Indian Statistical Institute

BSDS Ist Year

Academic Year 2024 - 2025: Semester I

Course: Probability Theory I

Instructor: Antar Bandyopadhyay

Assignment # 7

Date Given: October 16, 2024 **Date Due: October 24, 2024**
Total Points: 10

3.1.12 Suppose that counts $(N_1, N_2, \cdots, N_m)$ are the numbers of results in m categories in n repeated trials. So $(N_1, N_2, \cdots, N_m)$ has *multinomial distribution* with parameters n and $p_1, p_2, \cdots, p_m$, as in the box above Example 3.1.7. Let $1 \leq i < j \leq m$. Answer the following questions with an explanation, but no calculation.

(a) What is the distribution of N_i?

(b) What is the distribution of $N_i + N_j$?

(c) What is the joint distribution of N_i, N_j, and $n - N_i - N_j$?

3.1.24 Suppose a box contains tickets, each labeled by an integer. Let X, Y, and Z be the results of draws at random with replacement from the box: Show that, no matter what the distribution of numbers in the box,

(a) $\mathbf{P}(X + Y \text{ is even }) \geq 1/2$;

(b) $\mathbf{P}(X + Y + Z \text{ is a multiple of } 3) \geq 1/4$;

Problem 3 Suppose 10 cards are dealt (without replacemnt) from a standard deck of 52 cards which are *well shuffled*. Let $X :=$ number of J, Q or K; $Y :=$ number of Aces and $Z :=$ number of *number* cards among the 10 randomly selected cards. Find

(a) Find the distribution of X.

(b) Find the distribution of $X + Y$.

(c) Find the joint distribution of (X, Y, Z).