Indian Statistical Institute

BSDS Ist Year

Academic Year 2024 - 2025: Semester I

Course: Probability Theory I

Instructor: Antar Bandyopadhyay

Assignment # 11

Date Given: November 13, 2024   **Date Due: November 21, 2024**
Total Points: 10

4.4.4 Suppose X has uniform distribution on $(-1, 1)$. Find the density of $Y = X^2$.

4.4.6 Suppose that a particle is fired from the origin in the (x, y)-plane in a straight line in a direction at a random angle ϕ to the x-axis, and let Y be the y-coordinate of the place where the particle hits the line $x = 1$. Show that if ϕ has uniform $(-\pi/2, \pi/2)$ distribution, then Y has the density

$$f_Y(y) = \frac{1}{\pi(1 + y^2)}$$

This is called the *Cauchy distribution*. Show that the Cauchy distribution is symmetric around 0, and thus its *median* is 0, but that the expectation of a Cauchy random variable is undefined.

4.5.2 Find and sketch the cumulative distribution functions of:

(a) the Binomial $(3, 1/2)$ distribution;

(b) the Geometric $(1/2)$ distribution.

4.5.6 Let X be a random variable with c.d.f. $F(x) = x^3$ for $0 \leq x \leq 1$. Find:

(a) $\mathbf{P}(X \geq 1/2)$;

(b) the density function $f(x)$ of X; and

(c) $\mathbf{E}[X]$.