The LNM Institute of Information Technology
Jaipur, Rajasthan

End Semester Exam

MATH-III, 21st Nov, 2016

Part-I

Time: 45 minutes, Maximum Marks:10

Name:________________________ **Roll No.:**________________________

Note: Each question carry 2 marks. Overwriting will be treated as a wrong answer. Use only the last page of main answer sheet for rough work and calculation. Write down the final answer, no marks for formula or incomplete solution.

1. The PDE which characterizes the surfaces

$$F\left(z - xy, x^2 + y^2\right) = 0$$

is ________________________.

Solution: $yp - xq = y^2 - x^2$ (Here $u = z - xy,\ v = x^2 + y^2,\ P = \frac{\partial(u,v)}{\partial(y,z)} = -2y,\ Q = \frac{\partial(u,v)}{\partial(z,x)} = 2x,\ R = \frac{\partial(u,v)}{\partial(x,y)} = 2x^2 - 2y^2$)

2. (*a*) The classification (elliptic, parabolic, or hyperbolic) of the following PDE (with region)

$$xu_{xx} + (x - y)u_{xy} - yu_{yy} = 0$$

is ________________________ type.

(*b*) Tick Appropriately: The PDE $(u_{xxx})^4 + u_{yy} = 0$ is first/second/third/forth order and linear/semi-linear/quasi-linear/non-linear PDE.

Solution: (a) (i) Hyperarabolic for $x + y \neq 0$ (ii) Parabolic if $x + y = 0$ (Here $b^2 - 4ac = (x - y)^2 + 4xy = (x + y)^2$)
(b) Third order non-linear PDE

3. Solution for following PDE

$$\begin{aligned} u_{tt} - 4u_{xx} &= 0, & & 0 < x < \infty,\ t > 0 \\ u(x, 0) &= 0, & & u_t(x, 0) = \cos x, 0 \leq x < \infty, \\ u(0, t) &= 0, & & t \geq 0. \end{aligned}$$

is $\underline{\frac{1}{2}\cos x \sin 2t}$

4. Solution of the wave equation (Hint: Duhamel's principle):

$$u_{tt} = u_{xx} + x, \qquad -\infty < x < \infty, t > 0,$$

$$u(x, 0) = u_t(x, 0) = 0, \quad -\infty < x < \infty.$$

is $\underline{\frac{1}{2}xt^2}$

5. Solution for the following Laplace equation:

$$\begin{aligned} u_{xx} + u_{yy} &= 0, & & 0 < x, y < \pi, \\ u(x, 0) &= x,\ 0 \leq x \leq \pi, & & u(x, \pi) = 0,\ 0 \leq x \leq \pi, \\ u(0, y) = u(\pi, y) &= 0, & & 0 \leq y \leq \pi. \end{aligned}$$

is $\underline{u(x, y) = \sum_{n=1}^{\infty} \frac{(-1)^n 2 \sin nx \sinh[n(y - \pi)]}{n \sinh(n\pi)}}$