Probability Theory Quiz 1 , 3

October 6, 2025

1. For $t, a \in \mathbb{R}$ find $\mathbf{E}[\Phi(t+aX)]$ in terms of t, a and Φ, where X is a standard normal random variable and Φ is the distribution function of it. [10]

2. Let $X_n \sim \text{Bin}(n, p)$ for $0 < p < 1$. Prove that the sample proportion $\frac{X_n}{n}$ converges in probability to p.

   That is, show that:

$$\frac{X_n}{n} \xrightarrow{P} p$$