NDIAN STATISTICAL INSTITUTE
Assignment-1 (Mathematics III)
Bachelor of Statistical Data Science (BSDS)

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1. Write the matrix representations of the linear operators with respect to the ordered basis $\mathcal{B}$

   - Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $T(x, y) = (x, y)$, and let $\mathcal{B} = \{(1, 1), (1, -1)\}$ be the ordered basis.
   - Let $D : \mathbb{P}_n(\mathbb{R}) \to \mathbb{P}_n(\mathbb{R})$ be the differentiation operator:
   $$D\left(a_0 + a_1x + a_2x^2 + \cdots + a_nx^n\right) = a_1 + 2a_2x + \cdots + na_nx^{n-1},$$
   with $\mathcal{B} = \{1, x, x^2, \ldots, x^n\}$.
   - Let $T : M_2(\mathbb{F}) \to M_2(\mathbb{F})$ be given by
   $$T\left(\begin{bmatrix} x & y \\ z & w \end{bmatrix}\right) = \begin{bmatrix} x+w & z \\ z+w & x \end{bmatrix},$$
   and let the ordered basis be
   $$\mathcal{B} = \left\{ E_{11} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, E_{12} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, E_{21} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, E_{22} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}.$$

2. Which of the following is an inner product.

   (a) $\langle (x_1, y_1), (x_2, y_2) \rangle = x_1x_2 + y_1y_2 + 3$ on $\mathbb{R}^2$ over $\mathbb{R}$.
   (b) $\langle (x_1, y_1), (x_2, y_2) \rangle = x_1x_2 - y_1y_2$ on $\mathbb{R}^2$ over $\mathbb{R}$.
   (c) $\langle (x_1, y_1), (x_2, y_2) \rangle = y_1(x_1 + 2x_2) + y_2(2x_1 + 5x_2)$ on $\mathbb{R}^2$ over $\mathbb{R}$.
   (d) $\langle (x_1, y_1), (x_2, y_2) \rangle = x_1\overline{x_2} + y_1\overline{y_2}$ on $\mathbb{C}^2$ over $\mathbb{C}$.
   (e) $\langle (x_1, y_1), (x_2, y_2) \rangle = x_1\overline{x_2} - y_1\overline{y_2}$ on $\mathbb{C}^2$ over $\mathbb{C}$.
   (f) If $A, B \in M_n(\mathbb{C})$, define $\langle A, B \rangle = \mathrm{Trace}(A\overline{B})$.
   (g) Suppose $C[0, 1]$ is the space of continuous complex-valued functions on the interval $[0, 1]$, and for $f, g \in C[0, 1]$,
   $$\langle f, g \rangle := \int_0^1 f(t)\overline{g(t)}\, dt.$$

3. Suppose $A = \begin{bmatrix} a & b \\ b & d \end{bmatrix} \in M_2(\mathbb{R})$ is such that $a > 0$ and $\det(A) = ad - b^2 > 0$. Show that
$$\langle X, Y \rangle = X^tAY$$
is an inner product on $\mathbb{R}^2$.