Problem G
Riemann vs Mertens

Input: Standard Input
Output: Standard Output
Time Limit: 2 Seconds

One of the biggest, most mathematicians would call it THE biggest, unsolved problems in mathematics is the proof of the Riemann Hypothesis: "All non-trivial zeros of the zeta function have real part one-half". Now your task is simple: For any natural number N, give the Nth zero... nah, just kidding! That would be a much too complex problem for an online contest. We'll leave Riemann and the zeta function and concern ourselves with the closely related, but much easier to calcultate Mertens's function. For those interested in the subject I can heartly recommend Derbyshire's book (see the epilogue).

Every positive natural number greater than 1, can be uniquely decomposed into it's prime factors. Some numbers have only one factor, namely the number itself, like 2, 11 and 71, and are caled prime numbers. Others have more than one factor, like 4 (2x2), 15 (3x5) and 144 (2x2x2x2x3x3), and are called composite numbers. If a number contains all it's prime factors only once, it is called square free. All prime numbers are square free. Some composite numbers are square free, like 21 (3x7) and 187 (11x17), others are not, like 9 (3x3) and 98 (2x7x7).

Let's define the Mobius function mu(N), for all positive natural numbers N:

- mu(1)=1, by definition;
- if N is not square free, mu(N)=0;
- if N is square free and contains an even number of prime factors, mu(N)=1;
- if N is square free and contains an odd number of prime factors, mu(N)=-1.

Now we can define Mertens's function M(N) as the sum of all mu() for 1 up to and including N:

M(N) = mu(1) + mu(2) + ... + mu(N).

The first 20 values for both functions are in this table:

N	factors	mu(N)	M(N)
1	-	1	1
2	2	-1	0
3	3	-1	-1
4	2 2	0	-1
5	5	-1	-2
6	2 3	1	-1
7	7	-1	-2

8	2 2 2	0	-2
9	3 3	0	-2
10	2 5	1	-1
11	11	-1	-2
12	2 2 3	0	-2
13	13	-1	-3
14	2 7	1	-2
15	3 5	1	-1
16	2 2 2 2	0	-1
17	17	-1	-2
18	2 3 3	0	-2
19	19	-1	-3
20	2 2 5	0	-3

We want you to calculate mu(N) and M(N) for some values of N.

Input

Up to 1000 numbers between 1 and 1000000 (one million), both included, each on a line by itself. The numbers are in random order and can appear more than once. Input is terminated by a line, which contains a single zero. This line should not be processed.

Output

For each number in the input print that number, the value of mu() for that number and the value of M() for that number, all three on one line, right justified in fields of width 8. The input order must be preserved.

Sample Input

```
20
1
144
73
0
```

Output for Sample Input

```
      20       0      -3
       1       1       1
     144       0      -1
      73      -1      -4
```

Problemsetter: Joachim Wulff (aka Little Joey)
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EPILOGUE (Not required to solve the problem above)

The zeta function can be defined as the infinte sum:

zeta(s) = 1 + 2^(-s) + 3^(-s) + 4^(-s) + etc.

s can be any complex number.

The zeta function has many zeros (values of s for which zeta(s)=0). For some of them, the non-trivial ones, Riemann in 1859 conjectured that the real part of s should always be exactly one-half. The imaginary part can have any value. Many zeros have been found so far, all with real part one-half, but no one was ever able to prove (or disprove) the conjecture. This now famous Riemann Hypothesis has many corollaries in all fields of mathematics (and beyond); many theorems are based on the fact that the hypothesis is true, so whoever disproves it (one counter example is enough), would make a ruin of modern mathematics!

One way to prove the hypothesis, would be to prove that the Mertens's function is bounded by the square root of it's argument: M(N) = O(sqrt(N)), where O stands for the big-oh notation, which means that for big enough N, the absolute value of M(N) will never exceed sqrt(N). This is never proved, but if it's true, the Riemann Hypothesis is true (don't ask me why).

This, and much more, can be found in the excelent book "Prime Obsession" by John Derbyshire, a mathematical, biographical and historical account of the Riemann Hypothesis. Very readable, also for mathematical laymen like myself.