The LNM Institute of Information Technology
Jaipur, Rajsthan

MATH-II ■ Assignment 1

1. Given $A = (a_{ij})$ define $A^T = (b_{ij})$ (where $b_{ij} = a_{ji}$), then show that $(AB)^T = B^T A^T$ if AB is defined.

2. Let A and B are invertible matrices with same dimension,then show that $(AB)^{-1} = B^{-1}A^{-1}$.

3. Show that every square matrix can be written as a sum of a symmetric and a skew symmetric matrices. Further, show that if A and B are symmetric, then AB is also symmetric if and only if $AB = BA$.

4. If A is real orthogonal matrix (i.e., $AA^T = I$), then Show that $|A| = \pm 1$.

5. Let A be a nilpotent ($A^m = 0$, for some $m \geq 1$) matrix. Show that $I + A$ is invertible.

6. Use row operations to find the row echelon form and row reduce echelon form of the following matrices:

$$\text{(a)} \begin{bmatrix} 4 & 3 \\ -8 & -6 \\ 16 & 12 \end{bmatrix} \quad \text{(b)} \begin{bmatrix} 1 & 5 & 8 \\ 3 & 2 & 9 \end{bmatrix} \quad \text{(c)} \begin{bmatrix} 0 & 2 & -3 \\ 2 & 0 & 5 \\ -3 & 5 & 0 \end{bmatrix} \quad \text{(d)} \begin{bmatrix} 1 & 2 & -3 & 1 & 2 \\ 2 & 4 & -4 & 6 & 10 \\ 3 & 6 & -6 & 9 & 13 \end{bmatrix}$$

7. Apply Gauss elimination method to solve the following system:

$$\text{(I)} \begin{aligned} x - y + z &= 0 \\ -x + y - z &= 0 \\ 10y + 25z &= 90 \\ 20x + 10y &= 80 \end{aligned} \qquad \text{(II)} \begin{aligned} 3x + 2y + z &= 3 \\ 2x + y + z &= 0 \\ 3x + y + 2z &= 3 \end{aligned} \qquad \text{(III)} \begin{aligned} 3x + 2y + z &= 3 \\ 2x + y + z &= 0 \\ 6x + 2y + 4z &= -6 \end{aligned}$$

8. Consider the following linear non-homogenous system:

$$\begin{aligned} x + y + z &= 5 \\ 2x + 3y + 5z &= 8 \\ 4x + 5z &= 2. \end{aligned}$$

   (a) Find the coefficient matrix A and augmented matrix $[A|b]$, where b is the right hand side vector. What can we say about the existence of the solution for the given system.

   (b) Apply Gauss Jordan elimination method to find the solution.

9. Choose h and k such that the system has (a) no solution (b) a unique solution (c) infinitely many solution

$$\text{(I)} \begin{aligned} x + hy &= 2 \\ 4x + 8y &= k \end{aligned} \qquad \text{(II)} \begin{aligned} x + 3y &= 2 \\ 3x + hy &= k \end{aligned}$$

10. Find inverse of the following matrices by using Gauss-Jordan elimination method:

$$\text{(a)} \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} \quad \text{(b)} \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{bmatrix} \quad \text{(c)} \begin{bmatrix} 1 & 2 & -4 \\ -1 & -1 & 5 \\ 2 & 7 & -3 \end{bmatrix} \quad \text{(d)} \begin{bmatrix} 1 & 3 & -4 \\ 1 & 5 & -1 \\ 3 & 13 & -6 \end{bmatrix}$$