Indian Statistical Institute

BSDS Ist Year

Academic Year 2024 - 2025: Semester I

Course: Probability Theory I

Instructor: Antar Bandyopadhyay

Assignment # 8

Date Given: October 23, 2024 **Date Due: October 31, 2024**
Total Points: 10

3.2.14 A building has 10 floors above the basement. If 12 people get into an elevator at the basement, and each chooses a floor at random to get out, independently of the others, at how many floors do you expect the elevator to make a stop to let out one or more of these 12 people?

3.2.26 A standard deck of 52 cards is shuffled and dealt. Let X_1 be the number of cards appearing before the first ace, X_2 the number of cards between the first and second ace (not counting either ace), X_3 the number between the second and third ace, X_4 the number between the third and fourth ace, and X_5 the number after the last ace. It can be shown that each of these random variables X_i has the same distribution for $i = 1, 2, \cdots, 5$, and you can assume this to be true.

(a) Write down a formula for $\mathbf{P}(X_i = k)$, where $0 \leq k \leq 48$.

(b) Show that $\mathbf{E}[X_i] = 9.6$. [Hint: Do not use your answer to (a)]

(c) Are $X_1, X_2, \cdots, X_5$ *pairwise independent*? Prove your answer.

3.2.22 Consider a sequence of $n \geq 4$ independent trials, each resulting in success (S) with probability p, and failure (F) with probability $1 - p$. Say *run of three successes* occurs at the beginning of the sequence if the *first four* trials result in SSSF; a *run of three successes* occurs at the end of the sequence if the *last four* trials result in FSSS; and a *run of three successes* elsewhere in the sequence is the pattern FSSSF. Let $R_{3,n}$ denote the number of runs of three successes in the n trials.

(a) Find $\mathbf{E}[R_{3,n}]$.

(b) Define $R_{m,n}$, the number of success runs of length m in n trials, similarly for $1 \leq m \leq n$. Find $\mathbf{E}[R_{m,n}]$.

(c) Let R_n be the total number of non-overlapping success runs in n trials, counting runs of any length between 1 and n. Find $\mathbf{E}[R_n]$ by using the result of (b).

(d) Find $\mathbf{E}[R_n]$ another way by considering for each $1 \leq j \leq n$ the number of runs that start on the j-th trial. Check that the two methods give the same answer.