Divide and Conquer - Basic: Recurrence Relations

Directly solve, by unrolling the recurrence, the following relations to find their exact solutions.

1. $T(n) = T(n-1) + n$

2. $T(n) = T(\frac{n}{2}) + 1$

Use induction to show bounds on the following recurrence relations.

3. Show that $T(n) = 2T(\frac{n}{2}) + n \in \Omega(nlog(n))$. *Note: We are using big-omega here, so your inequality will use* $T(n) \geq c * n * log(n)$.

4. Show that $T(n) = 2T(\frac{n}{2} + 17) + n \in O(nlog(n))$.

5. Show that $T(n) = 2T(\sqrt{n}) + log(n) \in O(log(n) * log(log(n)))$. *Hint: Try creating a new variable m and substituting the equation for m to make it look like a common recurrence we've seen before. Then solve the easier recurrence and substitute n back in for m at the end.*

6. Show that $T(n) = 4T(\frac{n}{2}) + n \in \Theta(n^2)$. You'll need to subtract off a lower-order term to make the induction work here.

7. Show that $T(n) = 4T(\frac{n}{3}) + n \in \Theta(n^{log_3(4)})$. You'll need to subtract off a lower-order term to make the induction work here.

Use the master theorem (or main recurrence theorem if applicable) to solve the following recurrence relations. State which case of the theorem you are using and why.

8. $T(n) = 2T(\frac{n}{4}) + 1$

9. $T(n) = 2T(\frac{n}{4}) + \sqrt{n}$

10. $T(n) = 2T(\frac{n}{4}) + n$

11. $T(n) = 2T(\frac{n}{4}) + n^2$