INDIAN STATISTICAL INSTITUTE
Mathematics-3 (BSDS)
Midterm Examination

DATE: 6/10/2025 Total Marks: 30 Time: 11:00 – 1:00 pm

MARKS WILL BE AWARDED ONLY FOR THOSE ANSWERS WITH PROPER JUSTIFICATION

Question 1: Let A be an $n \times n$ real matrix. Then show that

- AA^T and A^TA are positive semidefinite.
- The nonzero eigenvalues of AA^T and A^TA coincide.

[2+3=5]

Question 2: Let V be a real inner product space with norm

$$\|x\| = \sqrt{\langle x, x\rangle}.$$

Prove that for all $x, y \in V$,

$$\langle x, y\rangle = \frac{1}{2}(\|x+y\|^2 - \|x\|^2 - \|y\|^2).$$

[5]

Question 3: What is the maximum number of vertices on a graph that has 35 edges and every vertex has degree ≥ 3. Justify your answer. [5]

Question 4: Let G be a simple graph of order 9. Suppose that for every pair of distinct vertices $u, v \in V(G)$,

$$\deg(u) + \deg(v) \geq 8.$$

Prove that G is connected. [5]

Question 5: Suppose T is a tree of order n that contains only vertices of degree 1 and 3. Prove that T contains $\frac{n-2}{2}$ vertices of degree 3. [5]

Question 6: Let P and Q be paths of maximum length in a connected graph G. Prove that $V(P) \cap V(Q) \neq \emptyset$. [5]

—ALL THE BEST—