Indian Statistical Institute

BSDS Ist Year

Academic Year 2024 - 2025: Semester I

Course: Probability Theory I

Instructor: Antar Bandyopadhyay

Assignment # 4

Date Given: September 11, 2024 **Date Due: September 19, 2024**
Total Points: 10

3.1.4 In a World Series, teams A and B play until one team has won four games. Assume that each game played is won by team A with probability p, independently of all previous games.

(a) For $g = 4$ through 7, find a formula in terms of p and $q = 1 - p$ for the probability that team A wins in g games.

(b) What is the probability that team A wins the World Series, in terms of p and q?

(c) Use your formula to evaluate this probability for $p = 1/2$ and $p = 2/3$.

(d) Let X be a Binomial$(7, p)$ random variable. Explain why $\mathbf{P}(A\text{wins}) = \mathbf{P}(X \geq 4)$ using an intuitive argument. Verify algebraically that this is true.

(e) Let G represent the number of games played. What is the distribution of G?

3.4.14 In independent repetation of a Bernoulli(p) trials let V_n be the number of trials required to produce either n successes or n failures, whichever comes first. Find the distribution of V_n.

3.6.2 A deck of standard 52 cards is shuffled and dealt. Find the probabilities of the following events and give reasons supporting your answers:

(a) the tenth card is a *queen*;

(b) the twentieth card is a spade;

(c) the last five cards are spades;

(d) The last king appears on the 48-th card.

2.2.10 A probability class has 30 students. As part of an assignment, each student tosses an *unbiased* coin 200 times and records the number of heads. What is the chance that no student gets exactly 100 heads? Write a formula and give reasons behind your answer. Can you use R-programming to give an approximation of what the value is?