Indian Statistical Institute

BSDS IInd Year

Academic Year 2025 - 2026: Semester I

Course: Probability II

Instructor: Antar Bandyopadhyay

Assignment # 1

Date Given: August 13, 2025 **Date Due: August 22, 2025**
Total Points: 10

6.5.4 Suppose X and Y are two standard normal variables. find an expression for $\mathbf{P}(x + 2Y \leq 3)$ in terms of the standard normal distribution function Φ,

(a) in case when X and Y are independent; and

(b) in case when X and Y have *bivariate normal* distribution with correlation $1/2$.

6.5.6 Let X and Y be independent standard normal variables.

(a) For a constant k, find $\mathbf{P}(X > kY)$.

(b) If $U = \sqrt{3}X + Y$ and $V = X - \sqrt{3}Y$, find $\mathbf{P}(U > kV)$.

(c) Find $\mathbf{P}\left(U^2 + V^2 < 1\right)$.

(d) find the conditional distribution of X given $V = v$.

6.5.10 Show that if V and W have a bivariate normal distribution then

(a) every linear combination $aV + bW$ has a normal distribution;

(b) every pair of linear combinations $(aV + bW, cV + dW)$ has a bivariate normal distribution;

(c) Find the parameters of the distributions obtained in (a) and (b) above in terms of the parameters of the joint distribution of V and W.

6.5.11 Show that for standard bivariate normal variables X and Y with correlation ρ,

$$\mathbf{E}[\max(X, Y)] = \sqrt{\frac{1-\rho}{\pi}}$$