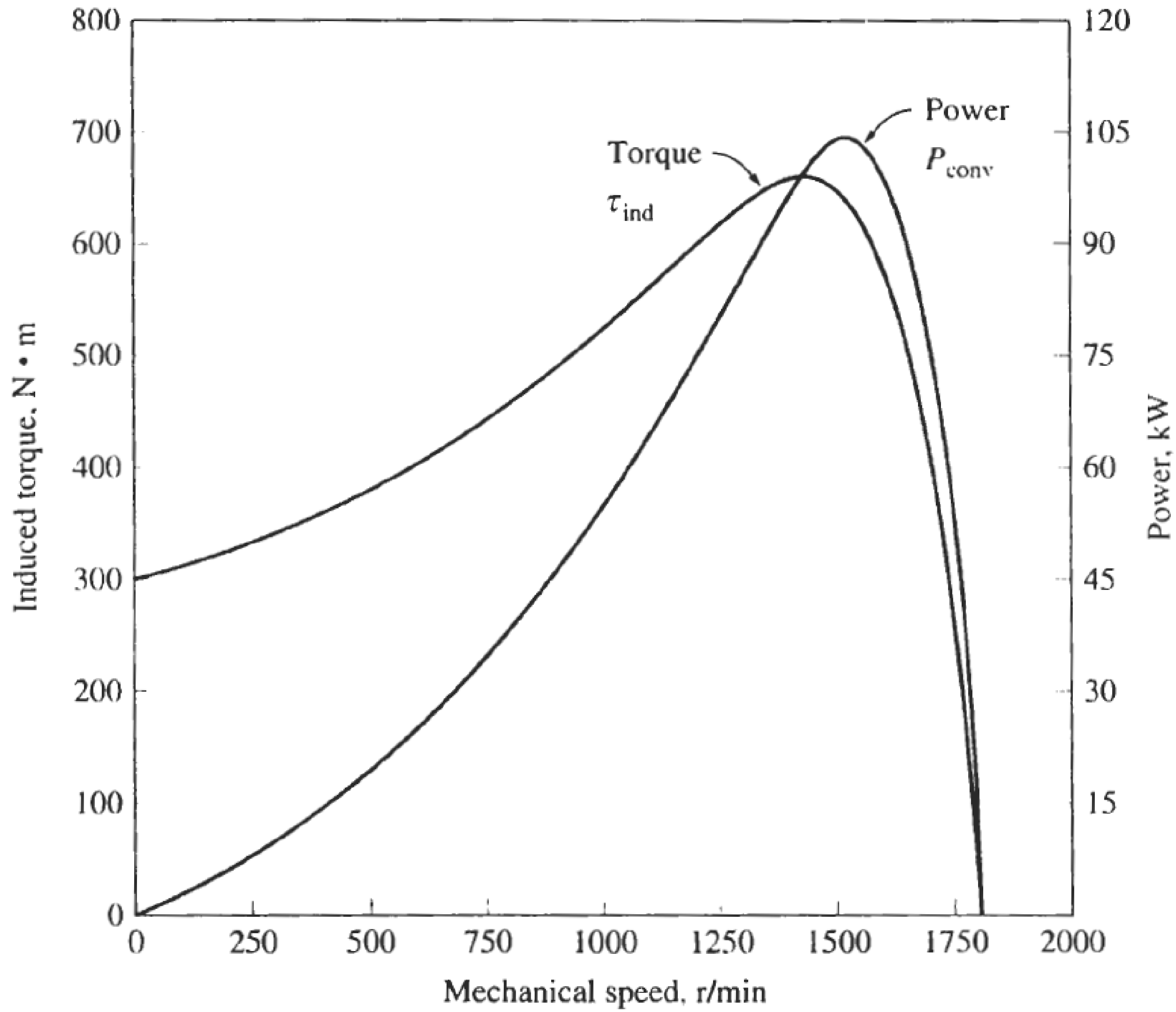
1. The induced torque of the motor is zero at synchronous speed. This fact has been discussed previously.
2. The torque–speed curve is nearly linear between no load and full load. In this range, the rotor resistance is much larger than the rotor reactance, so the rotor current, the rotor magnetic field, and the induced torque increase linearly with increasing slip.
3. There is a maximum possible torque that cannot be exceeded. This torque, called the *pullout torque* or *breakdown torque*, is 2 to 3 times the rated full-load torque of the motor. The next section of this chapter contains a method for calculating pullout torque.
4. The starting torque on the motor is slightly larger than its full-load torque, so this motor will start carrying any load that it can supply at full power.

Summarize of torque-speed curve

5. Notice that the torque on the motor for a given slip varies as the square of the applied voltage. This fact is useful in one form of induction motor speed control that will be described later.
6. If the rotor of the induction motor is driven faster than synchronous speed, then the direction of the induced torque in the machine reverses and the machine becomes a *generator*, converting mechanical power to electric power. The use of induction machines as generators will be described later.
7. If the motor is turning backward relative to the direction of the magnetic fields, the induced torque in the machine will stop the machine very rapidly and will try to rotate it in the other direction. Since reversing the direction of magnetic field rotation is simply a matter of switching any two stator phases, this fact can be used as a way to very rapidly stop an induction motor. The act of switching two phases in order to stop the motor very rapidly is called *plugging*.

Torque-speed and P_{conv}-speed curves

$$P_{conv} = \tau_{ind}\,\omega_m$$



Maximum (Pullout) torque of an induction motor

- The maximum induced torque will occur when the power consumed by the R_2/s resistor is maximum
- ***Maximum power transfer theorem***
  - When the ***magnitude*** of the impedance R_2/s is equal to the ***magnitude*** of the source impedance

$$Z_{\text{source}} = R_{\text{TH}} + jX_{\text{TH}} + jX_2$$

$$\frac{R_2}{s} = \sqrt{R_{\text{TH}}^2 + (X_{\text{TH}} + X_2)^2}$$

  - The maximum torque slip s_{max}

$$s_{\max} = \frac{R_2}{\sqrt{R_{\text{TH}}^2 + (X_{\text{TH}} + X_2)^2}}$$

Notably, the value of s_{max} is proportional to R_2 value II

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Pullout torque

- The pullout torque can be obtained by substituting s_{max} into torque equation

$$\boxed{\tau_{\max} = \frac{3V_{\text{TH}}^2}{2\omega_{\text{sync}}[R_{\text{TH}} + \sqrt{R_{\text{TH}}^2 + (X_{\text{TH}} + X_2)^2}]}}$$

- The maximum torque does not depend on rotor resistance R_2
- The value of s_{max} is proportional to R2 value

Example 7-8

Example 7–8. The following test data were taken on a 7.5-hp, four-pole, 208-V, 60-Hz, design A, Y-connected induction motor having a rated current of 28 A.

DC test:

$V_{DC} = 13.6\ \text{V}$ $\qquad$ $I_{DC} = 28.0\ \text{A}$

No-load test:

$V_T = 208\ \text{V}$ $\qquad$ $f = 60\ \text{Hz}$

$I_A = 8.12\ \text{A}$ $\qquad$ $P_{in} = 420\ \text{W}$

$I_B = 8.20\ \text{A}$

$I_C = 8.18\ \text{A}$

Locked-rotor test:

$V_T = 25\ \text{V}$ $\qquad$ $f = 15\ \text{Hz}$

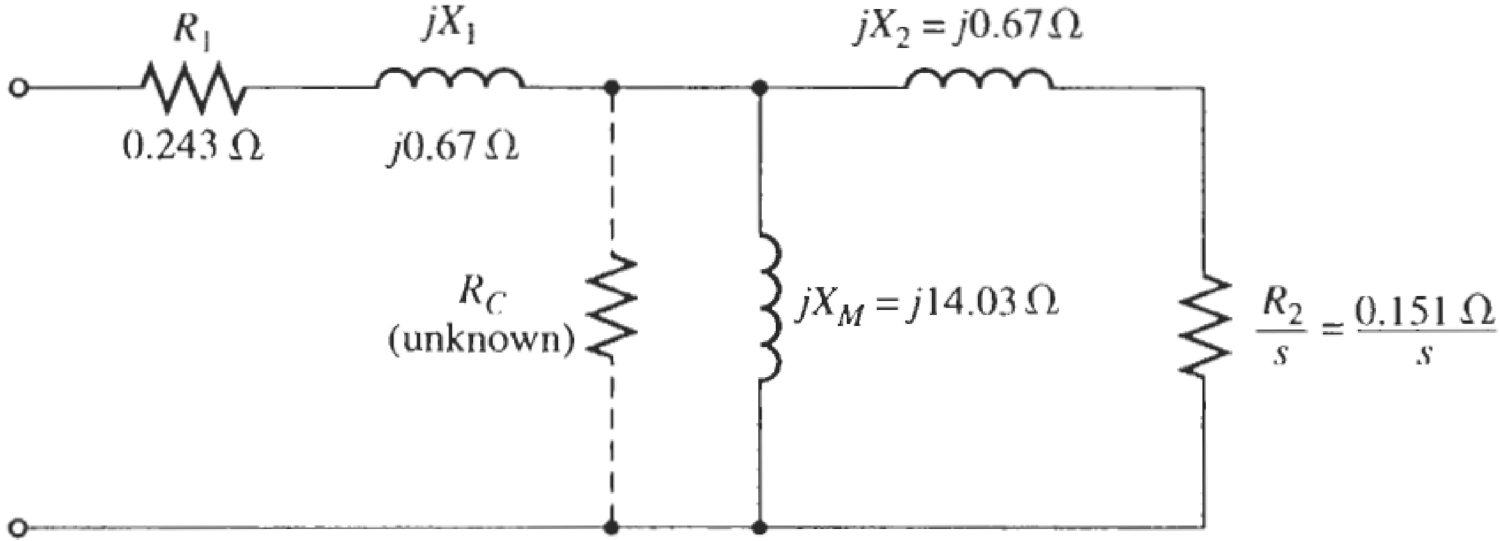
$I_A = 28.1\ \text{A}$ $\qquad$ $P_{in} = 920\ \text{W}$

$I_B = 28.0\ \text{A}$

$I_C = 27.6\ \text{A}$

(a) Sketch the per-phase equivalent circuit for this motor.

(b) Find the slip at the pullout torque, and find the value of the pullout torque itself.



$$\mathbf{V}_{TH} = \mathbf{V}_\phi \frac{Z_M}{Z_M + Z_1} = \mathbf{V}_\phi \frac{jX_M}{R_1 + jX_1 + jX_M}$$

$$Z_{TH} = R_{TH} + jX_{TH} = \frac{jX_M(R_1 + jX_1)}{R_1 + j(X_1 + X_M)}$$

$$s_{max} = \frac{R_2}{\sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}}$$

$$\boxed{\tau_{max} = \frac{3V_{TH}^2}{2\omega_{sync}[R_{TH} + \sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}]}}$$

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(b) For this equivalent circuit, the Thevenin equivalents are found from Equations (7–41b), (7–44), and (7–45) to be

$$V_{TH} = 114.6 \text{ V} \qquad R_{TH} = 0.221\ \Omega \qquad X_{TH} = 0.67\ \Omega$$

Therefore, the slip at the pullout torque is given by

$$s_{max} = \frac{R_2}{\sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}} \tag{7–53}$$

$$= \frac{0.151\ \Omega}{\sqrt{(0.243\ \Omega)^2 + (0.67\ \Omega + 0.67\ \Omega)^2}} = 0.111 = 11.1\%$$

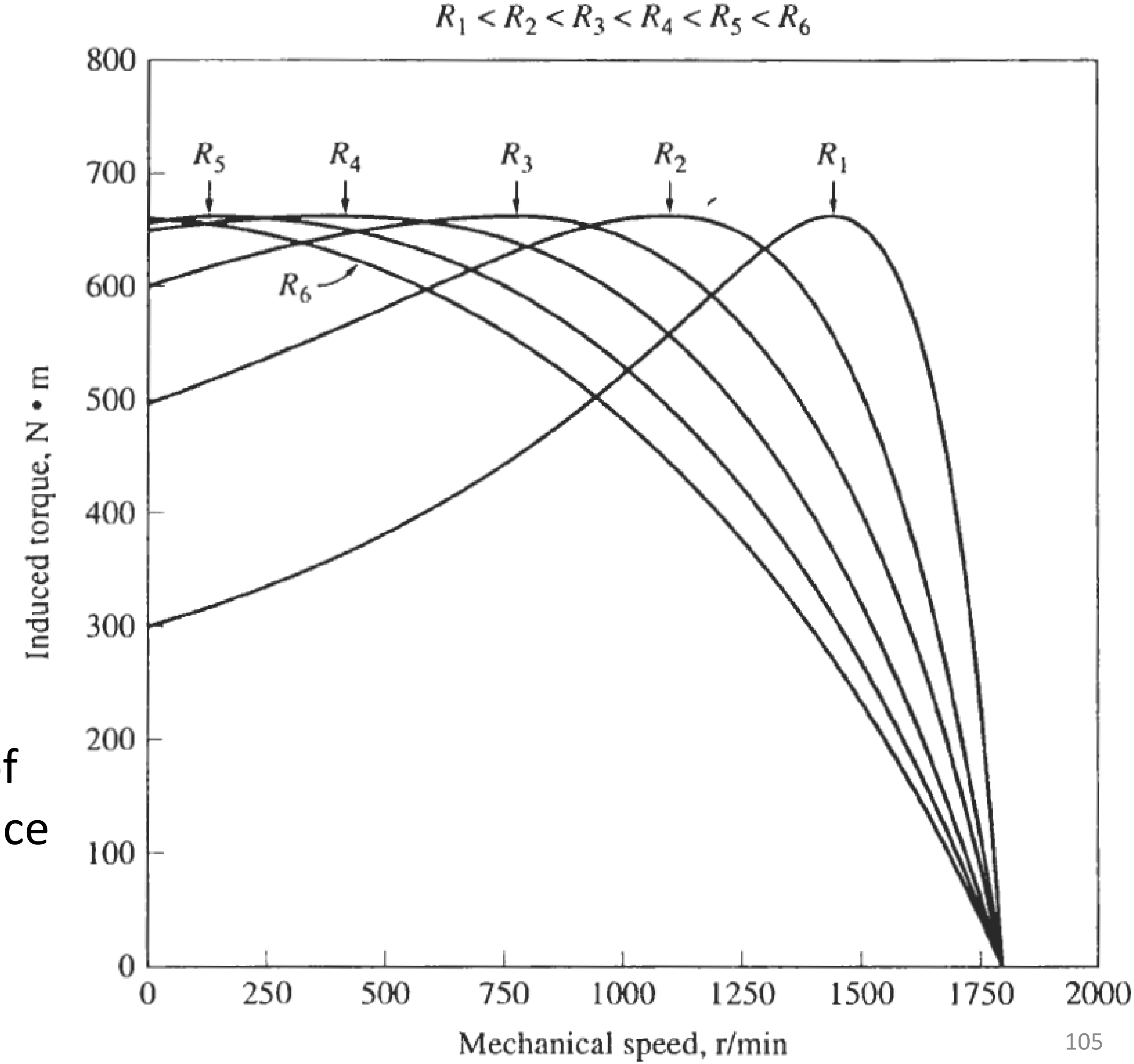
The maximum torque of this motor is given by

$$\tau_{max} = \frac{3V_{TH}^2}{2\omega_{sync}[R_{TH} + \sqrt{R_{TH}^2 + (X_{TH} + X^2)}]} \tag{7–54}$$

$$= \frac{3(114.6\text{ V})^2}{2(188.5\text{ rad/s})[0.221\ \Omega + \sqrt{(0.221\ \Omega)^2 + (0.67\ \Omega + 0.67\ \Omega)^2}]}$$

$$= 66.2\text{ N}\bullet\text{m}$$

The effect of rotor resistance



Rotor resistance tuning

- Rotor resistance R_2 only can be arbitrarily tuned in wound rotor induction motor
- For heavy load, large rotor resistance can be inserted in motor starting period to increase the starting torque

Example 7-4

Example 7–4. A two-pole, 50-Hz induction motor supplies 15 kW to a load at a speed of 2950 r/min.

(a) What is the motor's slip?
(b) What is the induced torque in the motor in N • m under these conditions?
(c) What will the operating speed of the motor be if its torque is doubled?
(d) How much power will be supplied by the motor when the torque is doubled?

(*a*) The synchronous speed of this motor is

$$n_{\text{sync}} = \frac{120 f_e}{P} = \frac{120(50 \text{ Hz})}{2 \text{ poles}} = 3000 \text{ r/min}$$

Therefore, the motor's slip is

$$s = \frac{n_{\text{sync}} - n_{\text{m}}}{n_{\text{sync}}} (\times\ 100\%) \tag{7–4}$$

$$= \frac{3000 \text{ r/min} - 2950 \text{ r/min}}{3000 \text{ r/min}} (\times\ 100\%)$$

$$= 0.0167 \text{ or } 1.67\%$$

(*b*) The induced torque in the motor must be assumed equal to the load torque, and P_{conv} must be assumed equal to P_{load}, since no value was given for mechanical losses. The torque is thus

$$\tau_{\text{ind}} = \frac{P_{\text{conv}}}{\omega_{\text{m}}}$$

$$= \frac{15 \text{ kW}}{(2950 \text{ r/min})(2\pi \text{ rad/r})(1 \text{ min}/60 \text{ s})}$$

$$= 48.6 \text{ N} \bullet \text{m}$$

(c) In the low-slip region, the torque–speed curve is linear, and the induced torque is directly proportional to slip. Therefore, if the torque doubles, then the new slip will be 3.33 percent. The operating speed of the motor is thus

$$n_m = (1 - s)n_{\text{sync}} = (1 - 0.0333)(3000 \text{ r/min}) = 2900 \text{ r/min}$$

(d) The power supplied by the motor is given by

$$P_{\text{conv}} = \tau_{\text{ind}}\omega_m$$

$$= (97.2 \text{ N} \bullet \text{m})(2900 \text{ r/min})(2\pi \text{ rad/r})(1 \text{ min/60 s})$$

$$= 29.5 \text{ kW}$$

Example 7-5

Example 7–5. A 460-V, 25-hp, 60-Hz, four-pole, Y-connected wound-rotor induction motor has the following impedances in ohms per phase referred to the stator circuit:

$$R_1 = 0.641\ \Omega \qquad R_2 = 0.332\ \Omega$$
$$X_1 = 1.106\ \Omega \qquad X_2 = 0.464\ \Omega \qquad X_M = 26.3\ \Omega$$

(a) What is the maximum torque of this motor? At what speed and slip does it occur?

(b) What is the starting torque of this motor?

(c) When the rotor resistance is doubled, what is the speed at which the maximum torque now occurs? What is the new starting torque of the motor?

The Thevenin voltage of this motor is

$$V_{TH} = V_\phi \frac{X_M}{\sqrt{R_1^2 + (X_1 + X_M)^2}} \tag{7–41a}$$

$$= \frac{(266 \text{ V})(26.3\ \Omega)}{\sqrt{(0.641\ \Omega)^2 + (1.106\ \Omega + 26.3\ \Omega)^2}} = 255.2 \text{ V}$$

The Thevenin resistance is

$$R_{TH} \approx R_1 \left(\frac{X_M}{X_1 + X_M}\right)^2 \tag{7–44}$$

$$\approx (0.641\ \Omega)\left(\frac{26.3\ \Omega}{1.106\ \Omega + 26.3\ \Omega}\right)^2 = 0.590\ \Omega$$

The Thevenin reactance is

$$X_{TH} \approx X_1 = 1.106\ \Omega$$

(a) The slip at which maximum torque occurs is given by Equation (7–53):

$$s_{max} = \frac{R_2}{\sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}} \tag{7–53}$$

$$= \frac{0.332\ \Omega}{\sqrt{(0.590\ \Omega)^2 + (1.106\ \Omega + 0.464\ \Omega)^2}} = 0.198$$

This corresponds to a mechanical speed of

$$n_m = (1 - s)n_{\text{sync}} = (1 - 0.198)(1800 \text{ r/min}) = 1444 \text{ r/min}$$

The torque at this speed is

$$\tau_{\max} = \frac{3V_{\text{TH}}^2}{2\omega_{\text{sync}}[R_{\text{TH}} + \sqrt{R_{\text{TH}}^2 + (X_{\text{TH}} + X_2)^2}]} \tag{7--54}$$

$$= \frac{3(255.2 \text{ V})^2}{2(188.5 \text{ rad/s})[0.590\ \Omega + \sqrt{(0.590\ \Omega)^2 + (1.106\ \Omega + 0.464\ \Omega)^2}]}$$

$$= 229 \text{ N} \bullet \text{m}$$

(b) The starting torque of this motor is found by setting $s = 1$ in Equation (7–50):

$$\tau_{\text{start}} = \frac{3V_{\text{TH}}^2 R_2}{\omega_{\text{sync}}[(R_{\text{TH}} + R_2)^2 + (X_{\text{TH}} + X_2)^2]}$$

$$= \frac{3(255.2 \text{ V})^2(0.332\ \Omega)}{(188.5 \text{ rad/s})[(0.590\ \Omega + 0.332\ \Omega)^2 + (1.106\ \Omega + 0.464\ \Omega)^2]}$$

$$= 104 \text{ N} \bullet \text{m}$$

(c) If the rotor resistance is doubled, then the slip at maximum torque doubles, too. Therefore,

$$s_{\max} = 0.396$$

and the speed at maximum torque is

$$n_m = (1 - s)n_{\text{sync}} = (1 - 0.396)(1800 \text{ r/min}) = 1087 \text{ r/min}$$

The maximum torque is still

$$\tau_{\max} = 229 \text{ N} \cdot \text{m}$$

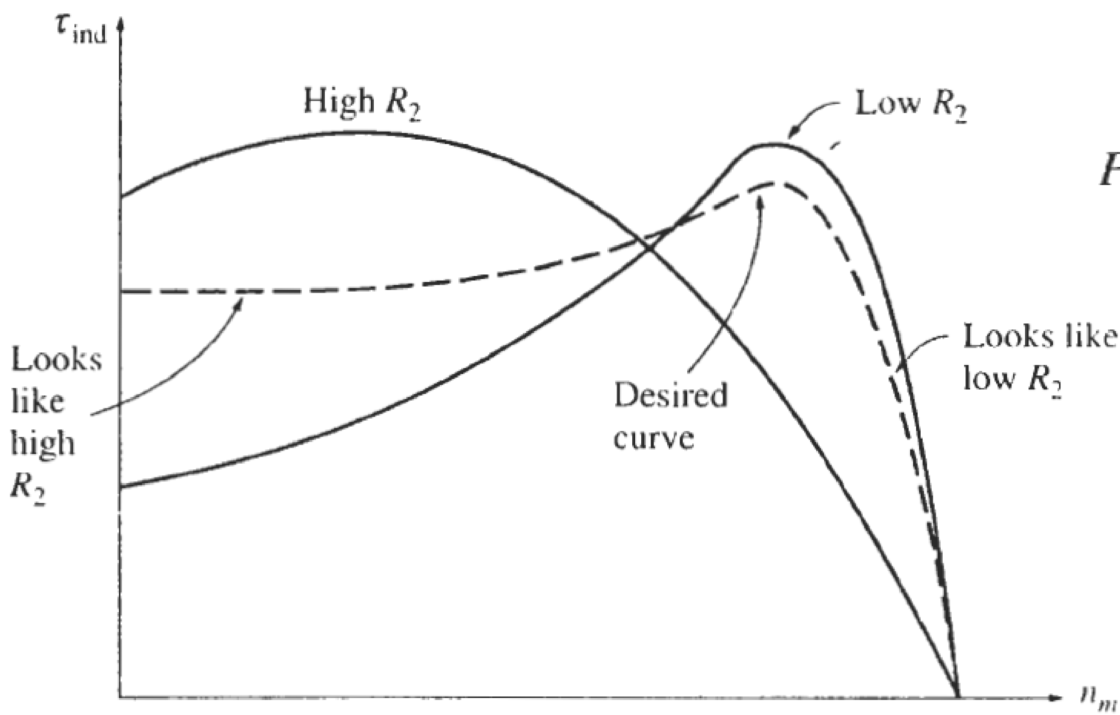
The starting torque is now

$$\tau_{\text{start}} = \frac{3(255.2 \text{ V})^2(0.664\ \Omega)}{(188.5 \text{ rad/s})[(0.590\ \Omega + 0.664\ \Omega)^2 + (1.106\ \Omega + 0.464\ \Omega)^2]}$$

$$= 170 \text{ N} \cdot \text{m}$$

Cage rotor torque-speed curve control

- However, there is no way to control the rotor resistance R2



$$P_{\text{conv}} = (1 - s)P_{\text{AG}}$$

Control by rotor bar structure

<u>Normal bar</u>

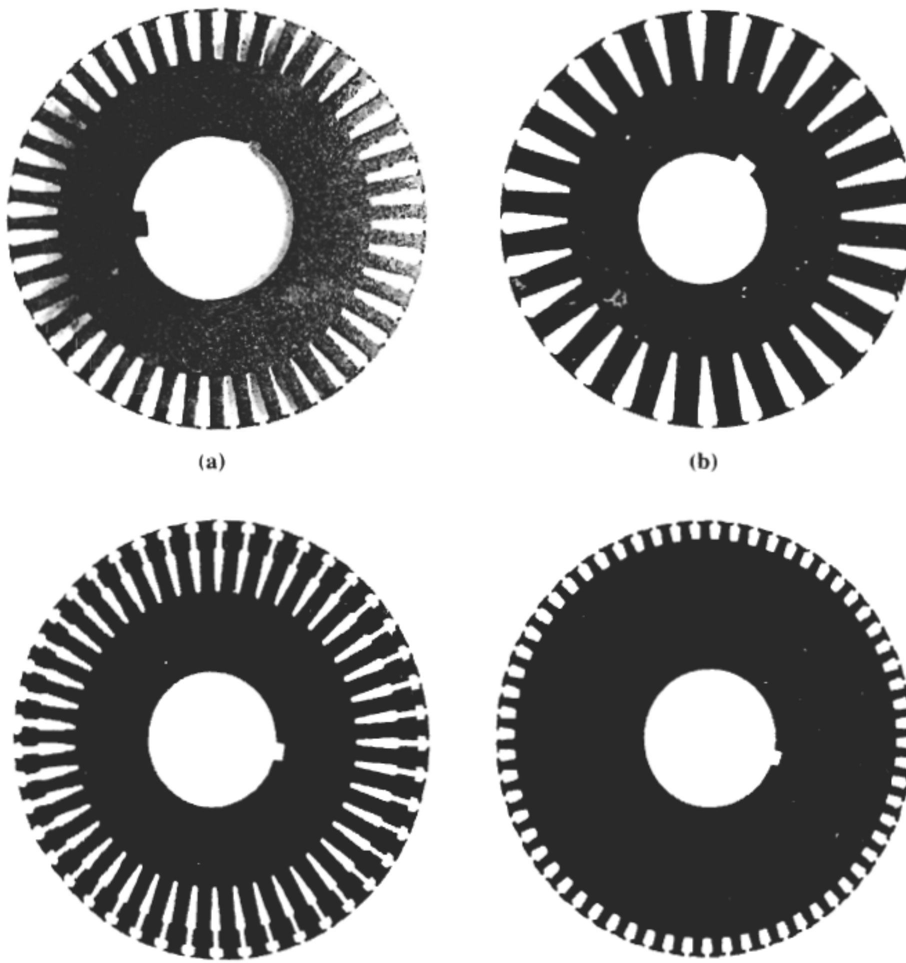
(Large area, Rr is small)

<u>Deep bar</u>

<u>Double Cage</u>

<u>Small bar</u>

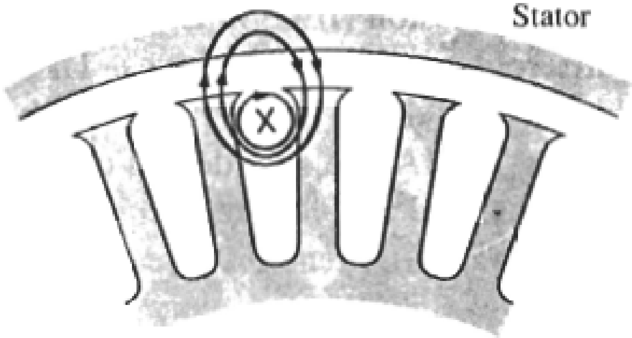
(Small area, Rr is large)



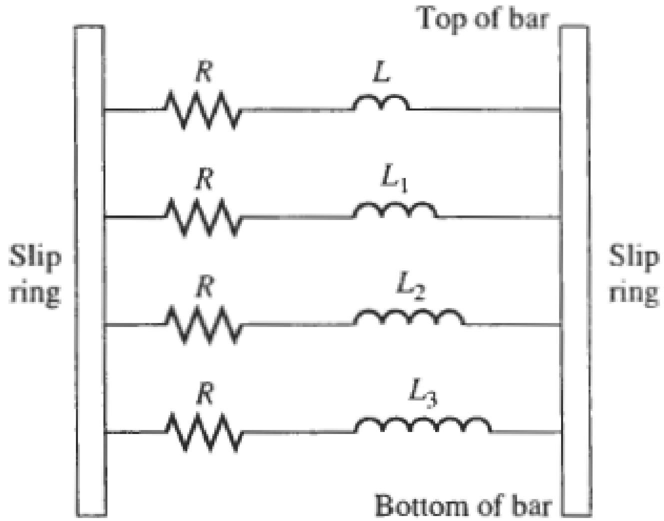
(a) (b) (c) (d)

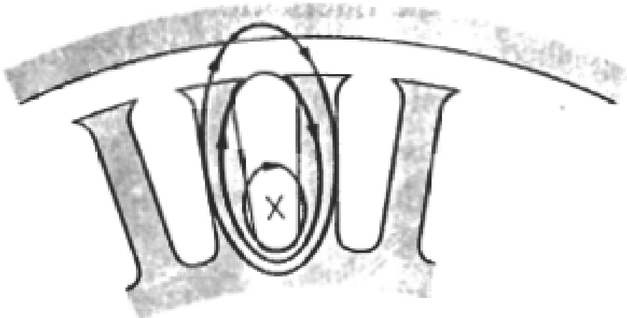
Variable resistance of cage rotor

- The variable resistance of cage rotor can be achieved by ***deep rotor bar*** or ***double cage***



(a)



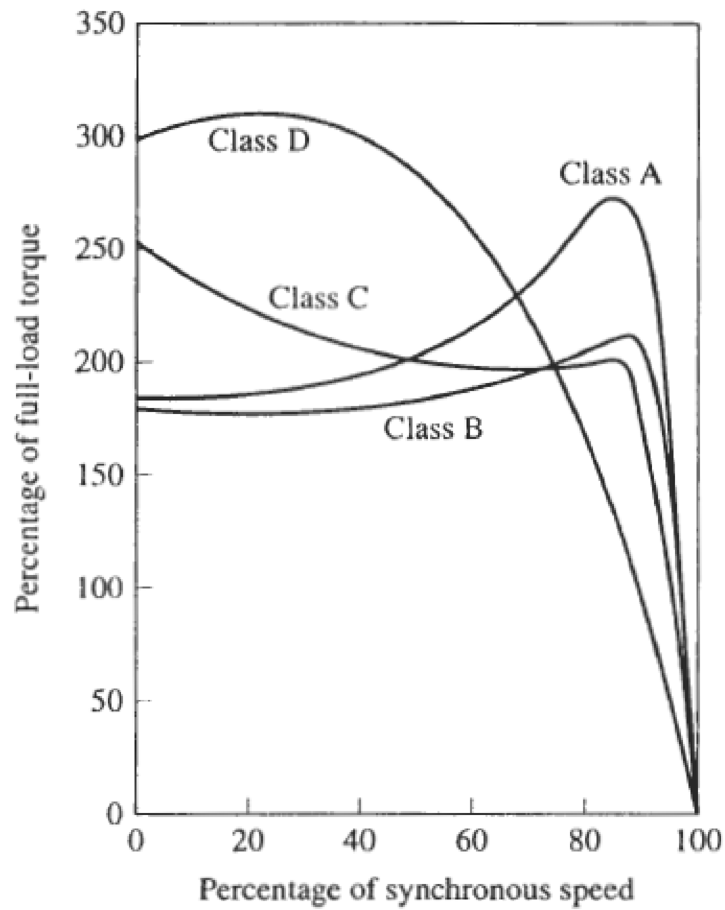


(b)

- At beginning, since ω_r is higher, than Xr is higher and the effective area is small. Thus, Rr is small
- At steady, since ω_r is smaller, than Xr is smaller and the effective area is large. Thus, Rr is large

NEMA design class A - D

- **Class A**
- the standard motor design
- **Class B**
- is similar to class A, but lower starting current, since R2 is lower
- **Class C**
- high starting torque with low starting current, low slip at full load. But pull-out torque is lower. The price is higher due to double-cage structure.
- **Class D**
- very high starting torque (low starting current) but high slip at full load.
  - Used for punch presses or shear applications
  - Used for slip power speed control



The way to determine X_1 and X_2

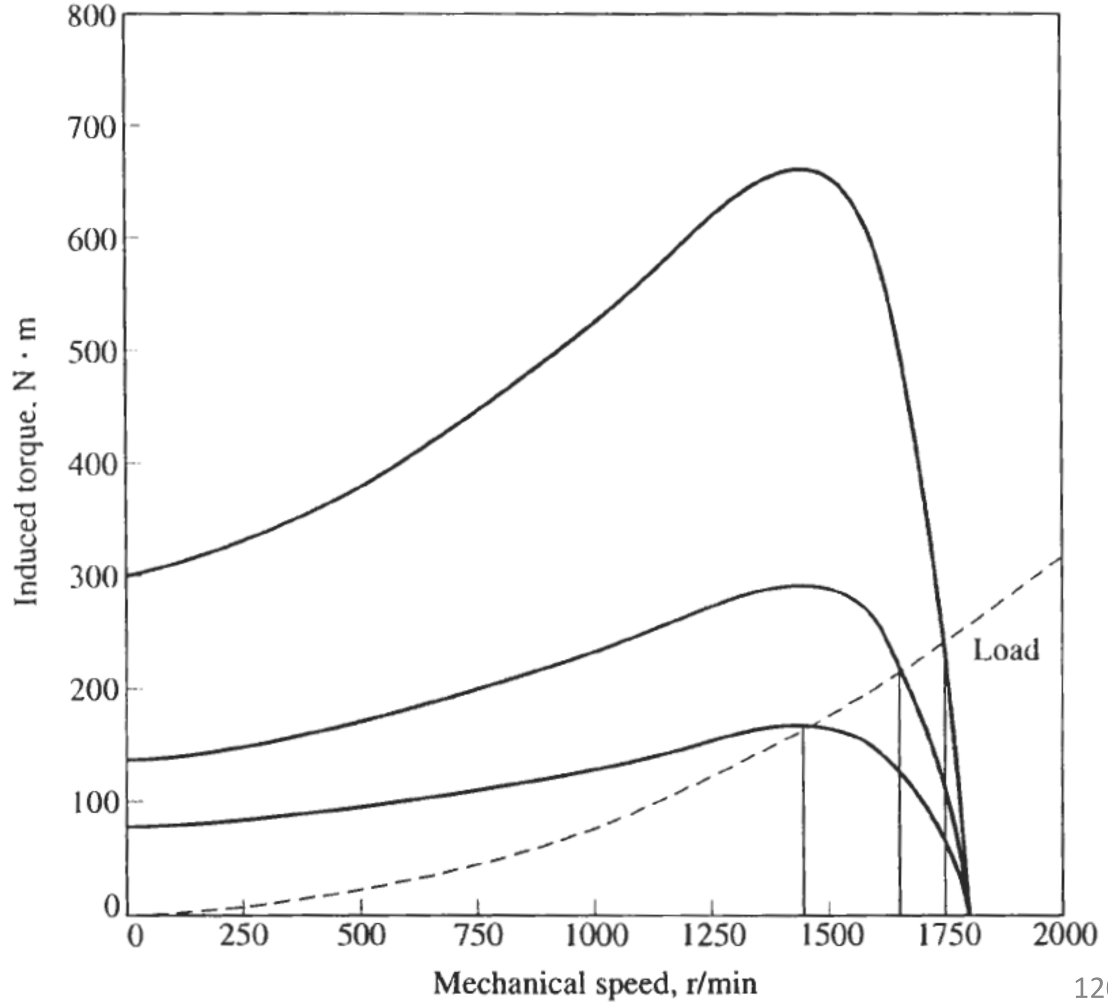
	X_1 and X_2 as functions of X_{LR}	
Rotor Design	X_1	X_2
Wound rotor	$0.5\ X_{LR}$	$0.5\ X_{LR}$
Design A	$0.5\ X_{LR}$	$0.5\ X_{LR}$
Design B	$0.4\ X_{LR}$	$0.6\ X_{LR}$
Design C	$0.3\ X_{LR}$	$0.7\ X_{LR}$
Design D	$0.5\ X_{LR}$	$0.5\ X_{LR}$

Speed Control of Induction Motors

- With the same load, how can we change the speed of induction motors
- Speed control by changing the line frequency
- Speed control by changing the line voltage
- Speed control by change the rotor resistance
- Speed control by change the number of poles

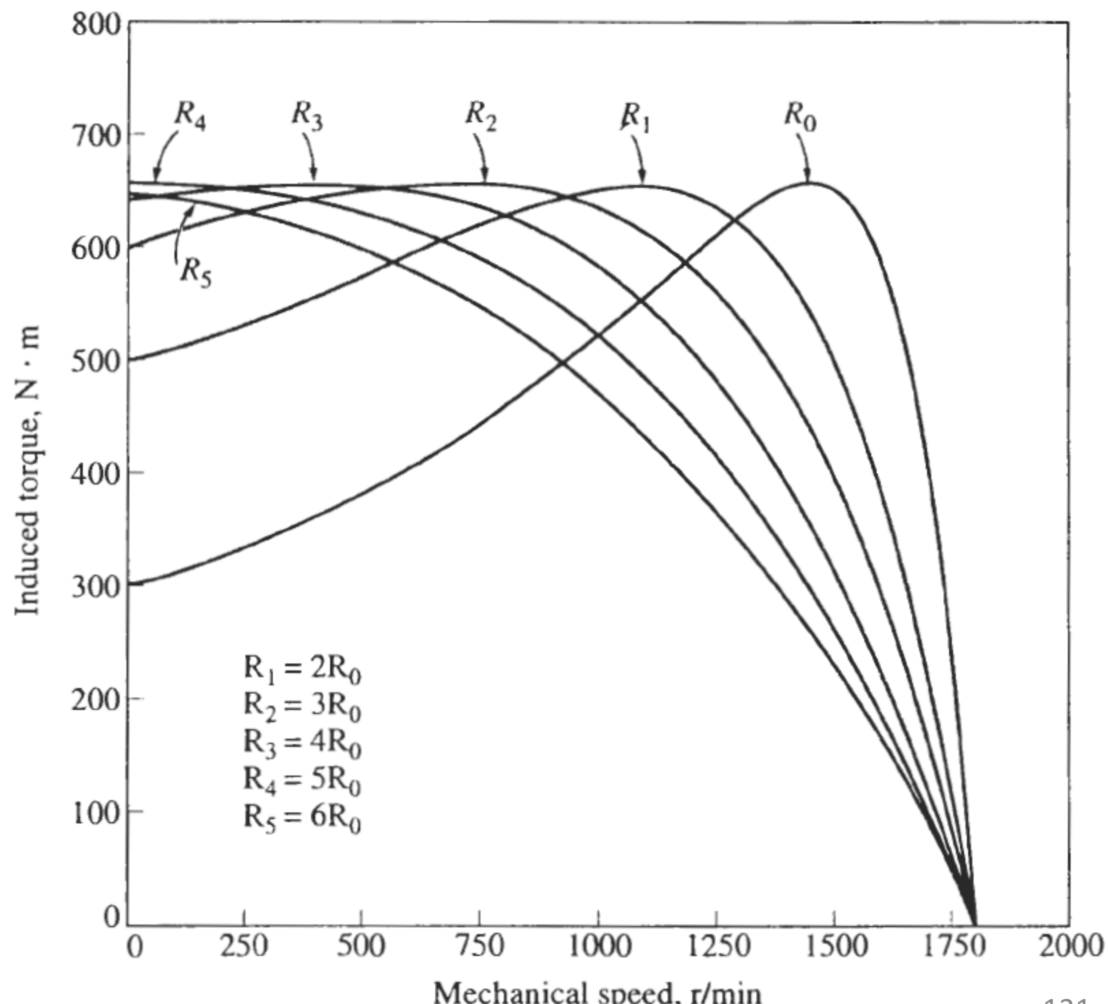
Speed Control by Changing the Line Voltage

- Speed control range is small
- Used for small motor driving fan

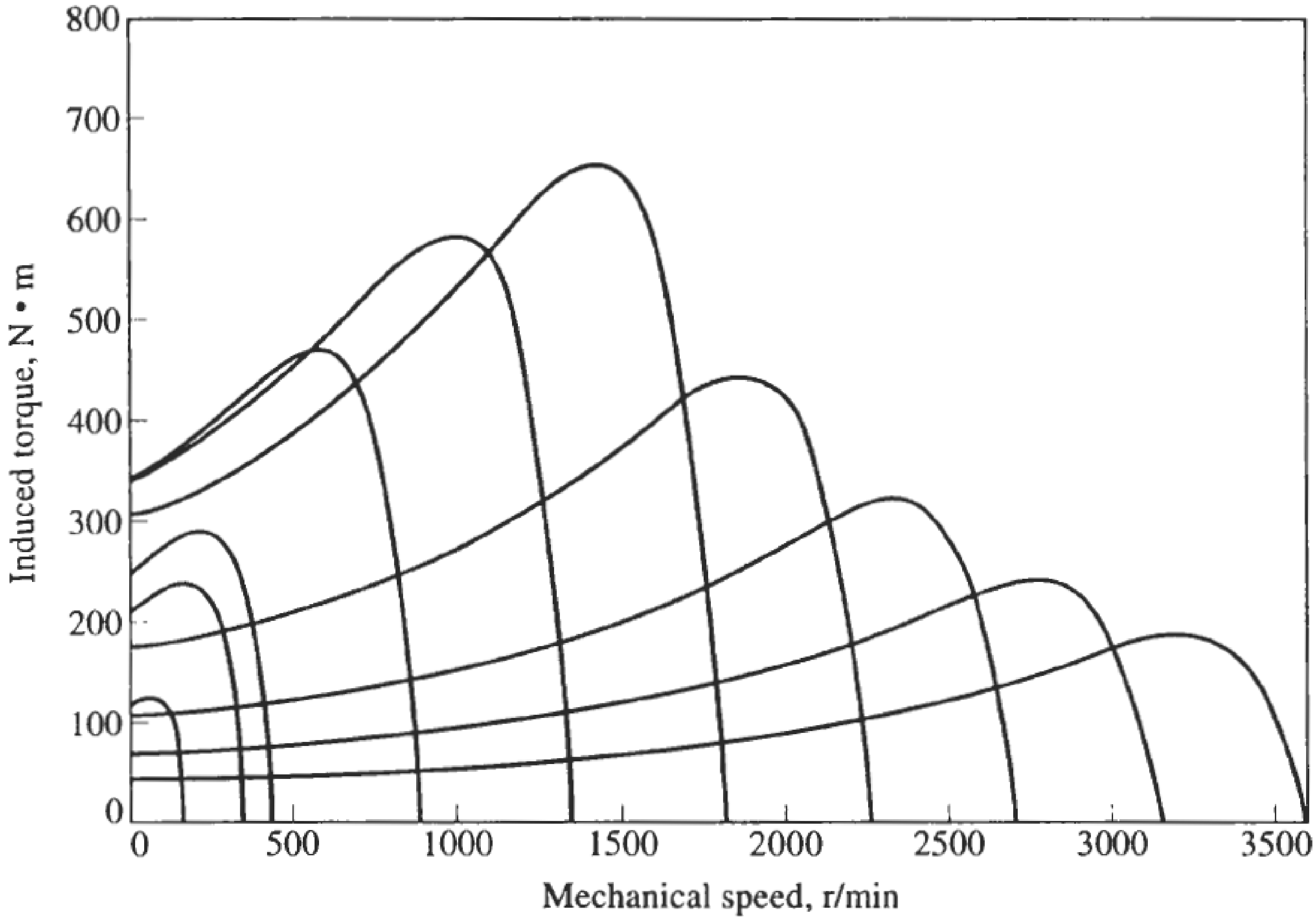


Speed Control by Changing the Rotor Resistance

- The efficiency is low
- Only used for short period due to low efficiency



Speed control by changing line frequency



Speed control by changing line frequency

- The range of speed control is large (5% base speed to double base speed)
- When running at speeds below the base speed, it is necessary to reduce the terminal voltage applied to the stator for preventing the flux **saturation**. Thus constant flux operation is presented
- When running above the base speed, it is necessary to maintain v(t) as constant to prevent **overvoltage**. Thus, constant power operation is presented.

$$v(t) = -N\frac{d\phi}{dt}$$

$$\phi(t) = \frac{1}{N_P}\int v(t)\,dt$$

$$= \frac{1}{N_P}\int V_M \sin \omega t\,dt$$

$$\boxed{\phi(t) = -\frac{V_M}{\omega N_P}\cos \omega t}$$

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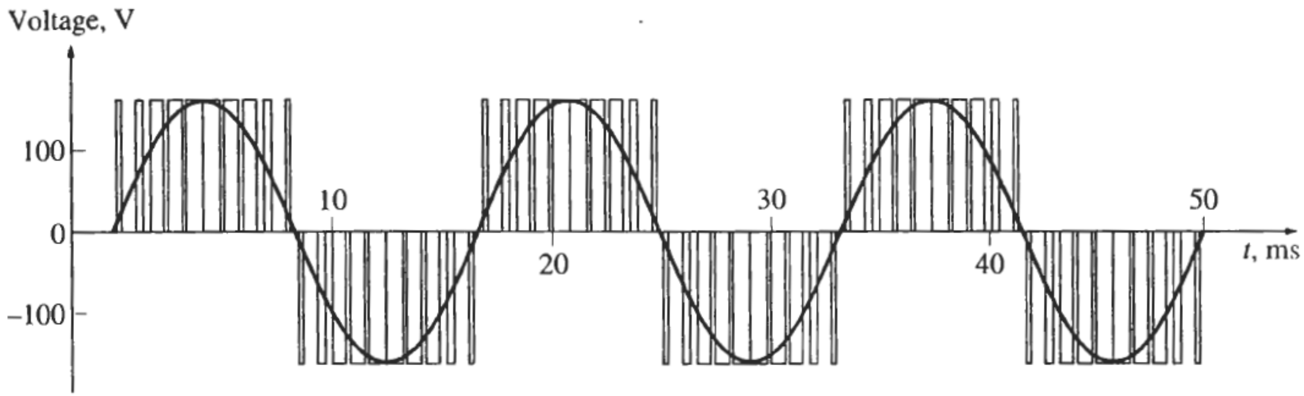
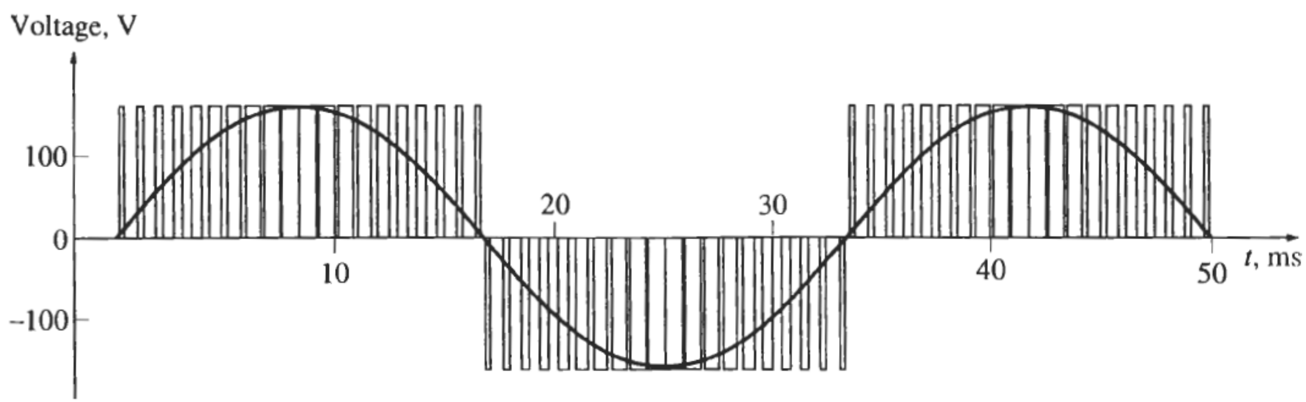
How to achieve the speed control

- Control the terminal voltage, frequency…
- All the above ideas can be achieved by solid state drives (power electronics techniques)

Solid state inverter drives



(a)



(b)

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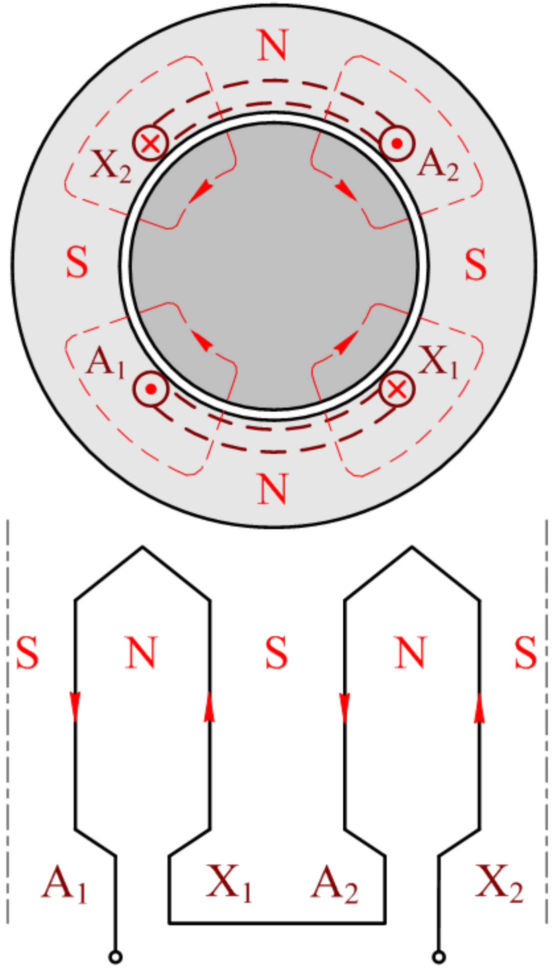
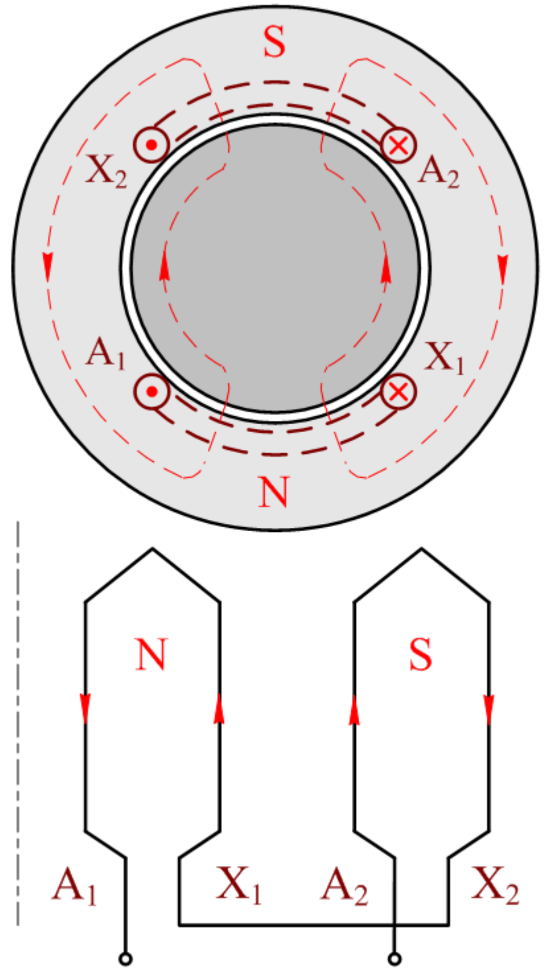
Speed control by change the number of poles

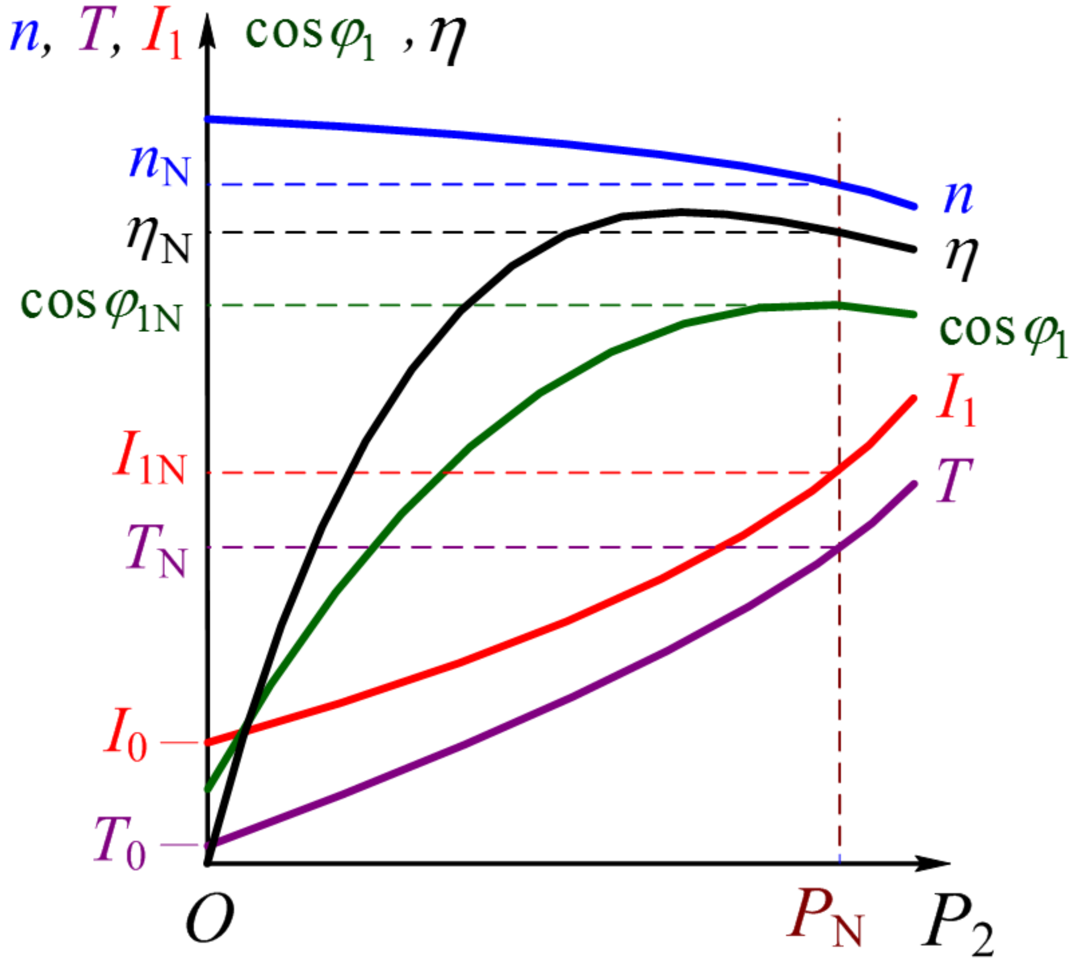


$p=2$



$p=1$

Load variation



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Starting of induction motor

- Starting current can be significant, but starting torque may not be sufficient

$$I_{\mathrm{s}} = \frac{U_1}{\sqrt{(R_1 + R_2')^2 + (X_{\sigma 1} + X'_{\sigma 2})^2}} = \frac{U_1}{|Z_{\mathrm{k}}|}$$

Starting of induction motor

- Reduced voltage start

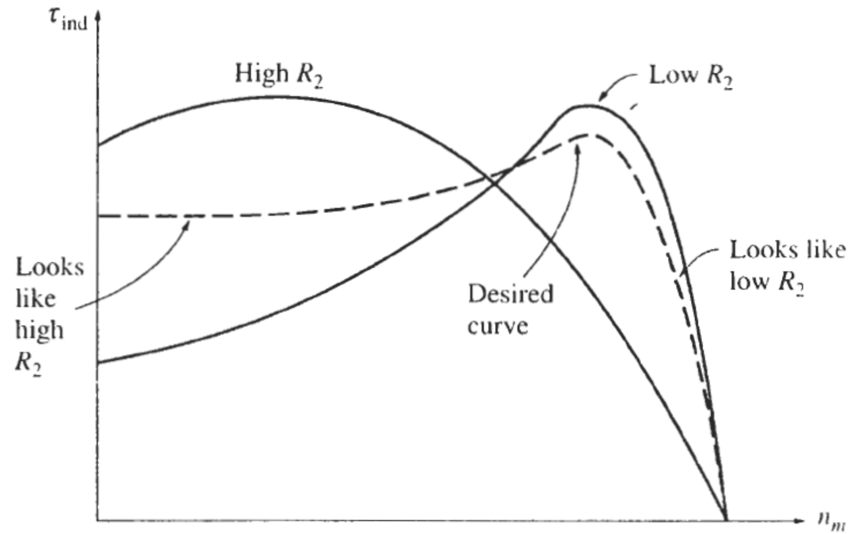
potential issue: $U_1 \downarrow \rightarrow\ T_s \propto U_1^2 \downarrow\downarrow$

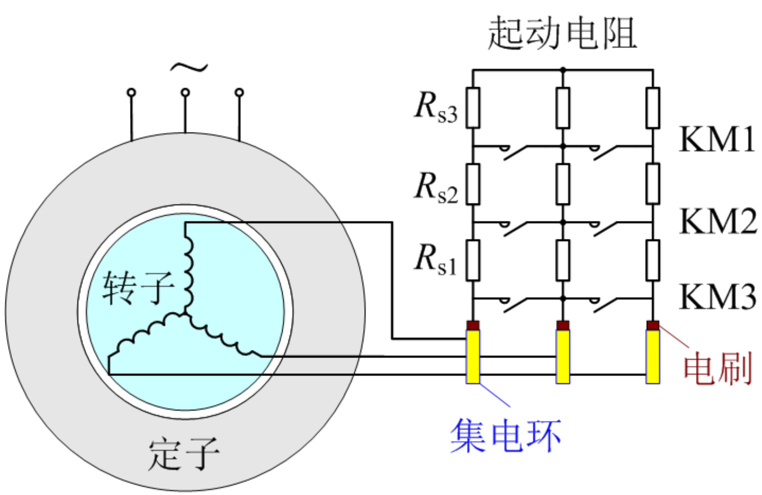
  - Only when starting torque is low

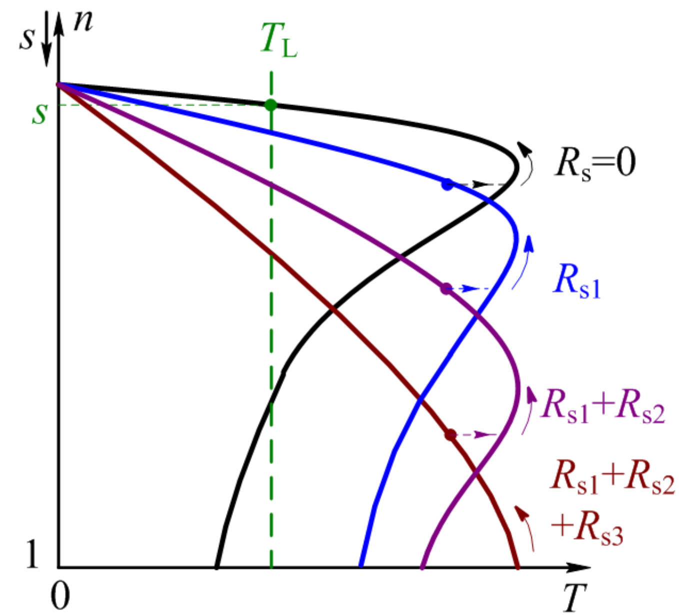
  - Use reactance in series with stator
  - Use autotransformer
  - Y- Δ start
  - Soft start

Starting of induction motor

- Double rotor cage start
- Use rotor resistance for wound rotor machine







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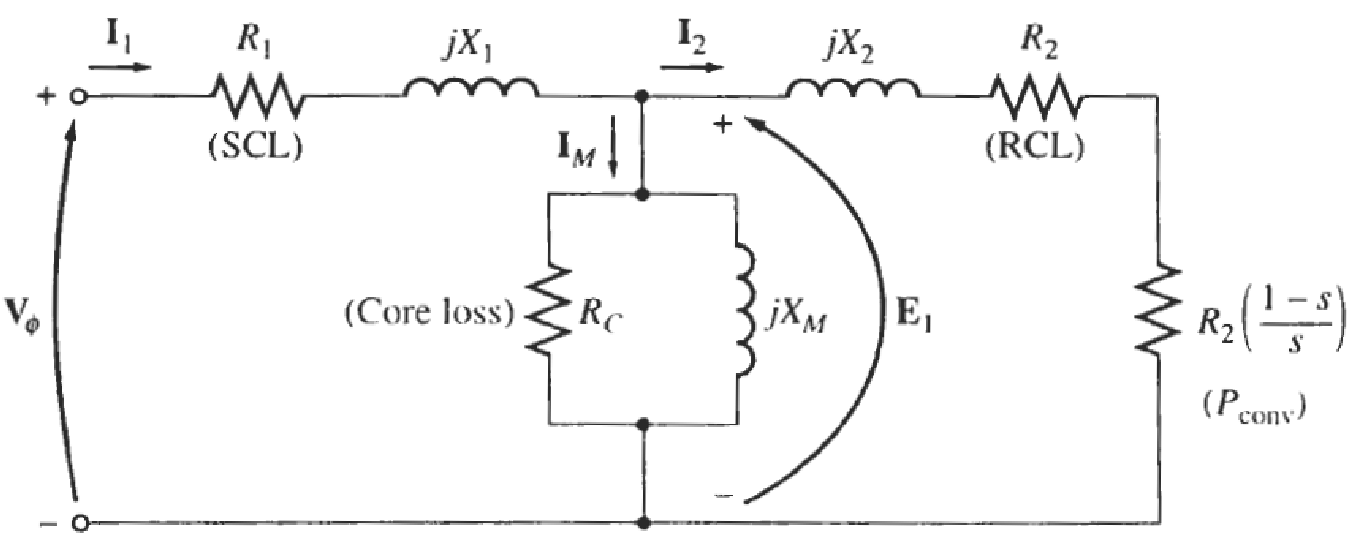
Induction generator

- Due to the simple structure
- How does the generator work?
  - $E_A = K\Phi\omega$
  - How can we build the Φ?

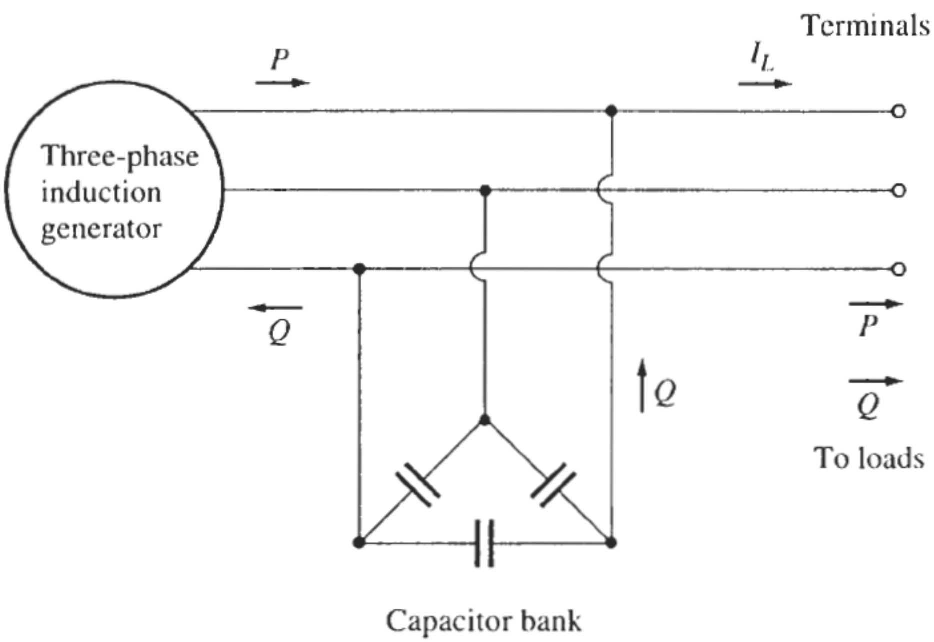
Induction generator

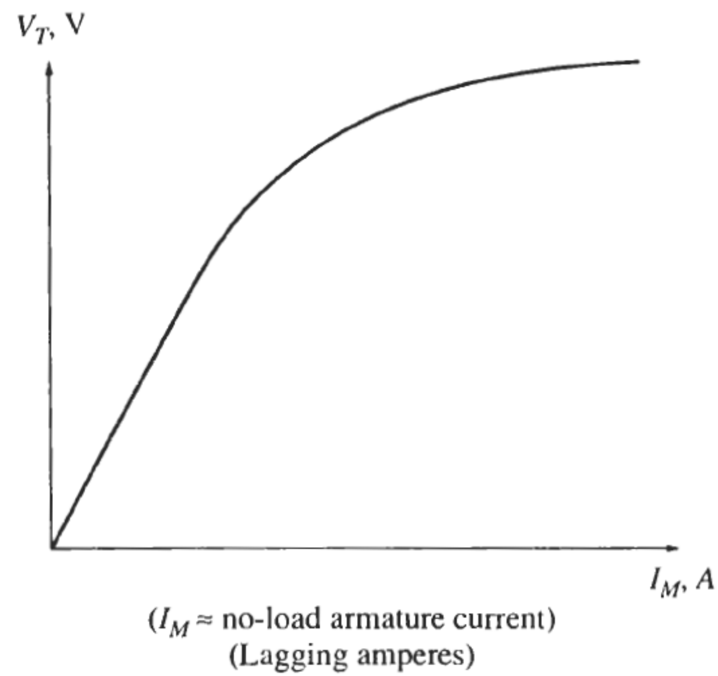
- We need the terminal voltage V_ϕ to build the flux (I_M)
- However, we also need the flux to build the terminal voltage …

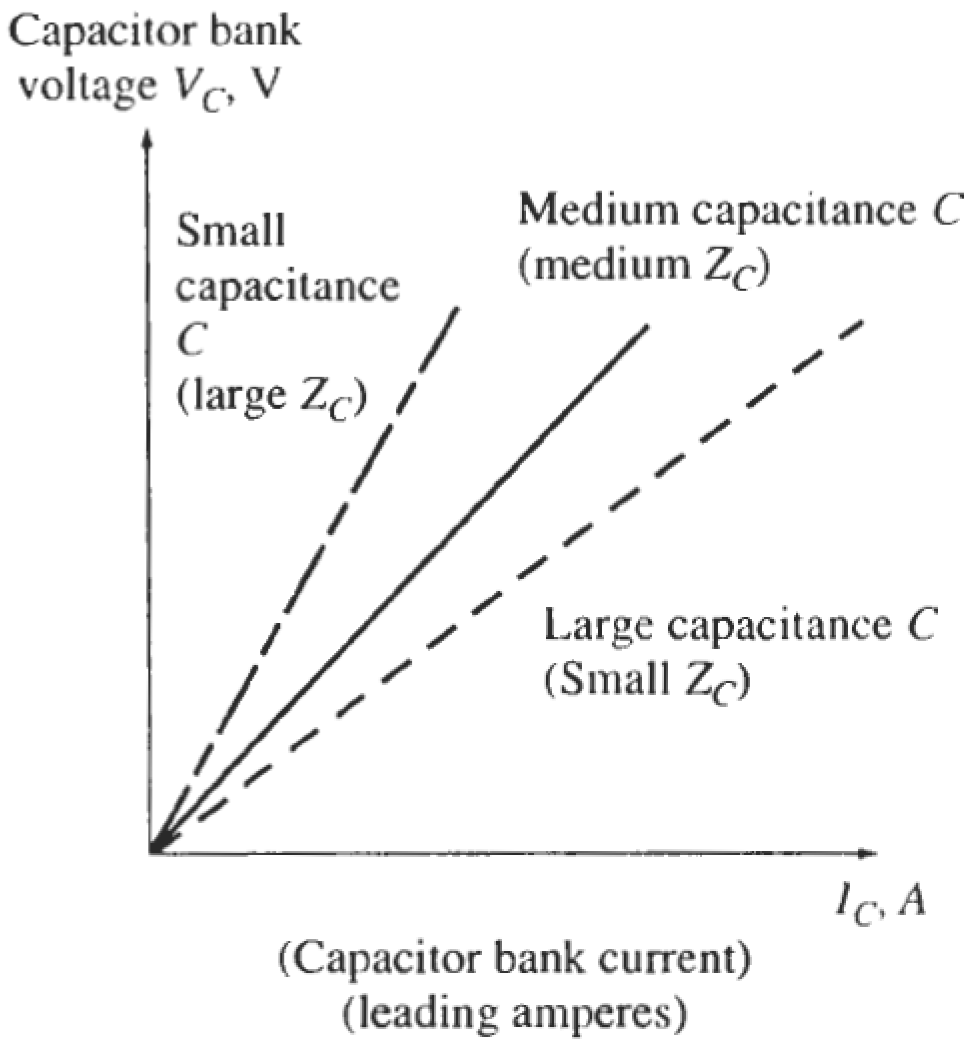


Induction generator

- The external capacitors are necessary for magnetizing current







Capacitor bank
voltage V_C, V
Small
capacitance
C
(large Z_C)
Medium capacitance C
(medium Z_C)
Large capacitance C
(Small Z_C)
I_C, A
(Capacitor bank current)
(leading amperes)

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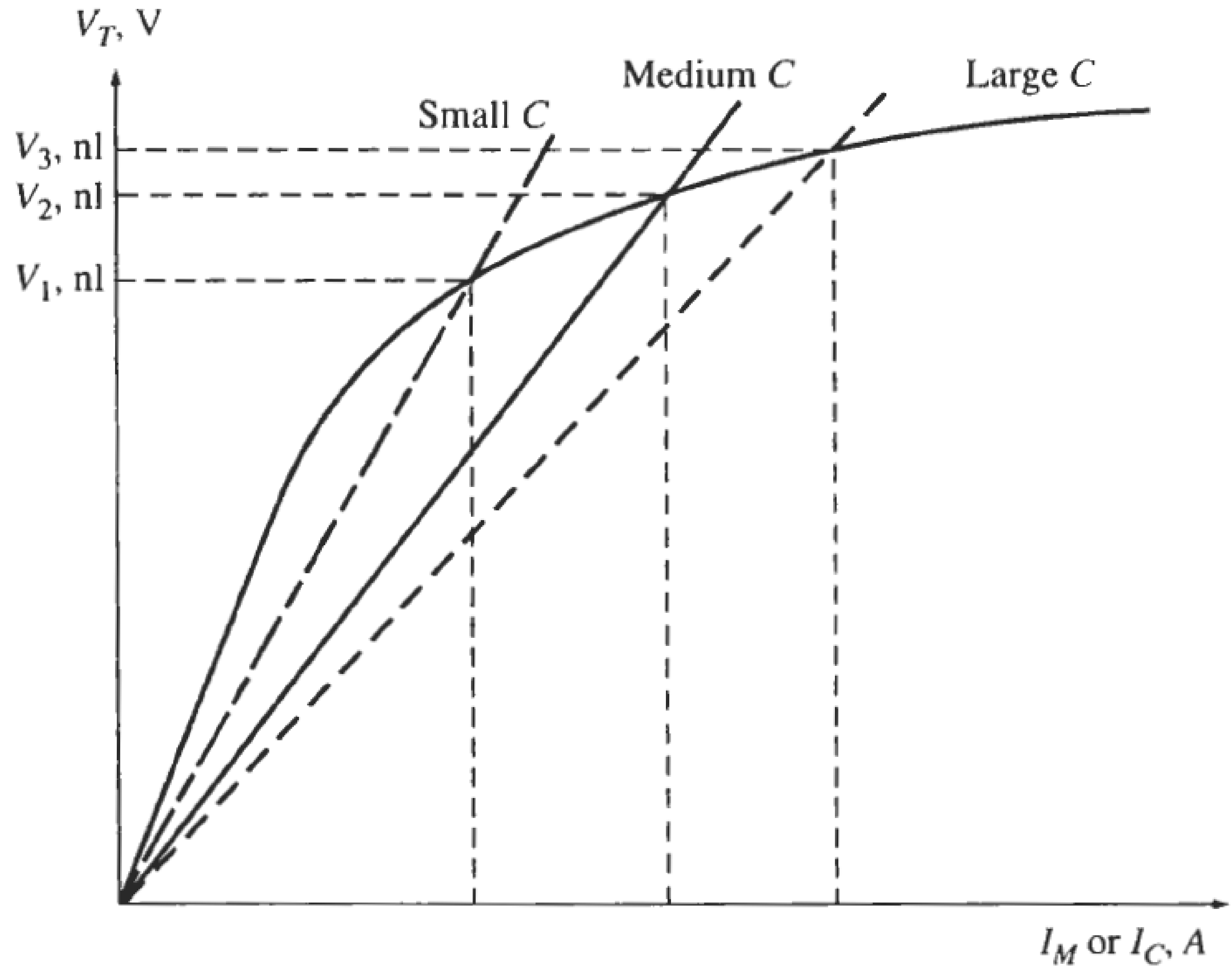
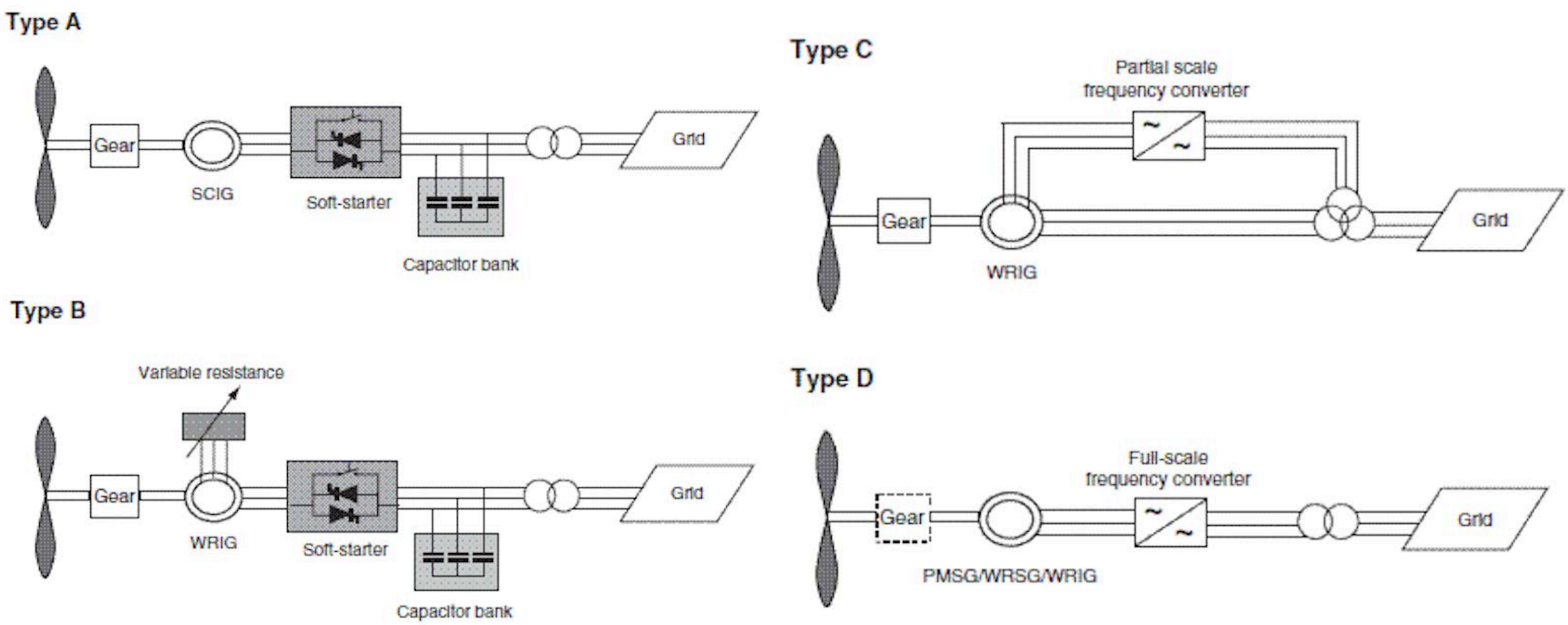
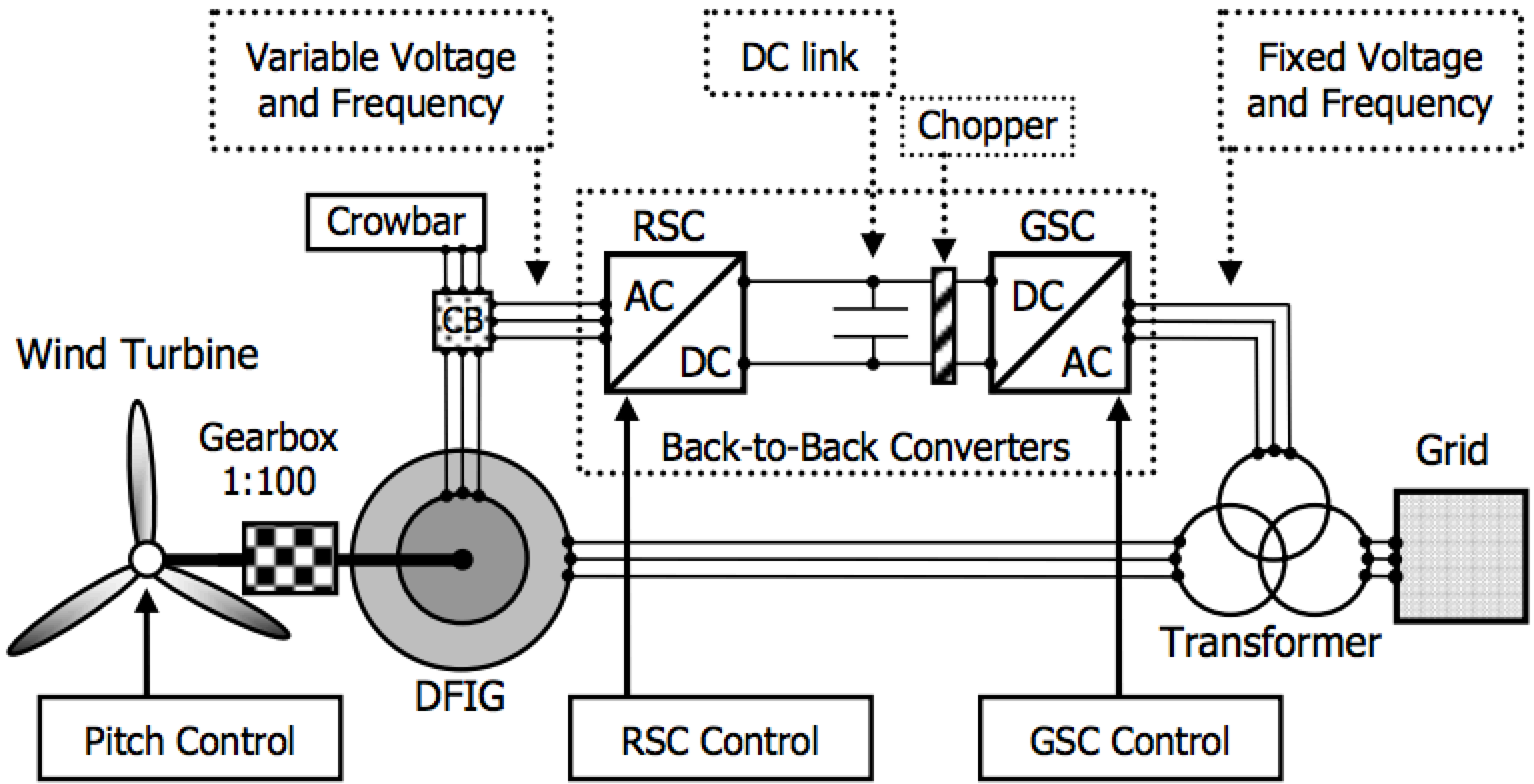

V_T, V
Medium C
Large C
Small C
V_3, nl
V_2, nl
V_1, nl
I_M or I_C, A

Wind Power Generator video

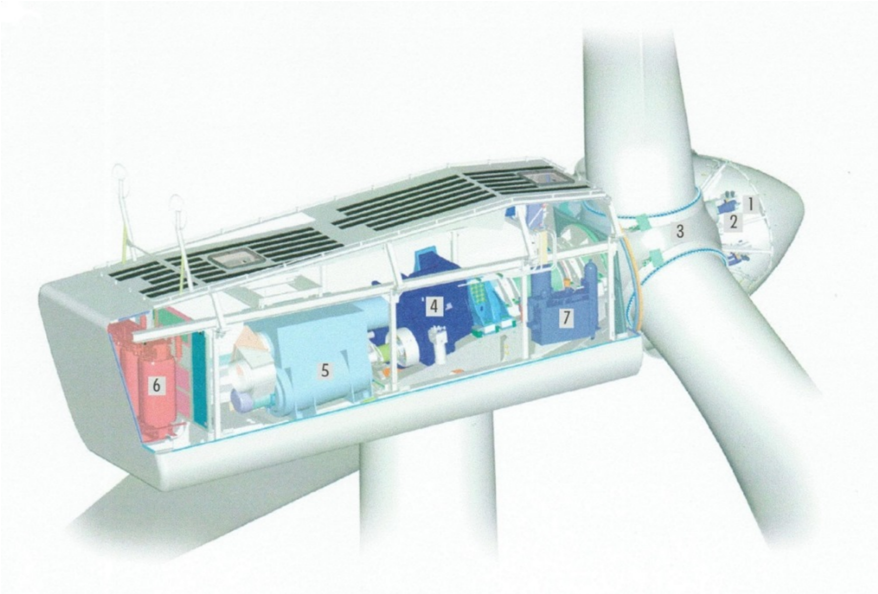
Types of Wind Power Generator



Double fed induction generator (DFIG)

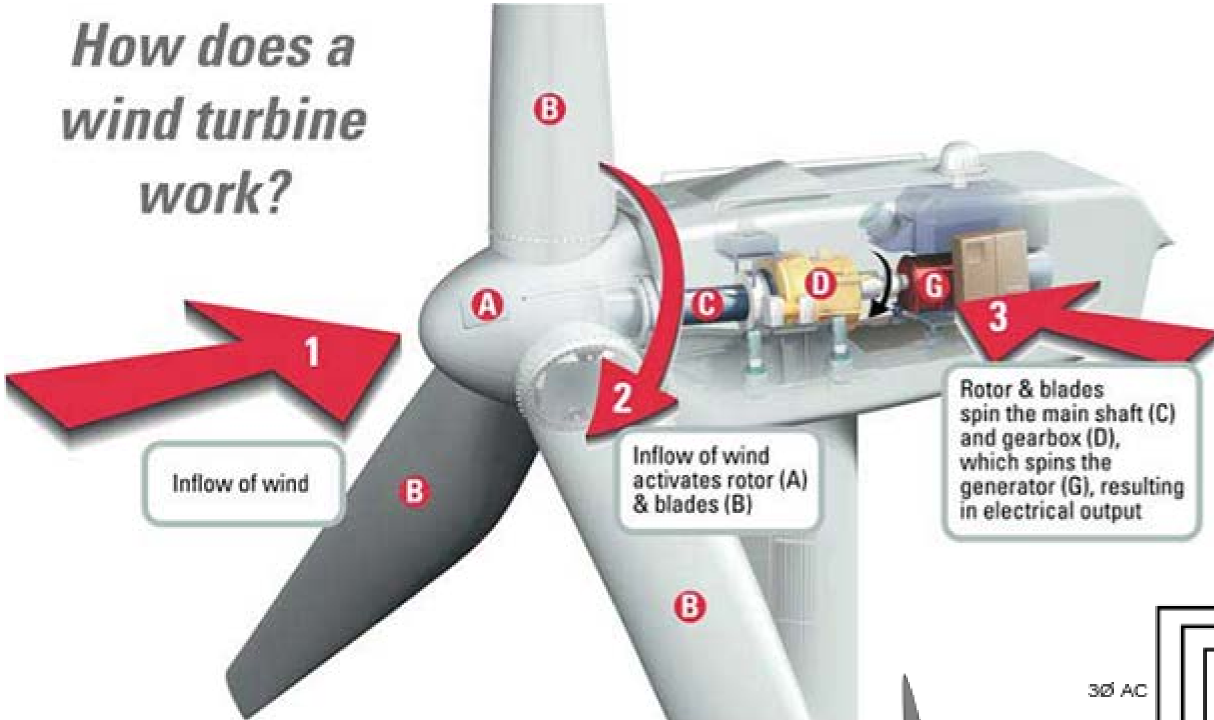


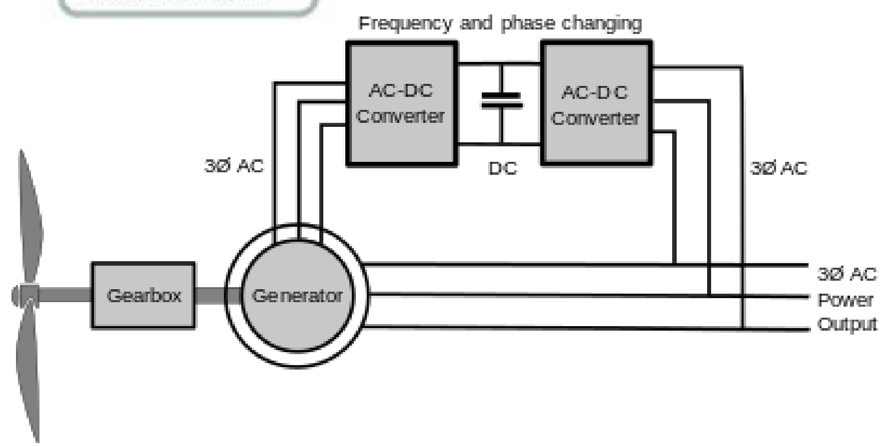
Inside a Wind Generator



1 – hub controller, 2 – “pitch” control cylinders, 3 – blade hub, 4 – gearbox, 5 – generator, 6 – high voltage transformer, 7 – hydraulic unit (Vestas, 2009)

Wind Generation Structure





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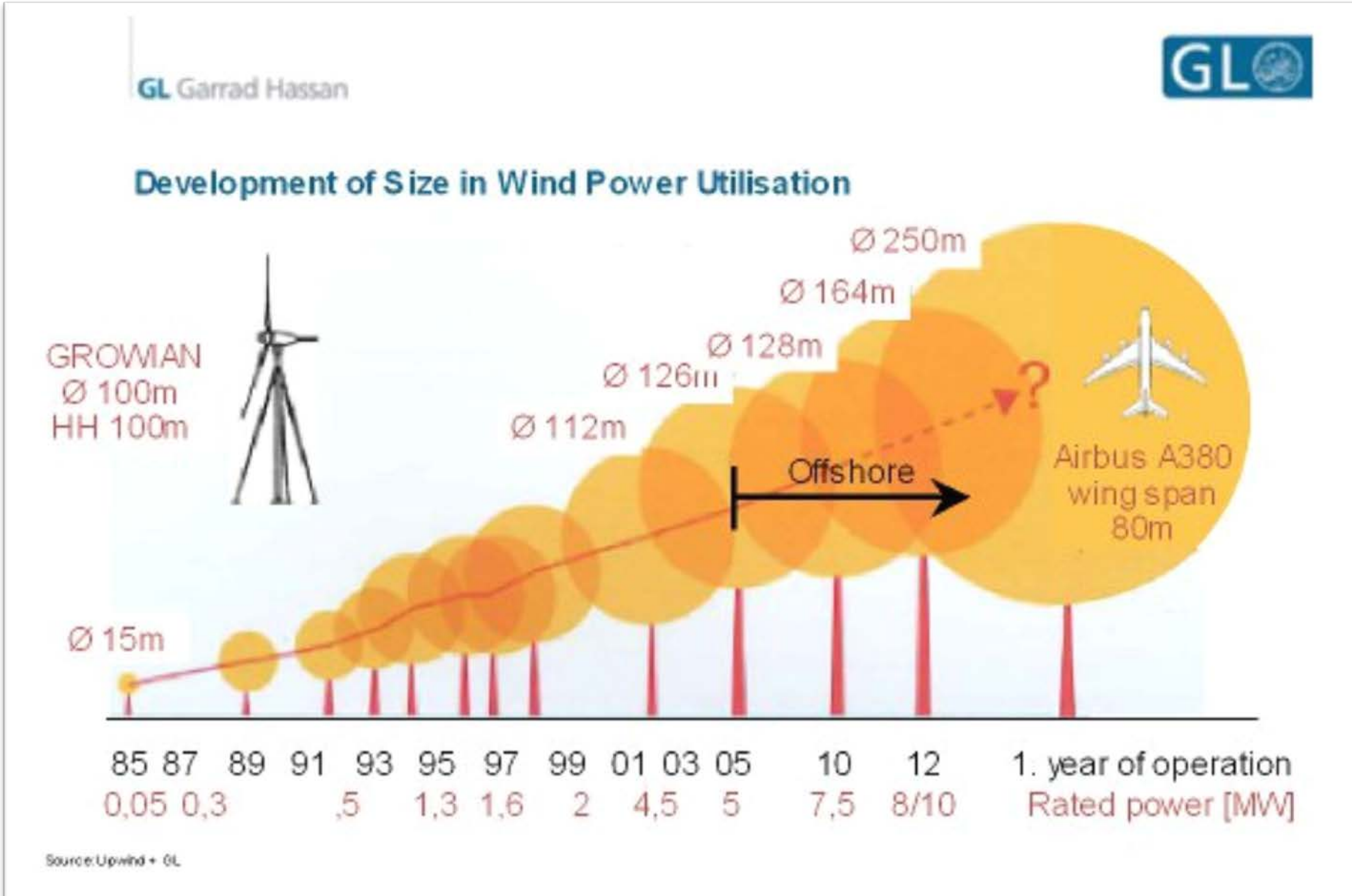
GL Garrad Hassan
GL
Development of Size in Wind Power Utilisation
GROWIAN
Ø 100m
HH 100m
Ø 15m
Ø 112m
Ø 126m
Ø 128m
Ø 164m
Ø 250m
?
Offshore
Airbus A380
wing span
80m
85 87 89 91 93 95 97 99 01 03 05 10 12 1. year of operation
0,05 0,3 ,5 1,3 1,6 2 4,5 5 7,5 8/10 Rated power [MW]
Source: Upwind + GL

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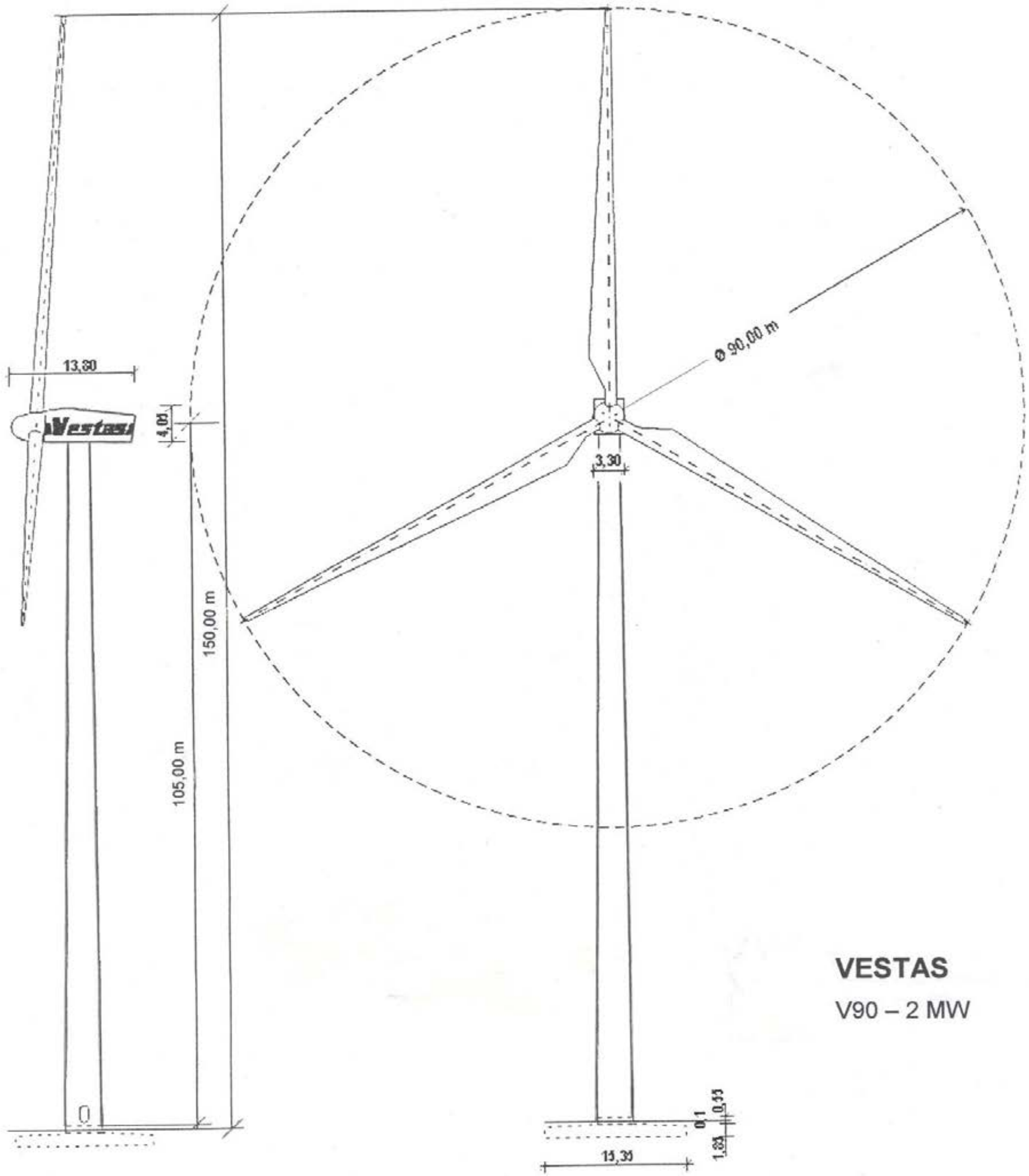
13,80
Vestas
4,05
3,30
Ø 90,00 m
150,00 m
105,00 m
15,30
VESTAS
V90 – 2 MW

Induction Motors

- Induction Motor Construction
- Basic Induction Motor Concepts
  - Induced Torque
  - Slip
  - Electrical Frequency on the Rotor
- Equivalent Circuit of an Induction Motor
- Induction Motor Equivalent Circuit Model Parameters
  - DC Test
  - No-Load Test
  - Locked-Rotor Test

Induction Motors

- Power and Torque in Induction Motor
- Torque-Speed Characteristics
- Torque-Speed Curve Variation
- Starting Induction Motor
- Speed Control of Induction Motor
- Solid-State Induction Motor Drives
- Induction Generator