

Homework # 1



1.1.3 *Sampling with replacement:*

A box contains tickets marked $1, 2, \dots, n$. A ticket is drawn at random from the box. Then this ticket is replaced in the box and a second ticket is drawn at random. Find the probabilities of the following events:

- a) the first ticket drawn is number 1 and the second ticket is number 2;
- b) the numbers on the two tickets are consecutive integers, meaning the first number drawn is one less than the second number drawn.
- c) the second number drawn is bigger than the first number drawn.

Sampling without replacement:

- d) Repeat a) through c) assuming instead that the first ticket drawn is not replaced, so the second ticket drawn must be different from the first.

1.1.6 Suppose a deck of 52 cards is shuffled and top card is dealt, it is then replaced and shuffled into the deck before the second card is dealt.

- a) How many ordered pairs of cards could possibly result as outcomes?
Assuming each of these pairs has the same chance, calculate:
- b) the chance that the first card is an ace;
- c) the chance that the second card is an ace (explain your answer by a symmetry argument as well as by counting);
- d) the chance that both cards are aces;
- e) the chance of at least one ace among the two cards.

1.1.8 Suppose two fair n -sided dice are rolled. Find the probabilities of the following events.

- a) the maximum of the two numbers rolled is less than or equal to 2;
- b) the maximum of the two numbers rolled is less than or equal to 3;
- c) the maximum of the two numbers rolled is exactly equal to 3.
- d) Repeat b) and c) for x instead of 3, for each x from 1 to 6.
- e) Denote by $P(x)$ the probability that the maximum number is exactly x . What should $P(1) + P(2) + P(3) + P(4) + P(5) + P(6)$ equal? Check this for your answers to d).

1.3.4 Let $\Omega = \{0, 1, 2\}$ be the outcome space in a model for tossing a coin twice and observing the total number of heads. Say if the following events can be represented as subsets of Ω . If you say “yes,” provide the subset; if you say “no,” explain why:

- a) the coin does not land heads both times;
- b) on one of the tosses the coin lands heads, and on the other toss it lands tails;
- c) on the first toss the coin lands heads, and on the second toss it lands tails;
- d) the coin lands heads at least once.

1.3.6 Suppose a word is picked at random from this sentence.

- a) What is the distribution of the length of the word picked?
- b) What is the distribution of the number of vowels in the word?