Indian Statistical Institute

BSDS Ist Year

Academic Year 2024 - 2025: Semester I

Course: Probability Theory I

Instructor: Antar Bandyopadhyay

Midterm Examination

Date: October 01, 2024 | **Total Points:30**
Time: 10:30 AM - 12:30 PM | **Duration: 120 minutes**

Note: There are four problems with a total of 36 points. Solve as many as you can. Maximum you can score is 30. Show all your works. Use only the pages provided. The rough works must be done on the pages provided for the same. Do not forget to write your name and roll number. Also circle the location/centre you are attending. Good luck!

Name: ______________________ **Roll No.:** ______________________

Location/Centre: Kolkata Delhi Bangalore

Please DO NOT write any thing below

For Instructor/TAI use only

Problems	1	2	3	4	Total
Points					

[Please Only Turn at 10:30 AM]

[Please DO NOT write anything on this page!]

1. Suppose $N \sim$ Negative-Binomial $(2, p)$ where $0 < p < 1$. Let X be a random variable such that the conditional distribution of X given $N = n$ is Uniform $(\{1, 2, \cdots, n-1\})$. Find the marginal distribution of X. What is the conditional distribution of N given $X = 10$. $[6 + 3 = 9]$

2. Ron was given the task of sending out the invitations for the wedding of Ginny and Harry. They wanted to invite 100 of their friends. Unfortunately, Ron did not realized that his charm made the 100 invitation letters to be randomly placed in the 100 addressed envelop. The outcome was a disaster, it may have been that an invitation has gone inside an envelop addressed to a different person, or an envelop containing more than one invitations or none at all. In all such cases the recipients were either confused or upset and they did not show up. Only the invitees who received the invitations inside a correctly addressed envelop came for the wedding. Let X be the total number of people who came for the wedding.

   (a) Find $\mathbf{P}\left(X = 100\right)$; and [4]

   (b) $\mathbf{P}\left(X = 0\right)$. [5]

3. Suppose $X \sim \text{Poisson}(\lambda)$ and $Y \sim \text{Poisson}(\mu)$ and they are independent. Find the distribution of $Z = X + Y$. For $k \geq 0$ find the *conditional distribution* of X given $Z = k$. $[4 + 5 = 9]$

4. (a) Cards are dealt one after the other from a well shuffled deck of 52 standard cards until the **first King** appears. Find a formula for $p(n) :=$ probability that exactly n cards are dealt. [4]

   (b) Suppose there are 10 drawers, each containing **two** coins. Drawers 1 and 2 contain only **gold** coins, while in drawers $3, 4$ and 5 one is a **gold** coin and the other one is a **silver** coin, and rest of the drawers contain only **silver** coins. Suppose you pick a drawer at random, and then a coin at random from it. Given that you choose a **gold** coin find the conditional probability that the other coin is also **gold**. [5]

Rough Work Page

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