Empirical Demand Analysis:
Part-I

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Introduction

- **Empirical Demand Analysis**
- The theory of consumption behaviour developed so far applies to single individual. In practice, consumption decisions are made by households, i.e., a collection of individuals.
- Data are either collected at a fixed time point from several individual households or at various time points from individual households. The first approach is known as cross-section approach and the data so collected are called Cross Section Data and the second approach is called the time series approach, and these data are known as Time Series Data.
- For consumption analysis the core data consist of consumptions of different commodities (y) and the associated prices (p) and the disposable income (x). In cross section analysis these data are collected from each household (y_{ij}, p_{ij}, x_j, where the suffix i stands for commodity and suffix j stands for household, say). For a small region, p_{ij} values will be the same. So, we may consider data on y_{ij} and x_j only.

Introduction (Continued)

- Panel data combines **cross-section (CS)** and **time series (TS)** data: the same individuals (persons, firms, cities, etc.) are observed at several points in time. I.e., here, we fix a set of households and collect data at various time points from the same set of households. This is the best type of data that can be envisaged. This is also known as Continuous Cross Section data. This procedure, being costly and laborious, is seldom adopted.
- Thus, we can make Cross-Sectional Studies, Time Series Studies or Panel Studies depending on the nature of data available for our purpose.

Analysis with Cross Section Data

- For a CS analysis we can write the demand functions as

$$y_i = f_i(p_1, p_2, \dots, p_n, x|u), i = 1,2, \dots, n,$$

- where i stands for ith commodity, u represents the tastes and preferences. Household size, occupation, regional particularities etc. condition this u. If the geological coverage is small in CS data, then we may expect homogeneous tastes and preferences.
- The household particulars may be classified into two groups.

(i) Quantitative: Household size, age and sex composition of the household, etc.

(ii) Qualitative: Occupation, region, race, etc.

- For a small geographical region, we have

$$y_i = f_i(x|u), i = 1,2, \dots, n.$$

- This is the general definition of an Engel curve. In case of homogeneous households, we have

$$y_i = f_i(x), i = 1,2, \dots, n,$$

- as the Engel curves.

Engel's Law and Engel Curves

- **Engel's law:** The proportion of income spent on a necessary good declines as income rises. $\Rightarrow 0 < \eta_{ix} < 1$.
- **Three Types of Engel's Relation:**

(I) As x increases, y_i decreases, i.e., $\frac{\partial y_i}{\partial x} < 0$. These are inferior goods. Examples: Coarse cloths, Bidi, etc.

(II) As x increases, y_i increases, but slowly, i.e., $\left[\frac{\partial y_i}{\partial x} > 0 \text{ and } \frac{\partial^2 y_i}{\partial x^2} < 0\right]$.

$$0 < \eta_{ix} < 1.$$

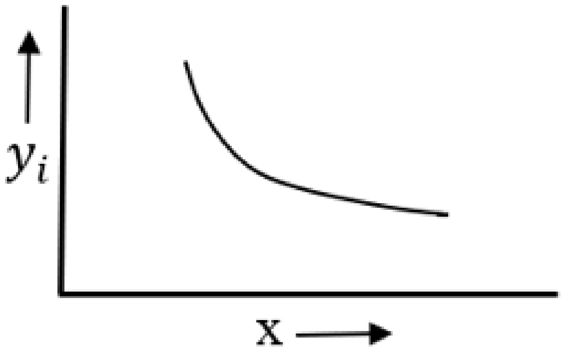
These are necessary goods. Examples: Cereals, casual dresses, etc.

(III) As x increases, y_i increases relatively faster, i.e., $\left[\frac{\partial y_i}{\partial x} > 0 \text{ and } \frac{\partial^2 y_i}{\partial x^2} > 0\right]$.

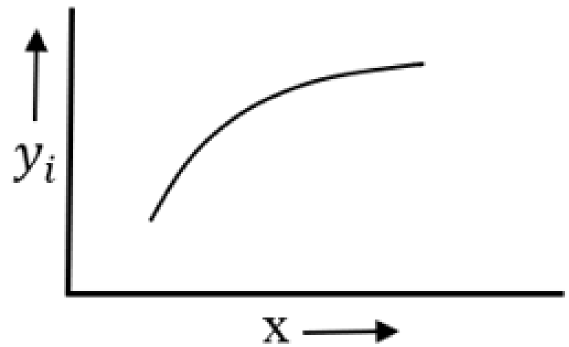
$$\eta_{ix} > 1.$$

These are luxury goods. Examples: Silk clothing, costly jewelry, etc.

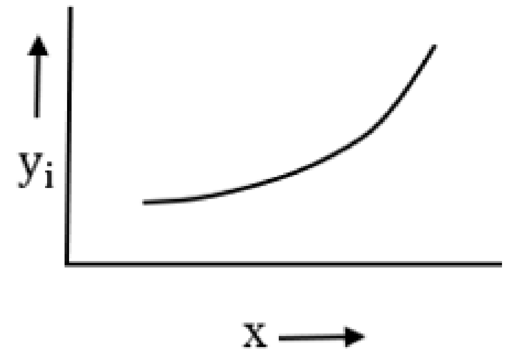
Three Types of Engel Curves



Inferior good



Necessary good



Luxury good

Remember that

$$\eta_{ix} = \frac{\partial \mathrm{Ln}(y_i)}{\partial \mathrm{Ln}(x)} = \frac{(\partial y_i / y_i) \times 100}{(\partial x / x) \times 100}$$

$$= \frac{\text{Percentage rise in } y_i}{\text{Percentage rise in x}}$$

$$= \text{Percentage change in } y_i \text{ for 1\% rise in x.}$$

Engel Elasticity in Value Terms

- If the consumption of a commodity is measured in value terms, then the

$$\text{Engel elasticity} = \frac{\partial \mathrm{Ln}(\mathrm{v_i})}{\partial \mathrm{Ln}(\mathrm{x})}.$$

Here, $$\frac{\partial \mathrm{Ln}(\mathrm{v_i}/\mathrm{x})}{\partial \mathrm{Ln}(\mathrm{x})} = \frac{\partial \mathrm{Ln}(\mathrm{v_i})}{\partial \mathrm{Ln}(\mathrm{x})} - 1.$$

So, if $$\frac{\partial \mathrm{Ln}(\mathrm{v_i})}{\partial \mathrm{Ln}(\mathrm{x})} > 1,$$

then as x increases $\mathrm{v_i}/\mathrm{x}$ also increases. Again,

$$\sum \mathrm{v_i} = \mathrm{x} \Rightarrow \sum \frac{\partial \mathrm{v_i}}{\partial \mathrm{x}} = 1.$$

$$\text{or,} \sum \frac{\partial \mathrm{v_i}}{\partial \mathrm{x}} . \frac{\mathrm{x}}{\mathrm{v_i}} . \frac{\mathrm{v_i}}{\mathrm{x}} = 1 \Rightarrow \sum \frac{\mathrm{v_i}}{\mathrm{x}} \eta_{\mathrm{ix}} = 1 \Rightarrow \sum \mathrm{w_i}\, \eta_{\mathrm{ix}} = 1.$$

- I.e., the weighted average (since $\sum \mathrm{w_i} = 1$) of Engel elasticities for all items = 1.

Quantity and Quality Elasticities

- Also, v = qp (q = quantity and p = price)

$$\mathrm{Ln(v)} = \mathrm{Ln(q)} + \mathrm{Ln(p)}.$$

$$\text{or,} \qquad \frac{\partial \mathrm{Ln(v)}}{\partial \mathrm{Ln(x)}} = \frac{\partial \mathrm{Ln(q)}}{\partial \mathrm{Ln(x)}} + \frac{\partial \mathrm{Ln(p)}}{\partial \mathrm{Ln(x)}}.$$

$$\text{or}, \eta_v = \eta_q + \eta_p.$$

- Value elasticity of income = quantity elasticity of income + price elasticity of income.
- As x increases consumer buys relatively expensive varieties and so p increases, i.e., $\eta_p > 0$. This is also called the quality elasticity. η_p denotes the rate at which consumer shifts to a finer quality with rising level of living.

Per Capita Formulation of Engel Relation

- Now, instead of considering a consumer, we consider one household, and then

$$q = f(x, n),$$

- where n is the numbers of members in the household. Assume that the function is homogeneous of degree one, i.e.,

$$f(cx, cn) = cf(x, n), c > 0,$$

- i.e., a household with x = 500 and n = 10 will consume exactly 10 times compared to a household with x = 50 and n =1. If it happens for an item, then we say that there is "no economy of scale" for the item. This is unrealistic for items like 'fuel and light', 'durables', 'house rent' etc.
- If there is "no economy of scale", then putting c = 1/n, we get

$$\frac{q}{n} = f\left(\frac{x}{n}, 1\right) = g\left(\frac{x}{n}\right),$$

$$\text{or}, y = g(x).$$

- This is the per capita formulation of Engel relation.

Analysis of EC: Cross Section Data

- **Analysis of Engel Curve: Cross Section Data**
- Let us recall that the cross-section data on household expenditure consists of the following:

$$y_{ij} = \text{Expenditure on commodity i by the jth household,}$$

$$j = 1,2, \dots, N; N = \text{No. of sample households.}$$

$$x_j = \text{income or total expenditure of the jth household.}$$

$$n_j = \text{household size of the jth household.}$$

- There are other particulars about the household characteristics such as occupation, age-sex composition etc.
- We assume

$$y_i = f_i(p_1, p_2, \dots, p_n, x|u), i = 1,2, \dots, n,$$

- where i stands for ith commodity, u represents the tastes and preferences. Since prices do not vary, we have

$$y_i = f_i(x|u), i = 1,2, \dots, n.$$

Problems of Cross-Section Analysis

- **Problems of Cross-Section Analysis:**

(I) Choice of Variables: The two main variables are y and x.

- The first problem is about taking appropriate y values, i.e., whether we should take quantity consumed or expenditure incurred and then whether we should take total or per capita.
- The second problem is about taking appropriate x values, i.e., whether we should take total expenditure or income and then whether we should take total or per capita.
- Total expenditure of the household (x) data is more reliable than income data. Total expenditure data is expected to be more stable and hence more directly related to the permanent level of living. Because, if income falls, the consumption may not fall due to level of living and consumption habit.

Problems of Cross-Section Analysis

- **Problems of Cross-Section Analysis:**

(II) Specification of the Form of Engel Curve: We have restrictions like

$$\sum y_i = x, \text{or} \sum w_i \eta_{ix} = 1 \text{ (Adding up criterion)}$$

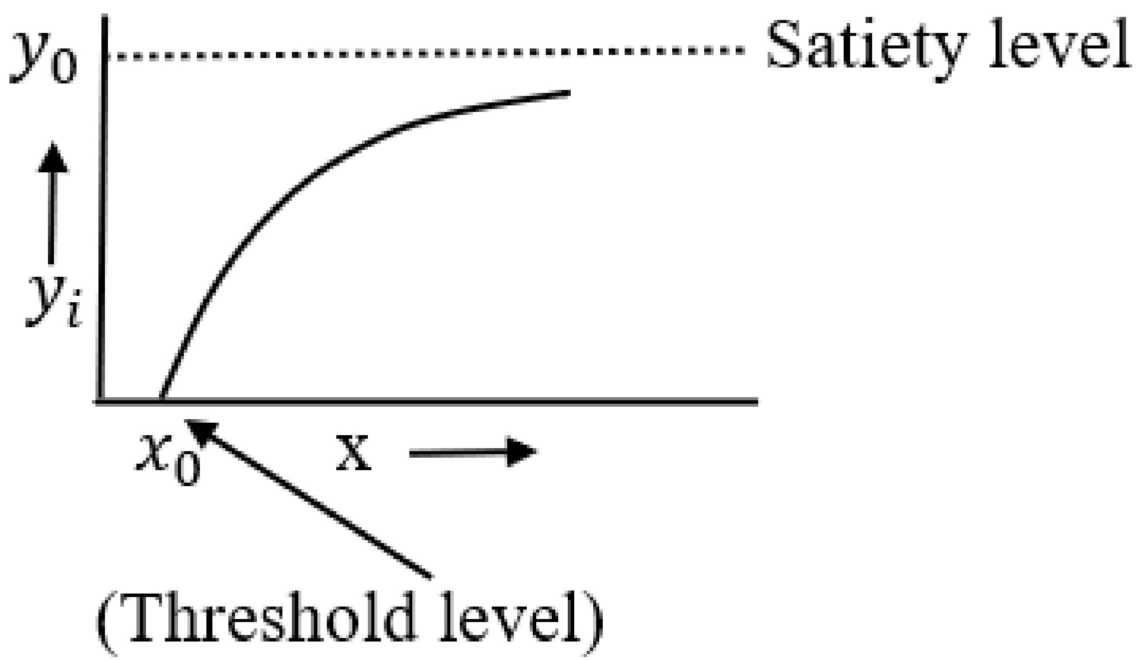
- Note that w_i's are known as Engel ratio.
- For the linear Engel curve the Adding up criterion (AUC) may be satisfied.
- In general, if we restrict the shape of the functions f_i such that they satisfy AUC then we may have poor fit of the Engel curve. So, we usually give up the AUC.

Criteria for Choosing Engel Curve

A. Economic Criteria:

A1. Adding up Criteria: Already discussed.

A2. Threshold and Satiety Levels: The form of the EC should have a (i) Threshold level of x and a (ii) satiety level of y; where y is physical consumption.

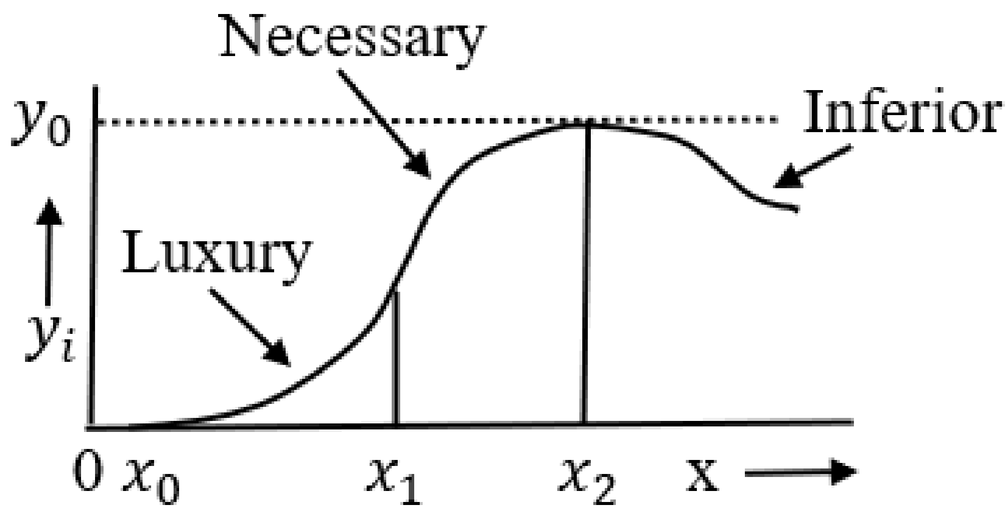


Threshold and Satiety Levels

(i) Threshold level exists mainly for luxury goods. Consumption of luxury goods starts only after a certain level of income x_0, which is the threshold level.

(ii) Satiety level exists mainly for necessary goods. There is an upper bound of consumption of necessary goods, viz. y_0, which is the satiety level.

- In general, we search for an Engel curve of the following shape.



Sigmoid Forms

- This has three parts in it. x_1 is the point of inflexion. At x_2, $y_0 = f(x_2)$ is the satiety level of y.

  For $x \in (x_0, x_1)$: It behaves as a luxury commodity.

  For $x \in (x_1, x_2)$: It behaves as a necessary commodity.

  For $x > x_2$: It behaves as an inferior commodity.

- Examples of functions behaving as in the above diagram:

  (i) Lognormal: $y = K\Lambda(x|\mu, \sigma^2)$

  (ii) $\log - \log -$ inverse: $Ln(y) = \alpha + \beta Ln(x) + \frac{\gamma}{x}$

- These functions have the sigmoid form. Sigmoid functions have domain of all real numbers, with return (response) value commonly monotonically increasing but could be decreasing. Log-log-inverse behaves like a sigmoid form depending on the values of α, β and γ.

Interpretation of Parameters

- **A3. Interpretation of Parameters:**

  If $\mathrm{y_i} = \mathrm{\beta_i x}$, then $\mathrm{\beta_i}$: mpc as well as apc of comm. i.

  If $\mathrm{y_i} = \mathrm{\alpha_i} + \mathrm{\beta_i x}$, then $\mathrm{\beta_i}$: mpc of comm. i.

  If $\mathrm{Ln(y_i)} = \mathrm{\alpha_i} + \mathrm{\beta_i Ln(x)}$, then $\mathrm{\beta_i}$: income elasticity of comm. i.

- We should have such forms of EC's that their parameters can be given nice meanings as above. But then we may get EC with a poor fit statistically. So, we may have to leave these criteria.

Statistical Criteria

B. Statistical Criteria:

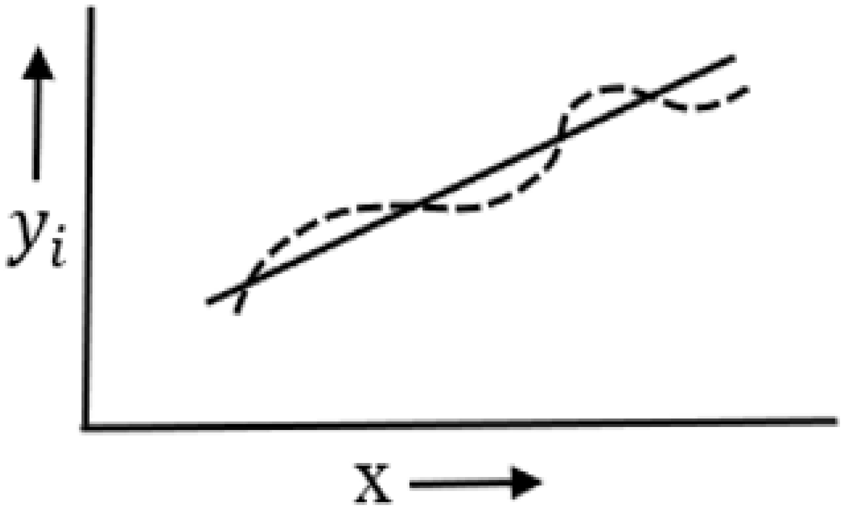
- **B1. Goodness of Fit:**
- Statistical criterion of goodness of fit in regression is given by squared multiple correlation coefficient (i.e., Coefficient of Determination) R^2.
- However, R^2 values keep on increasing as number of regressors are increased, i.e., number of parameters are increased. We must have a stopping rule.
- Adjusted R^2 can be used for this purpose. There are other criteria like the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC). These are applicable in any model and not necessarily in regression model.

AIC and BIC

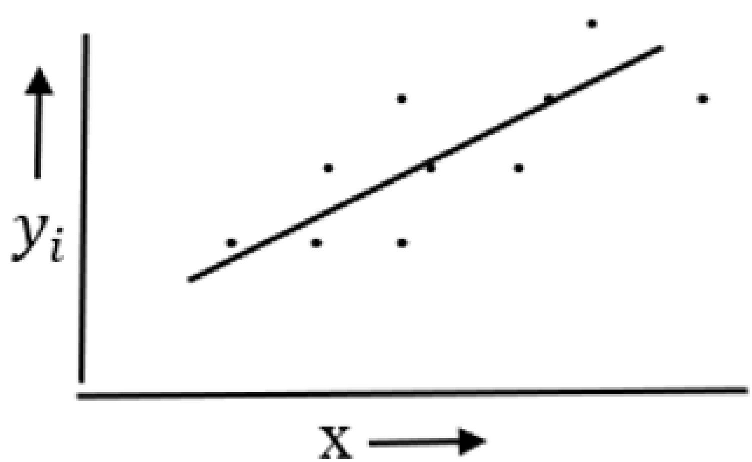
- **AIC:** Let k be the number of estimated parameters in the model. Let $\hat{L}$ be the maximum value of the Likelihood function for the model. Then the AIC value of the model is the following.
$$\mathrm{AIC} = 2\mathrm{k} - 2\mathrm{Ln}(\hat{\mathrm{L}}).$$
- Given a set of models for the data, the preferred model is the one with the minimum AIC value.
- **BIC:** The formula for the Bayesian information criterion (BIC) is similar to the formula for AIC, but with a different penalty for the number of parameters. With AIC the penalty is 2k, whereas with BIC the penalty is ln(n)×k. I.e.,
$$\mathrm{BIC} = \mathrm{k} \times \mathrm{Ln}(\mathrm{n}) - 2\mathrm{Ln}(\hat{\mathrm{L}}),$$
- where n is the number of observations. Here also the preferred model is the one with the minimum BIC value.
- One can easily see that the penalty is directly related with the number of parameters, k. That is why we seek for a parsimonious model. Parsimonious models are simple models with great explanatory predictive power. They explain data with a **minimum number of parameters**, or predictor variables. We should use no more “things” than necessary.

Randomness of Residuals

- **B2. Randomness of Residuals:**
- Consider the following two cases.



In this case there is autocorrelation between the residuals. The dotted line would have been a better fit.



In this case the residuals are distributed randomly. This is the ideal case.

Statistical Criteria

- The randomness of residuals is necessary. The determination of R^2 corresponding to a fitted Engel curve need not furnish the entire information about suitability of the specific form of EC. The regression residuals may show systematic pattern, and this indicates misspecification of the underlying true Engel curve. Besides, there may be heteroscedastic errors. Appropriate statistical methods should be used for estimation depending on the nature of residuals.
- **B3. Computational Ease:**
- The EC should show the desired curvature with as few parameters as possible. But very few parameters do not always imply that there is less difficulty in computation. It depends on the form of the curve.
- **B4. Tractability on Subsequent Analysis:**
- Due to the availability of computers and computer programming languages and statistical packages, complication in the functional form is not considered as a problem now a days. Sometimes, it may be necessary to derive something using the EC, which may be difficult if the form of the EC is complicated.

Popular Simple Forms of Engel Curves

- **Popular Simple Forms of Engel Curves:**

A. Two parameter Forms:

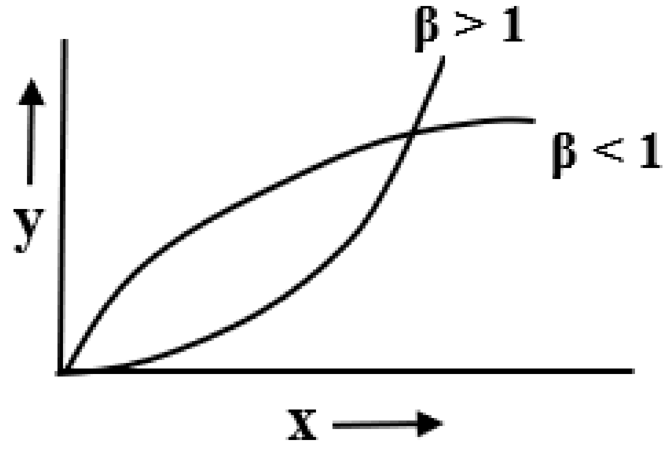
(i) Linear: $y = \alpha + \beta x$,

  - Here β has a very good interpretation. It is the marginal propensity to consume. Linear form may suffice when we compare two regions under same group, but for projection purposes it may be erroneous.
  - As we have already seen Sigmoid form is desirable in the whole range of $x \varepsilon (0, \infty)$. In a certain range (say $x \varepsilon (A, B)$) of x, the Sigmoid form may be well approximated by a linear form.

Double Log Form

(ii) Double log: $Ln(y) = \alpha + \beta Ln(x)$

- In this case, β is interpreted as the income elasticity of demand. The double log form, thus, has constant elasticity. This form fits well for luxurious goods compared to necessary goods. It does not satisfy AUC and satiety criterion. Threshold property also does not exist. But the form is still useful.



Other Two Parameter Forms

(iii) **Semi-log:** $y = \alpha + \beta Ln(x)$.

(iv) **Log-inverse:** $Ln(y) = \alpha + \beta/x$.

(v) **Hyperbola:** $y = \alpha + \beta/x$.

(vi) **Working-Leser:** $y/x = \alpha + \beta Ln(x)$.

- In all the above cases the LS theory is applicable since the forms are linear in parameters.

Three Parameter Forms

B. Three parameter Forms:

(vii) Log-log-inverse: $Ln(y) = \alpha + \beta Ln(x) + \gamma/x$.

(viii) Parabola (Quadratic): $y = \alpha + \beta x + \gamma x^2$.

(ix) Semi-log-inverse: $y = \alpha + \beta Ln(x) + \gamma/x$.

- Observe that in all the above three cases also the LS theory is applicable since the forms are linear in parameters.

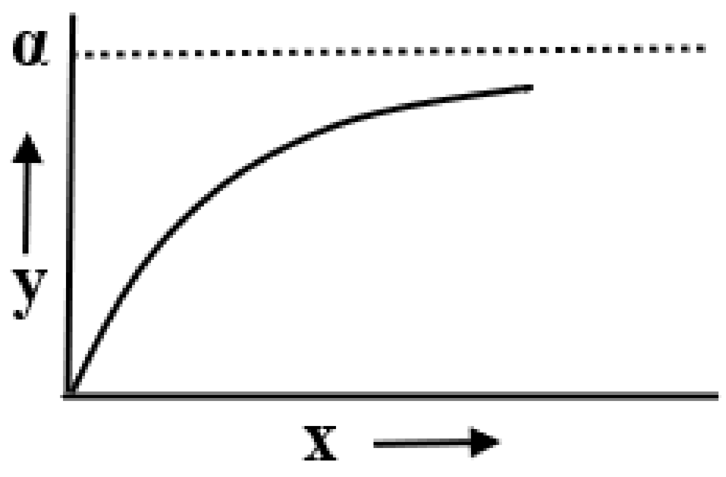
Tornquist's Forms

C. Tornquist's Forms:

- Tornquist's Forms are used for classification of commodities into necessary, luxury and inferior goods.

(x) For necessary goods:

$$y = \frac{\alpha x}{x + \beta}, \alpha \text{ and } \beta > 0.$$



$$y = \frac{\alpha x}{x + \beta}, \alpha \text{ and } \beta > 0.$$

$x = 0 \Rightarrow y = 0.$

$x \to \infty \Rightarrow y \to \alpha.$

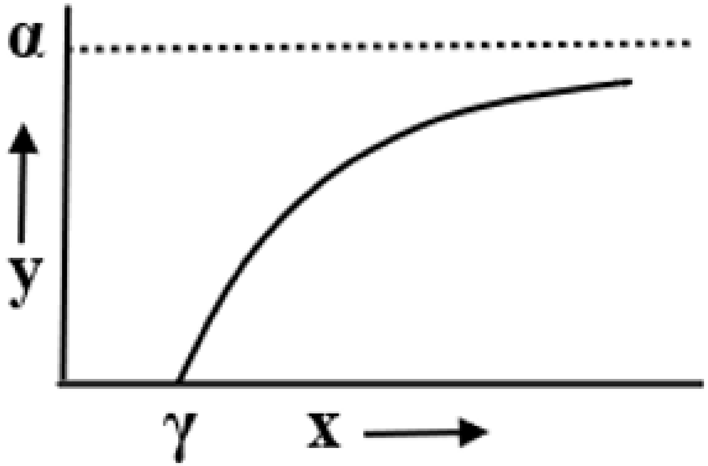
No Threshold.

Satiety level = α.

Tornquist's Form for Luxury Goods

(xi) For luxury goods:

$$y = \frac{\alpha(x-\gamma)}{x+\beta}, x > \gamma, \alpha, \gamma \text{ and } \beta > 0.$$



$$y = \frac{\alpha(x-\gamma)}{x+\beta}, x > \gamma, \alpha, \gamma \text{ and } \beta > 0.$$

$x < \gamma \Rightarrow y = 0.$

$x \to \infty \Rightarrow y \to \alpha.$

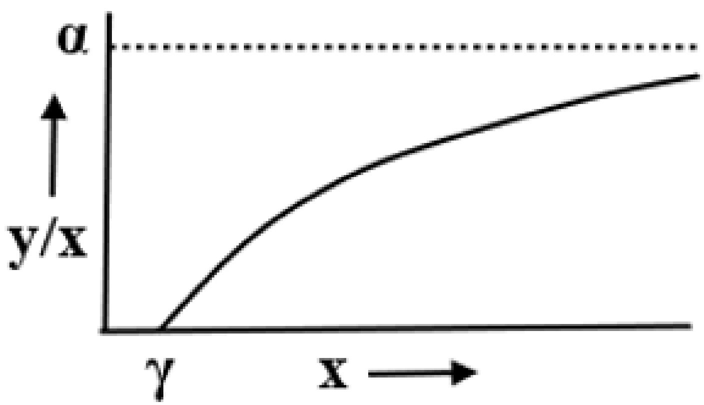
Threshold level $= \gamma.$

Satiety level $= \alpha.$

Tornquist's Form for Semi-Luxury Goods

(xii) For semi-luxury goods:

$$y/x = \frac{\alpha(x-\gamma)}{x+\beta}, x > \gamma, \alpha, \gamma \text{ and } \beta > 0.$$



$$\frac{y}{x} = \frac{\alpha(x-\gamma)}{x+\beta}, x > \gamma, \alpha, \gamma \text{ and } \beta > 0.$$

$x < \gamma \Rightarrow y = 0.$

$x \rightarrow \infty \Rightarrow y \rightarrow \alpha.$

Threshold level of $y/x = \gamma$.

Satiety level of $y/x = \alpha$.

Tornquist's Form for Inferior Goods

(xiii) For inferior goods:

$$y = \frac{\alpha x}{x - \beta}, x > \beta, \alpha \text{ and } \beta > 0.$$

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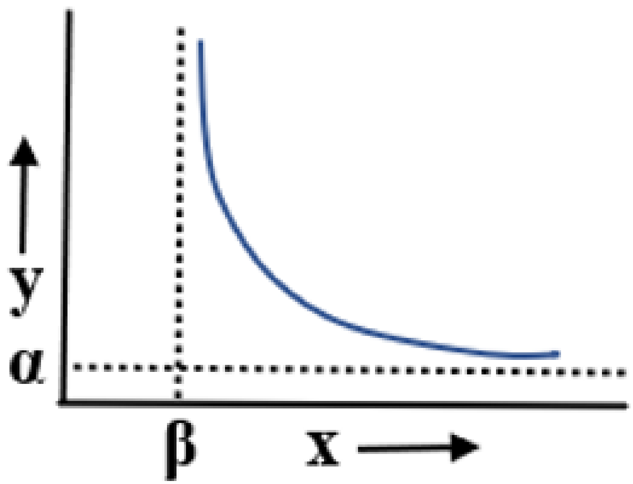
<td>
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$$y = \frac{\alpha x}{x - \beta}, x > \beta, \alpha \text{ and } \beta > 0.$$

$x \to \beta \Rightarrow y \to \infty.$
$x \to \infty \Rightarrow y \to \alpha.$

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Estimation of Tornquist's Forms

- Usual LS estimation is not possible in any of the Tornquist's models. Earlier, people used to take some transformation so that LS estimation is possible. E.g., let us take the EC as

$$y = \frac{\alpha x}{x + \beta} \Rightarrow \frac{x}{y} = \frac{\beta}{\alpha} + \frac{1}{\alpha} x.$$

- Find LS estimates of $\frac{\beta}{\alpha}$ and $\frac{1}{\alpha}$. To get the goodness of fit, we first get $\hat{y}$ and then $\rho^2(y, \hat{y})$. One can also use AIC or BIC.
- However, due to availability of statistical packages non-linear LS estimation is now done with the original equation.
- **Home Task:** Find Engel elasticities of each of the ECs stated above.

Specific Concentration Curve

- **Specific Concentration Curve:**
- Mahalanobis (1960) proposed a generalization of the LC, called Concentration Curves or Specific Concentration Curves, which have been found to yield measures of disparity called Concentration Ratios (CRs). These CRs help in the decomposition of income inequality by factor components (Kakwani, 1980). In the decomposition by factor components, one examines the influence of each component on the overall degree of inequality.
- Let x denote the income of a household and let $y_1, y_2, \dots, y_n$ denote n factor incomes such that

$$x = y_1 + y_2 + \cdots + y_n.$$

- Let $g_i(x)$ denote mean factor income y_i of households having the same total income x. Then

$$x = \sum_{i=1}^{n} g_i(x)\,.$$

Specific Concentration Curve (Continued)

- Arranging households in ascending order of total income x, one can construct concentration curves for the different factor incomes and also compute concentration ratios for all these incomes. It can be shown that

$$\mathrm{LR} = \frac{1}{\mu}\sum \mu_i C(g_i),$$

- where LR is the Lorenz Ratio of total income x, μ_i is the mean and $C(g_i)$ the concentration ratio of the ith factor income, and μ the mean of x. The following data illustrates it.

Decomposition of Income Inequality by Factor Components: An Illustration

Source of Income	Mean (μ_i)	$\left(\frac{\mu_i}{\mu}\right) \times 100$	Concentration Ratio (C_i)	Contribution to Overall Inequality	
				Actual	Percentage
Employment	3399	87.4	0.345	0.301	82.7
Unincorporated Business	276	7.1	0.585	0.042	11.4
Property	106	2.7	0.432	0.012	3.2
Regular annuity	37	1.0	0.054	0.0005	0.2
Capital Items	18	0.5	0.374	0.002	0.5
Capital Gains	39	1.0	0.774	0.008	2.2
Misc.	14	0.4	0.292	0.001	0.3
Total	μ=**3890**	**100.0**	**0.364**	**0.364**	**100.0**

Approach to Specific Concentration Curve

- Suppose X is a size distribution (say, income or total expenditure) with pdf f(x), $(0 < x < \infty)$. Define

$$F(x) = \int_0^x f(t)dt \text{ and } Q(x) = \frac{\int_0^x tf(t)dt}{\int_0^\infty tf(t)dt} = \frac{\int_0^x tf(t)dt}{E(X)}.$$

- Q(X) is the first moment distribution. Q as a function of F is the LC.
- The Engel function is

$$E(y|x) = g(x), \text{say}.$$

- Define,

$$Q_y(x) = \frac{\int_0^x E(y|t)f(t)dt}{\int_0^\infty E(y|t)f(t)dt} = \frac{\int_0^x E(y|t)f(t)dt}{E(y)}.$$

- $Q_y(x)$ as a function of F is called the Specific Concentration Curve (SCC).

Iyengar's Methods

- Method of estimating Engel elasticity from SCC was first proposed by Iyengar (1960, 1964).
- Let us assume that

$$(1)\ X \sim \Lambda(\mu, \sigma^2), \text{and } (2)\ E(y|x) = ax^b$$

- It is already known that

$$Q(x) = \Lambda(x|\mu + \sigma^2, \sigma^2) \text{ and } t_Q = t_F - \sigma.$$

- It can be proved that

$$Q_y(x) = \Lambda(x|\mu + b\sigma^2, \sigma^2) \text{ and } t_{Q_y} = t_F - b\sigma.$$

- **Iyengar** proposed the following two methods to find $\hat{b}$.
- **Method 1:** Put F = 0.5 $\Rightarrow$ $t_Q = -\sigma$ and $t_{Q_y} = -b\sigma$. $\therefore\ \hat{\mathbf{b}} = \dfrac{\mathbf{t_{Q_y}\ at\ F{=}0.5}}{\mathbf{t_Q\ at\ F{=}0.5}}$.
- **Method 2:** $\mathrm{LR} = 2\Phi\left(\frac{\sigma}{\sqrt{2}}\right) - 1$ and $\mathrm{CR} = 2\Phi\left(\frac{b\sigma}{\sqrt{2}}\right) - 1. \therefore\ \hat{\mathbf{b}} = \dfrac{\mathbf{t}_{\frac{\mathbf{CR+1}}{\mathbf{2}}}}{\mathbf{t}_{\frac{\mathbf{LR+1}}{\mathbf{2}}}}$.

Advantage of SCC Approach Over the Usual LS Approach

- **Advantage of SCC Approach Over the Usual LS Approach:**

I. Problem of Consistency

- In grouped data we have observations $(\bar{x}_i, \bar{y}_i), i = 1, 2, \ldots, K$, (K = number of class intervals).

$$\mathrm{Ln}\left(\overline{y_j}\right) = \alpha + \beta \mathrm{Ln}(\overline{x_j}) + \overline{\epsilon_j}; \text{ j indicates individual household.}$$

- For Least Squares we should have

$$\overline{\mathrm{Ln}\left(y_j\right)} = \alpha + \beta \overline{\mathrm{Ln}\left(x_j\right)} + \overline{\epsilon_j}.$$

- Thus, the LS estimate of β will be inconsistent. If we use Iyengar's estimate, $\frac{t_{Q_y}}{t_Q}$, then it is consistent.

Advantage of SCC Approach Over the Usual LS Approach (Continued)

II. Simplicity

- It goes without saying that the model is simple enough and the estimation is elegant.

III. Case of 0 values on y.

- $y = 0 \Rightarrow Ln(y) = -\infty$, so it cannot be used. There are some commodities for which the expenditure is zero for some households. If we neglect these observations, we end up with an inaccurate estimate of elasticities.

- **Note:** When elasticity is not constant for all x, we can define average elasticity $\bar{\eta}$ as

$$\bar{\eta} = \frac{\int_0^\infty \eta_x E(y|x)f(x)dx}{\int_0^\infty E(y|x)f(x)dx}.$$

Demand Projection from Cross-Section Data

- **Demand Projection from Cross-Section Data:**
- Suppose the income distribution changes from one period to the other period in a specified manner. Let us assume that the pdf of income variate in the base period is f(x), which changes to f*(x) in the target period. The EC is

$$E(y|x) = g(x) = ax^b, say,$$

- which remains same for both periods. The per capita demand in the target period is then

$$E^*(y) = \int_0^\infty E(y|x)\, f^*(x) dx.$$

- Assume that the population in the base and target periods are P and P^* respectively. The aggregate demands in the base and target periods are
- $P.E(y)$ and $P^*.E^*(y)$, respectively.

Demand Projection from Cross-Section Data (Continued)

- The ratio of the aggregate demand can be found by

$$\frac{P^*.E^*(y)}{P.E(y)}.$$

- The part P^*/P can be found from population growth rate which is easily known. If the growth rate is α, then $P^*/P = 1 + \alpha$, assuming that we consider to project demand for the immediately next period.

- To find the other part we make the following assumption.

- Assumption: Everybody's income is increased by c proportion. In other words, the income x becomes x(1 + c) in the next period. This implies

$$F^*\big(x(1+c)\big) = F(x).$$

- In other words,

$$F^*(x) = F\left(\frac{x}{(1+c)}\right) \Rightarrow\ f^*(x) = f\left(\frac{x}{1+c}\right).\frac{1}{1+c}.$$

Demand Projection from Cross-Section Data (Continued)

- Now,

$$E^*(y) = \int_0^\infty ax^b\, f\left(\frac{x}{1+c}\right).\frac{1}{1+c}\,dx.$$

$$= \int_0^\infty a\left(\frac{x}{1+c}\right)^b (1+c)^b\, f\left(\frac{x}{1+c}\right) d\left(\frac{x}{1+c}\right).$$

$$= \int_0^\infty a(x^*)^b(1+c)^b\, f(x^*)d(x^*).$$

$$= (1+c)^b E(y).$$

- Hence,

$$\frac{P^*.E^*(y)}{P.E(y)} = (1+c)^b(1 + \alpha).$$

Assumptions

- The following assumptions must be satisfied for the demand projection to be valid.
    1. Engel functions remain same over time.
    2. There is no change in the relative prices.
    3. Taste and preferences of the society do not vary.
- The economy (the composition of the market) is fixed.

Relaxing Some of the Assumptions

- Let us now make some changes in the assumption of $f(x)$ and $f^*(x)$. We now assume that

  (i) Both $f(x)$ and $f^*(x)$ are LN density functions.

  (ii) Mean income increases by a constant factor c.

  $$\text{i.e.,}\ e^{\mu^*+\frac{1}{2}\sigma^{*2}} = (1+c)e^{\mu+\frac{1}{2}\sigma^2}, (c \geq 0,) \qquad \text{... (A)}$$

  where $X \sim \Lambda(\mu, \sigma^2)$ for base period and $\sim \Lambda(\mu^*, \sigma^{*2})$ for the target period.

  (iii) Inequality (LR) of x also decreases by a constant factor.

  $$\text{i.e.,}\ 2\Phi\left(\frac{\sigma^*}{\sqrt{2}}\right) - 1 = (1-\beta)\left\{2\Phi\left(\frac{\sigma}{\sqrt{2}}\right) - 1\right\}, (\beta \geq 0) \qquad \text{... (B)}$$

Change in the Mean Consumption

$$E(y) = \int E(y|x)f(x)dx = \int ax^b f(x)dx = ae^{b\mu + \frac{1}{2}b^2\sigma^2}.$$

Similarly, $E^*(y) = \int E(y|x)f^*(x)dx = ae^{b\mu^* + \frac{1}{2}b^2\sigma^{*2}}.$

Thus, $$\frac{E^*(y)}{E(y)} = e^{b(\mu^*-\mu) + \frac{1}{2}b^2(\sigma^{*2}-\sigma^2)}. \quad \text{... (C)}$$

- Now, from (A)

$$e^{b(\mu^*-\mu)} = (1+c)^b e^{-\frac{1}{2}b(\sigma^{*2}-\sigma^2)}.$$

- Substituting it into (C), we get

$$\frac{E^*(y)}{E(y)} = (1+c)^b e^{-\frac{1}{2}b(\sigma^{*2}-\sigma^2)} e^{\frac{1}{2}b^2(\sigma^{*2}-\sigma^2)}.$$

or $$\frac{E^*(y)}{E(y)} = (1+c)^b e^{\frac{1}{2}b(b-1)(\sigma^{*2}-\sigma^2)}. \quad \text{(D)}$$

Direction of Per Capita Consumption

- Since LR* < LR according to the assumption (iii), we have $\sigma^{*2} < \sigma^2$.
- **Case 1:** $0 < b < 1 \Rightarrow \frac{E^*(y)}{E(y)} > 1$
- **Case 2:** $b < 0 \Rightarrow \frac{E^*(y)}{E(y)} < 1$
- **Case 3:** $b > 1 \Rightarrow \frac{E^*(y)}{E(y)}$ can be < 1, 1 or > 1. I.e., Nothing can be said about the direction of E(y).
- **Conclusion:** Direction of per capita consumption can be said in some cases even if we do not know the actual values of the parameters.

Thank you