
Statistical Inference

B. Statistical Data Science 2nd Year Indian Statistical Institute

Class Test - 1

9 September 2025

Total Marks: 10

Name:

Roll Number:

Instructions

- Answer as much as you can. The maximum you can score is 10.
 - Only write the answer in the space provided or tick the correct option. Any additional writing will not be graded. Rough works should be done on spare pages.
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Question 1. Let $X_1, \dots, X_n \stackrel{\text{i.i.d.}}{\sim} \text{Poisson}(\theta)$, $\theta > 0$. Define $\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$. Which of the following is correct? [1 mark]

- (a) \bar{X} is sufficient but not minimal sufficient.
(b) \bar{X} is minimal sufficient. (c) \bar{X} is not sufficient.

Question 2. Let $X_1, X_2, X_3, X_4 \stackrel{\text{i.i.d.}}{\sim} \text{Normal}(\theta)$, $\theta > 0$. Define $\mathbf{T} = (X_1 + X_2, X_3 + X_4)$. Which of the following is correct? [1 mark]

- (a) \mathbf{T} is sufficient but not minimal sufficient.
(b) \mathbf{T} is minimal sufficient. (c) \mathbf{T} is not sufficient.

Question 3. Let $X_1 \sim \text{Bernoulli}(\theta)$ and $X_2 \sim \text{Bernoulli}(\theta/5)$, $0 < \theta < 1$, and X_1, X_2 are independent. Which of the following is correct? [1 mark]

- (a) $X_1 + 5X_2$ is sufficient but not minimal sufficient.
(b) $X_1 + 5X_2$ is minimal sufficient. (c) $X_1 + 5X_2$ is not sufficient.

Question 4. Let $X_1, X_2, X_3 \stackrel{\text{i.i.d.}}{\sim} \text{Geometric}(\theta)$, $\theta \in (0, 1)$. Define $T = X_1 + 2X_2 + 3X_3$. Which of the following is correct? [1 mark]

- (a) T is sufficient but not minimal sufficient.
(b) T is minimal sufficient. (c) T is not sufficient.

Question 5. Let $X_1, \dots, X_n \stackrel{\text{i.i.d.}}{\sim} \text{Normal}(\theta, \theta)$, $\theta > 0$. Find a minimal sufficient statistic for θ . [1.5 marks]

Answer:

Question 6. Let $X_1, \dots, X_n \stackrel{\text{i.i.d.}}{\sim} f_\theta$, where $f_\theta(x) = \begin{cases} e^{-(x-\theta)} & \text{if } x \geq \theta, \\ 0 & \text{otherwise,} \end{cases} \quad \theta \in \mathbb{R}$. Find a minimal sufficient statistic for θ . [1.5 marks]

Answer:

Question 7. Let X_1, \dots, X_n be i.i.d. with pdf $f_\theta(x) = \begin{cases} \frac{2}{\theta} x e^{-\frac{x^2}{\theta}} & \text{if } x \geq 0, \\ 0 & \text{otherwise,} \end{cases} \quad \theta > 0$. Find a minimal sufficient statistic for θ . [1.5 marks]

Answer:

Question 8. Let X_1, X_2, X_3, X_4 be independent random variables with $X_1, X_2 \sim \text{Normal}(0, \theta)$ and $X_3, X_4 \sim \text{Laplace}(0, \theta)$. Find a minimal sufficient statistic for θ . [1.5 marks]

Answer:

Question 9. Let $X_1, \dots, X_n \stackrel{\text{i.i.d.}}{\sim} \text{Uniform}(-\theta, \theta)$. Find $\mathbb{E}(X_{(n)})$. [2 marks]

Answer: