Indian Statistical Institute

BSDS IInd Year

Academic Year 2025 - 2026: Semester I

Course: Probability II

Instructor: Antar Bandyopadhyay

Assignment # 4

Date Given: September 03, 2025 **Date Due: September 12, 2025**
Total Points: 10

Problem # 1 Suppose $X_1, X_2, X_3, \cdots, X_n$ are *i.i.d.* Normal random variables with mean $\mu \in \mathbb{R}$ and variance $\sigma^2 > 0$. Then show that the two *statistics*, $\bar{X}_n := \frac{1}{n} \sum\limits_{i=1}^{n} X_i$ and $s_n^2 := \frac{1}{n} \sum\limits_{i=1}^{n} \left(X_i - \bar{X}_n\right)^2$ are independent. Find the distributions of $\bar{X}_n$ and s_n^2.

Problem # 2 Give **five** different examples of non-negative integer valued discrete random variables with mean **infinity** and give **five** different examples of non-negative real valued continuous random variables with mean **infinity**.

6.3.24 A box contains four tickets, numbered $0, 1, 1$ and 2. Let S_n be the sum of the numbers obtained from n draws at random with replacement from the box.

(a) Display the distribution of S_n in a suitable table.

(b) Find $\mathbf{P}\left(S_{50} = 50\right)$ approximately.

(c) Find an exact formula for $\mathbf{P}\left(S_n = k\right)$ for $k = 0, 1, 2, \cdots$.

6.3.25 **Equality in Chebychev's Inequality:** Let μ, σ and k be three numbers, with $\sigma > 0$ and $k \geq 1$. Let X be a random variable with the following distribution:

$$\mathbf{P}\left(X = x\right) = \begin{cases} \frac{1}{2k^2} & \text{if } x = \mu + k\sigma \text{ or } x = \mu - k\sigma \\ 1 - \frac{1}{k^2} & \text{if } x = \mu \\ 0 & \text{otherwise.} \end{cases}$$

(a) Sketch the histogram of this distribution for $\mu = 0, \sigma = 10, k = 1, 2, 3$.

(b) Show that $\mathbf{E}\left[X\right] = \mu$, $\mathbf{Var}\left(X\right) = \sigma^2$ and $\mathbf{P}\left(|X - \mu| \geq k\sigma\right) = \frac{1}{k^2}$. (So there is an equality in Chebychev's inequality for this distribution of X. This means Chebychev's inequality cannot be improved without additional hypothesis on the distribution of X.)

(c) Show that if Y has $\mathbf{E}\left[Y\right] = \mu$, $\mathbf{Var}\left(Y\right) = \sigma^2$ and $\mathbf{P}\left(|Y - \mu| < \sigma\right) = 0$, then Y has the same distribution as X described above for $k = 1$.