LNMIIT/B.Tech/C/IC/2017-18/ODD/MTH102/E

LNMIIT
The LNM Institute of Information Technology

THE LNM INSTITUTE OF INFORMATION TECHNOLOGY
DEPARTMENT OF MATHEMATICS
MATHEMATICS-1 & MTH102
ESE

Time: 180 minutes | Date: 28/11/2017 | Maximum Marks: 50

**Note:** You should attempt all questions. Your writing should be legible and neat. Marks awarded are shown next to the question. Please make an index showing the question number and page number on the front page of your answer sheet in the following format, otherwise you may be penalized by the deduction of **2 marks.**

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1. (a) Let $f : [-1,1] \times [-1,1] \to \mathbb{R}$ be defined by

$$f(x,y) = \begin{cases} (x+y)^2 & \text{if } x+y \geq 0 \\ -(x+y)^2 & \text{if } x+y < 0. \end{cases}$$

Show that $f$ is continuous on $[-1,1] \times [-1,1]$. **[4 Marks]**

(b) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x,y) = \begin{cases} \dfrac{x^2y^2}{x^2y^2 + (x-y)^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

Find the iterated limits (if these exist)

$$\lim_{x\to 0}\left[\lim_{y\to 0} f(x,y)\right] \text{ and } \lim_{y\to 0}\left[\lim_{x\to 0} f(x,y).\right]$$

**[3 marks]**

2. (a) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x,y) = \begin{cases} \min\{|x|,|y|\} & \text{if } xy \geq 0 \\ -\min\{|x|,|y|\} & \text{if } xy < 0. \end{cases}$$

Determine all the points in the interior of first quadrant at which $\nabla f$ exists. **[4 Marks]**

(b) State and prove the mean value theorem for real-valued functions of two variables. **[3 Marks]**

3. (a) Consider $f : \mathbb{R}^2 \to \mathbb{R}$ defined by $f(x,y) = x^3y^5$. Find all the points at which $f$ has a local maximum, a local minimum, or a saddle point. **[3 Marks]**

(b) Find the absolute extreme values (if exist) of $f(x,y,z) = x^2yz + 1$ on the intersection of the plane $z = 1$ with the sphere $x^2 + y^2 + z^2 = 10$. **[4 Marks]**

4. Discuss the convergence/divergence of the improper integrals:

(a) $\displaystyle\int_1^\infty \frac{\cos^2 t}{t^2}\,dt.$ **[2 Marks]**

(b) $\displaystyle\int_1^\infty \frac{2+\sin t}{t}\,dt.$ **[2 Marks]**

5. Let $f : [-1, 1] \longrightarrow \mathbb{R}$ be twice differentiable and let

$$f(-1) = f'(-1) = f'(1) = 0 \text{ and } f(1) = \frac{1}{2}.$$

Show that there exist $x_1, x_2 \in (-1, 1)$ such that $f''(x_1) = 1 + f''(x_2)$. [4 Marks]

6. (a) Evaluate $\iint_A x^2 dxdy$, where $A$ is the region in the first quadrant bounded by the curve $xy = 16$ and the lines $y = x$, $y = 0$ and $x = 8$. [3 Marks]

(b) Using transformation $x + y = u$, $y = vu$, find $\int_0^1 \int_0^{1-x} e^{\frac{y}{x+y}} dydx$. [4 Marks]

7. (a) Using spherical coordinates, find the volume of the solid surrounded by the surface $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. [4 Marks]

(b) Compute triple integral $\iiint_D x \, dxdydz$ where $D$ is the region in space bounded by $x = 0$, $y = 0$, $z = 2$ and the surface $z = x^2 + y^2$. [3 Marks]

8. (a) Find the line integral of vector field $F = y\mathbf{i} - x\mathbf{j} + \mathbf{k}$ along each of the following path joining $(1, 0, 0)$ to $(1, 0, 1)$. Also interpret the solutions (why they are same or different ?)

$C_1 : \mathbf{r}_1(t) = \cos t\mathbf{i} + \sin t\mathbf{j} + \frac{t}{\pi}\mathbf{k}, 0 \leq t \leq \pi$
$C_2 : \mathbf{r}_2(t) = -\cos t\mathbf{i} + \sin t\mathbf{j} + \frac{t}{\pi}\mathbf{k}, \ 0 \leq t \leq \pi.$ [4 Marks]

(b) Find the line integral of $f(x, y, z) = x + \sqrt{y} - z^2$ over the path (Figure below) from $(0, 0, 0)$ to $(1, 1, 1)$ given by

$C_1 : \mathbf{r}_1(t) = t\mathbf{i} + t^2\mathbf{j}, \ 0 \leq t \leq 1$
$C_2 : \mathbf{r}_2(t) = \mathbf{i} + \mathbf{j} + t\mathbf{k}, \ 0 \leq t \leq 1.$ [3 Marks]