

Indian Statistical Institute

BSDS Ist Year

Academic Year 2024 - 2025: Semester I

Course: Probability Theory I

Instructor: Antar Bandyopadhyay

Assignment # 14

Date Given: November 27, 2024

Date Due: December 11, 2024
Total Points: 10

5.3.6 Let X and Y be independent *standard normal* variables. Find

- (a) $\mathbf{P}(3X + 2Y > 5)$;
- (b) $\mathbf{P}(\min(X, Y) < 1)$;
- (c) $\mathbf{P}(|\min(X, Y)| < 1)$;
- (d) $\mathbf{P}(\min(X, Y) > \max(X, Y) - 1)$.

6.3.6 Suppose X, Y are random variables with joint density

$$f_{X,Y}(x, y) = \frac{e^{-y/2}}{2\pi \sqrt{x(y-x)}}, \quad 0 < x < y < \infty.$$

- (a) Find the marginal distribution of Y .
- (b) Compute $\mathbf{E}[X \mid Y = 1]$.

5.2.12 An urn contains 1 black and 2 white balls. One ball is drawn at random and its color noted. The ball is replaced in the urn, together with an additional ball of its color. There are now four balls in the urn. Again, one ball is drawn at random from the urn, then replaced along with an additional ball of its color. The process continues in this way.

- (a) Let B_n be the number of black balls in the urn just before the n -th ball is drawn. (Thus B_1 is 1.) For $n \geq 1$, find $\mathbf{E}[B_{n+1} \mid B_n]$.
- (b) For $n \geq 1$, find $\mathbf{E}[B_n]$.
- (c) For $n \geq 1$, what is the expected proportion of black balls in the urn just before the n th ball is drawn?

Problem # 4 Suppose $X \sim \text{Beta}(a, b)$. Find the *probability density function (pdf)* of $Y = X/(1 - X)$.