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Q. 1 Let P_1, P_2 and P_3 be three products to be manufactured. The data of the problem can be summarized as follows.

Product	Product Ingredients A	B	C	Inert
P_1	5%	10%	5%	80%
P_2	5%	5%	10%	80%
P_3	20%	5%	10%	65%
Cost/kg (Rs)	80	20	50	20

Cost of P_1 $= 5\% \times 80 + 10\% \times 20 + 5\% \times 50 + 80\% \times 20$
$= 4 + 2 + 2.50 + 16 =$ Rs 24.50/kg.

Cost of P_2 $= 5\% \times 80 + 5\% \times 20 + 10\% \times 50 + 80\% \times 20$
$= 4 + 1 + 5 + 16 =$ Rs 26/kg.

Cost of P_3 $= 20\% \times 80 + 5\% \times 20 + 10\% \times 50 + 65\% \times 20$
$= 16 + 1 + 5 + 13 =$ Rs 35/kg.

(2)

Let x_1, x_2 and x_3 be the quantity (in kg) of P_1, P_2 and P_3 respectively to be manufactured.

The LP problem can be formulated as

Maximize (net profit) Z = (Selling price − Cost price) × (Quantity of product)

$$= (40.50 - 24.50)x_1 + (43-26)x_2 + (45-35)x_3$$

$$= 16x_1 + 17x_2 + 10x_3$$

Subject to the constraints

$$\frac{1}{20}x_1 + \frac{1}{20}x_2 + \frac{1}{5}x_3 \le 50$$

$$\Rightarrow x_1 + x_2 + 4x_3 \le 1000$$

$$\frac{1}{10}x_1 + \frac{1}{20}x_2 + \frac{1}{20}x_3 \le 90$$

$$\Rightarrow 2x_1 + x_2 + x_3 \le 1800$$

$$\frac{1}{20}x_1 + \frac{1}{10}x_2 + \frac{1}{10}x_3 \le 60$$

$$\Rightarrow x_1 + 2x_2 + 2x_3 \le 1200$$

$$x_1 \le 30, \quad x_1, x_2, x_3 \ge 0$$

Q. 2

$$\text{Max } Z = x_1 + 2x_2 + 3x_3 - x_4$$

Sub to

$$x_1 + 2x_2 + 3x_3 = 15$$
$$2x_1 + x_2 + 5x_3 = 20$$
$$x_1 + 2x_2 + x_3 + x_4 = 10$$

S.F.

$$\text{Max } z = x_1 + 2x_2 + 3x_3 - x_4 - MA_1 - MA_2$$

Sub to

$$x_1 + 2x_2 + 3x_3 + A_1 = 15$$
$$2x_1 + x_2 + 5x_3 + A_2 = 20$$
$$x_1 + 2x_2 + x_3 + x_4 = 10$$

		$c_j \rightarrow$	1	2	3	-1	-M	-M	
C_B	B	b (= x_B)	x_1	x_2	x_3	x_4	A_1	A_2	Min ratio x_B/x_3
-M	A_1	15	1	2	3	0	1	0	$15/3 = 5$
-M	A_2	20	2	1	5	0	0	1	$20/5 = 4$
-1	x_4	10	1	2	1	1	0	0	$10/1 = 10$
		z_j	-3M-1	-3M-2	-8M-1	-1	-M	-M	
		$c_j - z_j$	3M+2	3M+4	8M+4 ↑	0	0	0	

$R_2(\text{new}) = R_1(\text{old}) \times \frac{1}{5}$, $\quad R_1(\text{new}) = R_1(\text{old}) - 3R_2(\text{new})$

$R_3(\text{new}) = R_3(\text{old}) - R_2(\text{new})$

	C_j		1	2	3	-1	-M	
C_B	B	$b(=x_B)$	x_1	x_2	x_3	x_4	A_1	Min ratio x_B/x_2
$-M$	A_1	3	$-\frac{1}{5}$	$\frac{7}{5}$	0	0	1	$\frac{3}{7/5} = \frac{15}{7} \rightarrow$
3	x_3	4	$\frac{2}{5}$	$\frac{1}{5}$	1	0	0	$\frac{4}{1/5} = 20$
-1	x_4	6	$\frac{3}{5}$	$\frac{9}{5}$	0	1	0	$\frac{6}{9/5} = \frac{30}{9}$
		Z_j	$\frac{M}{5}+\frac{3}{5}$	$-\frac{7M}{5}-\frac{6}{5}$	3	-1	-M	
		$C_j - Z_j$	$-\frac{M}{5}-\frac{2}{5}$	$\frac{7M}{5}+\frac{16}{5}$	0	0	0	

↑

$R_1(\text{new}) = R_1(\text{old}) \times \frac{5}{7}$, $R_2(\text{new}) = R_2(\text{old}) - \frac{1}{5}(R_1)\text{new}$

$R_3(\text{new}) = R_3(\text{old}) - \frac{9}{5}(R_1)\text{ new}$

	$C_j \rightarrow$						
C_B	B	$b(=x_B)$	x_1	x_2	x_3	x_4	Min ratio x_B
2	x_2	$\frac{15}{7}$	$-\frac{1}{7}$	1	0	0	—
3	x_3	$\frac{25}{7}$	$\frac{3}{7}$	0	1	0	$\frac{25}{7} \times \frac{7}{3} = \frac{25}{3}$
-1	x_4	$\frac{15}{7}$	$\frac{6}{7}$	0	0	1	$\frac{15}{7} \times \frac{7}{6} = \frac{15}{6} = \frac{5}{2}$
$Z = \frac{90}{7}$	Z_j		$\frac{1}{7}$	2	3	-1	
	$C_j - Z_j$		$\frac{6}{7}$	0	0	0	

↑

$R_3\text{ new} = R_3(\text{old}) \times \frac{7}{6}$, $R_1(\text{new}) = R_1(\text{old}) + \frac{1}{7} \times R_3$

$R_2\text{ new} = R_2(\text{old}) - \frac{3}{7} R_3(\text{new})$

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		c_j	-1	2	3	-1.
C_B	B	$b\,(=x_B)$	x_1	x_2	x_3	x_4
2	x_2	$\frac{15}{6}$	0	1	0	$\frac{1}{6}$
3	x_3	$\frac{15}{6}$	0	0	1	$-\frac{3}{6}$
1	x_1	$\frac{15}{6}$	1	0	0	$\frac{7}{6}$
$Z=15$		Z_j	1	2	3	0
		$c_j - Z_j$	0	0	0	-1

All $c_j - Z_j \le 0$ Optimal Solⁿ has been arrived at with value

$x_1 = \frac{15}{6}$

$x_2 = \frac{15}{6}$

$x_3 = \frac{15}{6}$

Max $Z = 15$.

Q.3

Let x_1 and x_2 be the unit of model M_1 and M_2 ~~respectively~~ to be manufactured respectively.

$$\text{Max } Z = 400x_1 + 100x_2$$

Sub to
$$4x_1 + 2x_2 \le 1600$$
$$\frac{5}{2}x_1 + x_2 \le 1200$$
$$\frac{9}{2}x_1 + \frac{3}{2}x_2 \le 1600$$
$$x_1, x_2 \ge 0$$

Q.4 Dual problem

Primal LP can be written as

Minimize $Z_x = x_1 + 2x_2$

Sub to
$$-2x_1 - 4x_2 \ge -160$$
$$x_1 - x_2 \ge 30$$
$$x_1 - x_2 \le 30 \quad \text{or}$$
$$-x_1 + x_2 \ge -30$$
$$x_1 \ge 10$$

Let y_1, y_2, y_3 and y_4 be the dual variables.

$$\text{Max } Z_y = -160y_1 + 30y_2 - 30y_3 + 10y_4.$$

Sub to
$$-2y_1 + y_2 - y_3 + y_4 \le 1$$
$$-4y_1 - y_2 - y_3 \le 2$$
$$y_1, y_2, y_3, y_4 \ge 0.$$

Let $y = y_2 - y_3$

$$\text{Max } Z_y = -160y_1 + 30y + 10y_4$$

sub to

$$-2y_1 + y + y_4 \le 1$$
$$-4y_1 - y \le 2$$

and $y_1, y_4 \geq 0$

y being unrestricted in sign.

Q.5 $\text{Max } z^* = -x_1 - x_2$

sub to $-2x_1 - x_2 \le -2$

$x_1 + x_2 \le -1$

S.F

$$\text{Max } z^* = -x_1 - x_2 + 0s_1 + 0s_2$$

sub to $-2x_1 - x_2 + s_1 = -2$

$x_1 + x_2 + s_2 = -1$

Initial basic sol^n (infeasible is obtained) by setting

$x_1 = x_2 = 0$, $s_1 = -2$, $s_2 = -1$, $\text{Max } z^* = 0$

		$c_j \to$	-1	-1	0	0	
C_B	B	$b\,(=x_B)$	x_1	x_2	s_1	s_2	
0	s_1	-2	-2	-1	1	0	$\to$
0	s_2	-1	1	1	0	1	
		Z_j	0	0	0	0	
		$c_j - Z_j$	-1	-1	0	0	

Since all $(c_j - z_j) \le 0$ and sol^n value (x_{Bi}) are not non-negative, an optimal but infeasible sol^n has been obtained. Now in order to obtain a feasible sol^n, we select a basic variable to leave the basis and non-basic variable to enter the basis as follows:

key row $\text{Min}\{x_{Bi} : x_{Bi} < 0\} = \text{Min}\{-1, -2\} = -2$ (x_{B_1})

∴ the basic variable s_1 leave.

key column (variable to be enter the basis)

$$= \text{Min}\left\{\frac{c_j - z_j}{y_{rj}},\ y_{rj} < 0\right\}$$

$$= \text{Min}\left\{\frac{-1}{-2}, \frac{-1}{-1}\right\} = \frac{1}{2}$$

Iteration - I

C_B	B	$b\,(=x_B)$	x_1	x_2	s_1	s_2
-1	x_1	1	1	$\frac{1}{2}$	$-\frac{1}{2}$	0
0	s_2	-2	0	$\frac{1}{2}$	$\frac{1}{2}$	1 →
		z_j	1	$-\frac{1}{2}$	$-\frac{3}{2}$	-2
		$c_j - z_j$	-2	$\frac{-1}{2}$	$\frac{-3}{2}$	-2

Key row The only element (-2) in sol^n value column is negative, key row y_{r2}

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<u>Key Colum</u>

Since no element in the key and of x_2 is non-negative, the soln obtain is feasible with $x_1 = 1$, $x_2 = 0$.

$\text{Max } z^* = -1$

$\text{Min } z = 1$

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Q.6 Minimize total operating cost

$$Z = 3x_1 + 4x_2$$

sub to the constraints

$$15x_1 + 10x_2 \le 3000$$

$x_1 \le 150$ } due to the
$x_2 \le 100$ } policy of retaining at least one class-A truck with every two class-B truck in ~~garage~~ reserve.

~~A ship has three cargo loads ~~forward~~ after and centre.~~