

THE LNM INSTITUTE OF INFORMATION TECHNOLOGY
 DEPARTMENT OF MATHEMATICS
 MTH5152: GRAPH THEORY(GT)
 MID SEMESTER EXAM

14uecos1

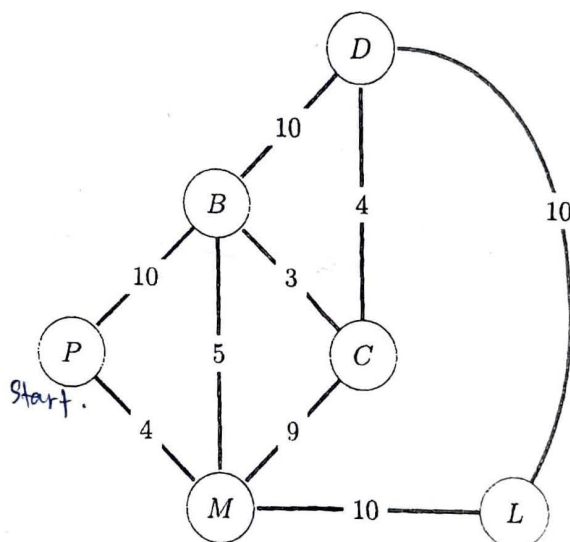
Maximum Time: 2PM-3.30PM

Date: 28/02/2020

Maximum Marks: 30

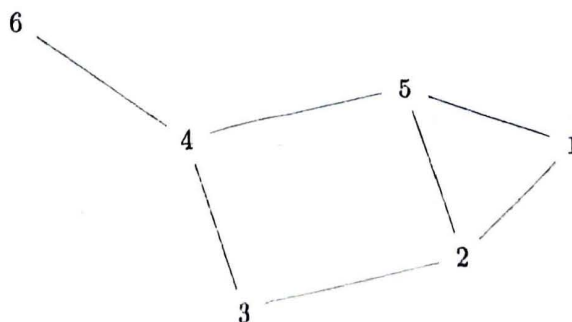
Instruction: You should attempt all questions. Your writing should be legible and neat. Marks awarded are shown next to the question.

1. Let v be a vertex of a connected graph G . If v is a cut-vertex, then prove that there exists a partition of $V - \{v\}$ into subsets U and W such that for any points $u \in U$ and $w \in W$, the point v is on every $u - w$ path. [2 marks]
2. Draw the 3-cube, Q_3 . Find a spanning tree of Q_3 and calculate the number of branches and chords. [2 marks]
3. Define weighted graph. Apply Dijkstra's algorithm on the following graph: [4 marks]

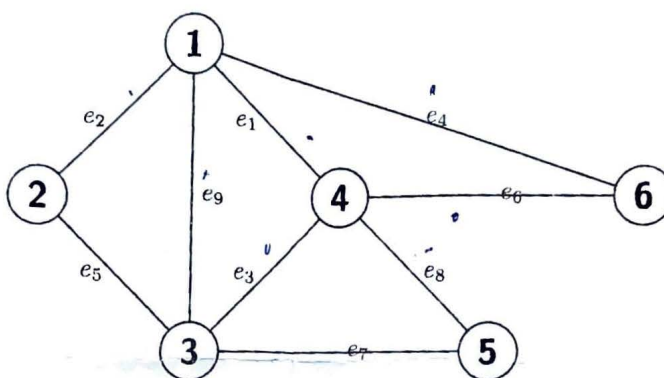


4. Define Hamiltonian circuit. If G is a simple graph with number of vertices $n (\geq 3)$, and if $\deg(v) + \deg(w) \geq n$ for every pair of non-adjacent vertices v and w , then prove that G is Hamiltonian. [3 marks]

5. Define line graph. Find line graph of the following graph: [2 marks]



6. Explain Konigsberg Bridge problem with figure, draw its Graph. Provide edge-disjoint Unicursal components of this graph which together covers all the edges of the graph. [3 marks]
7. Prove that the maximum number of edges in a complete bipartite graph of n vertices is $\frac{n^2}{4}$. [2 marks]
8. Let G be a connected graph. Prove that an edge e is a bridge in G if and only if e does not belong to any circuit of G . [2 marks]
9. A graph G is a tree if and only if there is one and only one path between any two vertices of G . [2 marks]
10. Is the following graph Eulerian?



Apply Fleury's algorithm and find an Euler circuit of this graph.

[5 marks]

11. Prove that every closed odd walk contains an odd circuit. Hence prove that, if a graph has no odd circuits, then it is always bipartite. [3 marks]

*****All the best.*****