

INDIAN STATISTICAL INSTITUTE
Assignment-3 (Mathematics III)
Bachelor of Statistical Data Science (BSDS)

1. A graph G is connected if and only if there is a path between every pair of vertices.
2. Suppose T is a tree with n vertices that contains only vertices of degree 1 and 3. Prove that T contains $\frac{(n-2)}{2}$ vertices of degree 3.
3. Does every Eulerian bipartite graph have an even number of edges ? Justify.
4. Does every Eulerian graph with an even number of vertices have an even number of edges? Explain.
5. Prove that every graph contains an even number of vertices of odd degree.
6. Show that there exists no graph $G = (V, E)$ with $|V| = 48$ vertices such that the degrees of 30 of the vertices are 16, the degree of 15 vertices is 9 and the degree of the remaining 3 vertices is 12.
7. Show that there are exactly three connected graphs with 4 vertices or less which admit an Euler circuit. In addition, list four different connected graphs with 5 vertices which admit Euler circuits, and find five different connected graphs with 6 vertices with an Euler circuits.
8. Let $G = (E, V)$ be a graph such that for all non-adjacent vertices, $x, y \in V$

$$\deg(x) + \deg(y) \geq |V| - 1.$$

Prove that G is connected.

9. Prove that if G is a graph with n vertices and n edges with no vertices of degree 0 or 1, then the degree of every vertex is 2.
10. Prove that it is impossible for every vertex of a graph to have a different degree.
11. If G_1 and G_2 are isomorphic graphs then they have the same number of vertices, the same number of edges, and the same degree sequence. What is the converse of this statement, and is it true or false? If true, prove it. If false, find a counterexample.
12. Find a Graph with 7 vertices such that both G and G^c contain Eulerian Circuit.