The LNM Institute of Information Technology
Jaipur, Rajsthan

Math-II (2014-15), Quiz-2: Section-B

Name: Roll No:

Time: 15 Minutes Maximum Marks: 10

Q1. If the vectors $\mathbf{x}$ and $\mathbf{y}$ are orthogonal, then by inner product theory show that the parallelogram spanned by vectors $\mathbf{x}$ and $\mathbf{y}$ is a rectangle. [5]

Sol. If the vectors $\mathbf{x}$ and $\mathbf{y}$ span a parallelogram then then the length of two diagonals of the parallelogram are given by $\|\mathbf{x}+\mathbf{y}\|$ and $\|\mathbf{x}-\mathbf{y}\|$.

Note that

$$\begin{aligned} \|\mathbf{x}+\mathbf{y}\|^{\mathbf{2}} - \|\mathbf{x}-\mathbf{y}\|^{\mathbf{2}} &= <\mathbf{x}+\mathbf{y},\mathbf{x}+\mathbf{y}> - <\mathbf{x}-\mathbf{y},\mathbf{x}-\mathbf{y}> \\ &= 4<\mathbf{x},\mathbf{y}>. \end{aligned}$$

Since $\mathbf{x}$ and $\mathbf{y}$ are orthogonal i.e. $<\mathbf{x},\mathbf{y}>=\mathbf{0}$, both the diagonals of the parallelogram are equal and the parallelogram is a rectangle.

Q2. If an $n\times n$ matrix A has n distinct eigen values then prove that A is diagonalizable. (No mark for giving examples) [5]

Sol. Since the matrix A has n distinct eigen values $\lambda_1, \lambda_2 \cdots, \lambda_n$, the correspoding eigen vectors $x_1, x_2, \cdots, x_n$ are LI.

Form a matrix P such that the n LI eigen vectors $x_1, x_2, \cdots, x_n$ are column vectors of the matrix P. Since rank(P) is n, P is invertible.

Since,

$$\begin{aligned} AP &= A[x_1\ x_2\ \cdots\ x_n] = [Ax_1\ Ax_2\ \cdots\ Ax_n] = [\lambda_1 x_1\ \lambda_2 x_2\ \cdots\ \lambda_n x_n] \\ &= [x_1\ x_2\ \cdots\ x_n]\begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \lambda_n \end{bmatrix} = PD, \end{aligned}$$

the matrix A is invertible.