The LNM Institute of Information Technology, Jaipur
Mathematics-II
Mid Term
February 21, 2018

Duration: 90 mins. Max.Marks: 30

[redacted] [redacted]

1. (a) For which values of g, h and k the following system of linear equations is consistent?

$$x - 4y + 7z = g$$
$$3y - 5z = h$$
$$-2x + 5y - 9z = k$$

[3]

(b) Prove that any square matrix A can be written as the sum of a symmetric matrix and a skew-symmetric matrix. **[2]**

2. (a) Let $V = C[0,1]$. Prove or disprove: $S = \{f \in V : f(\frac{1}{2}) = 0\}$ is a subspace of V. **[2]**

(b) Let V be a vector space over a field F. Let U and W be two subspaces of V. Prove that $U \cap W$ is also a subspace of V. **[3]**

3. (a) What is the span of $\{1\}$ in the vector space $\mathbb{C}$ over the field $\mathbb{C}$? What is the span of $\{1\}$ in the vector space $\mathbb{C}$ over the field $\mathbb{R}$. **[2]**

(b) Write the span of $\{(1,0,0),(0,-1,0)\}$ in $\mathbb{R}^3$. Explain geometrically. **[2]**

4. (a) Show that $B = \{(1,2,0),(1,3,2),(0,1,3)\}$ forms a basis for $\mathbb{R}^3$. **[3]**

(b) Determine the coordinate vector of $v = (2,7,-4)$ with respect to the ordered basis B given in Part (a). **[3]**

5. (a) Suppose $\{u_1, u_2\}$ is an orthogonal set in an inner product space V. Show that $\|u_1 + u_2\|^2 = \|u_1\|^2 + \|u_2\|^2$. **[2]**

(b) Use Gram-Schmidt process to transform the basis $\{(1,0,1),(1,0,-1),(0,3,4)\}$ for $\mathbb{R}^3$ into an orthogonal basis with the standard inner product. **[3]**

6. (a) Find all value/s of $k \in \mathbb{R}$ for which $A = \begin{pmatrix} 1 & k & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ is diagonalizable. **[3]**

(b) Let A be a square matrix. Prove that A and A^T have same set of eigen values. **[2]**