Indian Statistical Institute

BSDS IInd Year

Academic Year 2025 - 2026: Semester I

Course: Probability II

Instructor: Antar Bandyopadhyay

Assignment # 5

Date Given: September 10, 2025 **Date Due: September 19, 2025**
Total Points: 10

1. Suppose $A_1, A_2, A_3, \cdots, A_n, \cdots$ are events which are occurring *almost surely*. Then show that the events $\overset{n}{\underset{i=1}{\cap}} A_i$ also occur *almost surely* for all $n \geq 1$. Further, determine whether the event $\overset{\infty}{\underset{n=1}{\cap}} A_n$ also occur *almost surely*.

2. Give an example of a collection of events $\{A_\alpha\}_{\alpha \in \mathcal{I}}$, with some indexing set $\mathcal{I}$, such that, for each $\alpha \in \mathcal{I}$, the event A_α occurs almost surely, but $\underset{\alpha \in \mathcal{I}}{\cap} A_\alpha = \emptyset$.

3. Suppose $\varepsilon_1, \varepsilon_2, \varepsilon_3, \cdots, \varepsilon_n$ be *i.i.d. standard normal* random variables. Define

$$X_1 = \varepsilon_1; \text{ and } X_{i+1} = X_i + \phi\, \varepsilon_{i+1}, i \geq 1$$

for some $\phi \in \mathbb{R}$. Find the joint distribution of $(X_1, X_2, X_3, \cdots X_n)$.