A/LNMIIT/B.Tech/C/IC/2017-18/ODD/MTH102/MT

The LNM Institute of Information Technology, Jaipur
Mathematics-I
Mid Term

Duration: 30 mins. Max.Marks: 10

PART-A
Submit Part-A within 30 minutes of commencement of examination.

Name: ________________ Roll No.: ________________ Tutorial Section: ___________

NOTE: Encircle the most appropriate answer. There is a negative marking of 0.25 mark for each wrong answer. Each question carries 1 mark. Overwriting and cutting shall be treated as a wrong answer and hence there shall be negative marking for these as well.

1. Let (x_n) be a sequence such that the subsequences (x_{2n}) and (x_{2n+1}) converges to the same limit l. Then the sequence (x_n)
(a) is not bounded. (b)✓ converges to l. (c) converges to l only if $l = 0$. (d) may not converge to l.
2. The function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \begin{cases} 0, & x \in \mathbb{Q} \\ x, & x \notin \mathbb{Q}. \end{cases}$ is continuous on
(a) $\mathbb{R}$. (b) $\mathbb{Q}$. (c) ✓$\{0\}$. (d) the set of irrational numbers.
3. Let (x_n) be a sequence of real numbers. Then consider the statements
(p) (x_n) is convergent. (q) $(|x_n|)$ is convergent.
Then
(a) ✓$(p) \Rightarrow (q)$. (b) $(q) \Rightarrow (p)$. (c) $(p) \Longleftrightarrow (q)$ (d) None of the above.
4. Let
$$f(x) = \begin{cases} x|x|, & x \neq 0 \\ 0, & x = 0. \end{cases} .$$
Then
(a) f is discontinuous at $x = 0$. (b) f is not differentiable at $x = 0$.
(c)✓ f' is not continuous at $x = 0$. (d) f' is continuous at $x = 0$.
5. For a continuous function $f : [0, 1) \to [0, \infty)$ consider the following statements:
(i) $f([0, 1))$ must be an interval. (ii) $f([0, 1))$ must be a bounded subset of $\mathbb{R}$.
Then
(a)✓ (i) is true, (ii) is false. (b) (i) is false, (ii) is true. (c) (i), (ii) are true. (d) (i), (ii) are false.
6. Let (x_n) be a bounded above sequence of real numbers. Then (x_n) is convergent if it
(a) is bounded below. (b)✓ is increasing. (c) is decreasing. (d) has a convergent subsequence.
7. The series $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n}$ converges
(a) for all $x \in \mathbb{R}$. (b)✓ only for all $x \in [-1, 1)$. (c) only for all $x \in (-1, 1)$ (d) only at $x = 0$.
8. Suppose $f : \mathbb{R} \longrightarrow \mathbb{R}$ be a function such that $|f(x) - f(y)| \leq 5|x - y|$ for all $x, y \in \mathbb{R}$. Then f is
(a) ✓ continuous. (b) bounded. (c) increasing. (d) differentiable.
9. The equation $x^2 - \cos x = 0$ has
(a) no real roots. (b) exactly one real root. (c)✓ exactly two real roots. (d) infinitely many real roots.
10. Let (a_n) and (b_n) be two sequences of real numbers such that $\lim\limits_{n\to\infty} a_n = 1$ and $\lim\limits_{n\to\infty} b_n = -1$. Then the sequence (c_n) where $c_n = a_{2n} + b_{2n+1}$ $n \in \mathbb{N}$,
(a) converges to 1. (b) ✓ converges to 0. (c) converges to 2. (d) does not converge.