Indian Statistical Institute

BSDS Ist Year

Academic Year 2024 - 2025: Semester I

Course: Probability Theory I

Instructor: Antar Bandyopadhyay

Assignment # 10

Date Given: November 06, 2024 **Date Due: November 14, 2024**
Total Points: 10

3.3.30 Let X_i be the last digit of D_i^2, where D_i is a random digit between 0 and 9. For instance, if $D_i = 7$ then $D_i^2 = 49$ and $X_i = 9$. Let $\bar{X}_n := \left(X_1 + X_2 + \cdots + X_n\right)/n$ be the average of a large number n of such last digits, obtained from independent random digits $D_1, D_2, \cdots, D_n$.

(a) Predict the value of $\bar{X}_n$ for large n.

(b) Find a number ε such that for $n = 10,000$ the chance that your prediction is off by more than ε is about 1 in 200.

(c) Find approximately the least value of n such that your prediction of $\bar{X}_n$ is correct to within 0.01 with probability at least 0.99.

(d) Which can be predicted more accurately for large n: the value of $\bar{X}_n$, or the value of $\bar{D}_n := \left(D_1 + D_2 + \cdots + D_n\right)/n$?

(e) If you just had to predict the first digit of $\bar{X}_{100}$, what digit should you choose to maximize your chance of being correct, and what is that chance?

4.1.2 Suppose X has density $f(x) = c/x^4$ for $x > 1$, and $f(x) = 0$ otherwise, where c is a constant. Find

(a) the constant c;

(b) $\mathbf{E}[X]$; and

(c) $\text{Var}(X)$.

4.1.4 Suppose X with values in $(0, 1)$ has density $f(x) = cx^2(1-x)^2$ for $0 < x < 1$. Find

(a) the constant c;

(b) $\mathbf{E}[X]$; and

(c) $\text{Var}(X)$.

4.5.2 Find and sketch the cumulative distribution functions (CDF) of

(a) the Binomial $(3, 1/2)$ distribution; and

(b) the Geometric $(1/2)$ distribution;