Indian Statistical Institute

BSDS Ist Year

Academic Year 2024 - 2025: Semester I

Course: Probability Theory I

Instructor: Antar Bandyopadhyay

Assignment # 9

Date Given: October 30, 2024 **Date Due: November 07, 2024**
Total Points: 10

3.2.20 Show that the distribution of a random variable X with possible values $0, 1$, and 2 is determined by $\mu_1 := \mathbf{E}[X]$ and $\mu_2 := \mathbf{E}\left[X^2\right]$, by finding a formula for $\mathbf{P}(X = x)$ in terms of μ_1 and μ_2, $x = 0, 1, 2$.

3.3.8 Let A_1, A_2, and A_3 be events with probabilities $1/5, 1/4$, and $1/3$, respectively. Let N be the number of these events that occur.

(a) Write down a formula for N in terms of indicators.

(b) Find $\mathbf{E}[N]$.

In each of the following cases, calculate $\text{Var}(N)$:

(c) A_1, A_2, A_3 are disjoint;

(d) they are independent;

(e) $A_1 \subseteq A_2 \subseteq A_3$.

3.2.22 Let S be the number of successes in n independent Bernoulli trials, with possibly different probabilities $p_1, p_2, \cdots, p_n$ on different trials. Show that for fixed $\mu = \mathbf{E}[S]$, $\text{Var}(S)$ is largest in case the probabilities are all equal.

3.6.6 There are n balls labeled 1 through n, and n boxes labeled 1 through n. The balls are distributed randomly into the boxes, one in each box, so that all n permutations are equally likely. Say that a match occurs at place i if the ball labeled i happens to fall in the box labeled i. Let M be the total number of matches. Find $\mathbf{E}[M]$ and $\text{Var}(M)$.