SESSION 18

03 April 2025 12:29

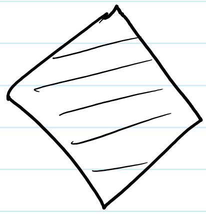
Quadratic Programing - a Special case of Optimization

minimise $x^T P x + q^T x + r$

Sub to $Gx \leq b$ where $P \in S_n^+$, $G \in \mathbb{R}^{m \times n}$, $A \in \mathbb{R}^{p \times n}$

$Ax = b$

The feasible set is a polyhedron.



Quadratically constrained quadratic program (QCQP)

The QCQP can be written as

$$\text{minimise} \quad \left(\tfrac{1}{2}\right) x^T P x + q_0^T x + r_0$$

$$\text{Subject to} \quad \left(\tfrac{1}{2}\right) x^T P_1 x + q_1^T x + r_1 \leq 0$$

$$Ax = b$$

feasible sets are the intersection of ellepsoids.

Least squares :-

minimise $\quad \|Ax - b\|_2^2 = (Ax-b)^T (Ax-b)$

$$f(x) = x^T A^T x - 2b^T A x + b^T b$$

$f : R^n \rightarrow R$

$$\frac{\partial f}{\partial \underset{\sim}{x}} = \begin{pmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \\ \vdots \end{pmatrix}_{n \times 1}$$

$f'(x) = 0$

$= 2A^T A x - 2b^T A = 0$

$= A^T A x = A^T b$

$$\boxed{\hat{x} = (A^T A)^{-1} A^T b} = A^{-1} (A^T)^{-1} A^T b$$

$$= A^{-1} (I_n) b$$

$$= A^{-1} b$$

Constrained Least Squares (Ref Pg- 152)

minimise $\quad \|Ax - b\|_2^2$

subject to $\quad l_i \leq x_i \leq u_i$

Example :- 2

Let us Consider Optimization variable as probablity distribution $P \in R^P$

Assume: we have some ~~know~~ knowledge about ϕ (constraints)

Let us consider a random variable 'f'.

Derive variance of f

$$Var(f) = \mathbb{E}[f^2] - (\mathbb{E}(f))^2$$

$$= \sum f_i^2 p_i - \left(\sum f_i p_i\right)^2$$

(where $f(u_i) = f_i$)

$\Rightarrow$ $Var(f)$ is a concave function of p.

Suppose we want to maximise the $Var(f)$ subject to prior info^n.

QP becomes maximise $\sum f_i^2 p_i - \left(\sum f_i p_i\right)^2$

subject to $p \geq 0$, $\mathbb{1}^T p = 1$ $\rightarrow$ $\alpha_1 \leq a_i^T p \leq \beta_1$

where $i = 1(1)m$