The LNM Institute of Information Technology
ECE and CCE

ECE 321: Control System Engineering (End-Term Examination)

Time: 3 Hours	**Date:** 05. 05. 2018	**Max. Marks: 50**

Instruction: *1) Start each answer on a fresh page of your answer book and highlight your answer number.*
2) Check that your Question paper has ***5 Questions****. All Questions are compulsory.*

Q1. (a) Using the Routh's criteria, check the stability of the following system and decide how many poles are on the right side of s-plane.

$$s^5 + s^4 + 2s^3 + 2s^2 + 3s + 15 = 0$$

(b) What is the phase margin (in degree) of a system having open loop transfer function as:

$$G(s)H(s) = \frac{2\sqrt{3}}{s(s+1)}$$

(c) What is the advantage of PID controller over PI and PD controllers?

(d) If the roots of a characteristic equation are given by $s_{1,2} = -3 \pm j2$, find the values of damping ratio and damped natural frequency (ω_d).

[2.5×4 = 10]

Q2. (a) Simplify the given block diagram (**Fig. 1**) using block diagram reduction method. Obtain the closed loop transfer function.

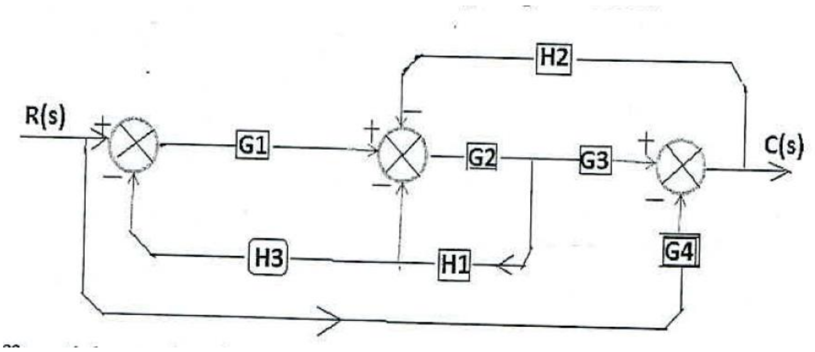


Fig. 1

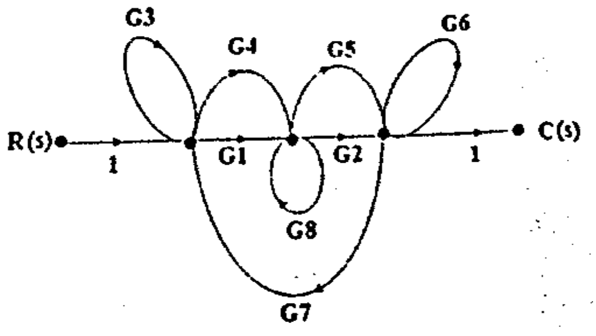


Fig. 2

(b) Find the transfer function of the signal flow graph shown in **Fig. 2** using Mason's Gain formula.

[5+5 = 10]

Q3. (a) For the feedback control system shown in **Fig. 3**, find the parameters *K* and *p* such that the following specifications will be satisfied:

(i) Peak overshoot of the response to a step input R(s) = 9.5%.

(ii) Steady state error to unit ramp disturbance W(s) = 0.01.

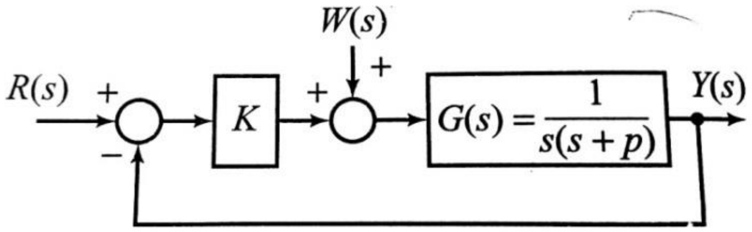


Fig. 3

(b) The open loop transfer function of a control system is given by:

$$G(s)H(s) = \frac{K}{s(s+4)(s^2+4s+13)}$$

Draw the root locus and determine

(i) The angle of asymptotes
(ii) The breakaway points
(iii) The angle of departure from complex poles
(iv) The value of K for marginal stability

(c) What is the effect of addition of poles and zeros to the closed loop transfer function?

[3+5+2 = 10]

Q4. (a) Write the state space model for the signal flow graph shown in **Fig. 4**.

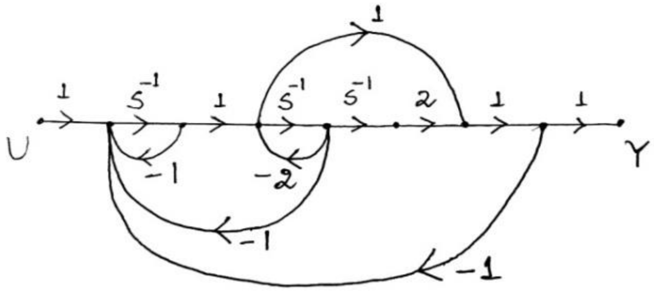


Fig. 4

(b) What will be the steady state error to a various standard inputs for type-2 non-unity feedback systems?

(c) For the given Bode plot (**Fig. 5**), determine the following:

(i) The transfer function $G(s)$ of the system.
(ii) The frequencies ω_{g1} and ω_{g2}.
(iii) Gain margin and Phase margin

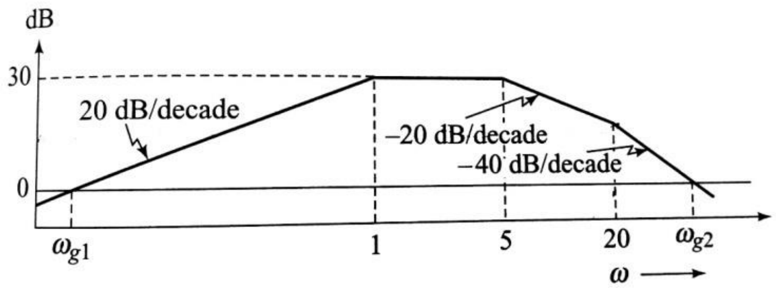


Fig. 5

[3+2+5=10]

Q5. (a) Sketch the Nyquist plot for a system with open loop transfer function

$$G(s)H(s) = \frac{K(s+3)(s+5)}{(s-2)(s-4)} \quad , \qquad K > 0$$

Find the range of K for which the system is stable.

(b) What are the advantages of state variable method over the transfer function based method?

(c) Define Nyquist Stability Criterion.

[6+2+2 = 10]

Solution to End term exam (CSE 2018)

Ans 1 (a)

Given,

$$s^5 + s^4 + 2s^3 + 2s^2 + 3s + 15 = 0$$

s^5	1	2	3
s^4	1	2	15
s^3	$\cancel{0}\ \epsilon$	-12	
s^2	$\frac{2\epsilon+12}{\epsilon}$ $+\infty$		
s^1	$\frac{-12\left(\frac{2\epsilon+12}{\epsilon}\right)-15\epsilon}{\left(2\epsilon+12/\epsilon\right)} = -12$		
s^0	15		

$1\frac{1}{2}$M

The system is unstable as their is sign change in 1st column of Routh's table.

There are 2 sign change, thus 2 poles in right side of s-plane.

1M

Ans 1 (b)

Given,
$$G(s)H(s) = \frac{2\sqrt{3}}{s(s+1)}$$

$$G(j\omega)H(j\omega) = \frac{2\sqrt{3}}{j\omega(j\omega+1)}$$

To find ω_{gc}.

$$|G(j\omega)H(j\omega)| = 1 \quad \text{———} \quad 1M$$

$$\Rightarrow \left|\frac{2\sqrt{3}}{j\omega((j\omega)+1)}\right| = 1$$

$$\Rightarrow \omega^4 + \omega^2 - 12 = 0$$

$\therefore \; \omega = \pm 2j, \pm\sqrt{3}$

We consider the real term i.e. $\omega_{gc} = +\sqrt{3}$ —— 0.5M

$$\text{Phase Margin (PM)} = 180^\circ + \angle G(j\omega)H(j\omega) \Big|_{\text{at } \omega = \omega_{gc}}$$

$$= 180^\circ + \left(-90^\circ - \tan^{-1}\frac{\omega_{gc}}{1}\right)$$

$$= 30^\circ$$ —— 1M

Ans. 1 (c)

PI	PD	PID
1. $TF = \frac{sK_p + K_i}{s}$	1. $TF = K_p + K_d s$	1. $TF = \frac{K_p s + K_D s^2 + K_i}{s}$
2. Addition of poles at origin → type ↑↑ $\boxed{\text{Thus } e_{ss} \downarrow\downarrow \text{ (decreases)}}$	2. Addition of a finite zero improves the stability of the system.	2. Addition of pole at origin ↓ $\boxed{e_{ss} \downarrow\downarrow \text{ (decreases)}}$
3. Addition of finite zero avoids the effect on stability. (1 pole - 1 zero) $\boxed{\text{Stability} \rightarrow \text{SAME}}$	3. No change in Type → $\boxed{e_{ss} = \text{same}}$	3. Addition of 2 finite zero. a) One zero avoids the effect on stability (for 1 pole) b) Another zero improves the stability

(2, 3 each point has 1 marks each, 1 → has ½ marks)

Ans 1 (d)   Given roots ⇒ $s_{1,2} = -3 \pm j2$

$$\text{Transfer function} = \frac{K}{(s+3-j2)(s+3+j2)}$$

$$= \frac{K}{s^2 + 6s + 13}$$ —— 0.5M

$\therefore$ Comparing with 2^{nd} order equations,

$$TF = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$2\xi\omega_n = 6$. and $\omega_n^2 = 13$

$\Rightarrow \omega_n = \sqrt{13}$

$\Rightarrow 2\xi\sqrt{13} = 6$

$\Rightarrow \boxed{\xi = \frac{3}{\sqrt{13}}}$ — 1M

Now, $\omega_d = \omega_n\sqrt{1-\xi^2}$

$= \sqrt{13}\sqrt{1-\frac{9}{13}}$

$= \sqrt{13-9} = 2$

$\therefore \boxed{\omega_d = 2}$ — 1M

Ans 3 (a) given, $M_p = 9.5\%$ (when i/p = R(s))

$$e^{-\pi\xi/\sqrt{1-\xi^2}} = 0.095$$

$$\Rightarrow \frac{-\pi\xi}{\sqrt{1-\xi^2}} = -2.35$$

$$\Rightarrow \frac{\xi}{\sqrt{1-\xi^2}} = 0.748$$ — 0.5M

$\Rightarrow \boxed{\xi = 0.6}$ — eq 1

Also, TF (when $R(s)$ is i/p)

$$\frac{Y(s)}{R(s)} = \frac{K}{s^2 + ps + K}$$

Comparing with $\dfrac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$

$\Rightarrow \omega_n^2 = K$ & $2\xi\omega_n = p$.

$\omega_n = \sqrt{K}$ — eq II

$2 \times 0.6\sqrt{K} = p$. (from eq I & II)

$\Rightarrow \sqrt{K} = \dfrac{p}{1.2}$

$\Rightarrow \boxed{p = 1.2\sqrt{K}}$ — 1M

Also, given

e_{ss} ($w(s)$ is i/p unit ramp) $= 0.01$

TF (when $w(s)$ is i/p)

$$\frac{Y(s)}{W(s)} = \frac{1}{s^2 + ps + K}$$

comparing with ~~$\dfrac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$~~

Given $w(s)$ = input unit ramp $= \dfrac{1}{s^2}$

formula

$$\boxed{e_{ss} = \frac{1}{\lim_{s \to 0} sG(s)H(s)}}$$

$$e_{ss} = \frac{1}{\lim_{s \to 0} s\left(\frac{1}{s(s+p)}\right)(K)}$$

$$\boxed{e_{ss} = p/K}$$

$$\boxed{e_{ss} = \frac{p}{K} = 0.01}$$ — 1M

Now, we found above

$$p = 1.2\sqrt{K}$$

$$\Rightarrow 0.01K = 1.2\sqrt{K}$$

$$\Rightarrow \boxed{\cancel{K = (0.008)^2}}$$ — 0.5M

$$\Rightarrow K = 14400$$

and $\boxed{p = 0.01K = 144}$

Ans 3(b)

Given,

$$G(s)H(s) = \frac{K}{s(s+4)(s^2+4s+13)}$$

i) Poles are $s = 0, -4, -2+3j, -2-3j$

ii) Angle of asymptotes

$$\theta = 45^\circ, 135^\circ, 225^\circ, 315^\circ$$ — 1M

iii) Centroid $= -2$

iv) Angle of departure from the complex pole $(-2+3j)$ — 1M

$$\phi_d = -90^\circ$$

v) characteristic eq^n

$$s(s+4)(s^2+4s+13) + K = 0$$

$$\Rightarrow K = -s(s+4)(s^2+4s+13)$$

$$\frac{dK}{ds} = 0$$

$$\Rightarrow s = -2, -2 \pm j1.58$$ (Break-away pts) — 1M

~~vi)~~

Ans 4 (

vi) Routh's table from characteristic eq^n

$$s^4 + 8s^3 + 29s^2 + 52s + K = 0$$

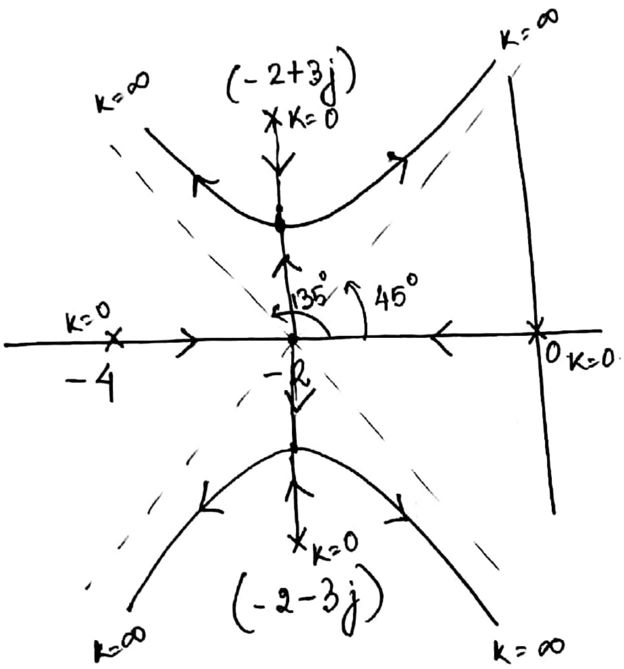
s^4	1	29	K
s^3	8	52	
s^2	22.5	K	
s^1	52 − 0.35K		
s^0	K		

for marginal stability,

$$52 - 0.35K = 0$$

$$\Rightarrow K = 148.6$$

(1M)



(1M)

Ans 3 (c)

Effect of addition of poles : As the poles moves towards the origin in s-plane, the rise time increases and the maximum overshoot decreases. The addition of pole to left half slow down the response.

Effect of addition of zeros :

- ✓ Makes the system overall response faster
- ✓ Rise time, peak time decreases but overshoot increases.

(2M)

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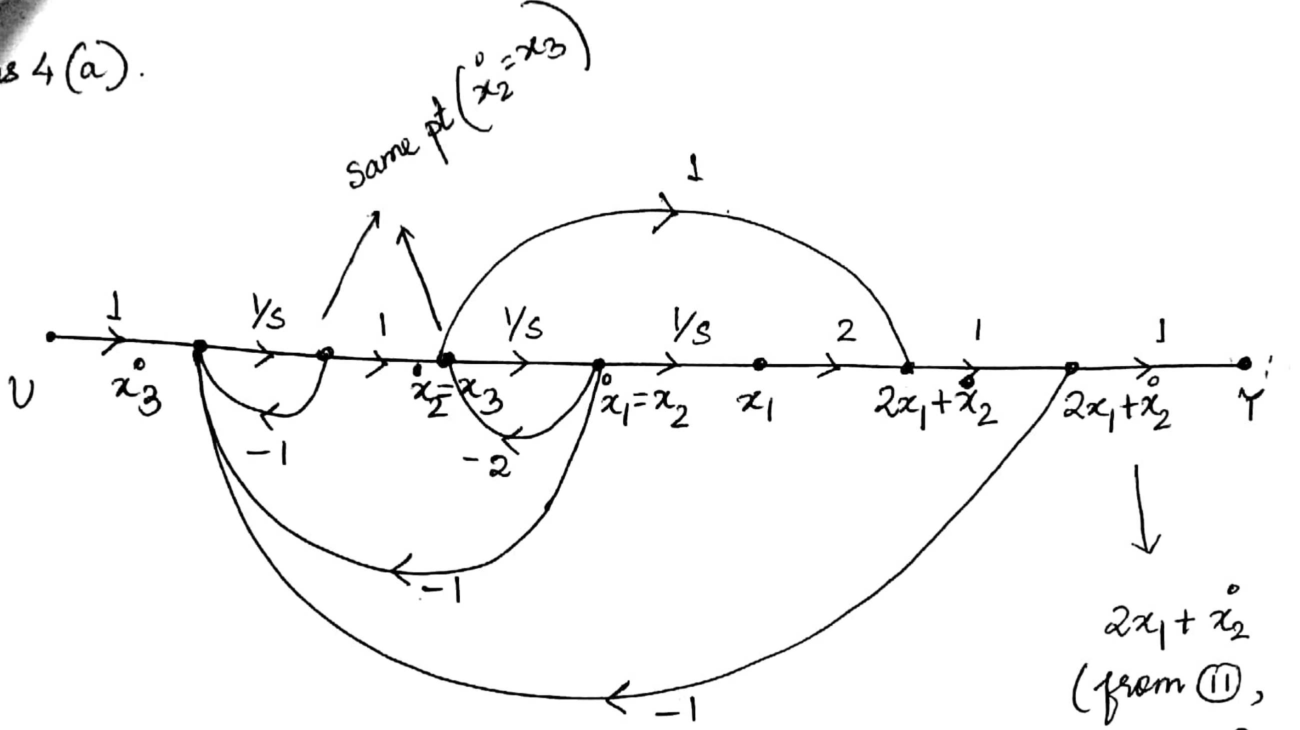
Ans 4 (a).



$2x_1 + \dot{x}_2$
(from (II), $= 2x_1 - 2x_2 + x_3$)

$$\dot{x}_1 = \frac{1}{s}\dot{x}_2 = x_2 \quad \text{—— (I)}$$

$$\dot{x}_2 = -2x_2 + \frac{1}{s}\dot{x}_3$$

$$= -2x_2 + x_3 \quad \text{—— (II)}$$

$$\dot{x}_3 = u - x_3 - x_2 - (2x_1 + \dot{x}_2)$$

$$= u - x_3 - x_2 - (2x_1 + (x_3) - 2x_2)$$

$$= u - x_3 - x_2 - 2x_1 - x_3 + 2x_2$$

$$= u - 2x_1 + x_2 - 2x_3 \quad \text{—— (III)}$$

$$y = 2x_1 + \dot{x}_2 = 2x_1 - 2x_2 + x_3 \quad \text{—— (IV)}$$

(2M)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -2 & 1 \\ -2 & 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

and $$y = \begin{bmatrix} 2 & -2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

(1M)

Ans 4(b)

Types of i/p	Steady state error Type - 2
Unit step	0
Unit ramp	0
Unit parabolic	$\frac{1}{K_a}$

$K_a = \lim_{s \to 0} s^2 G(s) H(s)$

— 2M

Ans 4 (c)

i) The first slope is 20 dB/decade. Thus Ks is a factor of the transfer function with $20 \log K = 30$

$\Rightarrow K = 31.623$

$\therefore$ $\boxed{\text{Transfer function} = \dfrac{31.623\, s}{(1+s)\left(1+\frac{s}{5}\right)\left(1+\frac{s}{20}\right)}}$

— 1M

ii)

$$20 = \frac{30 - 0}{\log 1 - \log \omega_{g_1}}$$

$\Rightarrow \boxed{\omega_{g_1} = 0.0316}$ — 1M

Similarly, $\boxed{\omega_{g_2} = 56.234}$ — 1M

(iii)

$$H(j\omega)G(j\omega) = \frac{31.623(j\omega)}{(1+j\omega)\left(1+\frac{j\omega}{5}\right)\left(1+\frac{j\omega}{20}\right)}$$

Phase Margin $= 180^\circ + \angle G(j\omega)H(j\omega)\Big|_{\text{at } \omega=\omega_{gc}}$

$$= 180^\circ + \left[90^\circ - \tan^{-1}\omega - \tan^{-1}\frac{\omega}{5} - \tan^{-1}\frac{\omega}{20}\right]_{\omega=\omega_{gc}}$$

ω_{gc} :-

$$|G(j\omega)H(j\omega)| = 1.$$

$$\Rightarrow \frac{31.623\,\omega}{\sqrt{(1+\omega^2)\left(1+\frac{\omega^2}{25}\right)\left(1+\frac{\omega^2}{400}\right)}} = 1$$

$$\Rightarrow (31.623\,\omega)^2 = (1+\omega^2)\left(1+\frac{\omega^2}{25}\right)\left(1+\frac{\omega^2}{400}\right).$$

$$\Rightarrow \omega_{gc} = \omega_{g1}/\omega_{g2}$$
(or)

1M

$\therefore$ PM $\simeq$ 192

1M

Similarly GM = 2.

Ans 5 (a):

Given,

$$G(s)H(s) = \frac{K(s+3)(s+5)}{(s-2)(s-4)}, \quad K>0.$$

$$M = G(j\omega)H(j\omega) = \frac{K(j\omega+3)(j\omega+5)}{(j\omega-2)(j\omega-4)}$$

~~At ω=0~~,

$$|G(j\omega)H(j\omega)| = \frac{K\sqrt{(\omega^2+9)(\omega^2+25)}}{\sqrt{(\omega^2+4)(\omega^2+16)}}$$

(1M)

$$\phi = \angle G(j\omega)H(j\omega) = \tan^{-1}\frac{\omega}{3} + \tan^{-1}\frac{\omega}{5} - \tan^{-1}\left(\frac{\omega}{-2}\right) - \tan^{-1}\left(\frac{\omega}{-4}\right)$$

$$= -360^\circ + \tan^{-1}\frac{\omega}{3} + \tan^{-1}\frac{\omega}{5} + \tan^{-1}\frac{\omega}{2} + \tan^{-1}\frac{\omega}{4}$$

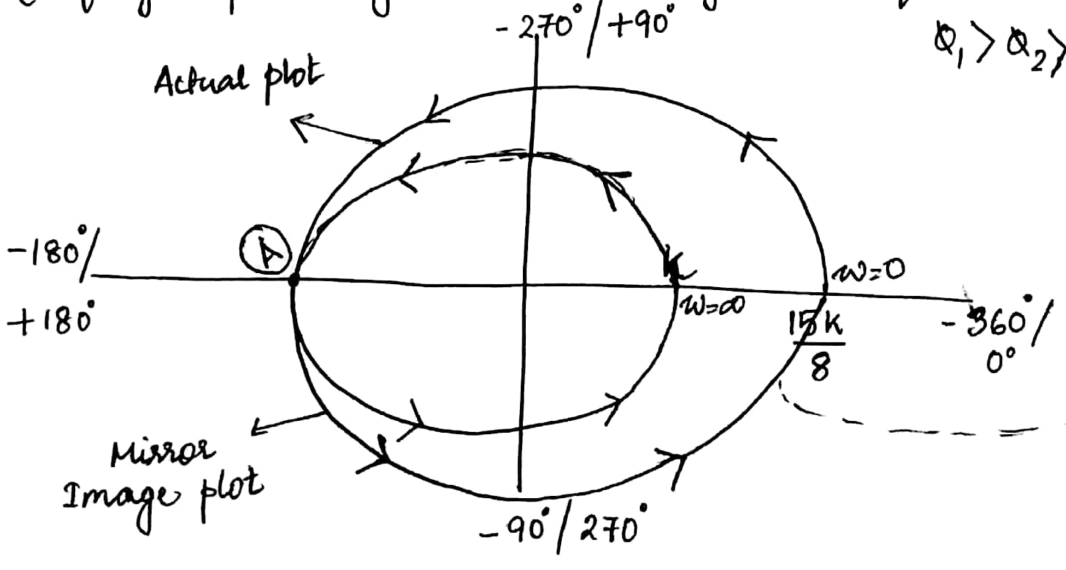
(1M)

At $\omega = 0$, $M = \frac{K\,15}{8}$, $\phi = -360^\circ$

and $\omega = \infty$, $M = K$, $\phi = 0^\circ$

(if you put any +ve ω value, you will find $\phi = -360^\circ, -\phi_1, -\phi_2, \ldots$ $\phi_1 > \phi_2 > \ldots$

(1M)



As $K > 0$

$\therefore \frac{15K}{8} > K$

let us find the ~~interectio~~ intersection point at real axis (A)

$\omega = \sqrt{11}$ | at $\phi = 180° / -180°$ (Putting in ϕ in previous page) — 1M

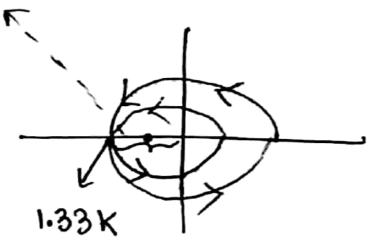
$$M\Big|_{at\ \omega=\sqrt{11}} = \frac{K\sqrt{20.36}}{\sqrt{15.27}} = 1.33K$$ — 1M

for the system to be stable :

~~1.33~~ $N = p$

$p = 2$ | as per given question

for N to be = 2, it has to enclose both circles



$\therefore$ $1.33K > 1$

$$\boxed{K > 0.75}$$ — 1M

Ans 5(b): Advantages of state variable method over the transfer function based method :

1. The method (state variable) is valid for linear, non-linear, Time variant, time invariant systems whereas Transfer function method is valid for only LTI system.
2. This method (SV) is valid for SISO as well as MIMO systems whereas TF method is valid for only SISO systems.
3. It describes internal state of system (SV method) whereas TF method donot.

(Any 2 pts) — 2M

Ans 5 (c)

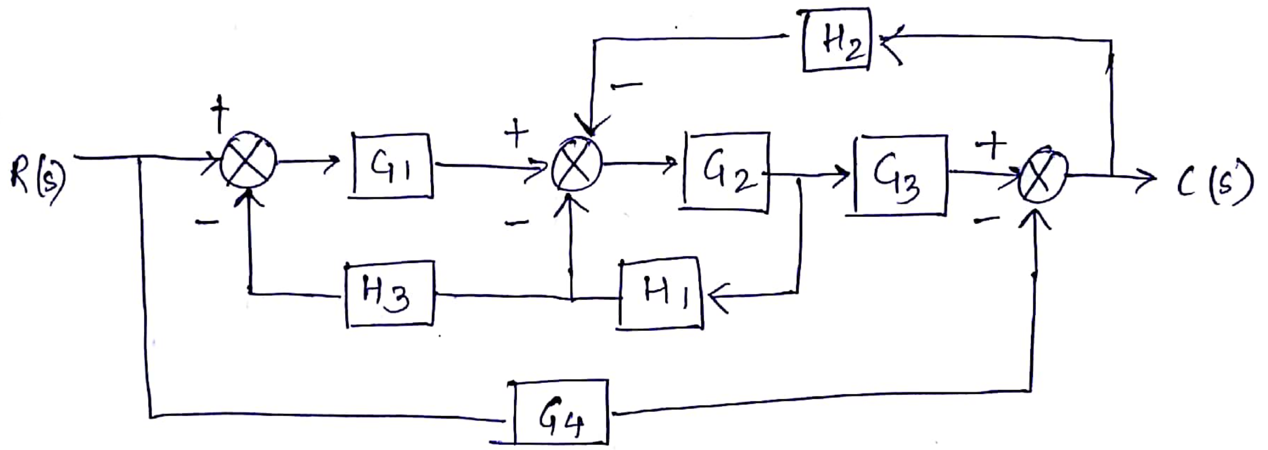
Nyquist stability criterion :-

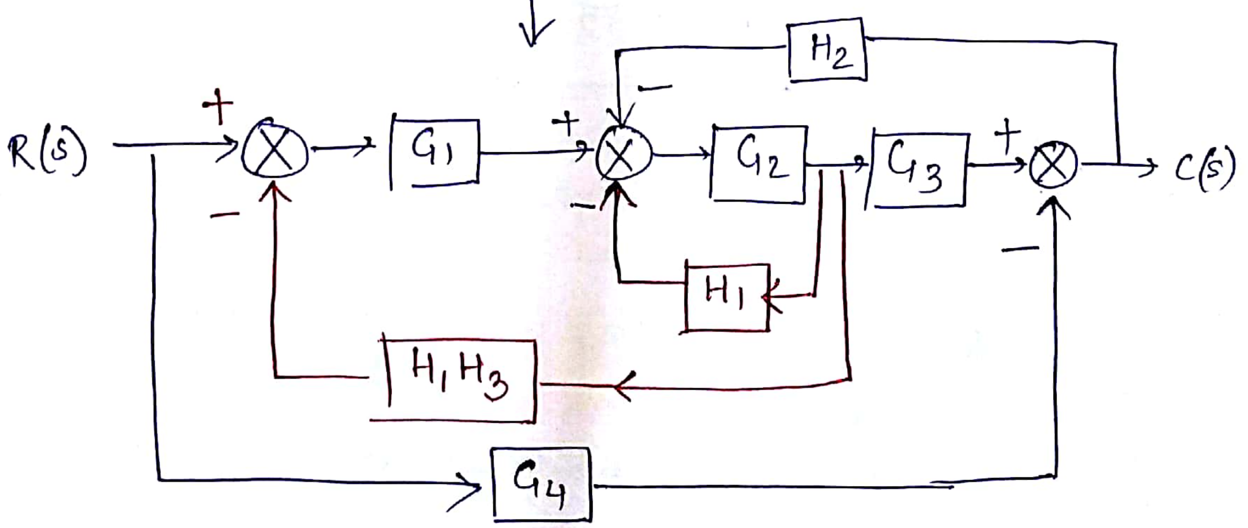
It states that the no. of encirclement about the critical point i.e. $-1+j0$ must be equal to poles of the characteristic equation which are the open loop transfer function poles in the right half s-plane

2M

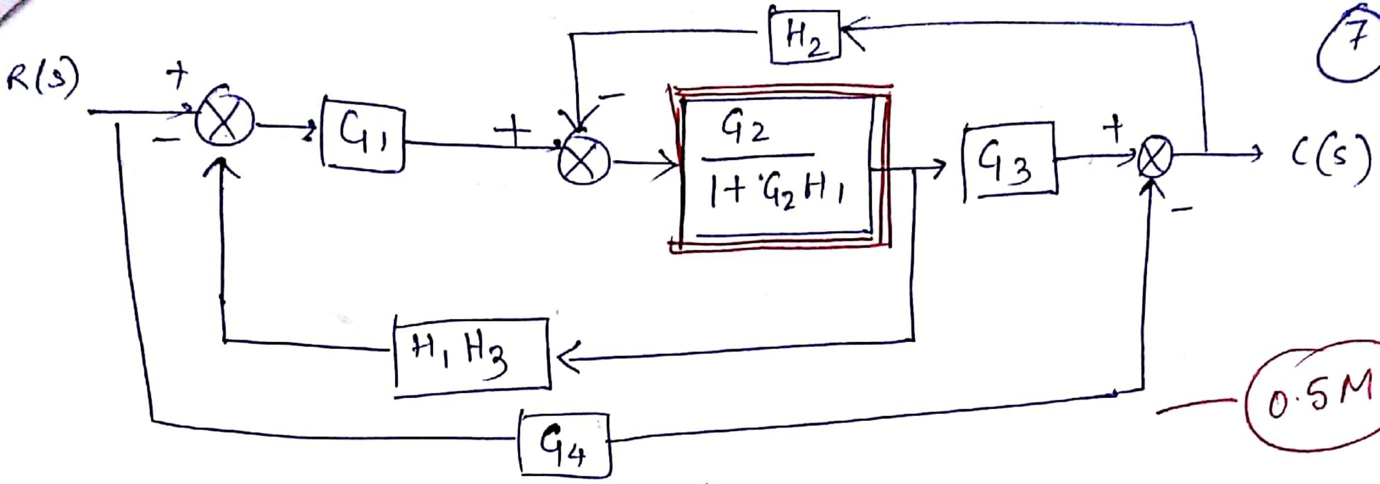
i.e $\boxed{N = P}$

Ans 2 (a)

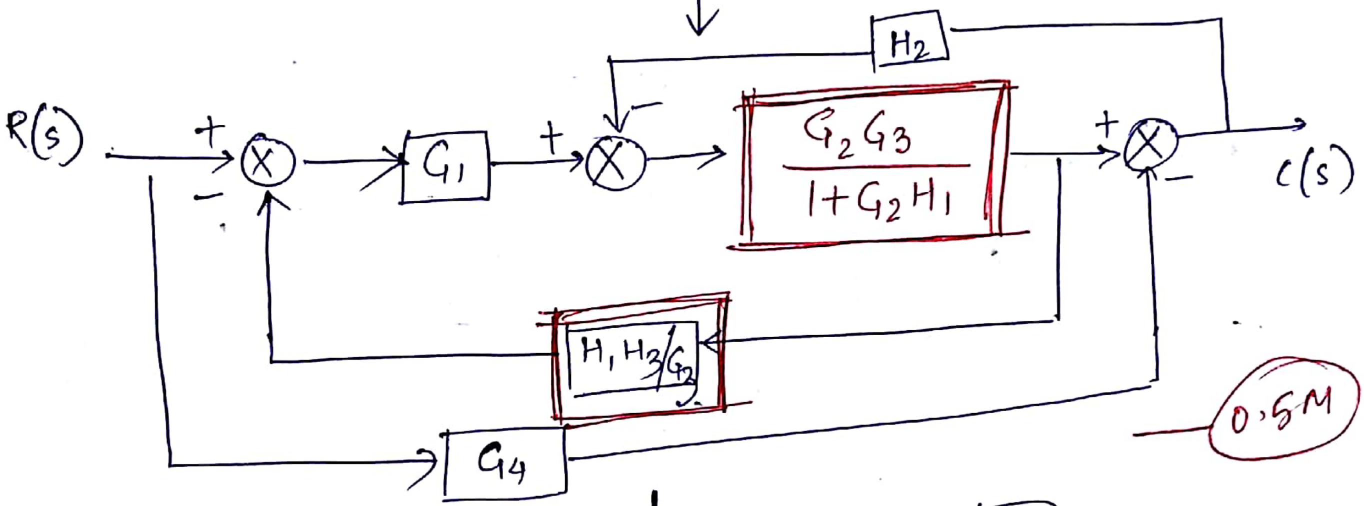




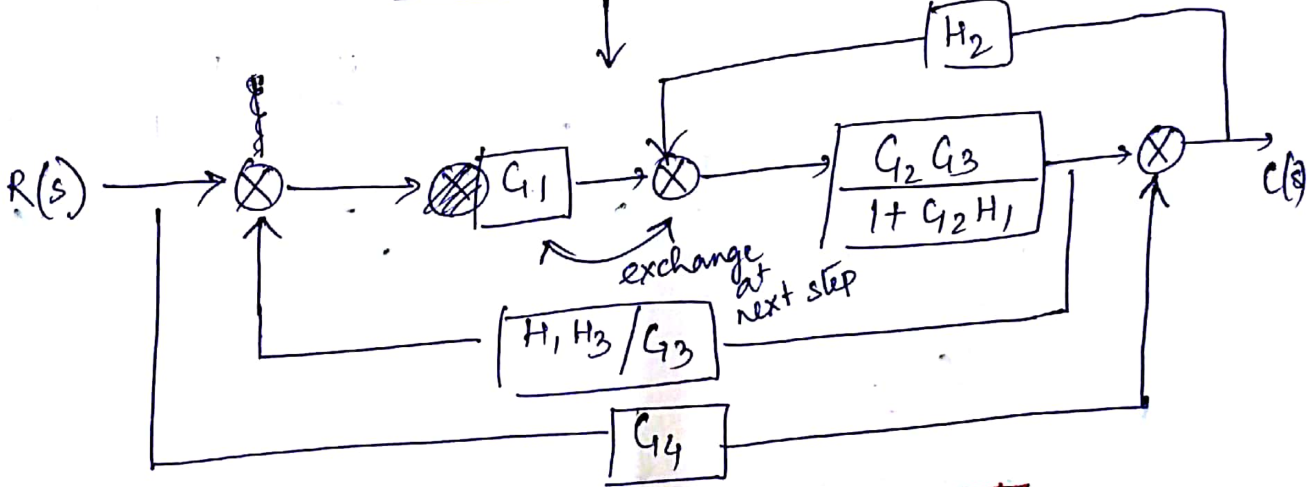
0.5 M

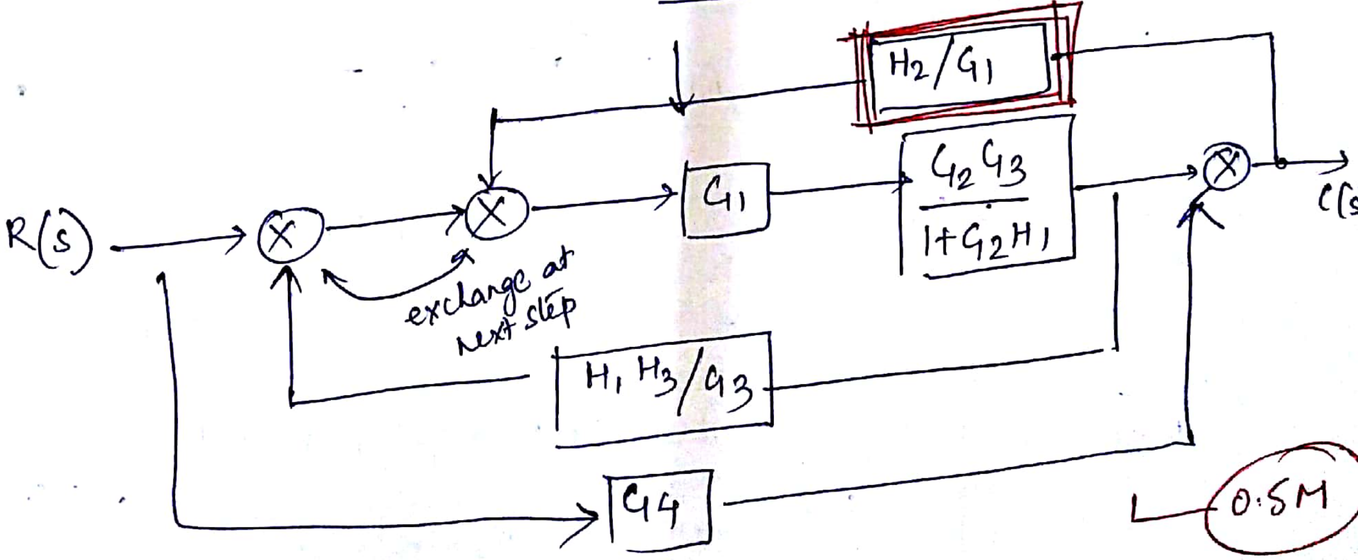

R(s)
+
−
G1
+
−
H2
$\frac{G_2}{1+G_2H_1}$
G3
+
−
C(s)
H1 H3
G4

0.5M

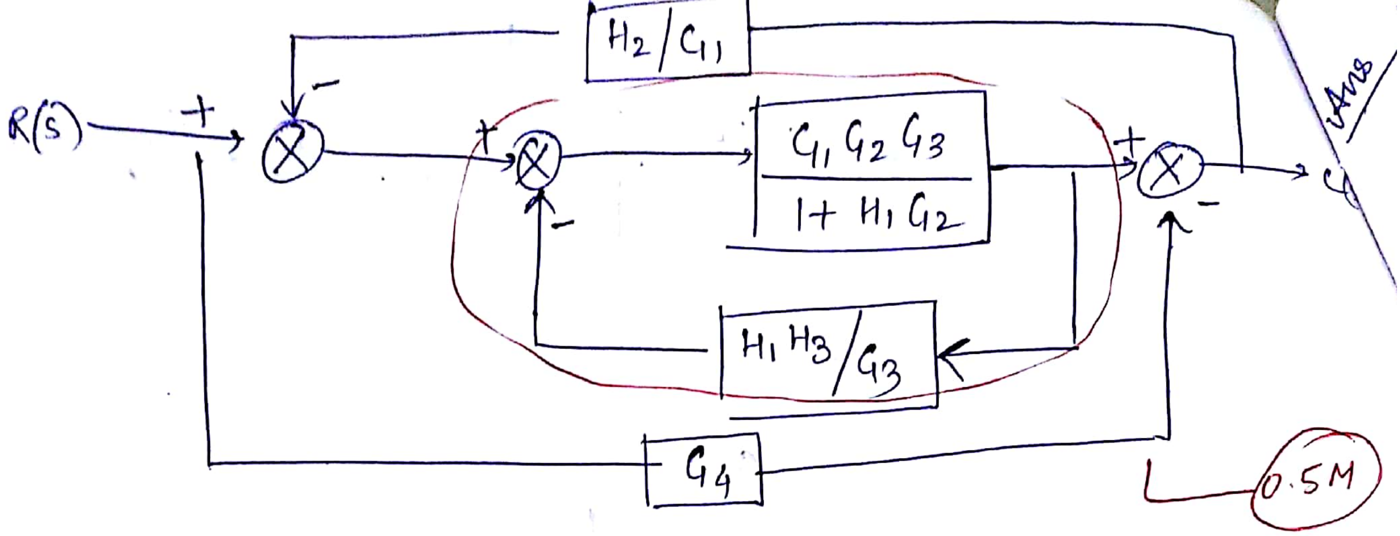

R(s)
+
−
G1
+
−
H2
$\frac{G_2G_3}{1+G_2H_1}$
+
−
C(s)
H1 H3/G3
G4

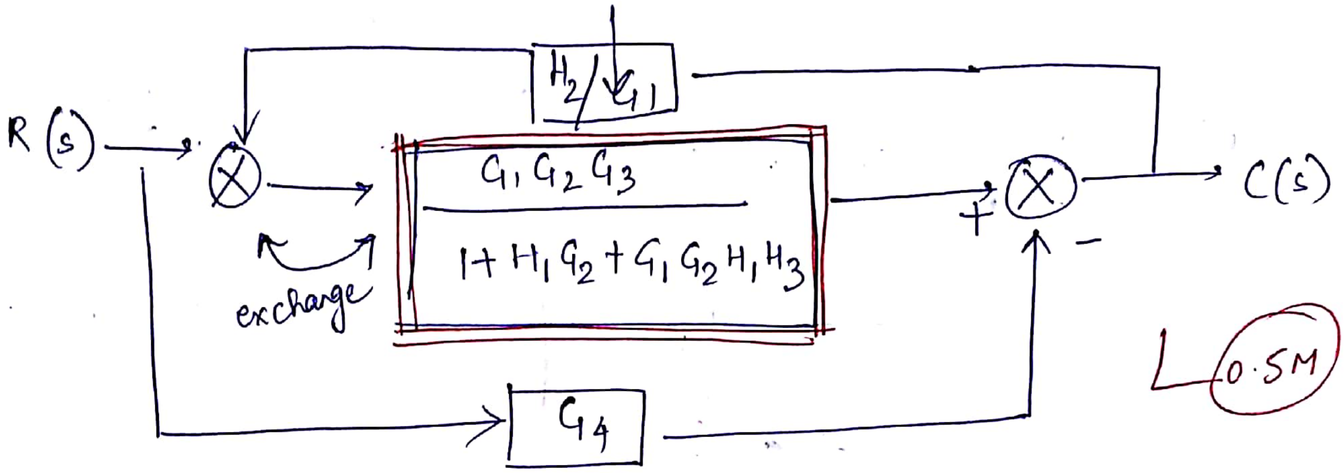
0.5M

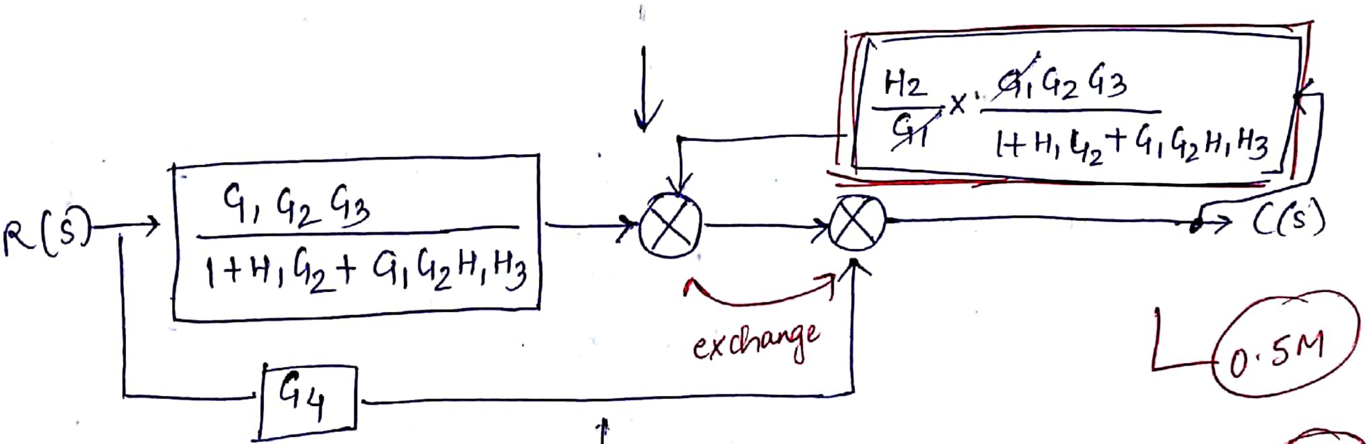

R(s)
G1
H2
$\frac{G_2G_3}{1+G_2H_1}$
C(s)
exchange at next step
H1 H3/G3
G4

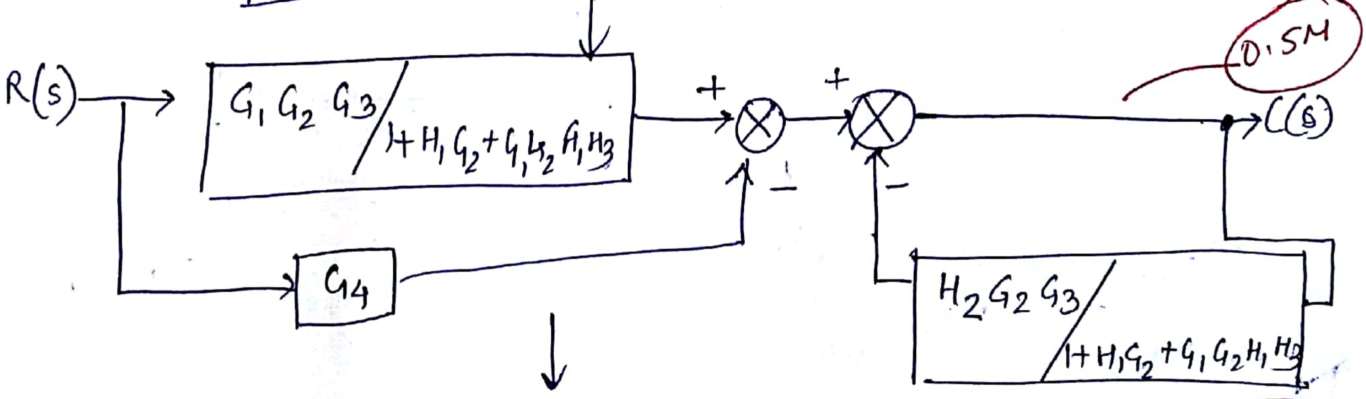

R(s)
H2/G1
G1
$\frac{G_2G_3}{1+G_2H_1}$
C(s)
exchange at next step
H1 H3/G3
G4

0.5M







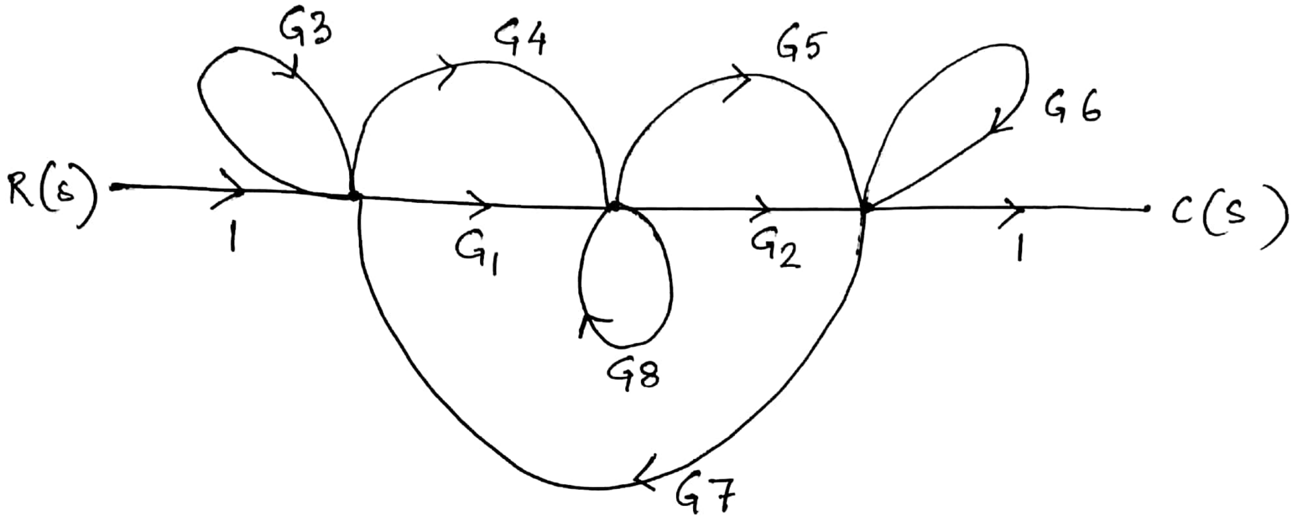


R(s) —
$$\frac{G_1G_2G_3 - G_4\left(1+H_1G_2+G_1G_2H_1H_3\right)}{1+H_1G_2+G_1G_2H_1H_3+H_2G_2G_3}$$
→ C(s)

1M

TF Ans

Ans 2 (b)



$$TF = \frac{G_1G_2 + G_4G_5 + G_4G_2 + G_1G_5}{1 - (G_3 + G_6 + G_8 + G_4G_5G_7 + G_4G_2G_7 + G_1G_2G_7 + G_1G_5G_7) + (G_3G_6 + G_3G_8 + G_6G_8) - G_3G_6G_8}$$

[forward path = (4) 1M

2 non-touching loops = 1M
+ (loops)
3 non-touching loops = 1M

Δ ——— 1M

TF ——— 1M