HW

Q1:- The model is given by
$$y_i = \beta x_i + e_i x_i$$
we can write $e_i = \frac{y_i - \beta x_i}{x_i}$

The sum of squared errors is given by
$$SSE = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} \left(\frac{y_i - \beta x_i}{x_i}\right)^2$$

Now, derivative of SSE with respect to β is taken and set to zero.

$$\frac{d\,SSE}{d\beta} = 0$$
$$\Rightarrow \sum_{i=1}^{n}\left(\frac{y_i}{x_i} - \beta\right) = 0$$
$$\Rightarrow \sum_{i=1}^{n}\frac{y_i}{x_i} - n\beta = 0$$
$$\Rightarrow \beta = \frac{1}{n}\sum_{i=1}^{n}\frac{y_i}{x_i}$$

(a) $\therefore$ least squares estimator, $\hat{\beta}_{LS} = \frac{1}{n}\sum_{i=1}^{n}\frac{y_i}{x_i}$

(b) The pdf of e_i is $f(u) = \frac{1}{2\lambda}\exp\left(-\frac{|u|}{\lambda}\right), -\infty < u < \infty$

Now, substituting $u = \frac{y_i - \beta x_i}{x_i}$, the joint likelihood function is given by.
$$L(\beta, \lambda) = \left(\frac{1}{2\lambda}\right)^n \exp\left(-\sum_{i=1}^{n}\frac{\left|\frac{y_i - \beta x_i}{x_i}\right|}{\lambda}\right).$$

The log-likelihood function is given by.
$$l(\beta, \lambda) = -n\log(2\lambda) - \frac{1}{\lambda}\sum_{i=1}^{n}\left|\frac{y_i - \beta x_i}{x_i}\right|$$
To find the maxi^n likelihood estimator of β, the log-likelihood

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function is maximized with respect to β.
This is equivalent to minimizing the term $\sum_{i=1}^{n} \left| \frac{y_i - \beta x_i}{x_i} \right|$.

Let, $t_i = \frac{y_i}{x_i}$, $i=1,2,\ldots,n$, then ~~us~~ we ~~maxim~~ minimize

$$\sum_{i=1}^{n} |t_i - \beta|.$$

~~We know that~~ Now the value of β that minimizes the sum of absolute deviations is the sample median of t_i values.

$\therefore$ Maximum likelihood estimator: $\hat{\beta}_{ML} = \text{median}(t_i)_{i=1,\ldots,n}$

$$= \text{median}\left(\frac{y_1}{x_1}, \ldots, \frac{y_n}{x_n}\right).$$