The LNM Institute of Information Technology Jaipur, Rajsthan

Math-II (2015-16), Quiz-2: Section-B

Name: Roll No:

Time: 15 Minutes Maximum Marks: 10

Q1. Verify the exactness of the ODE $(xy + x^2)dx + (y^2 + xy)dy = 0$ and then find an integrating factor if it is not exact. [5]

(Hint: Coefficients of dx and dy are homogeneous functions of degree 2)

Sol. Here, $M = xy + x^2$ and $N = y^2 + xy$. Since $\frac{\partial M}{\partial y} = x$ and $\frac{\partial N}{\partial x} = y$, The given DE is not exact.

Since $M = xy + x^2$ and $N = y^2 + xy$ are both homogeneous function of degree 2, the integrating factor is $\frac{1}{Mx+Ny}$ i.e. $\frac{1}{(x+y)(x^2+y^2)}$.

[5]

Q2. Solve the differential equation $yy'' + (y')^2 = 0$.

Sol. In this eqution independent variable x is missing explicitly. So introduce y'=p and $y''=\frac{dp}{dx}=\frac{dp}{dy}\frac{dy}{dx}=\frac{dp}{dy}p$.

Substituing in the given DE, we get a 1st order equation in p, i.e. $yp\frac{dp}{du} + p^2 = 0$.

By method of separation variables we get the solution as $py = c_1$.

By substituting y' = p and solving we get the solution as $y^2 - 2c_1x + 2c_2 = 0$ or $y^2 - c_1x + c_2 = 0$.