The LNM Institute of Information Technology

Dept. of Electronics and Communication Engineering, Cognitive Radio

Quiz-1, 10th February 2020, Time Duration: 60 minutes, Maximum Marks: 50

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13/50

Q1. Tick the appropriate answers:

a) Examples of unlicensed spectrum are:

i. ISM ✓ ii. 3G iii. UNII iv. 4G

1

b) Both short-term and long-term fading are well represented by:

i. Rician ii. Log-normal iii. Suzuki ✓

2

c) If T_p (the sensing periodicity) is short:

i) Lower is the secondary throughput
ii) Shorter will be the secondary interference on primary data
iii) Larger is the secondary throughput

d) If H_0 and H_1 represent the hypothesis that vacant spectrum and busy spectrum are true respectively then:

i) $P_{fa} = Prob(^{H_1}/_{H_0})$, True/ False: TRUE ✓

ii) $P_d = Prob(^{H_0}/_{H_1})$, True/ False: False ✓

iii) $P_m = Prob(^{H_1}/_{H_1})$, True/ False: False ✓

2

e) Hidden terminal problem arises:

i) due to channel fading and shadowing
ii) due to secondary interference ✓
iii) due to path-loss only

✗

Q2.

a) There are N number of D2D devices, each device is at a distance d_j from the primary receiver and emits P_j Watts. Find the SINR at the primary receiver with P_s as the received signal power and assuming a path-loss channel model, and N_0 being the AWGN power at the receiver: (deductions to be shown if required.)

✗

Ans:

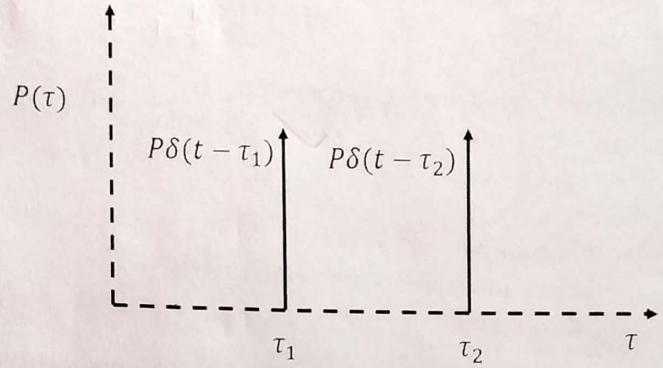
b) What is the expression for the interference temperature at the primary receiver due to aggregate interference due to secondary D2D devices for the above problem?

Ans: $T_i(f_c, B_i) = \dfrac{P(f_c, B_i)}{KB}$

1

Q3.

a) Find the delay spread σ_τ for the uniform power delay profile shown below given $\tau_1 = 1\ \mu sec$ and $\tau_2 = 2\ \mu sec$ is: (show calculations in rough sheet)



Ans:

i. $\cancel{\sigma_\tau} = 1.0\ \mu sec$ ii. $\sigma_\tau = 0.5\ \mu sec$ iii. $\sigma_\tau = 2.0\ \mu sec$

b) A received signal r(t) is Rayleigh distributed with $p(r) = \left(r/\sigma^2\right) e^{\left(-r^2/2\sigma^2\right)}$. Write the following values when $\sigma^2 = 1$.

i) DC power of r(t) = $\int_0^\infty r \cdot P(r)\,dr = \sqrt{\pi/2}\,\sigma = \sqrt{\pi/2}$

ii) Total power of r(t) = $\int_0^\infty r^2 P(r)\,dr = 2\sigma^2 = 2$

iii) AC power of r(t) = $Var(r) = \overline{r^2} - (\bar{r})^2 = 2\sigma^2 - (\sqrt{\pi/2}\,\sigma)^2 = \sigma^2(2-\pi/2) = 2-\pi/2$

2.5

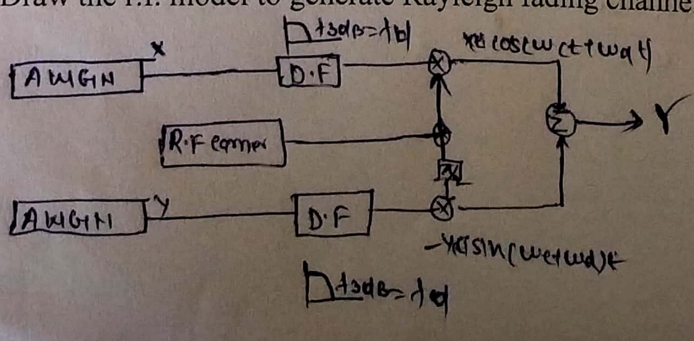
Q4.

a) A band-pass noise is given by $n(t) = n_c(t)\cos\omega_c t - n_s(t)\sin\omega_c t$. Write the expressions of:

i) the envelop of *n(t)* : $\sqrt{n_c^2(t) + n_s^2(t)}$

ii) the low-pass equivalent complex envelop of *n(t)*: $n_c(t) + j\,n_s(t)$

b) Draw the r.f. model to generate Rayleigh fading channel with Doppler.



2.5

Q5.

a) Rician parameter (K) is defined as:

$$K(dB) = 10\log_{10}\left({}^{a}/_{b}\right)$$

State one line physical interpretation each for the parameters a and b. To which distribution the Rician distribution will converge when $b \to 0$.

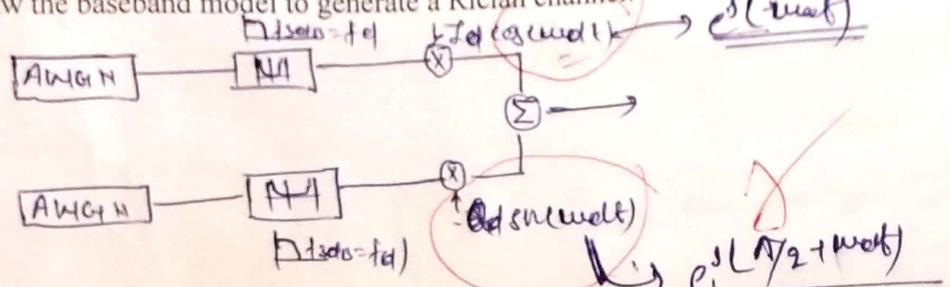
$a = s^2$

$b = \sum_{n=1}^{N} E(n)$ NLOS

When $b \to 0$, $K \Rightarrow \infty$, Gaussian channel (only direct path)

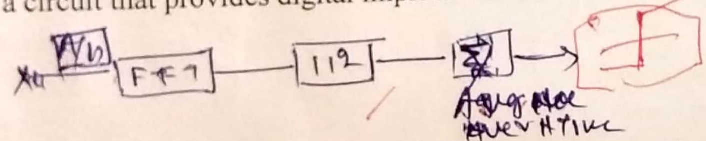
1.5

b) Draw the baseband model to generate a Rician channel.



Q6.

a) Draw a circuit that provides digital implementation of an energy detector for spectrum sensing.



b) The decision variable $T(y)$ for an energy detection based sensing is given by:

$$T(y) = \frac{1}{N}\sum_{i=1}^{N}\left|\frac{y(n)}{\sigma}\right|^2,$$

where, $y(n) = s(n) + w(n)$; $w(n)$ is a zero-mean Gaussian noise with variance σ^2. Find the distribution of $T(y)$ under the hypothesis H_0. (N is the total number of samples in the observation interval)

$E(T) = \frac{1}{N}\sum_{i=1}^{N}\left|\frac{w(n)}{\sigma}\right|^2$

$= \frac{1}{N}E\left[\frac{w(1)^2}{\sigma^2} + \frac{w(2)^2}{\sigma^2} + \cdots \frac{w(N)^2}{\sigma^2}\right]$

$= \frac{1}{N} \times N = 1$

$E[T^2] = \frac{1}{N^2}\left[E\left[\frac{w_1^4}{\sigma^4} + \cdots + \frac{w^4(N)}{\sigma^4}\right] + \frac{N(N-1)}{2} E\left(\frac{w_1^2}{\sigma^2}\right)\cdot E\left(\frac{w_2^2}{\sigma^2}\right)\right]$

$= \frac{1}{N^2}\left[3N\sigma^2 + N^2\sigma^2 - N\sigma^2\right] = \frac{1}{\sigma^2}\left(\frac{2}{N}\sigma^2 + \sigma^2\right)$

$Var = E[T^2] - (E[T])^2$

$= \frac{2}{N} + 1 - 1$

$= \frac{2}{N}$

$T(y) \simeq \text{Normal}\left(1, \frac{2}{N}\right)$

Q7.

a) Why cooperative spectrum sensing is needed? (Answer in one line)

Ans.: Due to provide reliable measure (spectrum estimate) when there is fading.

CR signal sent to fusion center

Diversity combining at fusion center.

b) Bring out the difference between 'data fusion' and 'decision fusion'. (Answer in one line)

Ans.: Data fusion is soft decision technique while decision fusion is hard decision technique.

c) In a cooperative spectrum sensing system, there are K number of CR users. If $P_{f,i}$ denotes the false alarm probability of the i^{th} CR in its local spectrum sensing, what will be the false alarm probability of the cooperative spectrum sensing system, using OR fusion logic?

Ans.: $1-(1-P_{f,i})^K = 1-(P_{d,i})^K$

Q8.

a) Draw the sketch of a baseband equivalent Doppler spectrum. How the same can be implemented in practice?

b) Express coherence bandwidth and coherence time in terms of relevant parameters such as rms spread and Doppler frequency spread as appropriate.

Q9.

a) A received signal at a wireless receiver is given by $y = hs + n$, where $s = \sqrt{P}, n = \mathcal{N}(0, \sigma_n^2)$ and h is Rayleigh distributed. Calculate the receiver SNR if the second moment of h is 8. $\mathcal{N}(0, \sigma_n^2)$ is a zero-mean normal distribution with variance σ_n^2.

$E[h^2] = 8, \quad \sigma^2 = 8, \quad \sigma^2 = 4, \quad SNR = \frac{h^2 s^2}{\sigma^2} = \frac{8P}{\sigma^2}$

b) Write down the pdf expression for the distribution that closely represents the shadowing effect.

$$\boxed{P_r = 10\log_{10}(P_t) + 10\log_{10}K - 10n\log_{10}\left(\frac{d}{d_0}\right) + \psi}$$

Q10.

a) Write mathematical expression of a received signal in a mobile fading channel environment that has suffered multipath fading, delay and Doppler shift.

b) Draw the ROC (Receiver Operating Characteristics) and on that, depict P_d, P_m and P_{fa}.

4 (a)

$n(t) = n_c(t)\cos\omega_c t - n_s(t)\sin\omega_c t$ [QAM signal]

↑ Inphase component ↑ quadrature component

Envelope $r = \sqrt{n_c^2(t) + n_s^2(t)}$

Low-pass equivalent

Complex envelope $= n_c(t) + j\,n_s(t)$

5 (a) a ⇒ signal power due to LOS component

b ⇒ ——— scattered multipath component.

6 (a) $y(t)$ → [BPF] — [A/D] — [$|\,|^2$] — [Σ_N] — [threshold] → decision

7 (c). PU ⟶ CR ⟶ BS/AP

H_0 H_0^{CR} H_0^{BS}

H_1 H_1^{CR} H_1^{BS}

8 (a)

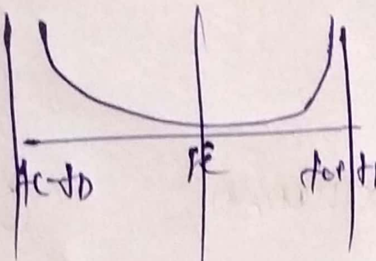
$$Q_f = \text{Prob}[H_1/H_0]$$

$$1 - \text{Prob}[H_0/H_0]$$

$$= 1 - \prod_{i=1}^{K}(1 - P_{fi}) = \boxed{1 - \prod_{i=1}^{K}(P_{di})}$$

9 (b) $$P(r) = \frac{1}{\sqrt{2\pi\sigma_w^2}}\exp\left(\frac{-P_{nr} - m_{dB}}{2\sigma_{dB}^2}\right)$$

(15)



$f_c - f_D \quad f_c \quad f_c + f_D$ → channel Doppler spec

$-f_D \quad 0 \quad f_D$ → (In base band eq

(16) a.

$$y(t) = \left[\sum_{n=1}^{N} a_n s_n(t - t_n)\, e^{j(w_c t + w t_n + \phi_{D,n})} \right]$$

Doppler

Fading + shadowing

lowpass eq ⇒ $= \sum \left[a_n s_n(t - t_n)\, e^{j\phi_n} \cdot e^{j w_c t} \right]$

$\phi_n = w t_n - \phi_{D,n}$ → Low pass equivalent