SESSION 19

04 April 2025 12:35

Quadratic Programming (QP)

Quadratically constrained QP (QCQP)

Boyd's book

Pg no. → 152

Sec → 4.4

e

Optimization p $1^T p = 1$

Random variable f

Problem

$$\max \quad Var(f)$$

$$\text{S.T} \quad 1^T p = 1, \quad p \geq 0$$

$$\alpha_i \leq a_i^T p \leq \beta_i \quad i = 1(1)m$$

H.W

$$\text{minimise} \quad E(f)$$

$$\text{S.T} \quad 1^T p = 1, \quad p \geq 0$$

$$\alpha_i \leq a_i^T p \leq \beta_i \quad i = 1(1)m$$

Linear programing with random cost

Consider an LP with variable $x \in \mathbb{R}^n$

$$\text{minimise} \quad C^T x$$

$$\text{S.T} \quad Gx \leq h$$

$$Ax = b$$

where c is the cost function (Vector) is random.

$E(c) = \mu$ | $c = (c_1, c_2, \ldots, c_n)$

$E(C) = \mu$ | $C = (c_1, c_2, \ldots, c_n)$

Covariance $(C) = \Sigma$

The optimization problem

minimize $E(C^T x) = \mu^T x$

$\text{Var-Cov}(C^T x) = x^T \text{Var}(C)\, x$

$= x^T \Sigma x$

Trade off b/w small expected cost and small variance of cost.

Take linear constraints

$$\boxed{\underbrace{E(C^T x)}_{\text{risk sensitive cost}} + \underbrace{\gamma}_{\text{risk aversion parameter}} \text{Var}(C^T x)}$$

S. Boyd Ref Pg - 154

Second - order cone programing (SOCP) Ref Pg - 156 S. Boyd

minimise $f^T x$

S.T $\boxed{\| A_i x + b_i \|_2 \leq c_i^T x + d_i}$

$Fx = g$

where $x \in \mathbb{R}^n$, $A_i \in \mathbb{R}^{n_i \times n}$, $F \in \mathbb{R}^{p \times n}$

Special cases (I) $c_i = 0$, $i = 1(1)m$, SOCP $\Rightarrow$ QCQP

(II) $A_i = 0$, $i = 1(1)m$, SOCP $\Rightarrow$ L.P

Reading refferences:

1. Robust Linear programming (Pg - 157)
2. LP with random constraints (Pg - 157)
3. Example 4.8 : Portfolio optimization.

$x = (x_1 \text{------}, x_n)$ $\qquad f: \mathbb{R}^n \to \mathbb{R}$

$$f(\underset{n\times 1}{\tilde{x}}) = \underset{1\times n}{b^T} \underset{n\times n}{A} x_{n\times 1}$$

$$\frac{\partial f}{\partial \tilde{x}} = \begin{pmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \\ \vdots \\ \frac{\partial f}{\partial x_n} \end{pmatrix}$$