Indian Statistical Institute

BSDS IInd Year

Academic Year 2025 - 2026: Semester I

Course: Probability II

Instructor: Antar Bandyopadhyay

Assignment # 3

Date Given: August 27, 2025 **Date Due: September 05, 2025**
Total Points: 10

1. Suppose $X_1, X_2, \cdots, X_d$ be *i.i.d.* standard normal random variables. Let $\mathbf{X}$ be the random (column) vector with components $X_1, X_2, \cdots, X_d$ stacked as a column with d entries. Let A be a $d \times d$ *non-singular* matrix with real entries. Let $\mathbf{Y} := A\mathbf{X}$. Show that $\mathbf{Y}$ is a *multivariate normal* random vector with *mean vector* $\mathbf{0}$ and *variance co-variance matrix* AA^T.

2. In the setup of **Problem # 1** above, suppose $B_{k \times d}$ is a *rectangular* matrix with real entries which is *full row rank* (that is, rows are linearly independent). Let $\mathbf{W} := B\mathbf{X}$, then show that $\mathbf{W}$ is a *multivariate normal* random vector with *mean vector* $\mathbf{0}$ and *variance co-variance matrix* BB^T.

3. Show that if Σ is a *positive definite (p.d.)* matrix of order $d \times d$, then it is necessarily a variance-co-variance matrix of some random vector of dimension d. [**Hint:** Use Problem # 1 above].

4. Determine whether $\Sigma := ((\sigma_{ij}))_{1 \leq i,j \leq d}$, where

$$\sigma_{ij} := \begin{cases} 1 & \text{if } i = j, \\ \rho & \text{otherwise} \end{cases}$$

and $|\rho| < 1$, is a p.d. matrix. [**Hint:** Use Problem # 3 above].