LNMIIT
The LNM Institute of Information Technology

LNMIIT/B.Tech/C/IC/2018-19/ODD/MTH213/ET

The LNM Institute of Information Technology, Jaipur
Mathematics-III
End Term Examination

Duration: 03 HOURS Max.Marks: 50

Name: [redacted] Roll No.: [redacted]

NOTE: You **should attempt all 8 questions.** Your writing should be legible and neat. Marks awarded are shown next to the question. **Start a new question on a new page and answer all its parts in the same place.** Please make an index showing the question number and page number on the front page of your answer sheet in the following format.

Question No.				
Page No.				

1. (a) Let f and g be entire functions which satisfy $|f(z)| < |g(z)|$ for all $z \in \mathbb{C}$. Show that there exists a constant $\lambda \in \mathbb{C}$ such that $f(z) = \lambda g(z)$ for all $z \in \mathbb{C}$. [3]

   (b) Find all the points where the function $f(z) = \text{Log } z$ satisfies the CR-equations in polar form. Then find $f'(z)$ wherever it exists. (Here Log z is the principal value of $\log z$.) [3]

2. (a) Evaluate $\displaystyle\int_C \frac{e^z}{(z^2+\pi^2)^2}dz$, where the contour $C : |z| = 4$ is taken in positive direction. [3]

   (b) Prove that $\left|e^{z^2}\right| \leq e^{|z|^2}$. [2]

3. (a) Form the first order PDE by eliminating the arbitrary function f from

$$f(x+y+z, x^2+y^2+z^2) = 0.$$

[3]

   (b) Find the singular solution (singular integral), if it exists for the following PDE

$$z = px + qy + p + q - pq.$$

[2]

   (c) Find the general integral of the PDE given by

$$(x^2 - yz)p + (y^2 - zx)q = z^2 - xy.$$

[3]

4. (a) Find the solution of the Cauchy's problem for the quasi linear PDE given by

$$z_x - zz_y + z = 0, \text{ for all } y \text{ and } x > 0,$$

with the initial data curve

$$C : x_0 = 0, y_0 = s, z_0 = -2s, -\infty < s < \infty$$

[3]

   (b) Reduce the following partial differential equation to a canonical form and hence solve it.

$$u_{xx} - 2\sin x \, u_{xy} - \cos^2 x \, u_{yy} - \cos x \, u_y = 0$$

[6]

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5. Using the D'Alemberts formula, solve the initial boundary value problem(IBVP) given by

$$\begin{array}{lll} PDE & : & u_{tt} - \alpha^2 u_{xx} = 0, & x > 0, t > 0 \\ IC & : & u(x,0) = g(x), u_t(x,0) = h(x), & x > 0 \\ BC & : & u(0,t) = 0, & t \geq 0. \end{array}$$

[5]

6. Solve the IBVP for the heat equation given by

$$\begin{array}{lll} PDE & : & u_t = u_{xx}, & 0 < x < 1, 0 < t < \infty \\ BC & : & u(0,t) = 1, u(1,t) = -2, & 0 < t < \infty \\ IC & : & u(x,0) = 2 - 3x, & 0 \leq x \leq 1. \end{array}$$

[5]

7. Use the Duhamel's principle to solve the IBVP given by

$$\begin{array}{lll} PDE & : & u_t - \alpha^2 u_{xx} = t \sin x, & 0 \leq x \leq \pi,\ 0 < t < \infty \\ BC & : & u(0,t) = 0, u(\pi,t) = 0, & t > 0 \\ IC & : & u(x,0) = 0, & 0 \leq x \leq \pi. \end{array}$$

[5]

8. (a) Use the separation of variables method to solve

$$\Delta u \equiv u_{xx} + u_{yy} = 0,\ 0 < x < a,\ 0 < y < b$$

with boundary conditions given by

$$u(x,0) = 0 = u(x,b) = u(0,y)$$

$$u(a,y) = g_1(y).$$

[5]

(b) Consider the BVP given by

$$(\star) \left\{ \begin{array}{lll} PDE & : & u_{xx} + u_{yy} = 0, & (x,y) \in R \\ BC & : & u(x,y) = K, & (x,y) \in \partial R \end{array} \right.$$

where $R = \{(x,y) : 0 < x < 1, 0 < y < 1\}$ and ∂R is its boundary; and K is a constant. Use Maximum principle to find the solution of $(\star)$ in R.

[2]