THE LNM INSTITUTE OF INFORMATION TECHNOLOGY
DEPARTMENT OF MATHEMATICS
MATHEMATICS-III & MTH213
MID TERM

Time: 90 minutes | Date: 21/09/2017 | Maximum Marks: 50

Note: Usual notations are used. Attempt all questions. Your writing should be legible and neat. **If you use a theorem, make sure to state it.** Marks awarded are shown next to the question. Please make an index showing the question number and page number on the front page of your answer sheet in the following format, otherwise you may be sanctioned by the deduction of **2 marks.**

Question No.				
Page No.				

1. (a) Simplify $\left(\frac{-1-i}{\sqrt{2}}\right)^{101}$ in terms of $a+ib$, where $a, b \in \mathbb{R}$. [3]

   (b) Find the principal value of $(1+i)^i$. [4]

2. (a) Discuss the continuity of the complex valued function $f(z)$ at $z=0$ defined as:

$$f(z) = \begin{cases} 0 & \texttt{if } z = 0 \\ \dfrac{(\mathrm{Re}(z))(\mathrm{Im}(z))}{2|z|^2} & \texttt{if } z \neq 0 \end{cases}$$

[3]

   (b) Show that $f(z) = i\cos z$ is an entire function. [4]

3. (a) **Prove or Disprove:** The complex valued function $f(z) = |Re(z)\ Im(z)|^{\frac{1}{2}}$ is differentiable at $z=0$. [3]

   (b) Find the region where the function $f(z) = x^2 + iy^2$ is (i) differentiable (ii) analytic. [4]

4. (a) Let $f(z) = z^3 + 2z + 4$ is defined on a simply connected domain D. Then, prove that integration of $f(z)$ is independent of the path in D. [4]

   (b) $f(z) = u(x,y) + iv(x,y)$ is analytic in a domain D and $u(x,y)$ is constant in D, than $f(z)$ is constant in D. [4]

5. (a) Use the ML-inequality to prove
$$\left|\int_{|z|=R} \frac{Logz}{2z^4}dz\right| \leq \pi\left(\frac{\pi + \ln R}{R^3}\right), \quad R > 1.$$
[4]

   (b) Find the value of the complex integration $I = \oint_C \frac{\tan\frac{z}{2}}{z^2+4z+3}dz$, where C is the square with vertices at (0, 0), (2, 0), (2, 2), (0, 2) traversed in the anti-clockwise direction. [3]

6. Define isolated singularity. Find the value of the complex integration $I = \oint_C \frac{2\cos z}{(z^2-\pi^2)}dz$, where C is the closed curve (a) $|z-1| = 3$ traversed in the clockwise direction (b) $|z+1| = 3$ traversed in the counter-clockwise direction. [7]

7. Define removable singularity and pole of a complex valued function. Find all the possible singular points and their categories (their types) of $f(z) = \frac{z\cos\frac{\pi z}{2}}{(z-1)\sin^4 z}$. [7]