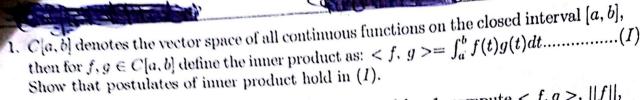


MATH-II

Assignment 3



- (a) For f(t) = t + 2, $g(t) = t^2 3t + 4$, a = -1 and b = 1, compute $\langle f, g \rangle$, ||f||, and ||g||.
- (b) For f(t) = 3t 5, $g(t) = t^2$, a = 0 and b = 1, compute $\langle f, g \rangle$, ||f||, and ||g||.
- (c) Verify the Cauchy-Schwarz inequality for the vector f and g in (a) and (b).

Verify that the following is an inner product on R^2 , where $\alpha=(x_1,x_2)$ and $\beta=$ (y_1, y_2) :

- (a) $<\alpha,\beta>=x_1y_1-x_1y_2-x_2y_1+4x_2y_2$
- (b) $\langle \alpha, \beta \rangle = 5x_1y_1 x_2y_2$
- (c) $<\alpha, \beta>=x_1y_1+x_2y_2+5$
- (d) $<\alpha,\beta>=x_1y_1+x_1y_2+x_2y_1+3x_2y_2$
- (e) $<\alpha,\beta>=x_1x_2+5y_1y_2$
- 3. Find the value of a so that the following is an inner product on R^2 , where $\alpha = (x_1, x_2)$ and $\beta = (y_1, y_2)$:

 $<\alpha,\beta>=x_1y_1-3x_1y_2-3x_2y_1+ax_2y_2$

- 4. Show that the norm of a vector in a vector space V has the following three properties
 - (a) $||v|| \ge 0$ and ||v|| = 0 if and only if v = 0.
 - (b) $\|\lambda v\| = |\lambda| \|v\|$ for all $\lambda \in \mathbb{R}$ and $v \in V$.
 - (c) $||v + w|| \le ||v|| + ||w||$ for all $v, w \in V$.

Note that $\|\cdot\| = \langle \cdot, \cdot \rangle^{1/2}$, where $\langle \cdot, \cdot \rangle$ is an inner product defined on V.

- 5. Let U be the subspace of \mathbb{R}^4 spanned by $\{(1,1,1,1), (1,-1,2,2), (1,2,-3,-4)\}$. Then by using Gram-Schmidt process find the orthonormal basis and orthonormal basis under the usual inner product on \mathbb{R}^4 .
- 6. Use Gram-Schmidt process to transform each of the following into an orthonormal basis:
 - (a) $\{(1,0,1),(1,0,-1),(0,3,4)\}$ for \mathbb{R}^3 with the standard inner product.
 - (b) $\{(1,0,1),(1,0,-1),(0,3,4)\}\$ for \mathbb{R}^3 using the inner product defined by $\langle (x, y, z), (x', y', z') \rangle = xx' + 2yy' + 3zz'.$
- 7. Consider the vector space $P(t) = \{a_0 + a_1t + a_2t^2 : a_0, a_1, a_2 \in R\}$ with inner product $\langle f, g \rangle = \int_0^1 f(t)g(t)dt$ Apply the Gram-Schmidt algorithm to the set $\{1, t, t^2\}$ to obtain an orthogonal set $\{f_0, f_1, f_2\}$ with integer coefficients.