LNMIIT/B.Tech/C/IC/2018-19/ODD/MTH213/MT

The LNM Institute of Information Technology, Jaipur
Mathematics-III
Mid Term

Duration: 90 mins. Max.Marks: 50

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NOTE: You should attempt all questions. Your writing should be legible and neat. Marks awarded are shown next to the question. **Start a new question on a new page and answer all its parts in the same place.** Please make an index showing the question number and page number on the front page of your answer sheet in the following format.

Question No.				
Page No.				

1. (a) For any two complex numbers z_1 and z_2, prove that $|z_1 + z_2| \leq |z_1| + |z_2|$. [4]
   (b) Find all the fifth roots of 32 and locate them geometrically. [4]

2. (a) Let $f(z) = x^3 + i(1-y)^3$. Find all the points where the function is differentiable. Also find the derivatives at all those points. For which value of z, $f(z)$ is analytic? [3]
   (b) Let $u(x, y) = 2x(1-y)$. Show that $u(x, y)$ is harmonic in some domain and find its harmonic conjugates. [4]

3. (a) Write all possible Laurent series expansion of $f(z) = \frac{1}{(z-1)^2(z-3)}$ in powers of $(z-1)$. [5]
   (b) For any two complex numbers z_1 and z_2, prove that

$$2\sin z_1 \cos z_2 = \sin(z_1 + z_2) + \sin(z_1 - z_2).$$

[2]

4. (a) Using Cauchy integral formula, evaluate the contour integrals $\int_C \frac{1}{(z^2+4)^2} dz$, where C is the circle $|z-i| = 2$ in positive directions. [3]
   (b) Find all the singular points of $f(z) = \frac{\text{Log}(z+2)}{(z-4)(z-5)}$. Classify them as non-isolated, isolated, poles, removable and essential singularity. [3]

5. (a) Suppose $f(z)$ is an entire function such that and $|f(z)| \leq |z|$. Using Cauchy's inequality prove that $|f'(z)|$ is bounded. Then prove that $f'(z)$ is constant. If $f(1) = 1$ and $f(i) = 2$, Find $f(2)$. [4]
   (b) If $f(z)$ is real-valued and analytic function defined on a domain, then prove that $f(z)$ is constant. [3]

6. (a) Using M-L Inequality, find an upper bound of

$$\left| \oint_C \frac{z^2 e^{(z+1)}}{z+1} dz \right|$$

where C is the circle $|z| = 4$. [4]
   (b) Find the radius of convergence of the series

$$\sum_{k=0}^{\infty} \frac{(z-4-3i)^k}{5^{2k}}$$

[3]

7. (a) Find $\int_C z e^{-\frac{1}{(z-2)}}$ where C is any positively oriented closed contour with $z = 2$ inside it. [3]
   (b) Evaluate $\int_0^{2\pi} \frac{1}{(2+\cos\theta)^2} d\theta$. [5]