Test the following series for convergence or divergence and give a reason for your decision in each case.

1. $$\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}.$$

9. $$\sum_{n=1}^{\infty} e^{-n^2}.$$

3. $$\sum_{n=1}^{\infty} \frac{2^n n!}{n^n}.$$

12. $$\sum_{n=1}^{\infty} \frac{n^{n+1/n}}{(n+1/n)^n}.$$

7 $$\sum_{n=2}^{\infty} \frac{1}{(\log n)^{1/n}}$$

In Exercises 1 through 32, determine convergence or divergence of the given series. In case of convergence, determine whether the series converges absolutely or conditionally.

10. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\log(e^n + e^{-n})}$.

4. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \left(\frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2 \cdot 4 \cdot 6 \cdots (2n)}\right)^3$.

5. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n(n-1)/2}}{2^n}$.

15. $\displaystyle\sum_{n=1}^{\infty} \sin(\log n)$.

49. If $a_n > 0$ and $\sum a_n$ converges, prove that $\sum 1/a_n$ diverges.