

对于自由表面采用直角坐标系

1. 表示势函数

散射波含有三种 p 波和两种 s 波

$$p \text{ 波: } \varphi_j^s(x, y) = \int_{-\infty}^{\infty} A_j^s(k) e^{ikx - v_{aj}y} dk \quad (j = 1, 2, 3) \quad (8a)$$

$$s \text{ 波: } \psi_m^s(x, y) = \int_{-\infty}^{\infty} B_m^s(k) e^{ikx - v_{bm}y} dk \quad (m = 1, 2) \quad (8b)$$

驻波含有三种 p 波和两种 s 波

$$p \text{ 波: } \varphi_j^R(x, y) = \int_{-\infty}^{\infty} A_j^R(k) e^{ikx - v_{aj}y} dk \quad (j = 1, 2, 3) \quad (8a)$$

$$s \text{ 波: } \psi_m^R(x, y) = \int_{-\infty}^{\infty} B_m^R(k) e^{ikx - v_{bm}y} dk \quad (m = 1, 2) \quad (8b)$$

2. 位移分量

固相

$$\text{总 } P \text{ 波位移分量: } u_x^{(p)} = u_x^{s(p)} + u_x^{R(p)} = \int_{-\infty}^{\infty} i k [A_j^s(k) + A_j^R(k)] e^{ikx - v_{aj}y} dk \quad (j = 1, 2, 3)$$

$$u_y^{(p)} = u_y^{s(p)} + u_y^{R(p)} = \int_{-\infty}^{\infty} (-v_{aj}) [A_j^s(k) + A_j^R(k)] e^{ikx - v_{aj}y} dk \quad (j = 1, 2, 3)$$

$$\text{总 } S \text{ 波位移分量: } u_x^{(s)} = u_x^{s(s)} + u_x^{R(s)} = \int_{-\infty}^{\infty} (-v_{bm}) [B_m^s(k) + B_m^R(k)] e^{ikx - v_{bm}y} dk \quad (m = 1, 2)$$

$$u_y^{(s)} = u_y^{s(s)} + u_y^{R(s)} = \int_{-\infty}^{\infty} (-ik) [B_m^s(k) + B_m^R(k)] e^{ikx - v_{bm}y} dk \quad (m = 1, 2)$$

总固相位移分量

$$u_x = u_x^{(p)} + u_x^{(s)} = \sum_{j=1}^3 \int_{-\infty}^{\infty} i k [A_j^s(k) + A_j^R(k)] e^{ikx - v_{aj}y} dk$$

$$+ \sum_{m=1}^2 \int_{-\infty}^{\infty} (-v_{bm}) [B_m^s(k) + B_m^R(k)] e^{ikx - v_{bm}y} dk$$

$$u_y = u_y^{(p)} + u_y^{(s)} = \sum_{j=1}^3 \int_{-\infty}^{\infty} (-v_{aj}) [A_j^s(k) + A_j^R(k)] e^{ikx - v_{aj}y} dk$$

$$+ \sum_{m=1}^2 \int_{-\infty}^{\infty} (-ik) [B_m^s(k) + B_m^R(k)] e^{ikx - v_{bm}y} dk$$

总液相位移分量

$$u_x = u_x^{(p)} + u_x^{(s)} = \sum_{j=1}^3 \alpha_j^L \int_{-\infty}^{\infty} i k [A_j^s(k) + A_j^R(k)] e^{ikx - v_{aj}y} dk$$

$$+ \sum_{m=1}^2 \beta_m^L \int_{-\infty}^{\infty} (-v_{bm}) [B_m^s(k) + B_m^R(k)] e^{ikx - v_{bm}y} dk$$

$$u_y = u_y^{(p)} + u_y^{(s)} = \sum_{j=1}^3 \alpha_j^L \int_{-\infty}^{\infty} (-v_{aj}) [A_j^s(k) + A_j^R(k)] e^{ikx - v_{aj}y} dk$$

$$+ \sum_{m=1}^2 \beta_m^L \int_{-\infty}^{\infty} (-ik) [B_m^s(k) + B_m^R(k)] e^{ikx - v_{bm}y} dk$$

总冰相位移分量

$$\begin{aligned}
 u_x &= u_x^{(p)} + u_x^{(s)} = \sum_{j=1}^3 \alpha_j^l \int_{-\infty}^{\infty} i k [A_j^s(k) + A_j^R(k)] e^{ikx - v_{aj}y} dk \\
 &\quad + \sum_{m=1}^2 \beta_m^l \int_{-\infty}^{\infty} (-v_{bm}) [B_m^s(k) + B_m^R(k)] e^{ikx - v_{bm}y} dk \\
 u_y &= u_y^{(p)} + u_y^{(s)} = \sum_{j=1}^3 \alpha_j^l \int_{-\infty}^{\infty} (-v_{aj}) [A_j^s(k) + A_j^R(k)] e^{ikx - v_{aj}y} dk \\
 &\quad + \sum_{m=1}^2 \beta_m^l \int_{-\infty}^{\infty} (-ik) [B_m^s(k) + B_m^R(k)] e^{ikx - v_{bm}y} dk
 \end{aligned}$$

3. 散度

$$\begin{aligned}
 \text{总固相位移散度: } \nabla \cdot \mathbf{u} &= - \sum_{j=1}^3 k_{aj}^2 \varphi_j(x, y) = - \sum_{j=1}^3 k_{aj}^2 \left(\int_{-\infty}^{\infty} [A_j^s(k) + A_j^R(k)] e^{ikx - v_{aj}y} dk \right) \\
 \text{总液相位移散度: } \nabla \cdot \mathbf{u}^f &= - \sum_{j=1}^3 \alpha_j^l k_{aj}^2 \varphi_j(x, y) = - \sum_{j=1}^3 \alpha_j^l k_{aj}^2 \left(\int_{-\infty}^{\infty} [A_j^s(k) + A_j^R(k)] e^{ikx - v_{aj}y} dk \right) \\
 \text{总冰相位移散度: } \nabla \cdot \mathbf{u}^i &= - \sum_{j=1}^3 \alpha_j^l k_{aj}^2 \varphi_j(x, y) = - \sum_{j=1}^3 \alpha_j^l k_{aj}^2 \left(\int_{-\infty}^{\infty} [A_j^s(k) + A_j^R(k)] e^{ikx - v_{aj}y} dk \right)
 \end{aligned}$$

4. 应变张量

固相

$$\begin{aligned}
 \text{正应变 } \varepsilon_{xx} &= \sum_{j=1}^3 \int_{-\infty}^{\infty} (-k^2) [A_j^s(k) + A_j^R(k)] e^{ikx - v_{aj}y} dk \\
 &\quad + \sum_{m=1}^2 \int_{-\infty}^{\infty} (-ikv_{bm}) [B_m^s(k) + B_m^R(k)] e^{ikx - v_{bm}y} dk \\
 \text{正应变 } \varepsilon_{yy} &= \sum_{j=1}^3 \int_{-\infty}^{\infty} v_{aj}^2 [A_j^s(k) + A_j^R(k)] e^{ikx - v_{aj}y} dk \\
 &\quad + \sum_{m=1}^2 \int_{-\infty}^{\infty} (ikv_{bm}) [B_m^s(k) + B_m^R(k)] e^{ikx - v_{bm}y} dk \\
 \text{剪应变 } \varepsilon_{xy} &= \sum_{j=1}^3 \int_{-\infty}^{\infty} (-ikv_{aj}) [A_j^s(k) + A_j^R(k)] e^{ikx - v_{aj}y} dk \\
 &\quad + \frac{1}{2} \sum_{m=1}^2 \int_{-\infty}^{\infty} [-(2k^2 - k_{\beta m}^2)] [B_m^s(k) + B_m^R(k)] e^{ikx - v_{bm}y} dk
 \end{aligned}$$

液相

$$\begin{aligned}
 \text{正应变 } \varepsilon_{xx}^l &= \sum_{j=1}^3 \alpha_j^l \int_{-\infty}^{\infty} (-k^2) [A_j^s(k) + A_j^R(k)] e^{ikx - v_{aj}y} dk \\
 &\quad + \sum_{m=1}^2 \beta_m^l \int_{-\infty}^{\infty} (-ikv_{bm}) [B_m^s(k) + B_m^R(k)] e^{ikx - v_{bm}y} dk \\
 \text{正应变 } \varepsilon_{yy}^l &= \sum_{j=1}^3 \alpha_j^l \int_{-\infty}^{\infty} v_{aj}^2 [A_j^s(k) + A_j^R(k)] e^{ikx - v_{aj}y} dk \\
 &\quad + \sum_{m=1}^2 \beta_m^l \int_{-\infty}^{\infty} (ikv_{bm}) [B_m^s(k) + B_m^R(k)] e^{ikx - v_{bm}y} dk
 \end{aligned}$$

$$\begin{aligned}
\text{剪应变 } \varepsilon_{xy}^l &= \sum_{j=1}^3 \alpha_j^l \int_{-\infty}^{\infty} (-ikv_{aj}) [A_j^s(k) + A_j^R(k)] e^{ikx - v_{aj}y} dk \\
&\quad + \frac{1}{2} \sum_{m=1}^2 \beta_m^l \int_{-\infty}^{\infty} [-(2k^2 - k_{bm}^2)] [B_m^s(k) + B_m^R(k)] e^{ikx - v_{bm}y} dk
\end{aligned}$$

冰相

$$\begin{aligned}
\text{正应变 } \varepsilon_{xx}^i &= \sum_{j=1}^3 \alpha_j^i \int_{-\infty}^{\infty} (-k^2) [A_j^s(k) + A_j^R(k)] e^{ikx - v_{aj}y} dk \\
&\quad + \sum_{m=1}^2 \beta_m^i \int_{-\infty}^{\infty} (-ikv_{bm}) [B_m^s(k) + B_m^R(k)] e^{ikx - v_{bm}y} dk \\
\text{正应变 } \varepsilon_{yy}^i &= \sum_{j=1}^3 \alpha_j^i \int_{-\infty}^{\infty} v_{aj}^2 [A_j^s(k) + A_j^R(k)] e^{ikx - v_{aj}y} dk \\
&\quad + \sum_{m=1}^2 \beta_m^i \int_{-\infty}^{\infty} (ikv_{bm}) [B_m^s(k) + B_m^R(k)] e^{ikx - v_{bm}y} dk \\
\text{剪应变 } \varepsilon_{xy}^i &= \sum_{j=1}^3 \alpha_j^i \int_{-\infty}^{\infty} (-ikv_{aj}) [A_j^s(k) + A_j^R(k)] e^{ikx - v_{aj}y} dk \\
&\quad + \frac{1}{2} \sum_{m=1}^2 \beta_m^i \int_{-\infty}^{\infty} [-(2k^2 - k_{bm}^2)] [B_m^s(k) + B_m^R(k)] e^{ikx - v_{bm}y} dk
\end{aligned}$$

5. 代入本构方程

$$\begin{aligned}
&(K_2 + C_{12} + C_{23}) \nabla^2 \varphi_u^L + \left(K_1 + C_{12} + C_{13} - \frac{2}{3} \mu_1 - \frac{1}{3} \mu_{13} \right) \nabla^2 \varphi_u^S \\
&+ (2\mu_1 + \mu_{13}) \left(\frac{\partial^2 \varphi_u^S}{\partial z^2} + \frac{\partial^2 \psi_u^S}{\partial z \partial x} \right) + (2\mu_3 + \mu_{13}) \left(\frac{\partial^2 \varphi_u^I}{\partial z^2} + \frac{\partial^2 \psi_u^I}{\partial z \partial x} \right) \\
&+ \left(K_3 + C_{13} + C_{23} - \frac{2}{3} \mu_3 - \frac{1}{3} \mu_{13} \right) \nabla^2 \varphi_u^I = 0 \quad (20j)
\end{aligned}$$

$$\begin{aligned}
&\left(\mu_1 + \frac{1}{2} \mu_{13} \right) \left(2 \frac{\partial^2 \varphi_u^S}{\partial x \partial z} + \frac{\partial^2 \psi_u^S}{\partial x^2} - \frac{\partial^2 \psi_u^S}{\partial z^2} \right) \\
&+ \left(\mu_3 + \frac{1}{2} \mu_{13} \right) \left(2 \frac{\partial^2 \varphi_u^I}{\partial x \partial z} + \frac{\partial^2 \psi_u^I}{\partial x^2} - \frac{\partial^2 \psi_u^I}{\partial z^2} \right) = 0 \quad (20k)
\end{aligned}$$

$$\frac{\partial (C_{12} \nabla^2 \varphi_u^S + K_2 \nabla^2 \varphi_u^L + C_{23} \nabla^2 \varphi_u^I)}{\partial z} = 0 \quad (20l)$$

这三个式子的本构方程势函数带入为

(20j)完整形式

$$\begin{aligned}
&(K_2 + C_{12} + C_{23}) \left(- \sum_{j=1}^3 \int_{-\infty}^{\infty} \alpha_j^l k_{aj}^2 [A_j^s(k) + A_j^R(k)] e^{ikx - v_{aj}y} dk \right) \\
&+ \left(K_1 + C_{12} + C_{13} - \frac{2}{3} \mu_1 - \frac{1}{3} \mu_{13} \right) \left(- \sum_{j=1}^3 \int_{-\infty}^{\infty} k_{aj}^2 [A_j^s(k) + A_j^R(k)] e^{ikx - v_{aj}y} dk \right) \\
&+ (2\mu_1 + \mu_{13}) \left(\sum_{j=1}^3 \int_{-\infty}^{\infty} v_{aj}^2 [A_j^s(k) + A_j^R(k)] e^{ikx - v_{aj}y} dk + \sum_{m=1}^2 \int_{-\infty}^{\infty} (-ikv_{bm}) [B_m^s(k) + B_m^R(k)] e^{ikx - v_{bm}y} dk \right) \\
&+ (2\mu_3 + \mu_{13}) \left(\sum_{j=1}^3 \int_{-\infty}^{\infty} \alpha_j^i v_{aj}^2 [A_j^s(k) + A_j^R(k)] e^{ikx - v_{aj}y} dk + \sum_{m=1}^2 \int_{-\infty}^{\infty} \beta_m^i (-ikv_{bm}) [B_m^s(k) + B_m^R(k)] e^{ikx - v_{bm}y} dk \right) \\
&+ \left(K_3 + C_{13} + C_{23} - \frac{2}{3} \mu_3 - \frac{1}{3} \mu_{13} \right) \left(- \sum_{j=1}^3 \int_{-\infty}^{\infty} \alpha_j^i k_{aj}^2 [A_j^s(k) + A_j^R(k)] e^{ikx - v_{aj}y} dk \right) = 0
\end{aligned}$$

方程(20k)完整形式

$$\begin{aligned} & \left(\mu_1 + \frac{1}{2} \mu_{13} \right) \left(2 \sum_{j=1}^3 \int_{-\infty}^{\infty} (-ikv_{aj}) [A_j^s(k) + A_j^R(k)] e^{ikx - v_{aj}y} dk \right. \\ & + \sum_{m=1}^2 \int_{-\infty}^{\infty} (-k^2) [B_m^s(k) + B_m^R(k)] e^{ikx - v_{bm}y} dk - \sum_{m=1}^2 \int_{-\infty}^{\infty} v_{bm}^2 [B_m^s(k) + B_m^R(k)] e^{ikx - v_{bm}y} dk \Big) \\ & + \left(\mu_3 + \frac{1}{2} \mu_{13} \right) \left(2 \sum_{j=1}^3 \int_{-\infty}^{\infty} \alpha_j^I (-ikv_{aj}) [A_j^s(k) + A_j^R(k)] e^{ikx - v_{aj}y} dk \right. \\ & \left. + \sum_{m=1}^2 \int_{-\infty}^{\infty} \beta_m^I (-k^2) [B_m^s(k) + B_m^R(k)] e^{ikx - v_{bm}y} dk - \sum_{m=1}^2 \int_{-\infty}^{\infty} \beta_m^I v_{bm}^2 [B_m^s(k) + B_m^R(k)] e^{ikx - v_{bm}y} dk \right) = 0 \end{aligned}$$

方程(20l)完整形式

$$\begin{aligned} & \frac{\partial}{\partial y} \left(C_{12} \left(- \sum_{j=1}^3 \int_{-\infty}^{\infty} k_{aj}^2 [A_j^s(k) + A_j^R(k)] e^{ikx - v_{aj}y} dk \right) \right. \\ & \left. + K_2 \left(- \sum_{j=1}^3 \int_{-\infty}^{\infty} \alpha_j^I k_{aj}^2 [A_j^s(k) + A_j^R(k)] e^{ikx - v_{aj}y} dk \right) \right) \\ & + C_{23} \left(- \sum_{j=1}^3 \int_{-\infty}^{\infty} \alpha_j^I k_{aj}^2 [A_j^s(k) + A_j^R(k)] e^{ikx - v_{aj}y} dk \right) = 0 \end{aligned}$$

$$\text{对 } y \text{ 求偏导后: } \sum_{j=1}^3 \int_{-\infty}^{\infty} [C_{12} + K_2 \alpha_j^I + C_{23} \alpha_j^I] k_{aj}^2 v_{aj} [A_j^s(k) + A_j^R(k)] e^{ikx - v_{aj}y} dk = 0$$

$$\frac{\partial \varphi_u^S}{\partial z} + \frac{\partial \psi_u^S}{\partial x} = \frac{\partial \varphi_u^I}{\partial z} + \frac{\partial \psi_u^I}{\partial x} \quad (20h)$$

$$\frac{\partial \varphi_u^S}{\partial x} - \frac{\partial \psi_u^S}{\partial z} = \frac{\partial \varphi_u^I}{\partial x} - \frac{\partial \psi_u^I}{\partial z} \quad (20i)$$

方程(20h)完整形式

$$\begin{aligned} & \left\{ \sum_{j=1}^3 \left[\int_{-\infty}^{\infty} A_j^s(k) (-v_{aj}) e^{ikx - v_{aj}y} dk + \int_{-\infty}^{\infty} A_j^R(k) (-v_{aj}) e^{ikx - v_{aj}y} dk \right] \right. \\ & + \sum_{m=1}^2 \left[\int_{-\infty}^{\infty} B_m^s(k) (ik) e^{ikx - v_{bm}y} dk + \int_{-\infty}^{\infty} B_m^R(k) (ik) e^{ikx - v_{bm}y} dk \right] \Big\} \\ & = \left\{ \sum_{j=1}^3 \alpha_j^I \left[\int_{-\infty}^{\infty} A_j^s(k) (-v_{aj}) e^{ikx - v_{aj}y} dk + \int_{-\infty}^{\infty} A_j^R(k) (-v_{aj}) e^{ikx - v_{aj}y} dk \right] \right. \\ & \left. + \sum_{m=1}^2 \beta_m^I \left[\int_{-\infty}^{\infty} B_m^s(k) (ik) e^{ikx - v_{bm}y} dk + \int_{-\infty}^{\infty} B_m^R(k) (ik) e^{ikx - v_{bm}y} dk \right] \right\} \end{aligned}$$

方程(20i)完整形式

$$\begin{aligned} & \left\{ \sum_{j=1}^3 \left[\int_{-\infty}^{\infty} A_j^s(k) (ik) e^{ikx - v_{aj}y} dk + \int_{-\infty}^{\infty} A_j^R(k) (ik) e^{ikx - v_{aj}y} dk \right] \right. \\ & - \sum_{m=1}^2 \left[\int_{-\infty}^{\infty} B_m^s(k) (-v_{bm}) e^{ikx - v_{bm}y} dk + \int_{-\infty}^{\infty} B_m^R(k) (-v_{bm}) e^{ikx - v_{bm}y} dk \right] \Big\} \\ & = \left\{ \sum_{j=1}^3 \alpha_j^I \left[\int_{-\infty}^{\infty} A_j^s(k) (ik) e^{ikx - v_{aj}y} dk + \int_{-\infty}^{\infty} A_j^R(k) (ik) e^{ikx - v_{aj}y} dk \right] \right. \\ & \left. - \sum_{m=1}^2 \beta_m^I \left[\int_{-\infty}^{\infty} B_m^s(k) (-v_{bm}) e^{ikx - v_{bm}y} dk + \int_{-\infty}^{\infty} B_m^R(k) (-v_{bm}) e^{ikx - v_{bm}y} dk \right] \right\} \end{aligned}$$

当 $y=0$ 时

$$\begin{aligned}
& \sum_{j=1}^3 \int_{-\infty}^{\infty} \left\{ (K_2 + C_{12} + C_{23}) \alpha_j^l k a_j^2 + \left(K_1 + C_{12} + C_{13} - \frac{2}{3} \mu_1 - \frac{1}{3} \mu_{13} \right) k a_j^2 \right. \\
& \quad \left. + (2\mu_1 + \mu_{13}) v_{aj}^2 + (2\mu_3 + \mu_{13}) \alpha_j^l v_{aj}^2 \right. \\
& \quad \left. + \left(K_3 + C_{13} + C_{23} - \frac{2}{3} \mu_3 - \frac{1}{3} \mu_{13} \right) \alpha_j^l k a_j^2 \right\} [A_j^S(k) + A_j^R(k)] e^{ikx} dk \\
& + \sum_{m=1}^2 \int_{-\infty}^{\infty} \{(-ikv_{bm}) + \beta_m^l (-ikv_{bm})\} [B_m^S(k) + B_m^R(k)] e^{ikx} dk = 0 \\
& \sum_{j=1}^3 \int_{-\infty}^{\infty} \left[\left(\mu_1 + \frac{1}{2} \mu_{13} \right) + \left(\mu_3 + \frac{1}{2} \mu_{13} \right) \alpha_j^l \right] \\
& \quad \times (-ikv_{aj}) [A_j^S(k) + A_j^R(k)] e^{ikx} dk \\
& + \sum_{m=1}^2 \int_{-\infty}^{\infty} \left[-(1 + \beta_m^l) (k^2 + v_{bm}^2) \right] \\
& \quad \times [B_m^S(k) + B_m^R(k)] e^{ikx} dk = 0 \\
& - \sum_{j=1}^3 \int_{-\infty}^{\infty} (C_{12} + K_2 \alpha_j^l + C_{23} \alpha_j^l) v_{aj} k a_j^2 [A_j^S(k) + A_j^R(k)] e^{ikx} dk = 0 \\
& \sum_{j=1}^3 \int_{-\infty}^{\infty} (-v_{aj}) [A_j^S(k) + A_j^R(k)] e^{ikx} dk \\
& + \sum_{m=1}^2 \int_{-\infty}^{\infty} (ik) [B_m^S(k) + B_m^R(k)] e^{ikx} dk \\
& = \sum_{j=1}^3 \alpha_j^l \int_{-\infty}^{\infty} (-v_{aj}) [A_j^S(k) + A_j^R(k)] e^{ikx} dk \\
& + \sum_{m=1}^2 \beta_m^l \int_{-\infty}^{\infty} (ik) [B_m^S(k) + B_m^R(k)] e^{ikx} dk \\
& \sum_{j=1}^3 \int_{-\infty}^{\infty} (ik) [A_j^S(k) + A_j^R(k)] e^{ikx} dk \\
& + \sum_{m=1}^2 \int_{-\infty}^{\infty} v_{bm} [B_m^S(k) + B_m^R(k)] e^{ikx} dk \\
& = \sum_{j=1}^3 \alpha_j^l \int_{-\infty}^{\infty} (ik) [A_j^S(k) + A_j^R(k)] e^{ikx} dk \\
& + \sum_{m=1}^2 \beta_m^l \int_{-\infty}^{\infty} v_{bm} [B_m^S(k) + B_m^R(k)] e^{ikx} dk
\end{aligned}$$

对于自由表面的矩阵方程是：

$$\begin{bmatrix} A_1^R \\ A_2^R \\ A_3^R \\ B_1^R \\ B_2^R \end{bmatrix} = [S]_{5 \times 5}^{-1} \cdot [T]_{5 \times 5} \cdot \begin{bmatrix} A_1^S \\ A_2^S \\ A_3^S \\ B_1^S \\ B_2^S \end{bmatrix}$$

$$\begin{aligned}
E_j &= (K_2 + C_{12} + C_{23}) \alpha_j^l k a_j^2 \\
s_{11} &= E_1 & t_{11} &= -E_1 \\
s_{12} &= E_2 & t_{12} &= -E_2 \\
s_{13} &= E_3 & t_{13} &= -E_3 \\
s_{14} &= F_1 & t_{14} &= -F_1 \\
s_{15} &= F_2 & t_{15} &= -F_2 \\
&+ \left(K_1 + C_{12} + C_{13} - \frac{2}{3} \mu_1 - \frac{1}{3} \mu_{13} \right) k a_j^2 \\
&+ (2\mu_1 + \mu_{13}) v_{aj}^2 & F_m &= (-ikv_{bm})(1 + \beta_m^l) \\
&+ (2\mu_3 + \mu_{13}) \alpha_j^l v_{aj}^2 \\
&+ \left(K_3 + C_{13} + C_{23} - \frac{2}{3} \mu_3 - \frac{1}{3} \mu_{13} \right) \alpha_j^l k a_j^2
\end{aligned}$$

$$\begin{aligned}
s_{21} &= G_1 & t_{21} &= -G_1 \\
s_{22} &= G_2 & t_{22} &= -G_2 \\
s_{23} &= G_3 & t_{23} &= -G_3 & G_j = \left[\left(\mu_1 + \frac{1}{2} \mu_{13} \right) + \left(\mu_3 + \frac{1}{2} \mu_{13} \right) \alpha_j^l \right] (-ikv_{aj}) & H_m = -(1 + \beta_m^l)(k^2 + v_{bm}^2) \\
s_{24} &= H_1 & t_{24} &= -H_1 \\
s_{25} &= H_2 & t_{25} &= -H_2 \\
s_{31} &= P_1 & t_{31} &= -P_1 \\
s_{32} &= P_2 & t_{32} &= -P_2 \\
s_{33} &= P_3 & t_{33} &= -P_3 & P_j = -(C_{12} + K_2 \alpha_j^l + C_{23} \alpha_j^l) v_{aj} k a_j^2 & \text{方程不涉及横波模式 (} B_m \text{ 项), 因此 } s_{34}, s_{35}, t_{34}, t_{35} \text{ 均为零。} \\
s_{34} &= 0 & t_{34} &= 0 \\
s_{35} &= 0 & t_{35} &= 0 \\
s_{41} &= Q_1 & t_{41} &= -Q_1 \\
s_{42} &= Q_2 & t_{42} &= -Q_2 & Q_j = (1 - \alpha_j^l)(-v_{aj}) = -(1 - \alpha_j^l)v_{aj} \\
s_{43} &= Q_3 & t_{43} &= -Q_3 & R_m = (1 - \beta_m^l)(ik) \\
s_{44} &= R_1 & t_{44} &= -R_1 \\
s_{45} &= R_2 & t_{45} &= -R_2 \\
s_{51} &= U_1 & t_{51} &= -U_1 \\
s_{52} &= U_2 & t_{52} &= -U_2 & U_j = (1 - \alpha_j^l)(ik) \\
s_{53} &= U_3 & t_{53} &= -U_3 & V_m = (1 - \beta_m^l)v_{bm} \\
s_{54} &= V_1 & t_{54} &= -V_1 \\
s_{55} &= V_2 & t_{55} &= -V_2
\end{aligned}$$