对于洞室表面，采用柱坐标

包含自由场（入射波+反射波）和散射场（散射波+驻波）

1. 表示势函数

入射波：

$$\varphi^i = \sum_{n=-\infty}^{\infty} \alpha_n^i J_n(k_{\alpha 1} r_1) e^{in\theta_{a1}}$$

反射波：

$$\varphi_j^r = \sum_{n=-\infty}^{\infty} \alpha_{jn}^r J_n(k_{\alpha j} r_1) e^{in\theta_{a1}} \quad (j=1,2,3)$$

$$\psi_m^r = \sum_{n=-\infty}^{\infty} b_{mn}^r J_n(k_{bm} r_1) e^{in\theta_{bm}} (m=1,2)$$

其中 $\alpha_n^i = e^{-ik_{\alpha 1} h \cos\theta_{\alpha 1}} e^{in\theta_{\alpha 1}}$, $\alpha_{jn}^r = (-1)^n A_j \, e^{ik_{\alpha j} h \cos\theta_{\alpha j}} e^{-in\theta_{\alpha j}}$, $b_{mn}^r = (-1)^n B_m e^{ik_{bm} h \cos\theta_{bm}} e^{-in\theta_{bm}}$

散射波：

$$\varphi_j^s = \sum_{n=-\infty}^{\infty} \alpha_{jn}^s H_n(k_{\alpha j} r_1) e^{in\theta_{a1}} \quad (j=1,2,3)$$

$$\psi_m^s = \sum_{n=-\infty}^{\infty} b_{mn}^s H_n(k_{\beta,m} r_1) e^{in\theta_{\beta,m}} \quad (m=1,2)$$

驻波：

$$\varphi_j^R = \sum_{n=-\infty}^{\infty} \alpha_{jn}^R J_n(k_{\alpha j} r_1) e^{in\theta_{aj}} \quad (j=1,2,3)$$

$$\psi_m^R = \sum_{n=-\infty}^{\infty} b_{mn}^R J_n(k_{bm} r_1) e^{in\theta_{bm}} \quad (m=1,2)$$

2. 求位移分量

固相

径向
位移
分量

$$u_r^S = \sum_{n=-\infty}^{\infty} \alpha_n^i k_{a1} J_n'(k_{a1} r_1) e^{in\theta_{a1}}$$
$$+ \sum_{j=1}^{3} \sum_{n=-\infty}^{\infty} \alpha_{jn}^r k_{aj} J_n'(k_{aj} r_1) e^{in\theta_{aj}}$$
$$+ \sum_{j=1}^{3} \sum_{n=-\infty}^{\infty} a_{jn}^s k_{aj} H_n'(k_{aj} r_1) e^{in\theta_{aj}}$$
$$+ \sum_{j=1}^{3} \sum_{n=-\infty}^{\infty} \alpha_{jn}^R k_{aj} J_n'(k_{aj} r_1) e^{in\theta_{aj}}$$
$$+ \sum_{m=1}^{2} \sum_{n=-\infty}^{\infty} b_{mn}^r \frac{in}{r_1} J_n(k_{bm} r_1) e^{in\theta_{bm}}$$
$$+ \sum_{m=1}^{2} \sum_{n=-\infty}^{\infty} b_{mn}^s \frac{in}{r_1} H_n(k_{bm} r_1) e^{in\theta_{bm}}$$
$$+ \sum_{m=1}^{2} \sum_{n=-\infty}^{\infty} b_{mn}^R \frac{in}{r_1} J_n(k_{bm} r_1) e^{in\theta_{bm}}$$

环向
位移
分量

$$u_\theta^S = \sum_{n=-\infty}^{\infty} \alpha_n^i \frac{in}{r_1} J_n(k_{a1} r_1) e^{in\theta_{a1}}$$
$$+ \sum_{j=1}^{3} \sum_{n=-\infty}^{\infty} \alpha_{jn}^r \frac{in}{r_1} J_n(k_{aj} r_1) e^{in\theta_{aj}}$$
$$+ \sum_{j=1}^{3} \sum_{n=-\infty}^{\infty} a_{jn}^s \frac{in}{r_1} H_n(k_{aj} r_1) e^{in\theta_{aj}}$$
$$+ \sum_{j=1}^{3} \sum_{n=-\infty}^{\infty} \alpha_{jn}^R \frac{in}{r_1} J_n(k_{aj} r_1) e^{in\theta_{aj}}$$
$$- \sum_{m=1}^{2} \sum_{n=-\infty}^{\infty} b_{mn}^r k_{bm} J_n'(k_{bm} r_1) e^{in\theta_{bm}}$$
$$- \sum_{m=1}^{2} \sum_{n=-\infty}^{\infty} b_{mn}^s k_{bm} H_n'(k_{bm} r_1) e^{in\theta_{bm}}$$
$$- \sum_{m=1}^{2} \sum_{n=-\infty}^{\infty} b_{mn}^R k_{bm} J_n'(k_{bm} r_1) e^{in\theta_{bm}}$$

液相

$$u_r^L = \sum_{j=1}^{3}\alpha_j^L\Bigg[\sum_{n=-\infty}^{\infty}\alpha_n^i\,k_{a1}J_n{}'(k_{a1}r_1)e^{in\theta_{a1}}$$

$$+\sum_{n=-\infty}^{\infty}\alpha_{jn}^r\,k_{aj}J_n{}'(k_{aj}r_1)e^{in\theta_{aj}}$$

$$+\sum_{n=-\infty}^{\infty}a_{jn}^s\,k_{aj}H_n{}'(k_{aj}r_1)e^{in\theta_{aj}}$$

径向
位移
分量
$$+\sum_{n=-\infty}^{\infty}\alpha_{jn}^R\,k_{aj}J_n{}'(k_{aj}r_1)e^{in\theta_{aj}}\Bigg]$$

$$+\sum_{m=1}^{2}\beta_m^L\Bigg[\sum_{n=-\infty}^{\infty}b_{mn}^r\frac{in}{r_1}J_n(k_{bm}r_1)e^{in\theta_{bm}}$$

$$+\sum_{n=-\infty}^{\infty}b_{mn}^s\frac{in}{r_1}H_n(k_{bm}r_1)e^{in\theta_{bm}}$$

$$+\sum_{n=-\infty}^{\infty}b_{mn}^R\frac{in}{r_1}J_n(k_{bm}r_1)e^{in\theta_{bm}}\Bigg]$$

$$u_\theta^L = \sum_{j=1}^{3}\alpha_j^L\Bigg[\sum_{n=-\infty}^{\infty}\alpha_n^i\frac{in}{r_1}J_n(k_{a1}r_1)e^{in\theta_{a1}}$$

$$+\sum_{n=-\infty}^{\infty}\alpha_{jn}^r\frac{in}{r_1}J_n(k_{aj}r_1)e^{in\theta_{aj}}$$

$$+\sum_{n=-\infty}^{\infty}a_{jn}^s\frac{in}{r_1}H_n(k_{aj}r_1)e^{in\theta_{aj}}$$

环向
位移
分量
$$+\sum_{n=-\infty}^{\infty}\alpha_{jn}^R\frac{in}{r_1}J_n(k_{aj}r_1)e^{in\theta_{aj}}\Bigg]$$

$$+\sum_{m=1}^{2}\beta_m^L\Bigg[-\sum_{n=-\infty}^{\infty}b_{mn}^r\,k_{bm}J_n{}'(k_{bm}r_1)e^{in\theta_{bm}}$$

$$-\sum_{n=-\infty}^{\infty}b_{mn}^s\,k_{bm}H_n{}'(k_{bm}r_1)e^{in\theta_{bm}}$$

$$-\sum_{n=-\infty}^{\infty}b_{mn}^R\,k_{bm}J_n{}'(k_{bm}r_1)e^{in\theta_{bm}}\Bigg]$$

冰相

$$u_r^I = \sum_{j=1}^{3}\alpha_j^I\Bigg[\sum_{n=-\infty}^{\infty}\alpha_n^i\,k_{a1}J_n{}'(k_{a1}r_1)e^{in\theta_{a1}}$$

$$+\sum_{n=-\infty}^{\infty}\alpha_{jn}^r\,k_{aj}J_n{}'(k_{aj}r_1)e^{in\theta_{aj}}$$

$$+\sum_{n=-\infty}^{\infty}a_{jn}^s\,k_{aj}H_n{}'(k_{aj}r_1)e^{in\theta_{aj}}$$

径向
位移
分量
$$+\sum_{n=-\infty}^{\infty}\alpha_{jn}^R\,k_{aj}J_n{}'(k_{aj}r_1)e^{in\theta_{aj}}\Bigg]$$

$$+\sum_{m=1}^{2}\beta_m^I\Bigg[\sum_{n=-\infty}^{\infty}b_{mn}^r\frac{in}{r_1}J_n(k_{bm}r_1)e^{in\theta_{bm}}$$

$$+\sum_{n=-\infty}^{\infty}b_{mn}^s\frac{in}{r_1}H_n(k_{bm}r_1)e^{in\theta_{bm}}$$

$$+\sum_{n=-\infty}^{\infty}b_{mn}^R\frac{in}{r_1}J_n(k_{bm}r_1)e^{in\theta_{bm}}\Bigg]$$

$$u_\theta^I = \sum_{j=1}^{3}\alpha_j^I\Bigg[\sum_{n=-\infty}^{\infty}\alpha_n^i\frac{in}{r_1}J_n(k_{a1}r_1)e^{in\theta_{a1}}$$

$$+\sum_{n=-\infty}^{\infty}\alpha_{jn}^r\frac{in}{r_1}J_n(k_{aj}r_1)e^{in\theta_{aj}}$$

$$+\sum_{n=-\infty}^{\infty}a_{jn}^s\frac{in}{r_1}H_n(k_{aj}r_1)e^{in\theta_{aj}}$$

环向
位移
分量
$$+\sum_{n=-\infty}^{\infty}\alpha_{jn}^R\frac{in}{r_1}J_n(k_{aj}r_1)e^{in\theta_{aj}}\Bigg]$$

$$+\sum_{m=1}^{2}\beta_m^I\Bigg[-\sum_{n=-\infty}^{\infty}b_{mn}^r\,k_{bm}J_n{}'(k_{bm}r_1)e^{in\theta_{bm}}$$

$$-\sum_{n=-\infty}^{\infty}b_{mn}^s\,k_{bm}H_n{}'(k_{bm}r_1)e^{in\theta_{bm}}$$

$$-\sum_{n=-\infty}^{\infty}b_{mn}^R\,k_{bm}J_n{}'(k_{bm}r_1)e^{in\theta_{bm}}\Bigg]$$

3. 求散度

$$\nabla\cdot u^S = -\sum_{n=-\infty}^{\infty}\alpha_n^i\,k_{a1}^2 J_n(k_{a1}r_1)e^{in\theta_{a1}}$$

$$-\sum_{j=1}^{3}\sum_{n=-\infty}^{\infty}\alpha_{jn}^r\,k_{aj}^2 J_n(k_{aj}r_1)e^{in\theta_{aj}}$$

$$-\sum_{j=1}^{3}\sum_{n=-\infty}^{\infty}a_{jn}^s\,k_{aj}^2 H_n(k_{aj}r_1)e^{in\theta_{aj}}$$

$$-\sum_{j=1}^{3}\sum_{n=-\infty}^{\infty}\alpha_{jn}^R\,k_{aj}^2 J_n(k_{aj}r_1)e^{in\theta_{aj}}$$

$$\nabla\cdot u^L = -\sum_{j=1}^{3}\alpha_j^L\begin{bmatrix}\sum_{n=-\infty}^{\infty}\alpha_n^i\,\delta_{j1}k_{a1}^2 J_n(k_{a1}r_1)e^{in\theta_{a1}}\\ +\sum_{n=-\infty}^{\infty}\alpha_{jn}^r\,k_{aj}^2 J_n(k_{aj}r_1)e^{in\theta_{aj}}\\ +\sum_{n=-\infty}^{\infty}a_{jn}^s\,k_{aj}^2 H_n(k_{aj}r_1)e^{in\theta_{aj}}\\ +\sum_{n=-\infty}^{\infty}\alpha_{jn}^R\,k_{aj}^2 J_n(k_{aj}r_1)e^{in\theta_{aj}}\end{bmatrix}$$

$$\nabla\cdot u^I = -\sum_{j=1}^{3}\alpha_j^I\begin{bmatrix}\sum_{n=-\infty}^{\infty}\alpha_n^i\,\delta_{j1}k_{a1}^2 J_n(k_{a1}r_1)e^{in\theta_{a1}}\\ +\sum_{n=-\infty}^{\infty}\alpha_{jn}^r\,k_{aj}^2 J_n(k_{aj}r_1)e^{in\theta_{aj}}\\ +\sum_{n=-\infty}^{\infty}a_{jn}^s\,k_{aj}^2 H_n(k_{aj}r_1)e^{in\theta_{aj}}\\ +\sum_{n=-\infty}^{\infty}\alpha_{jn}^R\,k_{aj}^2 J_n(k_{aj}r_1)e^{in\theta_{aj}}\end{bmatrix}$$

4.求应变张量

固相

$$\varepsilon_{rr}^S = \sum_{n=-\infty}^{\infty} \alpha_n^i \, k_{a1}^2 J_n{''}(k_{a1}r_1)e^{in\theta_{a1}}$$

$$+ \sum_{j=1}^{3}\sum_{n=-\infty}^{\infty} \alpha_{jn}^r \, k_{aj}^2 J_n{''}(k_{aj}r_1)e^{in\theta_{aj}}$$

$$+ \sum_{j=1}^{3}\sum_{n=-\infty}^{\infty} a_{jn}^s \, k_{aj}^2 H_n{''}(k_{aj}r_1)e^{in\theta_{aj}}$$

$$+ \sum_{j=1}^{3}\sum_{n=-\infty}^{\infty} \alpha_{jn}^R \, k_{aj}^2 J_n{''}(k_{aj}r_1)e^{in\theta_{aj}}$$

$$+ \sum_{m=1}^{2}\sum_{n=-\infty}^{\infty} b_{mn}^r \frac{in}{r_1} k_{bm} J_n{'}(k_{bm}r_1)e^{in\theta_{bm}}$$

$$- \sum_{m=1}^{2}\sum_{n=-\infty}^{\infty} b_{mn}^r \frac{in}{r_1^2} J_n(k_{bm}r_1)e^{in\theta_{bm}}$$

$$+ \sum_{m=1}^{2}\sum_{n=-\infty}^{\infty} b_{mn}^s \frac{in}{r_1} k_{bm} H_n{'}(k_{bm}r_1)e^{in\theta_{bm}}$$

$$- \sum_{m=1}^{2}\sum_{n=-\infty}^{\infty} b_{mn}^s \frac{in}{r_1^2} H_n(k_{bm}r_1)e^{in\theta_{bm}}$$

$$+ \sum_{m=1}^{2}\sum_{n=-\infty}^{\infty} b_{mn}^R \frac{in}{r_1} k_{bm} J_n{'}(k_{bm}r_1)e^{in\theta_{bm}}$$

$$- \sum_{m=1}^{2}\sum_{n=-\infty}^{\infty} b_{mn}^R \frac{in}{r_1^2} J_n(k_{bm}r_1)e^{in\theta_{bm}}$$

$$\varepsilon_{\theta\theta}^S = \sum_{n=-\infty}^{\infty} \alpha_n^i \left[\frac{1}{r_1}k_{a1}J_n{'}(k_{a1}r_1) - \frac{n^2}{r_1^2}J_n(k_{a1}r_1)\right]e^{in\theta_{a1}}$$

$$+ \sum_{j=1}^{3}\sum_{n=-\infty}^{\infty} \alpha_{jn}^r \left[\frac{1}{r_1}k_{aj}J_n{'}(k_{aj}r_1) - \frac{n^2}{r_1^2}J_n(k_{aj}r_1)\right]e^{in\theta_{aj}}$$

$$+ \sum_{j=1}^{3}\sum_{n=-\infty}^{\infty} a_{jn}^s \left[\frac{1}{r_1}k_{aj}H_n{'}(k_{aj}r_1) - \frac{n^2}{r_1^2}H_n(k_{aj}r_1)\right]e^{in\theta_{aj}}$$

$$+ \sum_{j=1}^{3}\sum_{n=-\infty}^{\infty} \alpha_{jn}^R \left[\frac{1}{r_1}k_{aj}J_n{'}(k_{aj}r_1) - \frac{n^2}{r_1^2}J_n(k_{aj}r_1)\right]e^{in\theta_{aj}}$$

$$+ \sum_{m=1}^{2}\sum_{n=-\infty}^{\infty} b_{mn}^r \left[-\frac{in}{r_1^2}J_n(k_{bm}r_1) - k_{bm}^2 J_n{''}(k_{bm}r_1)\right]e^{in\theta_{bm}}$$

$$+ \sum_{m=1}^{2}\sum_{n=-\infty}^{\infty} b_{mn}^s \left[-\frac{in}{r_1^2}H_n(k_{bm}r_1) - k_{bm}^2 H_n{''}(k_{bm}r_1)\right]e^{in\theta_{bm}}$$

$$+ \sum_{m=1}^{2}\sum_{n=-\infty}^{\infty} b_{mn}^R \left[-\frac{in}{r_1^2}J_n(k_{bm}r_1) - k_{bm}^2 J_n{''}(k_{bm}r_1)\right]e^{in\theta_{bm}}$$

$$\varepsilon_{r\theta}^S = \frac{1}{2}\left\{\sum_{n=-\infty}^{\infty} \alpha_n^i \left[\frac{2in}{r_1}k_{a1}J_n{'}(k_{a1}r_1) - \frac{in}{r_1^2}J_n(k_{a1}r_1)\right]e^{in\theta_{a1}}\right.$$

$$+ \sum_{j=1}^{3}\sum_{n=-\infty}^{\infty} \alpha_{jn}^r \left[\frac{2in}{r_1}k_{aj}J_n{'}(k_{aj}r_1) - \frac{in}{r_1^2}J_n(k_{aj}r_1)\right]e^{in\theta_{aj}}$$

$$+ \sum_{j=1}^{3}\sum_{n=-\infty}^{\infty} a_{jn}^s \left[\frac{2in}{r_1}k_{aj}H_n{'}(k_{aj}r_1) - \frac{in}{r_1^2}H_n(k_{aj}r_1)\right]e^{in\theta_{aj}}$$

$$+ \sum_{j=1}^{3}\sum_{n=-\infty}^{\infty} \alpha_{jn}^R \left[\frac{2in}{r_1}k_{aj}J_n{'}(k_{aj}r_1) - \frac{in}{r_1^2}J_n(k_{aj}r_1)\right]e^{in\theta_{aj}}$$

$$+ \sum_{m=1}^{2}\sum_{n=-\infty}^{\infty} b_{mn}^r \left[-\frac{2n^2}{r_1^2}J_n(k_{bm}r_1) - k_{bm}^2 J_n{''}(k_{bm}r_1) + \frac{1}{r_1}k_{bm}J_n{'}(k_{bm}r_1)\right]e^{in\theta_{bm}}$$

$$+ \sum_{m=1}^{2}\sum_{n=-\infty}^{\infty} b_{mn}^s \left[-\frac{2n^2}{r_1^2}H_n(k_{bm}r_1) - k_{bm}^2 H_n{''}(k_{bm}r_1) + \frac{1}{r_1}k_{bm}H_n{'}(k_{bm}r_1)\right]e^{in\theta_{bm}}$$

$$\left.+ \sum_{m=1}^{2}\sum_{n=-\infty}^{\infty} b_{mn}^R \left[-\frac{2n^2}{r_1^2}J_n(k_{bm}r_1) - k_{bm}^2 J_n{''}(k_{bm}r_1) + \frac{1}{r_1}k_{bm}J_n{'}(k_{bm}r_1)\right]e^{in\theta_{bm}}\right\}$$

液相

$$\varepsilon_{rr}^L = \sum_{j=1}^{3} \alpha_j^L \left[\sum_{n=-\infty}^{\infty} \alpha_n^i k_{a1}^2 J_n''(k_{a1}r_1)e^{in\theta_{a1}} \right.$$
$$+ \sum_{n=-\infty}^{\infty} \alpha_{jn}^r k_{aj}^2 J_n''(k_{aj}r_1)e^{in\theta_{aj}}$$
$$+ \sum_{n=-\infty}^{\infty} a_{jn}^s k_{aj}^2 H_n''(k_{aj}r_1)e^{in\theta_{aj}}$$
$$\left. + \sum_{n=-\infty}^{\infty} \alpha_{jn}^R k_{aj}^2 J_n''(k_{aj}r_1)e^{in\theta_{aj}} \right]$$
$$+ \sum_{m=1}^{2} \beta_m^L \left[\sum_{n=-\infty}^{\infty} b_{mn}^r \frac{in}{r_1} k_{bm} J_n'(k_{bm}r_1)e^{in\theta_{bm}} \right.$$
$$- \sum_{n=-\infty}^{\infty} b_{mn}^r \frac{in}{r_1^2} J_n(k_{bm}r_1)e^{in\theta_{bm}}$$
$$+ \sum_{n=-\infty}^{\infty} b_{mn}^s \frac{in}{r_1} k_{bm} H_n'(k_{bm}r_1)e^{in\theta_{bm}}$$
$$- \sum_{n=-\infty}^{\infty} b_{mn}^s \frac{in}{r_1^2} H_n(k_{bm}r_1)e^{in\theta_{bm}}$$
$$+ \sum_{n=-\infty}^{\infty} b_{mn}^R \frac{in}{r_1} k_{bm} J_n'(k_{bm}r_1)e^{in\theta_{bm}}$$
$$\left. - \sum_{n=-\infty}^{\infty} b_{mn}^R \frac{in}{r_1^2} J_n(k_{bm}r_1)e^{in\theta_{bm}} \right]$$

$$\varepsilon_{\theta\theta}^L = \sum_{j=1}^{3} \alpha_j^L \left[\sum_{n=-\infty}^{\infty} \alpha_n^i \left(\frac{1}{r_1} k_{a1} J_n'(k_{a1}r_1) - \frac{n^2}{r_1^2} J_n(k_{a1}r_1) \right) e^{in\theta_{a1}} \right.$$
$$+ \sum_{n=-\infty}^{\infty} \alpha_{jn}^r \left(\frac{1}{r_1} k_{aj} J_n'(k_{aj}r_1) - \frac{n^2}{r_1^2} J_n(k_{aj}r_1) \right) e^{in\theta_{aj}}$$
$$+ \sum_{n=-\infty}^{\infty} a_{jn}^s \left(\frac{1}{r_1} k_{aj} H_n'(k_{aj}r_1) - \frac{n^2}{r_1^2} H_n(k_{aj}r_1) \right) e^{in\theta_{aj}}$$
$$\left. + \sum_{n=-\infty}^{\infty} \alpha_{jn}^R \left(\frac{1}{r_1} k_{aj} J_n'(k_{aj}r_1) - \frac{n^2}{r_1^2} J_n(k_{aj}r_1) \right) e^{in\theta_{aj}} \right]$$
$$+ \sum_{m=1}^{2} \beta_m^L \left[\sum_{n=-\infty}^{\infty} b_{mn}^r \left(-\frac{in}{r_1^2} J_n(k_{bm}r_1) - k_{bm}^2 J_n''(k_{bm}r_1) \right) e^{in\theta_{bm}} \right.$$
$$+ \sum_{n=-\infty}^{\infty} b_{mn}^s \left(-\frac{in}{r_1^2} H_n(k_{bm}r_1) - k_{bm}^2 H_n''(k_{bm}r_1) \right) e^{in\theta_{bm}}$$
$$\left. + \sum_{n=-\infty}^{\infty} b_{mn}^R \left(-\frac{in}{r_1^2} J_n(k_{bm}r_1) - k_{bm}^2 J_n''(k_{bm}r_1) \right) e^{in\theta_{bm}} \right]$$

$$\varepsilon_{r\theta}^L = \frac{1}{2} \left\{ \sum_{j=1}^{3} \alpha_j^L \left[\sum_{n=-\infty}^{\infty} \alpha_n^i \left(\frac{2in}{r_1} k_{a1} J_n'(k_{a1}r_1) - \frac{in}{r_1^2} J_n(k_{a1}r_1) \right) e^{in\theta_{a1}} \right. \right.$$
$$+ \sum_{n=-\infty}^{\infty} \alpha_{jn}^r \left(\frac{2in}{r_1} k_{aj} J_n'(k_{aj}r_1) - \frac{in}{r_1^2} J_n(k_{aj}r_1) \right) e^{in\theta_{aj}}$$
$$+ \sum_{n=-\infty}^{\infty} a_{jn}^s \left(\frac{2in}{r_1} k_{aj} H_n'(k_{aj}r_1) - \frac{in}{r_1^2} H_n(k_{aj}r_1) \right) e^{in\theta_{aj}}$$
$$\left. + \sum_{n=-\infty}^{\infty} \alpha_{jn}^R \left(\frac{2in}{r_1} k_{aj} J_n'(k_{aj}r_1) - \frac{in}{r_1^2} J_n(k_{aj}r_1) \right) e^{in\theta_{aj}} \right]$$
$$+ \sum_{m=1}^{2} \beta_m^L \left[\sum_{n=-\infty}^{\infty} b_{mn}^r \left(-\frac{2n^2}{r_1^2} J_n(k_{bm}r_1) - k_{bm}^2 J_n''(k_{bm}r_1) + \frac{1}{r_1} k_{bm} J_n'(k_{bm}r_1) \right) e^{in\theta_{bm}} \right.$$
$$+ \sum_{n=-\infty}^{\infty} b_{mn}^s \left(-\frac{2n^2}{r_1^2} H_n(k_{bm}r_1) - k_{bm}^2 H_n''(k_{bm}r_1) + \frac{1}{r_1} k_{bm} H_n'(k_{bm}r_1) \right) e^{in\theta_{bm}}$$
$$\left. \left. + \sum_{n=-\infty}^{\infty} b_{mn}^R \left(-\frac{2n^2}{r_1^2} J_n(k_{bm}r_1) - k_{bm}^2 J_n''(k_{bm}r_1) + \frac{1}{r_1} k_{bm} J_n'(k_{bm}r_1) \right) e^{in\theta_{bm}} \right] \right\}$$

冰相

$$\varepsilon_{rr}^I = \sum_{j=1}^{3} \alpha_j^I \Bigg[\sum_{n=-\infty}^{\infty} \alpha_n^i \, k_{a1}^2 J_n''(k_{a1}r_1) e^{in\theta_{a1}}$$

$$+ \sum_{n=-\infty}^{\infty} \alpha_{jn}^r \, k_{aj}^2 J_n''(k_{aj}r_1) e^{in\theta_{aj}}$$

$$+ \sum_{n=-\infty}^{\infty} a_{jn}^s \, k_{aj}^2 H_n''(k_{aj}r_1) e^{in\theta_{aj}}$$

$$+ \sum_{n=-\infty}^{\infty} \alpha_{jn}^R \, k_{aj}^2 J_n''(k_{aj}r_1) e^{in\theta_{aj}} \Bigg]$$

$$+ \sum_{m=1}^{2} \beta_m^I \Bigg[\sum_{n=-\infty}^{\infty} b_{mn}^r \frac{in}{r_1} k_{bm} J_n'(k_{bm}r_1) e^{in\theta_{bm}}$$

$$- \sum_{n=-\infty}^{\infty} b_{mn}^r \frac{in}{r_1^2} J_n(k_{bm}r_1) e^{in\theta_{bm}}$$

$$+ \sum_{n=-\infty}^{\infty} b_{mn}^s \frac{in}{r_1} k_{bm} H_n'(k_{bm}r_1) e^{in\theta_{bm}}$$

$$- \sum_{n=-\infty}^{\infty} b_{mn}^s \frac{in}{r_1^2} H_n(k_{bm}r_1) e^{in\theta_{bm}}$$

$$+ \sum_{n=-\infty}^{\infty} b_{mn}^R \frac{in}{r_1} k_{bm} J_n'(k_{bm}r_1) e^{in\theta_{bm}}$$

$$- \sum_{n=-\infty}^{\infty} b_{mn}^R \frac{in}{r_1^2} J_n(k_{bm}r_1) e^{in\theta_{bm}} \Bigg]$$

$$\varepsilon_{\theta\theta}^I = \sum_{j=1}^{3} \alpha_j^I \Bigg[\sum_{n=-\infty}^{\infty} \alpha_n^i \left(\frac{1}{r_1} k_{a1} J_n'(k_{a1}r_1) - \frac{n^2}{r_1^2} J_n(k_{a1}r_1) \right) e^{in\theta_{a1}}$$

$$+ \sum_{n=-\infty}^{\infty} \alpha_{jn}^r \left(\frac{1}{r_1} k_{aj} J_n'(k_{aj}r_1) - \frac{n^2}{r_1^2} J_n(k_{aj}r_1) \right) e^{in\theta_{aj}}$$

$$+ \sum_{n=-\infty}^{\infty} a_{jn}^s \left(\frac{1}{r_1} k_{aj} H_n'(k_{aj}r_1) - \frac{n^2}{r_1^2} H_n(k_{aj}r_1) \right) e^{in\theta_{aj}}$$

$$+ \sum_{n=-\infty}^{\infty} \alpha_{jn}^R \left(\frac{1}{r_1} k_{aj} J_n'(k_{aj}r_1) - \frac{n^2}{r_1^2} J_n(k_{aj}r_1) \right) e^{in\theta_{aj}} \Bigg]$$

$$+ \sum_{m=1}^{2} \beta_m^I \Bigg[\sum_{n=-\infty}^{\infty} b_{mn}^r \left(-\frac{in}{r_1^2} J_n(k_{bm}r_1) - k_{bm}^2 J_n''(k_{bm}r_1) \right) e^{in\theta_{bm}}$$

$$+ \sum_{n=-\infty}^{\infty} b_{mn}^s \left(-\frac{in}{r_1^2} H_n(k_{bm}r_1) - k_{bm}^2 H_n''(k_{bm}r_1) \right) e^{in\theta_{bm}}$$

$$+ \sum_{n=-\infty}^{\infty} b_{mn}^R \left(-\frac{in}{r_1^2} J_n(k_{bm}r_1) - k_{bm}^2 J_n''(k_{bm}r_1) \right) e^{in\theta_{bm}} \Bigg]$$

$$\varepsilon_{r\theta}^I = \frac{1}{2} \Bigg\{ \sum_{j=1}^{3} \alpha_j^I \Bigg[\sum_{n=-\infty}^{\infty} \alpha_n^i \left(\frac{2in}{r_1} k_{a1} J_n'(k_{a1}r_1) - \frac{in}{r_1^2} J_n(k_{a1}r_1) \right) e^{in\theta_{a1}}$$

$$+ \sum_{n=-\infty}^{\infty} \alpha_{jn}^r \left(\frac{2in}{r_1} k_{aj} J_n'(k_{aj}r_1) - \frac{in}{r_1^2} J_n(k_{aj}r_1) \right) e^{in\theta_{aj}}$$

$$+ \sum_{n=-\infty}^{\infty} a_{jn}^s \left(\frac{2in}{r_1} k_{aj} H_n'(k_{aj}r_1) - \frac{in}{r_1^2} H_n(k_{aj}r_1) \right) e^{in\theta_{aj}}$$

$$+ \sum_{n=-\infty}^{\infty} \alpha_{jn}^R \left(\frac{2in}{r_1} k_{aj} J_n'(k_{aj}r_1) - \frac{in}{r_1^2} J_n(k_{aj}r_1) \right) e^{in\theta_{aj}} \Bigg]$$

$$+ \sum_{m=1}^{2} \beta_m^I \Bigg[\sum_{n=-\infty}^{\infty} b_{mn}^r \left(-\frac{2n^2}{r_1^2} J_n(k_{bm}r_1) - k_{bm}^2 J_n''(k_{bm}r_1) + \frac{1}{r_1} k_{bm} J_n'(k_{bm}r_1) \right) e^{in\theta_{bm}}$$

$$+ \sum_{n=-\infty}^{\infty} b_{mn}^s \left(-\frac{2n^2}{r_1^2} H_n(k_{bm}r_1) - k_{bm}^2 H_n''(k_{bm}r_1) + \frac{1}{r_1} k_{bm} H_n'(k_{bm}r_1) \right) e^{in\theta_{bm}}$$

$$+ \sum_{n=-\infty}^{\infty} b_{mn}^R \left(-\frac{2n^2}{r_1^2} J_n(k_{bm}r_1) - k_{bm}^2 J_n''(k_{bm}r_1) + \frac{1}{r_1} k_{bm} J_n'(k_{bm}r_1) \right) e^{in\theta_{bm}} \Bigg] \Bigg\}$$

5.代入本构方程

$$(\lambda_S + 2\mu_S)\left(\frac{\partial^2 \varphi^S}{\partial r^2} + \frac{1}{r}\frac{\partial^2 \psi^S}{\partial r \partial \theta}\right) + \lambda_S\left(\frac{1}{r}\frac{\partial \varphi^S}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \varphi^S}{\partial \theta^2} - \frac{1}{r^2}\frac{\partial \psi^S}{\partial \theta} + \frac{1}{r}\frac{\partial^2 \psi^S}{\partial r \partial \theta}\right)$$
$$-K_L\left[\frac{\partial^2 \varphi^L}{\partial r^2} + \frac{1}{r}\frac{\partial \varphi^L}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \varphi^L}{\partial \theta^2} + \frac{1}{r}\frac{\partial^2 \psi^L}{\partial r \partial \theta} - \frac{1}{r^2}\frac{\partial \psi^L}{\partial \theta} + \frac{1}{r}\frac{\partial^2 \psi^L}{\partial r \partial \theta}\right]$$
$$+(\lambda_I + 2\mu_I)\left(\frac{\partial^2 \varphi^I}{\partial r^2} + \frac{1}{r}\frac{\partial^2 \psi^I}{\partial r \partial \theta}\right) + \lambda_I\left(\frac{1}{r}\frac{\partial \varphi^I}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \varphi^I}{\partial \theta^2} - \frac{1}{r^2}\frac{\partial \psi^I}{\partial \theta} + \frac{1}{r}\frac{\partial^2 \psi^I}{\partial r \partial \theta}\right) = 0$$

$$\mu_S\left(\frac{1}{r}\frac{\partial^2 \varphi^S}{\partial r \partial \theta} - \frac{1}{r^2}\frac{\partial \varphi^S}{\partial \theta} + \frac{\partial^2 \psi^S}{\partial r^2} - \frac{1}{r}\frac{\partial \psi^S}{\partial r} - \frac{1}{r^2}\frac{\partial^2 \psi^S}{\partial \theta^2}\right)$$
$$+\mu_I\left(\frac{1}{r}\frac{\partial^2 \varphi^I}{\partial r \partial \theta} - \frac{1}{r^2}\frac{\partial \varphi^I}{\partial \theta} + \frac{\partial^2 \psi^I}{\partial r^2} - \frac{1}{r}\frac{\partial \psi^I}{\partial r} - \frac{1}{r^2}\frac{\partial^2 \psi^I}{\partial \theta^2}\right) = 0$$

$$K_L\left(\frac{\partial^2 \varphi^L}{\partial r^2} + \frac{1}{r}\frac{\partial \varphi^L}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \varphi^L}{\partial \theta^2} + \frac{1}{r}\frac{\partial^2 \psi^L}{\partial r \partial \theta} - \frac{1}{r^2}\frac{\partial \psi^L}{\partial \theta} + \frac{1}{r}\frac{\partial^2 \psi^L}{\partial r \partial \theta}\right) = 0$$

$$\frac{\partial \varphi^S}{\partial r} + \frac{1}{r}\frac{\partial \psi^S}{\partial \theta} = \frac{\partial \varphi^I}{\partial r} + \frac{1}{r}\frac{\partial \psi^I}{\partial \theta}$$

$$\frac{1}{r}\frac{\partial \varphi^S}{\partial \theta} - \frac{\partial \psi^S}{\partial r} = \frac{1}{r}\frac{\partial \varphi^I}{\partial \theta} - \frac{\partial \psi^I}{\partial r}$$

具体分解一下
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$$(\lambda_S + 2\mu_S)\left[\frac{\partial^2 \varphi^S}{\partial r^2} + \frac{1}{r}\frac{\partial^2 \psi^S}{\partial r \partial \theta}\right] + \lambda_S\left[\frac{1}{r}\frac{\partial \varphi^S}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \varphi^S}{\partial \theta^2} - \frac{1}{r^2}\frac{\partial \psi^S}{\partial \theta} + \frac{1}{r}\frac{\partial^2 \psi^S}{\partial r \partial \theta}\right]$$

$$= (\lambda_S + 2\mu_S)\left[\frac{\partial^2}{\partial r^2}\left(\sum_{n=-\infty}^{\infty} \alpha_n^i J_n(k_{\alpha 1} r_1)e^{in\theta_{\alpha 1}} + \sum_{j=1}^{3}\sum_{n=-\infty}^{\infty} \alpha_{jn}^r J_n(k_{\alpha j} r_1)e^{in\theta_{\alpha j}}\right.\right.$$

$$\left(+\sum_{j=1}^{3}\sum_{n=-\infty}^{\infty} a_{jn}^s H_n(k_{\alpha j} r_1)e^{in\theta_{\alpha j}} + \sum_{j=1}^{3}\sum_{n=-\infty}^{\infty} \alpha_{jn}^R J_n(k_{\alpha j} r_1)e^{in\theta_{\alpha j}}\right)\right)$$

$$\left(+\frac{1}{r}\frac{\partial^2}{\partial r \partial \theta}\left(\sum_{m=1}^{2}\sum_{n=-\infty}^{\infty} b_{mn}^r J_n(k_{\beta m} r_1)e^{in\theta_{\beta m}} + \sum_{m=1}^{2}\sum_{n=-\infty}^{\infty} b_{mn}^s H_n(k_{\beta m} r_1)e^{in\theta_{\beta m}}\right.\right.$$

$$\left.\left.+\sum_{m=1}^{2}\sum_{n=-\infty}^{\infty} b_{mn}^R J_n(k_{\beta m} r_1)e^{in\theta_{\beta m}}\right)\right]$$

$$+\lambda_S\left[\frac{1}{r}\frac{\partial}{\partial r}\left(\sum_{n=-\infty}^{\infty} \alpha_n^i J_n(k_{\alpha 1} r_1)e^{in\theta_{\alpha 1}} + \sum_{j=1}^{3}\sum_{n=-\infty}^{\infty} \alpha_{jn}^r J_n(k_{\alpha j} r_1)e^{in\theta_{\alpha j}}\right.\right.$$

$$\left(+\sum_{j=1}^{3}\sum_{n=-\infty}^{\infty} a_{jn}^s H_n(k_{\alpha j} r_1)e^{in\theta_{\alpha j}} + \sum_{j=1}^{3}\sum_{n=-\infty}^{\infty} \alpha_{jn}^R J_n(k_{\alpha j} r_1)e^{in\theta_{\alpha j}}\right)\right)$$

$$+\frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\left(\sum_{n=-\infty}^{\infty} \alpha_n^i J_n(k_{\alpha 1} r_1)e^{in\theta_{\alpha 1}} + \sum_{j=1}^{3}\sum_{n=-\infty}^{\infty} \alpha_{jn}^r J_n(k_{\alpha j} r_1)e^{in\theta_{\alpha j}}\right.$$

$$\left.+\sum_{j=1}^{3}\sum_{n=-\infty}^{\infty} a_{jn}^s H_n(k_{\alpha j} r_1)e^{in\theta_{\alpha j}} + \sum_{j=1}^{3}\sum_{n=-\infty}^{\infty} \alpha_{jn}^R J_n(k_{\alpha j} r_1)e^{in\theta_{\alpha j}}\right)$$

$$-\frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\sum_{m=1}^{2}\sum_{n=-\infty}^{\infty} b_{mn}^r J_n(k_{\beta m} r_1)e^{in\theta_{\beta m}} + \sum_{m=1}^{2}\sum_{n=-\infty}^{\infty} b_{mn}^s H_n(k_{\beta m} r_1)e^{in\theta_{\beta m}}\right.$$

$$\left.+\sum_{m=1}^{2}\sum_{n=-\infty}^{\infty} b_{mn}^R J_n(k_{\beta m} r_1)e^{in\theta_{\beta m}}\right)$$

$$\left(+\frac{1}{r}\frac{\partial^2}{\partial r \partial \theta}\left(\sum_{m=1}^{2}\sum_{n=-\infty}^{\infty} b_{mn}^r J_n(k_{\beta m} r_1)e^{in\theta_{\beta m}} + \sum_{m=1}^{2}\sum_{n=-\infty}^{\infty} b_{mn}^s H_n(k_{\beta m} r_1)e^{in\theta_{\beta m}}\right.\right.$$

$$\left.\left.+\sum_{m=1}^{2}\sum_{n=-\infty}^{\infty} b_{mn}^R J_n(k_{\beta m} r_1)e^{in\theta_{\beta m}}\right)\right]$$

$$-K_L\left[\frac{\partial^2\varphi^L}{\partial r^2}+\frac{1}{r}\frac{\partial\varphi^L}{\partial r}+\frac{1}{r^2}\frac{\partial^2\varphi^L}{\partial\theta^2}+\frac{1}{r}\frac{\partial^2\psi^L}{\partial r\partial\theta}-\frac{1}{r^2}\frac{\partial\psi^L}{\partial\theta}+\frac{1}{r}\frac{\partial^2\psi^L}{\partial r\partial\theta}\right]$$

$$=-K_L\left[\frac{\partial^2}{\partial r^2}\left(\sum_{j=1}^{3}\alpha_j^L\left[\sum_{n=-\infty}^{\infty}\alpha_n^i\,\delta_{j1}J_n(k_{\alpha 1}r_1)e^{in\theta_{\alpha 1}}+\sum_{n=-\infty}^{\infty}\alpha_{jn}^r J_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\right.\right.\right.$$

$$\left.\left.\left(+\sum_{n=-\infty}^{\infty}a_{jn}^s H_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}+\sum_{n=-\infty}^{\infty}\alpha_{jn}^R J_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\right]\right)\right)$$

$$+\frac{1}{r}\frac{\partial}{\partial r}\left(\sum_{j=1}^{3}\alpha_j^L\left[\sum_{n=-\infty}^{\infty}\alpha_n^i\,\delta_{j1}J_n(k_{\alpha 1}r_1)e^{in\theta_{\alpha 1}}+\sum_{n=-\infty}^{\infty}\alpha_{jn}^r J_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\right.\right.$$

$$\left.\left.+\sum_{n=-\infty}^{\infty}a_{jn}^s H_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}+\sum_{n=-\infty}^{\infty}\alpha_{jn}^R J_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\right]\right)$$

$$+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\left(\sum_{j=1}^{3}\alpha_j^L\left[\sum_{n=-\infty}^{\infty}\alpha_n^i\,\delta_{j1}J_n(k_{\alpha 1}r_1)e^{in\theta_{\alpha 1}}+\sum_{n=-\infty}^{\infty}\alpha_{jn}^r J_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\right.\right.$$

$$\left.\left.+\sum_{n=-\infty}^{\infty}a_{jn}^s H_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}+\sum_{n=-\infty}^{\infty}\alpha_{jn}^R J_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\right]\right)$$

$$+\frac{1}{r}\frac{\partial^2}{\partial r\partial\theta}\left(\sum_{m=1}^{2}\beta_m^L\left[\sum_{n=-\infty}^{\infty}b_{mn}^r J_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}+\sum_{n=-\infty}^{\infty}b_{mn}^s H_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\right.\right.$$

$$\left.\left.+\sum_{n=-\infty}^{\infty}b_{mn}^R J_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\right]\right)$$

$$-\frac{1}{r^2}\frac{\partial}{\partial\theta}\left(\sum_{m=1}^{2}\beta_m^L\left[\sum_{n=-\infty}^{\infty}b_{mn}^r J_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}+\sum_{n=-\infty}^{\infty}b_{mn}^s H_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\right.\right.$$

$$\left.\left.+\sum_{n=-\infty}^{\infty}b_{mn}^R J_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\right]\right)$$

$$\left(+\frac{1}{r}\frac{\partial^2}{\partial r\partial\theta}\left(\sum_{m=1}^{2}\beta_m^L\left[\sum_{n=-\infty}^{\infty}b_{mn}^r J_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}+\sum_{n=-\infty}^{\infty}b_{mn}^s H_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\right.\right.\right.$$

$$\left.\left.\left.+\sum_{n=-\infty}^{\infty}b_{mn}^R J_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\right]\right)\right)\right]$$

$$+(\lambda_I + 2\mu_I)\left[\frac{\partial^2 \varphi^I}{\partial r^2} + \frac{1}{r}\frac{\partial^2 \psi^I}{\partial r \partial \theta}\right] + \lambda_I\left[\frac{1}{r}\frac{\partial \varphi^I}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \varphi^I}{\partial \theta^2} - \frac{1}{r^2}\frac{\partial \psi^I}{\partial \theta} + \frac{1}{r}\frac{\partial^2 \psi^I}{\partial r \partial \theta}\right]$$

$$= (\lambda_I + 2\mu_I)\left[\frac{\partial^2}{\partial r^2}\left(\sum_{j=1}^{3}\alpha_j^I\left[\sum_{n=-\infty}^{\infty}\alpha_n^i\,\delta_{j1}J_n(k_{\alpha 1}r_1)e^{in\theta_{\alpha 1}} + \sum_{n=-\infty}^{\infty}\alpha_{jn}^r J_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\right.\right.\right.$$

$$\left.\left.\left(+\sum_{n=-\infty}^{\infty}a_{jn}^s H_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}} + \sum_{n=-\infty}^{\infty}\alpha_{jn}^R J_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\right]\right)\right)$$

$$\left(+\frac{1}{r}\frac{\partial^2}{\partial r\partial\theta}\left(\sum_{m=1}^{2}\beta_m^I\left[\sum_{n=-\infty}^{\infty}b_{mn}^r J_n(k_{\beta m}r_1)e^{in\theta_{\beta m}} + \sum_{n=-\infty}^{\infty}b_{mn}^s H_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\right.\right.\right.$$

$$\left.\left.\left.+\sum_{n=-\infty}^{\infty}b_{mn}^R J_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\right]\right)\right)\right]$$

$$+\lambda_I\left[\frac{1}{r}\frac{\partial}{\partial r}\left(\sum_{j=1}^{3}\alpha_j^I\left[\sum_{n=-\infty}^{\infty}\alpha_n^i\,\delta_{j1}J_n(k_{\alpha 1}r_1)e^{in\theta_{\alpha 1}} + \sum_{n=-\infty}^{\infty}\alpha_{jn}^r J_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\right.\right.\right.$$

$$\left.\left.\left(+\sum_{n=-\infty}^{\infty}a_{jn}^s H_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}} + \sum_{n=-\infty}^{\infty}\alpha_{jn}^R J_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\right]\right)\right)$$

$$+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\left(\sum_{j=1}^{3}\alpha_j^I\left[\sum_{n=-\infty}^{\infty}\alpha_n^i\,\delta_{j1}J_n(k_{\alpha 1}r_1)e^{in\theta_{\alpha 1}} + \sum_{n=-\infty}^{\infty}\alpha_{jn}^r J_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\right.\right.$$

$$\left.\left.+\sum_{n=-\infty}^{\infty}a_{jn}^s H_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}} + \sum_{n=-\infty}^{\infty}\alpha_{jn}^R J_n(k_{\alpha j}r_1)e^{in\theta_{\alpha j}}\right]\right)$$

$$-\frac{1}{r^2}\frac{\partial}{\partial\theta}\left(\sum_{m=1}^{2}\beta_m^I\left[\sum_{n=-\infty}^{\infty}b_{mn}^r J_n(k_{\beta m}r_1)e^{in\theta_{\beta m}} + \sum_{n=-\infty}^{\infty}b_{mn}^s H_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\right.\right.$$

$$\left.\left.+\sum_{n=-\infty}^{\infty}b_{mn}^R J_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\right]\right)$$

$$\left(+\frac{1}{r}\frac{\partial^2}{\partial r\partial\theta}\left(\sum_{m=1}^{2}\beta_m^I\left[\sum_{n=-\infty}^{\infty}b_{mn}^r J_n(k_{\beta m}r_1)e^{in\theta_{\beta m}} + \sum_{n=-\infty}^{\infty}b_{mn}^s H_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\right.\right.\right.$$

$$\left.\left.\left.+\sum_{n=-\infty}^{\infty}b_{mn}^R J_n(k_{\beta m}r_1)e^{in\theta_{\beta m}}\right]\right)\right)\right]$$

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$$\sigma_{r\theta}^S = \mu_S \left[\frac{2}{r_1} \frac{\partial^2 \varphi^S}{\partial r_1 \partial \theta_1} - \frac{2}{r_1^2} \frac{\partial \varphi^S}{\partial \theta_1} + \frac{1}{r_1^2} \frac{\partial^2 \psi^S}{\partial \theta_1^2} - \frac{\partial^2 \psi^S}{\partial r_1^2} + \frac{1}{r_1} \frac{\partial \psi^S}{\partial r_1} \right]$$

$$= \mu_S \left[\frac{2}{r_1} \frac{\partial^2}{\partial r_1 \partial \theta_1} \left(\sum_{n=-\infty}^{\infty} \alpha_n^i J_n(k_{\alpha 1} r_1) e^{in\theta_{\alpha 1}} \right. \right.$$

$$+ \sum_{j=1}^{3} \sum_{n=-\infty}^{\infty} \alpha_{jn}^r J_n(k_{\alpha j} r_1) e^{in\theta_{\alpha j}}$$

$$+ \sum_{j=1}^{3} \sum_{n=-\infty}^{\infty} a_{jn}^s H_n(k_{\alpha j} r_1) e^{in\theta_{\alpha j}}$$

$$+ \sum_{j=1}^{3} \sum_{n=-\infty}^{\infty} \alpha_{jn}^R J_n(k_{\alpha j} r_1) e^{in\theta_{\alpha j}} \Bigg)$$

$$- \frac{2}{r_1^2} \frac{\partial}{\partial \theta_1} \left(\sum_{n=-\infty}^{\infty} \alpha_n^i J_n(k_{\alpha 1} r_1) e^{in\theta_{\alpha 1}} \right.$$

$$+ \sum_{j=1}^{3} \sum_{n=-\infty}^{\infty} \alpha_{jn}^r J_n(k_{\alpha j} r_1) e^{in\theta_{\alpha j}}$$

$$+ \sum_{j=1}^{3} \sum_{n=-\infty}^{\infty} a_{jn}^s H_n(k_{\alpha j} r_1) e^{in\theta_{\alpha j}}$$

$$+ \sum_{j=1}^{3} \sum_{n=-\infty}^{\infty} \alpha_{jn}^R J_n(k_{\alpha j} r_1) e^{in\theta_{\alpha j}} \Bigg)$$

$$+ \frac{1}{r_1^2} \frac{\partial^2}{\partial \theta_1^2} \left(\sum_{m=1}^{2} \sum_{n=-\infty}^{\infty} b_{mn}^r J_n(k_{\beta m} r_1) e^{in\theta_{\beta m}} \right.$$

$$+ \sum_{m=1}^{2} \sum_{n=-\infty}^{\infty} b_{mn}^s H_n(k_{\beta m} r_1) e^{in\theta_{\beta m}}$$

$$+ \sum_{m=1}^{2} \sum_{n=-\infty}^{\infty} b_{mn}^R J_n(k_{\beta m} r_1) e^{in\theta_{\beta m}} \Bigg)$$

$$- \frac{\partial^2}{\partial r_1^2} \left(\sum_{m=1}^{2} \sum_{n=-\infty}^{\infty} b_{mn}^r J_n(k_{\beta m} r_1) e^{in\theta_{\beta m}} \right.$$

$$+ \sum_{m=1}^{2} \sum_{n=-\infty}^{\infty} b_{mn}^s H_n(k_{\beta m} r_1) e^{in\theta_{\beta m}}$$

$$+ \sum_{m=1}^{2} \sum_{n=-\infty}^{\infty} b_{mn}^R J_n(k_{\beta m} r_1) e^{in\theta_{\beta m}} \Bigg)$$

$$+ \frac{1}{r_1} \frac{\partial}{\partial r_1} \left(\sum_{m=1}^{2} \sum_{n=-\infty}^{\infty} b_{mn}^r J_n(k_{\beta m} r_1) e^{in\theta_{\beta m}} \right.$$

$$+ \sum_{m=1}^{2} \sum_{n=-\infty}^{\infty} b_{mn}^s H_n(k_{\beta m} r_1) e^{in\theta_{\beta m}}$$

$$\left. \left. + \sum_{m=1}^{2} \sum_{n=-\infty}^{\infty} b_{mn}^R J_n(k_{\beta m} r_1) e^{in\theta_{\beta m}} \right) \right]$$

$$\sigma_{r\theta}^I = \mu_I \left[\frac{2}{r_1} \frac{\partial^2 \varphi^I}{\partial r_1 \partial \theta_1} - \frac{2}{r_1^2} \frac{\partial \varphi^I}{\partial \theta_1} + \frac{1}{r_1^2} \frac{\partial^2 \psi^I}{\partial \theta_1^2} - \frac{\partial^2 \psi^I}{\partial r_1^2} + \frac{1}{r_1} \frac{\partial \psi^I}{\partial r_1} \right]$$

$$= \mu_I \left[\frac{2}{r_1} \frac{\partial^2}{\partial r_1 \partial \theta_1} \left(\sum_{j=1}^{3} \alpha_j^I \left[\sum_{n=-\infty}^{\infty} \alpha_n^i \, \delta_{j1} J_n(k_{\alpha 1} r_1) e^{in\theta_{\alpha 1}} \right.\right.\right.$$

$$+ \sum_{n=-\infty}^{\infty} \alpha_{jn}^r J_n(k_{\alpha j} r_1) e^{in\theta_{\alpha j}}$$

$$+ \sum_{n=-\infty}^{\infty} a_{jn}^s H_n(k_{\alpha j} r_1) e^{in\theta_{\alpha j}}$$

$$\left.\left. + \sum_{n=-\infty}^{\infty} \alpha_{jn}^R J_n(k_{\alpha j} r_1) e^{in\theta_{\alpha j}} \right]\right)$$

$$- \frac{2}{r_1^2} \frac{\partial}{\partial \theta_1} \left(\sum_{j=1}^{3} \alpha_j^I \left[\sum_{n=-\infty}^{\infty} \alpha_n^i \, \delta_{j1} J_n(k_{\alpha 1} r_1) e^{in\theta_{\alpha 1}} \right.\right.$$

$$+ \sum_{n=-\infty}^{\infty} \alpha_{jn}^r J_n(k_{\alpha j} r_1) e^{in\theta_{\alpha j}}$$

$$+ \sum_{n=-\infty}^{\infty} a_{jn}^s H_n(k_{\alpha j} r_1) e^{in\theta_{\alpha j}}$$

$$\left.\left. + \sum_{n=-\infty}^{\infty} \alpha_{jn}^R J_n(k_{\alpha j} r_1) e^{in\theta_{\alpha j}} \right]\right)$$

$$+ \frac{1}{r_1^2} \frac{\partial^2}{\partial \theta_1^2} \left(\sum_{m=1}^{2} \beta_m^I \left[\sum_{n=-\infty}^{\infty} b_{mn}^r J_n(k_{\beta m} r_1) e^{in\theta_{\beta m}} \right.\right.$$

$$+ \sum_{n=-\infty}^{\infty} b_{mn}^s H_n(k_{\beta m} r_1) e^{in\theta_{\beta m}}$$

$$\left.\left. + \sum_{n=-\infty}^{\infty} b_{mn}^R J_n(k_{\beta m} r_1) e^{in\theta_{\beta m}} \right]\right)$$

$$- \frac{\partial^2}{\partial r_1^2} \left(\sum_{m=1}^{2} \beta_m^I \left[\sum_{n=-\infty}^{\infty} b_{mn}^r J_n(k_{\beta m} r_1) e^{in\theta_{\beta m}} \right.\right.$$

$$+ \sum_{n=-\infty}^{\infty} b_{mn}^s H_n(k_{\beta m} r_1) e^{in\theta_{\beta m}}$$

$$\left.\left. + \sum_{n=-\infty}^{\infty} b_{mn}^R J_n(k_{\beta m} r_1) e^{in\theta_{\beta m}} \right]\right)$$

$$+ \frac{1}{r_1} \frac{\partial}{\partial r_1} \left(\sum_{m=1}^{2} \beta_m^I \left[\sum_{n=-\infty}^{\infty} b_{mn}^r J_n(k_{\beta m} r_1) e^{in\theta_{\beta m}} \right.\right.$$

$$+ \sum_{n=-\infty}^{\infty} b_{mn}^s H_n(k_{\beta m} r_1) e^{in\theta_{\beta m}}$$

$$\left.\left.\left. + \sum_{n=-\infty}^{\infty} b_{mn}^R J_n(k_{\beta m} r_1) e^{in\theta_{\beta m}} \right]\right)\right]$$

（3）

$$K_L \sum_{j=1}^{3} \alpha_j^L \Bigg[\sum_{n=-\infty}^{\infty} \alpha_n^i \, \delta_{j1} \left(k_{\alpha 1}^2 J_n''(k_{\alpha 1}r) + \frac{k_{\alpha 1}}{r} J_n'(k_{\alpha 1}r) - \frac{n^2}{r^2} J_n(k_{\alpha 1}r) \right) e^{in\theta_{\alpha 1}}$$

$$+ \sum_{n=-\infty}^{\infty} \alpha_{jn}^r \left(k_{\alpha j}^2 J_n''(k_{\alpha j}r) + \frac{k_{\alpha j}}{r} J_n'(k_{\alpha j}r) - \frac{n^2}{r^2} J_n(k_{\alpha j}r) \right) e^{in\theta_{\alpha j}}$$

$$+ \sum_{n=-\infty}^{\infty} a_{jn}^s \left(k_{\alpha j}^2 H_n''(k_{\alpha j}r) + \frac{k_{\alpha j}}{r} H_n'(k_{\alpha j}r) - \frac{n^2}{r^2} H_n(k_{\alpha j}r) \right) e^{in\theta_{\alpha j}}$$

$$+ \sum_{n=-\infty}^{\infty} \alpha_{jn}^R \left(k_{\alpha j}^2 J_n''(k_{\alpha j}r) + \frac{k_{\alpha j}}{r} J_n'(k_{\alpha j}r) - \frac{n^2}{r^2} J_n(k_{\alpha j}r) \right) e^{in\theta_{\alpha j}} \Bigg]$$

$$+ 2K_L \sum_{m=1}^{2} \beta_m^L \Bigg[\sum_{n=-\infty}^{\infty} b_{mn}^r \frac{ink_{\beta m}}{r} J_n'(k_{\beta m}r) e^{in\theta_{\beta m}}$$

$$+ \sum_{n=-\infty}^{\infty} b_{mn}^s \frac{ink_{\beta m}}{r} H_n'(k_{\beta m}r) e^{in\theta_{\beta m}}$$

$$+ \sum_{n=-\infty}^{\infty} b_{mn}^R \frac{ink_{\beta m}}{r} J_n'(k_{\beta m}r) e^{in\theta_{\beta m}} \Bigg]$$

$$- K_L \sum_{m=1}^{2} \beta_m^L \Bigg[\sum_{n=-\infty}^{\infty} b_{mn}^r \frac{in}{r^2} J_n(k_{\beta m}r) e^{in\theta_{\beta m}}$$

$$+ \sum_{n=-\infty}^{\infty} b_{mn}^s \frac{in}{r^2} H_n(k_{\beta m}r) e^{in\theta_{\beta m}}$$

$$+ \sum_{n=-\infty}^{\infty} b_{mn}^R \frac{in}{r^2} J_n(k_{\beta m}r) e^{in\theta_{\beta m}} \Bigg] = 0$$

$$\frac{\partial \varphi^S}{\partial r} + \frac{1}{r}\frac{\partial \psi^S}{\partial \theta} = \sum_{n=-\infty}^{\infty} \alpha_n^i \, k_{\alpha 1} J_n'(k_{\alpha 1}r_1) e^{in\theta_{\alpha 1}}$$

$$+ \sum_{j=1}^{3} \sum_{n=-\infty}^{\infty} \alpha_{jn}^r \, k_{\alpha j} J_n'(k_{\alpha j}r_1) e^{in\theta_{\alpha j}}$$

$$+ \sum_{j=1}^{3} \sum_{n=-\infty}^{\infty} a_{jn}^s \, k_{\alpha j} H_n'(k_{\alpha j}r_1) e^{in\theta_{\alpha j}}$$

$$+ \sum_{j=1}^{3} \sum_{n=-\infty}^{\infty} \alpha_{jn}^R \, k_{\alpha j} J_n'(k_{\alpha j}r_1) e^{in\theta_{\alpha j}}$$

$$+ \sum_{m=1}^{2} \sum_{n=-\infty}^{\infty} b_{mn}^r \frac{in}{r_1} J_n(k_{\beta m}r_1) e^{in\theta_{\beta m}}$$

$$+ \sum_{m=1}^{2} \sum_{n=-\infty}^{\infty} b_{mn}^s \frac{in}{r_1} H_n(k_{\beta m}r_1) e^{in\theta_{\beta m}}$$

$$+ \sum_{m=1}^{2} \sum_{n=-\infty}^{\infty} b_{mn}^R \frac{in}{r_1} J_n(k_{\beta m}r_1) e^{in\theta_{\beta m}}$$

$$\frac{\partial \varphi^I}{\partial r} + \frac{1}{r}\frac{\partial \psi^I}{\partial \theta} = \sum_{j=1}^{3} \alpha_j^I \Bigg[\sum_{n=-\infty}^{\infty} \alpha_n^i \, \delta_{j1} k_{\alpha 1} J_n'(k_{\alpha 1}r_1) e^{in\theta_{\alpha 1}}$$

$$+ \sum_{n=-\infty}^{\infty} \alpha_{jn}^r \, k_{\alpha j} J_n'(k_{\alpha j}r_1) e^{in\theta_{\alpha j}}$$

$$+ \sum_{n=-\infty}^{\infty} a_{jn}^s \, k_{\alpha j} H_n'(k_{\alpha j}r_1) e^{in\theta_{\alpha j}}$$

$$+ \sum_{n=-\infty}^{\infty} \alpha_{jn}^R \, k_{\alpha j} J_n'(k_{\alpha j}r_1) e^{in\theta_{\alpha j}} \Bigg]$$

$$+ \sum_{m=1}^{2} \beta_m^I \Bigg[\sum_{n=-\infty}^{\infty} b_{mn}^r \frac{in}{r_1} J_n(k_{\beta m}r_1) e^{in\theta_{\beta m}}$$

$$+ \sum_{n=-\infty}^{\infty} b_{mn}^s \frac{in}{r_1} H_n(k_{\beta m}r_1) e^{in\theta_{\beta m}}$$

$$+ \sum_{n=-\infty}^{\infty} b_{mn}^R \frac{in}{r_1} J_n(k_{\beta m}r_1) e^{in\theta_{\beta m}} \Bigg]$$

$$\sum_{n=-\infty}^{\infty} \alpha_n^i \, k_{\alpha 1} J_n'(k_{\alpha 1}r_1) e^{in\theta_{\alpha 1}}$$

$$+ \sum_{j=1}^{3} \sum_{n=-\infty}^{\infty} \alpha_{jn}^r \, k_{\alpha j} J_n'(k_{\alpha j}r_1) e^{in\theta_{\alpha j}}$$

$$+ \sum_{j=1}^{3} \sum_{n=-\infty}^{\infty} a_{jn}^s \, k_{\alpha j} H_n'(k_{\alpha j}r_1) e^{in\theta_{\alpha j}}$$

$$+ \sum_{j=1}^{3} \sum_{n=-\infty}^{\infty} \alpha_{jn}^R \, k_{\alpha j} J_n'(k_{\alpha j}r_1) e^{in\theta_{\alpha j}}$$

$$+ \sum_{m=1}^{2} \sum_{n=-\infty}^{\infty} b_{mn}^r \frac{in}{r_1} J_n(k_{\beta m}r_1) e^{in\theta_{\beta m}}$$

$$+ \sum_{m=1}^{2} \sum_{n=-\infty}^{\infty} b_{mn}^s \frac{in}{r_1} H_n(k_{\beta m}r_1) e^{in\theta_{\beta m}}$$

$$+ \sum_{m=1}^{2} \sum_{n=-\infty}^{\infty} b_{mn}^R \frac{in}{r_1} J_n(k_{\beta m}r_1) e^{in\theta_{\beta m}}$$

$$= \sum_{j=1}^{3} \alpha_j^I \Bigg[\sum_{n=-\infty}^{\infty} \alpha_n^i \, \delta_{j1} k_{\alpha 1} J_n'(k_{\alpha 1}r_1) e^{in\theta_{\alpha 1}}$$

$$+ \sum_{n=-\infty}^{\infty} \alpha_{jn}^r \, k_{\alpha j} J_n'(k_{\alpha j}r_1) e^{in\theta_{\alpha j}}$$

$$+ \sum_{n=-\infty}^{\infty} a_{jn}^s \, k_{\alpha j} H_n'(k_{\alpha j}r_1) e^{in\theta_{\alpha j}}$$

$$+ \sum_{n=-\infty}^{\infty} \alpha_{jn}^R \, k_{\alpha j} J_n'(k_{\alpha j}r_1) e^{in\theta_{\alpha j}} \Bigg]$$

$$+ \sum_{m=1}^{2} \beta_m^I \Bigg[\sum_{n=-\infty}^{\infty} b_{mn}^r \frac{in}{r_1} J_n(k_{\beta m}r_1) e^{in\theta_{\beta m}}$$

$$+ \sum_{n=-\infty}^{\infty} b_{mn}^s \frac{in}{r_1} H_n(k_{\beta m}r_1) e^{in\theta_{\beta m}}$$

$$+ \sum_{n=-\infty}^{\infty} b_{mn}^R \frac{in}{r_1} J_n(k_{\beta m}r_1) e^{in\theta_{\beta m}} \Bigg]$$

$$\frac{1}{r}\frac{\partial \varphi^S}{\partial \theta} - \frac{\partial \psi^S}{\partial r} = \sum_{n=-\infty}^{\infty} \alpha_n^i \frac{in}{r_1} J_n(k_{\alpha 1} r_1) e^{in\theta_{\alpha 1}}$$
$$+ \sum_{j=1}^{3} \sum_{n=-\infty}^{\infty} \alpha_{jn}^r \frac{in}{r_1} J_n(k_{\alpha j} r_1) e^{in\theta_{\alpha j}}$$
$$+ \sum_{j=1}^{3} \sum_{n=-\infty}^{\infty} a_{jn}^s \frac{in}{r_1} H_n(k_{\alpha j} r_1) e^{in\theta_{\alpha j}}$$
$$+ \sum_{j=1}^{3} \sum_{n=-\infty}^{\infty} \alpha_{jn}^R \frac{in}{r_1} J_n(k_{\alpha j} r_1) e^{in\theta_{\alpha j}}$$
$$- \sum_{m=1}^{2} \sum_{n=-\infty}^{\infty} b_{mn}^r k_{\beta m} J_n{'}(k_{\beta m} r_1) e^{in\theta_{\beta m}}$$
$$- \sum_{m=1}^{2} \sum_{n=-\infty}^{\infty} b_{mn}^s k_{\beta m} H_n{'}(k_{\beta m} r_1) e^{in\theta_{\beta m}}$$
$$- \sum_{m=1}^{2} \sum_{n=-\infty}^{\infty} b_{mn}^R k_{\beta m} J_n{'}(k_{\beta m} r_1) e^{in\theta_{\beta m}}$$

$$\frac{1}{r}\frac{\partial \varphi^I}{\partial \theta} - \frac{\partial \psi^I}{\partial r} = \sum_{j=1}^{3} \alpha_j^I \left[\sum_{n=-\infty}^{\infty} \alpha_n^i \, \delta_{j1} \frac{in}{r_1} J_n(k_{\alpha 1} r_1) e^{in\theta_{\alpha 1}} \right.$$
$$+ \sum_{n=-\infty}^{\infty} \alpha_{jn}^r \frac{in}{r_1} J_n(k_{\alpha j} r_1) e^{in\theta_{\alpha j}}$$
$$+ \sum_{n=-\infty}^{\infty} a_{jn}^s \frac{in}{r_1} H_n(k_{\alpha j} r_1) e^{in\theta_{\alpha j}}$$
$$\left. + \sum_{n=-\infty}^{\infty} \alpha_{jn}^R \frac{in}{r_1} J_n(k_{\alpha j} r_1) e^{in\theta_{\alpha j}} \right]$$
$$- \sum_{m=1}^{2} \beta_m^I \left[\sum_{n=-\infty}^{\infty} b_{mn}^r k_{\beta m} J_n{'}(k_{\beta m} r_1) e^{in\theta_{\beta m}} \right.$$
$$- \sum_{n=-\infty}^{\infty} b_{mn}^s k_{\beta m} H_n{'}(k_{\beta m} r_1) e^{in\theta_{\beta m}}$$
$$\left. - \sum_{n=-\infty}^{\infty} b_{mn}^R k_{\beta m} J_n{'}(k_{\beta m} r_1) e^{in\theta_{\beta m}} \right]$$

$$\sum_{n=-\infty}^{\infty} \alpha_n^i \frac{in}{r_1} J_n(k_{\alpha 1} r_1) e^{in\theta_{\alpha 1}}$$
$$+ \sum_{j=1}^{3} \sum_{n=-\infty}^{\infty} \alpha_{jn}^r \frac{in}{r_1} J_n(k_{\alpha j} r_1) e^{in\theta_{\alpha j}}$$
$$+ \sum_{j=1}^{3} \sum_{n=-\infty}^{\infty} a_{jn}^s \frac{in}{r_1} H_n(k_{\alpha j} r_1) e^{in\theta_{\alpha j}}$$
$$+ \sum_{j=1}^{3} \sum_{n=-\infty}^{\infty} \alpha_{jn}^R \frac{in}{r_1} J_n(k_{\alpha j} r_1) e^{in\theta_{\alpha j}}$$
$$- \sum_{m=1}^{2} \sum_{n=-\infty}^{\infty} b_{mn}^r k_{\beta m} J_n{'}(k_{\beta m} r_1) e^{in\theta_{\beta m}}$$
$$- \sum_{m=1}^{2} \sum_{n=-\infty}^{\infty} b_{mn}^s k_{\beta m} H_n{'}(k_{\beta m} r_1) e^{in\theta_{\beta m}}$$
$$- \sum_{m=1}^{2} \sum_{n=-\infty}^{\infty} b_{mn}^R k_{\beta m} J_n{'}(k_{\beta m} r_1) e^{in\theta_{\beta m}}$$
$$= \sum_{j=1}^{3} \alpha_j^I \left[\sum_{n=-\infty}^{\infty} \alpha_n^i \, \delta_{j1} \frac{in}{r_1} J_n(k_{\alpha 1} r_1) e^{in\theta_{\alpha 1}} \right.$$
$$+ \sum_{n=-\infty}^{\infty} \alpha_{jn}^r \frac{in}{r_1} J_n(k_{\alpha j} r_1) e^{in\theta_{\alpha j}}$$
$$+ \sum_{n=-\infty}^{\infty} a_{jn}^s \frac{in}{r_1} H_n(k_{\alpha j} r_1) e^{in\theta_{\alpha j}}$$
$$\left. + \sum_{n=-\infty}^{\infty} \alpha_{jn}^R \frac{in}{r_1} J_n(k_{\alpha j} r_1) e^{in\theta_{\alpha j}} \right]$$
$$- \sum_{m=1}^{2} \beta_m^I \left[\sum_{n=-\infty}^{\infty} b_{mn}^r k_{\beta m} J_n{'}(k_{\beta m} r_1) e^{in\theta_{\beta m}} \right.$$
$$- \sum_{n=-\infty}^{\infty} b_{mn}^s k_{\beta m} H_n{'}(k_{\beta m} r_1) e^{in\theta_{\beta m}}$$
$$\left. - \sum_{n=-\infty}^{\infty} b_{mn}^R k_{\beta m} J_n{'}(k_{\beta m} r_1) e^{in\theta_{\beta m}} \right]$$

洞室表面方程组：

$$[\bar{E}^S]_{5\times5}\begin{bmatrix} a_{jn}^S \\ b_{mn}^S \end{bmatrix} + [\bar{E}^R]_{5\times5}\begin{bmatrix} a_{jn}^R + a_{jn}^1 \\ b_{mn}^R + b_{mn}^1 \end{bmatrix} = 0$$

对于(1)

系数定义

$$A_j^P = (\lambda_S + 2\mu_S) - K_L\alpha_j^L + (\lambda_I + 2\mu_I)\alpha_j^I$$
$$B_j^P = \lambda_S - K_L\alpha_j^L + \lambda_I\alpha_j^I$$
$$C_m^S = 2(\lambda_S + \mu_S) - 2K_L\beta_m^L + 2(\lambda_I + \mu_I)\beta_m^I$$
$$D_m^S = \lambda_S - K_L\beta_m^L + \lambda_I\beta_m^I$$

E^S矩阵第一行系数

$$E_{11}^S(n) = A_1^P k_{\alpha1}^2 H_n''(k_{\alpha1}r_1) + B_1^P\left[\frac{k_{\alpha1}}{r_1}H_n'(k_{\alpha1}r_1) - \frac{n^2}{r_1^2}H_n(k_{\alpha1}r_1)\right]e^{in\theta_{\alpha1}}$$

$$E_{12}^S(n) = A_2^P k_{\alpha2}^2 H_n''(k_{\alpha2}r_1) + B_2^P\left[\frac{k_{\alpha2}}{r_1}H_n'(k_{\alpha2}r_1) - \frac{n^2}{r_1^2}H_n(k_{\alpha2}r_1)\right]e^{in\theta_{\alpha2}}$$

$$E_{13}^S(n) = A_3^P k_{\alpha3}^2 H_n''(k_{\alpha3}r_1) + B_3^P\left[\frac{k_{\alpha3}}{r_1}H_n'(k_{\alpha3}r_1) - \frac{n^2}{r_1^2}H_n(k_{\alpha3}r_1)\right]e^{in\theta_{\alpha3}}$$

$$E_{14}^S(n) = C_1^S\frac{in}{r_1}k_{\beta1}H_n'(k_{\beta1}r_1) - D_1^S\frac{in}{r_1^2}H_n(k_{\beta1}r_1)e^{in\theta_{\beta1}}$$

$$E_{15}^S(n) = C_2^S\frac{in}{r_1}k_{\beta2}H_n'(k_{\beta2}r_1) - D_2^S\frac{in}{r_1^2}H_n(k_{\beta2}r_1)e^{in\theta_{\beta2}}$$

系数定义

$$A_j^P = (\lambda_S + 2\mu_S) - K_L\alpha_j^L + (\lambda_I + 2\mu_I)\alpha_j^I$$
$$B_j^P = \lambda_S - K_L\alpha_j^L + \lambda_I\alpha_j^I$$
$$C_m^S = 2(\lambda_S + \mu_S) - 2K_L\beta_m^L + 2(\lambda_I + \mu_I)\beta_m^I$$
$$D_m^S = \lambda_S - K_L\beta_m^L + \lambda_I\beta_m^I$$

E^R矩阵第一行系数

$$E_{11}^R(n) = A_1^P k_{\alpha1}^2 J_n'' + B_1^P\left(\frac{k_{\alpha1}}{r_1}J_n' - \frac{n^2}{r_1^2}J_n\right)e^{in\theta_{\alpha1}}$$

$$E_{12}^R(n) = A_2^P k_{\alpha2}^2 J_n'' + B_2^P\left(\frac{k_{\alpha2}}{r_1}J_n' - \frac{n^2}{r_1^2}J_n\right)e^{in\theta_{\alpha2}}$$

$$E_{13}^R(n) = A_3^P k_{\alpha3}^2 J_n'' + B_3^P\left(\frac{k_{\alpha3}}{r_1}J_n' - \frac{n^2}{r_1^2}J_n\right)e^{in\theta_{\alpha3}}$$

$$E_{14}^R(n) = C_1^S\frac{in}{r_1}k_{\beta1}J_n' - D_1^S\frac{in}{r_1^2}J_ne^{in\theta_{\beta1}}$$

$$E_{15}^R(n) = C_2^S\frac{in}{r_1}k_{\beta2}J_n' - D_2^S\frac{in}{r_1^2}J_ne^{in\theta_{\beta2}}$$

对于（2）

系数定义

$$F_j^\alpha(n) = \frac{in}{r_1}k_{\alpha j}H_n'(k_{\alpha j}r_1) - \frac{in}{r_1^2}H_n(k_{\alpha j}r_1)$$

$$F_m^\beta(n) = k_{\beta m}^2 H_n''(k_{\beta m}r_1) - \frac{k_{\beta m}}{r_1}H_n'(k_{\beta m}r_1) + \frac{n^2}{r_1^2}H_n(k_{\beta m}r_1)$$

E^S矩阵第二行系数

$$E_{21}^S(n) = (\mu_S + \mu_I\alpha_1^I)F_1^\alpha(n)e^{in\theta_{\alpha1}}$$
$$E_{22}^S(n) = (\mu_S + \mu_I\alpha_2^I)F_2^\alpha(n)e^{in\theta_{\alpha2}}$$
$$E_{23}^S(n) = (\mu_S + \mu_I\alpha_3^I)F_3^\alpha(n)e^{in\theta_{\alpha3}}$$
$$E_{24}^S(n) = (\mu_S + \mu_I\beta_1^I)F_1^\beta(n)e^{in\theta_{\beta1}}$$
$$E_{25}^S(n) = (\mu_S + \mu_I\beta_2^I)F_2^\beta(n)e^{in\theta_{\beta2}}$$

系数定义

$$C_1 = \mu_S + \mu_I\alpha_1^I$$
$$C_2 = \mu_S + \mu_I\alpha_2^I$$
$$C_3 = \mu_S + \mu_I\alpha_3^I$$
$$D_1 = \mu_S + \mu_I\beta_1^I$$
$$D_2 = \mu_S + \mu_I\beta_2^I$$

E^R矩阵第二行系数

$$E_{21}^R(n) = C_1 \left[\frac{in}{r_1} k_{\alpha 1} J_n{}'(k_{\alpha 1} r_1) - \frac{in}{r_1^2} J_n(k_{\alpha 1} r_1) \right] e^{in\theta_{\alpha 1}}$$

$$E_{22}^R(n) = C_2 \left[\frac{in}{r_1} k_{\alpha 2} J_n{}'(k_{\alpha 2} r_1) - \frac{in}{r_1^2} J_n(k_{\alpha 2} r_1) \right] e^{in\theta_{\alpha 2}}$$

$$E_{23}^R(n) = C_3 \left[\frac{in}{r_1} k_{\alpha 3} J_n{}'(k_{\alpha 3} r_1) - \frac{in}{r_1^2} J_n(k_{\alpha 3} r_1) \right] e^{in\theta_{\alpha 3}}$$

$$E_{24}^R(n) = D_1 \left[k_{\beta 1}^2 J_n{}''(k_{\beta 1} r_1) - \frac{k_{\beta 1}}{r_1} J_n{}'(k_{\beta 1} r_1) + \frac{n^2}{r_1^2} J_n(k_{\beta 1} r_1) \right] e^{in\theta_{\beta 1}}$$

$$E_{25}^R(n) = D_2 \left[k_{\beta 2}^2 J_n{}''(k_{\beta 2} r_1) - \frac{k_{\beta 2}}{r_1} J_n{}'(k_{\beta 2} r_1) + \frac{n^2}{r_1^2} J_n(k_{\beta 2} r_1) \right] e^{in\theta_{\beta 2}}$$

<mark>对于（3）</mark>

E^S矩阵第三行系数

$$E_{31}^S(n) = H_n{}'(k_{\alpha 1} r_1) + \frac{1}{r_1} H_n(k_{\alpha 1} r_1) - \frac{n^2}{r_1^2} H_n(k_{\alpha 1} r_1)$$

$$E_{32}^S(n) = H_n{}'(k_{\alpha 2} r_1) + \frac{1}{r_1} H_n(k_{\alpha 2} r_1) - \frac{n^2}{r_1^2} H_n(k_{\alpha 2} r_1)$$

$$E_{33}^S(n) = H_n{}'(k_{\alpha 3} r_1) + \frac{1}{r_1} H_n(k_{\alpha 3} r_1) - \frac{n^2}{r_1^2} H_n(k_{\alpha 3} r_1)$$

$$E_{34}^S(n) = \frac{in}{r_1} H_n(k_{\beta 1} r_1)$$

$$E_{35}^S(n) = \frac{in}{r_1} H_n(k_{\beta 2} r_1)$$

E^R矩阵第三行系数

$$E_{31}^R(n) = J_n{}'(k_{\alpha 1} r_1) + \frac{1}{r_1} J_n(k_{\alpha 1} r_1) - \frac{n^2}{r_1^2} J_n(k_{\alpha 1} r_1)$$

$$E_{32}^R(n) = J_n{}'(k_{\alpha 2} r_1) + \frac{1}{r_1} J_n(k_{\alpha 2} r_1) - \frac{n^2}{r_1^2} J_n(k_{\alpha 2} r_1)$$

$$E_{33}^R(n) = J_n{}'(k_{\alpha 3} r_1) + \frac{1}{r_1} J_n(k_{\alpha 3} r_1) - \frac{n^2}{r_1^2} J_n(k_{\alpha 3} r_1)$$

$$E_{34}^R(n) = \frac{in}{r_1} J_n(k_{\beta 1} r_1)$$

$$E_{35}^R(n) = \frac{in}{r_1} J_n(k_{\beta 2} r_1)$$

<mark>对于（4）</mark>

E^S矩阵第四行系数

$$E_{41}^S(n) = k_{\alpha 1} H_n{}'(k_{\alpha 1} r_1)$$
$$E_{42}^S(n) = k_{\alpha 2} H_n{}'(k_{\alpha 2} r_1)$$
$$E_{43}^S(n) = k_{\alpha 3} H_n{}'(k_{\alpha 3} r_1)$$
$$E_{44}^S(n) = \frac{in}{r_1} H_n(k_{\beta 1} r_1)$$
$$E_{45}^S(n) = \frac{in}{r_1} H_n(k_{\beta 2} r_1)$$

E^R矩阵第四行系数

$$E_{41}^R(n) = k_{\alpha 1} J_n{}'(k_{\alpha 1} r_1)$$
$$E_{42}^R(n) = k_{\alpha 2} J_n{}'(k_{\alpha 2} r_1)$$
$$E_{43}^R(n) = k_{\alpha 3} J_n{}'(k_{\alpha 3} r_1)$$
$$E_{44}^R(n) = \frac{in}{r_1} J_n(k_{\beta 1} r_1)$$
$$E_{45}^R(n) = \frac{in}{r_1} J_n(k_{\beta 2} r_1)$$

<mark>对于（5）</mark>

E^S矩阵第五行系数

$$E_{51}^S(n) = \frac{in}{r_1} H_n(k_{\alpha 1} r_1)$$

$$E_{52}^S(n) = \frac{in}{r_1} H_n(k_{\alpha 2} r_1)$$

$$E_{53}^S(n) = \frac{in}{r_1} H_n(k_{\alpha 3} r_1)$$

$$E_{54}^S(n) = -k_{\beta 1} H_n{}'(k_{\beta 1} r_1)$$
$$E_{55}^S(n) = -k_{\beta 2} H_n{}'(k_{\beta 2} r_1)$$

E^R矩阵第五行系数

$$E_{51}^R(n) = \frac{in}{r_1} J_n(k_{\alpha 1} r_1)$$

$$E_{52}^R(n) = \frac{in}{r_1} J_n(k_{\alpha 2} r_1)$$

$$E_{53}^R(n) = \frac{in}{r_1} J_n(k_{\alpha 3} r_1)$$

$$E_{54}^R(n) = -k_{\beta 1} J_n{}'(k_{\beta 1} r_1)$$

$$E_{55}^R(n) = -k_{\beta 2} J_n{}'(k_{\beta 2} r_1)$$