

End Term Exam (Part-1)

MATHEMATICS-I

SOLUTIONS (Series: A)

1. Consider the sequence  $a_n = \frac{n}{n+2}$ . The minimum  $n_0$  such that  $|a_n - 1| < \frac{1}{100}$  for  $n \geq n_0$ , is  
(A) 198, (B) 199, (C) 200, (D) 201.

Ans. (B)

2. Given  $\epsilon > 0$ , the largest  $\delta$  which fits the definition of continuity of the function

$$f(x) = \begin{cases} \frac{x+3}{2} & \text{if } x \leq 1 \\ \frac{7-x}{3} & \text{if } 1 \leq x \end{cases} \quad \text{at } x = 1 \text{ is}$$

(A)  $\epsilon/2$ , (B)  $\epsilon/3$ , (C)  $2\epsilon$ , (D)  $3\epsilon$ .

Ans. (C)

3. The function  $f(x) = e^{-\frac{1}{|x|}}$  for  $x \neq 0$  and  $f(0) = 0$  is concave up in  $m$  open intervals and concave down in  $n$  open intervals. Then  $(m, n)$  equals.

(A)  $(0, 1)$ , (B)  $(1, 0)$ , (C)  $(1, 2)$ , (D)  $(2, 1)$ .

Ans. (C)

4. The sequence  $a_n = \sum_{k=1}^n (-1)^k$  is

(A) bounded but not convergent, (B) both bounded and convergent,  
(C) convergent but not bounded, (D) neither bounded nor convergent.

Ans. (A)

5. Let  $f : [0, 1] \rightarrow \mathbb{R}$  be a continuous function. Let  $P = \left\{0, \frac{1}{10}, \frac{2}{10}, \dots, 1\right\}$  and  $Q = \left\{0, \frac{1}{100}, \frac{2}{100}, \dots, 1\right\}$

be the partitions of  $[0, 1]$ . Then

(A)  $L(P, f) \leq L(Q, f) \leq U(Q, f) \leq U(P, f)$ , (B)  $L(Q, f) \leq L(P, f) \leq U(P, f) \leq U(Q, f)$ ,  
(C)  $L(Q, f) \leq L(P, f)$  and  $U(Q, f) \leq U(P, f)$ , (D) None of above.

Ans. (A)

6. The curve  $r = 2 \cos \theta, 0 \leq \theta \leq \pi$  represents a

(A) circle, (B) cardioid, (C) lemniscate, (D) ray.

Ans. (A)

7. Parametric equations of the line through  $P(-1, 4, 2)$  and in the direction of  $\vec{v} = (1, 2, 3)$  is:

\_\_\_\_\_.

Ans.  $x = -1 + t, y = 4 + 2t, z = 2 + 3t$ .

8. Let  $f$  be a scalar field defined on  $R^2$  and suppose directional derivatives of  $f$  exist for all directions. Then  $f$  is continuous. TRUE or FALSE \_\_\_\_\_.

Ans. FALSE

9. The value of the double integral  $\int \int_D e^{-(x^2+y^2)} dA$ , where  $D$  is the region between the two circles  $x^2 + y^2 = 1$  and  $x^2 + y^2 = 4$  is  
(A)  $\pi(e^{-1} - e^{-3})$ , (B)  $\pi(e^{-1} - e^{-4})$ , (C)  $\pi(e^{-2} - e^{-3})$ , (D)  $\pi(e^{-2} - e^{-4})$ .

Ans. (B)

10. The value of the line integral  $\int_C x - y \, ds$ , where  $C$  is the line segment from  $(1, 3)$  to  $(5, -2)$  is: \_\_\_\_\_.

Ans.  $\frac{5}{2}\sqrt{41}$ .