BSDS QUIZ 3

Date: March 10th, 2025

(1) Describe the circle of convergence (that is, it's radius and centre) of the power series $\sum_{n=0}^{\infty} \frac{(z+i)^n}{(n+1)5^n}$ (here z is a complex variable). Give some details on how you arrived at the answer.

Answer: $\lim_{n\to\infty}(\frac{|z+i|^n}{5^n(n+1)})^{\frac{1}{n}} = \frac{|z+i|}{5}$, so the centre is $-i$ and the radius is 5.

(2) For the real power series $\sum_{n=0}^{\infty} \frac{(x-1)^n}{(n+2)!}$ determine the set of all real x for which it converges and compute the sum of this series.

Answer: Let the series be $f(x)$, then $(x-1)^2 f(x)+1+(x-1) = \exp(x-1)$, so this makes sense for all real numbers and $f(x) = \frac{\exp(x-1)-x}{(x-1)^2}$ if $x \neq 1$ and $f(1) = \frac{1}{2}$.