SESSION 21

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Unconstrained minimization (Ch. 9 Boyds Book)

Objective is minimise $f(x)$ where f is convex and twice differentiable Assumption.

(i) the problem is solvable and $\exists$ an optimal point x^*.

(ii) $\inf_x f(x) = f(x^*) = p^*$, optimal value

Note that since f is convex and twice differentiable the necessary and sufficent condition for x^* to be optimal.

$$\nabla f(x^*) = 0 \quad \text{---} \quad (2)$$

So (1) and (2) are equivalent

Analytical solution of (2) can only be obtained in few cases. More common and practiced method is iterative algorithm.

Algorithm: Compute a sequence of points $x^{(0)}, x^{(1)} \in \text{dom}(f)$ with $f(x^{(k)}) \to p^*$ as $k \to \infty$

$\Rightarrow$ $x^{(0)}, x^{(1)}, \text{----}, x^{(k)}$, are minimizing sequence of points.
The algorithm is terminated where

$$|f(x^*) - p^*| \leq \epsilon, \quad \epsilon > 0 \text{ is a speified tolerance level.}$$

Initial point and sub level set

Descent method

Consider a minimizing sequence $x^{(k)}$ such that

$$x^{(k+1)} = x^{(k)} + t^{(k)} \Delta x^{(k)} \quad \text{and} \quad t^{(k)} > 0$$

Δx: Step or Search direction
$t^{(k)}$: Step size

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$k: 0, 1, \ldots$ iteration no.

This is a descent method as $f(x^{(k+1)}) < f(x^{(k)})$

From convexity we know that $\nabla f(x^{(k)})^T (y - x^{(k)}) \geq 0$

implies $f(y) \geq f(x^k)$ Revisit the convex (H.W)

So the search direction in a descent method must satisfy

$$\nabla f(x^{(k)}) \Delta x^{(k)} < 0$$

So, it is an acute angle with negative gradient
we call such a direction descant direction.

Gradient descant method (Ref 9.3 Boyds Book)

Search direction $\Delta x = -\nabla f(x)$

Given a starting point $x \in \text{dom}(f)$

Algorithm Step 1: $\Delta x = -\nabla f(x)$
Step 2: choose step size by exact or back tracking line search

Step 3: Update $x = x + t\Delta x$.