# Indian Statistical Institute

**BSDS Ist Year**

**Academic Year 2024 - 2025: Semester I**

**Course: Probability Theory I**

**Instructor: Antar Bandyopadhyay**

**Assignment # 6**

**Date Given: September 25, 2024** **Date Due[1]: October 01, 2024**
**Total Points: 10**

**3.1.6** A fair coin is tossed three times. Let $X$ be the number of heads on the first two tosses, $Y$ be the number of heads on the last two tosses.

(a) Make a table showing the joint distribution of $X$ and $Y$.

(b) Are $X$ and $Y$ independent?

(c) Find the distribution of $X + Y$.

**3.4.12** Let $W_1$ and $W_2$ be independent Geometric random variables (counting the number of trails before the first success) variables with parameters $p_1$ and $p_2$. Find

(a) $\mathbf{P}(W_1 = W_2)$.

(b) $\mathbf{P}(W_1 < W_2)$.

(c) $\mathbf{P}(W_1 > W_2)$.

(d) the distribution of $\min(W_1, W_2)$;

(e) the distribution of $\max(W_1, W_2)$.

**3.4.14** In a independent repetition of Bernoulli $(p)$ trials let $V_n$ be the number of trials required to produce either $n$ successes or $n$ failures, whichever comes first. Find the distribution of $V_n$.

**3.1.24** Suppose a box contains tickets, each labeled by an integer. Let $X, Y$, and $Z$ be the results of draws at random *with replacement* from the box. Show that, no matter what the distribution of numbers in the box,

(a) $\mathbf{P}(X + Y \text{ is even }) \geq 1/2$; and

(b) $\mathbf{P}(X + Y + Z \text{ is a multiple of } 3) \geq 1/4$.

[1]**NOTE:** It is the date of the Midterm Examination when you should handover your homework solutions to your respective TAs at your respective locations.