The LNM Institute of Information Technology
Jaipur, Rajasthan

End Term Exam (Part-1)

MATHEMATICS-I

SOLUTIONS (Series: B)

1. Let $f : [0,1] \to \mathbb{R}$ be a continuous function. Let $P = \left\{0, \frac{1}{10}, \frac{2}{10}, \cdots, 1\right\}$ and $Q = \left\{0, \frac{1}{100}, \frac{2}{100}, \cdots, 1\right\}$ be the partitions of $[0,1]$. Then
(**A** $L(P,f) \le L(Q,f) \le U(Q,f) \le U(P,f)$, (**B**) $L(Q,f) \le L(P,f) \le U(P,f) \le U(Q,f)$,
(**C**) $L(Q,f) \le L(P,f)$ and $U(Q,f) \le U(P,f)$, (**D**) None of above.

Ans. (A)

2. The function $f(x) = e^{-\frac{1}{|x|}}$ for $x \neq 0$ and $f(0) = 0$ is concave up in m open intervals and concave down in n open intervals. Then (m,n) equals.
(**A**) $(0,1)$, (**B**) $(1,0)$, (**C**) $(1,2)$, (**D**) $(2,1)$.

Ans. (C)

3. Consider the sequence $a_n = \dfrac{n}{n+2}$. The minimum n_0 such that $|a_n - 1| < \frac{1}{100}$ for $n \ge n_0$, is
(A) 198, **(B)** 199, **(C)** 200, **(D)** 201.

Ans. (B)

4. The sequence $a_n = \sum_{k=1}^{n} (-1)^k$ is
(A) bounded but not convergent, **(B)** both bounded and convergent,
(C) convergent but not bounded, **(D)** neither bounded nor convergent.

Ans. (A)

5. Given $\epsilon > 0$, the largest δ which fits the definition of continuity of the function

$$f(x) = \begin{cases} \frac{x+3}{2} & \text{if } x \le 1 \\ \frac{7-x}{3} & \text{if } 1 \le x \end{cases} \quad \text{at } x = 1 \text{ is}$$

(**A**) $\epsilon/2$, (**B**) $\epsilon/3$, (**C**) 2ϵ, (**D**) 3ϵ.

Ans. (C)

6. Parametric equations of the line through $P(-1,4,2)$ and in the direction of $\overrightarrow{v} = (1,2,3)$ is: _____.

Ans. $x = -1 + t$, $y = 4 + 2t$, $z = 2 + 3t$.

7. The curve $r = 2\cos\theta, 0 \le \theta \le \pi$ represents a
(A) circle, (B) cardioid, (C) lemniscate, (D) ray.

Ans. (A)

8. **Let f be a scalar field defined on R^2 and suppose directional derivatives of f exist for all directions. Then f is continuous. TRUE or FALSE _____.**

Ans. FALSE

9. **The value of the line integral $\int_C x - y\, ds$, where C is the line segment from $(1, 3)$ to $(5, -2)$ is: _____.**

Ans. $\frac{5}{2}\sqrt{41}$.

10. **The value of the double integral $\int\int_D e^{-(x^2+y^2)} dA$, where D is the region between the two circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$ is**
(A) $\pi(e^{-1} - e^{-3})$, **(B)** $\pi(e^{-1} - e^{-4})$, **(C)** $\pi(e^{-2} - e^{-3})$, **(D)** $\pi(e^{-2} - e^{-4})$.

Ans. (B)