Quiz 2 Solution

1. A factory that produces batches of 1000 laptops each finds that on average, two laptops per batch are defective. Find the minimum probability that fewer than five laptops in the next batch will be defective.

Let X be the number of defective laptops. We are looking for the minimum value of $P(X < 5)$. Given:

$$E[X] = 2$$

Using: Markov's Inequality

$$P(X \geq a) \leq \frac{E[X]}{a}$$

Method 1: Using the event $X \geq 5$

To find the minimum for $P(X < 5)$, we find the maximum for its complement, $P(X \geq 5)$. Let $a = 5$:

$$P(X \geq 5) \leq \frac{E[X]}{5} = \frac{2}{5}$$

Now, we find the lower bound for the original event:

$$P(X < 5) = 1 - P(X \geq 5)$$

$$P(X < 5) \geq 1 - \frac{2}{5}$$

$$\implies P(X < 5) \geq \frac{3}{5}$$

Method 2: Using the event $X > 4$

For a discrete variable, the event $X \geq 5$ is identical to $X > 4$. Using: Markov's Inequality

$$P(X > a) \leq \frac{E[X]}{a}$$

$$P(X > 4) \leq \frac{E[X]}{4} = \frac{2}{4} = \frac{1}{2}$$

Now, we find the lower bound for the original event, noting $P(X < 5) = P(X \leq 4)$:

$$P(X \leq 4) = 1 - P(X > 4)$$

$$P(X \leq 4) \geq 1 - \frac{1}{2}$$

$$\implies P(X \leq 4) \geq \frac{1}{2}$$
