The LNM Institute of Information Technology, Jaipur

First Mid Sem Examination 2011

Mathematics II

Date: 3^{rd} Feb 2011 Full Mark 40 Duration: 1 hour

1. (i) In the electric field between two concentric cylinders the equipotential lines are circles given by $x^2+y^2=c$ (volts). Find the curves of electric force which are orthogonal to equipotential lines. (4)

   (ii) Show that if $(\partial M/\partial y - \partial N/\partial x)/(Ny - Mx)$ is a function $g(z)$ of the product $z = xy$, then
$$\mu = e^{\int g(z)dz}$$
   is an integrating factor for the differential equation $M(x,y)dx+N(x,y)dy = 0$. (5)

2. (i)Suppose that the function $F(x,y)$ is continuously differentiable. Show that the initial value problem (IVP)
$$\frac{dy}{dx} = F(x,y), \qquad y(0) = y_0.$$
   has at most one solution in a neighborhood of the origin. (5)

   (ii) Apply Euler's method to solve the IVP
$$y' = x+y, \qquad y(0) = 1.$$
   and describe what happens if you are using a computer with ordinary precision and a very small step size, say $h = 10^{-10}$. (4)

3. (i) Let $y_1(x)$ and $y_2(x)$ be two linearly independent solution of
$$y'' + p(x)y' + q(x)y = 0. \qquad (A)$$
   Then prove that the zeros of the y_1 and y_2 interlace. Hence or otherwise prove that the zeros of the functions $a\sin x + b\cos x$ and $c\sin x + d\cos x$ are distinct and occur alternately provided $ad \neq bc$. (5+3)

   (ii) Let $p(x)$, $q(x)$ and $r(x)$ be continuous functions defined on an interval I. Let $y_1(x)$ and $y_2(x)$ be two linearly independent solution of (A) on I. Then prove that a particular solution y_p of
$$y'' + p(x)y' + q(x)y = r(x)$$

is given by

$$y_p = -y_1 \int \frac{y_2 r(x)}{W(y_1, y_2)} dx + y_2 \int \frac{y_1 r(x)}{W(y_1, y_2)} dx,$$

where $W = W(y_1, y_2)$ be the Wronksian of y_1 and y_2. (6)

4. Consider the Euler-Cauchy equation

$$x^n \frac{d^n y}{dx^n} + a_{n-1} x^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + ... + a_1 x \frac{dy}{dx} + a_0 y = 0, \ \ x \in I \ (x > 0),$$

where $a_0, a_1, ..., a_n \in \mathbb{R}$ are constants. Let $x = e^t$, and let $D = \dfrac{d}{dt}$, $d = \dfrac{d}{dx}$. Then

(i) Show that $xd(y) = Dy(t)$.

(ii) Using mathematical induction show that

$$x^n d^n y = (D(D-1) \cdots (D-n+1))y(t).$$

(2+6)

End of paper