Indian Statistical Institute

BSDS IInd Year

Academic Year 2025 - 2026: Semester I

Course: Probability II

Instructor: Antar Bandyopadhyay

Assignment # 2

Date Given: August 20, 2025 **Date Due: August 29, 2025**
Total Points: 10

Problem # 1 Use **R** software to present *contour plots* of the *bivariate normal density function* with five different choices of the mean vector and the variance-co-variance matrix. Printout of the plots preferably in colour should be submitted.

Problem # 2 Suppose (X, Y) be distributed as bivariate normal with means μ_X and μ_Y; variances σ_X^2 and σ_Y^2 respectively and correlation ρ. For $x, y \in \mathbb{R}$, find the conditional distribution of Y given $X = x$, and the conditional distribution of X given $Y = y$.

6.5.7(e) In the setup of the **Problem # 2** above, if $\mu_X = \mu_Y = 0$, then show that $X\cos\theta + Y\sin\theta$ and $Y\cos\theta - X\sin\theta$ are two independent random variables, if

$$\theta = \frac{1}{2}\cot^{-1}\left(\frac{\sigma_X^2 - \sigma_Y^2}{2\rho\sigma_X\sigma_Y}\right)$$

6.5.9 Suppose that $W \sim \text{Normal}\left(\mu, \sigma^2\right)$ distribution. Given that $W = w$, suppose Z has conditional distribution as Normal $\left(aw + b, \tau^2\right)$.

(a) Show that the joint distribution of W and Z is bivariate normal, and find its parameters.

(b) What is the marginal distribution of Z.

(c) What is the conditional distribution of W given $Z = z$.