LNMIIT
The LNM Institute of Information Technology

LNMIIT/B.Tech/C/IC/2017-18/ODD/MTH102/MT

The LNM Institute of Information Technology, Jaipur
Mathematics-I
Mid Term

Duration: 60 mins.

Max.Marks: 20

Name: [redacted]

Roll No.: [redacted]

PART-B

NOTE: You should attempt all questions. Your writing should be legible and neat. Marks awarded are shown next to the question. **Start a new question on a new page and answer all its parts in the same place.** Please make an index showing the question number and page number on the front page of your answer sheet in the following format.

Question No.				
Page No.				

1. (a) State Archimedean Property. If three real numbers a, x, and y satisfy the inequalities
$$a \le x \le a + \frac{y}{n}$$
for every integer $n \ge 1$, using Archimedean property show that $x = a$. [2]

   (b) If $0 < \alpha < 1$ and (x_n) is a sequence satisfying
$$|x_{n+2} - x_{n+1}| \le \alpha |x_{n+1} - x_n| \text{ for all } n \in \mathbb{N},$$
then show that (x_n) is a Cauchy sequence. [3]

2. (a) Establish the convergence/divergence of the series $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2}$. [2]

   (b) Let (a_n) and (b_n) be two sequences such that
$$a_n = b_{n+1} - b_n \text{ for all } n \ge 1,$$
then show that the series $\displaystyle\sum_{n=1}^{\infty} a_n$ converges if and only if $\displaystyle\lim_{n\to\infty} b_n$ exists and
$$\sum_{n=1}^{\infty} a_n = -b_1 + \lim_{n\to\infty} b_n.$$
[3]

3. (a) Suppose the third derivative of f is less than 3 in absolute value. Write the second degree Taylor polynomial $P_2(x)$ for f with center 0. If we approximate $f(x)$ by $P_2(x)$, then estimate the error in the approximation when $|x| \le 0.1$. [2]

   (b) Let $f(x)$ and $g(x)$ be continuous on $[a, b]$, differentiable on (a, b) and let $f(a) = f(b) = 0$. Then show that $g'(c)f(c) + f'(c) = 0$ for some $c \in (a, b)$. [3]

4. (a) Define lower and upper Riemann integrals of a bounded function $f : [a, b] \longrightarrow \mathbb{R}$ from a closed interval $[a, b]$ to $\mathbb{R}$. Let $f : [0, 1] \to \mathbb{R}$ defined by $f(x) = \begin{cases} 1, & x \in \mathbb{Q} \\ 0, & x \notin \mathbb{Q} \end{cases}$. Show that $f(x)$ is not Riemann integrable. [2]

   (b) Let $f(x) = \dfrac{x^2 - 4}{x - 1}$, for $x \neq 1$. Find the intervals where the function f is decreasing / increasing. Find local maxima/minima (if any). Find the intervals where the graph of the function is concave up / concave down (convex / concave). Find the points of inflection (if any). [3]