Mid Semester Exam (Make up) (b)

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Subject: Cognitive Radio

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Ans 1(a)

$$Y_i = \sum_{j=1}^{N} g(r_j) P_j h_j$$

r_j = distance between the j^{th} active secondary Tx & the primary Rx

$(R \le r_1 \le r_2 \le \dots \le r_\infty)$

$g(r)$ = Path loss power gain at a distance r from the transmitter of the signal

h_j = normalized composite shadowing and Nakagami fading with pdf $f_h(x)$

P_j = Transmitter power

(b) (i)

$\because P = kTB$.

$\Rightarrow T_I(f_c, B) = \frac{P_I(f_c, B)}{kB}$

$P_I(f_c, B)$ = Avg interference power

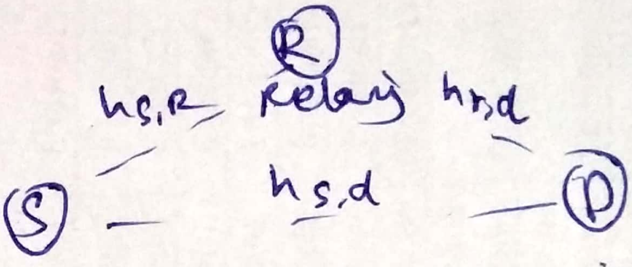
k = Boltzmann's const = 1.38×10^{-23} J/K.

(ii) if N_0 is tx power of secondary user & γ is interference, then

$N_0 = (k/\gamma) B (T_L(f_c) - T_I(f_c, B))$

→ P.T.O.

Ans 2(a) 3-node relayed system



Assuming both S & R transmitting same power i.e. $P_1 = P_2 = P$.

Phase 1 : Source sends s/g to destination & the relay.

$$y_{s,d} = \sqrt{P} \cdot h_{s,d} \cdot x + n_{s,d}$$

$$\& \quad y_{s,r} = \sqrt{P} \cdot h_{s,r} \cdot x + n_{s,r}$$

Given that; $h_{s,d}$ & $h_{s,r}$ are channel coefficients as zero mean gaussian r.v. with variances $\delta_{s,d}^2$ & $\delta_{s,r}^2$ resp.

And; $n_{s,d}$, $n_{s,r}$ are complex gaussian r.v. with variance N_0

Phase 2 : Relay sends signal to destination.

$$y_{r,d} = h_{r,d} \cdot \underbrace{q(y_{s,r})}_{\text{depends on what type of processing is employed at the relay. (A/F or D/F)}} + n_{r,d}$$

As given in question, for D commⁿ using AF protocol
then,

$$\beta_r = \frac{\sqrt{P}}{\sqrt{P \cdot |h_{s,r}|^2 + N_0}}$$

Transmitted signal from relay is $\beta_r \cdot y_{s,r}$

Therefore,

$$y_{r,d} = \beta_r \cdot y_{s,r} \cdot h_{r,d} + n_{r,d}$$

$$= \frac{\sqrt{P}}{\sqrt{P \cdot |h_{s,r}|^2 + N_0}} y_{s,r} \cdot h_{r,d} + n_{r,d}$$

$$= \frac{\sqrt{P}}{\sqrt{P \cdot |h_{s,r}|^2 + N_0}} \sqrt{P} \cdot h_{s,r} \cdot x \cdot h_{r,d} + n'_{r,d}.$$

where $n'_{r,d} = \frac{\sqrt{P}}{\sqrt{P \cdot |h_{s,0}|^2 + N_0}} \cdot h_{r,d} \cdot n_{s,r} + n_{r,d}$

Ans 2(b) Capacity = ? if $SNR_{P1} = SNR_{P2} = 10dB$.

~~$C = W \log_2 (1 + S/N)$.~~

Phase 1

$$SNR_{s,d} = T\, |h_{s,d}|^2 , \quad T = \frac{P}{N_0}$$

Phase 2 SNR.

$r = r_1 + r_2$ $10dB = 10 \log(x)$

$x = 10.$

$\text{Capacity} = w \log_2 (1 + S/N)$

$= w \log (1+1)$

$= w \log (2)$

$= 0.3010\, w.$

$\longrightarrow$ P.T.O.

Ans 3(a) The complex envelope of a Rician fading channel is given by

$$E = E_0 + \sum_{n=1}^{N} E_n e^{j\theta_n}$$

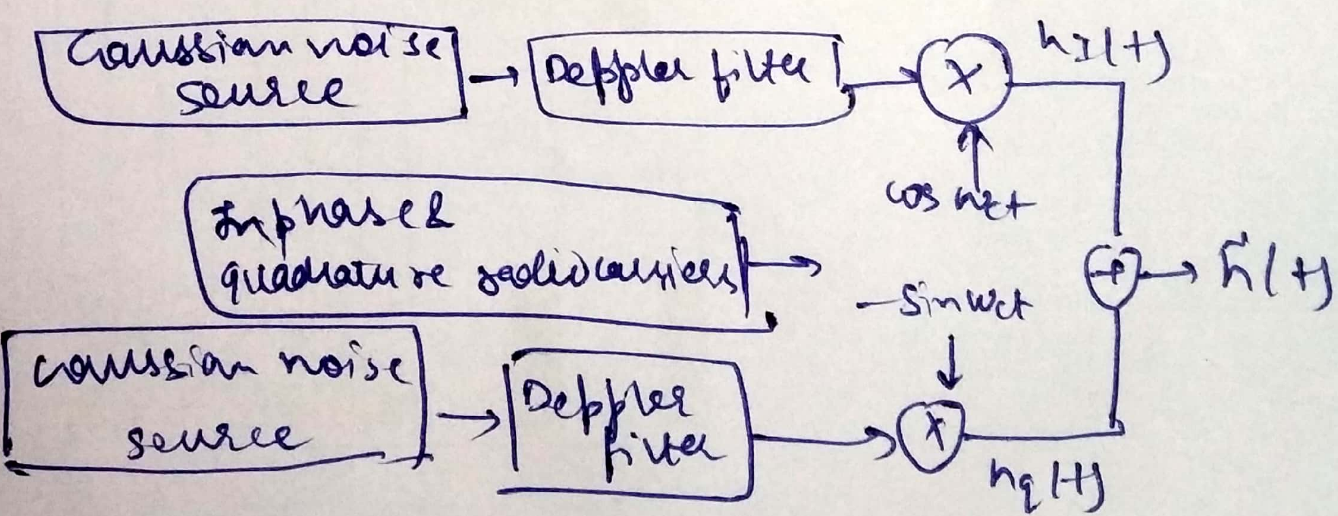
This rician channel is characterized by the Rician factor K which represent the strength of the direct path, can be defined as

$$K = \frac{|E_0|^2}{\text{Power in the scattered paths } \left(\sum_{n=1}^{N} |E_n|^2\right)}$$

$$K_{dB} = 10\log_{10}\left(\frac{|E_0|^2}{\sum_{n=1}^{N} |E_n|^2}\right)$$

Ans 3(b) :- Lowpass (baseband) equivalent model of a Rayleigh channel

Baseband in-phase channel impulse response



Ans 5(a) Considering spectrum sensing as a binary hypothesis problem

Key parameters

False alarm (FA) = an idle ch is detected as busy

Missed detection (MD) = busy channel detected as idle

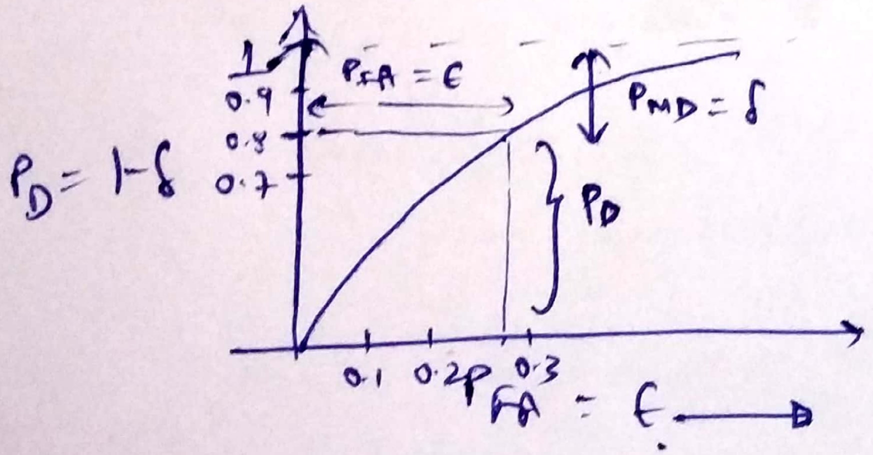
therefore

$P_{FA} = \epsilon = \text{Prob}\,[\text{decide } H1/H0]$

$P_{MD} = \delta = \text{Prob}\,[\text{decide } H0/H1]$

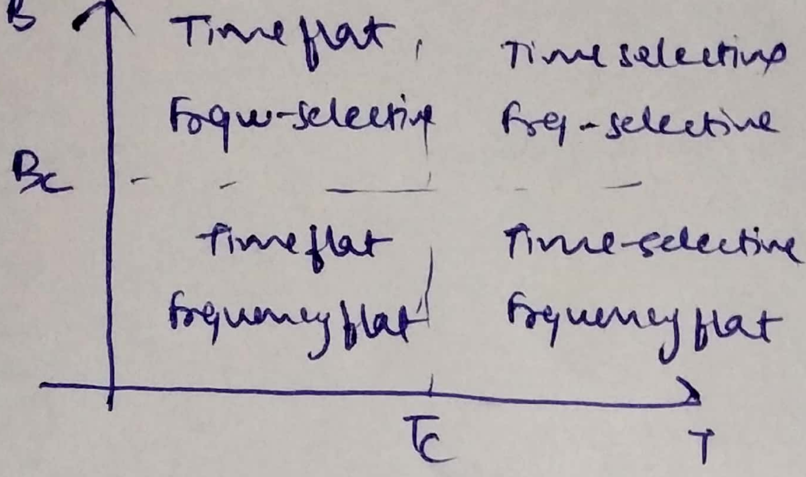
$P_D = \text{Prob of detection} = 1-\delta$

ROC (Receiver operating characterstics)



→ P.T.O.

Ans 5(b) Radio channel classification



we have two quantities here, namely coherance bandwidth (B_c) & coherance time (T_c) which describe channel behaviour for the tx s/g.

Hence, condition for both frequency flat & time flat is

if $\underbrace{B < B_c \ \& \ T < T_c}$.

Here $B_c \triangleq \dfrac{1}{2\pi\sigma_T}$

and $T_c \triangleq \dfrac{1}{f_m}$.

Ans 4(a) :-

~~Let~~ given,

$$x(t) = A_{cm}\cos(\omega_c t + \theta) - A_{sm}\sin(\omega_c t + \theta) \quad \text{— (1)}$$

If $x(t) = Re\{g(t)e^{j\omega_c t}\}$ — (2)

Then, $g(t)$ = complex envelope of $x(t)$

$= y(t) + j z(t)$ (let assume). — (3)

~~$\therefore g(t)e^{j\omega_c t} = y(t)(\cos\omega_c t + j\sin\omega_c t) + jz(t)(\cos\omega_c t + j\sin\omega_c t)$~~

~~$\therefore Re\{g(t)e^{j\omega_c t}\} = y(t)\cos\omega_c t - z(t)\sin(\omega_c t)$~~

~~or~~

$$x(t) = Re\{A_{cm}(\cos(\omega_c t + \theta) + j\sin\omega_c t) + jA_{sm}(\cos(\omega_c t + \theta) + j\sin\omega_c t)\}$$

$$= Re\{(A_{cm} + jA_{sm})e^{j\omega_c t}.e^{j\theta}\}$$

$$= Re\{\{(A_{cm} + jA_{sm})e^{j\theta}\}e^{j\omega_c t}\}. \quad \text{— (4)}$$

Comparing (2) with (4), we get

$$\boxed{\begin{aligned} g(t) &= (A_{cm} + jA_{sm})e^{j\theta} \\ &= \text{complex envelope of } x(t) \end{aligned}}$$

Ans 4(b)

$y = hs + n$, $s = \sqrt{P}$,

$n \sim N(0, \sigma_n^2)$

$h \rightarrow$ Rayleigh distributed

Received SNR = ? if second moment of $h = 8$