Normative Measures of Inequality

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Normative Measure

- Normative measures of inequality is based on Social Welfare Function Approach. This was first proposed by Atkinson in 1970. In his pioneering work ("Measures of Inequality", Journal of Economic Theory, 1970), Atkinson started with individual utility function

$$U(x) = \text{utility of income } x \text{ for an individual},$$

- and assumed that it is an increasing and concave function of x. This implies that

  (i) $U' > 0$, (marginal utility of money).

  (ii) $U'' < 0$, (law of diminishing marginal utility).

- The social welfare function is the sum of individual utility functions, i.e.,

Social Welfare Function

$$W = \sum_{i=1}^{n} U(x_i)\,.$$

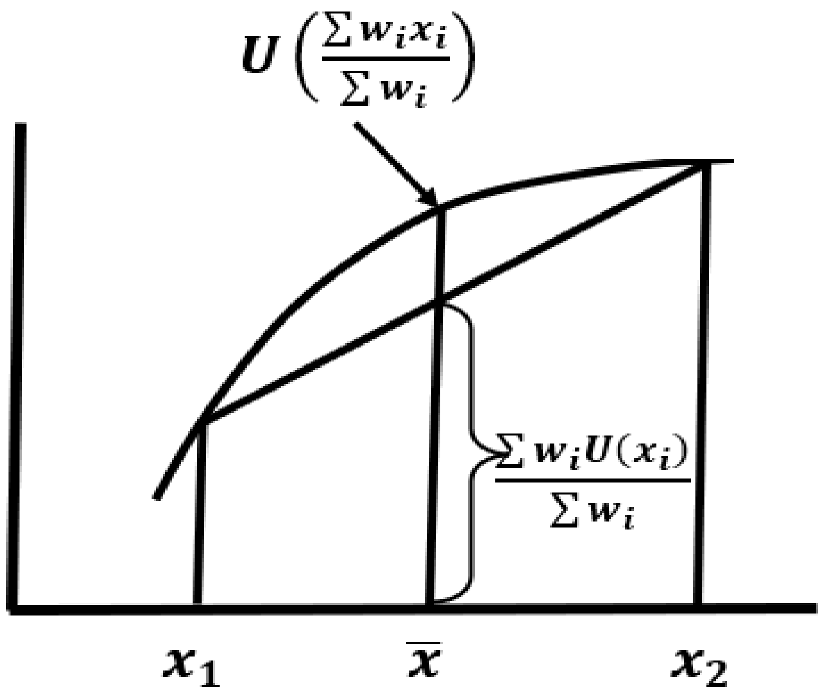
- i.e., Social Welfare Function is an additively separable and symmetric function of the individual utilities.
- **A Result:** (It follows from Jensen's inequality) If U is concave then

$$\frac{\sum w_i U(x_i)}{\sum w_i} \leq U\left(\frac{\sum w_i x_i}{\sum w_i}\right),$$

- i.e., Weighted mean of utilities is less than or equal to the utility of the weighted mean.

An Illustration of Jensen's inequality

- An Illustration of Jensen's inequality when n = 2.



Dalton's Measure

- As a special case when $w_i = \frac{1}{n}$ for all i,

$$\frac{\sum U(x_i)}{n} \le U\left(\frac{\sum x_i}{n}\right),$$

or,

$$\sum U(x_i) \le nU(\bar{x}).$$

- In other words, if the total income $n\bar{x}$ is distributed equally to n persons, then Social Welfare Function (SWF) attains the maximum value.
- Dalton defined inequality as

$$I_D = 1 - \frac{\sum U(x_i)}{nU(\bar{x})}.$$

- By Jensen's inequality $0 \le I \le 1$. $I = 0$ when all incomes are equal and $I = 1$ in the limit as $n \to \infty$ for increasing inequality.

Atkinson's Measure

- Atkinson modified I to obtain a measure which will be invariant under positive linear (affine) transformation of U(.), I is the same for U and a+bU with b > 0.
- Define x_{EDE} (equally distributed equivalent income) as follows.

$$U(x_{EDE}) = \frac{1}{n}\sum U(x_i)\,,$$

Or,
$$nU(x_{EDE}) = \sum U(x_i) = W,$$

As,
$$U(x_{EDE}) = \frac{1}{n}\sum U(x_i) \leq U(\bar{x}) \text{ by Jensen's inequality,}$$

we have
$$x_{EDE} \leq \bar{x}, \text{ since U is an increasing function.}$$

- Atkinson's measure is

$$I_A = 1 - \frac{x_{EDE}}{\bar{x}}.$$

Properties of I_A

- Obviously, $0 \le I_A \le 1$. $I_A = 0$ if $x_{EDE} = \bar{x}$, i.e., all incomes are equal and $I_A = 1$ if $x_{EDE} = 0$ (compared to $\bar{x}$). This will be the case in the limit as $n \to \infty$ and all but one gets zero income and only one gets all income.
- **Result:** I_A is invariant under positive linear transformation of U.
- **Proof:** Let us take U and the positive linear transformation of U as $a+bU=U_1$.

$$U(x_{EDE}) = \frac{1}{n}\sum U(x_i) \text{ and } U_1(x^*_{EDE}) = \frac{1}{n}\sum U_1(x_i)\,.$$

$$U_1(x^*_{EDE}) = a + bU(x^*_{EDE}).$$

$$\text{Also } \frac{1}{n}\sum U_1(x_i) = \frac{1}{n}\sum\big(a + bU(x_i)\big)$$

$$= a + b\frac{1}{n}\sum U(x_i) = a + bU(x_{EDE}).$$

$$\therefore\ x^*_{EDE} = x_{EDE}.$$

Hence, $\quad I_A(U) = I_A(U_1). \qquad\qquad$ Q. E. D.

Utility Function

- **Result:** If $I_A = 1 - \frac{x_{EDE}}{\bar{x}}$ is invariant under proportional shifts in the x-values.

$$\text{i.e.}, I(x_1, x_2, \dots, x_n) = I(cx_1, cx_2, \dots, cx_n), \forall\, c > 0,$$

- then U(.) must have the following form.

$$\mathbf{U(x) = A + B\frac{x^{1-\varepsilon}}{1-\varepsilon}\ , \varepsilon \neq 1}$$

$$\mathbf{= Ln(x), \qquad \varepsilon = 1.}$$

- Assume $\varepsilon \geq 0$ for concavity of U(.). We omit this proof.

Specific Values of ε

- **Specific values of ε**

(1) $\varepsilon = 0$, $U(x) = A + Bx$;

$$W = \sum (A + Bx_i) = nA + B \sum x_i .$$

- i.e., W depends on the total income and not on distribution. Since, it is not desirable, we do not take $\varepsilon = 0$.

(2) $\varepsilon = 1$, $U(x) = A + B\,Ln(x)$; $U'(x) = \frac{B}{x}$.

$$W = \sum \left(A + BLn(x_i)\right) = nA + B \sum Ln(x_i) .$$

(3) $\varepsilon = 2$, $U(x) = A - \frac{B}{x}$, $U'(x) = \frac{B}{x^2}$.

$$W = \sum \left(A + \frac{B}{x_i^2}\right) = nA + B \sum \frac{1}{x_i^2} .$$

- I.e., at $\varepsilon = 2$, U(x) decreases faster than $\varepsilon = 1$.
- ε represents degree of inequality aversion or sensitiveness of W to inequality. Reasonable range of ε: 1 to 2.5.

Atkinson's Final Measure

- If the above form of U(.) is accepted, then

$$U(x_{EDE}) = \frac{1}{n}\sum U(x_i)\,.$$

$$\Rightarrow \qquad A + B\frac{x_{EDE}^{1-\varepsilon}}{1-\varepsilon} = \frac{1}{n}\sum\left(A + B\frac{x_i^{1-\varepsilon}}{1-\varepsilon}\right)$$

$$\text{Or} \qquad x_{EDE} = \left(\frac{1}{n}\sum x_i^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}$$

$$\text{Or} \qquad I_A = 1 - \frac{1}{\bar{x}}\left(\left(\frac{1}{n}\sum x_i^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}\right)$$

$$= 1 - \left(\frac{1}{n}\sum\left(\frac{x_i}{\bar{x}}\right)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}.$$

- This is Atkinson's equally distributed equivalent (EDE) measure of inequality.

Specific values of ε

- **Specific values of ε**

$$(1)\ \varepsilon = 0, U(x) = A + Bx, \qquad I_A = 1 - \frac{AM}{AM} = 0.$$

- The measure is insensitive to the distribution of X.

$$(2)\ \varepsilon = 1, U(x) = A + B\,Ln(x), \qquad I_A = 1 - \frac{GM}{AM}.$$

$$(3)\ \varepsilon = 2, U(x) = A - \frac{B}{x}, \qquad I_A = 1 - \frac{HM}{AM}.$$

$$(4)\varepsilon \to \infty, \qquad I_A \to 1 - \frac{Min(x_1, x_2, \dots, x_n)}{AM}.$$

- Observe that AM > GM > HM > Minimum value.
- As ε increases the measure becomes more and more sensitive to the transfer of income among the poorer persons in the community. The value of the inequality measure increases.

Some Results

- **Theorem:** Suppose two income distributions have the same mean. Then if the LC of the first distribution is uniformly above (or as high as) the LC of the second distribution, then $W_1 > W_2$ for any concave utility function (and vice versa).

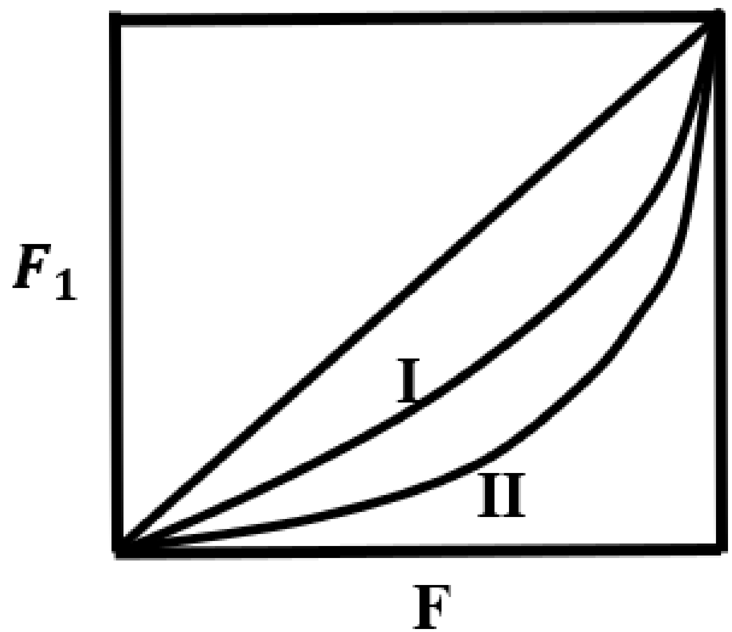
- **Hardy, Littlewood and Polya's version:**

- Let $(x_1, x_2, \ldots, x_n)$ and $(y_1, y_2, \ldots, y_n)$ be the vectors such that $\sum_1^n x_i = \sum_1^n y_i$, and $\sum_1^k x_i \leq \sum_1^k y_i$ for all k < n and $\sum_1^k x_i < \sum_1^k y_i$ for at least one k, then

$$W_x = \sum\nolimits_1^n U(x_i) < W_y = \sum\nolimits_1^n U(y_i),$$

- for every strictly concave U(.). The converse is also true.

Illustration



- Distribution (I) is more egalitarian according to EDE measure, whatever U(.) may be. Conversely, if the LC's intersect, one cannot rank the two distribution by inequality without assuming a utility (welfare) function. It is possible to find out two utility functions giving opposite rankings.

Examination of Conventional Measures

- **Examination of Conventional Measures:**
- Relative mean deviation (RMD) has the deficit that it is insensitive to transfers on the same side of mean.
- CV, LR and SD(Ln) satisfy the criterion of Principle of Transfer.
- CV does not satisfy the principle of diminishing transfer. In fact, the reduction of inequality (ΔI) is independent of x.
- LR does not satisfy the principle of diminishing transfer. ΔI is greater for transfer affecting middle income groups.
- SD(LN) satisfies the principle of diminishing transfer. ΔI is a decreasing function of x.
- I_A satisfies the principle of diminishing transfer. ΔI is a decreasing function of x for $\varepsilon > 0$.

Empirical Results

- **Empirical Results**
- **Simon Kuznets:** "Quantitative aspects of the economic growth of nations, VIII: The distribution of income by size", *Economic Development and Cultural Change*, 11, pp. 1–92. (1963).
- Broad Consensus:

  (1) Inequality decreased in the developed countries in the 1900's after World War I.

  (2) Inequality in developing countries in 1960's was as pronounced as it was in the developed countries around 1900.

- **Atkinson** re-examined Kuznets' data on 7 developed and 5 developing (Including India) countries.
- In 16 out of 66 cases of pairs of countries the LC's did not intersect.
- India more equal at the bottom and less equal at the top compared with US. This is typical for all developing countries.

Thank you