Indian Statistical Institute

BSDS Ist Year

Academic Year 2024 - 2025: Semester I

Course: Probability Theory I

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Assignment # 3

Date Given: September 04, 2024 **Date Due: September 12, 2024**
Total Points: 10

1.4.10 Suppose electric power is supplied from two independent sources which work with probabilities $0.4, 0.5$, respectively. If both sources are providing power enough power will be available with probability 1. If exactly one of them works there will be enough power with probability 0.6. Of course, if none of them works the probability that there will be sufficient supply is 0.

(a) What are the probabilities that exactly k sources work for $k = 0, 1, 2$?

(b) Compute the probability that enough power will be available.

1.6.6 Suppose you roll a fair six-sided die repeatedly until the first time you roll a number that you have rolled before.

(a) For each $r = 1, 2, \cdots$ calculate the probability p_r that you roll exactly r times.

(b) Without calculation, write down the value of $p_1 + p_2 + \cdots + p_{10}$. Give explanation of your answer.

(c) Check that your calculated values of p_r have this value for their sum.

1.6.8 Suppose that the birthday of each of three people is equally likely to be any one of the 365 days of the year, independently of others. Let B_{ij} denote the event that person i has the same birthday as person j, where the labels i and j may be $1, 2$, or 3.

(a) Are the events B_{12} and B_{23} independent?

(b) Are the events B_{12}, B_{23}, and B_{13} independent?

(c) Are the events B_{12}, B_{23}, and B_{13} *pairwise independent*?

2.1.4 A die is rolled 8 times. Given that there were 3 sixes in the 8 rolls, what is the probability that there were 2 sixes in the *first five* rolls?

2.5.12 Consider a well shuffled standard deck of cards and 5 cards are dealt from there which will be called a *hand*. Assume that all $\binom{52}{5}$ hands are equally likely. Find the probability of being dealt:

(a) a *straight flush* (5 consecutive cards of the same suit);

(b) *four of a kind* (ranks a, a, a, a, b);

(c) a *full house* (ranks a, a, a, b, b);

(d) a *flush* (5 of the same suit, but not a *straight flush*);

(e) a *straight* (5 consecutive ranks, but not a *flush*);

(f) *three of a kind* (ranks a, a, a, b, c);

(g) *two pairs* (ranks a, a, b, b, c);

(h) a *pair* (ranks a, a, b, c, d);

(i) none of the above.