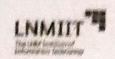


Duration: 90 mins.



Max.Marks: 50

LNMHT/B.Tech/C/IC/2018-19/ODD/MTH213/MT

The LNM Institute of Information Technology, Jaipur Mathematics-III Mid Term

Name: Mayank Roll No.: 17DCS606	
NOTE: You should attempt all questions. Your writing should be legible and neat. Marks awarded ar next to the question. Start a new question on a new page and answer all its parts in the same	
Please make an index showing the question number and page number on the front page of your answer sheet	
following format.	
Question No. Page No.	
Tage No.	
	F41
1. (a) For any two complex numbers z_1 and z_2 , prove that $ z_1 + z_2 \le z_1 + z_2 $.	[4]
Find all the fifth roots of 32 and locate them geometrically.	[4]
2. (a) Let $f(z) = x^3 + i(1-y)^3$. Find all the points where the function is differentiable. Also find the derest at all those points. For which value of z , $f(z)$ is analytic?	
at all those points. For which value of z , $f(z)$ is analytic? Let $u(x,y) = 2x(1-y)$. Show that $u(x,y)$ is harmonic in some domain and find its harmonic con [4]	jugates.
7 [4]	
Write all possible Laurent series expansion of $f(z) = \frac{1}{(z-1)^2(z-3)}$ in powers of $(z-1)$.	[5]
For any two complex numbers z_1 and z_2 , prove that	
$2\sin z_1\cos z_2 = \sin(z_1+z_2) + \sin(z_1-z_2).$	
	[2]
4. (a) Using Cauchy integral formula, evaluate the contour integrals $\int_C \frac{1}{(z^2+4)^2} dz$, where C is the circle	z-i =2
in positive directions.	[3]
(b) Find all the singular points of $f(z) = \frac{\text{Log}(z+2)}{(z-4)(z-5)}$. Classify them as non-isolated, isolated, poles, re-	movable
and essential singularity.	[3]
5. (a) Suppose $f(z)$ is an entire function such that and $ f(z) \leq z $. Using Cauchy's inequality p	rove that
f'(z) is bounded. Then prove that $f'(z)$ is constant. If $f(1) = 1$ and $f(i) = 2$, Read $f(z)$. (b) If $f(z)$ is real-valued and analytic function defined on a domain, then prove that $f(z)$ is constant.	[4]
	ant. [3]
6. (a) Using M-L Inequality, find an upper bound of	
$\left \oint_C \frac{z^2 e^{(z+1)}}{z+1} dz\right $	
where C is the circle $ z =4$.	[4]
where C is the circle $ z = 4$. (b) Find the radius of convergence of the series	ידו
$\sum_{i=1}^{\infty}\frac{(z-4-3i)^k}{5^{2k}}$	
k=0	
	[3]
Find $\int_C ze^{-\frac{1}{(z-2)}}$ where C is any positively oriented closed contour with $z=2$ inside it.	
Evaluate $\int_0^{2\pi} \frac{1}{(2+\cos\theta)^2} d\theta$.	[3]
$J_0 = (2 + \cos \theta)^2 u v$	[5]
1	