

Indian Statistical Institute

BSDS Ist Year

Academic Year 2024 - 2025: Semester I

Course: Probability Theory I

Instructor: Antar Bandyopadhyay

Assignment # 12

Date Given: November 20, 2024

Date Due: November 28, 2024
Total Points: 10

4.4.8 Suppose that Y has the *Cauchy distribution* as in Exercise. Let $Z = 1/(1 + Y^2)$.

- (a) Find the density of Z .
- (b) Find $\mathbf{E}[Z]$.
- (c) Find $\text{Var}(Z)$.

5.4.10 Find the density of $Y = U/V$ for independent Uniform $(0, 1)$ variables U and V .

5.4.18 Let f_n be the density function and F_n the CDF of the sum S_n of n independent Uniform $(0, 1)$ random variables.

- (a) Show that $f_n(x) = F_{n-1}(x) - F_{n-1}(x - 1)$.
- (b) Show that on each of the n intervals $(i - 1, i)$ for $i = 1$ to n , f_n is equal to a polynomial of degree $n - 1$, and F_n is equal to a polynomial of degree n .
- (c) Find $f_n(x)$ and $F_n(x)$ for $0 \leq x \leq 1$.
- (d) Find $f_n(x)$ and $F_n(x)$ for $n - 1 \leq x \leq n$.
- (e) Find $\mathbf{P}(0 \leq S_4 \leq 1)$.
- (f) Find $\mathbf{P}(1 \leq S_4 \leq 2)$.
- (g) Find $\mathbf{P}(1.5 \leq S_4 \leq 2)$.