

$$2. \quad W = \{(x, y) \in \mathbb{R}^2 : x + y = 0\}.$$

$$(ii) \quad \langle (x_1, y_1), (x_2, y_2) \rangle = 2x_1x_2 + y_1y_2$$

Let  $(a, b) \in \mathbb{R}^2$ . The shortest distance from  $(a, b)$  to  $W$  is  $\|(a, b) - \text{proj}_W(a, b)\|$ .

An orthonormal basis for  $W$  is  $\beta = \left\{ \left( \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}} \right) \right\}$

$$\text{Since } \left\langle \left( \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}} \right), \left( \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}} \right) \right\rangle = \frac{2}{3} + \frac{1}{3} = 1$$

$$\text{and } \left( \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}} \right) \in W, \text{ and } \dim W = 1.$$

$$\begin{aligned} \text{Now, } \text{proj}_W(a, b) &= \left\langle (a, b), \left( \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}} \right) \right\rangle \left( \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}} \right) \\ &= \frac{2a - b}{\sqrt{3}} \left( \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}} \right) \end{aligned}$$

$$\|(a, b) - \text{proj}_W(a, b)\| = \left\| (a, b) - \left( \frac{2a - b}{3}, -\frac{2a + b}{3} \right) \right\|$$

$$= \left\| \left( \frac{a + b}{3}, \frac{2a + 2b}{3} \right) \right\|$$

$$= \left\langle \left( \frac{a + b}{3}, \frac{2a + 2b}{3} \right), \left( \frac{a + b}{3}, \frac{2a + 2b}{3} \right) \right\rangle^{1/2}$$

$$= \left\langle (a + b) \left( \frac{1}{3}, \frac{2}{3} \right), (a + b) \left( \frac{1}{3}, \frac{2}{3} \right) \right\rangle^{1/2}$$

$$= (a + b) \left\langle \left( \frac{1}{3}, \frac{2}{3} \right), \left( \frac{1}{3}, \frac{2}{3} \right) \right\rangle^{1/2}$$

$$= (a + b) \left( \frac{2}{9} + \frac{4}{9} \right)^{1/2}$$

$$= (a + b) \frac{\sqrt{6}}{3}$$