MATH 3: End-Semester Examination: Part-A
(To be returned after 30 mins..)

R.No.: **Section:** **Name:**

xxxxxxxxxxxxxxxx

Instructions:

- Attempt all questions. Use the main sheet for rough work. Only the answers should be written on this sheet.
- ***Answers will be rejected if there is any overwriting or cutting.*** No partial credits. Each question carries 4 marks.
- **Calculations (Rough work) should be clearly demonstrated in the main sheet.**

Fill in the Blanks

1. The subset of all points in C for which $f(z) = 3\overline{z}(z+\overline{z})$ is differentiable: ________________

Ans $f(z) = 3\overline{z}(z+\overline{z}) = 3z\overline{z} + 3\overline{z}^2 = 3x^2+3y^2+3x^2-3y^2-6ixy = 6x^2 - i6xy$. Thus $u = 6x^2$ and $v = -6xy$. Now $u_y = 0 \& v_x = -6y$ and $u_x = 12x \& v_y = -6x$. Condition of Cauchy-Riemann equations $u_y = -v_x$ and $u_x = v_y$ doe not satisfy, other $z = 0$, so $f(z)$ is differentiable if and only if $z = 0$.

2. The function $f(z) = \cosh z$ is conformal except at $z = \{k\pi i : k = 0, 1, 2, 3...\}$

Ans $f(z) = \cosh z$ is analytic on C and $f'(z) = \sinh z \neq 0$ except the points $z = k\pi i$ for $k = 0, 1, 2, 3....$ Thus , function $f(z) = \cosh z$ is conformal except at $z = k\pi i$ for $k = 0, 1, 2, 3...$

3. For the function $\dfrac{1}{Z^2+3z+2}$, find out all possible regions of Taylor's and Laurent series expansions about the point $z = 1$ ________________

Ans The function is not analytic at the points z= -1 and z = -2. The distance between the point $z = 1$ and $z = -1$ is 2, and between the point $z = 1$ and $z = -2$ is 3. Thus, we consider the regions, (i) $|z-1| < 2$ (ii) $2 < |z-1| < 3$ (iii) $|z-1| > 3$.
In the region, $|z-1| < 2$ the function is analytic, hence, we obtain Tayler series expansion. In other regions $2 < |z-1| < 3$ and $|z-1| > 3$, we obtain Laurent series expansions.

4. Solution for following PDE

$$\begin{aligned} u_{tt} - 16u_{xx} &= 0, \quad 0 < x < 20,\ t > 0 \\ u(x,0) &= 2\sin\frac{\pi x}{4} - 9\sin\pi x, u_t(x,0) = 0, \quad 0 \leq x \leq 20, \\ u(0,t) &= u(20,t) = 0, \quad t \geq 0 \end{aligned}$$

is $\underline{u(x,t) = 2\cos\pi t\sin\frac{\pi x}{4} - 9\cos 4\pi t\sin\pi x}$

5. Solution of the wave equation (Hint: Duhamel's principle):

$$u_{tt} = u_{xx} + xt, \qquad -\infty < x < \infty,\ t > 0,$$

$$u(x,0) = u_t(x,0) = 0, \quad -\infty < x < \infty$$

is $\underline{xt^3/6}$.

6. The following Laplace equation

$$\triangle u = 0 \text{ in } \Omega = \{0 < x < 1, 0 < y < 1\},\ u(x,y) = 0 \text{ on boundary } \partial\Omega$$

has unique solution. TRUE/FALSE (give justification)
TRUE by uniquness of Laplace equation in a bounded domain.