对于洞室表面，采用柱坐标

包含自由场（入射波+反射波）和散射场（散射波+驻波）

1. 表示势函数

入射波：

$$\varphi^i = \sum_{n=-\infty}^{\infty} \alpha_n^i J_n(k_{\alpha 1} r_1) e^{in\theta_{a1}}$$

反射波：

$$\varphi_j^r = \sum_{n=-\infty}^{\infty} \alpha_{jn}^r J_n(k_{\alpha j} r_1) e^{in\theta_{a1}} \quad (j = 1,2,3)$$

$$\psi_m^r = \sum_{n=-\infty}^{\infty} b_{mn}^r J_n(k_{bm} r_1) e^{in\theta_{bm}} \text{（m=1,2）}$$

其中$\alpha_n^i = e^{-ik_{a1}h\cos\theta_{a1}} e^{in\theta_{a1}}$，$\alpha_{jn}^r = (-1)^n A_j\ e^{ik_{aj}h\cos\theta_{aj}} e^{-in\theta_{aj}}$，$b_{mn}^r = (-1)^n B_m e^{ik_{bm}h\cos\theta_{bm}} e^{-in\theta_{bm}}$

散射波：

$$\varphi_j^s = \sum_{n=-\infty}^{\infty} \alpha_{jn}^s H_n(k_{\alpha j} r_1) e^{in\theta_{a1}} \quad (j = 1,2,3)$$

$$\psi^s{}_m = \sum_{n=-\infty}^{\infty} b_{mn}^s H_n(k_{\beta,m} r_1) e^{in\theta_{\beta,m}} \quad \text{（m=1,2）}$$

驻波：

$$\varphi_j^R = \sum_{n=-\infty}^{\infty} \alpha_{jn}^R J_n(k_{\alpha j} r_1) e^{in\theta_{aj}} \quad (j = 1,2,3)$$

$$\psi_m^R = \sum_{n=-\infty}^{\infty} b_{mn}^R J_n(k_{bm} r_1) e^{in\theta_{bm}} \quad \text{（m=1,2）}$$

2. 求位移分量

固相

径向位移分量

$$\begin{aligned} u_r^S = & \sum_{n=-\infty}^{\infty} \alpha_n^i k_{a1} J_n'(k_{a1} r_1) e^{in\theta_{a1}} \\ & + \sum_{j=1}^{3} \sum_{n=-\infty}^{\infty} \alpha_{jn}^r k_{aj} J_n'(k_{aj} r_1) e^{in\theta_{aj}} \\ & + \sum_{j=1}^{3} \sum_{n=-\infty}^{\infty} \alpha_{jn}^s k_{aj} H_n'(k_{aj} r_1) e^{in\theta_{aj}} \\ & + \sum_{j=1}^{3} \sum_{n=-\infty}^{\infty} \alpha_{jn}^R k_{aj} J_n'(k_{aj} r_1) e^{in\theta_{aj}} \\ & + \sum_{m=1}^{2} \sum_{n=-\infty}^{\infty} b_{mn}^r \frac{in}{r_1} J_n(k_{bm} r_1) e^{in\theta_{bm}} \\ & + \sum_{m=1}^{2} \sum_{n=-\infty}^{\infty} b_{mn}^s \frac{in}{r_1} H_n(k_{bm} r_1) e^{in\theta_{bm}} \\ & + \sum_{m=1}^{2} \sum_{n=-\infty}^{\infty} b_{mn}^R \frac{in}{r_1} J_n(k_{bm} r_1) e^{in\theta_{bm}} \end{aligned}$$

环向位移分量

$$\begin{aligned} u_\theta^S = & \sum_{n=-\infty}^{\infty} \alpha_n^i \frac{in}{r_1} J_n(k_{a1} r_1) e^{in\theta_{a1}} \\ & + \sum_{j=1}^{3} \sum_{n=-\infty}^{\infty} \alpha_{jn}^r \frac{in}{r_1} J_n(k_{aj} r_1) e^{in\theta_{aj}} \\ & + \sum_{j=1}^{3} \sum_{n=-\infty}^{\infty} \alpha_{jn}^s \frac{in}{r_1} H_n(k_{aj} r_1) e^{in\theta_{aj}} \\ & + \sum_{j=1}^{3} \sum_{n=-\infty}^{\infty} \alpha_{jn}^R \frac{in}{r_1} J_n(k_{aj} r_1) e^{in\theta_{aj}} \\ & - \sum_{m=1}^{2} \sum_{n=-\infty}^{\infty} b_{mn}^r k_{bm} J_n'(k_{bm} r_1) e^{in\theta_{bm}} \\ & - \sum_{m=1}^{2} \sum_{n=-\infty}^{\infty} b_{mn}^s k_{bm} H_n'(k_{bm} r_1) e^{in\theta_{bm}} \\ & - \sum_{m=1}^{2} \sum_{n=-\infty}^{\infty} b_{mn}^R k_{bm} J_n'(k_{bm} r_1) e^{in\theta_{bm}} \end{aligned}$$

液相

径向位移分量

$$
\begin{aligned}
u_r^L = \sum_{j=1}^{3}\alpha_j^L\Bigg[&\sum_{n=-\infty}^{\infty}\alpha_n^i\,k_{a1}J_n'(k_{a1}r_1)e^{in\theta_{a1}}\\
&+\sum_{n=-\infty}^{\infty}\alpha_{jn}^r\,k_{aj}J_n'(k_{aj}r_1)e^{in\theta_{aj}}\\
&+\sum_{n=-\infty}^{\infty}a_{jn}^s\,k_{aj}H_n'(k_{aj}r_1)e^{in\theta_{aj}}\\
&+\sum_{n=-\infty}^{\infty}\alpha_{jn}^R\,k_{aj}J_n'(k_{aj}r_1)e^{in\theta_{aj}}\Bigg]\\
+\sum_{m=1}^{2}\beta_m^L\Bigg[&\sum_{n=-\infty}^{\infty}b_{mn}^r\frac{in}{r_1}J_n(k_{bm}r_1)e^{in\theta_{bm}}\\
&+\sum_{n=-\infty}^{\infty}b_{mn}^s\frac{in}{r_1}H_n(k_{bm}r_1)e^{in\theta_{bm}}\\
&+\sum_{n=-\infty}^{\infty}b_{mn}^R\frac{in}{r_1}J_n(k_{bm}r_1)e^{in\theta_{bm}}\Bigg]
\end{aligned}
$$

环向位移分量

$$
\begin{aligned}
u_\theta^L = \sum_{j=1}^{3}\alpha_j^L\Bigg[&\sum_{n=-\infty}^{\infty}\alpha_n^i\frac{in}{r_1}J_n(k_{a1}r_1)e^{in\theta_{a1}}\\
&+\sum_{n=-\infty}^{\infty}\alpha_{jn}^r\frac{in}{r_1}J_n(k_{aj}r_1)e^{in\theta_{aj}}\\
&+\sum_{n=-\infty}^{\infty}a_{jn}^s\frac{in}{r_1}H_n(k_{aj}r_1)e^{in\theta_{aj}}\\
&+\sum_{n=-\infty}^{\infty}\alpha_{jn}^R\frac{in}{r_1}J_n(k_{aj}r_1)e^{in\theta_{aj}}\Bigg]\\
+\sum_{m=1}^{2}\beta_m^L\Bigg[&-\sum_{n=-\infty}^{\infty}b_{mn}^r\,k_{bm}J_n'(k_{bm}r_1)e^{in\theta_{bm}}\\
&-\sum_{n=-\infty}^{\infty}b_{mn}^s\,k_{bm}H_n'(k_{bm}r_1)e^{in\theta_{bm}}\\
&-\sum_{n=-\infty}^{\infty}b_{mn}^R\,k_{bm}J_n'(k_{bm}r_1)e^{in\theta_{bm}}\Bigg]
\end{aligned}
$$

冰相

径向位移分量

$$
\begin{aligned}
u_r^l = \sum_{j=1}^{3}\alpha_j^l\Bigg[&\sum_{n=-\infty}^{\infty}\alpha_n^i\,k_{a1}J_n'(k_{a1}r_1)e^{in\theta_{a1}}\\
&+\sum_{n=-\infty}^{\infty}\alpha_{jn}^r\,k_{aj}J_n'(k_{aj}r_1)e^{in\theta_{aj}}\\
&+\sum_{n=-\infty}^{\infty}a_{jn}^s\,k_{aj}H_n'(k_{aj}r_1)e^{in\theta_{aj}}\\
&+\sum_{n=-\infty}^{\infty}\alpha_{jn}^R\,k_{aj}J_n'(k_{aj}r_1)e^{in\theta_{aj}}\Bigg]\\
+\sum_{m=1}^{2}\beta_m^l\Bigg[&\sum_{n=-\infty}^{\infty}b_{mn}^r\frac{in}{r_1}J_n(k_{bm}r_1)e^{in\theta_{bm}}\\
&+\sum_{n=-\infty}^{\infty}b_{mn}^s\frac{in}{r_1}H_n(k_{bm}r_1)e^{in\theta_{bm}}\\
&+\sum_{n=-\infty}^{\infty}b_{mn}^R\frac{in}{r_1}J_n(k_{bm}r_1)e^{in\theta_{bm}}\Bigg]
\end{aligned}
$$

环向位移分量

$$
\begin{aligned}
u_\theta^l = \sum_{j=1}^{3}\alpha_j^l\Bigg[&\sum_{n=-\infty}^{\infty}\alpha_n^i\frac{in}{r_1}J_n(k_{a1}r_1)e^{in\theta_{a1}}\\
&+\sum_{n=-\infty}^{\infty}\alpha_{jn}^r\frac{in}{r_1}J_n(k_{aj}r_1)e^{in\theta_{aj}}\\
&+\sum_{n=-\infty}^{\infty}a_{jn}^s\frac{in}{r_1}H_n(k_{aj}r_1)e^{in\theta_{aj}}\\
&+\sum_{n=-\infty}^{\infty}\alpha_{jn}^R\frac{in}{r_1}J_n(k_{aj}r_1)e^{in\theta_{aj}}\Bigg]\\
+\sum_{m=1}^{2}\beta_m^l\Bigg[&-\sum_{n=-\infty}^{\infty}b_{mn}^r\,k_{bm}J_n'(k_{bm}r_1)e^{in\theta_{bm}}\\
&-\sum_{n=-\infty}^{\infty}b_{mn}^s\,k_{bm}H_n'(k_{bm}r_1)e^{in\theta_{bm}}\\
&-\sum_{n=-\infty}^{\infty}b_{mn}^R\,k_{bm}J_n'(k_{bm}r_1)e^{in\theta_{bm}}\Bigg]
\end{aligned}
$$

3. 求散度

$$
\begin{aligned}
\nabla\cdot u^S = &-\sum_{n=-\infty}^{\infty}\alpha_n^i\,k_{a1}^2J_n(k_{a1}r_1)e^{in\theta_{a1}}\\
&-\sum_{j=1}^{3}\sum_{n=-\infty}^{\infty}\alpha_{jn}^r\,k_{aj}^2J_n(k_{aj}r_1)e^{in\theta_{aj}}\\
&-\sum_{j=1}^{3}\sum_{n=-\infty}^{\infty}a_{jn}^s\,k_{aj}^2H_n(k_{aj}r_1)e^{in\theta_{aj}}\\
&-\sum_{j=1}^{3}\sum_{n=-\infty}^{\infty}\alpha_{jn}^R\,k_{aj}^2J_n(k_{aj}r_1)e^{in\theta_{aj}}
\end{aligned}
$$

$$
\nabla\cdot u^L = -\sum_{j=1}^{3}\alpha_j^L\left[\begin{aligned}
&\sum_{n=-\infty}^{\infty}\alpha_n^i\,\delta_{j1}k_{a1}^2J_n(k_{a1}r_1)e^{in\theta_{a1}}\\
&+\sum_{n=-\infty}^{\infty}\alpha_{jn}^r\,k_{aj}^2J_n(k_{aj}r_1)e^{in\theta_{aj}}\\
&+\sum_{n=-\infty}^{\infty}a_{jn}^s\,k_{aj}^2H_n(k_{aj}r_1)e^{in\theta_{aj}}\\
&+\sum_{n=-\infty}^{\infty}\alpha_{jn}^R\,k_{aj}^2J_n(k_{aj}r_1)e^{in\theta_{aj}}
\end{aligned}\right]
$$

$$
\nabla\cdot u^l = -\sum_{j=1}^{3}\alpha_j^l\left[\begin{aligned}
&\sum_{n=-\infty}^{\infty}\alpha_n^i\,\delta_{j1}k_{a1}^2J_n(k_{a1}r_1)e^{in\theta_{a1}}\\
&+\sum_{n=-\infty}^{\infty}\alpha_{jn}^r\,k_{aj}^2J_n(k_{aj}r_1)e^{in\theta_{aj}}\\
&+\sum_{n=-\infty}^{\infty}a_{jn}^s\,k_{aj}^2H_n(k_{aj}r_1)e^{in\theta_{aj}}\\
&+\sum_{n=-\infty}^{\infty}\alpha_{jn}^R\,k_{aj}^2J_n(k_{aj}r_1)e^{in\theta_{aj}}
\end{aligned}\right]
$$

4.求应变张量

固相

$$
\begin{aligned}
\varepsilon_{rr}^{S} = & \sum_{n=-\infty}^{\infty} \alpha_n^i k_{a1}^2 J_n''(k_{a1}r_1) e^{in\theta_{a1}} \\
& + \sum_{j=1}^{3}\sum_{n=-\infty}^{\infty} \alpha_{jn}^r k_{aj}^2 J_n''(k_{aj}r_1) e^{in\theta_{aj}} \\
& + \sum_{j=1}^{3}\sum_{n=-\infty}^{\infty} a_{jn}^s k_{aj}^2 H_n''(k_{aj}r_1) e^{in\theta_{aj}} \\
& + \sum_{j=1}^{3}\sum_{n=-\infty}^{\infty} \alpha_{jn}^R k_{aj}^2 J_n''(k_{aj}r_1) e^{in\theta_{aj}} \\
& + \sum_{m=1}^{2}\sum_{n=-\infty}^{\infty} b_{mn}^r \frac{in}{r_1} k_{bm} J_n'(k_{bm}r_1) e^{in\theta_{bm}} \\
& - \sum_{m=1}^{2}\sum_{n=-\infty}^{\infty} b_{mn}^r \frac{in}{r_1^2} J_n(k_{bm}r_1) e^{in\theta_{bm}} \\
& + \sum_{m=1}^{2}\sum_{n=-\infty}^{\infty} b_{mn}^s \frac{in}{r_1} k_{bm} H_n'(k_{bm}r_1) e^{in\theta_{bm}} \\
& - \sum_{m=1}^{2}\sum_{n=-\infty}^{\infty} b_{mn}^s \frac{in}{r_1^2} H_n(k_{bm}r_1) e^{in\theta_{bm}} \\
& + \sum_{m=1}^{2}\sum_{n=-\infty}^{\infty} b_{mn}^R \frac{in}{r_1} k_{bm} J_n'(k_{bm}r_1) e^{in\theta_{bm}} \\
& - \sum_{m=1}^{2}\sum_{n=-\infty}^{\infty} b_{mn}^R \frac{in}{r_1^2} J_n(k_{bm}r_1) e^{in\theta_{bm}}
\end{aligned}
$$

$$
\begin{aligned}
\varepsilon_{\theta\theta}^{S} = & \sum_{n=-\infty}^{\infty} \alpha_n^i \left[\frac{1}{r_1} k_{a1} J_n'(k_{a1}r_1) - \frac{n^2}{r_1^2} J_n(k_{a1}r_1)\right] e^{in\theta_{a1}} \\
& + \sum_{j=1}^{3}\sum_{n=-\infty}^{\infty} \alpha_{jn}^r \left[\frac{1}{r_1} k_{aj} J_n'(k_{aj}r_1) - \frac{n^2}{r_1^2} J_n(k_{aj}r_1)\right] e^{in\theta_{aj}} \\
& + \sum_{j=1}^{3}\sum_{n=-\infty}^{\infty} a_{jn}^s \left[\frac{1}{r_1} k_{aj} H_n'(k_{aj}r_1) - \frac{n^2}{r_1^2} H_n(k_{aj}r_1)\right] e^{in\theta_{aj}} \\
& + \sum_{j=1}^{3}\sum_{n=-\infty}^{\infty} \alpha_{jn}^R \left[\frac{1}{r_1} k_{aj} J_n'(k_{aj}r_1) - \frac{n^2}{r_1^2} J_n(k_{aj}r_1)\right] e^{in\theta_{aj}} \\
& + \sum_{m=1}^{2}\sum_{n=-\infty}^{\infty} b_{mn}^r \left[-\frac{in}{r_1^2} J_n(k_{bm}r_1) - k_{bm}^2 J_n''(k_{bm}r_1)\right] e^{in\theta_{bm}} \\
& + \sum_{m=1}^{2}\sum_{n=-\infty}^{\infty} b_{mn}^s \left[-\frac{in}{r_1^2} H_n(k_{bm}r_1) - k_{bm}^2 H_n''(k_{bm}r_1)\right] e^{in\theta_{bm}} \\
& + \sum_{m=1}^{2}\sum_{n=-\infty}^{\infty} b_{mn}^R \left[-\frac{in}{r_1^2} J_n(k_{bm}r_1) - k_{bm}^2 J_n''(k_{bm}r_1)\right] e^{in\theta_{bm}}
\end{aligned}
$$

$$
\begin{aligned}
\varepsilon_{r\theta}^{S} = \frac{1}{2}\Bigg\{ & \sum_{n=-\infty}^{\infty} \alpha_n^i \left[\frac{2in}{r_1} k_{a1} J_n'(k_{a1}r_1) - \frac{in}{r_1^2} J_n(k_{a1}r_1)\right] e^{in\theta_{a1}} \\
& + \sum_{j=1}^{3}\sum_{n=-\infty}^{\infty} \alpha_{jn}^r \left[\frac{2in}{r_1} k_{aj} J_n'(k_{aj}r_1) - \frac{in}{r_1^2} J_n(k_{aj}r_1)\right] e^{in\theta_{aj}} \\
& + \sum_{j=1}^{3}\sum_{n=-\infty}^{\infty} a_{jn}^s \left[\frac{2in}{r_1} k_{aj} H_n'(k_{aj}r_1) - \frac{in}{r_1^2} H_n(k_{aj}r_1)\right] e^{in\theta_{aj}} \\
& + \sum_{j=1}^{3}\sum_{n=-\infty}^{\infty} \alpha_{jn}^R \left[\frac{2in}{r_1} k_{aj} J_n'(k_{aj}r_1) - \frac{in}{r_1^2} J_n(k_{aj}r_1)\right] e^{in\theta_{aj}} \\
& + \sum_{m=1}^{2}\sum_{n=-\infty}^{\infty} b_{mn}^r \left[-\frac{2n^2}{r_1^2} J_n(k_{bm}r_1) - k_{bm}^2 J_n''(k_{bm}r_1) + \frac{1}{r_1} k_{bm} J_n'(k_{bm}r_1)\right] e^{in\theta_{bm}} \\
& + \sum_{m=1}^{2}\sum_{n=-\infty}^{\infty} b_{mn}^s \left[-\frac{2n^2}{r_1^2} H_n(k_{bm}r_1) - k_{bm}^2 H_n''(k_{bm}r_1) + \frac{1}{r_1} k_{bm} H_n'(k_{bm}r_1)\right] e^{in\theta_{bm}} \\
& + \sum_{m=1}^{2}\sum_{n=-\infty}^{\infty} b_{mn}^R \left[-\frac{2n^2}{r_1^2} J_n(k_{bm}r_1) - k_{bm}^2 J_n''(k_{bm}r_1) + \frac{1}{r_1} k_{bm} J_n'(k_{bm}r_1)\right] e^{in\theta_{bm}} \Bigg\}
\end{aligned}
$$

液相

$$\varepsilon_{rr}^{L}=\sum_{j=1}^{3}\alpha_{j}^{L}\left[\sum_{n=-\infty}^{\infty}\alpha_{n}^{i}\,k_{a1}^{2}J_{n}''(k_{a1}r_{1})e^{in\theta_{a1}}+\sum_{n=-\infty}^{\infty}\alpha_{jn}^{r}\,k_{aj}^{2}J_{n}''(k_{aj}r_{1})e^{in\theta_{aj}}+\sum_{n=-\infty}^{\infty}\alpha_{jn}^{s}\,k_{aj}^{2}H_{n}''(k_{aj}r_{1})e^{in\theta_{aj}}+\sum_{n=-\infty}^{\infty}\alpha_{jn}^{R}\,k_{aj}^{2}J_{n}''(k_{aj}r_{1})e^{in\theta_{aj}}\right]+\sum_{m=1}^{2}\beta_{m}^{L}\left[\sum_{n=-\infty}^{\infty}b_{mn}^{r}\frac{in}{r_{1}}k_{bm}J_{n}'(k_{bm}r_{1})e^{in\theta_{bm}}-\sum_{n=-\infty}^{\infty}b_{mn}^{r}\frac{in}{r_{1}^{2}}J_{n}(k_{bm}r_{1})e^{in\theta_{bm}}+\sum_{n=-\infty}^{\infty}b_{mn}^{s}\frac{in}{r_{1}}k_{bm}H_{n}'(k_{bm}r_{1})e^{in\theta_{bm}}-\sum_{n=-\infty}^{\infty}b_{mn}^{s}\frac{in}{r_{1}^{2}}H_{n}(k_{bm}r_{1})e^{in\theta_{bm}}+\sum_{n=-\infty}^{\infty}b_{mn}^{R}\frac{in}{r_{1}}k_{bm}J_{n}'(k_{bm}r_{1})e^{in\theta_{bm}}-\sum_{n=-\infty}^{\infty}b_{mn}^{R}\frac{in}{r_{1}^{2}}J_{n}(k_{bm}r_{1})e^{in\theta_{bm}}\right]$$

$$\varepsilon_{\theta\theta}^{L}=\sum_{j=1}^{3}\alpha_{j}^{L}\left[\sum_{n=-\infty}^{\infty}\alpha_{n}^{i}\left(\frac{1}{r_{1}}k_{a1}J_{n}'(k_{a1}r_{1})-\frac{n^{2}}{r_{1}^{2}}J_{n}(k_{a1}r_{1})\right)e^{in\theta_{a1}}+\sum_{n=-\infty}^{\infty}\alpha_{jn}^{r}\left(\frac{1}{r_{1}}k_{aj}J_{n}'(k_{aj}r_{1})-\frac{n^{2}}{r_{1}^{2}}J_{n}(k_{aj}r_{1})\right)e^{in\theta_{aj}}+\sum_{n=-\infty}^{\infty}\alpha_{jn}^{s}\left(\frac{1}{r_{1}}k_{aj}H_{n}'(k_{aj}r_{1})-\frac{n^{2}}{r_{1}^{2}}H_{n}(k_{aj}r_{1})\right)e^{in\theta_{aj}}+\sum_{n=-\infty}^{\infty}\alpha_{jn}^{R}\left(\frac{1}{r_{1}}k_{aj}J_{n}'(k_{aj}r_{1})-\frac{n^{2}}{r_{1}^{2}}J_{n}(k_{aj}r_{1})\right)e^{in\theta_{aj}}\right]+\sum_{m=1}^{2}\beta_{m}^{L}\left[\sum_{n=-\infty}^{\infty}b_{mn}^{r}\left(-\frac{in}{r_{1}^{2}}J_{n}(k_{bm}r_{1})-k_{bm}^{2}J_{n}''(k_{bm}r_{1})\right)e^{in\theta_{bm}}+\sum_{n=-\infty}^{\infty}b_{mn}^{s}\left(-\frac{in}{r_{1}^{2}}H_{n}(k_{bm}r_{1})-k_{bm}^{2}H_{n}''(k_{bm}r_{1})\right)e^{in\theta_{bm}}+\sum_{n=-\infty}^{\infty}b_{mn}^{R}\left(-\frac{in}{r_{1}^{2}}J_{n}(k_{bm}r_{1})-k_{bm}^{2}J_{n}''(k_{bm}r_{1})\right)e^{in\theta_{bm}}\right]$$

$$\varepsilon_{r\theta}^{L}=\frac{1}{2}\left\{\sum_{j=1}^{3}\alpha_{j}^{L}\left[\sum_{n=-\infty}^{\infty}\alpha_{n}^{i}\left(\frac{2in}{r_{1}}k_{a1}J_{n}'(k_{a1}r_{1})-\frac{in}{r_{1}^{2}}J_{n}(k_{a1}r_{1})\right)e^{in\theta_{a1}}+\sum_{n=-\infty}^{\infty}\alpha_{jn}^{r}\left(\frac{2in}{r_{1}}k_{aj}J_{n}'(k_{aj}r_{1})-\frac{in}{r_{1}^{2}}J_{n}(k_{aj}r_{1})\right)e^{in\theta_{aj}}+\sum_{n=-\infty}^{\infty}\alpha_{jn}^{s}\left(\frac{2in}{r_{1}}k_{aj}H_{n}'(k_{aj}r_{1})-\frac{in}{r_{1}^{2}}H_{n}(k_{aj}r_{1})\right)e^{in\theta_{aj}}+\sum_{n=-\infty}^{\infty}\alpha_{jn}^{R}\left(\frac{2in}{r_{1}}k_{aj}J_{n}'(k_{aj}r_{1})-\frac{in}{r_{1}^{2}}J_{n}(k_{aj}r_{1})\right)e^{in\theta_{aj}}\right]+\sum_{m=1}^{2}\beta_{m}^{L}\left[\sum_{n=-\infty}^{\infty}b_{mn}^{r}\left(-\frac{2n^{2}}{r_{1}^{2}}J_{n}(k_{bm}r_{1})-k_{bm}^{2}J_{n}''(k_{bm}r_{1})+\frac{1}{r_{1}}k_{bm}J_{n}'(k_{bm}r_{1})\right)e^{in\theta_{bm}}+\sum_{n=-\infty}^{\infty}b_{mn}^{s}\left(-\frac{2n^{2}}{r_{1}^{2}}H_{n}(k_{bm}r_{1})-k_{bm}^{2}H_{n}''(k_{bm}r_{1})+\frac{1}{r_{1}}k_{bm}H_{n}'(k_{bm}r_{1})\right)e^{in\theta_{bm}}+\sum_{n=-\infty}^{\infty}b_{mn}^{R}\left(-\frac{2n^{2}}{r_{1}^{2}}J_{n}(k_{bm}r_{1})-k_{bm}^{2}J_{n}''(k_{bm}r_{1})+\frac{1}{r_{1}}k_{bm}J_{n}'(k_{bm}r_{1})\right)e^{in\theta_{bm}}\right]\right\}$$

冰相

$$\begin{aligned}\varepsilon_{rr}^{I} = \sum_{j=1}^{3}\alpha_j^I\Bigg[&\sum_{n=-\infty}^{\infty}\alpha_n^i\,k_{a1}^2 J_n''(k_{a1}r_1)e^{in\theta_{a1}}\\&+\sum_{n=-\infty}^{\infty}\alpha_{jn}^r\,k_{aj}^2 J_n''(k_{aj}r_1)e^{in\theta_{aj}}\\&+\sum_{n=-\infty}^{\infty}\alpha_{jn}^s\,k_{aj}^2 H_n''(k_{aj}r_1)e^{in\theta_{aj}}\\&+\sum_{n=-\infty}^{\infty}\alpha_{jn}^R\,k_{aj}^2 J_n''(k_{aj}r_1)e^{in\theta_{aj}}\Bigg]\\+\sum_{m=1}^{2}\beta_m^I\Bigg[&\sum_{n=-\infty}^{\infty}b_{mn}^r\frac{in}{r_1}k_{bm}J_n'(k_{bm}r_1)e^{in\theta_{bm}}\\&-\sum_{n=-\infty}^{\infty}b_{mn}^r\frac{in}{r_1^2}J_n(k_{bm}r_1)e^{in\theta_{bm}}\\&+\sum_{n=-\infty}^{\infty}b_{mn}^s\frac{in}{r_1}k_{bm}H_n'(k_{bm}r_1)e^{in\theta_{bm}}\\&-\sum_{n=-\infty}^{\infty}b_{mn}^s\frac{in}{r_1^2}H_n(k_{bm}r_1)e^{in\theta_{bm}}\\&+\sum_{n=-\infty}^{\infty}b_{mn}^R\frac{in}{r_1}k_{bm}J_n'(k_{bm}r_1)e^{in\theta_{bm}}\\&-\sum_{n=-\infty}^{\infty}b_{mn}^R\frac{in}{r_1^2}J_n(k_{bm}r_1)e^{in\theta_{bm}}\Bigg]\end{aligned}$$

$$\begin{aligned}\varepsilon_{\theta\theta}^{I} = \sum_{j=1}^{3}\alpha_j^I\Bigg[&\sum_{n=-\infty}^{\infty}\alpha_n^i\left(\frac{1}{r_1}k_{a1}J_n'(k_{a1}r_1)-\frac{n^2}{r_1^2}J_n(k_{a1}r_1)\right)e^{in\theta_{a1}}\\&+\sum_{n=-\infty}^{\infty}\alpha_{jn}^r\left(\frac{1}{r_1}k_{aj}J_n'(k_{aj}r_1)-\frac{n^2}{r_1^2}J_n(k_{aj}r_1)\right)e^{in\theta_{aj}}\\&+\sum_{n=-\infty}^{\infty}\alpha_{jn}^s\left(\frac{1}{r_1}k_{aj}H_n'(k_{aj}r_1)-\frac{n^2}{r_1^2}H_n(k_{aj}r_1)\right)e^{in\theta_{aj}}\\&+\sum_{n=-\infty}^{\infty}\alpha_{jn}^R\left(\frac{1}{r_1}k_{aj}J_n'(k_{aj}r_1)-\frac{n^2}{r_1^2}J_n(k_{aj}r_1)\right)e^{in\theta_{aj}}\Bigg]\\+\sum_{m=1}^{2}\beta_m^I\Bigg[&\sum_{n=-\infty}^{\infty}b_{mn}^r\left(-\frac{in}{r_1^2}J_n(k_{bm}r_1)-k_{bm}^2J_n''(k_{bm}r_1)\right)e^{in\theta_{bm}}\\&+\sum_{n=-\infty}^{\infty}b_{mn}^s\left(-\frac{in}{r_1^2}H_n(k_{bm}r_1)-k_{bm}^2H_n''(k_{bm}r_1)\right)e^{in\theta_{bm}}\\&+\sum_{n=-\infty}^{\infty}b_{mn}^R\left(-\frac{in}{r_1^2}J_n(k_{bm}r_1)-k_{bm}^2J_n''(k_{bm}r_1)\right)e^{in\theta_{bm}}\Bigg]\end{aligned}$$

$$\begin{aligned}\varepsilon_{r\theta}^{I} = \frac{1}{2}\Bigg\{\sum_{j=1}^{3}\alpha_j^I\Bigg[&\sum_{n=-\infty}^{\infty}\alpha_n^i\left(\frac{2in}{r_1}k_{a1}J_n'(k_{a1}r_1)-\frac{in}{r_1^2}J_n(k_{a1}r_1)\right)e^{in\theta_{a1}}\\&+\sum_{n=-\infty}^{\infty}\alpha_{jn}^r\left(\frac{2in}{r_1}k_{aj}J_n'(k_{aj}r_1)-\frac{in}{r_1^2}J_n(k_{aj}r_1)\right)e^{in\theta_{aj}}\\&+\sum_{n=-\infty}^{\infty}\alpha_{jn}^s\left(\frac{2in}{r_1}k_{aj}H_n'(k_{aj}r_1)-\frac{in}{r_1^2}H_n(k_{aj}r_1)\right)e^{in\theta_{aj}}\\&+\sum_{n=-\infty}^{\infty}\alpha_{jn}^R\left(\frac{2in}{r_1}k_{aj}J_n'(k_{aj}r_1)-\frac{in}{r_1^2}J_n(k_{aj}r_1)\right)e^{in\theta_{aj}}\Bigg]\\+\sum_{m=1}^{2}\beta_m^I\Bigg[&\sum_{n=-\infty}^{\infty}b_{mn}^r\left(-\frac{2n^2}{r_1^2}J_n(k_{bm}r_1)-k_{bm}^2J_n''(k_{bm}r_1)+\frac{1}{r_1}k_{bm}J_n'(k_{bm}r_1)\right)e^{in\theta_{bm}}\\&+\sum_{n=-\infty}^{\infty}b_{mn}^s\left(-\frac{2n^2}{r_1^2}H_n(k_{bm}r_1)-k_{bm}^2H_n''(k_{bm}r_1)+\frac{1}{r_1}k_{bm}H_n'(k_{bm}r_1)\right)e^{in\theta_{bm}}\\&+\sum_{n=-\infty}^{\infty}b_{mn}^R\left(-\frac{2n^2}{r_1^2}J_n(k_{bm}r_1)-k_{bm}^2J_n''(k_{bm}r_1)+\frac{1}{r_1}k_{bm}J_n'(k_{bm}r_1)\right)e^{in\theta_{bm}}\Bigg]\Bigg\}\end{aligned}$$

5.代入本构方程

$$(\lambda_S+2\mu_S)\left(\frac{\partial^2\varphi^S}{\partial r^2}+\frac{1}{r}\frac{\partial^2\psi^S}{\partial r\partial\theta}\right)+\lambda_S\left(\frac{1}{r}\frac{\partial\varphi^S}{\partial r}+\frac{1}{r^2}\frac{\partial^2\varphi^S}{\partial\theta^2}-\frac{1}{r^2}\frac{\partial\psi^S}{\partial\theta}+\frac{1}{r}\frac{\partial^2\psi^S}{\partial r\partial\theta}\right)$$
$$-K_L\left[\frac{\partial^2\varphi^L}{\partial r^2}+\frac{1}{r}\frac{\partial\varphi^L}{\partial r}+\frac{1}{r^2}\frac{\partial^2\varphi^L}{\partial\theta^2}+\frac{1}{r}\frac{\partial^2\psi^L}{\partial r\partial\theta}-\frac{1}{r^2}\frac{\partial\psi^L}{\partial\theta}+\frac{1}{r}\frac{\partial^2\psi^L}{\partial r\partial\theta}\right]$$
$$+(\lambda_l+2\mu_l)\left(\frac{\partial^2\varphi^l}{\partial r^2}+\frac{1}{r}\frac{\partial^2\psi^l}{\partial r\partial\theta}\right)+\lambda_l\left(\frac{1}{r}\frac{\partial\varphi^l}{\partial r}+\frac{1}{r^2}\frac{\partial^2\varphi^l}{\partial\theta^2}-\frac{1}{r^2}\frac{\partial\psi^l}{\partial\theta}+\frac{1}{r}\frac{\partial^2\psi^l}{\partial r\partial\theta}\right)=0$$

$$\mu_S\left(\frac{1}{r}\frac{\partial^2\varphi^S}{\partial r\partial\theta}-\frac{1}{r^2}\frac{\partial\varphi^S}{\partial\theta}+\frac{\partial^2\psi^S}{\partial r^2}-\frac{1}{r}\frac{\partial\psi^S}{\partial r}-\frac{1}{r^2}\frac{\partial^2\psi^S}{\partial\theta^2}\right)$$
$$+\mu_l\left(\frac{1}{r}\frac{\partial^2\varphi^l}{\partial r\partial\theta}-\frac{1}{r^2}\frac{\partial\varphi^l}{\partial\theta}+\frac{\partial^2\psi^l}{\partial r^2}-\frac{1}{r}\frac{\partial\psi^l}{\partial r}-\frac{1}{r^2}\frac{\partial^2\psi^l}{\partial\theta^2}\right)=0$$

$$K_L\left(\frac{\partial^2\varphi^L}{\partial r^2}+\frac{1}{r}\frac{\partial\varphi^L}{\partial r}+\frac{1}{r^2}\frac{\partial^2\varphi^L}{\partial\theta^2}+\frac{1}{r}\frac{\partial^2\psi^L}{\partial r\partial\theta}-\frac{1}{r^2}\frac{\partial\psi^L}{\partial\theta}+\frac{1}{r}\frac{\partial^2\psi^L}{\partial r\partial\theta}\right)=0$$

$$\frac{\partial\varphi^S}{\partial r}+\frac{1}{r}\frac{\partial\psi^S}{\partial\theta}=\frac{\partial\varphi^l}{\partial r}+\frac{1}{r}\frac{\partial\psi^l}{\partial\theta}$$

$$\frac{1}{r}\frac{\partial\varphi^S}{\partial\theta}-\frac{\partial\psi^S}{\partial r}=\frac{1}{r}\frac{\partial\varphi^l}{\partial\theta}-\frac{\partial\psi^l}{\partial r}$$

具体分解一下

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$$(\lambda_S+2\mu_S)\left[\frac{\partial^2\varphi^S}{\partial r^2}+\frac{1}{r}\frac{\partial^2\psi^S}{\partial r\partial\theta}\right]+\lambda_S\left[\frac{1}{r}\frac{\partial\varphi^S}{\partial r}+\frac{1}{r^2}\frac{\partial^2\varphi^S}{\partial\theta^2}-\frac{1}{r^2}\frac{\partial\psi^S}{\partial\theta}+\frac{1}{r}\frac{\partial^2\psi^S}{\partial r\partial\theta}\right]$$
$$=(\lambda_S+2\mu_S)\left[\frac{\partial^2}{\partial r^2}\left(\sum_{n=-\infty}^{\infty}\alpha_n^i J_n(k_{\alpha1}r_1)e^{in\theta_{\alpha1}}+\sum_{j=1}^{3}\sum_{n=-\infty}^{\infty}\alpha_{jn}^r J_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\right.\right.$$
$$\left.\left(+\sum_{j=1}^{3}\sum_{n=-\infty}^{\infty}a_{jn}^s H_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}+\sum_{j=1}^{3}\sum_{n=-\infty}^{\infty}\alpha_{jn}^R J_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\right)\right)$$
$$\left(+\frac{1}{r}\frac{\partial^2}{\partial r\partial\theta}\left(\sum_{m=1}^{2}\sum_{n=-\infty}^{\infty}b_{mn}^r J_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}+\sum_{m=1}^{2}\sum_{n=-\infty}^{\infty}b_{mn}^s H_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\right)\right.$$
$$\left.+\sum_{m=1}^{2}\sum_{n=-\infty}^{\infty}b_{mn}^R J_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\right)\Bigg]$$
$$+\lambda_S\left[\frac{1}{r}\frac{\partial}{\partial r}\left(\sum_{n=-\infty}^{\infty}\alpha_n^i J_n(k_{\alpha1}r_1)e^{in\theta_{\alpha1}}+\sum_{j=1}^{3}\sum_{n=-\infty}^{\infty}\alpha_{jn}^r J_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\right.\right.$$
$$\left.\left(+\sum_{j=1}^{3}\sum_{n=-\infty}^{\infty}a_{jn}^s H_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}+\sum_{j=1}^{3}\sum_{n=-\infty}^{\infty}\alpha_{jn}^R J_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\right)\right)$$
$$+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\left(\sum_{n=-\infty}^{\infty}\alpha_n^i J_n(k_{\alpha1}r_1)e^{in\theta_{\alpha1}}+\sum_{j=1}^{3}\sum_{n=-\infty}^{\infty}\alpha_{jn}^r J_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\right.$$
$$\left.+\sum_{j=1}^{3}\sum_{n=-\infty}^{\infty}a_{jn}^s H_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}+\sum_{j=1}^{3}\sum_{n=-\infty}^{\infty}\alpha_{jn}^R J_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\right)$$
$$-\frac{1}{r^2}\frac{\partial}{\partial\theta}\left(\sum_{m=1}^{2}\sum_{n=-\infty}^{\infty}b_{mn}^r J_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}+\sum_{m=1}^{2}\sum_{n=-\infty}^{\infty}b_{mn}^s H_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\right.$$
$$\left.+\sum_{m=1}^{2}\sum_{n=-\infty}^{\infty}b_{mn}^R J_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\right)$$
$$\left(+\frac{1}{r}\frac{\partial^2}{\partial r\partial\theta}\left(\sum_{m=1}^{2}\sum_{n=-\infty}^{\infty}b_{mn}^r J_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}+\sum_{m=1}^{2}\sum_{n=-\infty}^{\infty}b_{mn}^s H_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\right)\right.$$
$$\left.+\sum_{m=1}^{2}\sum_{n=-\infty}^{\infty}b_{mn}^R J_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\right)\Bigg]$$

$$
\begin{aligned}
&-K_L\left[\frac{\partial^2\varphi^L}{\partial r^2}+\frac{1}{r}\frac{\partial\varphi^L}{\partial r}+\frac{1}{r^2}\frac{\partial^2\varphi^L}{\partial\theta^2}+\frac{1}{r}\frac{\partial^2\psi^L}{\partial r\partial\theta}-\frac{1}{r^2}\frac{\partial\psi^L}{\partial\theta}+\frac{1}{r}\frac{\partial^2\psi^L}{\partial r\partial\theta}\right]\\
&=-K_L\left[\frac{\partial^2}{\partial r^2}\left(\sum_{j=1}^{3}\alpha_j^L\left[\sum_{n=-\infty}^{\infty}\alpha_n^i\,\delta_{j1}J_n(k_{\alpha1}r_1)e^{in\theta_{\alpha1}}+\sum_{n=-\infty}^{\infty}\alpha_{jn}^r J_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\right.\right.\right.\\
&\left.\left.\left.\left(+\sum_{n=-\infty}^{\infty}a_{jn}^s H_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}+\sum_{n=-\infty}^{\infty}\alpha_{jn}^R J_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\right]\right)\right)\right.\\
&\left.+\frac{1}{r}\frac{\partial}{\partial r}\left(\sum_{j=1}^{3}\alpha_j^L\left[\sum_{n=-\infty}^{\infty}\alpha_n^i\,\delta_{j1}J_n(k_{\alpha1}r_1)e^{in\theta_{\alpha1}}+\sum_{n=-\infty}^{\infty}\alpha_{jn}^r J_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\right.\right.\right.\\
&\left.\left.\left.+\sum_{n=-\infty}^{\infty}a_{jn}^s H_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}+\sum_{n=-\infty}^{\infty}\alpha_{jn}^R J_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\right]\right)\right.\\
&\left.+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\left(\sum_{j=1}^{3}\alpha_j^L\left[\sum_{n=-\infty}^{\infty}\alpha_n^i\,\delta_{j1}J_n(k_{\alpha1}r_1)e^{in\theta_{\alpha1}}+\sum_{n=-\infty}^{\infty}\alpha_{jn}^r J_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\right.\right.\right.\\
&\left.\left.\left.+\sum_{n=-\infty}^{\infty}a_{jn}^s H_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}+\sum_{n=-\infty}^{\infty}\alpha_{jn}^R J_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\right]\right)\right.\\
&\left.+\frac{1}{r}\frac{\partial^2}{\partial r\partial\theta}\left(\sum_{m=1}^{2}\beta_m^L\left[\sum_{n=-\infty}^{\infty}b_{mn}^r J_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}+\sum_{n=-\infty}^{\infty}b_{mn}^s H_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\right.\right.\right.\\
&\left.\left.\left.+\sum_{n=-\infty}^{\infty}b_{mn}^R J_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\right]\right)\right.\\
&\left.-\frac{1}{r^2}\frac{\partial}{\partial\theta}\left(\sum_{m=1}^{2}\beta_m^L\left[\sum_{n=-\infty}^{\infty}b_{mn}^r J_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}+\sum_{n=-\infty}^{\infty}b_{mn}^s H_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\right.\right.\right.\\
&\left.\left.\left.+\sum_{n=-\infty}^{\infty}b_{mn}^R J_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\right]\right)\right.\\
&\left.\left(+\frac{1}{r}\frac{\partial^2}{\partial r\partial\theta}\left(\sum_{m=1}^{2}\beta_m^L\left[\sum_{n=-\infty}^{\infty}b_{mn}^r J_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}+\sum_{n=-\infty}^{\infty}b_{mn}^s H_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\right)\right.\right.\right.\\
&\left.\left.\left.+\sum_{n=-\infty}^{\infty}b_{mn}^R J_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\right]\right)\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
&+(\lambda_l+2\mu_l)\left[\frac{\partial^2\varphi^l}{\partial r^2}+\frac{1}{r}\frac{\partial^2\psi^l}{\partial r\partial\theta}\right]+\lambda_l\left[\frac{1}{r}\frac{\partial\varphi^l}{\partial r}+\frac{1}{r^2}\frac{\partial^2\varphi^l}{\partial\theta^2}-\frac{1}{r^2}\frac{\partial\psi^l}{\partial\theta}+\frac{1}{r}\frac{\partial^2\psi^l}{\partial r\partial\theta}\right]\\
&=(\lambda_l+2\mu_l)\left[\frac{\partial^2}{\partial r^2}\left(\sum_{j=1}^{3}\alpha_j^l\left[\sum_{n=-\infty}^{\infty}\alpha_n^i\,\delta_{j1}J_n(k_{\alpha1}r_1)e^{in\theta_{\alpha1}}+\sum_{n=-\infty}^{\infty}\alpha_{jn}^r J_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\right.\right.\right.\\
&\quad\left.\left.\left.\left(+\sum_{n=-\infty}^{\infty}a_{jn}^s H_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}+\sum_{n=-\infty}^{\infty}\alpha_{jn}^R J_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\right]\right)\right)\right.\\
&\quad\left(+\frac{1}{r}\frac{\partial^2}{\partial r\partial\theta}\left(\sum_{m=1}^{2}\beta_m^l\left[\sum_{n=-\infty}^{\infty}b_{mn}^r J_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}+\sum_{n=-\infty}^{\infty}b_{mn}^s H_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\right)\right.\right.\\
&\quad\left.\left.+\sum_{n=-\infty}^{\infty}b_{mn}^R J_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\right]\right)\Bigg]\\
&+\lambda_l\left[\frac{1}{r}\frac{\partial}{\partial r}\left(\sum_{j=1}^{3}\alpha_j^l\left[\sum_{n=-\infty}^{\infty}\alpha_n^i\,\delta_{j1}J_n(k_{\alpha1}r_1)e^{in\theta_{\alpha1}}+\sum_{n=-\infty}^{\infty}\alpha_{jn}^r J_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\right.\right.\right.\\
&\quad\left.\left.\left.\left(+\sum_{n=-\infty}^{\infty}a_{jn}^s H_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}+\sum_{n=-\infty}^{\infty}\alpha_{jn}^R J_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\right]\right)\right)\right.\\
&\quad+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\left(\sum_{j=1}^{3}\alpha_j^l\left[\sum_{n=-\infty}^{\infty}\alpha_n^i\,\delta_{j1}J_n(k_{\alpha1}r_1)e^{in\theta_{\alpha1}}+\sum_{n=-\infty}^{\infty}\alpha_{jn}^r J_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\right.\right.\\
&\quad\left.\left.+\sum_{n=-\infty}^{\infty}a_{jn}^s H_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}+\sum_{n=-\infty}^{\infty}\alpha_{jn}^R J_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\right]\right)\\
&\quad-\frac{1}{r^2}\frac{\partial}{\partial\theta}\left(\sum_{m=1}^{2}\beta_m^l\left[\sum_{n=-\infty}^{\infty}b_{mn}^r J_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}+\sum_{n=-\infty}^{\infty}b_{mn}^s H_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\right.\right.\\
&\quad\left.\left.+\sum_{n=-\infty}^{\infty}b_{mn}^R J_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\right]\right)\\
&\quad\left(+\frac{1}{r}\frac{\partial^2}{\partial r\partial\theta}\left(\sum_{m=1}^{2}\beta_m^l\left[\sum_{n=-\infty}^{\infty}b_{mn}^r J_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}+\sum_{n=-\infty}^{\infty}b_{mn}^s H_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\right)\right.\right.\\
&\quad\left.\left.+\sum_{n=-\infty}^{\infty}b_{mn}^R J_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\right]\right)\Bigg]
\end{aligned}
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\begin{aligned}
\sigma_{r\theta}^{S} &= \mu_S\left[\frac{2}{r_1}\frac{\partial^2\varphi^S}{\partial r_1\partial\theta_1}-\frac{2}{r_1^2}\frac{\partial\varphi^S}{\partial\theta_1}+\frac{1}{r_1^2}\frac{\partial^2\psi^S}{\partial\theta_1^2}-\frac{\partial^2\psi^S}{\partial r_1^2}+\frac{1}{r_1}\frac{\partial\psi^S}{\partial r_1}\right]\\
&= \mu_S\left[\frac{2}{r_1}\frac{\partial^2}{\partial r_1\partial\theta_1}\left(\sum_{n=-\infty}^{\infty}\alpha_n^i J_n(k_{\alpha1}r_1)e^{in\theta_{\alpha1}}\right.\right.\\
&\quad+\sum_{j=1}^{3}\sum_{n=-\infty}^{\infty}\alpha_{jn}^r J_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\\
&\quad+\sum_{j=1}^{3}\sum_{n=-\infty}^{\infty}a_{jn}^s H_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\\
&\quad\left.+\sum_{j=1}^{3}\sum_{n=-\infty}^{\infty}\alpha_{jn}^R J_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\right)\\
&\quad-\frac{2}{r_1^2}\frac{\partial}{\partial\theta_1}\left(\sum_{n=-\infty}^{\infty}\alpha_n^i J_n(k_{\alpha1}r_1)e^{in\theta_{\alpha1}}\right.\\
&\quad+\sum_{j=1}^{3}\sum_{n=-\infty}^{\infty}\alpha_{jn}^r J_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\\
&\quad+\sum_{j=1}^{3}\sum_{n=-\infty}^{\infty}a_{jn}^s H_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\\
&\quad\left.+\sum_{j=1}^{3}\sum_{n=-\infty}^{\infty}\alpha_{jn}^R J_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\right)\\
&\quad+\frac{1}{r_1^2}\frac{\partial^2}{\partial\theta_1^2}\left(\sum_{m=1}^{2}\sum_{n=-\infty}^{\infty}b_{mn}^r J_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\right.\\
&\quad+\sum_{m=1}^{2}\sum_{n=-\infty}^{\infty}b_{mn}^s H_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\\
&\quad\left.+\sum_{m=1}^{2}\sum_{n=-\infty}^{\infty}b_{mn}^R J_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\right)\\
&\quad-\frac{\partial^2}{\partial r_1^2}\left(\sum_{m=1}^{2}\sum_{n=-\infty}^{\infty}b_{mn}^r J_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\right.\\
&\quad+\sum_{m=1}^{2}\sum_{n=-\infty}^{\infty}b_{mn}^s H_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\\
&\quad\left.+\sum_{m=1}^{2}\sum_{n=-\infty}^{\infty}b_{mn}^R J_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\right)\\
&\quad+\frac{1}{r_1}\frac{\partial}{\partial r_1}\left(\sum_{m=1}^{2}\sum_{n=-\infty}^{\infty}b_{mn}^r J_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\right.\\
&\quad+\sum_{m=1}^{2}\sum_{n=-\infty}^{\infty}b_{mn}^s H_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\\
&\quad\left.\left.+\sum_{m=1}^{2}\sum_{n=-\infty}^{\infty}b_{mn}^R J_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
\sigma_{r\theta}^{I} &= \mu_I\left[\frac{2}{r_1}\frac{\partial^2\varphi^I}{\partial r_1\partial\theta_1}-\frac{2}{r_1^2}\frac{\partial\varphi^I}{\partial\theta_1}+\frac{1}{r_1^2}\frac{\partial^2\psi^I}{\partial\theta_1^2}-\frac{\partial^2\psi^I}{\partial r_1^2}+\frac{1}{r_1}\frac{\partial\psi^I}{\partial r_1}\right]\\
&= \mu_I\left[\frac{2}{r_1}\frac{\partial^2}{\partial r_1\partial\theta_1}\left(\sum_{j=1}^{3}\alpha_j^I\left[\sum_{n=-\infty}^{\infty}\alpha_n^i\,\delta_{j1}J_n(k_{\alpha1}r_1)e^{in\theta_{\alpha1}}\right.\right.\right.\\
&\quad+\sum_{n=-\infty}^{\infty}\alpha_{jn}^r J_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\\
&\quad+\sum_{n=-\infty}^{\infty}a_{jn}^s H_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\\
&\quad\left.\left.\left.+\sum_{n=-\infty}^{\infty}\alpha_{jn}^R J_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\right]\right)\right.\\
&\quad-\frac{2}{r_1^2}\frac{\partial}{\partial\theta_1}\left(\sum_{j=1}^{3}\alpha_j^I\left[\sum_{n=-\infty}^{\infty}\alpha_n^i\,\delta_{j1}J_n(k_{\alpha1}r_1)e^{in\theta_{\alpha1}}\right.\right.\\
&\quad+\sum_{n=-\infty}^{\infty}\alpha_{jn}^r J_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\\
&\quad+\sum_{n=-\infty}^{\infty}a_{jn}^s H_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\\
&\quad\left.\left.+\sum_{n=-\infty}^{\infty}\alpha_{jn}^R J_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\right]\right)\\
&\quad+\frac{1}{r_1^2}\frac{\partial^2}{\partial\theta_1^2}\left(\sum_{m=1}^{2}\beta_m^I\left[\sum_{n=-\infty}^{\infty}b_{mn}^r J_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\right.\right.\\
&\quad+\sum_{n=-\infty}^{\infty}b_{mn}^s H_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\\
&\quad\left.\left.+\sum_{n=-\infty}^{\infty}b_{mn}^R J_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\right]\right)\\
&\quad-\frac{\partial^2}{\partial r_1^2}\left(\sum_{m=1}^{2}\beta_m^I\left[\sum_{n=-\infty}^{\infty}b_{mn}^r J_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\right.\right.\\
&\quad+\sum_{n=-\infty}^{\infty}b_{mn}^s H_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\\
&\quad\left.\left.+\sum_{n=-\infty}^{\infty}b_{mn}^R J_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\right]\right)\\
&\quad+\frac{1}{r_1}\frac{\partial}{\partial r_1}\left(\sum_{m=1}^{2}\beta_m^I\left[\sum_{n=-\infty}^{\infty}b_{mn}^r J_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\right.\right.\\
&\quad+\sum_{n=-\infty}^{\infty}b_{mn}^s H_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\\
&\quad\left.\left.\left.+\sum_{n=-\infty}^{\infty}b_{mn}^R J_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\right]\right)\right]
\end{aligned}
$$

（3）

$$
\begin{aligned}
&K_L\sum_{j=1}^{3}\alpha_j^L\left[\sum_{n=-\infty}^{\infty}\alpha_n^i\,\delta_{j1}\left(k_{\alpha1}^2J_n''(k_{\alpha1}r)+\frac{k_{\alpha1}}{r}J_n'(k_{\alpha1}r)-\frac{n^2}{r^2}J_n(k_{\alpha1}r)\right)e^{in\theta_{\alpha1}}\right.\\
&+\sum_{n=-\infty}^{\infty}\alpha_{jn}^r\left(k_{\alpha j}^2J_n''(k_{\alpha j}r)+\frac{k_{\alpha j}}{r}J_n'(k_{\alpha j}r)-\frac{n^2}{r^2}J_n(k_{\alpha j}r)\right)e^{in\theta_{\alpha j}}\\
&+\sum_{n=-\infty}^{\infty}a_{jn}^s\left(k_{\alpha j}^2H_n''(k_{\alpha j}r)+\frac{k_{\alpha j}}{r}H_n'(k_{\alpha j}r)-\frac{n^2}{r^2}H_n(k_{\alpha j}r)\right)e^{in\theta_{\alpha j}}\\
&\left.+\sum_{n=-\infty}^{\infty}\alpha_{jn}^R\left(k_{\alpha j}^2J_n''(k_{\alpha j}r)+\frac{k_{\alpha j}}{r}J_n'(k_{\alpha j}r)-\frac{n^2}{r^2}J_n(k_{\alpha j}r)\right)e^{in\theta_{\alpha j}}\right]\\
&+2K_L\sum_{m=1}^{2}\beta_m^L\left[\sum_{n=-\infty}^{\infty}b_{mn}^r\frac{ink_{\beta m}}{r}J_n'(k_{\beta m}r)e^{in\theta_{\beta m}}\right.\\
&+\sum_{n=-\infty}^{\infty}b_{mn}^s\frac{ink_{\beta m}}{r}H_n'(k_{\beta m}r)e^{in\theta_{\beta m}}\\
&\left.+\sum_{n=-\infty}^{\infty}b_{mn}^R\frac{ink_{\beta m}}{r}J_n'(k_{\beta m}r)e^{in\theta_{\beta m}}\right]\\
&-K_L\sum_{m=1}^{2}\beta_m^L\left[\sum_{n=-\infty}^{\infty}b_{mn}^r\frac{in}{r^2}J_n(k_{\beta m}r)e^{in\theta_{\beta m}}\right.\\
&+\sum_{n=-\infty}^{\infty}b_{mn}^s\frac{in}{r^2}H_n(k_{\beta m}r)e^{in\theta_{\beta m}}\\
&\left.+\sum_{n=-\infty}^{\infty}b_{mn}^R\frac{in}{r^2}J_n(k_{\beta m}r)e^{in\theta_{\beta m}}\right]=0
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial\varphi^S}{\partial r}+\frac{1}{r}\frac{\partial\psi^S}{\partial\theta}=&\sum_{n=-\infty}^{\infty}\alpha_n^i\,k_{\alpha1}J_n'(k_{\alpha1}r_1)e^{in\theta_{\alpha1}}\\
&+\sum_{j=1}^{3}\sum_{n=-\infty}^{\infty}\alpha_{jn}^r\,k_{\alpha j}J_n'(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\\
&+\sum_{j=1}^{3}\sum_{n=-\infty}^{\infty}a_{jn}^s\,k_{\alpha j}H_n'(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\\
&+\sum_{j=1}^{3}\sum_{n=-\infty}^{\infty}\alpha_{jn}^R\,k_{\alpha j}J_n'(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\\
&+\sum_{m=1}^{2}\sum_{n=-\infty}^{\infty}b_{mn}^r\frac{in}{r_1}J_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\\
&+\sum_{m=1}^{2}\sum_{n=-\infty}^{\infty}b_{mn}^s\frac{in}{r_1}H_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\\
&+\sum_{m=1}^{2}\sum_{n=-\infty}^{\infty}b_{mn}^R\frac{in}{r_1}J_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial\varphi^I}{\partial r}+\frac{1}{r}\frac{\partial\psi^I}{\partial\theta}=&\sum_{j=1}^{3}\alpha_j^I\left[\sum_{n=-\infty}^{\infty}\alpha_n^i\,\delta_{j1}k_{\alpha1}J_n'(k_{\alpha1}r_1)e^{in\theta_{\alpha1}}\right.\\
&+\sum_{n=-\infty}^{\infty}\alpha_{jn}^r\,k_{\alpha j}J_n'(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\\
&+\sum_{n=-\infty}^{\infty}a_{jn}^s\,k_{\alpha j}H_n'(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\\
&\left.+\sum_{n=-\infty}^{\infty}\alpha_{jn}^R\,k_{\alpha j}J_n'(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\right]\\
&+\sum_{m=1}^{2}\beta_m^I\left[\sum_{n=-\infty}^{\infty}b_{mn}^r\frac{in}{r_1}J_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\right.\\
&+\sum_{n=-\infty}^{\infty}b_{mn}^s\frac{in}{r_1}H_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\\
&\left.+\sum_{n=-\infty}^{\infty}b_{mn}^R\frac{in}{r_1}J_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\right]
\end{aligned}
$$

$$
\begin{aligned}
&\sum_{n=-\infty}^{\infty}\alpha_n^i\,k_{\alpha1}J_n'(k_{\alpha1}r_1)e^{in\theta_{\alpha1}}\\
&+\sum_{j=1}^{3}\sum_{n=-\infty}^{\infty}\alpha_{jn}^r\,k_{\alpha j}J_n'(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\\
&+\sum_{j=1}^{3}\sum_{n=-\infty}^{\infty}a_{jn}^s\,k_{\alpha j}H_n'(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\\
&+\sum_{j=1}^{3}\sum_{n=-\infty}^{\infty}\alpha_{jn}^R\,k_{\alpha j}J_n'(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\\
&+\sum_{m=1}^{2}\sum_{n=-\infty}^{\infty}b_{mn}^r\frac{in}{r_1}J_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\\
&+\sum_{m=1}^{2}\sum_{n=-\infty}^{\infty}b_{mn}^s\frac{in}{r_1}H_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\\
&+\sum_{m=1}^{2}\sum_{n=-\infty}^{\infty}b_{mn}^R\frac{in}{r_1}J_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\\
=&\sum_{j=1}^{3}\alpha_j^I\left[\sum_{n=-\infty}^{\infty}\alpha_n^i\,\delta_{j1}k_{\alpha1}J_n'(k_{\alpha1}r_1)e^{in\theta_{\alpha1}}\right.\\
&+\sum_{n=-\infty}^{\infty}\alpha_{jn}^r\,k_{\alpha j}J_n'(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\\
&+\sum_{n=-\infty}^{\infty}a_{jn}^s\,k_{\alpha j}H_n'(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\\
&\left.+\sum_{n=-\infty}^{\infty}\alpha_{jn}^R\,k_{\alpha j}J_n'(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\right]\\
&+\sum_{m=1}^{2}\beta_m^I\left[\sum_{n=-\infty}^{\infty}b_{mn}^r\frac{in}{r_1}J_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\right.\\
&+\sum_{n=-\infty}^{\infty}b_{mn}^s\frac{in}{r_1}H_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\\
&\left.+\sum_{n=-\infty}^{\infty}b_{mn}^R\frac{in}{r_1}J_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\right]
\end{aligned}
$$

$$
\begin{aligned}
\frac{1}{r}\frac{\partial \varphi^{S}}{\partial \theta}-\frac{\partial \psi^{S}}{\partial r} = & \sum_{n=-\infty}^{\infty} \alpha_{n}^{i} \frac{in}{r_1} J_n(k_{\alpha 1} r_1) e^{in\theta_{\alpha 1}} \\
& + \sum_{j=1}^{3} \sum_{n=-\infty}^{\infty} \alpha_{jn}^{r} \frac{in}{r_1} J_n(k_{\alpha j} r_1) e^{in\theta_{\alpha j}} \\
& + \sum_{j=1}^{3} \sum_{n=-\infty}^{\infty} a_{jn}^{s} \frac{in}{r_1} H_n(k_{\alpha j} r_1) e^{in\theta_{\alpha j}} \\
& + \sum_{j=1}^{3} \sum_{n=-\infty}^{\infty} \alpha_{jn}^{R} \frac{in}{r_1} J_n(k_{\alpha j} r_1) e^{in\theta_{\alpha j}} \\
& - \sum_{m=1}^{2} \sum_{n=-\infty}^{\infty} b_{mn}^{r} k_{\beta m} J_n'(k_{\beta m} r_1) e^{in\theta_{\beta m}} \\
& - \sum_{m=1}^{2} \sum_{n=-\infty}^{\infty} b_{mn}^{s} k_{\beta m} H_n'(k_{\beta m} r_1) e^{in\theta_{\beta m}} \\
& - \sum_{m=1}^{2} \sum_{n=-\infty}^{\infty} b_{mn}^{R} k_{\beta m} J_n'(k_{\beta m} r_1) e^{in\theta_{\beta m}}
\end{aligned}
$$

$$
\begin{aligned}
\frac{1}{r}\frac{\partial \varphi^{I}}{\partial \theta}-\frac{\partial \psi^{I}}{\partial r} = & \sum_{j=1}^{3} \alpha_{j}^{I} \left[\sum_{n=-\infty}^{\infty} \alpha_{n}^{i} \delta_{j1} \frac{in}{r_1} J_n(k_{\alpha 1} r_1) e^{in\theta_{\alpha 1}} \right. \\
& + \sum_{n=-\infty}^{\infty} \alpha_{jn}^{r} \frac{in}{r_1} J_n(k_{\alpha j} r_1) e^{in\theta_{\alpha j}} \\
& + \sum_{n=-\infty}^{\infty} a_{jn}^{s} \frac{in}{r_1} H_n(k_{\alpha j} r_1) e^{in\theta_{\alpha j}} \\
& \left. + \sum_{n=-\infty}^{\infty} \alpha_{jn}^{R} \frac{in}{r_1} J_n(k_{\alpha j} r_1) e^{in\theta_{\alpha j}} \right] \\
& - \sum_{m=1}^{2} \beta_{m}^{I} \left[\sum_{n=-\infty}^{\infty} b_{mn}^{r} k_{\beta m} J_n'(k_{\beta m} r_1) e^{in\theta_{\beta m}} \right. \\
& - \sum_{n=-\infty}^{\infty} b_{mn}^{s} k_{\beta m} H_n'(k_{\beta m} r_1) e^{in\theta_{\beta m}} \\
& \left. - \sum_{n=-\infty}^{\infty} b_{mn}^{R} k_{\beta m} J_n'(k_{\beta m} r_1) e^{in\theta_{\beta m}} \right]
\end{aligned}
$$

$$
\begin{aligned}
& \sum_{n=-\infty}^{\infty} \alpha_{n}^{i} \frac{in}{r_1} J_n(k_{\alpha 1} r_1) e^{in\theta_{\alpha 1}} \\
& + \sum_{j=1}^{3} \sum_{n=-\infty}^{\infty} \alpha_{jn}^{r} \frac{in}{r_1} J_n(k_{\alpha j} r_1) e^{in\theta_{\alpha j}} \\
& + \sum_{j=1}^{3} \sum_{n=-\infty}^{\infty} a_{jn}^{s} \frac{in}{r_1} H_n(k_{\alpha j} r_1) e^{in\theta_{\alpha j}} \\
& + \sum_{j=1}^{3} \sum_{n=-\infty}^{\infty} \alpha_{jn}^{R} \frac{in}{r_1} J_n(k_{\alpha j} r_1) e^{in\theta_{\alpha j}} \\
& - \sum_{m=1}^{2} \sum_{n=-\infty}^{\infty} b_{mn}^{r} k_{\beta m} J_n'(k_{\beta m} r_1) e^{in\theta_{\beta m}} \\
& - \sum_{m=1}^{2} \sum_{n=-\infty}^{\infty} b_{mn}^{s} k_{\beta m} H_n'(k_{\beta m} r_1) e^{in\theta_{\beta m}} \\
& - \sum_{m=1}^{2} \sum_{n=-\infty}^{\infty} b_{mn}^{R} k_{\beta m} J_n'(k_{\beta m} r_1) e^{in\theta_{\beta m}} \\
= & \sum_{j=1}^{3} \alpha_{j}^{I} \left[\sum_{n=-\infty}^{\infty} \alpha_{n}^{i} \delta_{j1} \frac{in}{r_1} J_n(k_{\alpha 1} r_1) e^{in\theta_{\alpha 1}} \right. \\
& + \sum_{n=-\infty}^{\infty} \alpha_{jn}^{r} \frac{in}{r_1} J_n(k_{\alpha j} r_1) e^{in\theta_{\alpha j}} \\
& + \sum_{n=-\infty}^{\infty} a_{jn}^{s} \frac{in}{r_1} H_n(k_{\alpha j} r_1) e^{in\theta_{\alpha j}} \\
& \left. + \sum_{n=-\infty}^{\infty} \alpha_{jn}^{R} \frac{in}{r_1} J_n(k_{\alpha j} r_1) e^{in\theta_{\alpha j}} \right] \\
& - \sum_{m=1}^{2} \beta_{m}^{I} \left[\sum_{n=-\infty}^{\infty} b_{mn}^{r} k_{\beta m} J_n'(k_{\beta m} r_1) e^{in\theta_{\beta m}} \right. \\
& - \sum_{n=-\infty}^{\infty} b_{mn}^{s} k_{\beta m} H_n'(k_{\beta m} r_1) e^{in\theta_{\beta m}} \\
& \left. - \sum_{n=-\infty}^{\infty} b_{mn}^{R} k_{\beta m} J_n'(k_{\beta m} r_1) e^{in\theta_{\beta m}} \right]
\end{aligned}
$$

洞室表面方程组：

$$[\bar{E}^S]_{5\times5}\begin{bmatrix}a_{jn}^S\\ b_{mn}^S\end{bmatrix}+[\bar{E}^R]_{5\times5}\begin{bmatrix}a_{jn}^R+a_{jn}^1\\ b_{mn}^R+b_{mn}^1\end{bmatrix}=0$$

对于(1)

系数定义

$$\begin{aligned}A_j^P &= (\lambda_S+2\mu_S)-K_L\alpha_j^L+(\lambda_I+2\mu_I)\alpha_j^I\\ B_j^P &= \lambda_S-K_L\alpha_j^L+\lambda_I\alpha_j^I\\ C_m^S &= 2(\lambda_S+\mu_S)-2K_L\beta_m^L+2(\lambda_I+\mu_I)\beta_m^I\\ D_m^S &= \lambda_S-K_L\beta_m^L+\lambda_I\beta_m^I\end{aligned}$$

E^S矩阵第一行系数

$$\begin{aligned}E_{11}^S(n) &= A_1^Pk_{\alpha1}^2H_n''(k_{\alpha1}r_1)+B_1^P\left[\frac{k_{\alpha1}}{r_1}H_n'(k_{\alpha1}r_1)-\frac{n^2}{r_1^2}H_n(k_{\alpha1}r_1)\right]e^{in\theta_{\alpha1}}\\ E_{12}^S(n) &= A_2^Pk_{\alpha2}^2H_n''(k_{\alpha2}r_1)+B_2^P\left[\frac{k_{\alpha2}}{r_1}H_n'(k_{\alpha2}r_1)-\frac{n^2}{r_1^2}H_n(k_{\alpha2}r_1)\right]e^{in\theta_{\alpha2}}\\ E_{13}^S(n) &= A_3^Pk_{\alpha3}^2H_n''(k_{\alpha3}r_1)+B_3^P\left[\frac{k_{\alpha3}}{r_1}H_n'(k_{\alpha3}r_1)-\frac{n^2}{r_1^2}H_n(k_{\alpha3}r_1)\right]e^{in\theta_{\alpha3}}\\ E_{14}^S(n) &= C_1^S\frac{in}{r_1}k_{\beta1}H_n'(k_{\beta1}r_1)-D_1^S\frac{in}{r_1^2}H_n(k_{\beta1}r_1)e^{in\theta_{\beta1}}\\ E_{15}^S(n) &= C_2^S\frac{in}{r_1}k_{\beta2}H_n'(k_{\beta2}r_1)-D_2^S\frac{in}{r_1^2}H_n(k_{\beta2}r_1)e^{in\theta_{\beta2}}\end{aligned}$$

系数定义

$$\begin{aligned}A_j^P &= (\lambda_S+2\mu_S)-K_L\alpha_j^L+(\lambda_I+2\mu_I)\alpha_j^I\\ B_j^P &= \lambda_S-K_L\alpha_j^L+\lambda_I\alpha_j^I\\ C_m^S &= 2(\lambda_S+\mu_S)-2K_L\beta_m^L+2(\lambda_I+\mu_I)\beta_m^I\\ D_m^S &= \lambda_S-K_L\beta_m^L+\lambda_I\beta_m^I\end{aligned}$$

E^R矩阵第一行系数

$$\begin{aligned}E_{11}^R(n) &= A_1^Pk_{\alpha1}^2J_n''+B_1^P\left(\frac{k_{\alpha1}}{r_1}J_n'-\frac{n^2}{r_1^2}J_n\right)e^{in\theta_{\alpha1}}\\ E_{12}^R(n) &= A_2^Pk_{\alpha2}^2J_n''+B_2^P\left(\frac{k_{\alpha2}}{r_1}J_n'-\frac{n^2}{r_1^2}J_n\right)e^{in\theta_{\alpha2}}\\ E_{13}^R(n) &= A_3^Pk_{\alpha3}^2J_n''+B_3^P\left(\frac{k_{\alpha3}}{r_1}J_n'-\frac{n^2}{r_1^2}J_n\right)e^{in\theta_{\alpha3}}\\ E_{14}^R(n) &= C_1^S\frac{in}{r_1}k_{\beta1}J_n'-D_1^S\frac{in}{r_1^2}J_ne^{in\theta_{\beta1}}\\ E_{15}^R(n) &= C_2^S\frac{in}{r_1}k_{\beta2}J_n'-D_2^S\frac{in}{r_1^2}J_ne^{in\theta_{\beta2}}\end{aligned}$$

对于（2）

系数定义

$$\begin{aligned}F_j^\alpha(n) &= \frac{in}{r_1}k_{\alpha j}H_n'(k_{\alpha j}r_1)-\frac{in}{r_1^2}H_n(k_{\alpha j}r_1)\\ F_m^\beta(n) &= k_{\beta m}^2H_n''(k_{\beta m}r_1)-\frac{k_{\beta m}}{r_1}H_n'(k_{\beta m}r_1)+\frac{n^2}{r_1^2}H_n(k_{\beta m}r_1)\end{aligned}$$

E^S矩阵第二行系数

$$\begin{aligned}E_{21}^S(n) &= (\mu_S+\mu_I\alpha_1^I)F_1^\alpha(n)e^{in\theta_{\alpha1}}\\ E_{22}^S(n) &= (\mu_S+\mu_I\alpha_2^I)F_2^\alpha(n)e^{in\theta_{\alpha2}}\\ E_{23}^S(n) &= (\mu_S+\mu_I\alpha_3^I)F_3^\alpha(n)e^{in\theta_{\alpha3}}\\ E_{24}^S(n) &= (\mu_S+\mu_I\beta_1^I)F_1^\beta(n)e^{in\theta_{\beta1}}\\ E_{25}^S(n) &= (\mu_S+\mu_I\beta_2^I)F_2^\beta(n)e^{in\theta_{\beta2}}\end{aligned}$$

系数定义

$$\begin{aligned}C_1 &= \mu_S+\mu_I\alpha_1^I\\ C_2 &= \mu_S+\mu_I\alpha_2^I\\ C_3 &= \mu_S+\mu_I\alpha_3^I\\ D_1 &= \mu_S+\mu_I\beta_1^I\\ D_2 &= \mu_S+\mu_I\beta_2^I\end{aligned}$$

E^R矩阵第二行系数

$$E_{21}^{R}(n) = C_1\left[\frac{in}{r_1}k_{\alpha1}J_n'(k_{\alpha1}r_1) - \frac{in}{r_1^2}J_n(k_{\alpha1}r_1)\right]e^{in\theta_{\alpha1}}$$
$$E_{22}^{R}(n) = C_2\left[\frac{in}{r_1}k_{\alpha2}J_n'(k_{\alpha2}r_1) - \frac{in}{r_1^2}J_n(k_{\alpha2}r_1)\right]e^{in\theta_{\alpha2}}$$
$$E_{23}^{R}(n) = C_3\left[\frac{in}{r_1}k_{\alpha3}J_n'(k_{\alpha3}r_1) - \frac{in}{r_1^2}J_n(k_{\alpha3}r_1)\right]e^{in\theta_{\alpha3}}$$
$$E_{24}^{R}(n) = D_1\left[k_{\beta1}^2J_n''(k_{\beta1}r_1) - \frac{k_{\beta1}}{r_1}J_n'(k_{\beta1}r_1) + \frac{n^2}{r_1^2}J_n(k_{\beta1}r_1)\right]e^{in\theta_{\beta1}}$$
$$E_{25}^{R}(n) = D_2\left[k_{\beta2}^2J_n''(k_{\beta2}r_1) - \frac{k_{\beta2}}{r_1}J_n'(k_{\beta2}r_1) + \frac{n^2}{r_1^2}J_n(k_{\beta2}r_1)\right]e^{in\theta_{\beta2}}$$

对于（3）

E^S矩阵第三行系数

$$E_{31}^{S}(n) = H_n'(k_{\alpha1}r_1) + \frac{1}{r_1}H_n(k_{\alpha1}r_1) - \frac{n^2}{r_1^2}H_n(k_{\alpha1}r_1)$$
$$E_{32}^{S}(n) = H_n'(k_{\alpha2}r_1) + \frac{1}{r_1}H_n(k_{\alpha2}r_1) - \frac{n^2}{r_1^2}H_n(k_{\alpha2}r_1)$$
$$E_{33}^{S}(n) = H_n'(k_{\alpha3}r_1) + \frac{1}{r_1}H_n(k_{\alpha3}r_1) - \frac{n^2}{r_1^2}H_n(k_{\alpha3}r_1)$$
$$E_{34}^{S}(n) = \frac{in}{r_1}H_n(k_{\beta1}r_1)$$
$$E_{35}^{S}(n) = \frac{in}{r_1}H_n(k_{\beta2}r_1)$$

E^R矩阵第三行系数

$$E_{31}^{R}(n) = J_n'(k_{\alpha1}r_1) + \frac{1}{r_1}J_n(k_{\alpha1}r_1) - \frac{n^2}{r_1^2}J_n(k_{\alpha1}r_1)$$
$$E_{32}^{R}(n) = J_n'(k_{\alpha2}r_1) + \frac{1}{r_1}J_n(k_{\alpha2}r_1) - \frac{n^2}{r_1^2}J_n(k_{\alpha2}r_1)$$
$$E_{33}^{R}(n) = J_n'(k_{\alpha3}r_1) + \frac{1}{r_1}J_n(k_{\alpha3}r_1) - \frac{n^2}{r_1^2}J_n(k_{\alpha3}r_1)$$
$$E_{34}^{R}(n) = \frac{in}{r_1}J_n(k_{\beta1}r_1)$$
$$E_{35}^{R}(n) = \frac{in}{r_1}J_n(k_{\beta2}r_1)$$

对于（4）

E^S矩阵第四行系数

$$E_{41}^{S}(n) = k_{\alpha1}H_n'(k_{\alpha1}r_1)$$
$$E_{42}^{S}(n) = k_{\alpha2}H_n'(k_{\alpha2}r_1)$$
$$E_{43}^{S}(n) = k_{\alpha3}H_n'(k_{\alpha3}r_1)$$
$$E_{44}^{S}(n) = \frac{in}{r_1}H_n(k_{\beta1}r_1)$$
$$E_{45}^{S}(n) = \frac{in}{r_1}H_n(k_{\beta2}r_1)$$

E^R矩阵第四行系数

$$E_{41}^{R}(n) = k_{\alpha1}J_n'(k_{\alpha1}r_1)$$
$$E_{42}^{R}(n) = k_{\alpha2}J_n'(k_{\alpha2}r_1)$$
$$E_{43}^{R}(n) = k_{\alpha3}J_n'(k_{\alpha3}r_1)$$
$$E_{44}^{R}(n) = \frac{in}{r_1}J_n(k_{\beta1}r_1)$$
$$E_{45}^{R}(n) = \frac{in}{r_1}J_n(k_{\beta2}r_1)$$

对于（5）

E^S矩阵第五行系数

$$E_{51}^{S}(n) = \frac{in}{r_1}H_n(k_{\alpha1}r_1)$$
$$E_{52}^{S}(n) = \frac{in}{r_1}H_n(k_{\alpha2}r_1)$$
$$E_{53}^{S}(n) = \frac{in}{r_1}H_n(k_{\alpha3}r_1)$$
$$E_{54}^{S}(n) = -k_{\beta1}H_n'(k_{\beta1}r_1)$$
$$E_{55}^{S}(n) = -k_{\beta2}H_n'(k_{\beta2}r_1)$$

E^R矩阵第五行系数

$$E_{51}^R(n) = \frac{in}{r_1} J_n(k_{\alpha 1} r_1)$$
$$E_{52}^R(n) = \frac{in}{r_1} J_n(k_{\alpha 2} r_1)$$
$$E_{53}^R(n) = \frac{in}{r_1} J_n(k_{\alpha 3} r_1)$$
$$E_{54}^R(n) = - k_{\beta 1} J_n'(k_{\beta 1} r_1)$$
$$E_{55}^R(n) = - k_{\beta 2} J_n'(k_{\beta 2} r_1)$$