LNMIIT/B.Tech/C/IC/2019-20/ODD/MTH213/MT

LNMIIT

The LNM Institute of Information Technology, Jaipur
Department of Mathematics
Mathematics-III MTH213
Mid Term

Duration: 90 mins. September 26, 2019 Max Marks: 30

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**NOTE:** You should attempt all questions. Your writing should be legible and neat. Marks awarded are shown next to the question. **Start a new question on a new page and answer all its parts in the same place.** Please make an index showing the question number and page number on the front page of your answer sheet in the following format.

| Question No. | | | | |
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| Page No. | | | | |

1. (a) Prove that for any complex number $z \neq 1$, we have [3]

$$1 + z + z^2 + \cdots + z^n = \frac{1 - z^{n+1}}{1 - z}.$$

and then use it to derive *Lagrange's trigonometric identity*:

$$1 + \cos\theta + \cos 2\theta + \cdots + \cos n\theta = \frac{1}{2} + \frac{\sin[(2n+1)\theta/2]}{2\sin(\theta/2)}, \qquad 0 < \theta < 2\pi$$

(b) Let $f : \mathbf{C} \longrightarrow \mathbf{C}$ be an analytic function such that $|f(z)| \geq 1$ for all $z \in \mathbf{C}$. Show that $f$ is constant. [3]

2. (a) Show that the function $f(z) = \begin{cases} \frac{x^3(1+i) - y^3(1-i)}{x^2+y^2} & z \neq 0 \\ 0 & z = 0 \end{cases}$ satisfies the CR equations at $(0,0)$ but is not differentiable there. [3]

(b) Suppose $f$ is an analytic function on a domain $D$. Show that if $|f|$ is harmonic, then $f$ is constant on $D$. [3]

3. (a) Show that $|\cos z|^2 = \cos^2 x + \sinh^2 y$. Conclude that the cosine function is unbounded in $\mathbf{C}$. [3]

(b) Let $f(z) = \frac{1}{z^2}$. Evaluate $\int_C f(z)dz$ where $C$ is any simple closed contour in counterclockwise direction not passing through 0. [3]

4. (a) Let $C_R$ denote the upper half circle $|z| = R$ $(R > 2)$, taken in the counterclockwise direction. Find an upperbound of

$$\left| \int_{C_R} \frac{2z^2 - 1}{z^4 + 5z^2 + 4} dz \right|.$$

Hence show that $\lim_{R\to\infty} \int_{C_R} \frac{2z^2 - 1}{z^4 + 5z^2 + 4} dz = 0$. [3]

(b) Find all the possible Laurent series expansion of $f(z) = \frac{2z}{z^2 - 9}$ about $z = 3$ and using that find the residue of $f(z)$ at $z = 3$. [3]

5. (a) Evaluate $\int_C z^2 e^{\frac{1}{z}}$ where $C$ is any simple closed contour in counterclockwise direction with $z = 0$ inside it. [2]

(b) Using contour integral, evaluate $\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)^2}$. [4]