Demand Analysis: An Introduction

Manoranjan Pal

Syllabus: Demand Analysis

- **Syllabus: Demand analysis**
- Introduction, Popular Simple Forms of Engel Curves, Classification of commodities, Engel curve analysis using cross-section and time series data, Engel curves incorporating household characteristics, demand projection, specific concentration curves.

Demand Analysis

- **Demand Analysis**
- Consider a consumer in a given accounting period. Suppose x = disposable income of the person. There are n goods (including services) with fixed prices $p_1, p_2, \dots, p_n$. The consumer can buy any bundle $q_1, q_2, \dots, q_n$ subject to the condition

$$\sum_{1}^{n} p_i q_i \leq x.$$

- We assume that the consumer will buy that bundle for which his/her preference function $u(q_1, q_2, \dots, q_n)$ is maximized subject to the budget constraint $\sum p_i q_i \leq x$.

The Optimum Quantities

- We use Lagrange multiplier technique to get the optimum quantities.
- $We\ maximize\ L = u + \lambda(\sum_1^n p_i q_i - x)$.

$$\frac{\partial L}{\partial q_i} = u_i + \lambda p_i = 0\ \forall i$$

$$\begin{Bmatrix} \dfrac{\partial L}{\partial q_i} = u_i + \lambda p_i = 0\ \forall i \\ \dfrac{\partial L}{\partial \lambda} = \displaystyle\sum_1^n p_i q_i - x = 0. \end{Bmatrix} \qquad \dots (I)$$

- We solve the system of equations to get

$$\begin{Bmatrix} q_i = f_i(p_1, p_2, \dots, p_n, x) \\ \lambda = f_\lambda(p_1, p_2, \dots, p_n, x). \end{Bmatrix} \qquad \dots (II)$$

Demand Function

- From (I) we get

$$\frac{u_1}{p_1} = \frac{u_2}{p_2} = \cdots = \frac{u_n}{p_n} = -\lambda.$$

- (Observe that $u_i = \frac{\partial u}{\partial q_i}$ = marginal utility of commodity i.)

$$q_i = f_i(p_1, p_2, \dots, p_n, x)$$

- is called the demand function for the i^{th} good.
- The set of all demand functions thus derived is known as Marshallian demand function.

Demand Curve and Engel Curve

- Special cases of demand function in which all but one of the parameters $p_1, p_2, \ldots, p_n, x$ are held constant, are widely used in the literature on partial equilibrium analysis. Thus, holding $p_1, p_2, \ldots, p_{i-1}, p_{i+1}, \ldots, p_n$ and x constant in the above equation gives the demand curve for the ith good.

$$q_i = f_i(p_i). \qquad \ldots \text{(III)}$$

- The demand curve indicates the effect of a change in the price of a good on the quantity demanded on the same good, holding other prices and income constant. (i.e., the ceteris paribus effect of a change in own price.)
- A second partial equilibrium approach holds $p_1, p_2, \ldots, p_n$ constant to yield

$$\left\{\begin{matrix} \bar{p}_i q_i = g_i^*(x) \\ \text{or } q_i = g_i^{**}(x). \end{matrix}\right\} \qquad \ldots \text{(IV)}$$

- This is known as Engel Curve. It indicates the effect of a change in income on the expenditure/quantity for ith good at fixed prices. This is also known as income-consumption relation.

Own Price Elasticity of Demand

- Elasticities of demand can be defined using the demand function, demand curve or Engel curve. These elasticities are convenient summaries of the responsiveness of the quantity demanded to factors influencing it, because these are independent of the units of measurement of the good, prices or income.
- The own price elasticity of demand for good j is defined as

$$\eta_{jj} = \frac{\partial f_j(p_1, p_2, \dots, p_n, x)}{\partial p_j} . \frac{p_j}{q_j}$$

$$= \frac{\partial \mathrm{Ln}\left(f_j\right)}{\partial \mathrm{Ln}\left(p_j\right)} .$$

- It gives us the percentage change in the quantity demanded for 1% change in the price of the good. Usually, the price elasticity so defined is negative. The good is said to be price elastic if $\left|\eta_{jj}\right| > 1$ and price inelastic if $\left|\eta_{jj}\right| < 1$.

Income Elasticity of Demand

- The income elasticity of demand for good j is

$$\eta_{jx} = \frac{\partial f_j(p_1, p_2, \dots, p_n, x)}{\partial x} . \frac{x}{q_j}$$

$$= \frac{\partial Ln\left(f_j\right)}{\partial Ln(x)} .$$

- It gives us the percentage change in the quantity demanded for 1% change in the income. Usually, the income elasticity is positive. The good is said to be income elastic if $\eta_{jx} > 1$ and income inelastic if $\eta_{jx} < 1$.

Cross-price Elasticity of Demand

- Finally, the cross-price elasticity of demand is defined as

$$\eta_{jj'} = \frac{\partial f_j(p_1, p_2, \dots, p_n, x)}{\partial p_{j'}} . \frac{p_{j'}}{f_j}$$

$$= \frac{\partial Ln\left(f_j\right)}{\partial Ln\left(p_{j'}\right)} .$$

- It indicates the effect of a change in the price of j'th good on the demand for the jth good ($j \neq j'$). (If $j = j'$ then it becomes own price elasticity.)

Engel's law, Giffen and Inferior Goods

- **Engel's law:** Engel's law refers to the property of the Engel curve for good. The law states that the proportion of income consumers spend on food decreases as their income increases. If q_2 is food, then $\frac{q_2 p_2}{x}$ decreases as x increases. This property is however true for all income inelastic goods. Income inelastic goods are referred as necessary goods.
- A good is said to be **normal** if it has a downward sloping demand curve or equivalently a negative own price elasticity. Otherwise, it is a **Giffen** good.
- A good is said to be **superior** if it has an upward sloping Engel curve, or equivalently a positive income elasticity. Otherwise, it is an **inferior** good.
- A Giffen good cannot be a superior good.

Budget Constraint

- Economic theory suggests that the demand functions should satisfy certain restrictions.

1. Budget Constraint: First, they must satisfy the budget constraint.

$$\sum p_i q_i = x.$$

- If we divide both sides of the constraint by x, then we get

$$\sum \frac{p_i q_i}{x} = 1$$

$$\text{or } \sum s_i = 1,$$

- where s_i is called the budget share. So, s_i is the proportion of income spent on ith commodity.

Adding up Criterion

2. Adding up Criterion: If we differentiate the budget constraint with respect to income, we get

$$\sum p_i \frac{\partial q_i}{\partial x} = 1.$$

$$\text{or} \sum p_i . \frac{\partial q_i / q_i}{\partial x / x} . \frac{q_i}{x} = 1.$$

$$\text{or} \sum \frac{p_i q_i}{x} \eta_{ix} = 1.$$

$$\text{or} \sum s_i \eta_{ix} = 1.$$

- Weighted mean of income elasticities, where weights are budget shares, add up to 1. This is also known as Engel Aggregation Condition.

Cournot Aggregation Conditions

3. **Cournot Aggregation Conditions:** Another set of aggregation conditions, known as Cournot Aggregation Conditions, can be found by differentiating budget constraint with respect to p_i, i = 1, 2, …, n. Thus, we have n Cournot aggregation conditions.

$$\sum_{j=1}^{n} p_j \frac{\partial q_j}{\partial p_i} + q_i = 0.$$

$$\text{or } \sum_{j=1}^{n} \frac{\partial q_j / q_j}{\partial p_i / p_i} \frac{q_j p_j}{p_i} + q_i = 0.$$

$$\text{or } \sum_{j=1}^{n} \eta_{ji} \frac{q_j p_j}{x} / \frac{q_i p_i}{x} + 1 = 0.$$

$$\text{or } \sum_{j=1}^{n} \eta_{ji} \frac{s_j}{s_i} + 1 = 0.$$

Homogeneity Condition

4. Homogeneity Condition: A scaling of all prices and income from $(\mathrm{p}_1, \mathrm{p}_2, \dots, \mathrm{p}_\mathrm{n}, \mathrm{x})$ to $(\alpha\mathrm{p}_1, \alpha\mathrm{p}_2, \dots, \alpha\mathrm{p}_\mathrm{n}, \alpha\mathrm{x})$ does not change the demand. Or,

$$\mathrm{f}_\mathrm{i}(\alpha\mathrm{p}_1, \alpha\mathrm{p}_2, \dots, \alpha\mathrm{p}_\mathrm{n}, \alpha\mathrm{x}) = \mathrm{f}_\mathrm{i}(\mathrm{p}_1, \mathrm{p}_2, \dots, \mathrm{p}_\mathrm{n}, \mathrm{x}).$$

- Thus, the demand function should be homogeneous of degree zero. It follows from the Euler's theorem of homogeneous function that

$$\frac{\partial \mathrm{f}_\mathrm{i}}{\partial \mathrm{p}_1}\mathrm{p}_1 + \frac{\partial \mathrm{f}_\mathrm{i}}{\partial \mathrm{p}_2}\mathrm{p}_2 + \cdots + \frac{\partial \mathrm{f}_\mathrm{i}}{\partial \mathrm{p}_\mathrm{n}}\mathrm{p}_\mathrm{n} + \frac{\partial \mathrm{f}_\mathrm{i}}{\partial \mathrm{x}}\mathrm{x} = 0.$$

$$\text{or } \eta_{\mathrm{i}1} + \eta_{\mathrm{i}2} + \cdots + \eta_{\mathrm{in}} + \eta_{\mathrm{ix}} = 0$$

- ***Euler's Theorem:***

$$f(\alpha x_1, \alpha x_2, \dots, \alpha x_n) = \alpha^h f(x_1, x_2, \dots, x_n)$$

$$\Rightarrow \sum_{j=1}^{n} \frac{\partial f}{\partial x_j} x_j = hf(x_1, x_2, \dots, x_n).$$

Thank you