INDIAN STATISTICAL INSTITUTE
Assignment-4 (Mathematics III)
Bachelor of Statistical Data Science (BSDS)

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1. If a graph G does not contain a patth of length more than 2, show that it's connected components are all star graphs.
2. Prove that a graph G is bipartite if and only if every subgraph H of G has an independent set consisting of atleast halpf of $V(H)$.
3. For $k \geq 1$, a $k-$regular **bipartite graph** has a perfect matching.
4. For $k \geq 1$, show that there are $k-$ regular graphs with no perfect matching.
5. What is a necessary and sufficient condition for a tree to be a complete bipartite graph ? Justify.
6. Let G be an $n-$ vertex simple graph with $n \geq 2$. Determine the maximum possible number of edges in G under each of the following conditions
   - G has an independent set of size k.
   - G has exactly k components.
   - G is disconnected.
7. Is it true that every tree has at most one perfect matching ? Justify.
8. Let T be a tree on n vertices such that $\alpha(T) = k$. Can you determine $\alpha'(T)$ in terms of n, k?