LNMIIT
The LNM Institute of Information Technology

The LNM Institute of Information Technology
Department of Mathematics
Mathematics-1 & MTH 102
MID TERM

Time: 9.30AM-11.00AM | Date: 03/10/2018 | Maximum Marks: 30

Note: You should attempt all questions. Your writing should be legible and neat. Marks awarded are shown next to the question. Number all pages of your answerbook. Please make an index showing the question number and page number(of answerbook) on the front page of your answer sheet in the following format.

Question No.	1	2	3	4	5	6	7	8
Page No.								

1. Prove or disprove: the sequence (x_n) is convergent if for each $n \geq 1$,

$$x_n = \frac{1}{\pi} + \frac{1}{2\pi} + \frac{1}{3\pi} + \ldots + \frac{1}{n\pi}.$$

[3 Marks]

2. Prove or disprove: The following function is differentiable at $x = 0$

$$f(x) = \begin{cases} x^3 \cos \frac{1}{x} & for\ x \neq 0 \\ 0 & for\ x = 0. \end{cases}$$

Check the continuity of the function $f'(x)$ at $x = 0$ (Justify your claim). [5 Marks]

3. Find all the values of x, for which the series $\sum_{n=2}^{\infty} \frac{x^n}{n \log\, n}$ converges (Interval of convergence). [3 Marks]

4. Prove that the equation $5x^{13} - 2e^{-x} + 3x - 7 = 0$ has one real root. Also show that it can not have more than one real root. [4 Marks]

5. Prove that a bounded function $f : [a, b] \to \mathbb{R}$ is Riemann integrable if and only if for each $\epsilon > 0$, there exists a partition P such that $U(P, f) - L(P, f) < \epsilon$. [4 Marks]

6. Let the function $f : \mathbb{R} \to \mathbb{R}$ and suppose that $|f(x) - f(y)| = 3(x - y)^2$, $\forall x, y \in \mathbb{R}$. Then, prove that f is a ~~constant~~ function. [3 Marks]
continous

7. Prove that the function $f : [0, 3] \to \mathbb{R}$ is Riemann Integrable if

$$f(x) = \begin{cases} 3 & for\ 0 \leq x < 1, \\ 1 & for\ 1 \leq x < 2, \\ 2 & for\ 2 \leq x \leq 3. \end{cases}$$

$\frac{1}{n}$

[4 Marks]

8. Determine all the values of p for which given improper integral converge: $\int_0^{\infty} \frac{(1 - e^{-x})}{x^{p-1}} dx$. [4 Marks]