

Indian Statistical Institute

BSDS IInd Year

Academic Year 2025 - 2026: Semester I

Course: Probability II

Instructor: Antar Bandyopadhyay

Assignment # 7

Date Given: September 24, 2025

Date Due: October 07, 2025
Total Points: 10

1. Suppose X_1, X_2, \dots, X_n are i.i.d. outcomes of an *unbiased coin toss*. Show that

$$\mathbf{P}(X_n = H, X_{n+1} = H, \dots, X_{n+99} = H \text{ i.o.}) = 1.$$

In other words, probability of having 100 heads in succession infinitely often from repeated but independent unbiased coin toss is one.

2. For $(A_n)_{n \geq 1}$ a sequence of events show that

$$\liminf_{n \rightarrow \infty} \mathbf{P}(A_n) \leq \mathbf{P}(A_n \text{ eventually}) \leq \mathbf{P}(A_n \text{ i.o.}) \leq \limsup_{n \rightarrow \infty} \mathbf{P}(A_n).$$

3. Suppose we have a printed book with very large number of pages. Let X_n be the number of misprints in the book till page n . Show that there exists a constant $c > 0$, such that,

$$\frac{X_n}{n} \longrightarrow c \text{ a.s. as } n \rightarrow \infty$$

State all your assumptions clearly and interpret the constant c .

4. Let $(U_n)_{n \geq 1}$ be i.i.d. Uniform $(0, 1)$ and $M_n := \max(U_1, U_2, \dots, U_n)$. Show that $M_n \xrightarrow{\mathbf{P}} 1$ as $n \rightarrow \infty$. Hence or otherwise show that, in fact, $M_n \longrightarrow 1$ a.s. as $n \rightarrow \infty$.