Data Structures and Algorithms
ISI-Delhi
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Problems Session 1

Asymptotic Notations

Problem 1

For each row i, indicate whether A_i is in $\mathcal{O}$, Ω, or Θ of B_i. Mark all that apply. The first two rows are examples.

A	**B**	$\mathcal{O}$	Ω	Θ
$5n$	n	✓	✓	✓
5	n	✓		
n^2	$2^{10}n$			
$(7/8)^n$	$(8/7)^n$			
8^n	4^n			
n^5	$32^{\log n}$			
n^{2025}	2025^n			
$n^{0.8}$	$(0.8)^n$			
$n^{\log 3}$	$3^{\log n}$			
$\log(n!)$	$\log(n^n)$			
$n^{1/\log n}$	1			
$\log^7 n$	$n^{0.7}$			
$\log \sqrt{n}$	$4^{\sqrt{n}}$			
$\log(\sqrt{n})$	$\sqrt{\log n}$			

Problem 2

Prove the following:

a) $3n^3 + 4n^2 = \mathcal{O}(n^3)$

b) 16^n is **not** $\mathcal{O}(2^n)$

Problem 3

Use a recursion tree to determine a good asymptotic upper bound on the recurrence

$$T(n) = 3\,T\left(\left\lfloor \frac{n}{2} \right\rfloor\right) + n.$$

Then use the substitution method to verify your bound. Clearly state your final asymptotic result in Big-Theta notation.

Problem 4

Let $f(n)$ and $g(n)$ be asymptotically positive functions. Prove or disprove each of the following conjectures.

a) $f(n) = O(g(n))$ implies $g(n) = O(f(n))$.

b) $f(n) + g(n) = \Theta(\min(f(n), g(n)))$.

c) $f(n) = O(g(n))$ implies $\lg(f(n)) = O(\lg(g(n)))$, where $\lg(g(n)) \geq 1$ and $f(n) \geq 1$ for all sufficiently large n.

d) $f(n) = O(g(n))$ implies $2^{f(n)} = O(2^{g(n)})$.

e) $f(n) = O((f(n))^2)$.

f) $f(n) = O(g(n))$ implies $g(n) = \Omega(f(n))$.

g) $f(n) = \Theta(f(n/2))$.

h) $f(n) + o(f(n)) = \Theta(f(n))$.

Problem 5

Let

$$p(n) = \sum_{i=0}^{d} a_i n^i,$$

where $a_d > 0$, be a degree-d polynomial in n, and let k be a constant. Use the definitions of the asymptotic notations to prove the following properties:

a) If $k \geq d$, then $p(n) = O(n^k)$.

b) If $k \leq d$, then $p(n) = \Omega(n^k)$.

c) If $k = d$, then $p(n) = \Theta(n^k)$.

d) If $k > d$, then $p(n) = o(n^k)$.

e) If $k < d$, then $p(n) = \omega(n^k)$.