Endterm

Discrete Mathematical Structures
B.Tech 1st Year, The LNMIIT Jaipur

(Attempt all questions)

Q-1: Answer the following questions with proper justifications:

(a) Check which of the following are posets. In case of poset, draw the Hasse diagram as well.
$(D_{231}, \ |), (D_{525}, \ |), (D_{385}, \ |)$ **3**

(b) Consider the following Boolean matrices:

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}, \ \mathbf{B} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}, \ \mathbf{C} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$$

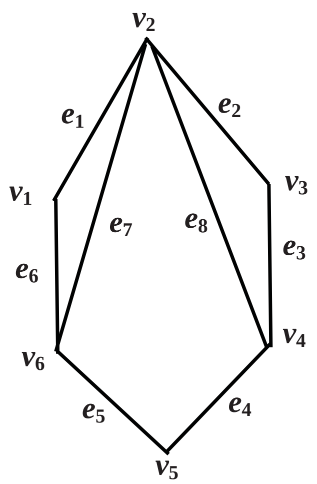
Now compute $\mathbf{A} \odot \mathbf{B} \odot \mathbf{C}$ **2**

(c) Define the *Symmetric Difference* between two sets and depict it through *Venn diagram.* **2**

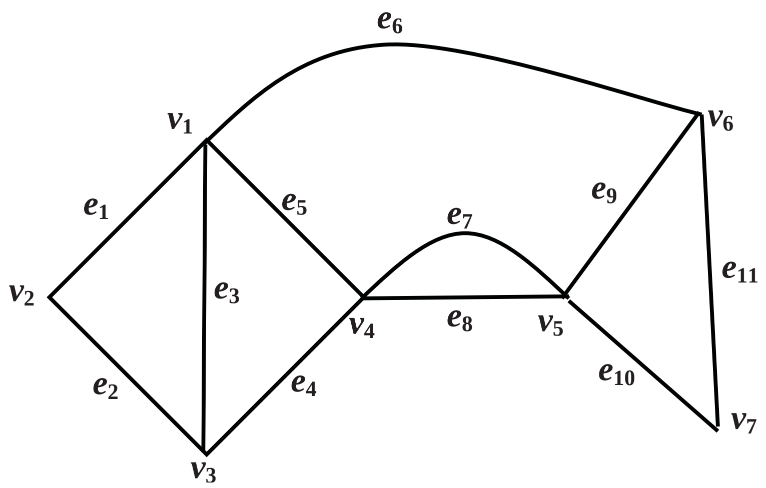
(d) Compute the last two digits of the number: $17^{5005} + 23^{3003}$ by using Euler's generalization of *Fermat's little theorem.* **3**

(e) Find the reminder when $42! + 43!$ is divided by 47 **2**

(f) Check with justification if the following graph is an *Euler* or *Unicursal* graph. In either case, provide the particular Euler or Unicursal line. **2**



(g) For the following graph, provide the incidence matrix: **2**



(h) Compute the number of subsets with an odd number of elements for a set having 13 elements. **2**

(i) Compute the form of a_n explicitly in a closed form for the following recurrence relation: **2**

$$\left.\begin{array}{l} a_n = 5a_{n-1} + 3, \quad \forall n \geq 2 \\ a_1 = 3 \end{array}\right\}$$

Q-2: Use truth table to verify if the following two compound statements are equivalent or not: $\neg(p \Leftrightarrow q)$ and $(p \wedge \neg q) \vee (q \wedge \neg p)$ **5**

Q-3: State and prove *Fermat's Little Theorem (FLT)*. Use this theorem to show that if p and q are two distinct primes, then

$$p^{q-1} + q^{p-1} \equiv 1 \pmod{pq}$$

1+2+2=5

Q-4: Write the set U_{20}. Using the *operation table*, prove that $(U_{20}, \odot_{20})$ is a group. Specify the identity element of this group and find the inverse of each element.

1+2+2=5

Q-5: Consider the set: $A = \{a, b, c, d, e\}$ and let R be a relation on $A \times A$, as defined below:

$$R = \{(a,a), (a,d), (b,b), (c,d), (c,e), (d,a), (d,b), (d,d)\}$$

Write down the Boolean matrix $\mathbf{M}_R$ for this relation and then compute the relation R^2. Find the transitive closure of R using the *Warshall's algorithm.* **1+1+3=5**