Q.1

(a) Isoquant: Locus of combinations of inputs, which yield a specific value of output

(b) $Q = K^{\alpha} L^{\beta}$; $\alpha, \beta > 0$ & $\alpha + \beta = 1$

$dQ = f_K dK + f_L dL$

$f_K = \frac{\partial Q}{\partial K} = \alpha K^{\alpha-1} L^{\beta}$

$f_L = \frac{\partial Q}{\partial L} = \beta K^{\alpha} L^{\beta-1}$

$dQ = (\alpha K^{\alpha-1} L^{\beta}) dK + (\beta K^{\alpha} L^{\beta-1}) dL$

Along an iso-quant $dQ = 0$

$\Rightarrow (\alpha K^{\alpha-1} L^{\beta}) dK = -(\beta K^{\alpha} L^{\beta-1}) dL$

$\frac{dK}{dL} = -\frac{\beta}{\alpha} \cdot \frac{K}{L} < 0$ [Slope is negative]

↳ Iso-quant is downward sloping.

$f_{LL} = \beta(\beta-1) K^{\alpha} L^{\beta-2}$

$f_{KK} = \alpha(\alpha-1) K^{\alpha-2} L^{\beta-1}$

$f_{LK} = \beta \alpha K^{\alpha-1} L^{\beta-1}$

$f_{KL} = \alpha \beta K^{\alpha-1} L^{\beta-1}$

Convexity

$\frac{d^2}{dL}\left(\frac{dK}{dL}\right)$

$f_K^2 f_{LL} - 2 f_K f_L f_{KL} + f_L^2 f_{KK}$

$(+)(-) - 2(+)(+)(+) + (+)(-)$

$= (-) - 2(+) + (-)$

$= (-) - 2(-) - < 0 \Rightarrow$ Convex to the origin.

(c). Downward sloping : Marginal Rate of Tech Substitution.

Convexity of Iso-quant : Diminishing marginal rate of technical substitution.

Q.2

$$Z = 1.5LK - 0.3L^2 - 0.15K^2 + \lambda[1000 - 60L - 74K]$$

$$\frac{\partial Z}{\partial L} = 0$$

$$\Rightarrow 1.5K - 0.6L - \lambda 60 = 0$$

$$\Rightarrow 1.5K - 0.6L = \lambda 60 \quad -(1)$$

$$\frac{\partial Z}{\partial K} = 0$$

$$\Rightarrow 1.5L - 0.3K - \lambda 74 = 0$$

$$\Rightarrow 1.5L - 0.3K = \lambda 74 \quad -(2)$$

$$\frac{\partial Z}{\partial \lambda} = 0$$

$$\Rightarrow 1000 - 60L - 74K = 0 \quad -(3)$$

Technical

$Q^* = 1.5(10.38)(5.08) + 0.3(10.38)^2 - 0.15(5.08)^2$

$= 42.90$

Q.3

$$\text{Max } Q = f(K, L) \quad \text{s.t. } \bar{C} = wL + rK$$

$$\text{Min } C = wL + rK \quad \text{s.t. } \bar{Q} = f(K, L)$$

$Z = f(K,L) + \lambda_0[C - wL - rK]$

$\frac{\partial Z}{\partial K} = 0$

$\Rightarrow \frac{\partial f}{\partial K} - \lambda_0 r = 0$ —(1)

$\frac{\partial Z}{\partial L} = 0$

$\Rightarrow \frac{\partial f}{\partial L} - \lambda_0 w = 0$ —(2)

$\frac{\partial Z}{\partial \lambda_0} = 0$

$\Rightarrow C - wL - rK = 0$ —(3)

$\mathcal{L} = wL + rK + \lambda_1[\bar{Q} - f(K, L)]$

$\frac{\partial \mathcal{L}}{\partial L} = 0$

$\Rightarrow w - \lambda_1 \frac{\partial f}{\partial L} = 0$ —(4)

$\frac{\partial \mathcal{L}}{\partial K} = 0$

$\Rightarrow r - \lambda_1 \frac{\partial f}{\partial K} = 0$ —(5)

$\frac{\partial \mathcal{L}}{\partial \lambda_1} = 0$

$\Rightarrow \bar{Q} - f(K, L) = 0$ —(6)

$\left(\frac{1}{2}\right) \Rightarrow \frac{\partial f/\partial K}{\partial f/\partial L} = \frac{r}{w}$

$\left(\frac{5}{4}\right) \Rightarrow \frac{\partial f/\partial K}{\partial f/\partial L} = \frac{r}{w}$

from (1) $\lambda_0 = \frac{\partial f/\partial K}{r}$

(2) $\lambda_0 = \frac{\partial f/\partial L}{w}$

(5) (6) $\lambda_1 = \frac{1}{\lambda_0}$

(4) $\lambda_2 = \frac{1}{\lambda_0}$

Reciprocal

$$\frac{1.5K - 0.6L}{1.5L - 0.3K} = \frac{60}{74}$$

$$\Rightarrow 74(1.5K - 0.6L) = 60(1.5L - 0.3K)$$

$$= 111K - 44.4L = 90L - 18K$$

$$\Rightarrow 93K = 45.6L$$

$$K = \frac{45.6}{93} L$$

$$60L + 74K = 1\cancel{0}500$$

$$60L + 74\left(\frac{45.6}{93} L\right) = 1\cancel{0}500$$

$$\Rightarrow 60L + \frac{3374.4L}{93} = 1\cancel{0}500$$

$$\Rightarrow 5580L + 3374.4L = \cancel{93000}\ 139500$$

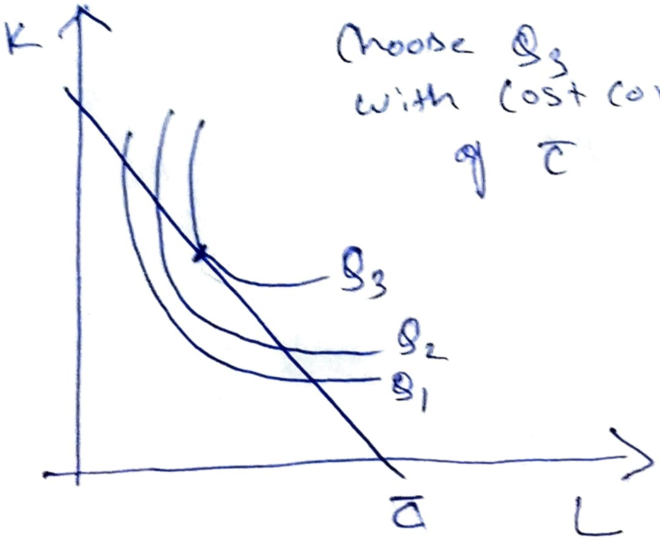
$$\Rightarrow 8954.4L = \cancel{93000}\ 139500$$

$$\boxed{\begin{array}{l} \cancel{L^* = 10.38} \\ K^* = 5.08 \end{array}}$$

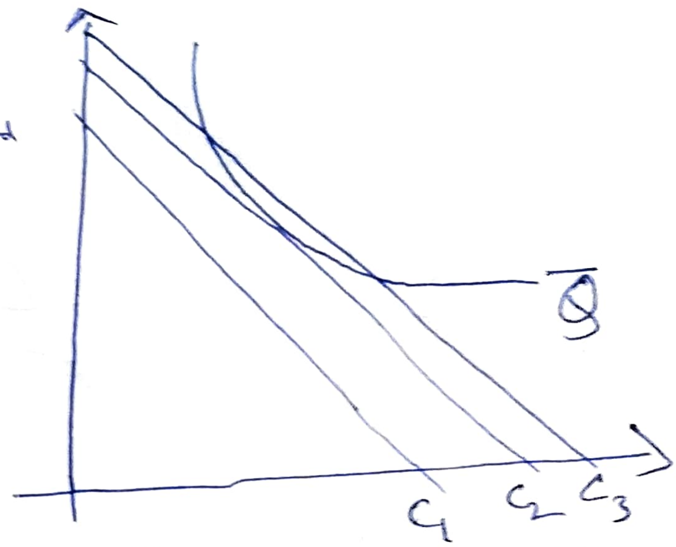
$\cancel{15.57}$ $K = 11.39$

$L = 10.93$

$Q = 131.43$



Choose Q_3 with cost constraint of $\bar{C}$



Choose C_2 with $\bar{Q}$ o/p level.

Q.4 $TC = 100 + q^2 + q$.

$Q = 500 - \frac{P}{2}$ [DD]

$Q = P - 100$ [SS]

(a) $500 - \frac{P}{2} = P - 100$

$\Rightarrow P = 400$

and $Q = 300$

(b) $P = MC$

$400 = 2q + 1$

$\Rightarrow q = 199.5$

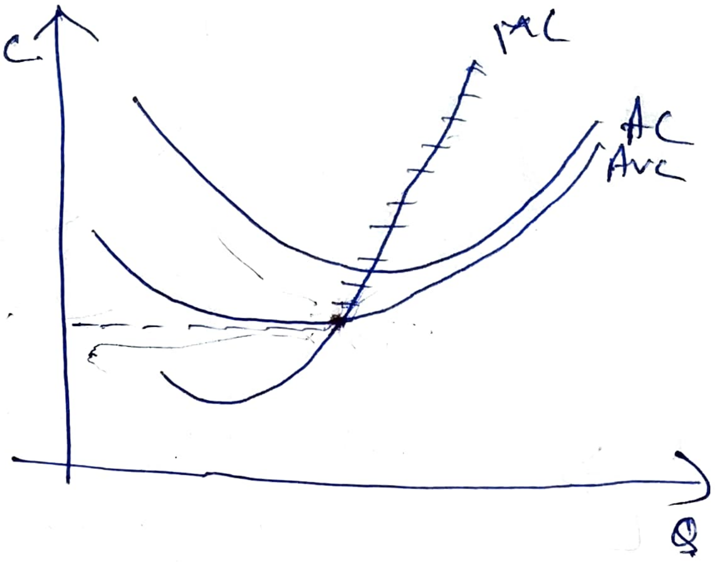
$TR = 400 \times 199.5 = 79800$

$TC = 100 + (199.5)^2 + 199.5 = 40099.75$

$\pi = 39700.5$; $\because \pi > 0$ short-run

Q.5

(a)



1. Shut-down Q
Condⁿ

2. $p^* \leq AVC_{min}$

3. $p^* \geq AVC_{min}$

4. MC SS curve.
↳ $P = MC$

(b)

$$C_i = 0.04 q_i^3 - 0.8 q_i^2 + 10 q_i + 10000$$

$$MC_i = 0.12 q_i^2 - 1.6 q_i + 10$$

$$AVC_i = 0.04 q_i^2 - 0.8 q_i + 10$$

$$\frac{d(AVC_i)}{dq_i} = 0$$

$$\Rightarrow 0.08 q_i - 0.8 = 0$$

$$\Rightarrow q_i = 10$$

$$\frac{d^2(AVC)}{dq_i^2} = 0.08 > 0$$

$$AVC_{i,min.} = 0.04 (10)^2 - 0.8 (10) + 10$$

$$= 4 - 8 + 10$$

$$= 6$$

$P = MC_i$

$P = 0.12 q_i^2 - 1.6 q_i + 10$

$$q_i = \frac{1.6 \pm \sqrt{2.56 - 4(0.12)(10-P)}}{0.24}$$

$$= \frac{1.6 \pm \sqrt{2.56 - 0.48(10-P)}}{0.24}$$

$P < 6 \Rightarrow SS = 0$

$P \geq 6 \Rightarrow SS = \dfrac{1.6 \pm \sqrt{2.56 - 0.48(10-P)}}{0.24}$

Q.6 $e = -2$

$MC = 20$

$$\frac{P_0 - MC}{P_0} = -\frac{1}{e}$$

$$\frac{P_0 - 20}{P_0} = -\frac{1}{2}$$

$P_0 = 40/3$

$$\frac{P_1 - 25}{P_1} = -\frac{1}{2}$$

$P_1 = 50/3$

$$\frac{\Delta P}{P_0} \times 100 = \frac{10/3}{40/3} \times 100$$

$= 25\%$ [Yes, True]

Or,

$$\frac{P_0 - MC}{P_0} = \frac{1}{2}$$

$$\frac{P_0 - 20}{P_0} = \frac{1}{2} \Rightarrow P_0 = 40$$

$$\frac{P_1 - 25}{P_1} = \frac{1}{2} \Rightarrow P_1 = 50$$

$$\frac{\Delta P}{P} \times 100 = \frac{10}{40} \times 100 = 25\%$$

$\boxed{\text{Yes, True}}$

8.7 $P_1 = 55 - x_1$ $\qquad$ $P_2 = 35 - \frac{x_2}{2}$

$MR_1 = 55 - 2x_1$ $\qquad$ $MR_2 = 35 - x_2$

In market (1) $MR_1 = MC$

$x_1 = 25$

$P_1 = 30$ (Substituting value of x_1)

In market (2)

$MR_2 = MC$

$x_2 = 30$

$P_2 = 20$

a) $x_1 + x_2 = x$

$\Rightarrow 25 + 30 = 55$ (Total o/p)

b) $x_1 = 25 \quad x_2 = 30$

c) $P_1 = 30 \quad P_2 = 20$

d) $\pi = (30 \times 25) + (20 \times 30) - 5(55)$

$= 750 + 600 - 275 = 1075$

ealt with accordingly

2

$$Z = 1.5LK - 0.3L^2 - 0.15K^2 + \lambda[1500 - 60L - 74K]$$

$\frac{\partial Z}{\partial L} = 0 \Rightarrow 1.5K - 0.6L - \lambda 60 = 0$

$\frac{\partial Z}{\partial K} = 0 \Rightarrow 1.5L - 0.3K - \lambda 74 = 0$

$$\frac{1.5K - 0.6L}{1.5L - 0.3K} = \frac{60}{74}$$

$\Rightarrow 74(1.5K - 0.6L) = 60(1.5L - 0.3K)$

$= 111K - 44.4L = 90L - 18K$

$\Rightarrow 129K = 134.4L$

$K = \frac{134.4}{129}L$

$K = 1.04L$

$1500 = 60L + 74(1.04L)$

$60L + 76.96L$

~~193500 = 7740L + 3374.4L~~

$1500 = 136.96L$

~~193500 = 11114.4L~~

$L = 10.95$

$K = 11.38$

~~L = 17.40~~ ~~K = 6.15~~

$Q = 131.43$

(8.)

$$MR_i = P_i\left(1 - \frac{1}{e_i}\right) \; ; \quad i = 1, 2$$

$$MR_1 = MR_2$$

$$P_1\left(1 - \frac{1}{e_1}\right) = P_2\left(1 - \frac{1}{e_2}\right)$$

$$\Rightarrow \frac{P_1}{P_2} = \frac{(e_2 - 1)e_1}{(e_1 - 1)e_2}.$$

Case I $\quad P_1 = P_2 \Rightarrow e_1 = e_2$

Case II $\quad P_1 > P_2 \Rightarrow e_1 < e_2$

Case III $\quad P_1 < P_2 \Rightarrow e_1 > e_2$

Statement is False.