LNMIIT
The LNM Institute of Information Technology

The LNM Institute of Information Technology
Department Name: CSE

Course Name & Course Code DMS & CSE119
Exam Type(ET)

Time: 180 Minutes **Date: 04/05/2018** **Max. Marks: 40**

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Instructions: *Write appropriate justification for each problem whenever required. Simply Yes/No (without proper explanation) will not be accepted. No query will be handled during exam. Your writing should be legible and neat. If you are making any assumptions (Assumptions should be appropriate), then state it before solving the problem. Please solve all the parts of any question sequentially. After solving all questions, please make the following Index Table on the front page of answer sheet otherwise you may be penalized by the deduction of 3 marks.*

Questions	1	2	3	4	5	6
Page No. (To and from)						

Q1. [Graph Theory] Solve all parts [1+1.5+1.5+4+2=10]

I. Suppose that a connected planar graph has eight vertices, each of degree 3. Into how many regions is the plane divided by a planar representation of this graph? (Justification required)

II. Which of these nonplanar graphs have the property that the removal of any vertex and all edges incident with that vertex produces a planar graph? (a) K_5 (b) K_6 (c) $K_{3,3}$ (d) $K_{3,4}$ (Justify too)

III. In the usual representation of Konigsberg's Seven Bridge problem (As discussed in the class), I am adding two additional bridges, besides the seven that were present in the usual problem. These new bridges connect regions A & C and C & D, respectively. Can someone cross all nine bridges in this city exactly once and return to the starting point? (Justification required)

IV. Find out the Spanning Tree for $K_{2,3}$ (Proper explanation is required including steps)

V. Are the following graphs Isomorphic? (Justification required)

Figure 1: Graph G1 and G2 Repectively

Q2. [Partially Ordered Sets (poset)] Solve all parts [2+3=5]

I. Find two incomparable elements in the following posets. (Justification required)
(a) $(P(\{0,1,2\}),\subseteq)$ (b) $(\{1,2,4,6,8\}, |)$

II. Draw the Hesse diagram for the partial ordering $\{(A,B) \mid A \subseteq B\}$ on the power set P(S) where S={a,b,c}. Determine also whether there is a greatest element and a least element in the poset $(P(S), \subseteq)$ (Explain completely)

Q3. [Relations] Solve all parts [2+2=4]

I. Let R be the relation on the set of integers such that aRb if and only if a=b or a=-b. Is it an equivalence relation? If the relation is equivalence, then compute the equivalence class of an integer otherwise add some conditions so that R would be an equivalence relation. (Justify too)

II. Let R be the relation on the set of all people in the world that contains (a,b) if a has met b. What is its transitive closure? (Proper explanation is required)

Q4 [Counting] Solve all parts [1+4+1=6]

I. Find the least number of cables required to connect 100 computers to 20 printers to guarantee that 20 computers can directly access 20 different printers. Justify your answer with explanation.

II. Deduce the formula for D_n (a collection of derangements of n distinct objects) using Generalized Exclusion-Inclusion Principle. (Proper explanation & steps are required)

III. How many square matrices can be formed from a given matrix of order m*n where m<n? (Explanation required)

Q5 [Recursion & PMI] Solve all parts [4+4+3=11]

I. Design a recursive algorithm for Bubble Sort and establish its correctness. (Explain completely)

II. Using generating functions solve the following recurrence relation (Explain completely)

$$a_n = a_{n-1} + 2, \quad where\ a_1 = 1$$

III. Solve the following recurrence relation by using any one of the methods. (Explain completely)

$$a_n = 7a_{n-1} - 12\,a_{n-2} + 3^n, \quad where\ a_0 = 0, a_1 = 2$$

Q6 [Discrete Probability]. Solve the following question [4]

Let p denotes the probability of success in a Bernoulli trial. Prove that the expected number of successes in a sequence of n Bernoulli trials is np.

OR

Analyze the average case complexity for the Linear Search Algorithm

Note: Usual graph for Seven Bridges problem. Please consider it for the Q 1 (iii) and redraw accordingly.

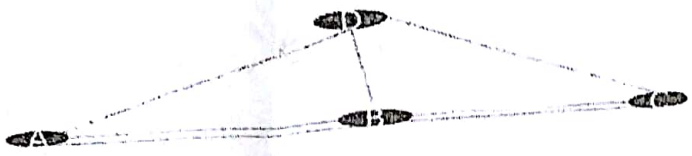

Figure 2: Usual Graph for Seven Bridge Problem