Q1 $R(Q)$, $C(Q)$, $R(R)$ are vector space, but not $R(C)$

as $2 \in R$ & $2i \in C$, but $2 \cdot 2i = 4i \notin R$

Q2 addition of two differentiable functions is differentiable and scalar multiplication of differentiable function is also diff..

Q3. (a) not a vector space as $(2,3,5) \in W$. & $-2 \in R$, but $a.v = (-4,-6,-10) \notin W$.

(b) yes as (x,y,z): $x+y-z=0$

(c) Not, as $(1,1,1) \in W$, but $2(1,1,1) = (2,2,2) \notin W$ Not

(d) $(1,0,0), (0,1,0) \in W$ but $\alpha+\beta = (1,1,0) \notin W$ Not

(4) $x^2+4x-3 = a(x^2-2x+5) + b(2x^2-3x) + c(x-1)$

so $\Rightarrow$
$$\begin{aligned} a+2b &= 1 \\ -2a-3b+c &= 4 \\ 5a-c &= -3 \end{aligned}$$
gives
$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 1 \\ -2 & -3 & 1 & 4 \\ 5 & 0 & -1 & -3 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 6 \\ 0 & -10 & -1 & -8 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 6 \\ 0 & 0 & 9 & 52 \end{array}\right]$$

gives $c = \frac{52}{9}$, $b = \frac{2}{9}$, $a = \frac{5}{9}$, $5/9 = a$

Q5 (a) $\begin{vmatrix} 1 & 0 & 2 & 1 \\ 1 & 3 & 2 & 1 \\ 4 & 1 & 2 & 2 \end{vmatrix} \sim \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 3 & 0 & 0 \\ 0 & 1 & -6 & -2 \end{bmatrix}$ so Independent

(b) $\begin{bmatrix} 1 & 2 & 6 \\ -1 & 3 & 4 \\ -1 & -4 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 6 \\ 0 & 5 & 10 \\ 0 & -2 & -4 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 6 \\ 0 & 5 & 10 \\ 0 & 0 & 0 \end{bmatrix}$ LD

(c) $a(u+v) + b(v+w) + c(w+u) = (a+c)u + (a+b)v + (b+c)w = 0$

$\Rightarrow \left\{ \begin{aligned} a+c &= 0 \\ a+b &= 0 \\ b+c &= 0 \end{aligned} \right\}$ $a=b=c=0$ so LI

(d) $\begin{bmatrix} 1 & 2 & 0 \\ 3 & -1 & 1 \\ 4 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 0 \\ 0 & -7 & 1 \\ 0 & -7 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 0 \\ 0 & -7 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ so LD.

(6) As $a \sin x + b e^x + c x^2 = 0 \Rightarrow a = b = c = 0$ so L.I

(7) If $\{\alpha_1, \dots \alpha_n\}$ are L.D then $a_1\alpha_1 + \dots + a_n\alpha_n = 0$, where some a_i are non zero.

$\Rightarrow$ let $a_k \neq 0$, then $\alpha_k = \frac{1}{a_k}(a_1\alpha_1 \dots + a_{k-1}\alpha_{k-1} + a_{k+1}\alpha_{k+1} + \dots + a_n\alpha_n)$

which shows that α_k is linear combination of.

(8) ~~for~~ As set is L.I & span R^4 so Basis.

(9) (a) basis, (b) not basis (c) not basis as L.D (d) basis

(10) (a) dim = 6

(b) dim = 6 as upper-triangular matrix = $\begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix}$

(c) dim = 6

(d) dim = 1 as basis is $\left\{ \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \right\}$

(11) $A = \begin{bmatrix} 1 & 0 & 3 & 1 \\ 2 & 2 & 0 & 1 \\ 5 & 4 & 3 & k \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 2 & -6 & -1 \\ 0 & 4 & -12 & k-5 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 2 & -6 & -1 \\ 0 & 0 & 0 & k-7 \end{bmatrix}$ so for $k = 7$

$B = \begin{bmatrix} 1 & 0 & 3 & 1 \\ 3 & 0 & 9 & 3 \\ 2 & 2 & 0 & 1 \\ 4 & 2 & 6 & k \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & -6 & -1 \\ 0 & 2 & -6 & k-4 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 2 & -6 & -1 \\ 0 & 0 & 0 & k-5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ for $k = 5$

(12) $\begin{bmatrix} 1 & 2 & 1 & 3 & 1 & 2 \\ 2 & 4 & 3 & 7 & 5 & 6 \\ 1 & 2 & 3 & 5 & 7 & 6 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 1 & 3 & 1 & 2 \\ 0 & 0 & 1 & 1 & 3 & 2 \\ 0 & 0 & 2 & 2 & 6 & 4 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 2 & 1 & 3 & 1 & 2 \\ 0 & 0 & 1 & 1 & 3 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

dim W = 2 basis: $\left\{ \begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 1 & 3 & 2 \end{bmatrix} \right\}$