Indian Statistical Institute

BSDS IInd Year

Academic Year 2025 - 2026: Semester I

Course: Probability II

Instructor: Antar Bandyopadhyay

Assignment # 6

Date Given: September 17, 2025 **Date Due: September 26, 2025**
Total Points: 10

Suppose $A_1, A_2, A_3, \cdots, A_n, \cdots$ are events and B be the event that $[A_n \text{ i.o. }]$ and C is the event that $[A_n$ eventually]. Then show that

1. $C \subseteq B$

2. $\mathbf{P}(B) = \lim_{N\to\infty} \mathbf{P}\left(\bigcup_{n=N}^{\infty} A_n\right)$, where the limit is *decreasing*.

3. $\mathbf{P}(C) = \lim_{N\to\infty} \mathbf{P}\left(\bigcap_{n=N}^{\infty} A_n\right)$, where the limit is *increasing*.

4. If f_n be the indicator function of the event A_n, then $\limsup_{n\to\infty} f_n = \mathbf{1}_B$, and $\liminf_{n\to\infty} f_n = \mathbf{1}_C$.