Indian Statistical Institute

BSDS Ist Year

Academic Year 2024 - 2025: Semester I

Course: Probability Theory I

Instructor: Antar Bandyopadhyay

Assignment # 13

Date Given: November 27, 2024 **Date Due: December 11, 2024**
Total Points: 10

1. Suppose $U_1, U_2, \cdots, U_n$ be i.i.d. Uniform $(0,1)$. Let $X := \min_{1 \le i \le n} U_i$ and $Y := \max_{1 \le i \le n} U_i$. Find the marginal CDFs and pdfs of X and Y. Find the joint CDF and joint pdf of (X, Y).

5.1.9 Suppose a straight stick of unit length is broken in three at two points chosen independently at random along its length. What is the chance that the three sticks so formed can be made into the sides of a triangle?

5.2.16 Suppose X_1, X_2, X_3 are independent *exponential* random variables with parameters $\lambda_1, \lambda_2, \lambda_3$ respectively. Evaluate $\mathbf{P}(X_1 < X_2 < X_3)$.

4.6.4 Let $X := \min(S, T)$ and $Y := \max(S, T)$ for independent random variables S and T with a common probability density function f. Let Z denote the indicator of the event $[S < T]$.

(a) What is the distribution of Z?

(b) Are X and Z independent? Are Y and Z independent? Are (X, Y) and Z independent?