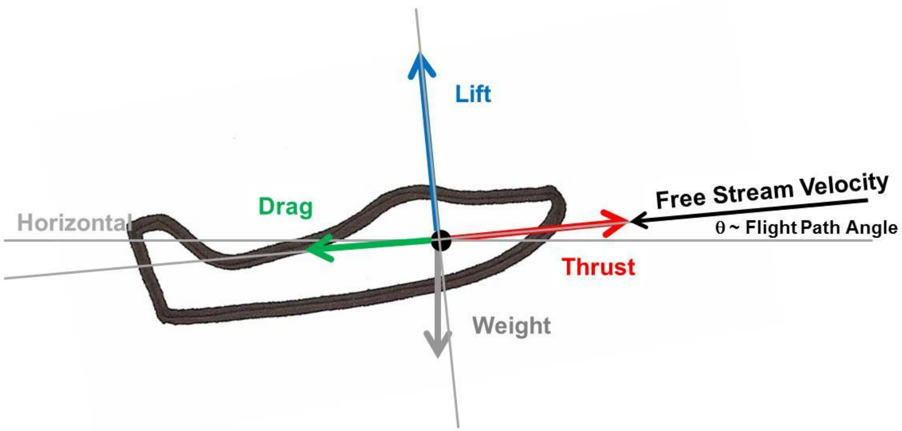
Aircraft Performance
Equations of Motion
Time, Fuel, and Distance
to Climb/Descend/Accel/Decel

UNIVERSITY OF Cincinnati

Block 2 Schedule – Aircraft Performance

Block 2 – Aircraft Performance		
Tuesday, January 31		Table Lookup, Flight Envelope
Thursday, February 2		Thrust Required
Tuesday, February 7		Energy, Min & Max Velocities
Thursday, February 9		Rate of Climb & Ceilings
Tuesday, February 14		Material Review
Thursday, February 16		Homework Review
Tuesday, February 21	**Exam #2a**	
Thursday, February 23		Climb & Accel
Tuesday, February 28		Maneuver
Thursday, March 2		Airfield
Tuesday, March 7		Material Review
Thursday, March 9	**Exam #2b**	

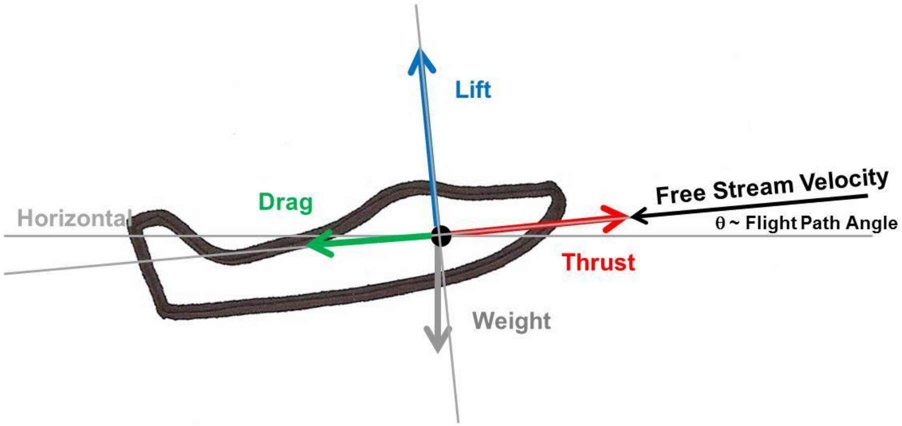
Aircraft Performance

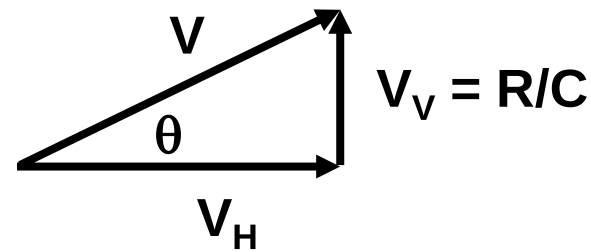


Climb = How long does it take an aircraft to get to a higher altitude? How <u>fast</u> can it climb?

How much <u>time</u> does it take to climb?
How much <u>fuel</u> does it take to climb?
How much <u>distance</u> will the aircraft travel during the climb?

Steady Climbing Flight



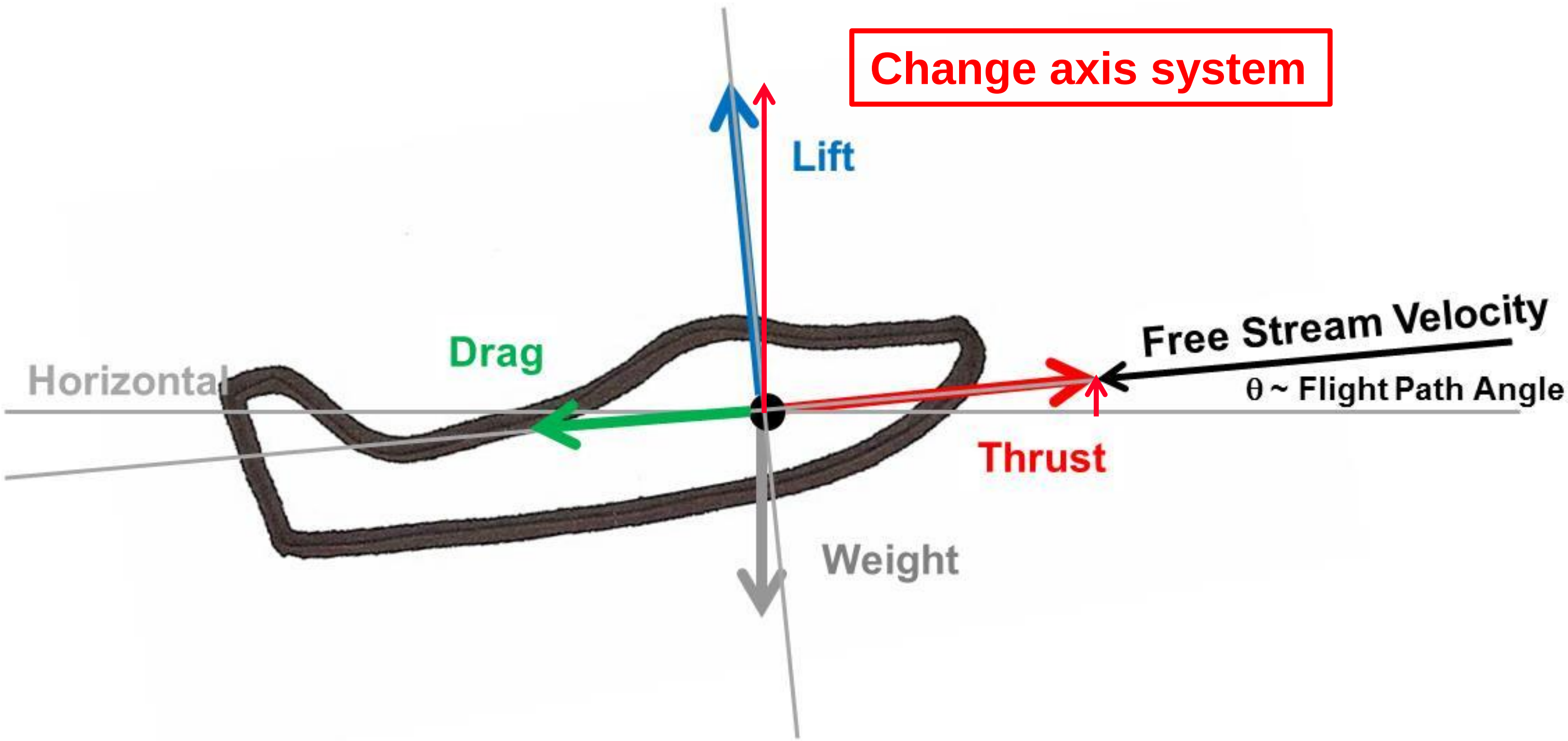


$$\sin\theta = \frac{T-D}{W} \longrightarrow V\sin\theta = \frac{(T-D)\,V}{W} = R/C$$

for R/C > 0 $\boxed{T > D}$

$$\cos\theta = \frac{L}{W} \longrightarrow L = W\cos\theta \quad \boxed{L < W}$$

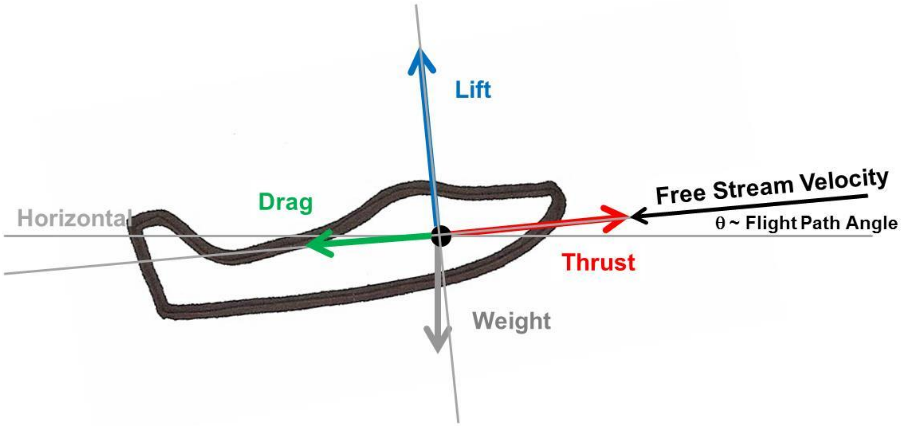
Steady Climbing Flight

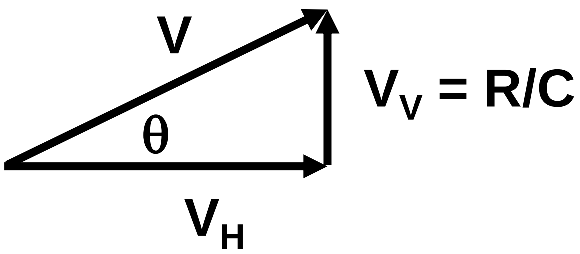


✓ $\boxed{L < W\ ??}$

$$W = L\cos\theta + (T - D)\sin\theta$$

Steady Climbing Flight





$$V \sin\theta = \frac{(T - D)\,V}{W} = R/C$$

$$(T - D)\,V = T\,V - D\,V = \text{Excess Power}$$

$$\frac{(T - D)\,V}{W} = \text{Specific Excess Power} = R/C$$

University of Cincinnati

Specific Excess Power

$$\boxed{\frac{(T-D)\,V}{W} = \frac{dh}{dt} + \frac{V}{g}\frac{dV}{dt} = P_s}$$

$$\frac{(T-D)\,V}{W} = \frac{dh}{dt} + \frac{V}{g}\frac{dV}{dt}$$

Rate of Climb

Acceleration capability

Specific Excess Power

$$\boxed{\frac{(T-D)\,V}{W} = \frac{dh}{dt} + \frac{V}{g}\frac{dV}{dt} = P_s}$$

$$\frac{(T-D)\,V}{W} = \underbrace{\frac{dh}{dt}}_{\text{Rate of Climb}} + \frac{V}{g}\underbrace{\cancel{\frac{dV}{dt}}}_{0}$$

Rate
of
Climb

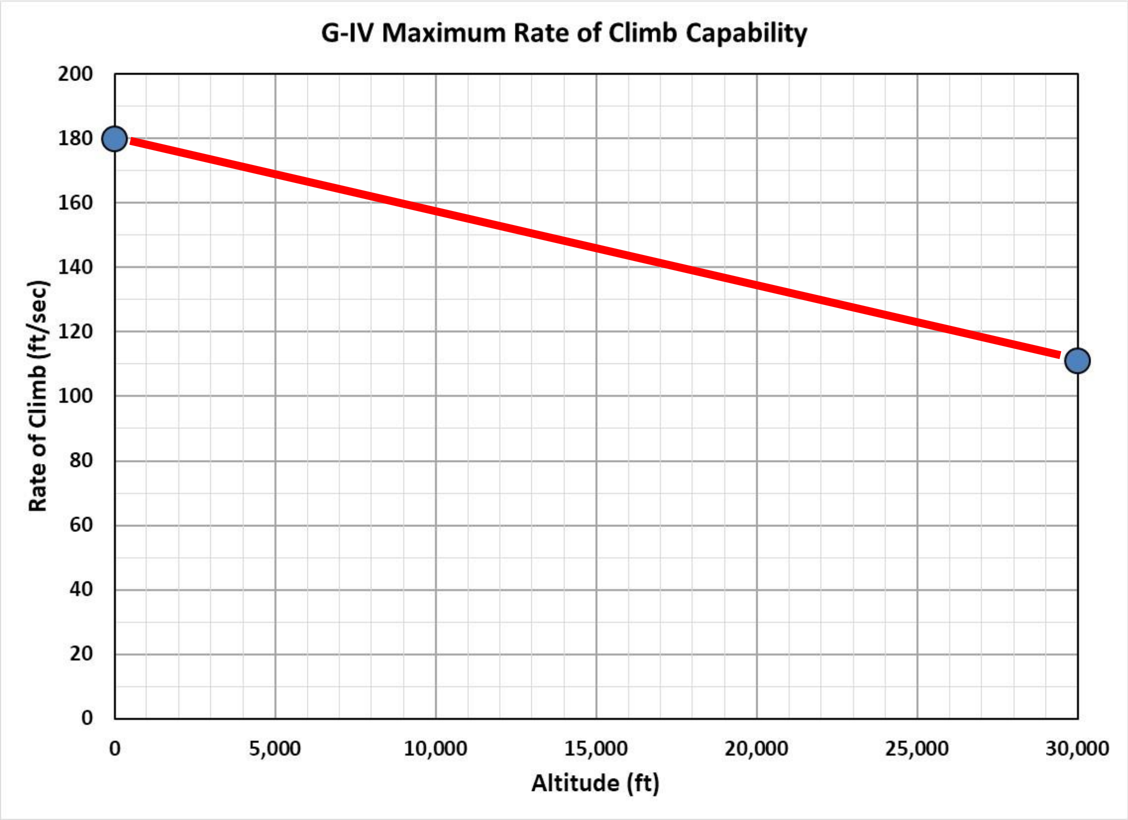
0

$\boxed{\text{Climbing Flight}}$

Minimum Time to Climb

Easier Method – assume linear R/C_{max} vs Altitude

Calculate the maximum Rate of Climb at Sea Level
Calculate the maximum Rate of Climb at 30,000 ft
Calculate the average Rate of Climb



Minimum Time to Climb

Easier Method

$$t_{min} = \frac{\Delta h}{(R/C)_{avg}}$$

$$(R/C)_{max} = \left[\left(\frac{W}{S}\right)\frac{Z}{3\,\rho\,C_{D_0}}\right]^{1/2}\left(\frac{T}{W}\right)^{3/2}\left[1-\frac{Z}{6}-\frac{3}{2\,Z\,\left(T/W\right)^2\left(L/D\right)^2_{max}}\right]$$

$$\text{where}\quad Z = 1+\sqrt{1+\frac{3}{\left(T/W\right)^2\left(L/D\right)^2_{max}}}$$

At Sea Level: $(R/C)_{max}$ = 179.8 ft/sec
At 30,000 ft: $(R/C)_{max}$ = 111.0 ft/sec

Minimum Time to Climb

Easier Method

$$\mathbf{t_{min}} = \frac{\Delta\,\mathbf{h}}{\mathbf{(R/C)_{avg}}}$$

At Sea Level: $(R/C)_{max}$ **= 179.8 ft/sec**
At 30,000 ft: $(R/C)_{max}$ **= 111.0 ft/sec**

$$\mathbf{t_{min}} = \frac{\mathbf{(30{,}000\ - 0)}}{\mathbf{(179.8 + 111.0)/2}}$$

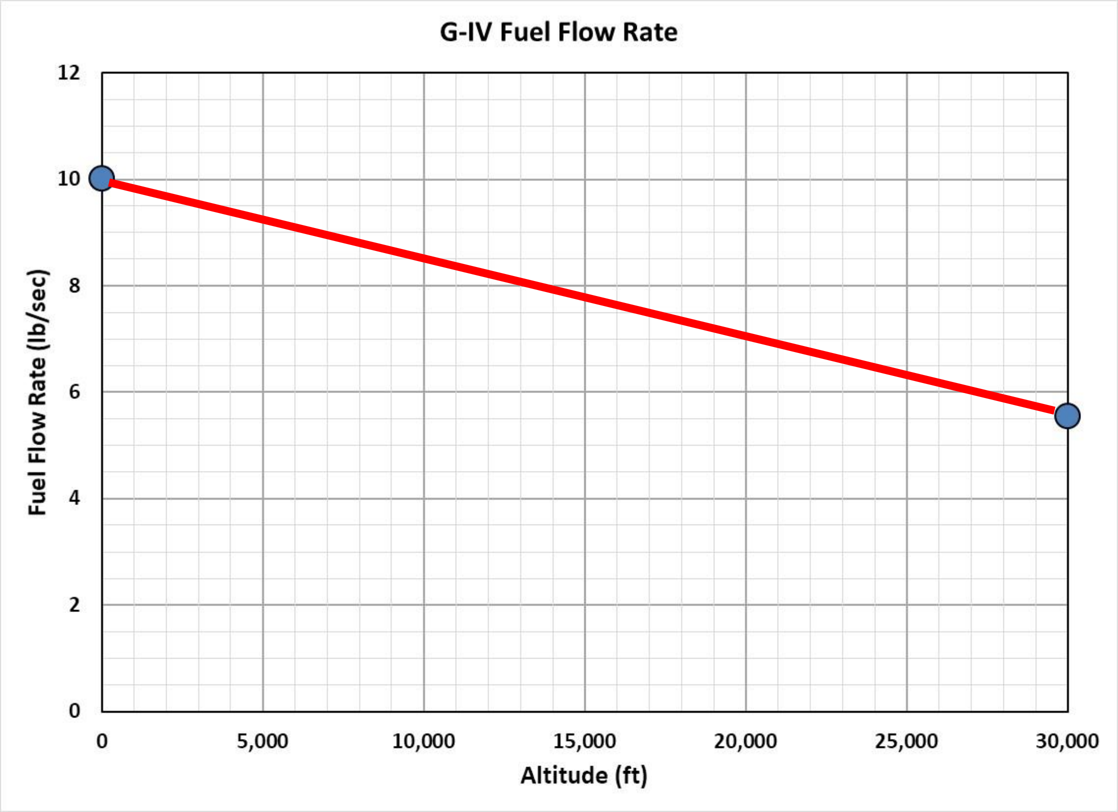
$$\boxed{\mathbf{t_{min} = 206.3\ sec = 3.44\ min}}$$

What about fuel and distance?

Fuel to Climb

<u>Easier Method</u> – assume linear Fuel Flow vs Altitude

Calculate the Fuel Flow at Sea Level
Calculate the Fuel Flow at 30,000 ft
Calculate the average Fuel Flow



UNIVERSITY OF Cincinnati

Fuel to Climb

Easier Method

$$\mathbf{w_{fuel}} = \mathbf{\dot{w}_{avg}}\ \Delta\,\mathbf{t}$$

At Sea Level: $\dot{w}$ **= 10.0028 lb/sec**
At 30,000 ft: $\dot{w}$ **= 5.5454 lb/sec**

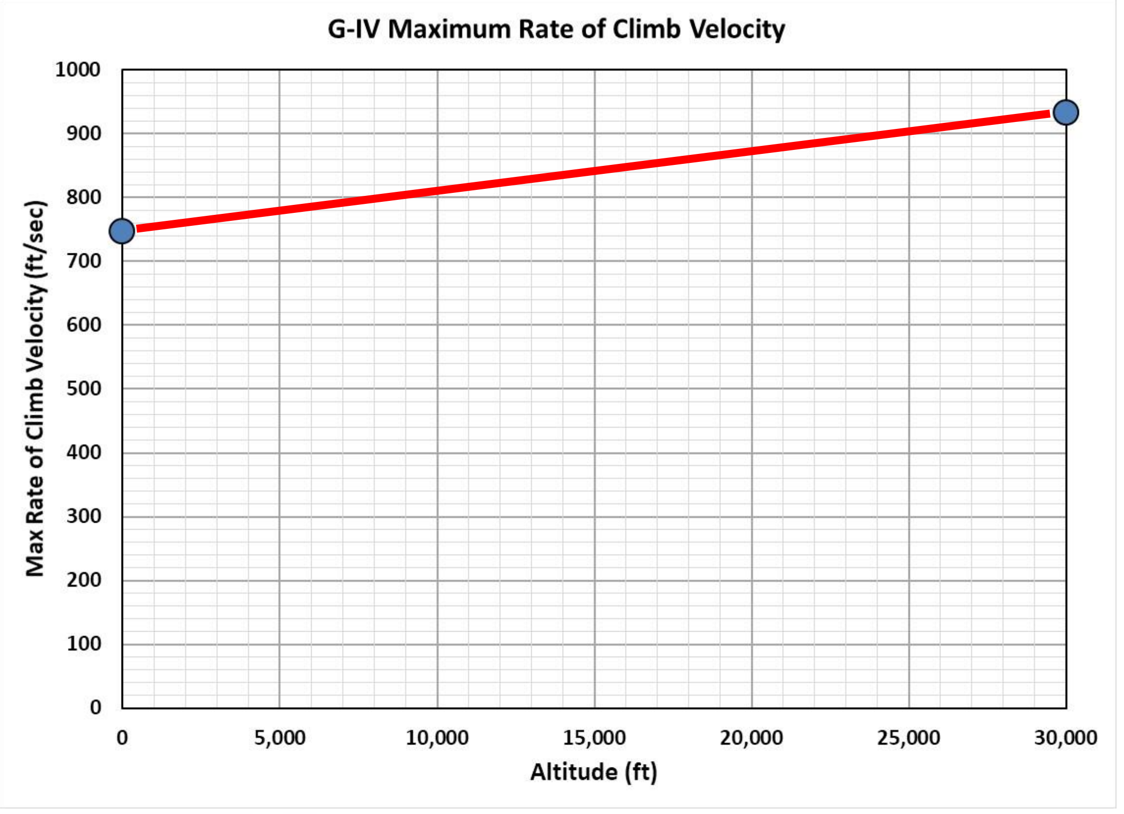
$$\mathbf{w_{fuel}} = \frac{(\mathbf{10.0028 + 5.5454})}{\mathbf{2}}\ \mathbf{206.3}$$

$$\boxed{\mathbf{w_{fuel} = 1{,}603.8\ lb}}$$

Distance to Climb

<u>Easier Method</u> – assume linear Velocity vs Altitude

Calculate the Max Rate of Climb Velocity at Sea Level
Calculate the Max Rate of Climb Velocity at 30,000 ft
Calculate the average Velocity



Distance to Climb

Easier Method

$$\mathbf{s = V_{avg}\ \Delta t}$$

$$\mathbf{V_{(R/C)_{max}} = \left[\left(\frac{T}{W}\right) \left(\frac{W}{S}\right) \frac{Z}{3\, \rho\, C_{D_0}} \right]^{1/2}}$$

$$\mathbf{\text{where}\ \ Z = 1 + \sqrt{1 + \frac{3}{(T/W)^2\, (L/D)^2_{max}}}}$$

At Sea Level: V = 747.3 ft/sec
At 30,000 ft: V = 932.2 ft/sec

Distance to Climb

Easier Method

$$s = V_{avg}\ \Delta t$$

At Sea Level: V = 747.3 ft/sec
At 30,000 ft: V = 932.2 ft/sec

$$s = \frac{(747.3+932.2)}{2}\ 206.3$$

$$\boxed{s = 173,240\ \text{ft} = 28.51\ \text{NM}}$$

UNIVERSITY OF Cincinnati

Climb Summary

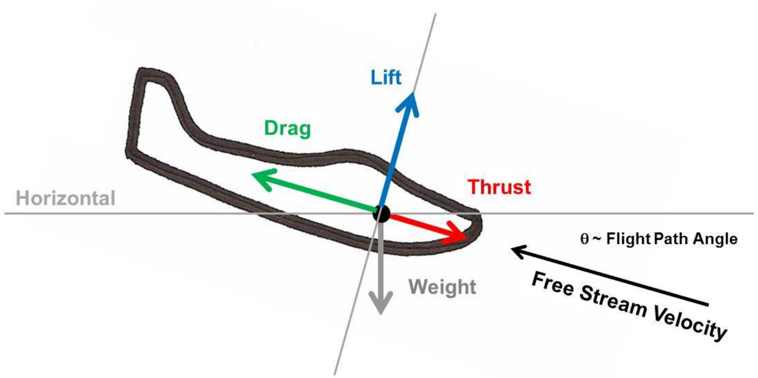
Easier Method

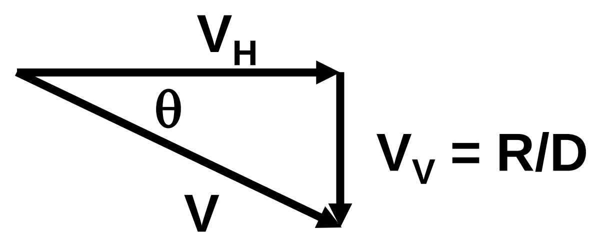
$$t_{min} = \frac{\Delta h}{(R/C)_{avg}} \qquad w_{fuel} = \dot{w}_{avg}\ \Delta t \qquad s = V_{avg}\ \Delta t$$

Sea Level			30,000 ft		
R/C	FFR	Velocity	R/C	FFR	Velocity
(ft/sec)	(lb/sec)	(ft/sec)	(ft/sec)	(lb/sec)	(ft/sec)
179.8	10.0028	747.3	111.0	5.5454	932.2

Time	Fuel	Distance
3.44 min	1,603.8 lb	28.51 NM

Steady Descending Flight





$$\sin\theta = \frac{D-T}{W} \longrightarrow V\sin\theta = \frac{(D-T)\,V}{W} = R/D$$

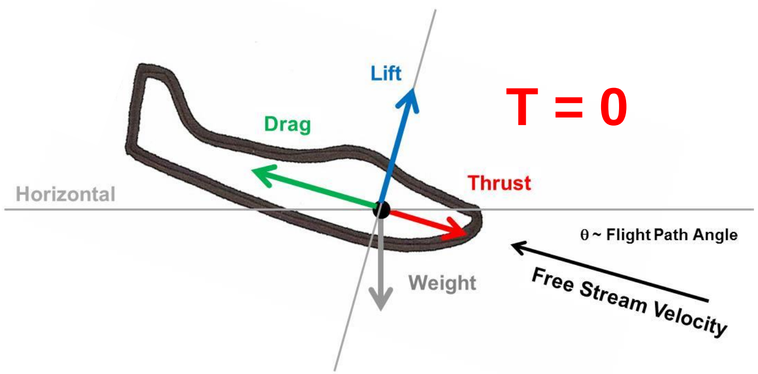
for R/D > 0 $\boxed{D > T}$

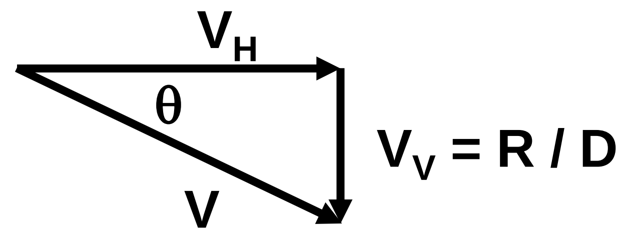
$$\cos\theta = \frac{L}{W} \longrightarrow L = W\cos\theta \qquad \boxed{L < W}$$

$$\tan\theta = \frac{D-T}{L} = \frac{1-\left({}^{T}/_{D}\right)}{\left({}^{L}/_{D}\right)}$$

UNIVERSITY OF Cincinnati

Steady Gliding Flight





$$\sin\theta = \frac{D - \cancelto{0}{T}}{W} \longrightarrow V\sin\theta = \frac{D\,V}{W} = R/D$$

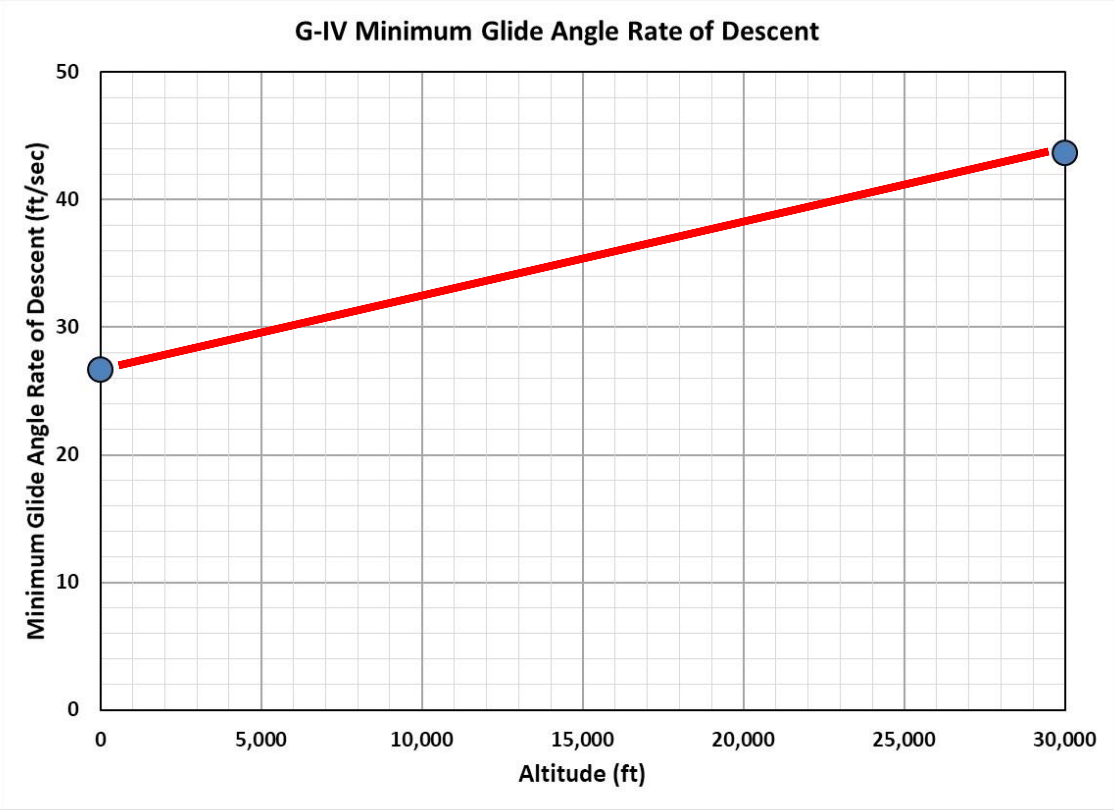
$$\cos\theta = \frac{L}{W} \longrightarrow L = W\cos\theta$$

$$\tan\theta = \frac{D - \cancelto{0}{T}}{L} = \frac{1 - \cancelto{0}{\left(T/D\right)}}{\left(L/D\right)} = \frac{1}{\left(L/D\right)}$$

Time to Descend

Easier Method – assume linear Minimum Glide Angle vs Altitude

Calculate the Minimum Glide Angle and Velocity at Sea Level
Calculate the Minimum Glide Angle and Velocity at 30,000 ft
Calculate the average Glide Angle, average Velocity, and R/D



Time to Descend

Easier Method

$$t_{\theta min} = \frac{\Delta h}{(R/D)_{avg}}$$

$$\tan\theta_{min} = \frac{1}{\left(L/D\right)_{max}} \qquad R/D = V\sin\theta$$

$$V_{\theta_{min}} = \sqrt{\frac{2}{\rho}\left(\frac{K}{C_{D_0}}\right)^{1/2}\left(\frac{W}{S}\right)\cos\theta}$$

At Sea Level: $\theta_{min} = 3.96°$ $V_{\theta_{min}} = 385.1$ ft/sec

At 30,000 ft: $\theta_{min} = 3.96°$ $V_{\theta_{min}} = 631.0$ ft/sec

Example 5.18

Time to Descend

Easier Method

$$t_{\theta min} = \frac{\Delta h}{(R/D)_{avg}}$$

At Sea Level: $\theta_{min} = 3.96°$ $V_{\theta_{min}} = 385.1$ ft/sec R/D = 26.7 ft/sec

At 30,000 ft: $\theta_{min} = 3.96°$ $V_{\theta_{min}} = 631.0$ ft/sec R/D = 43.6 ft/sec

$$t_{\theta min} = \frac{(30,000 - 0)}{(43.6 + 26.7)/2}$$

$$\boxed{t_{\theta min} = 853.5 \text{ sec} = 14.22 \text{ min}}$$

UNIVERSITY OF Cincinnati

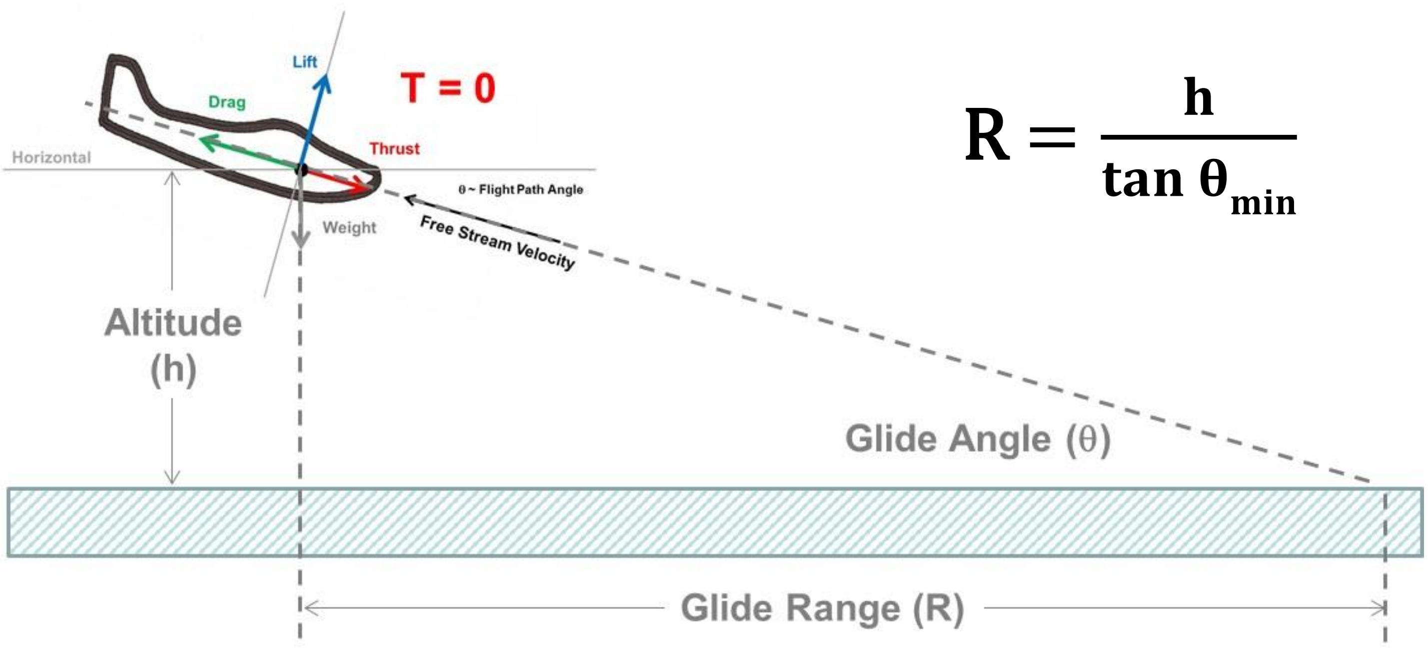
Fuel to Descend

Easier Method – assume linear Fuel Flow vs Altitude

Since this is "gliding flight" T = 0 and FFR = 0

Distance to Descend

<u>Easier Method</u> – assume linear Glide Angle vs Altitude



$$R = \frac{h}{\tan \theta_{min}}$$

$$R = \frac{h}{\tan \theta_{min}} = \frac{(30{,}000 - 0)}{\tan 3.96°}$$

$$\boxed{R = 433{,}013 \text{ ft} = 71.26 \text{ NM}}$$

Descent Summary

Easier Method

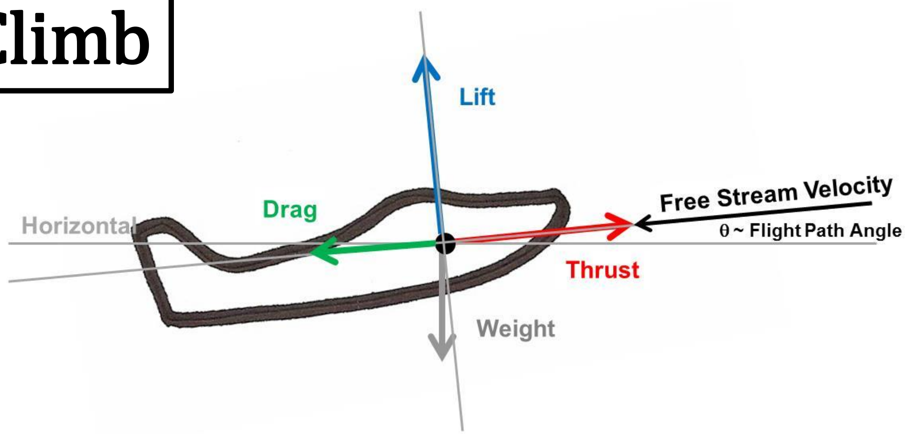
$$t_{\theta min} = \frac{\Delta h}{(R/D)_{avg}} \qquad w_{fuel} = 0 \qquad R = \frac{h}{\tan \theta_{min}}$$

Sea Level			30,000 ft		
θ_{min}	Velocity	R/D	θ_{min}	Velocity	R/D
(deg)	(ft/sec)	(ft/sec)	(deg)	(ft/sec)	(ft/sec)
3.96	385.1	26.7	3.96	631.0	43.6

Time	Fuel	Distance
14.22 min	0.0 lb	71.26 NM

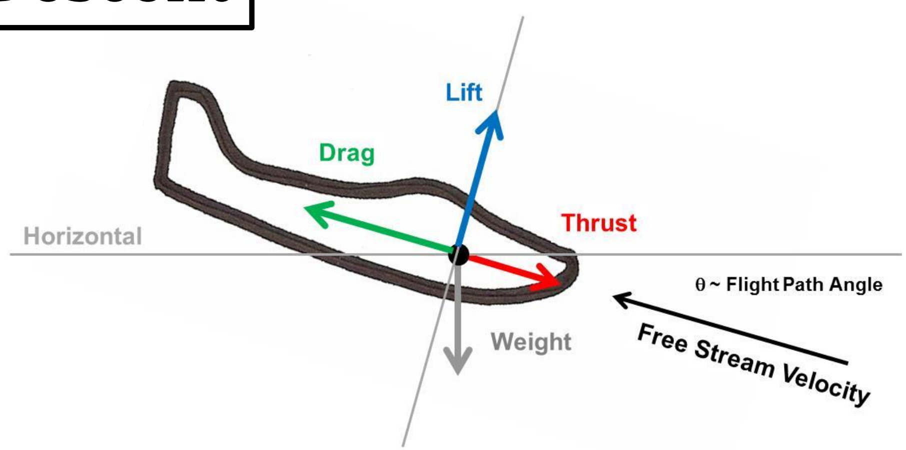
Aircraft Performance

Climb



$T > D$ $L < W$

Descent

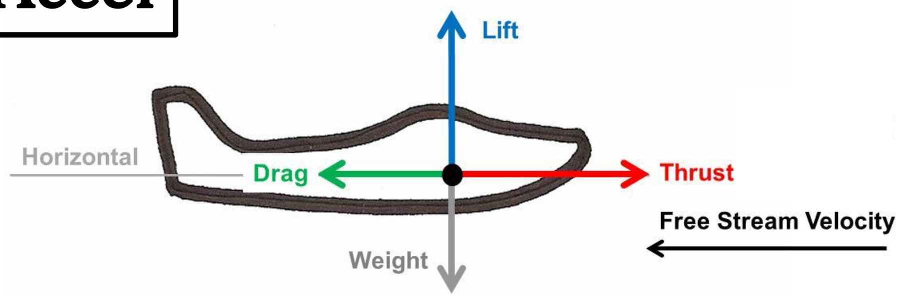


$T < D$ $L < W$

UNIVERSITY OF Cincinnati

Aircraft Performance

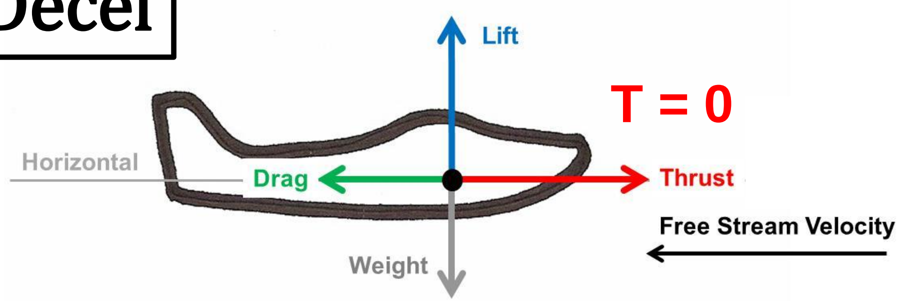
Accel



$T > D$

$L = W$

Decel



$T < D$

$L = W$

UNIVERSITY OF Cincinnati

Specific Excess Power

$$\boxed{\frac{(T-D)\,V}{W} = \frac{dh}{dt} + \frac{V}{g}\frac{dV}{dt} = P_s}$$

$$\frac{(T-D)\,V}{W} = \cancel{\frac{dh}{dt}} + \frac{V}{g}\frac{dV}{dt}$$

0

Acceleration capability

Level Acceleration

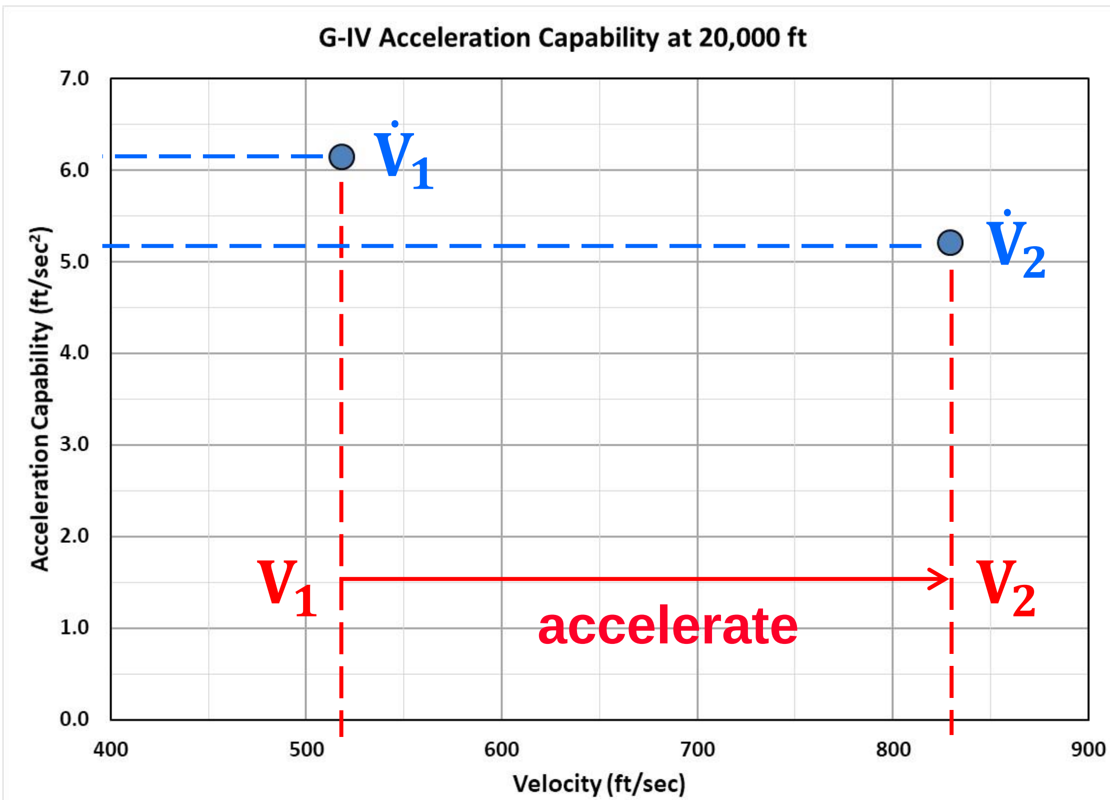
Example 5.17

Time to Accelerate

<u>Textbook Method</u> (at a specified altitude)

$$t_{accel} = \int_{V_1}^{V_2} \frac{dV}{\dot{V}}$$

Accelerate from 0.50 to 0.80 M at 20,000 ft



UNIVERSITY OF Cincinnati

Example 5.17

Time to Accelerate

<u>Textbook Method</u> (at a specified altitude)

Calculate the acceleration capability ($\dot{V}_1$) at initial velocity (V_1)
Calculate the acceleration capability ($\dot{V}_2$) at final velocity (V_2)
Derive the linear relationship $\dot{V}$ = a + b ΔV = a + b (V_2 - V_1)

$$\mathbf{t_{accel}} = \int_{V_1}^{V_2} \frac{\mathbf{dV}}{\dot{\mathbf{V}}} = \int_{V_1}^{V_2} \frac{\mathbf{dV}}{\mathbf{a + b\,\Delta V}}$$

$$\mathbf{t_{accel}} = \frac{\mathbf{1}}{\mathbf{b}}\left[\mathbf{ln(a + b\,\Delta V) - ln\,a}\right]$$

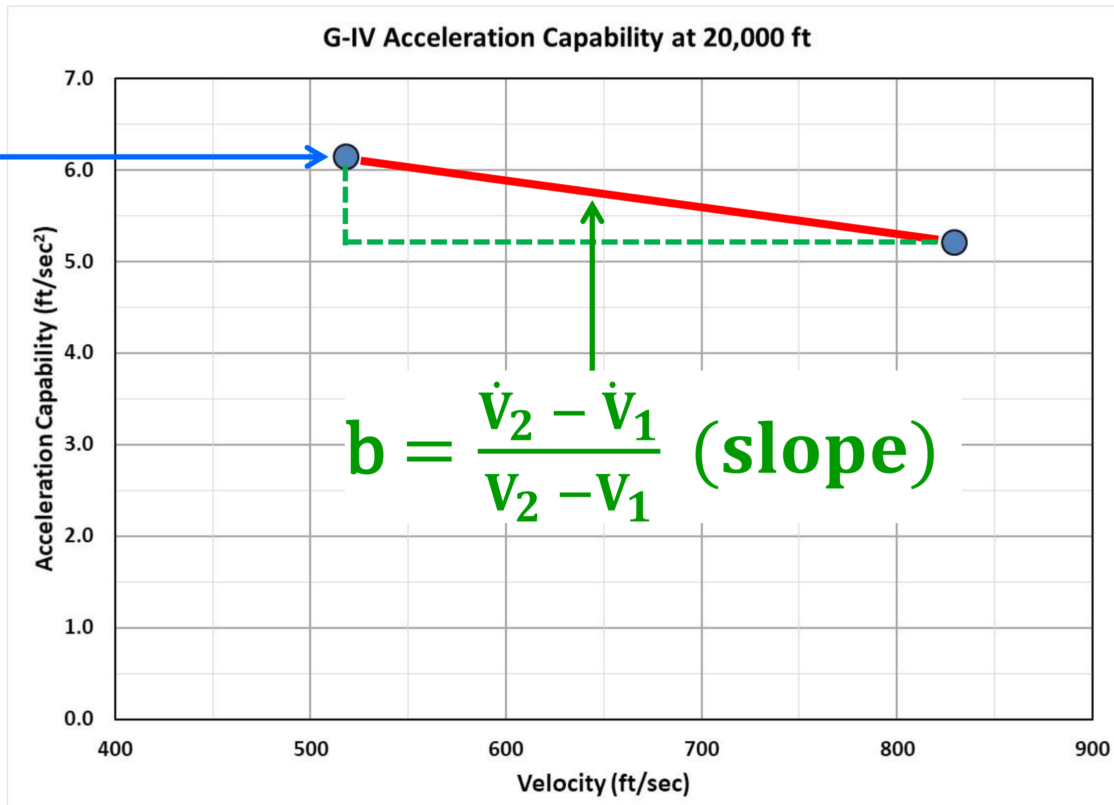
Time to Accelerate

<u>Textbook Method</u> (at a specified altitude)

Calculate the acceleration capability ($\dot{V}_1$) at initial velocity (V_1)
Calculate the acceleration capability ($\dot{V}_2$) at final velocity (V_2)
Derive the linear relationship $\dot{V} = a + b\,\Delta V = a + b\,(V_2 - V_1)$

$a = \dot{V}_1$

$\dot{V}$



$$b = \frac{\dot{V}_2 - \dot{V}_1}{V_2 - V_1} \text{ (slope)}$$

V

Time to Accelerate

<u>**Textbook Method**</u> **(at a specified altitude)**

$$\mathbf{t_{accel} = \frac{1}{b}\left[\ln(a + b\ \Delta V) - \ln a\right]}$$

since $\dot{V}$ = a + b ΔV = a + b (V_2 - V_1)

$$\mathbf{t_{accel} = \frac{1}{b}\left[\ln \dot{V}_2 - \ln \dot{V}_1\right]}$$

UNIVERSITY OF Cincinnati

Time to Accelerate

<u>Textbook Method</u> (at a specified altitude)

$$\mathbf{t_{accel}} = \frac{\mathbf{1}}{\mathbf{b}}\left[\mathbf{\ln \dot{V}_2 - \ln \dot{V}_1}\right]$$

Solve for b

At initial velocity (518.4 ft/sec): $\dot{V}$ = 6.1366 ft/sec^2
At final velocity (829.5 ft/sec): $\dot{V}$ = 5.1774 ft/sec^2

a = 6.1366 b = (5.1774 – 6.1366) / (829.5 – 518.4) = -0.003083

$\dot{V}$ = 6.1366 – 0.003083 ΔV

$$\mathbf{t_{accel}} = \frac{\mathbf{1}}{\mathbf{-0.003083}}\left[\mathbf{\ln(5.1774) - \ln(6.1366)}\right]$$

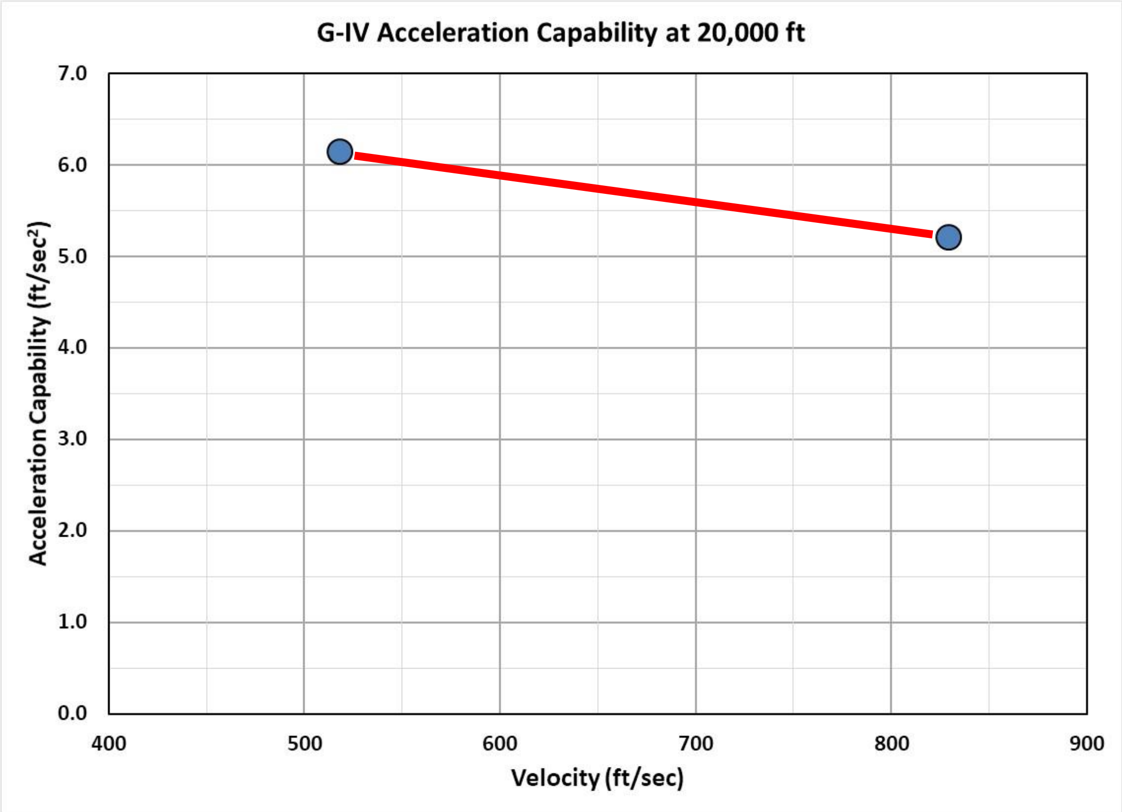
$$\boxed{\mathbf{t_{min} = 55.1\ sec = 0.92\ min}}$$

University of Cincinnati

Time to Accelerate

<u>Easier Method</u> – assume linear Acceleration Capability vs Velocity

Calculate the Acceleration Capability ($\dot{V}_1$) at initial Velocity (V_1)
Calculate the Acceleration Capability ($\dot{V}_2$) at final Velocity (V_2)
Calculate the average Acceleration Capability



Level Acceleration

$$\frac{(T - D)\,V}{W} = \underbrace{\cancel{\frac{dh}{dt}}}_{0} + \frac{V}{g}\frac{dV}{dt}$$

Acceleration capability

Rearranging terms:

$$\frac{dV}{dt} = \frac{(T - D)\,g}{W}$$

UNIVERSITY OF Cincinnati

Level Acceleration

$$\frac{dV}{dt} = \frac{(T - D)\,g}{W}$$

Accelerate from 0.50 to 0.80 M at 20,000 ft & 73,000 lb

$$\text{At } V_1 \text{ (518.4 ft/sec)}: T = 18,985.5 \text{ lb}; D = 5,062.0 \text{ lb}$$
$$\dot{V}_1 = 6.1366 \text{ ft/sec}^2$$

$$\text{At } V_2 \text{ (829.5 ft/sec)}: T = 18,985.5 \text{ lb}; D = 7,238.4 \text{ lb}$$
$$\dot{V}_2 = 5.1774 \text{ ft/sec}^2$$

Time to Accelerate

Easier Method

$$t_{accel} = \frac{\Delta V}{(\dot{V})_{avg}}$$

At 20,000 ft & 518.4 ft/sec: $\dot{V}$ **= 6.1366 ft/sec²**

At 20,000 ft & 829.5 ft/sec: $\dot{V}$ **= 5.1774 ft/sec²**

$$t_{accel} = \frac{(829.5 - 518.4)}{(5.1774 + 6.1366)/2}$$

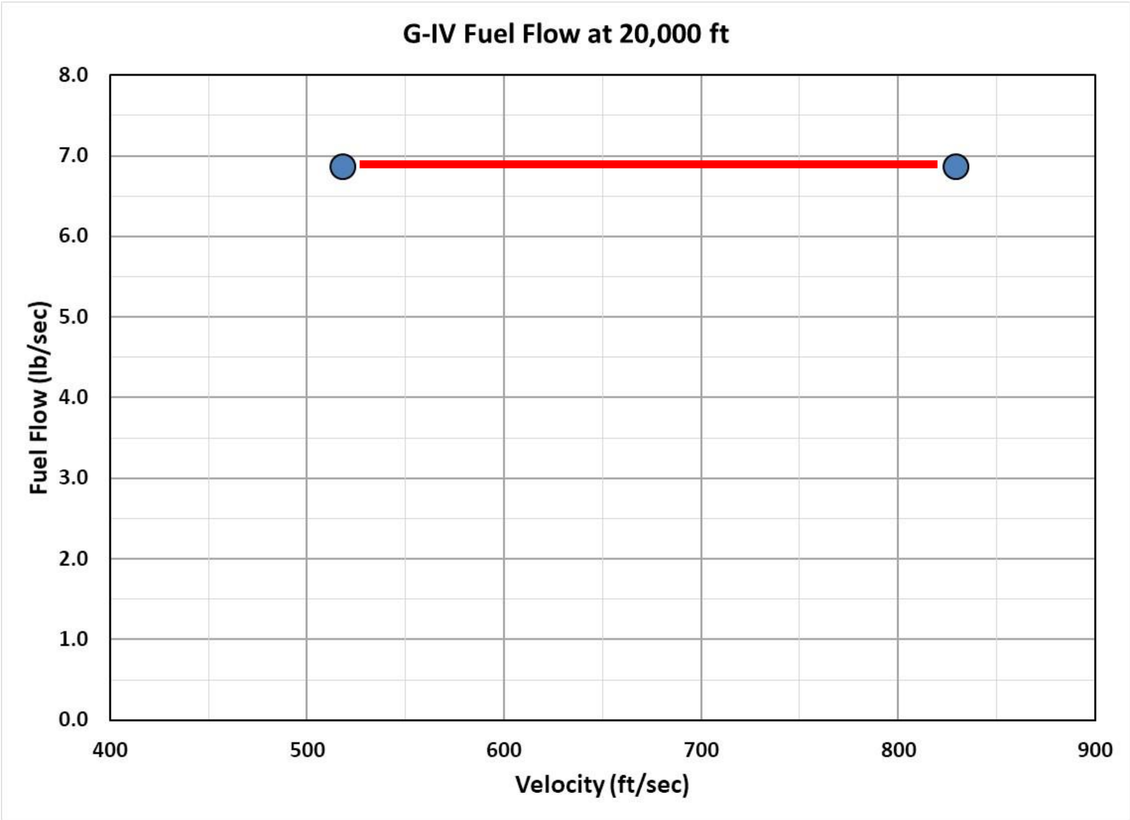
$$\boxed{t_{accel} = 55.0\ \mathrm{sec} = 0.92\ \mathrm{min}}$$

What about fuel and distance?

Fuel to Accelerate

Easier Method – assume linear Fuel Flow vs Velocity

Calculate the Fuel Flow at the initial Velocity
Calculate the Fuel Flow at the final Velocity
Calculate the average Fuel Flow



Fuel to Accelerate

Easier Method

$$\mathbf{w_{fuel} = \dot{w}_{avg}\ \Delta\, t}$$

At V_1 (518.4 ft/sec): **$\dot{w}$ = 6.8559 lb/sec**
At V_2 (829.5 ft/sec): **$\dot{w}$ = 6.8559 lb/sec**

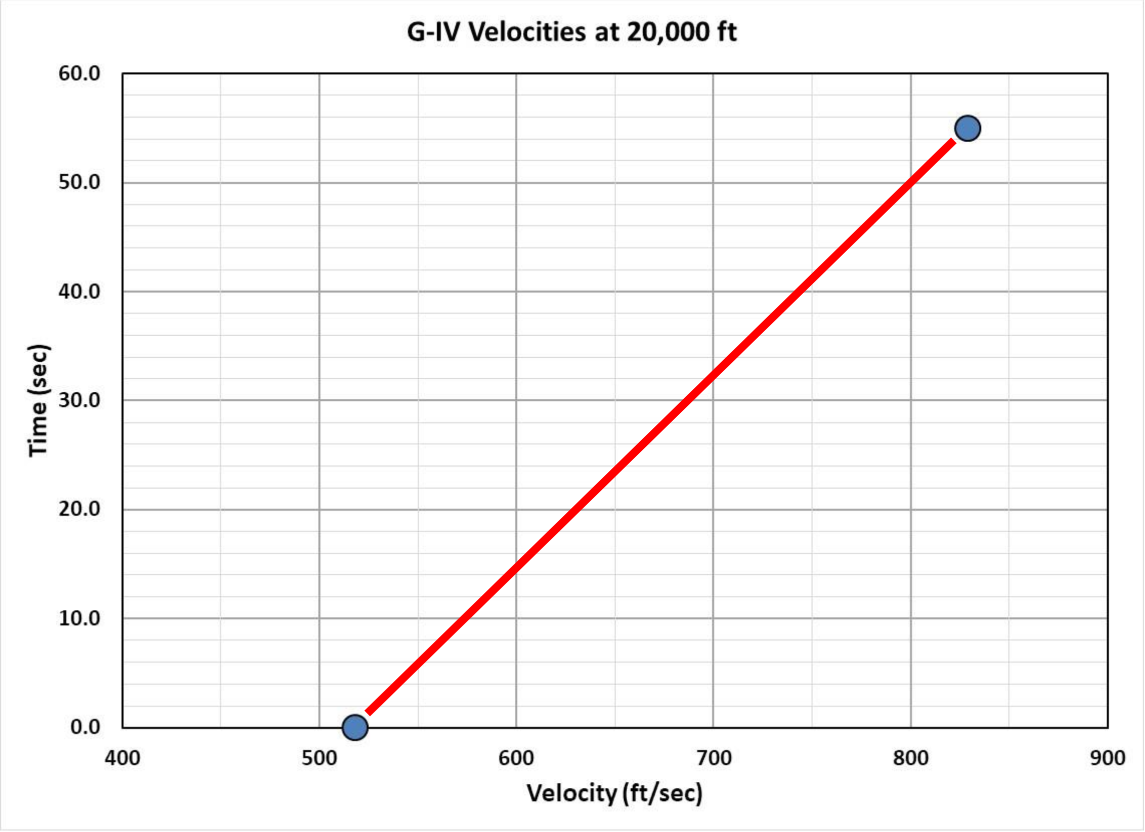
$$\mathbf{w_{fuel} = \frac{(6.8559 + 6.8559)}{2}\ 55.0}$$

$$\boxed{\mathbf{w_{fuel} = 377.1\ lb}}$$

Distance to Accelerate

Easier Method – assume linear Velocity vs Time

Calculate the average Velocity during the Acceleration



Example 5.18

Distance to Accelerate

Easier Method

$$s = V_{avg}\ \Delta t$$

$$s = \frac{(518.4 + 829.5)}{2}\ 55.0$$

$$\boxed{s = 37,067\ \text{ft} = 6.10\ \text{NM}}$$

UNIVERSITY OF Cincinnati

Acceleration Summary

Easier Method

$$\mathbf{t_{accel}} = \frac{\Delta\,\mathbf{V}}{(\dot{\mathbf{V}})_{\mathbf{avg}}} \qquad \mathbf{w_{fuel}} = \dot{\mathbf{w}}_{\mathbf{avg}}\ \Delta\,\mathbf{t} \qquad \boldsymbol{s} = \mathbf{V_{avg}}\ \Delta\,\mathbf{t}$$

0.50 M / 20,000 ft			0.80 M / 20,000 ft		
$\dot{V}$	FFR	Velocity	$\dot{V}$	FFR	Velocity
(ft/sec^2)	(lb/sec)	(ft/sec)	(ft/sec^2)	(lb/sec)	(ft/sec)
6.1366	6.8559	518.4	5.1811	6.8559	829.5

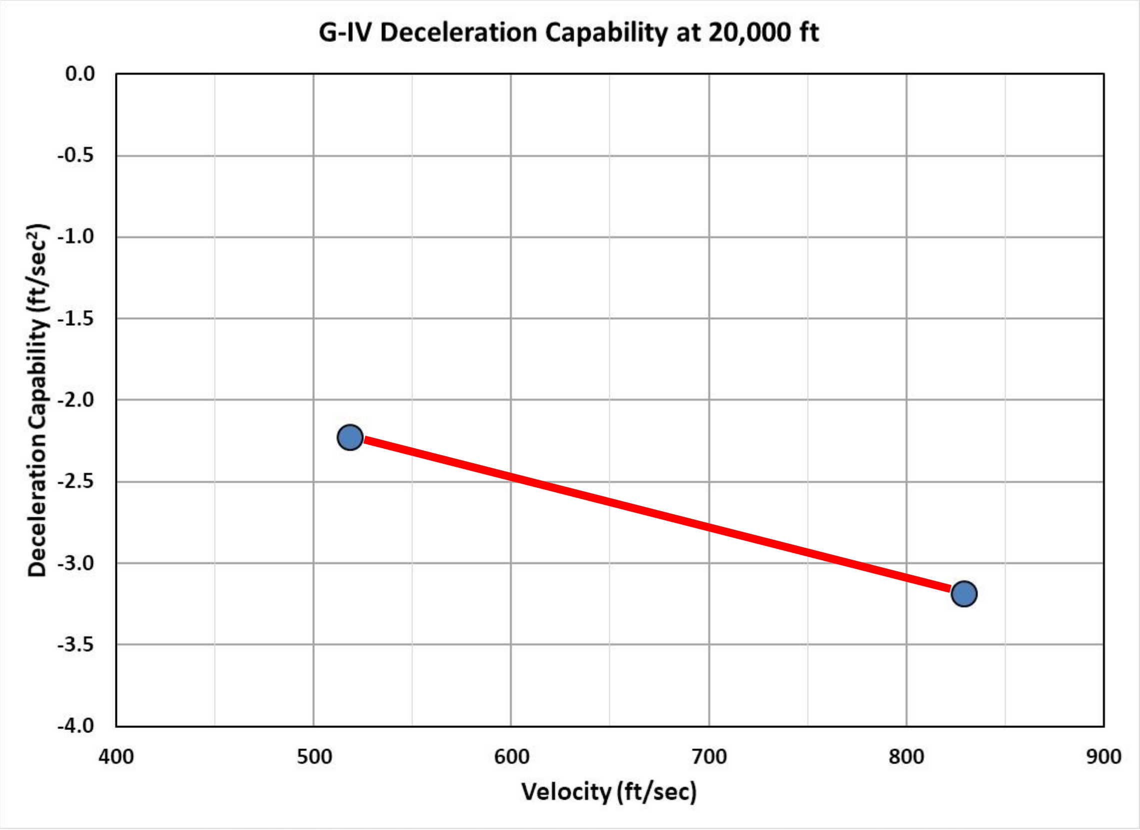
Time	Fuel	Distance
55.0 sec	377.1 lb	6.10 NM

UNIVERSITY OF Cincinnati

Time to Decelerate

<u>Easier Method</u> – assume linear Deceleration Capability vs Velocity

Calculate the Deceleration Capability ($\dot{V}_1$) at initial Velocity (V_1)
Calculate the Deceleration Capability ($\dot{V}_2$) at final Velocity (V_2)
Calculate the average Deceleration Capability



UNIVERSITY OF Cincinnati

Level Deceleration

$$\frac{(T-D)\,V}{W} = \cancel{\frac{dh}{dt}} + \frac{V}{g}\,\frac{dV}{dt}$$

0

Deceleration capability

Rearranging terms:

$$\frac{dV}{dt} = \frac{(T-D)\,g}{W}$$

Level Deceleration

$$\frac{dV}{dt} = \frac{(T - D)\, g}{W}$$

Accelerate from 0.50 to 0.80 M at 20,000 ft & 73,000 lb

At V_1 (829.5 ft/sec): $T = 0$ lb; $D = 7,238.4$ lb

$\dot{V}_1 =$ -3.1902 ft/sec^2

At V_2 (518.4 ft/sec): $T = 0$ lb; $D = 5,062.0$ lb

$\dot{V}_2 =$ -2.2310 ft/sec^2

Example 5.18

Time to Decelerate

Easier Method

$$\mathbf{t_{decel}} = \frac{\Delta\,\mathbf{V}}{(\dot{\mathbf{V}})_{\mathbf{avg}}}$$

At 20,000 ft & 829.5 ft/sec: $\dot{V}$ **= -3.1902 ft/sec^2**
At 20,000 ft & 518.4 ft/sec: $\dot{V}$ **= -2.2310 ft/sec^2**

$$\mathbf{t_{decel}} = \frac{(\mathbf{518.4 - 829.5})}{(\mathbf{-3.1902 - 2.2310})/\mathbf{2}}$$

$$\boxed{\mathbf{t_{decel} = 114.8\ sec = 1.91\ min}}$$

What about fuel and distance?

UNIVERSITY OF Cincinnati

Fuel to Decelerate

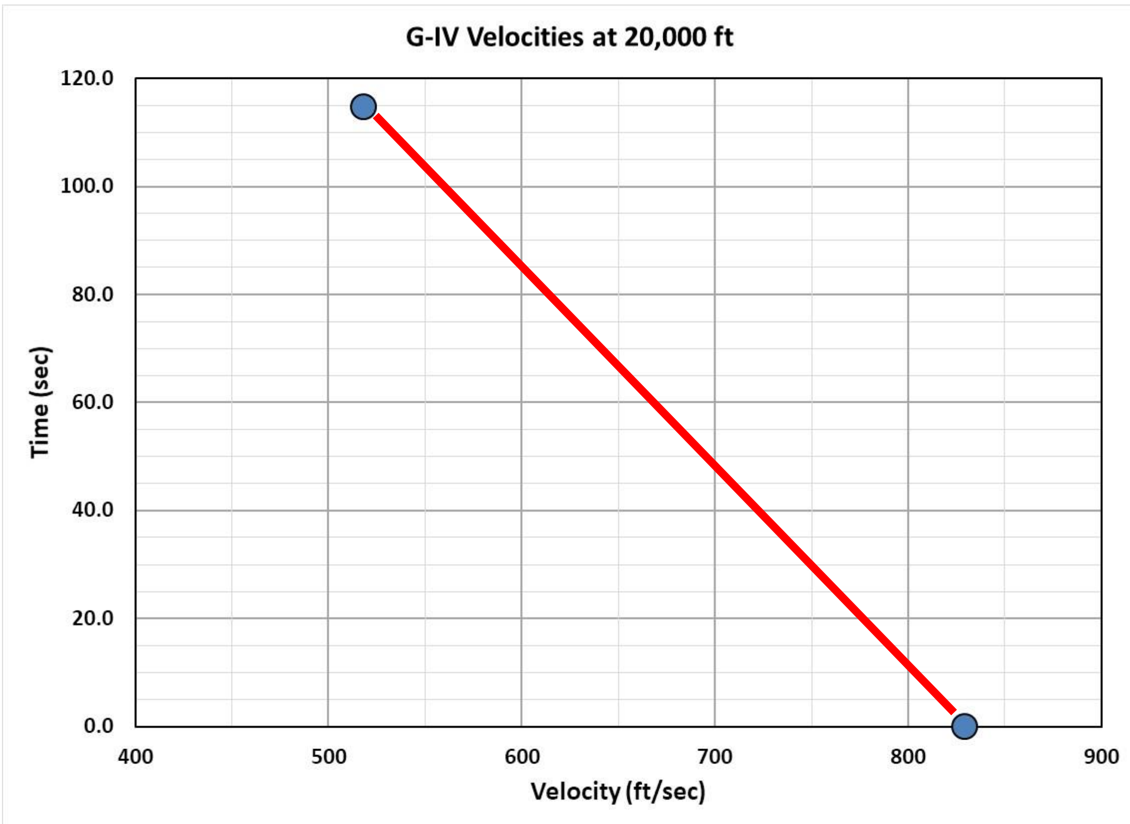
<u>Easier Method</u> – assume linear Fuel Flow vs Velocity

Since this is "decelerating flight" T = 0 and FFR = 0

Distance to Decelerate

Easier Method – assume linear Velocity vs Time

Calculate the average Velocity during the Acceleration



Distance to Decelerate

Easier Method

$$\mathbf{s = V_{avg}\ \Delta\, t}$$

$$\mathbf{s = \frac{(518.4 + 829.5)}{2}\ 114.8}$$

$$\boxed{\mathbf{s = 77{,}369\ ft = 12.73\ NM}}$$

Deceleration Summary

Easier Method

$$t_{decel} = \frac{\Delta V}{(\dot{V})_{avg}} \qquad w_{fuel} = 0 \qquad s = V_{avg}\ \Delta t$$

0.80 M / 20,000 ft			0.50 M / 20,000 ft		
$\dot{V}$	FFR	Velocity	$\dot{V}$	FFR	Velocity
(ft/sec^2)	(lb/sec)	(ft/sec)	(ft/sec^2)	(lb/sec)	(ft/sec)
-3.1902	0.0	829.5	-2.2310	0.0	518.4

Time	Fuel	Distance
114.8 sec	0 lb	12.73 NM

Homework Assignments

HW #12 – Climb & Descent Time, Fuel & Distance
<u>HW #13 – Accel & Decel Time, Fuel & Distance</u>
(due by 11:59 pm ET on Monday)
Reading – Chapter 6.6 - 6.4

<u>HW Help Session</u>
Monday 4:00 – 5:00 pm ET

<u>Posted on Canvas</u>
HW #12 & 13 Assignments with instructions, tips, and checklists
HW #12 & 13 Templates for data table in Excel

UNIVERSITY OF Cincinnati

Questions?

UNIVERSITY OF Cincinnati