LNMIIT
The LNM Institute of Information Technology

# The LNM Institute of Information Technology

## Computer Science & Engineering

## Introduction to Simulation & Modeling( CSE3171)

Exam Type: Mid Term

Time: 90 minutes Date: 26/09/2019 Max. Marks: 50

Name: ______________________ Enrollment No: ______________

**Instructions:**

- Attempt all the questions.
- Marks for each question are written against them.
- *Do not write anything* in *question-paper* except Name and Enrolment Number.
- Though careful proof reading has been done for question paper. Even then if you have any doubt/confusion regarding the question you can make your assumption. You must write your assumption clearly before you start attempting that question. If Instructor thinks that your doubt/confusion and assumption is genuine then only he will entertain that assumption and check the question based on assumption otherwise your doubt/confusion/assumption will be ignored.

Q1. Discuss the algorithm to generate uniformly distributed random numbers. Let a random generator generates numbers as mention in the below table, suggest & implement different test required to perform on the numbers in order to ensure that they follows the properties of random numbers i.e. Uniformity & Independence null hypothesis is not rejected, the significance value $\alpha$=0.05, $Z_{\alpha/2}$=1.96, $\chi^2_{0.05,9}$ =16.9 and the critical value of D, obtained from K-S table for the given $\alpha$ and N=25 is 0.27. [4+6 Marks]

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.74 | 0.39 | 0.66 | 0.17 | 0.03 | 0.05 | 0.82 | 0.32 | 0.03 | 0.38 | 0.8 | 0.49 | 0.65 |
| 0.71 | 0.75 | 0.28 | 0.71 | 0.28 | 0.1 | 0.69 | 0.95 | 0.44 | 0.77 | 0.19 | 0.45 | |

Q2. Consider a drive-in restaurant where carhops take orders and bring food to the car. Car arrival & service are shown in the manner given in the table, [10 Marks]

<table>
<tr><th>S.No</th><th>Inter-Arrival Time</th><th>Service Time if Able Served</th><th>Service Time if Baker Served</th><th>There are two carhops- Able and Baker. Consider that the average performance of Able and Baker are same. Construct simulation table for Carhops and find</th></tr>
<tr><td>1</td><td>-</td><td>3</td><td>4</td><td rowspan="10">a. Average waiting time<br>b. Probability of Able idle.<br>c. Probability of Baker idle.<br>d. Average service time.<br>e. Probability of wait for customers.<br>f. Average time customer spends in the system<br>g. Average waiting time of those who waits<br>h. Average time between arrivals. ξ(N-1)</td></tr>
<tr><td>2</td><td>3</td><td>6</td><td>6</td></tr>
<tr><td>3</td><td>6</td><td>2</td><td>5</td></tr>
<tr><td>4</td><td>5</td><td>5</td><td>8</td></tr>
<tr><td>5</td><td>4</td><td>1</td><td>2</td></tr>
<tr><td>6</td><td>2</td><td>3</td><td>2</td></tr>
<tr><td>7</td><td>7</td><td>9</td><td>5</td></tr>
<tr><td>8</td><td>1</td><td>5</td><td>4</td></tr>
<tr><td>9</td><td>9</td><td>9</td><td>6</td></tr>
<tr><td>10</td><td>6</td><td>2</td><td>3</td></tr>
</table>

Q3. Scheme for numerical solution of the differential equations $\frac{dy}{dt} = \int_0^t \eta^2 (t-\tau)^2 e^{-\eta(t-\tau)} y(\tau) d\tau + \sin(\omega t) e^{-\beta t}$, apply Runge-Kutta second order method to simulate the given differential equation. [10 Marks]

Q4. Let $X_1, X_2, \ldots, X_n$, such that $\forall X_i \sim U(0,1)$, are IID random numbers, If $Z_n = X_1 + X_2 + \ldots + X_n$, prove that $\lim_{n\to\infty} Z_n \sim$ Gaussian distribution. [10 Marks]

Q5. Find first four central moments of Poisson distributed random numbers. If N and M are Poisson distributed random numbers with mean μ and λ respectively then find the probability distribution of S, where S=N+M. [10 Marks]

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Formulae for testing NULL hypothesis:

Runs up and runs down:

$$\mu_a = \frac{2N-1}{3},$$

$$\sigma_a^2 = \frac{16N-29}{90}$$

Runs above and below mean:

$$\mu_a = \frac{2n_1 n_2}{N} + \frac{1}{2},$$

$$\sigma_a^2 = \frac{2n_1 n_2 (2n_1 n_2 - N)}{N^2 (N-1)}$$

Autocorrelation test:

$$Z_0 = \frac{\hat{\rho}_{im}}{\sigma_{\hat{\rho}_{im}}},$$

$$\hat{\rho}_{im} = \frac{1}{M+1}\left[\sum_{k=0}^{M} R_{i+km} R_{i+(k+1)m}\right] - 0.25,$$

$$\sigma_{\hat{\rho}_{im}} = \frac{\sqrt{13M+7}}{12(M+1)}$$