The LNM Institute of Information Technology
Jaipur, Rajasthan

# End Semester Exam

MATH-III,, 21st Nov, 2016

**Part-I**

Time: 45 minutes,, Maximum Marks:10

**Name:**________________________ **Roll No.:**________________________

**Note:** Each question carry 2 marks. Overwriting will be treated as a wrong answer. Use only the last page of main answer sheet for rough work and calculation. Write down the final answer, no marks for formula or incomplete solution.

1. The PDE which characterizes the surfaces

$$F(xy, x+y-z) = 0$$

is ________________________.

Solution: $xp - yq = x - y$ (Here $u = xy,\ v = x+y-z,\ P = \frac{\partial(u,v)}{\partial(y,z)} = -x,\ Q = \frac{\partial(u,v)}{\partial(z,x)} = y,\ R = \frac{\partial(u,v)}{\partial(x,y)} = y - x$)

2. $(a)$ The classification (elliptic, parabolic, or hyperbolic) of the PDE

$$xu_{xx} + (x-y)u_{xy} - yu_{yy} = 0 \text{ (i) along x = y and (ii) x = 0, y = 0}$$

is ________________________________________________________________ type.

$(b)$ Tick Appropriately: The PDE $u_{xx}u_{xxx} + (u_{yy})^4 = 0$ is first/second/third/forth order and linear/semi-linear/quasi-linear/non-linear PDE.

Solution: (a) (i) Hyperarabolic if $(x, y) \neq (0, 0)$ and parabolic for $(x, y) = (0, 0)$ (ii) Parabolic if $x = 0,\ y = 0$ (Here $b^2 - 4ac = (x-y)^2 + 4xy = (x+y)^2$)
(b) Third order quasi-linear PDE

3. Solution for following PDE

$$\begin{aligned} u_{tt} - 4u_{xx} &= 0, & 0 < x < \infty,\ t > 0 \\ u(x,0) &= \cos x, & u_t(x,0) = 0, 0 \leq x < \infty, \\ u(0,t) &= 0, & t \geq 0. \end{aligned}$$

is $u(x,t) = \begin{cases} \cos x \cos 2t & x \geq 2t \\ -\sin x \sin 2t & x \leq 2t \end{cases}$

4. Solution of the wave equation (Hint: Duhamel's principle):

$$\begin{aligned} u_{tt} &= u_{xx} + t, & -\infty < x < \infty, t > 0, \\ u(x,0) &= u_t(x,0) = 0, & -\infty < x < \infty. \end{aligned}$$

is $\frac{t^3}{6}$

5. Solution for the following Laplace equation:

$$\begin{aligned} u_{xx} + u_{yy} &= 0, & 0 < x, y < 2\pi, \\ u(x,0) = x,\ 0 \leq x &\leq 2\pi, & u(x, 2\pi) = 0,\ 0 \leq x \leq 2\pi, \\ u(0,y) = u(2\pi, y) &= 0, & 0 \leq y \leq 2\pi. \end{aligned}$$

is $u(x,y) = \displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n 4 \sin(\frac{nx}{2}) \sinh[\frac{n(y-2\pi)}{2}]}{n \sinh(n\pi)}$