The LNM Institute of Information Technology

ECE and CCE

ECE 321: Control System Engineering

Mid Term

Time: 1.5 hours	**Date:** 24/02/2018	**Max. Marks: 30**

Instruction: *1) Start each answer on a fresh page of your answer book and highlight your answer number.*
*2) Check that your Question paper has **3 Questions**.*

Q1. Attempt any **two** questions from **(a)/(b)/(c)**. Rest is compulsory. [(3x2) + (2x2) = 10]

(a) In the given signal flow graph (**Fig. 1**), find (a) total number of loops, (b) total number of forward paths, and (c) the linear algebraic equations for each node.

(b) Draw the analogous electrical circuit of the given system (**Fig. 2**) using Force-Current analogy. (f(t) is the external force applied)

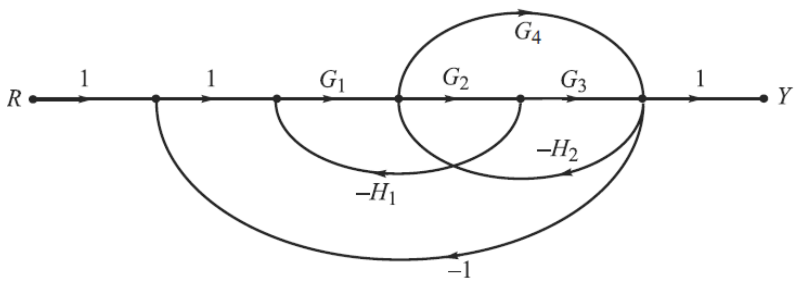


Fig. 1

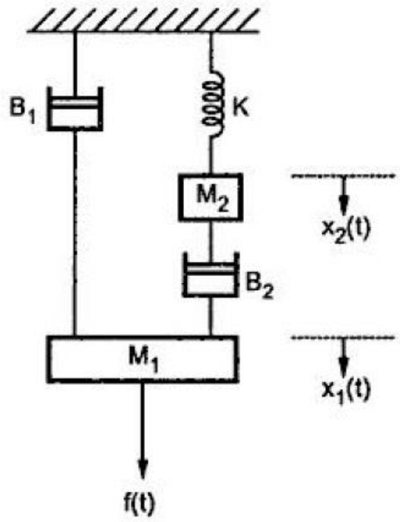


Fig. 2

(c) A system with two inputs and two outputs is shown in **Fig. 3**. Find $\frac{C_1(s)}{R_1(s)}, \frac{C_1(s)}{R_2(s)}, \frac{C_2(s)}{R_2(s)}$.

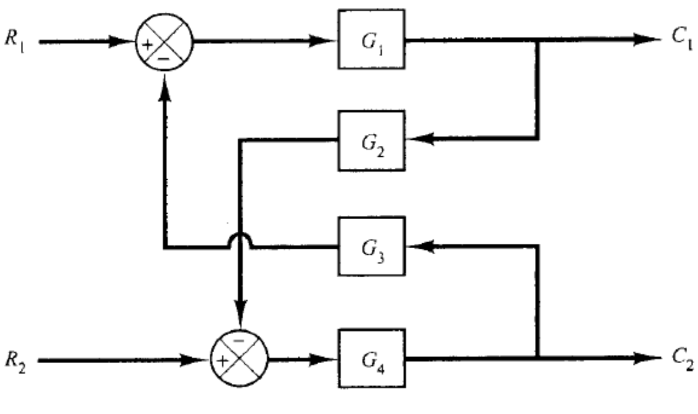


Fig. 3

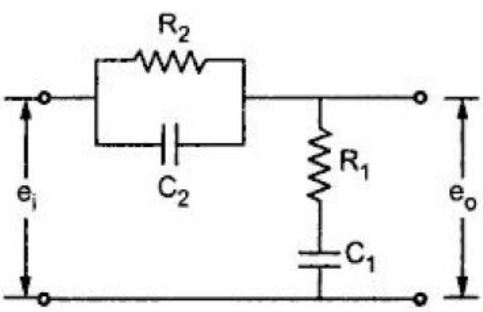


Fig. 4

(d) Find the transfer function $\frac{E_0(s)}{E_i(s)}$ of the system given in **Fig. 4**.

(e) The transfer function of a system is given as 100/ (s^2+20s+100). Comment about the type of the system (undamped/ underdamped/ critically damped/ overdamped). Draw the poles/zeros, whichever is applicable, in the complex frequency domain.

Q2. [4+4+2=10]

(a) Simplify the given system (**Fig. 5**) using block diagram reduction method and obtain the transfer function.

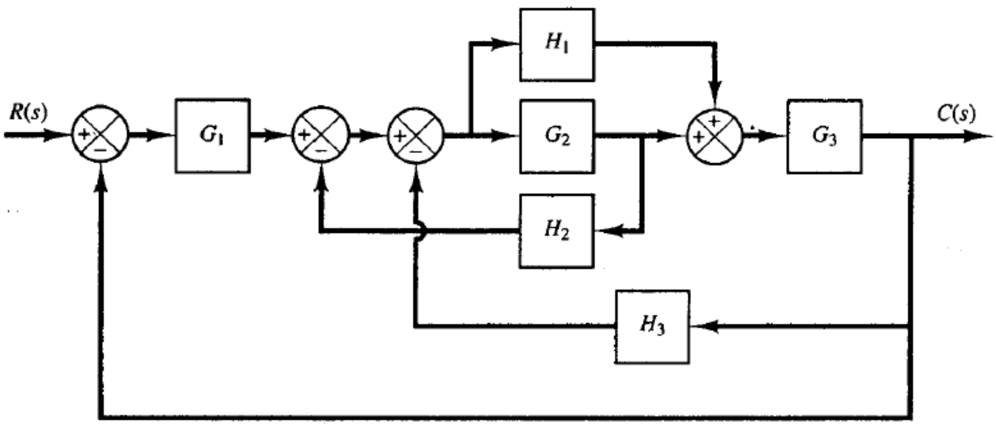


Fig. 5

(b) The signal flow graph of a system is shown in **Fig. 6**. Find the transfer function $\frac{C(s)}{R(s)}$ of the system.

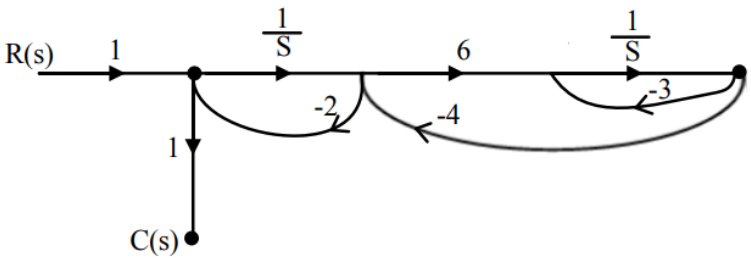


Fig. 6

(c) For the system having transfer function of 2/(s+1), find the time required for a step response to reach 98% of its final value.

Q3. [5+5=10]

(a) For a unity feedback control system, the open loop transfer function is G(s) = $\frac{10(s+2)}{s^2(s+1)}$, find

i) position, velocity and acceleration error constants.

ii) the steady state error when $r(t) = 3 - 2t + (t^2/6)$

(b) The forward path transfer function of a unity feedback system has type-1, 2nd order with pole at -2. The damping ratio is 0.4. Find the time response specifications of the above system. (t_p, t_d, t_r, M_p, t_s - all the notations have its usual significance).

Ans 1(a).

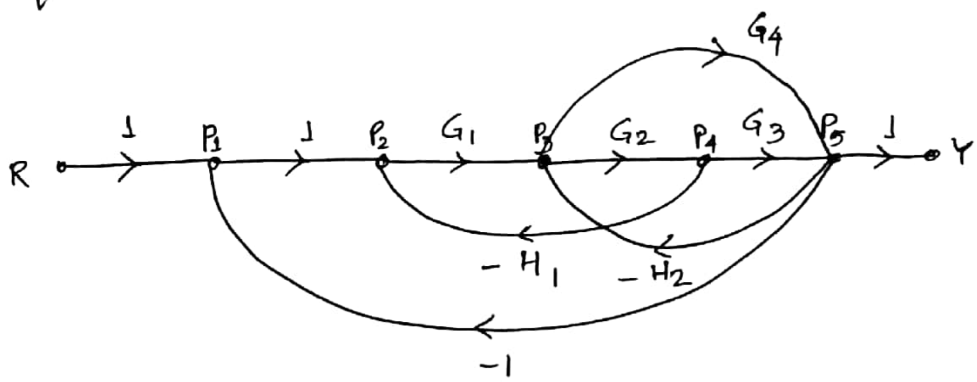
→ Total no. of loops : 4 $(-G_1G_2H_1, -G_2G_3H_2, -G_1G_4, -G_1G_2G_3)$ — 1M

→ Total no. of forward path : 2.

$(G_1G_2G_3, G_1G_2G_4)$. — 1M

→ ~~Total no. of~~ Equations :-

given

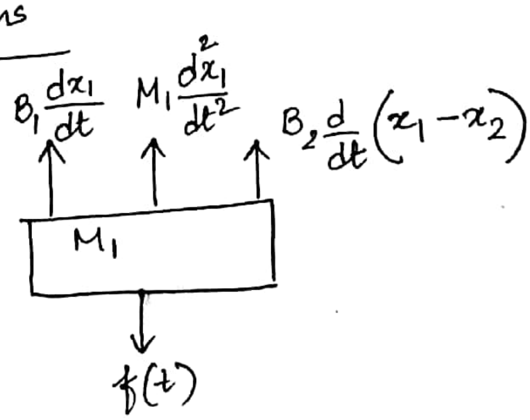


$$P_1 = R - P_5$$
$$P_2 = P_1 - H_1P_4$$
$$P_3 = G_1P_2 - H_2P_5$$
$$P_4 = G_2P_3$$
$$P_5 = G_3P_4 + G_4P_3$$
$$Y = P_5$$

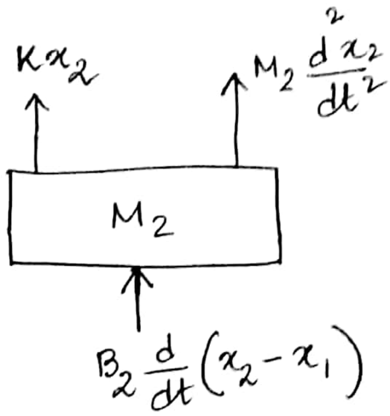
— 1M

Ans 1(b) Free body diagrams

for M_1



for M_2,



The mechanical equations are :-

for M_1 $$f(t) = B_1 \frac{dx_1}{dt} + M_1 \frac{d^2x_1}{dt^2} + B_2 \frac{d}{dt}(x_1 - x_2) \quad \text{——— (I)}$$

for M_2 $$Kx_2 + M_2 \frac{d^2x_2}{dt^2} + B_2 \frac{d}{dt}(x_2 - x_1) = 0 \quad \text{——— (II)}$$

1M

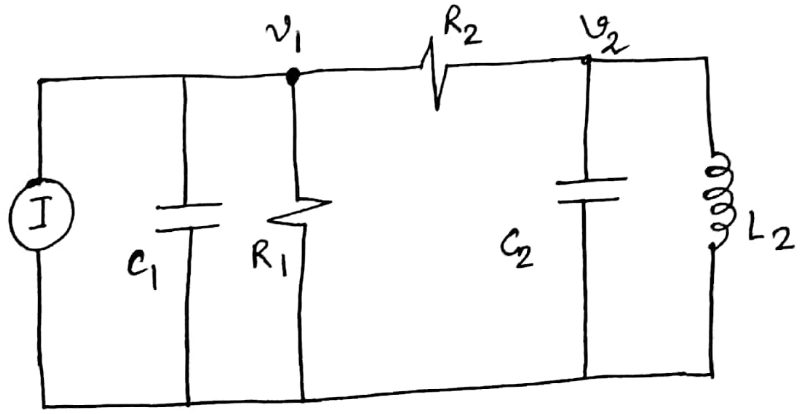
The corresponding electrical equations are : (Force - Current analogy)

$$I = \frac{1}{R_1} v_1 + C_1 \frac{dv_1}{dt} + \frac{1}{R_2}(v_1 - v_2) \quad \text{——— (III)}$$

and $$0 = \frac{1}{L} \int v_2 \, dt + C_2 \frac{dv_2}{dt} + \frac{1}{R_2}(v_2 - v_1) \quad \text{——— (IV)}$$

1M

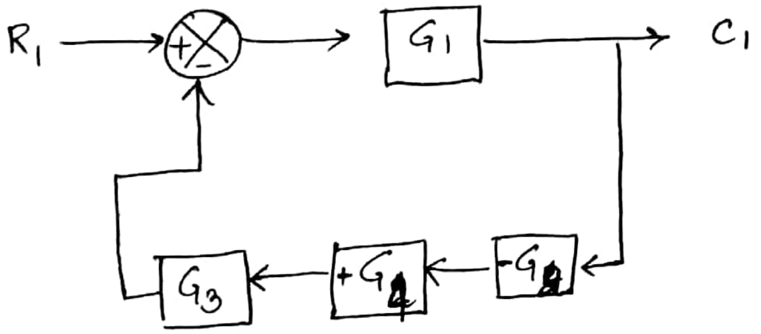
from eq. (III) & (IV),



——— 1M

Ans 1 (c)

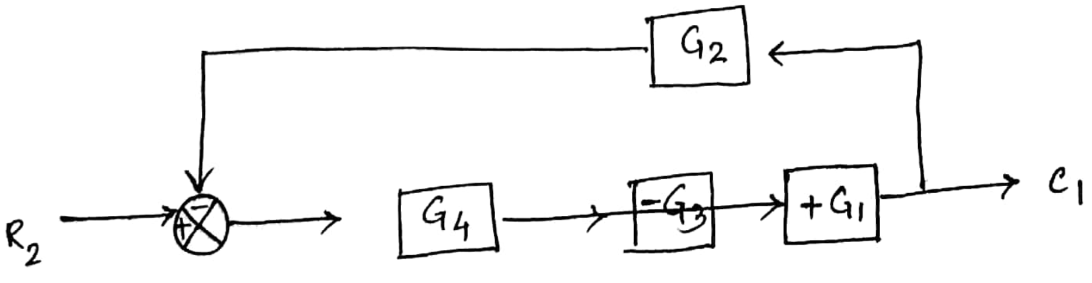
→ for $\frac{C_1(s)}{R_1(s)}$, $R_2(s) = 0$



$\therefore,\quad \frac{C_1}{R_1} = \frac{G_1}{1 + G_1 G_4 G_3 (-G_2)}$

$$\boxed{\frac{C_1}{R_1} = \frac{G_1}{1 - G_1 G_2 G_3 G_4}}$$ — 1M

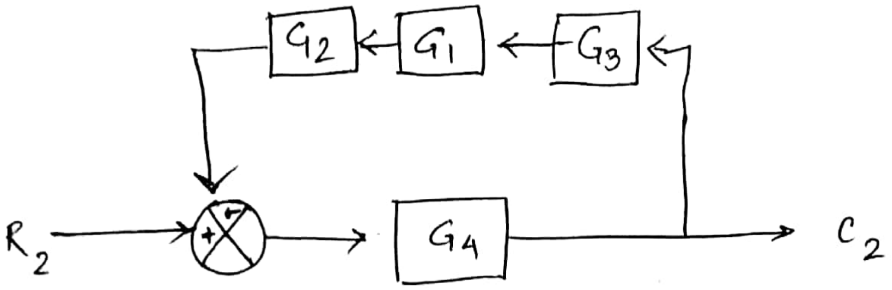
→ for $\frac{C_1(s)}{R_2(s)}$, $R_1(s) = 0$.



$$\frac{C_1}{R_2} = \frac{-G_1 G_3 G_4}{1 + (-G_3) G_2 G_1 G_4}$$

$$\boxed{\frac{C_1}{R_2} = \frac{-G_1 G_3 G_4}{1 - G_1 G_2 G_3 G_4}}$$ — 1M

for $\frac{C_2}{R_2}$, $R_1 = 0$



$$\boxed{\frac{C_2}{R_2} = \frac{G_4}{1 - G_1 G_2 G_3 G_4}}$$ — 1M

Ans 1(d)

R_2 is in parallel with C_2

∴, in s-domain,

$$X_1 = \frac{R_2 \times \frac{1}{C_2 s}}{R_2 + \frac{1}{C_2 s}} \quad \text{(equivalent)}.$$

$$= \frac{R_2}{1 + R_2 C_2 s}$$

Let us consider $I(s)$ as the current across the circuit,

$$E_i(s) = \frac{R_2}{1 + R_2 C_2 s} I(s) + \left(R_1 + \frac{1}{C_1 s}\right) I(s)$$

$$= I(s)\left[\frac{R_2}{1 + R_2 C_2 s} + R_1 + \frac{1}{C_1 s}\right] \quad \text{— (1)}$$

— 1M

Also,

$$E_0(s) = I(s)\left[R_1 + \frac{1}{C_1 s}\right]$$

$$\Rightarrow I(s) = \frac{E_0(s)}{R_1 + \frac{1}{C_1 s}} \quad \text{——— (11)}$$

Putting eq (11) in eq (1),

$$E_i(s) = \frac{E_0(s)}{R_1 + \frac{1}{C_1 s}}\left[\frac{R_2}{1+R_2C_2s} + R_1 + \frac{1}{C_1 s}\right]$$

$$= \frac{E_0(s)}{\frac{R_1C_1s+1}{\cancel{C_1s}}}\left[\frac{R_2C_1s + R_1(1+R_2C_2s)C_1s + (1+R_2C_2s)}{(1+R_2C_2s)(\cancel{C_1s})}\right]$$

$$\boxed{\frac{E_0(s)}{E_i(s)} = \frac{(1+R_1C_1s)(1+R_2C_2s)}{R_2C_1s + R_1C_1s(1+R_2C_2s) + (1+R_2C_2s)}}$$

(1M)

Ans 1(e) Given,

$$H(s) = \frac{100}{s^2+20s+100}$$

Comparing the given eq^n with generalized equation.

$$\frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$2\xi\omega_n = 20 \quad \text{and} \quad \omega_n^2 = 100$$

$$\Rightarrow \boxed{\omega_n = 10} \quad \text{———} \quad (1M)$$

$$\therefore \boxed{\xi = 1} \rightarrow \text{CRITICALLY DAMPED.}$$

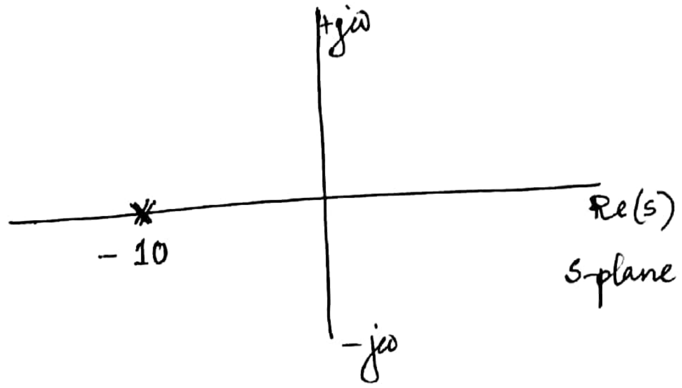
from the transfer function (given)

$$s^2 + 20s + 100 = 0$$

$$(s+10)(s+10) = 0$$

(No zeros).

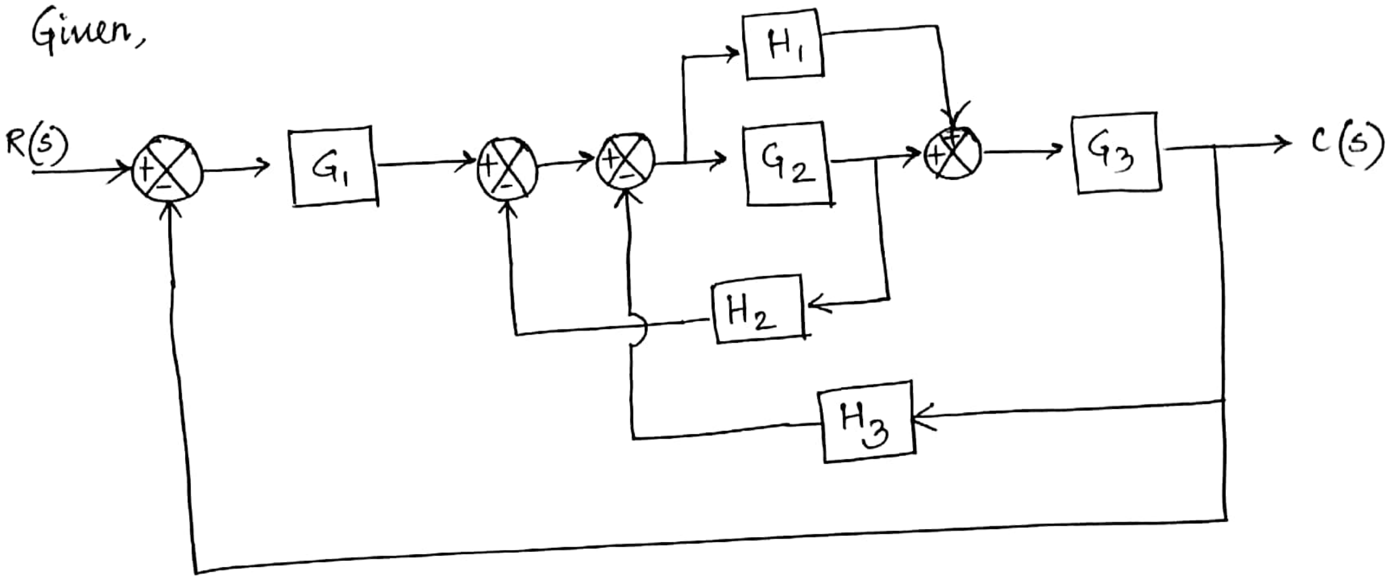
$$s = -10, -10.$$

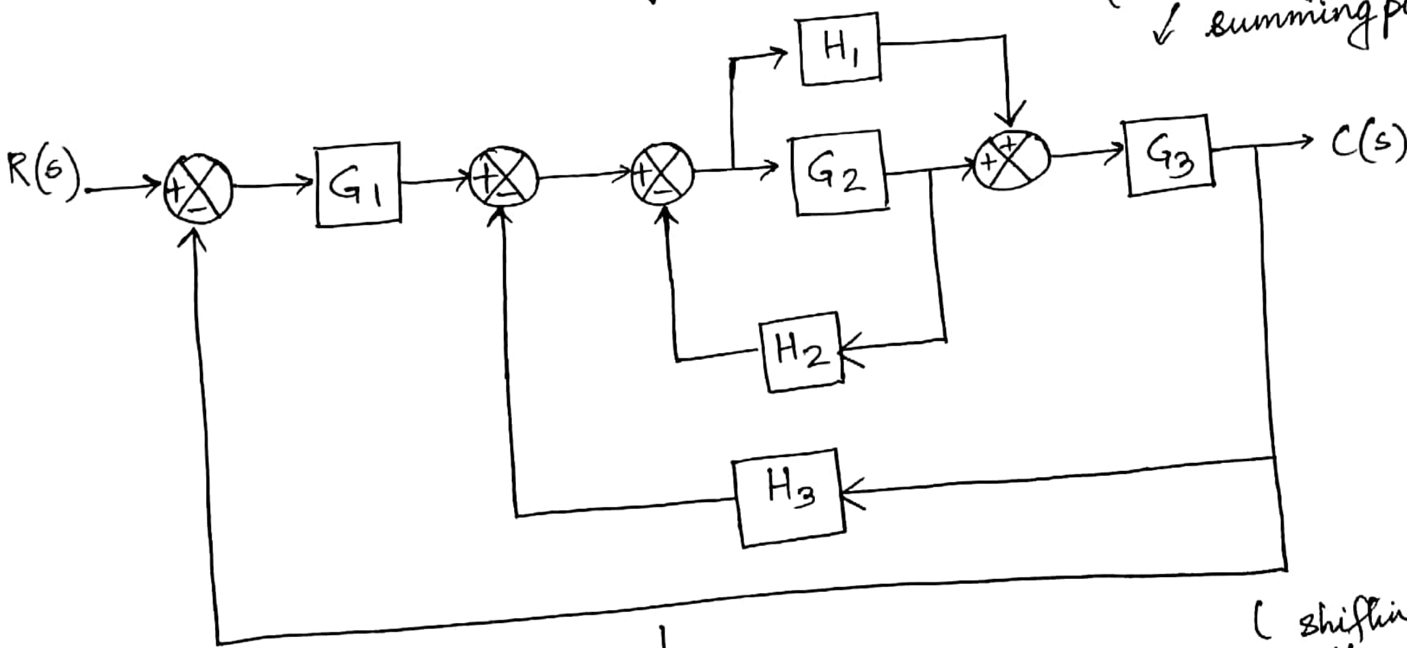


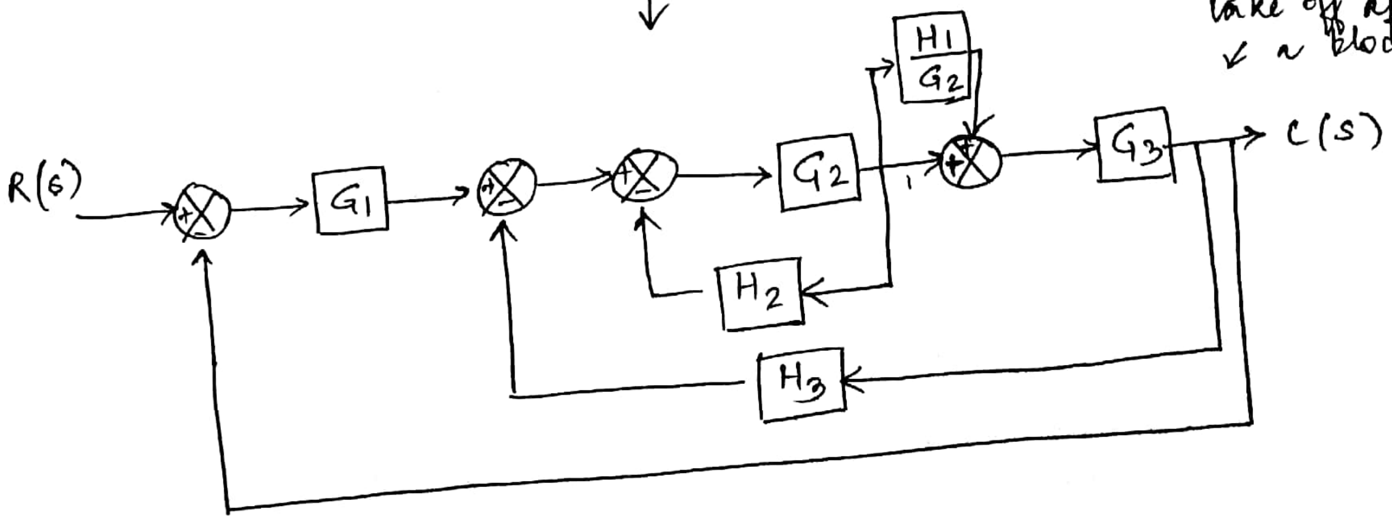
1M

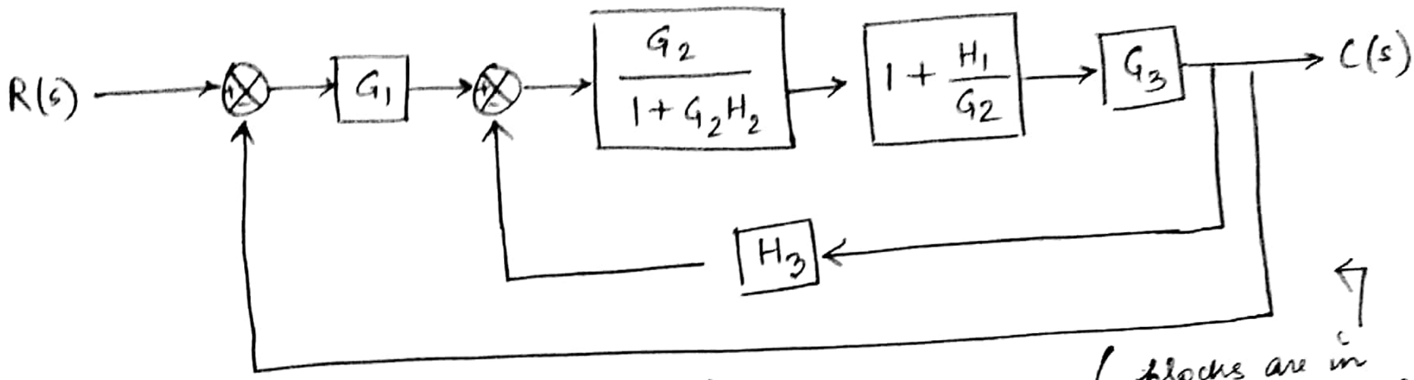
Ans 2 (a) (This question can be solved multiple ways but TF should come out same)

Given,

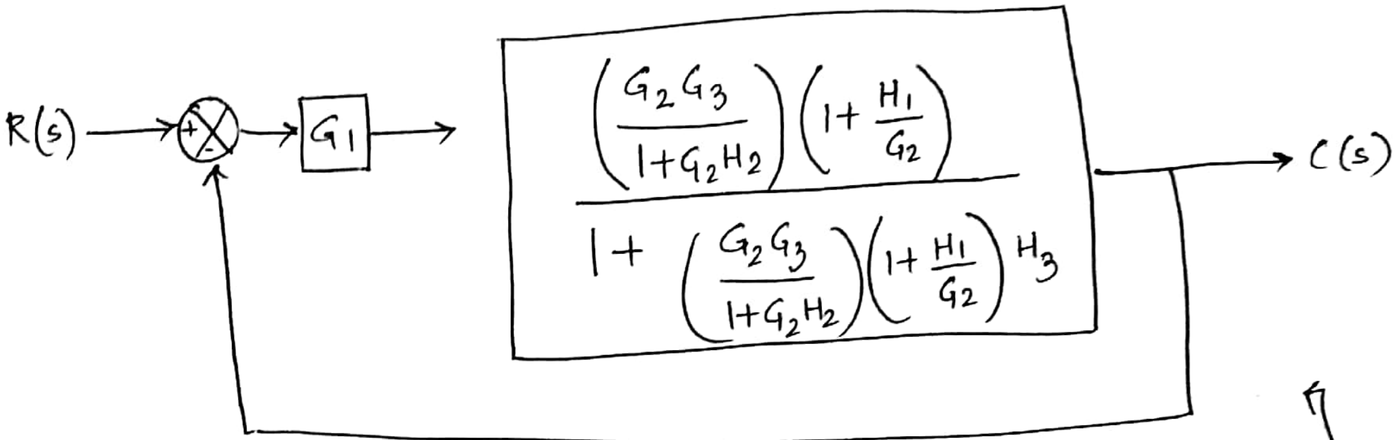




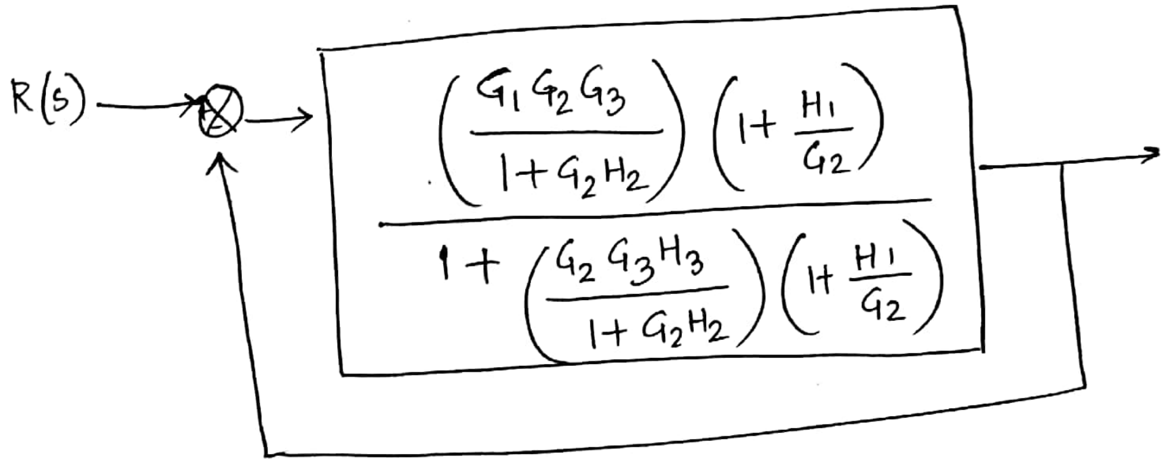




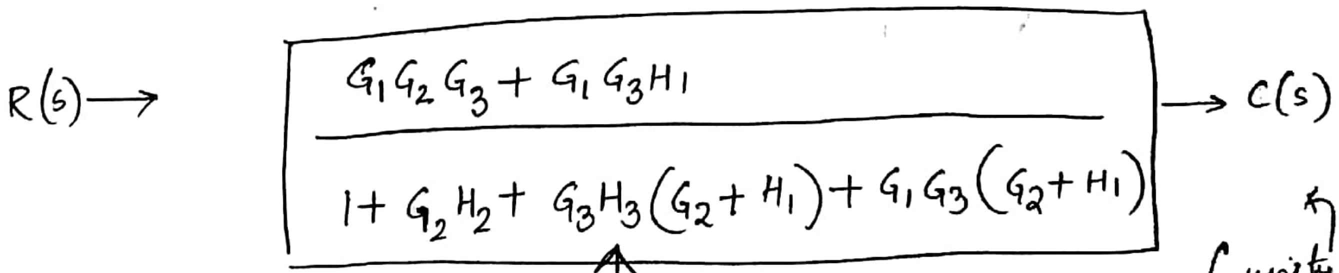
(blocks are in feedback and parallel)



(blocks are in series followed by feedback)



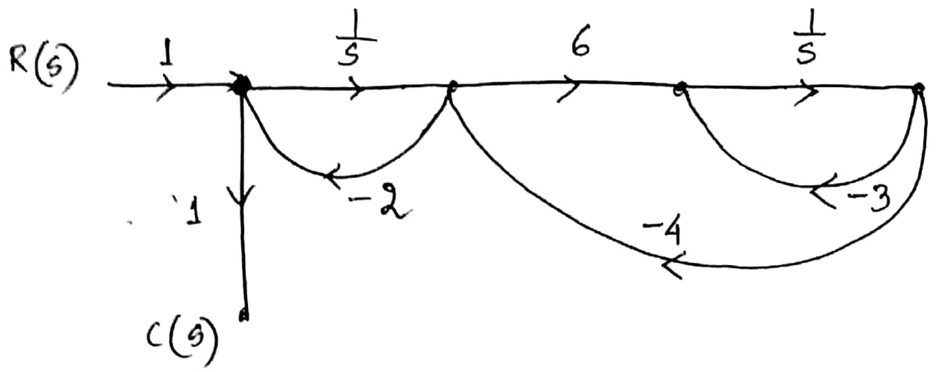
(blocks are in series)



(unity feedback)

Transfer function of the system

Ans 2(b) Given, SFG



(As in Question no specific method is mentioned, you can use equation or Mason's Gain formula to solve it.)

✓ no. of forward path :- 1, $\boxed{P_1 = 1}$ —— 0.5M

✓ No. of loops :- 3, $\boxed{L_1 = -\frac{3}{s}, \ L_2 = -4 \times \frac{6}{s}, \ L_3 = -\frac{2}{s}}$ —— 0.5M

✓ No. of non-touching loops :- 1, i.e. (L_1 and L_3)
(pair of 2) $\boxed{NL_1 = \frac{6}{s^2}}$ —— 0.5M

(There are no 3 or more pairs of non-touching loops).

✓ $$\Delta = 1 - \left(-\frac{3}{s} - \frac{24}{s} - \frac{2}{s}\right) + \frac{6}{s^2}$$

$$\boxed{\Delta = 1 + \frac{29}{s} + \frac{6}{s^2}} = \cancel{1 + \frac{35}{s}}$$ —— 1M

✓ $$\Delta_1 = 1 - L_1 - L_2 = 1 - \left(-\frac{3}{s}\right) - \left(-\frac{24}{s}\right)$$

$$\boxed{\Delta_1 = \frac{s+27}{s}}$$ —— 0.5M

A/ Mason's gain formula,

$$\frac{C(s)}{R(s)} = \sum_{K=1} \frac{P_K \Delta_K}{\Delta} = \frac{P_1 \Delta_1}{\Delta}$$

$$= \frac{1 \cdot \left(\frac{s+27}{s}\right)}{1 + \frac{29}{s} + \frac{6}{s^2}}$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{s(s+27)}{s^2 + 29s + 6}}$$ — 1M

Ans.

Ans. 2 (c)

Given, system has TF

$$H(s) = \frac{2}{s+1}$$

Also given i/p = step signal.

$$\therefore, \quad R(s) = \frac{1}{s}$$

Now, $$H(s) = \frac{C(s)}{R(s)}$$

$$\Rightarrow \quad C(s) = \frac{2}{s+1} \cdot \frac{1}{s}$$

$$= \frac{2}{s} - \frac{2}{s+1}$$ (using partial fraction) — 0.5M

Taking inverse laplace,

step response $c(t) = \left(2 - 2e^{-t}\right) u(t)$

final value of $c(t) = \lim_{t \to \infty} c(t) = 2$ (final value theorem) — 0.5M

Let t_s be time taken to reach 98% of final value (i.e. 2)

$\therefore$, $2 - 2e^{-t_s} = 98\%$ of 2.

$\Rightarrow 2(1 - e^{-t_s}) = 2 \times 0.98$

$\Rightarrow t_s = 3.91$ sec.

$\boxed{t_s \simeq 4 \text{ sec}}$ —— 1M

Ans.

Ans. 3(a)

i) from the given open loop transfer function, we can say that the system is of Type-2.

Thus,

position & velocity error constants $= 0$. —— 1M

acceleration error constant $(K_a) = \lim_{s \to 0} s^2 G(s)$

$= \lim_{s \to 0} \frac{10(s+2)}{s+1}$

$\boxed{K_a = 20}$ —— 1M

ii) Given, $r(t) = 3 - 2t + (t^2/6)$

$\therefore$, $R(s) = \frac{3}{s} - \frac{2}{s^2} + \frac{1}{3s^3}$ —— 1M

$L\left[\frac{t^2}{6}\right] = \frac{2}{s^3} \cdot \frac{1}{6}$

$e_{ss} = \lim_{s \to 0} \frac{sR(s)}{1 + G(s)}$ (for unity feedback).

$$= \lim_{s \to 0} \frac{s\left(\frac{3}{s} - \frac{2}{s^2} + \frac{1}{3s^3}\right)}{1 + \frac{10(s+2)}{s^2(s+1)}}$$

$$= \cancel{\lim_{s \to 0}} \quad \cancel{\left(3s^2\right.} = \lim_{s \to 0} \frac{\left(3 - \frac{2}{s} + \frac{1}{3s^2}\right)\left(s^2(s+1)\right)}{s^2(s+1) + 10(s+2)}$$

$$= \frac{\frac{1}{3}}{20} = \frac{1}{60}$$

$$\therefore \boxed{e_{ss} = \frac{1}{60} = 0.0167}$$ — 2M

Ans

Ans 3 (b).

Given.

$$G(s) = \frac{K}{s(s+2)} \quad \text{— (1)}$$

$$\xi = 0.4$$

~~To find the value of K~~, compare eq(1) with generalized 2^{nd} order system of G(s)

(K → not required to find)

$$\frac{K}{s(s+2)} = \frac{\omega_n^2}{s(s+2\xi\omega_n)}$$

$$\Rightarrow K = \omega_n^2 \quad \& \quad \xi\omega_n = 1$$

$$\omega_n = \frac{1}{\xi} = \frac{1}{0.4} = 2.5$$

$$\boxed{K = 6.25}$$

$$\therefore \boxed{\omega_n = 2.5}, \text{ So, } \omega_d = \omega_n\sqrt{1-\xi^2}$$

$$= 2.5\sqrt{1-0.4^2}$$

$$\boxed{\omega_d = 2.29}$$

✓ $t_d = \dfrac{1+0.7\xi}{\omega_n} = \dfrac{1+(0.7\times0.4)}{2.5}$

$\boxed{t_d = 0.512}$ → 1M

✓ $t_r = \dfrac{\pi-\theta}{\omega_d} = 0.87$, $\boxed{t_r = 0.87}$ — 1M

✓ $t_p = \dfrac{\pi}{\omega_d} = 1.372$, $\boxed{t_p = 1.372}$ — 1M

✓ $M_p = e^{-\pi\xi/\sqrt{1-\xi^2}}$

$\boxed{M_p = 0.254}$ — 1M

✓ $t_s = \dfrac{4}{\xi\omega_n} = \dfrac{4}{0.4\times2.5} = 4$ (2% tolerance)

$t_s = \dfrac{3}{\xi\omega_n} = \dfrac{3}{1} = 3$ (5% tolerance).

$\boxed{t_s = \begin{cases} 4, & \text{for } 2\% \text{ tol.} \\ 3, & \text{for } 5\% \text{ tol.} \end{cases}}$ — 1M

NOTE: There are 2 corrections to the solutions provided above:

Q1(a) total no.of loops is 5 (i missed -G4H2)

Q3(a) position and velocity error constants = infinity.