MATH 3: End-Semester Examination: Part-A
(To be returned after 45 mins..)

R.No.: **Branch:** **Name:**

Instructions:

- Attempt all questions. Use the main sheet for rough work. Only the answers should be written on this sheet.
- ***Answers will be rejected if there is any overwriting or cutting.*** No partial credits. Each question carries 4 marks.
- **Calculations (Rough work) should be clearly demonstrated in the main sheet.**

Fill in the Blanks

1. The subset of all points in $\mathcal{C}$ for which $f(z) = 2iz\overline{z}$ is differentiable $= \{0\}$

Ans Let $z = x + iy$. Then, $f(z) = 2iz\overline{z} = 2i(x + iy)\overline{(x + iy)} = 2i(x + iy)(x - iy) = 2i(x2 + y2)$ To check if a function $f(z)$ is analytic, we apply Cauchy-Riemann equations for $f(z) = u(x, y) + iv(x, y)$, i.e.,
$u_x = v_y$ and $u_y = -v_x$ However, we have $u = 0$ and $v = 2x^2 + 2y^2$, so $u_x = 0 \neq 2y = v_y$.
Obviously, at only at 0 C-R equations holds. Hence, $f(z)$ is not differentiable at any zero.

2. The analytic function $f(z) = \sinh z$ is conformal except at $z = \{\frac{(2k+1)\pi i}{2}$: k = 0, 1, 2, 3 ...}

Ans $f(z) = \sinh z$ is analytic on C and $f'(z) = \cosh z \neq 0$ except the points $z = \frac{(2k+1)\pi i}{2}$ for $k = 0, 1, 2, 3....$ Thus , function $f(z) = \sinh z$ is conformal except at $\frac{(2k+1)\pi i}{2}$.

3. For the function $\dfrac{1}{Z^2 + 4z + 3}$, find out all possible regions of Taylor's and Laurent series expansions at $z = 1$ ________________

Ans The function is not analytic at the points z= -1 and z = -3. The distance between the point $z = 1$ and $z = -1$ is 2, and between the point $z = 1$ and $z = -3$ is 4. Thus, we consider the regions, (i) $|z - 1| < 2$ (ii) $2 < |z - 1| < 4$ (iii) $|z - 1| > 4$.
In the region, $|z - 1| < 2$ the function is analytic, hence, we obtain Tayler series expansion. In other regions $2 < |z - 1| < 4$ and $|z - 1| > 4$, we obtain Laurent series expansions.

4. Solution for the following PDE

$$\begin{aligned} u_{tt} - 4u_{xx} &= 0, \quad 0 < x < 20,\ t > 0 \\ u(x, 0) &= 5\sin\frac{\pi x}{10} - \sin\frac{3\pi x}{2}, u_t(x, 0) = 0, \quad 0 \leq x \leq 20, \\ u(0, t) &= u(20, t) = 0, \quad t \geq 0 \end{aligned}$$

is $\underline{u(x, t) = 5\cos(\frac{\pi t}{5})\sin\frac{\pi x}{10} - \cos(3\pi t)\sin\frac{3\pi x}{2}}$

5. Solution of the wave equation (Hint: Duhamel's principle):

$$u_{tt} = u_{xx} + x, \qquad -\infty < x < \infty,\ t > 0,$$

$$u(x,0) = u_t(x,0) = 0, \quad -\infty < x < \infty$$

is $\underline{xt^2/2}$.

6. The following Laplace equation

$$\triangle u = 0\ \forall\ x \in \mathcal{R}, 0 < y < \infty,\ u(x,0) = 0\ \forall\ x \in \mathcal{R}$$

has unique solution. TRUE/FALSE (give justification)<u>FALSE as domain is not bounded</u>.