NDIAN STATISTICAL INSTITUTE
Assignment-2 (Mathematics III)
Bachelor of Statistical Data Science (BSDS)

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1. Find an orthonormal basis of

$$P_3 := \{f(x) \in \mathbb{R}[x] : \deg f(x) < 3\}$$

with respect to the inner product

$$\langle f, g\rangle := \int_0^1 f(t)\, g(t)\, dt.$$

2. Suppose

$$W = \{(x, y) \in \mathbb{R}^2 : x + y = 0\}.$$

Find the shortest distance of $(a, b) \in \mathbb{R}^2$ from W with respect to

(i) the standard inner product $\langle (x_1, y_1), (x_2, y_2)\rangle = x_1x_2 + y_1y_2$,
(ii) the inner product $\langle (x_1, y_1), (x_2, y_2)\rangle = 2x_1x_2 + y_1y_2$.

3. Suppose W is a subspace of the finite dimensional inner product space V. Define

$$W^\perp := \{v \in V : \langle w, v\rangle = 0 \ \ \forall w \in W\}.$$

Show the following statements:

(a) $W^\perp$ is a subspace of V.
(b) $W \cap W^\perp = \{0\}$.
(c) $V = W \oplus W^\perp$.
(d) $(W^\perp)^\perp = W$.

4. Find a singular value decomposition of the following matrices:

$$A = \begin{pmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \end{pmatrix}, \quad A = \begin{pmatrix} 1 & 1 & 1 \\ -1 & 0 & -2 \\ 1 & 2 & 0 \end{pmatrix}.$$

5. If A is a real $n \times n$ matrix, a factorization

$$A = GQ$$

where G is positive (symmetric positive semidefinite) and Q is orthogonal, is called a *polar decomposition* of A. Prove that every square real matrix has a polar form.

6. If A is $n \times n$ show that AA^T and A^TA are similar.