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16UCC118 · Quiz 3

Q 1(a) $H = \sum_{i=1}^{M} p(m_i) \log_2 \frac{1}{p(m_i)}$

$p(x_i) = \frac{2^{M-1}}{2M} \quad i = 1, \ldots, M$

$= \frac{1}{2^{M}}$

$H = \sum_{i=1}^{M} \frac{1}{2^{M}} \log_2 (2^{M}) = \sum_{i=1}^{M} \frac{1}{2}$

$H = \cancel{\frac{M(M-1)}{2^{M-1}}}$ ~~bits/symbol~~ $\frac{M}{2}$ bits/symbol.

(b) $p(x_i) = \frac{1}{27} \cdot \quad i = 1, \ldots, 27$

$H = \sum_{i=1}^{27} \frac{1}{27} \log_2 (27)$

$H = \log_2 (27)$ bits/symbol

(c) (a) $p = 0.1, \quad p(x_1) = p(x_2) = 0.5$

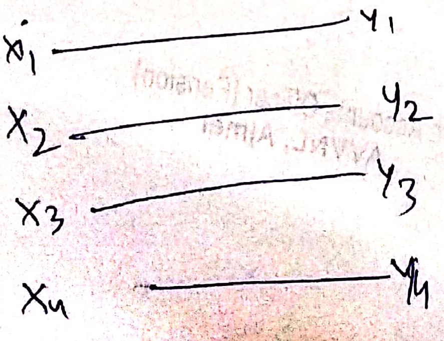
$= p(x) \times p(y/x)$

$H(X,Y) = H(X) + H(Y/X) = \frac{1}{2} \log_2 (2) + 0.25 \times \log_2 5$

$\boxed{H(X,Y) = 0.5 + 0.25 \log_2 5}$

$\boxed{I(X,Y) = 0.5 - 0.25 \log_2 5}$

(e) 4x4 Noiseless channel model



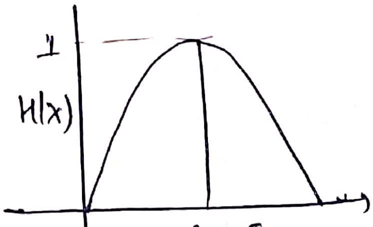
Equivocation

= Avg amt of info that has been lost on the channel is called equivocation

Ans ② (a) $H(X) = p\log_2\left(\frac{1}{p}\right) + (1-p)\log_2\left(\frac{1}{1-p}\right)$

it will be maximum when

$$p = 1-p \Rightarrow p = 0.5$$



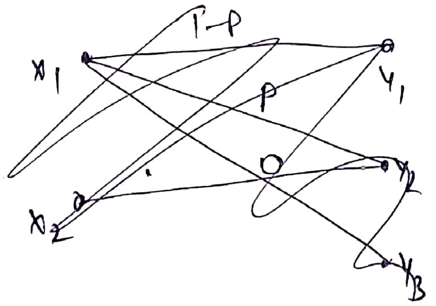
BSC achieve max value ~~at~~ 1 at $p = 0.5 = q$

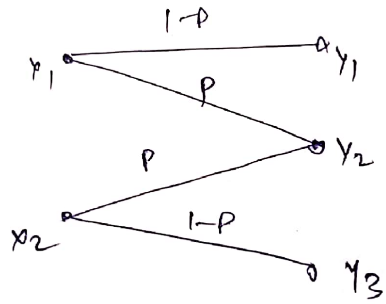
(b) $$p(Y/X) = \begin{bmatrix} 1-p & p & 0 \\ 0 & p & 1-p \end{bmatrix}$$

means $p(Y_1/X_1) = 1-p$ | $p(Y_1/X_2) = 0$

$p(Y_2|X_1) = p$ | $p(Y_2/X_2) = p$

$p(Y_3/X_1) = 0$ | $p(Y_3/X_2) = 1-p$





Channel diagram

Ans ③ (a) Ergodic capacity is the upper bound of the cap of on the statistic channel.

Ergodic channel cap

$$C = B\log_2\left(1 + \frac{S}{N}\right)$$

$$= B\log_2\left(1 + \frac{P_{av}}{N_0 B}\right).$$

For fading channel, $C_{Fading} < C_{AWGN}$ is true.

Ans 5) (b) $C = B\log_2(1+S/N)$

$S/N = 10000$

$C = 3000 \log_2(1+10000)$

$\boxed{C = 39864 \text{ bits/s}}$

(a) $I(X,Y) = H(X) + H(Y) - H(X,Y)$

$H(X) = \frac{1}{2}\log_2 2 + \frac{1}{2}\log_2 2 = 1$

$H(Y) = 1$

$H(X,Y) = \sum_i \sum_j P(x_i, y_j) \log \frac{1}{p(x_i, y_j)}$

$= \sum\sum p(x,y) \log_2 (p(x) \cdot p(y/x))$

$= \sum\sum 0.25 \log_2 (0.1)$

$= \log_2(0.1)$

$\therefore I(X,Y) = 2 - \log_2(0.1)$