Nam

Instructions: **No query will be entertained during the exam.** Attempt all questions. Total 3 questions. Usual notations are used.

1. (a) Determine and sketch the region in the complex plane for $Re(\frac{4}{z}) < 1$. [3 Marks]

   (b) Find all possible solutions of $z^5 + 1 - i = 0$. [3 Marks]

2. Find out the region of analyticity of the function $f(z) = Log(z + 4 - i\sqrt{2})$, where $Logz$ denotes the the principal value of the logarithm. **Justify your claim.** [6 Marks]

3. What are the value of the integer n, $u(x, y) = x^n - y^n$ is harmonic? The value of $n > 1$ for which $u(x, y)$ is harmonic, find the conjugate harmonic of $u(x, y)$. Construct $f(z) = u(x, y) + iv(x, y)$. Finally, find the function $f(z)$ in terms of z. [8 Marks]