Assignment

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(16UEC110)

Ans 1: Mutual information:

The measure of the information transferred from a transmitter to a receiver in presence of noise.
Mutual information can be calculated as

$$\boxed{I(x_i, y_j) = \log \frac{P\left(\frac{x_i}{y_j}\right)}{P(x_i)}}$$

Channel capacity

It is a tight upper bound on the rate at which information can be reliably transmitted over a communications channel w/o any constraints on encoding or decoder complexity and delay.

Shannon defined the capacity of a single user channel in terms of the mutual information b/w the I/P & O/P of the channel.
If X & Y be the random variables for I/P & O/P of channel.
Then M.I. is $\quad I(X;Y) = \int P(x,y) \log\left(\frac{P(x,y)}{P(x)P(y)}\right)$

Shannon capacity. $\boxed{C = \max_{P(x)} I(X;Y)}$

Joint entropy

It measures the avg. infor. content of a pair of I/P & O/P symbols or the avg uncertainity of the commn system.

$$\boxed{H(X,Y) = \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} P(x_i, y_j) \log_2\left(\frac{1}{P(x_i, y_j)}\right) \text{ bits/symbol pair}}$$

Ans2 To find entropy of continuous random variable "X", s.t.

$$X \sim \mathcal{N}(0,N) \qquad \text{means } \mu=0,\ \sigma^2=N$$

Entropy is given by $H(X) = -\int_I p(x)\ln(p(x))\,dx$, $I=(-\infty,\infty)$

Therefore,

$$H(X) = -\int_{-\infty}^{\infty} p(x)\ln(p(x))\,dx ,$$

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}} = \frac{1}{\sqrt{2\pi N}}\, e^{-\frac{(x^2)}{2N}}$$

$$H(x) = -\int_{-\infty}^{\infty} \frac{e^{-\frac{x^2}{2N}}}{\sqrt{2\pi N}} \ln\left(\frac{e^{-\frac{x^2}{2N}}}{\sqrt{2\pi N}}\right) dx$$

from simple calculus

$$= \int_{-\infty}^{\infty} \frac{e^{-x^2/2N}}{\sqrt{2\pi N}} \left[\frac{1}{2}\log(2\pi N) + \frac{1}{2N}\cdot(x)^2\right] dx$$

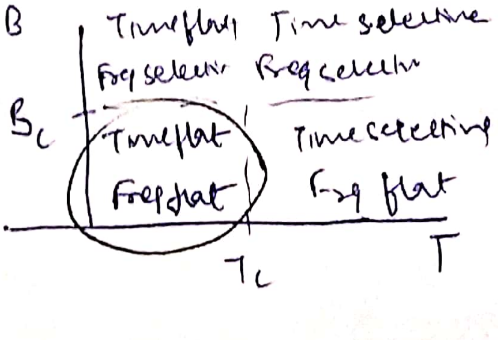
$$= \frac{1}{2}\log(2\pi N)\underbrace{\int_{-\infty}^{\infty} \frac{e^{-x^2/2N}}{\sqrt{2\pi N}}\,dx}_{=1} + \frac{1}{2N}\underbrace{\int_{-\infty}^{\infty} x^2\, \frac{e^{-x^2/2N}}{\sqrt{2\pi N}}\,dx}_{=N}$$

$$= \frac{1}{2}\log(2\pi N) + \frac{1}{2}$$

$$\boxed{H(x) = \frac{1}{2}\left[\log(2\pi N) + 1\right]}$$

Ans3 → If $T << T_c$ (coherence time), there is no time selective fading channel is flat in time

here also if $B << B_c$ (coherence BW), then freq flat



Ans 4 Doppler spread (Bd)

Time varying fading due to the motion of a scatter or the motion of the transmitter or receiver or both results in Doppler spread.

$$B_d = \frac{1}{T_c}$$, T_c = coherence time

$$T_c = \frac{9}{16\pi f_{rms}}$$, f_{rms} is rms value of max doppler shift.

$$f_{rms} = \frac{f_m}{\sqrt{2}}$$

$$f_m = \frac{v}{c} f_c \Rightarrow f_{rms} = \frac{v}{c\sqrt{2}} f_c$$

$f_c \Rightarrow$ ~~signal at~~ freq at which signal is sent

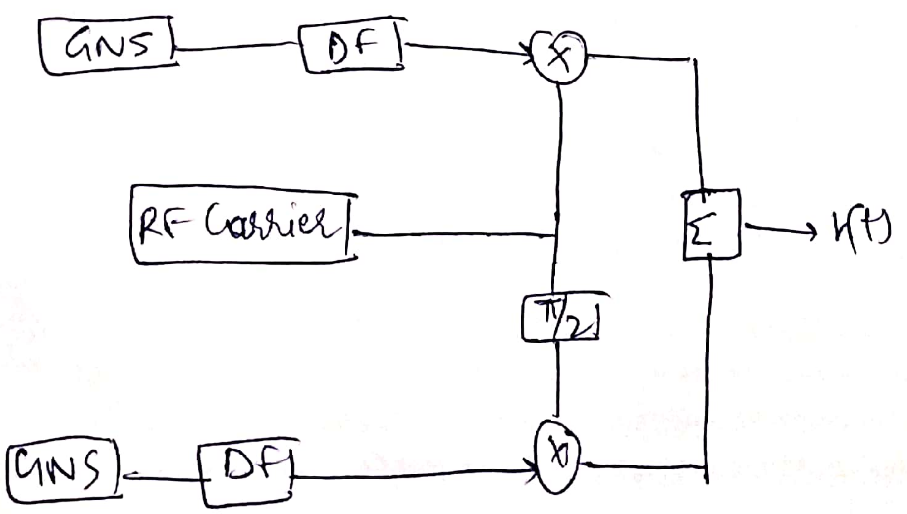
$c \Rightarrow$ speed of light

$v \Rightarrow$ speed of mobile

Therefore $B_d = \frac{16\pi f_{rms}}{9}$

$$\boxed{B_{d\,max} = \frac{16\pi\, v \cdot f_c}{9\sqrt{2}\, c}}$$

Ans 5 ⇒ baseband equivalent model of Rayleigh channel with Doppler.



Ans 6: Consider a SIMO channel with one transmit antenna & L recieving antenna.

$$y_\ell[m] = h_\ell\, x[m] + w_\ell[m] \qquad \ell = 1, 2, \dots, L$$

here, h_ℓ = find complex gain from transmit antenna to ℓ^{th} receiver antenna & $w_\ell[m]$ = Additive gaussian noise independent across antennas

P = avg energy / transmit symbol

$$\therefore \boxed{C = \log\left(1 + \frac{P\|h\|^2}{N_0}\right) \text{ bits/s/Hz}}$$

Ans 7:- $z = x(t) + j\,y(t)$, $x(t) \sim N(0,1)$, $y(t) \sim N(0,1)$

To find mean square value & variance of $|z|$

$$\boxed{\sigma_n^2 = 1}$$

$$T = \Sigma |w(n)|^2$$

$$E(T_z) = \Sigma |w_x(n) + w_y(n)|^2$$

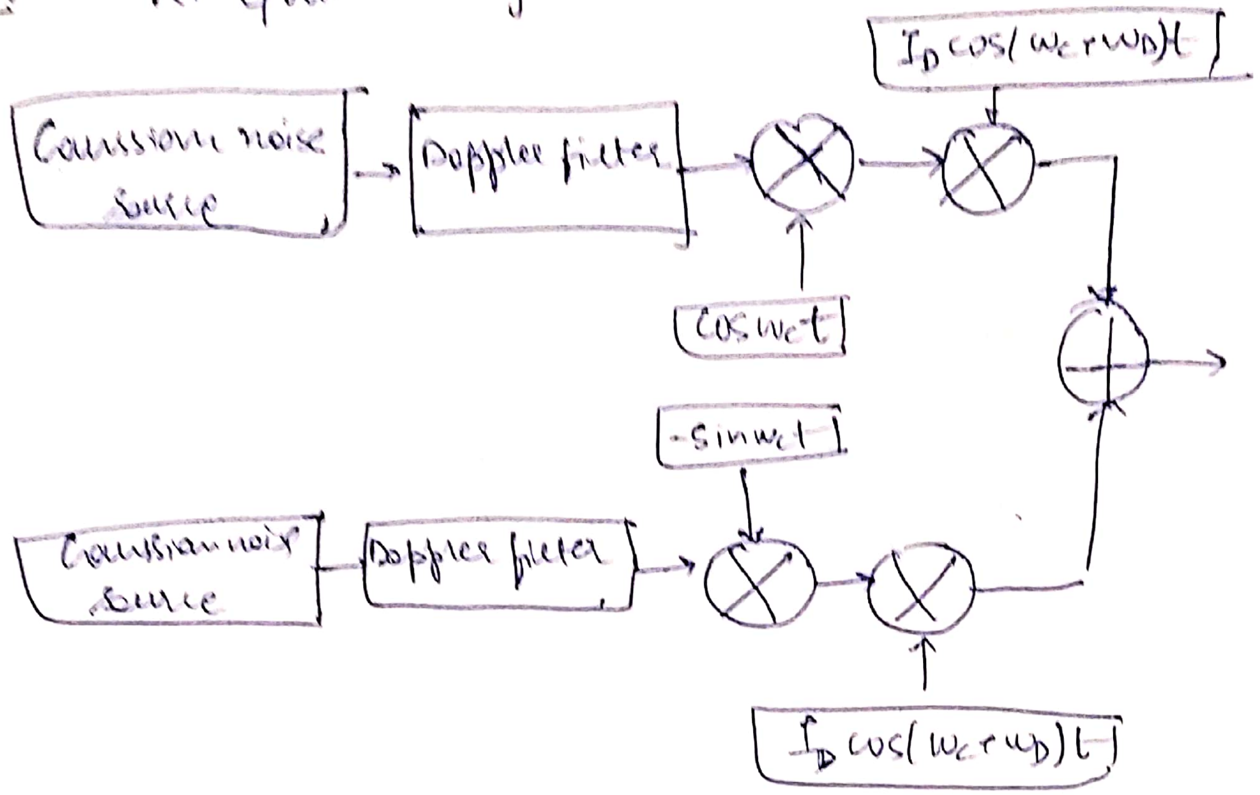
$$= \Sigma |w_x(n)|^2 + \Sigma |w_y(n)|^2$$

$$= N + N$$

$$\boxed{E(T_z) = 2N}$$

$$\text{var}(T_z) = E(T_z^2) - [E(T)]^2$$

Ans 8:- RF equivalent of a Rician channel model with Doppler



Ans 9: Mean delay of the power delay

$$P(\tau) = \delta(\tau - 10\,\mu sec) + 0.3\,\delta(\tau - 17\,\mu sec)$$

$P(\tau)$ power delay profile gives distribution of signal power received over a multipath channel as a fn of propagation delay. It is obtained as the spatial average of complex baseband channel impulse response as

$$P(\tau) = R_u(0,\tau) = E[|h(t,\tau)|^2]$$

mean delay is given by for discrete channel.

$$\bar{\tau} = \Sigma \tau p(\tau) / \Sigma p(\tau)$$

Ans 12 ?

$SNR = 2dB$

$h_1 = h_2 = h_3 = h_4 = 3$

$$SNR = 10 \log_2 \left(\frac{P\|h\|^2}{N_0} \right)$$

$$2 = 10 \log_2 \left(\frac{P\|h\|^2}{N_0} \right)$$

$$2^{0.2} = \frac{P\|h\|^2}{N_0}$$

$$\Rightarrow \text{Capacity} = \frac{1}{2} \log_2 \left(1 + \frac{P\|h_1\|^2}{N_0} + \frac{P\|h_2\|^2}{N_0} + \frac{P\|h_3\|^2}{N_0} + \frac{P\|h_4\|^2}{N_0} \right)$$

$$= \frac{1}{2} \log_2 \left(1 + 2^{0.2} \cancel{\times 4} \right)$$

$$= \frac{1}{2} \log_2 (5.594)$$

$$\boxed{\text{capacity} = 1.241 \text{ bits/second}}$$

Ans 13 ? capacity of a Rayleigh faded. $E[|h(m)|^2] = 0.8575$

$SNR = 4$

$C \cancel{SISO} = \cancel{\log_2(1+SNR_1)}$ $\mathbb{E}[\log(1+|H|^2 SNR)]$

$\cancel{= \log_2(5)} \approx \mathbb{E}[|h|^2 SNR] \log_2 e$

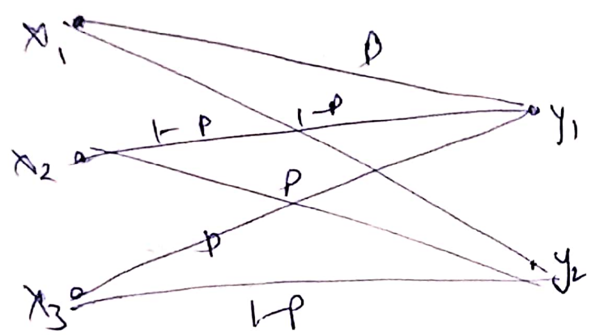
~~$C_{SISO} = 2.3219$ bits/second~~

$\boxed{C = 4.94 \text{ bit/s}}$

...3. possible SNR ...

Ans 14 Channel matrix of 3x2 MIMO system

Channel matrix $[P(Y/X)] = \begin{bmatrix} p & 1-p \\ 1-p & p \\ p & 1-p \end{bmatrix}$



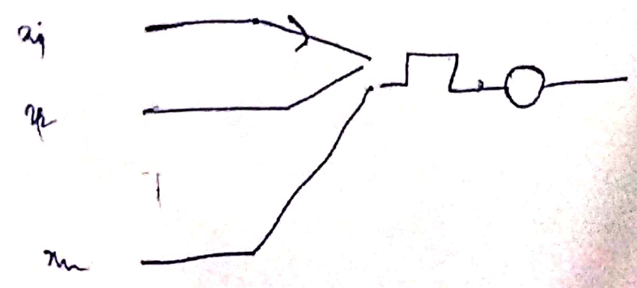
Ans 15 → channel capacity of MISO s/s.

$$C = \log_2\left(1 + \frac{P\|h\|^2}{N_0}\right) \quad b/s/Hz$$

or $$C = \log_2\left(1 + \alpha/n_t \sum_{i=1}^{n_i} |y_i|\right) \quad b/s/Hz$$

in eqn y_i, $i = 1, 2, \ldots, n_t$ represents the constant gain of channel, which is established b/n i^{th} transmitter antenna & single receiver antenna over a symbol period

α = SNR



Ans 16 :- BW = 30KHz, possible SNR $\gamma_1 = 30dB, \gamma_2 = 10dB$

$P(\gamma_1) = 0.1, \quad P(\gamma_2) = 0.4$

$10\log(x) = 30$
$x = 10^3 = 1000$

$$C = \sum_{i=1}^{n} B\log_2(1+\gamma_i)P(\gamma_i)$$

$$= 30{,}000 \log_2(1+1000)0.1 + 30{,}000 \log_2(1+10)0.4$$

$$= 30{,}000(100.1) + 30{,}000[4.4]$$

$$= 30{,}000[104.5]$$

Ans 17 :- Binary data is tx at rate of R bits, Bandwidth = B

$SNR = 3dB = 10\log_2 SNR \Rightarrow SNR = 2^{0.3}$

$R = B\log_2(1+SNR)$ bit/sec
$= B\log_2(1+2^{0.3})$
$R = B\log_2(2.231)$ bit/sec originally

$\Rightarrow \frac{R}{B} = \log_2(2.231) \longleftarrow (1)$

Now, given that $R \Rightarrow 2.65R$ & $B \Rightarrow 1.75B$, means

$\Rightarrow 2.65R = 1.75B\log_2(1+SNR_{new})$

$\Rightarrow \log_2(1+SNR_{new}) = \frac{2.65}{1.75} \times \log_2(2.231)$ from (1).

$\Rightarrow 1+SNR_{new} = 2^{1.753}$

$\Rightarrow \boxed{SNR_{new} = 2.37058 \text{ approx}}$

or $SNR_{dB} = 10\log(2.37058) = 3.7486\,dB$

A

Ans 18 :- $z = x(t) + jy(t)$ $\quad x \sim N(0,1)$, $y \sim N(0,1)$

mean = ?, rms = ?

$r = \sqrt{x^2+y^2}$

$E[r] = \text{mean} = \sigma\sqrt{\pi/2} = 1.253\sigma$

$E[r^2] = 2\sigma^2$ $\quad \boxed{r_{RMS} = \sqrt{2}\sigma}$

Ans 19 → $E(\tau^2) = 8\ \mu sec^2$, $E(\tau) = 2\ \mu sec$

variance $= E(\tau^2) - [E(\tau)]^2$

$= 8 - 4\ \mu sec^2$

$= 4\ \mu sec^2$

delay spread $(\sigma_\tau) = \sqrt{4}\ \mu sec = 2\ \mu sec$

for conventional size cell in urban area results in coherence BW of size 80 KHz

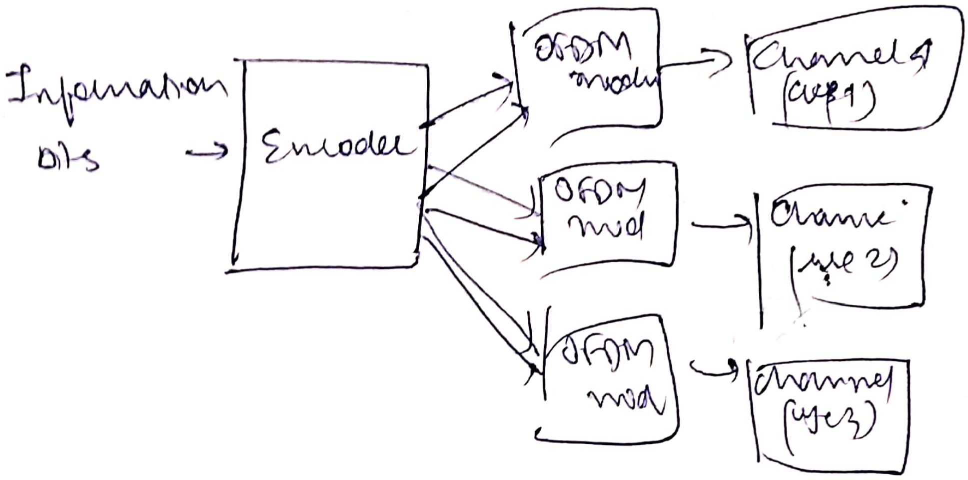
Ans 20 → frequency-selective block fading,

3 sub carriers each of BW = 1MHz

SNRs $[\gamma_1 = 10dB,\ \gamma_2 = 20dB,\ \gamma_3 = 30dB]$

$$\boxed{C_{N_c} = \max_{P_0 \ldots P_{N_c-1}} \sum_{n=0}^{N_c-1} \log\left(1 + \frac{P_n|\bar{h}_n|^2}{N_0}\right)}$$

Ans 10 → essential constraints in optimum bit loading OFDM



Ans 11 →

$$P(\tau) = 0.01\delta(\tau) + 0.01\delta(\tau-1) + \delta(\tau-2) + 0.1\delta(\tau-3) + 0.01\,\delta(\tau-4).$$

$$E(\tau) = \bar{\tau} = \frac{\int_0^\infty \tau P_n(\tau)\,d\tau}{\int_0^\infty P_n(\tau)\,d\tau}.$$

$\boxed{\bar{\tau} = \;\;}$

Ans 23 →

SNO	Gain
1	0.4412 0.7662
2	0.6681 0.2760
3	−0.8365 + j0.2766
4	0.7134 + j0.3141,

capacity = --