Obj fn $\quad C^T x = (c_1, c_2, \dots, c_n)\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \sum_{i=1}^{n} c_i x_i$

s.t.

$$Ax \ (\le, =, \ge)\ b \qquad A \in M_{m \times n}$$

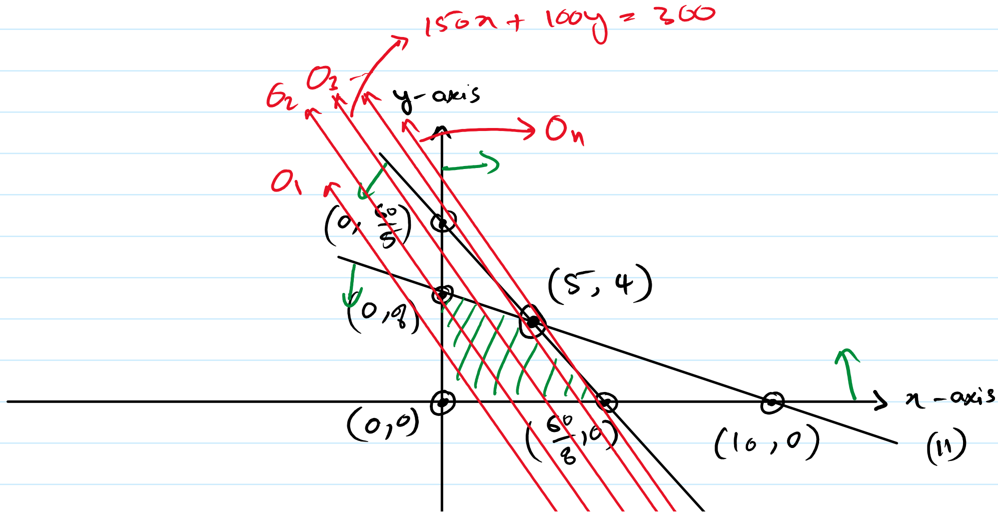
$$x_j \ge 0 \quad \forall\ j \quad \{j = 1, 2, \dots, n\}$$

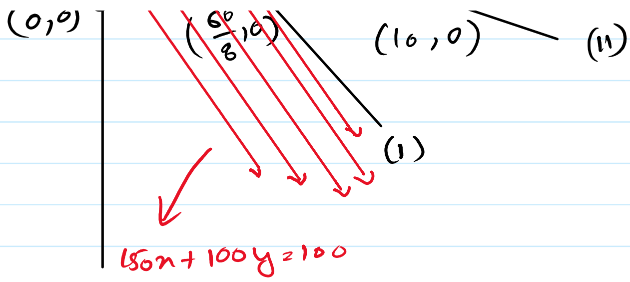
Max $\quad Z = 150x + 100y$ —— (i)

s.t. $\quad 8x + 5y \le 60$ —— (ii)

$\quad 4x + 5y \le 40$ —— (iii)

$\quad x, y \ge 0$





i^{th} optimal function

O_i as i is increasin in the feasible region the value of O_i also increases.

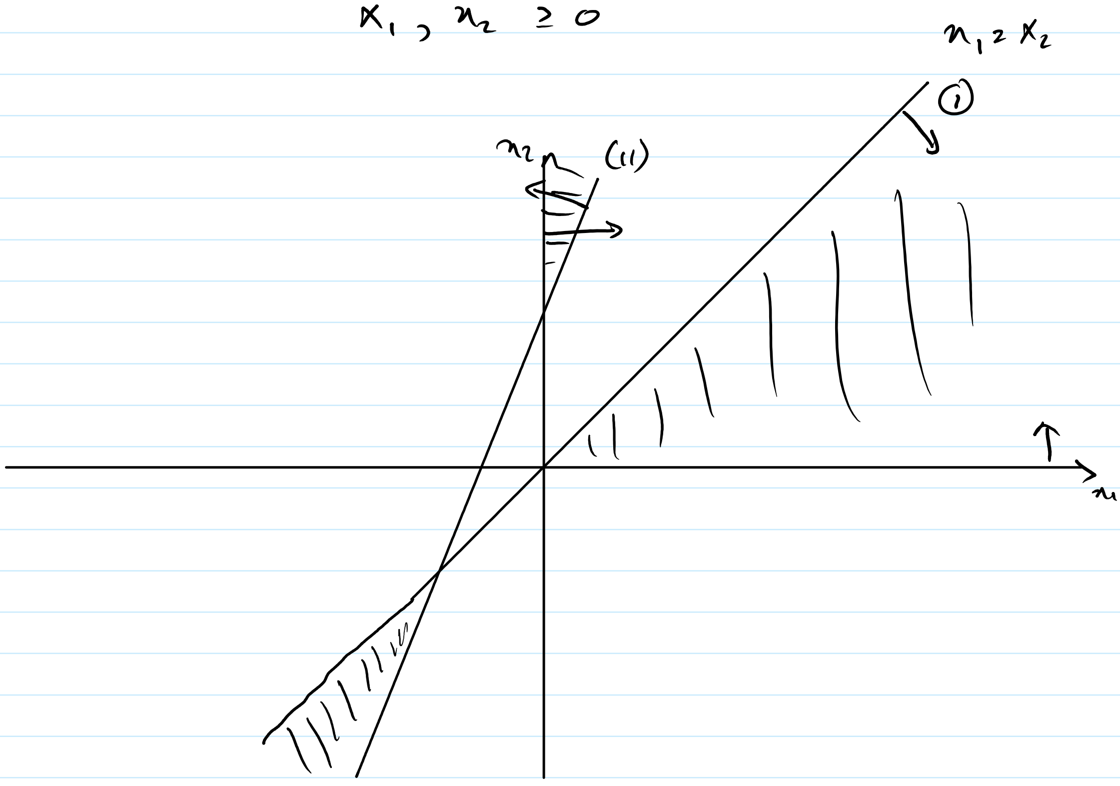
Q2

Max $z = x_1 + x_2$

s.t. $x_1 - x_2 \geq 0$

$2x_1 - x_2 \leq -2$

$x_1, x_2 \geq 0$



$\therefore$ No common feasible sol^n region

Q3 Max $z = 3x_1 + 4x_2$

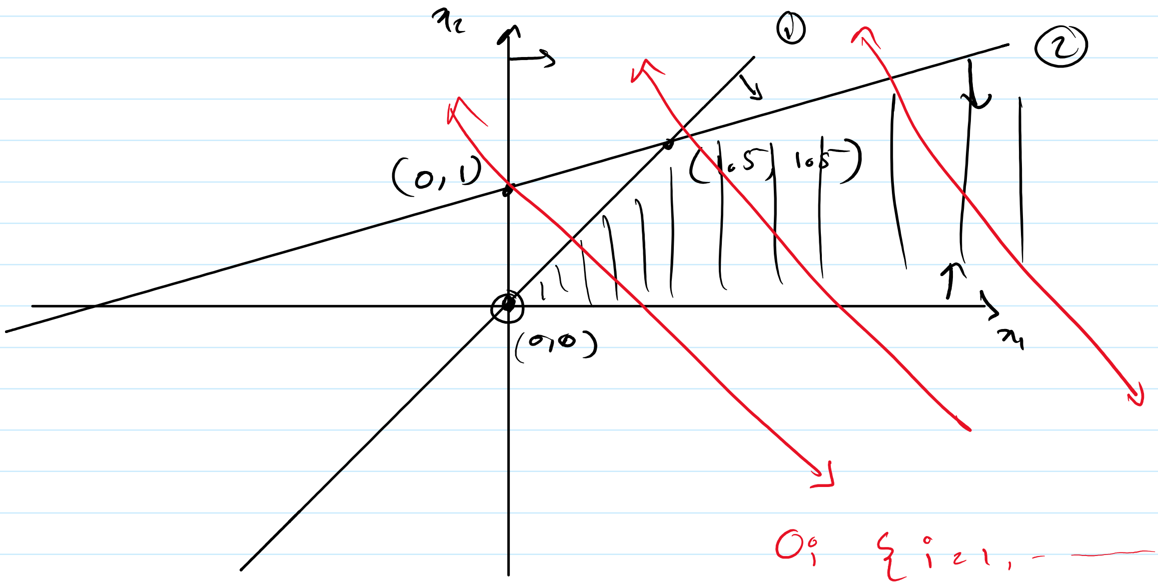
s.t. $x_1 - x_2 \geq 0$ — ①

$-x_1 + 3x_2 \leq 3$ — ②

$x_1, x_2 \geq 0$

$x_2 \geq x_1$ — ①

$x_2 \leq \frac{x_1}{3} + 1$ — ②



$O_i \; \{i = 1, \ldots, n\}$

Q4

$\boxed{\text{How}}$

Max $z = 6x_1 + 4x_2$

s.t. $7x_1 + 5x_2 \leq 35$

$5x_1 + 7x_2 \leq 35$

$4x_1 + 3x_2 \geq 12$

$3x_1 + x_2 > 3$

$x_1, x_2 \geq 0$

$Ax = b \quad A_{m\times n} \quad b \in R^m \quad x \in R^n$

$m \to$ no. of constraints
$n \to$ " " variable

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{n1} & \cdots & \cdots & a_{nn} \end{bmatrix} \quad R(A) = m \quad A = [\underset{m}{B} \ \underset{n-m}{N}]$$

(Basic) $B = (\quad)_{m\times m}$

x_B = basic variable
x_N = Non-basic variable

$$[B \quad N] \begin{bmatrix} X_B \\ X_N \end{bmatrix} = b$$

$$B x_B + \cancel{N X_n}^{\,0} = b$$

$$X_B = B^{-1} b$$

$$x = (B^{-1} b, 0)$$

Ex $\to$

$$2x_1 + 4x_2 - 2x_3 = 10$$

$$10x_1 + 3x_2 + 7x_3 = 33$$

$$A = \begin{array}{ccc} \begin{bmatrix} 2 & 4 & -2 \\ 10 & 3 & 7 \end{bmatrix} \\ a_1 \quad a_2 \quad a_3 \end{array}$$

$$x_1, x_2, x_3 \geq 0 \qquad R(A) = m = 2$$

$$B_1 (a_1 a_2) = \begin{pmatrix} 2 & 4 \\ 10 & 3 \end{pmatrix}, \ B_2 (a_1, a_3), \ B_3 (a_2, a_3)$$

$$\det(B_1) = -34$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = X_B = B_1^{-1} b = \begin{pmatrix} & \end{pmatrix}\begin{pmatrix} \\ \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$$x = (x_1, x_2, x_3)$$

$$x = (3, 1, 0) \checkmark \quad \text{as } x_1, x_2, x_3 \geq 0$$

- All BFS (Basic feasible sol^n) of $\underbrace{Ax = b, \; x \geq 0}_{\text{LPP}}$ are extreme pt. of convex set of f.s & conversely

- If LPP admits an optimal sol^n then the obj fn attains the optimum value at an extreme pt. of the convex set generated by all f.S.