LNMIIT

THE LNM INSTITUTE OF INFORMATION TECHNOLOGY
DEPARTMENT OF COMPUTER SCIENCE & ENGINEERING

Optimization Techniques and Applications
End-Term Examination

Time: 180 minutes (04:00-07:00PM)
Date: 03/12/2018

Maximum Marks: 40

Answer all questions
Note: Calculator is allowed for simple computation not scientific.

FORMULATION & MODELING

[3 × 5 = 15]

No mark will be awarded, if you have not follow the following instructions.
Instruction:

- Analyze the problem properly and write all the steps of your analysis that the formulation/modeling needs.
- Select proper parameters, identify the constraints exist in the model with proper justifications and construct the objective function.
- Specify all steps of your formulation with proper justification.

1. ABC printing company is facing a tight financial squeeze and is attempting to cut costs wherever possible. At present it has only one printing contract, and luckily the book is selling well in both the hardcover and paperback editions. It has just received a request to print more copies of this book in either the hardcover and paperback form. The printing cost for the hardcover books is Rs 600 per 100 books while that for paperback is only Rs 500 per 100. Although the company is attempting to economize, it does not wish to lay off any employee. Therefore, it feels obliged to run its two printing presses-I and II, at least 80 and 60 hours per week, respectively. Press-I can produce 100 hardcover books in 2 hours or100 paperback books in 1 hours. Press-II can produce 100 hardcover books in 1 hour or 100 paperback books in 2 hours.

   Formulate the problem to determine how many books for each type should be printed in order to minimize costs. [5]

2. A company manufactures two products, A and B. It takes 30 minutes to process one unit of product A and 15 minutes to process each unit of B. The maximum machine time available is 35 hours per week. Product A an B require 2 KG and 3 KG of raw materials per unit respectively. The available quantity of raw material is envisaged to be 180 KG per week.

   The products A and B which have unlimited market potential, sell for Rs 200 and 500 per unit respectively. If the manufacturing cost for product A and B are $2x^2$ and $3y^2$ respectively, where x and y represent the quantity of products A and B to be produce respectively. Formulate the problem to find how much of each product should be produced per week. [5]

Please turn ↪

3. A company XYZ have a production unit. The manufacturing and raw material cost to manufacture each of two products P_1 and P_2 is proportional to the square of quantity made. The products are made from a limited supply of a particular raw material and both are process on the same machine. It takes 40 minutes to process one unit of product P_1 and 30 minutes to process each unit of P_2. The machine operates for a maximum of 40 hours a week. Product P_1 needs 2 kg and P_2 needs 3 kg of raw material per unit which is in limited supply of 200 kg per week.

If net incomes from the products are Rs 250 and Rs 650 per unit and the manufacturing costs are $2x^2$ and $3y^2$ respectively. Where x and y represent the number units of product P_1 and P_2 respectively to be produced. Formulate this problem in a NLP model. [5]

LINEAR PROGRAMMING

4. Maximize $Z = 3x_1 + 2x_2 + 2x_3$
Subject to constraints

(i) $5x_1 + 7x_2 + 4x_3 \leq 7$

(ii) $-4x_i + 7x_2 + 5x_3 \geq -2$

(iii) $3x_1 + 4x_2 - 6x_3 \geq 29/7$

and $x_i \geq 0, i = 1, 2, 3$

Solve this LP problem using **two-phase Simplex method.** [5]

QUADRATIC PROGRAMMING

5. Solve the following quadratic programming problem by using Wolf's method.
Maximize $Z = 2x_1 + x_2 - x_1^2$
subject to the constraints

(i) $2x_1 + 3x_2 \leq 6$

(ii) $2x_1 + x_2 \leq 4$

and $x_1, x_2 \geq 0$ [5]

NON-LINEAR PROGRAMMING

6. Consider the following non-linear programming problem

Maximize $Z = 12x_1 + 21x_2 + 2x_1x_2 - 2x_1^2 - 2x_2^2$
subject to constraints

(i) $x_2 \leq 8$

(ii) $x_1 + x_2 \leq 10$

and $x_1, x_2 \geq 0$ Obtain **Kuhn-Tucker necessary conditions** of the given problem. Take all possible cases of the conditions and find the optimum solution. [5]

Please turn ↪

INTEGER PROGRAMMING

$[2 \times 5 = 10]$

7. Maximize $Z = 2x_1 + x_2$
subject to the constraints

(i) $2x_1 + 5x_2 \leq 17$

(ii) $3x_1 + 2x_2 \leq 10$

$x_1, x_2 \geq 0$ and are integers
Let the slack variables are s_1 and s_2 for constraint-1 and 2 respectively. Use **Gomory's cutting plane algorithm**, Obtain the solution of the above Integer LP problem.
The initial non-integer basic feasible and optimal solution have been obtained from the following Simplex table.

C_B	Variables in Basis B	Solutions Values $b(= x_B)$	x_1	x_2	s_1	s_2
0	s_1	31/3	0	1	1/2	0
2	x_1	10/3	1	2/3	0	1/3
$Z = \frac{20}{3}$		$c_j - z_j$	0	-1/3	0	-2/3

8. Maximize $Z = 3x_1 + 12x_2$
subject to the constraints

(i) $2x_1 + 4x_2 \leq 7$

(ii) $5x_1 + 3x_2 \leq 15$

$x_1, x_2 \geq 0$ and are integers
Use **Gomory's cutting plane algorithm**, Obtain the solution of the above Integer LP problem.
Assume the slack variables for constraint-1 and 2 are s_1 and s_2 respectively
The initial non-integer basic feasible solution is $x_1 = 0, x_2 = 7/4$ and Max $Z = 21$.

C_B	Variables in Basis B	Solutions Values $b(= x_B)$	x_1	x_2	s_1	s_2
12	x_2	7/4	1/2	1	1/4	0
0	s_2	39/4	7/2	0	-3/4	1
$Z = 21$		$c_j - z_j$	-3	0	-3	0

****END****