Aircraft Design
Wing Loading

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Aircraft Design Process

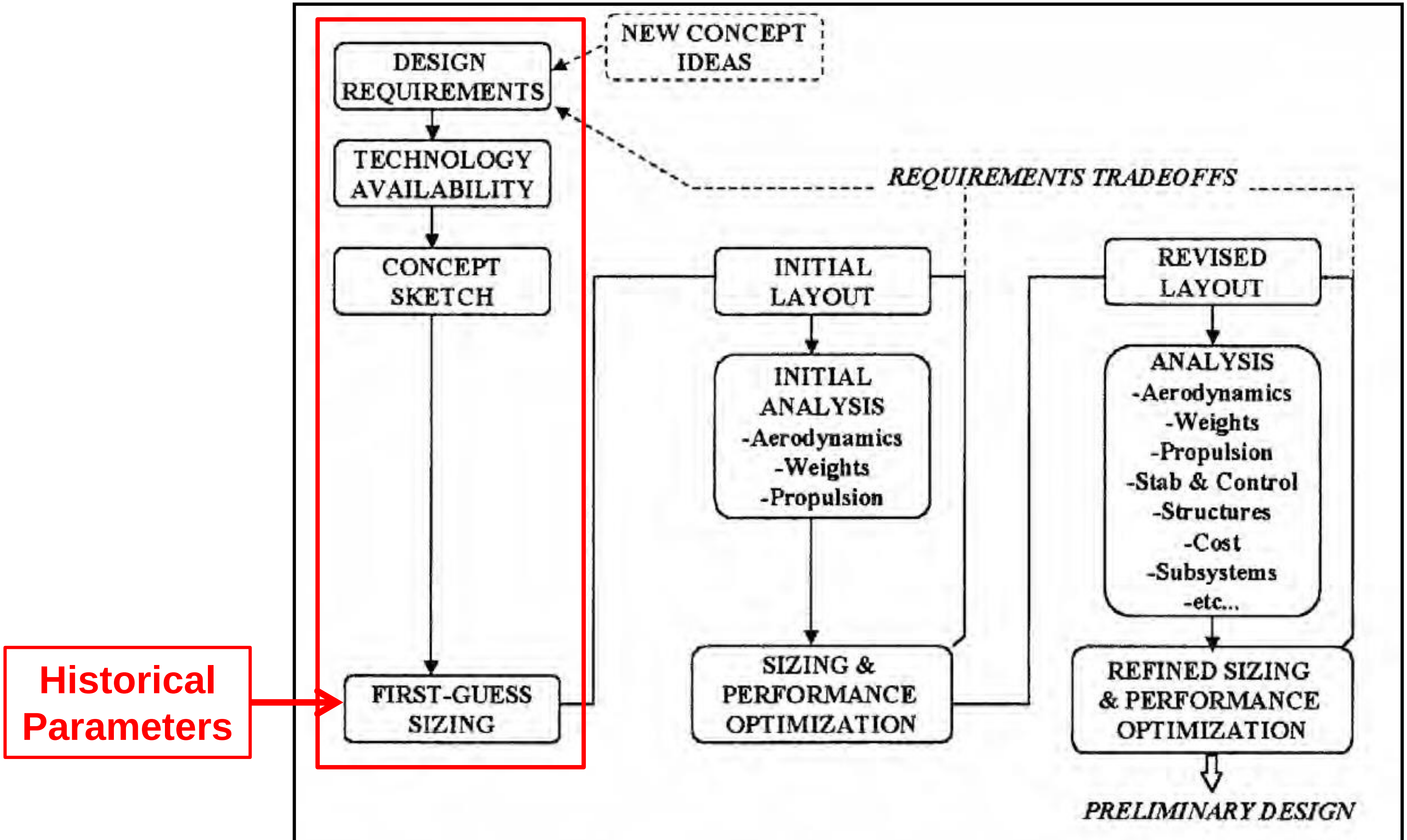

Fig. 2.4 Aircraft conceptual design process.

"Aircraft Design: A Conceptual Approach" by Daniel P. Raymer, page 9

Historical Wing Loading

	$(W/S)_{TO}$
Long Range	**110 - 130**
Short/Medium Range	**80 - 110**
Short Takeoff & Landing	**40 - 90**
Light Civil	**10 - 30**
Combat Fighter	**40 - 70**
Combat Intercept	**120 - 150**
High Altitude	**30 - 60**

↑ **Transports**

↓ **Fighters**

↑ **Interceptors**

↓ **Spy Planes**

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Wing Loading (W/S)

Wing Loading is a primary input into these flight phases:

- ⇩ Takeoff
- ⇩ Landing
- ⇧ ⇩ Climb
- ⇧ ⇩ Acceleration
- ⇧ ⇩ Range
- ⇩ Combat
- ⇩ Ceiling
- ⇩ Glide Rate

⇩ Low W/S (large wing)

⇧ High W/S (small wing)

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Wing Loading (W/S)

Takeoff

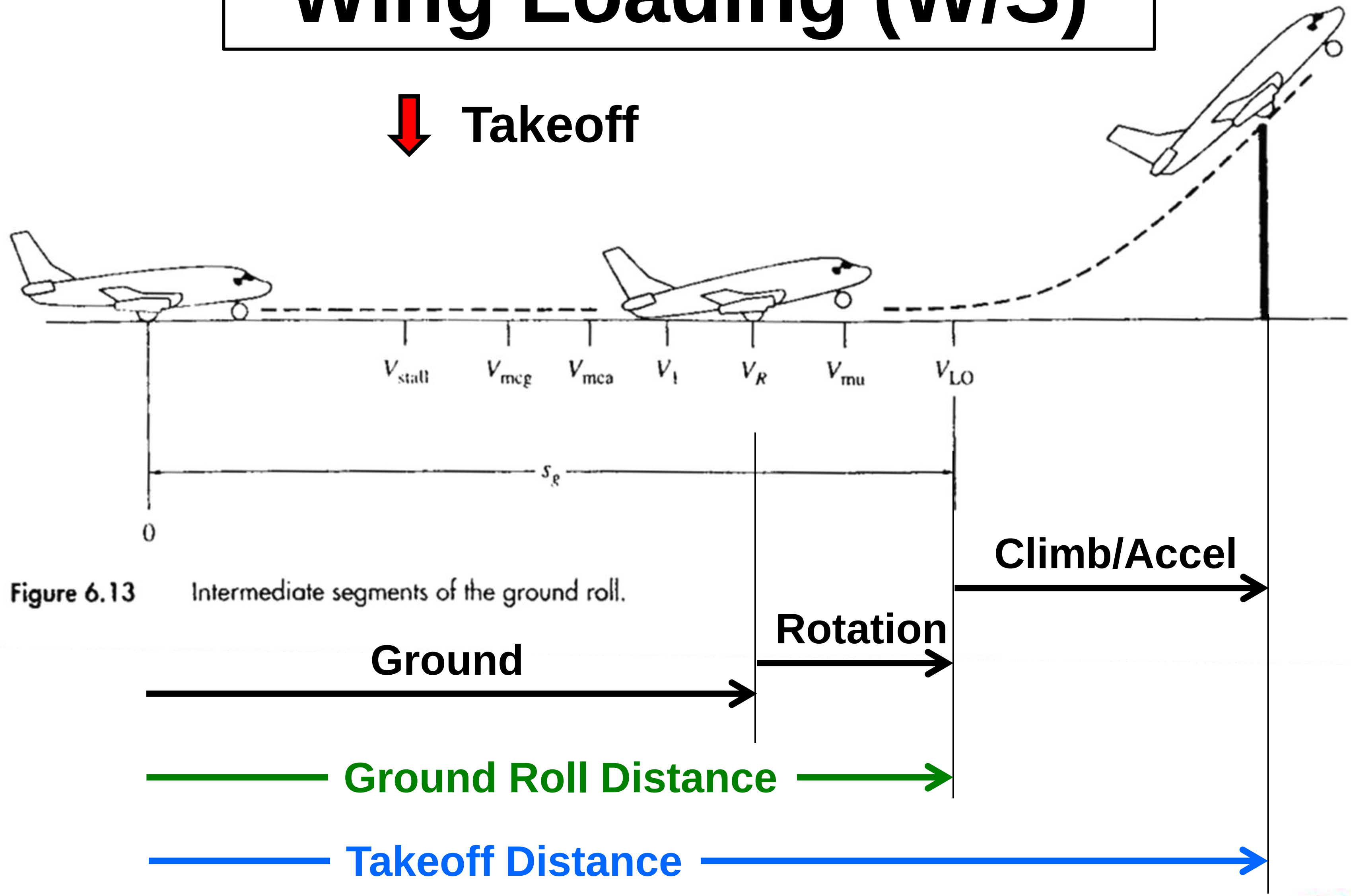
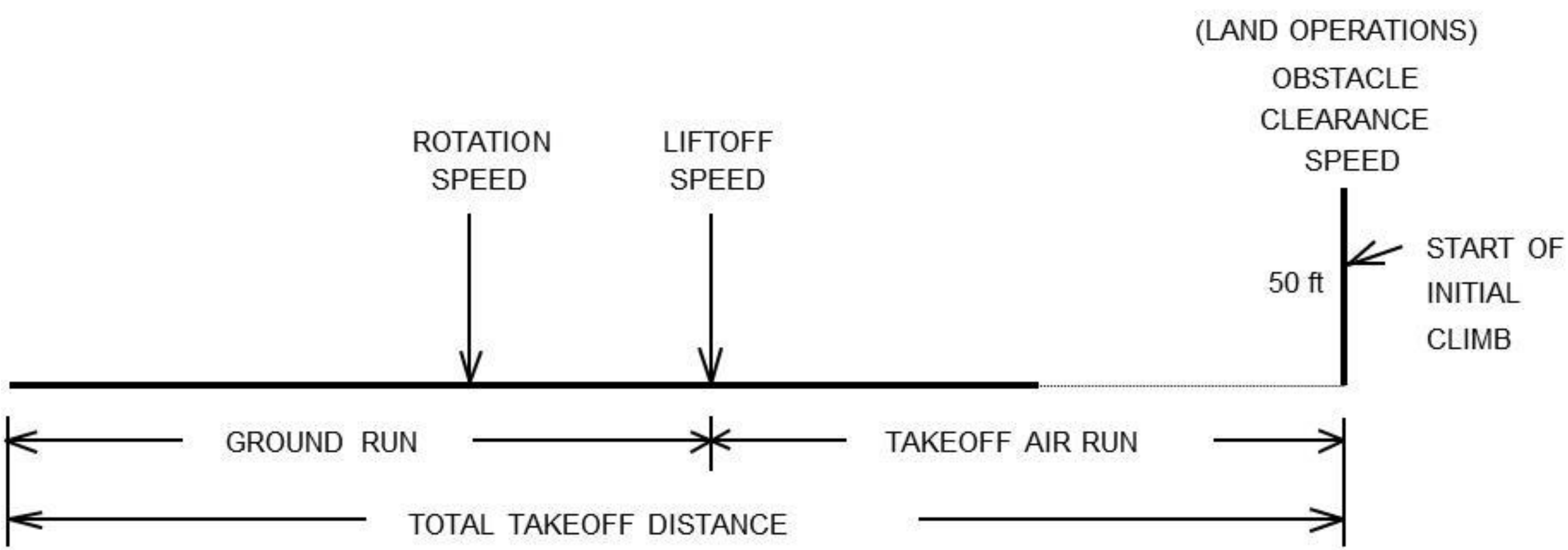


Figure 6.13 Intermediate segments of the ground roll.

Wing Loading (W/S)

Takeoff

$$V_{stall} = \sqrt{\frac{2\,(W/S)}{\rho\, C_{L_{max}}}} \qquad s_g = \frac{V_{TO}^{\;2}}{2\,a}$$



Rotation Speed
> 1.05 V_{stall}

Takeoff Speed
> 1.10 V_{stall}

Obstacle Speed
> 1.20 V_{stall}

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Wing Loading (W/S)

Takeoff

Takeoff Parameter (TOP) – correlates takeoff distances for a wide variety of aircraft

$$\mathbf{TOP} = \left(\frac{\mathbf{W}}{\mathbf{S}}\right)_{\mathbf{TO}} \frac{\mathbf{1}}{\mathbf{C_{L_{max}}}\ (\mathbf{T}/\mathbf{W})_{\mathbf{TO}}\ \boldsymbol{\sigma}}$$

$$\boldsymbol{\sigma} = \frac{\boldsymbol{\rho}_{\mathbf{TO}}}{\boldsymbol{\rho}_{\mathbf{SL}}}$$

$$\mathbf{s_{TO}} = \mathbf{20.9\ TOP} + \mathbf{87}\sqrt{\mathbf{TOP}\ (\mathbf{T}/\mathbf{W})}$$

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Wing Loading (W/S)

Example

$$\mathbf{TOP} = \left(\frac{\mathbf{W}}{\mathbf{S}}\right)_{\mathbf{TO}} \frac{\mathbf{1}}{\mathbf{C_{L_{max}}\,(T/W)_{TO}\,\sigma}} \qquad \boldsymbol{\sigma} = \frac{\boldsymbol{\rho}_{\mathbf{TO}}}{\boldsymbol{\rho}_{\mathbf{SL}}}$$

$$\mathbf{s_{TO} = 20.9\ TOP + 87\sqrt{TOP\,(T/W)}}$$

For the 747-400 airliner at Sea Level:

$$\left(\frac{\mathbf{T}}{\mathbf{W}}\right)_{\mathbf{TO}} = \mathbf{0.289} \qquad \left(\frac{\mathbf{W}}{\mathbf{S}}\right)_{\mathbf{TO}} = \mathbf{154.9} \qquad \mathbf{C_{L_{max}} = 1.6}$$

$$\mathbf{TOP} = \frac{\mathbf{154.9}}{\mathbf{(1.6)(0.289)(1)}} = \mathbf{335.0}$$

$$\mathbf{s_{TO} = (20.9)(335.0) + (87)\sqrt{(335.0)(0.289)} = 7,858\ ft}$$

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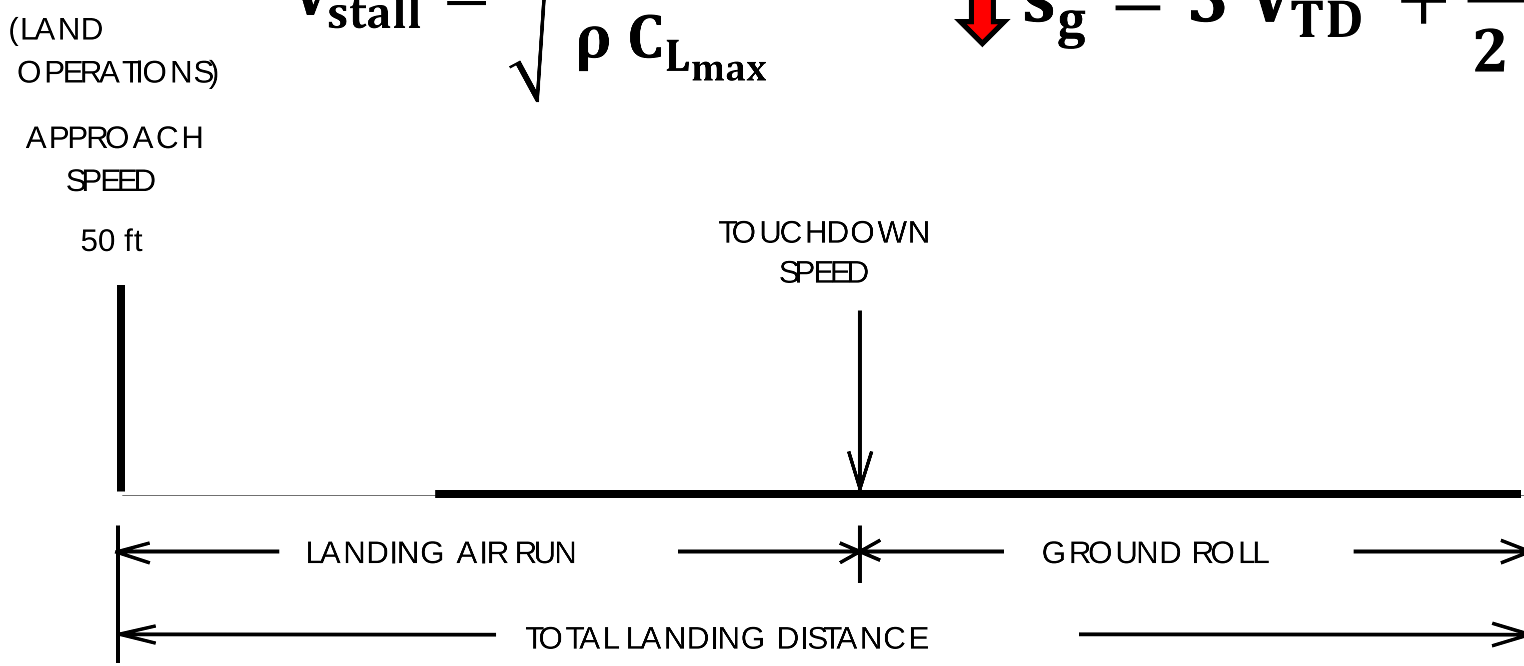
Wing Loading (W/S)

⬇ Landing

$$V_{stall} = \sqrt{\frac{2\,(W/S)}{\rho\, C_{L_{max}}}} \downarrow \qquad \downarrow s_g = 3\, V_{TD} + \frac{{V_{TD}}^2}{2\, g\, \mu}$$



Approach Speed V_{app} > 1.20 or 1.30 V_{stall}

Touchdown Speed V_{TD} > 1.10 or 1.15 V_{stall}

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Wing Loading (W/S)

Landing

Landing Parameter (LP) – correlates landing distances for a wide variety of aircraft

$$\mathbf{TOP} = \left(\frac{\mathbf{W}}{\mathbf{S}}\right)_{\mathbf{L}} \frac{\mathbf{1}}{\mathbf{C_{L_{max}}\,\sigma}} \qquad\qquad \boldsymbol{\sigma} = \frac{\boldsymbol{\rho}_{\mathbf{TO}}}{\boldsymbol{\rho}_{\mathbf{SL}}}$$

$$\mathbf{s_L = 118\,LP + 400}$$

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Example

Wing Loading (W/S)

$$\mathbf{TOP} = \left(\frac{\mathbf{W}}{\boldsymbol{S}}\right)_{\mathbf{L}} \frac{\mathbf{1}}{\mathbf{C}_{\mathbf{L}_{\mathbf{max}}}\,\boldsymbol{\sigma}} \qquad \boldsymbol{\sigma} = \frac{\boldsymbol{\rho}_{\mathbf{TO}}}{\boldsymbol{\rho}_{\mathbf{SL}}}$$

$$\mathbf{s}_{\mathbf{L}} = \mathbf{118\ LP + 400}$$

For the 747-400 airliner at 630,000 lb (MLW):

$$\left(\frac{\mathbf{W}}{\mathbf{S}}\right)_{\mathbf{L}} = \mathbf{111.5} \qquad \mathbf{C}_{\mathbf{L}_{\mathbf{max}}} = \mathbf{1.6}$$

$$\mathbf{LP} = \frac{\mathbf{111.5}}{(\mathbf{1.6})(\mathbf{1})} = \mathbf{69.7}$$

$$\mathbf{s}_{\mathbf{L}} = (\mathbf{118})(\mathbf{69.7}) + \mathbf{400} = \mathbf{8{,}625\ ft}$$

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Wing Loading (W/S)

Climb & Acceleration

$$\frac{(T-D)\,V}{W} = \frac{dh}{dt} + \frac{V}{g}\frac{dV}{dt} = P_s$$

For best climb & accel rate, need to minimize D / W

$$C_D = C_{D_0} + \frac{1}{\pi\, AR\, e}\, C_L^{\ 2} \longrightarrow \downarrow \frac{D}{W} = \frac{q\, C_{D_0}}{\uparrow (W/S)} + \frac{(W/S)\downarrow}{q\, \pi\, AR\, e}$$

	$(W/S)_{TO}$
Long Range	**110 - 130**
Short/Medium Range	**80 - 110**
Short Takeoff & Landing	**40 - 90**
Light Civil	**10 - 30**
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High Altitude	**30 - 60**

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Wing Loading (W/S)

⇧ ⇩ Range

$$R = \frac{V}{c_t}\frac{L}{D}\ln\frac{W_0}{W_1}$$ Breguet Range Equation

		Propeller Aircraft	Jet Aircraft
R = 1 / 3	$\text{Max}\ \frac{C_L{}^{3/2}}{C_D}$	Maximum Endurance	
R = 1	$\text{Max}\ \frac{C_L}{C_D}$	Maximum Range	Maximum Endurance
R = 3	$\text{Max}\ \frac{C_L{}^{1/2}}{C_D}$		Maximum Range

$$R = 3 \qquad C_{D_0} = 3\,C_{D_L} = 3\,K\,C_L{}^2$$

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Wing Loading (W/S)

⇧ ⇩ Range

$$R = 3 \qquad C_{D_0} = 3\,C_{D_L} = 3\,k\,C_L{}^2$$

$$C_D = C_{D_0} + k\,C_L{}^2 = 4\,k\,C_L{}^2$$

$$\frac{V\,L}{D} = \frac{V\,C_L}{C_D} = \frac{V\,C_L}{4\,k\,C_L{}^2} = \frac{V}{4\,k\,C_L}$$

$$\text{Since } L = W \longrightarrow \uparrow \frac{V\,L}{D} = \frac{V\,q\,S}{4\,k\,W} = \frac{q}{4\,k}\,\frac{V}{(W/S)}\downarrow$$

Wing Loading (W/S)

Range

$$\frac{V\,L}{D} = \frac{q\,S}{4\,k\,W} = \frac{q}{4\,k}\frac{V}{(W/S)}$$

Lower W/S = bigger wing = less α to generate required lift = less induced drag = less thrust required = less fuel flow

Higher W/S = smaller wing = more α to generate required lift = less skin friction drag = less fuel flow

Higher W/S is desirable for a smoother ride at higher speeds, less area for a gust to act upon

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Wing Loading (W/S)

Constant Altitude Cruise

As the aircraft burns off fuel during the cruise, the aircraft will naturally want to slow down to maintain a constant L / D

Calculate C_L at beginning of cruise leg: $C_L = W_1 / (0.5\ \rho\ V^2\ S)$
C_L = constant for the entire cruise leg
$W_2 = W_1$ – fuel burned, calculate V at the end of cruise leg

747-400 Example

Weight at beginning of cruise = 824,176 lb at 36,000 ft & 834.36 ft/sec

$$\mathbf{C_L = \frac{W_1}{0.5\ \rho\ V^2\ S} = \frac{824{,}176\ (W1)}{(0.5)(0.00070856)(834.36)^2(5650)} = 0.5915}$$

$$\mathbf{V = \sqrt{\frac{W_2}{0.5\ \rho\ C_L\ S}} = \sqrt{\frac{756{,}416\ (W2)}{(0.5)(0.00070856)(0.5915)(5650)}} = 799.33\ ft/sec}$$

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Wing Loading (W/S)

Cruise Climb

As the aircraft burns off fuel during the cruise, the aircraft will naturally want to climb to maintain a constant L / D

Calculate W_1 / δ at beginning of cruise leg (δ = density ratio)
W / δ = constant for the entire cruise leg
W_2 = W_1 – fuel burned, calculate δ at the end of cruise leg

747-400 Example

Weight at beginning of cruise = 824,176 lb at 36,000 ft

$$\mathbf{W_1}/\boldsymbol{\delta_1} = \frac{\mathbf{824,176}}{\mathbf{0.2981}} = \mathbf{2,764,764} = \mathbf{W_2}/\boldsymbol{\delta_2}$$

$$\boldsymbol{\delta_2} = \frac{\mathbf{W_2}}{(\mathbf{W_1}/\boldsymbol{\delta_1})} = \frac{\mathbf{756,416}}{\mathbf{2,764,764}} = \mathbf{0.2736} \longrightarrow \mathbf{h_2} = \mathbf{37,806\ ft}$$

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Wing Loading (W/S)

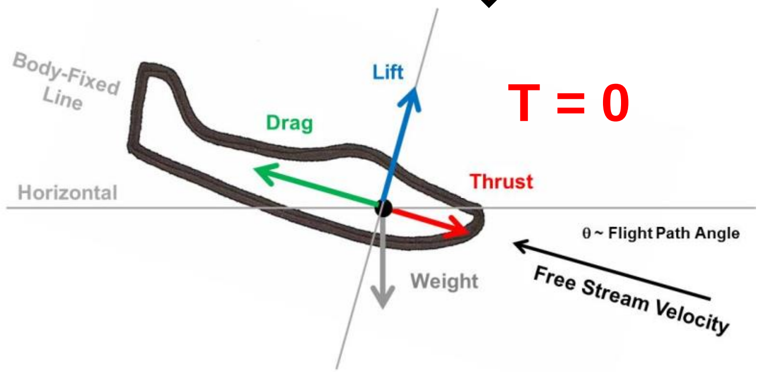
↓ Combat

Instantaneous Load Factor ↑ $n = \dfrac{q\, C_{L_{max}}}{(W/S)}$ ↓

Sustained Load Factor ↑ $n = \sqrt{\dfrac{q\, \pi\, AR\, e}{(W/S)} \left[\left(\dfrac{T}{W} \right)_{max} - \dfrac{q\, C_{D_0}}{(W/S)} \right]}$

Wing Loading (W/S)

Glide



T = 0

$$\tan\theta_{min} = \frac{1}{\left({}^{L}/_{D}\right)_{max}}$$

Minimum descent angle
= Maximum glide range

$$R = 1 \qquad C_{D_0} = C_{D_L} = k\,C_L^{\;2}$$

$$C_D = C_{D_0} + k\,C_L^{\;2} = 2\,k\,C_L^{\;2}$$

$$\frac{L}{D} = \frac{C_L}{C_D} = \frac{C_L}{2\,k\,C_L^{\;2}} = \frac{1}{2\,k\,C_L}$$

$$\text{Since } L = W \longrightarrow \uparrow\frac{L}{D} = \frac{q\,S}{2\,k\,W} = \frac{q}{2\,k}\,\frac{1}{(W/S)}\downarrow$$

HW #21 - Wing Loading (W/S)

Take-off			Landing			Weights				
H (f)	0		H (f)	0		TOGW	874,786			
CL(max)	1.6		CL(max)	1.6		Start-up & Takeoff	852,916			
T(max) (lb)	253,200		T(max) (lb)	253,200		Climb	823,917			
W_TO (lb)	874,786		W_L (lb)	622,754		Cruise Out	721,904			
S (f^2)	5650		S (f^2)	5650		Loiter	622,754			
W/S (lb/f^2)	154.83		W/S (lb/f^2)	110.22						
SIGMA	1.0000		SIGMA	1.0000						
T/W	0.289		T/W	0.407						
TOP	334.33		LP	68.89						
S_TO (f)	7,843		S_TO (f)	8,529						
CRUISE CLIMB						CONSTANT ALTITUDE CRUISE				
Cruise Out						Cruise Out				
Cruise Start			Cruise End			Cruise Start			Cruise End	
CD_0	0.0189		CD_0	0.0189		CD_0	0.0189		CD_0	0.0189
A	7.9		A	7.9		A	7.9		A	7.9
H (f)	26,180		H (f)	29,130		H (f)	26,180		H (f)	26,180
Cruise Mach	0.8500		Cruise Mach	0.8500		Cruise Mach	0.8500		Cruise Mach	0.7956
W (lb)	823,917		W (lb)	721,904		W (lb)	823,917		W (lb)	721,904
Lift (lb)	823,957		Lift (lb)	721,891		Lift (lb)	823,957		Lift (lb)	721,866
Delta	-40		Delta	13		Delta	-40		Delta	38
Cruise CL	0.3537		Cruise CL	0.3537		Cruise CL	0.3537		Cruise CL	0.3537
k	0.0504		k	0.0504		k	0.0504		k	0.0504
V (f/s)	862.74		V (f/s)	854.21		V (f/s)	862.74		V (f/s)	807.52
rho (lbm/f^3)	0.0356762		rho (lbm/f^3)	0.0318839		rho (lbm/f^3)	0.0356762		rho (lbm/f^3)	0.0356762
q (lbf/f^2)	412.34		q (lbf/f^2)	361.26		q (lbf/f^2)	412.34		q (lbf/f^2)	361.25
W/S (lb/f^2)	145.83		W/S_optimum	127.77		W/S (lb/f^2)	145.83		W/S_optimum	127.76
W/S_actual	145.83		W/S_actual	127.77		W/S_actual	145.83		W/S_actual	127.77

WINGLOAD.XLS

Iteration on Cruise Altitude

"Design of Aircraft" - Thomas C. Corke

HW #21 - Wing Loading (W/S)

Take-off			Landing			Weights	
H (f)	0		H (f)	0		TOGW	874,786
CL(max)	1.6		CL(max)	1.6		Start-up & Takeoff	852,916
T(max) (lb)	253,200		T(max) (lb)	253,200		Climb	823,917
W_TO (lb)	874,786		W_L (lb)	622,754		Cruise Out	721,904
S (f^2)	5650		S (f^2)	5650		Loiter	622,754
W/S (lb/f^2)	154.83		W/S (lb/f^2)	110.22			
SIGMA	1.0000		SIGMA	1.0000			
T/W	0.289		T/W	0.407			
TOP	334.33		LP	68.89			
S_TO (f)	7,843		S_TO (f)	8,529			

Input data from fact sheet & historical parameters

Input weights from HW #19 results in ITERTOW.XLS

Takeoff Altitude
Takeoff C_{Lmax}
Total Thrust
Wing Area

Landing Altitude
Landing C_{Lmax}

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HW #21 - Wing Loading (W/S)

CRUISE CLIMB						CONSTANT ALTITUDE CRUISE				
Cruise Out						**Cruise Out**				
Cruise Start			**Cruise End**			**Cruise Start**			**Cruise End**	
CD_0	0.0189		CD_0	0.0189		CD_0	0.0189		CD_0	0.0189
A	7.9		A	7.9		A	7.9		A	7.9
H (f)	26,180		H (f)	29,130		H (f)	26,180		H (f)	26,180
Cruise Mach	0.8500		Cruise Mach	0.8500		Cruise Mach	0.8500		Cruise Mach	0.7956
W (lb)	823,917		W (lb)	721,904		W (lb)	823,917		W (lb)	721,904
Lift (lb)	823,957		Lift (lb)	721,891		Lift (lb)	823,957		Lift (lb)	721,866
Delta	-40		Delta	13		Delta	-40		Delta	38
Cruise CL	0.3537		Cruise CL	0.3537		Cruise CL	0.3537		Cruise CL	0.3537
k	0.0504		k	0.0504		k	0.0504		k	0.0504
V (f/s)	862.74		V (f/s)	854.21		V (f/s)	862.74		V (f/s)	807.52
rho (lbm/f^3)	0.0356762		rho (lbm/f^3)	0.0318839		rho (lbm/f^3)	0.0356762		rho (lbm/f^3)	0.0356762
q (lbf/f^2)	412.34		q (lbf/f^2)	361.26		q (lbf/f^2)	412.34		q (lbf/f^2)	361.25
W/S (lb/f^2)	145.83		W/S_optimum	127.77		W/S (lb/f^2)	145.83		W/S_optimum	127.76
W/S_actual	145.83		W/S_actual	127.77		W/S_actual	145.83		W/S_actual	127.77

Calculate C_{D_0}
Input Aspect Ratio
Input Cruise Mach
Iterate on altitude so W = L

Use same starting conditions as Cruise Climb
Set end altitude = start altitude
Iterate on Mach Number so W = L

$$C_{D_0} = \frac{S_{wet}}{S_{ref}} C_{f_e}$$

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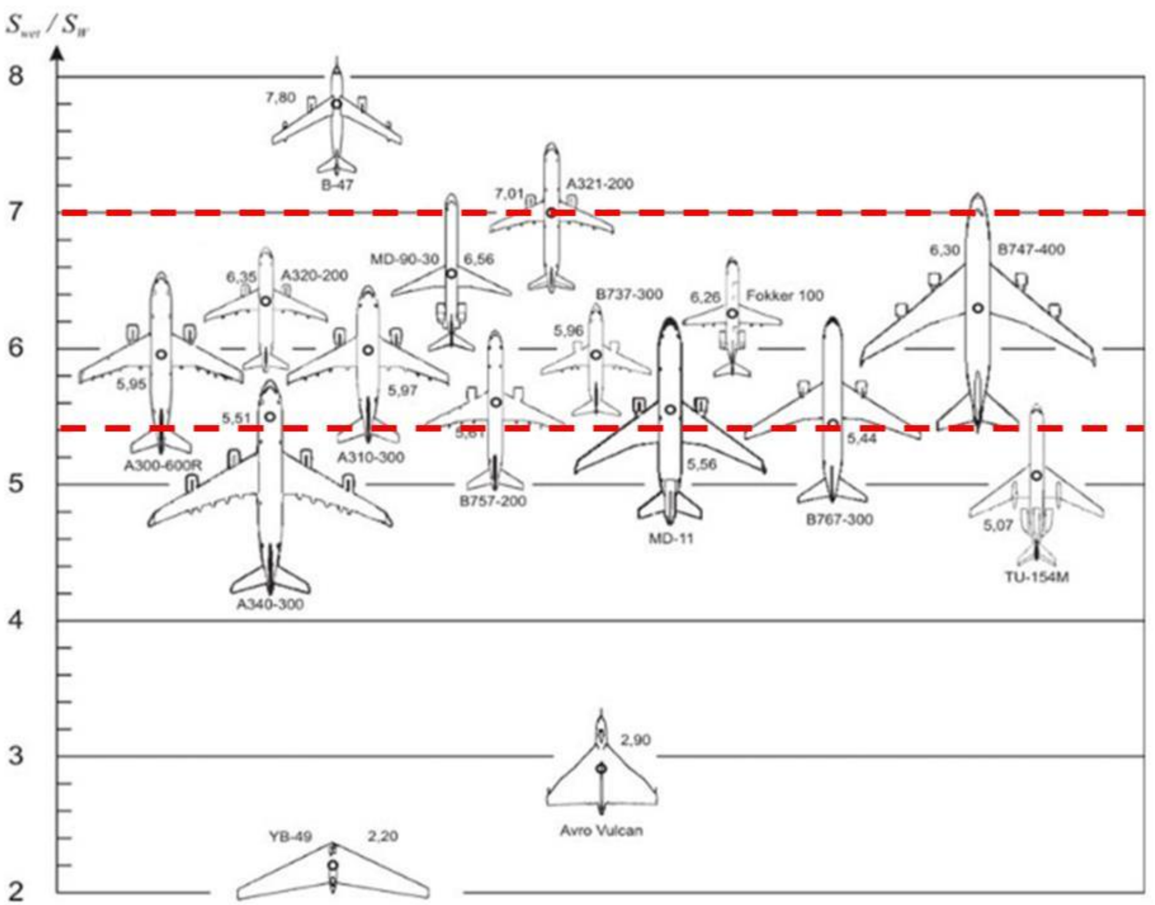
HW #21 - Wing Loading (W/S)

	C_{fe} - subsonic
Bomber and Civil Transport	**0.0030**
Military Cargo	**0.0035**
Air Force Fighter	**0.0035**
Navy Fighter	**0.0040**
Clean Supersonic Cruise	**0.0025**
Light Aircraft – Single Engine	**0.0055**
Light Aircraft – Twin Engine	**0.0045**

$$C_{D_0} = \frac{S_{wet}}{S_{ref}} C_{f_e}$$



HW #21 - Wing Loading (W/S)

CRUISE CLIMB							CONSTANT ALTITUDE CRUISE					
Cruise Out							**Cruise Out**					
Cruise Start			**Cruise End**				**Cruise Start**			**Cruise End**		
CD_0	0.0189		CD_0	0.0189			CD_0	0.0189		CD_0	0.0189	
A	7.9		A	7.9			A	7.9		A	7.9	
H (f)	26,180		H (f)	29,130			H (f)	26,180		H (f)	26,180	
Cruise Mach	0.8500		Cruise Mach	0.8500			Cruise Mach	0.8500		Cruise Mach	0.7956	
W (lb)	823,917		W (lb)	721,904			W (lb)	823,917		W (lb)	721,904	
Lift (lb)	823,957		Lift (lb)	721,891			Lift (lb)	823,957		Lift (lb)	721,866	
Delta	-40		Delta	13			Delta	-40		Delta	38	
Cruise CL	0.3537		Cruise CL	0.3537			Cruise CL	0.3537		Cruise CL	0.3537	
k	0.0504		k	0.0504			k	0.0504		k	0.0504	
V (f/s)	862.74		V (f/s)	854.21			V (f/s)	862.74		V (f/s)	807.52	
rho (lbm/f^3)	0.0356762		rho (lbm/f^3)	0.0318839			rho (lbm/f^3)	0.0356762		rho (lbm/f^3)	0.0356762	
q (lbf/f^2)	412.34		q (lbf/f^2)	361.26			q (lbf/f^2)	412.34		q (lbf/f^2)	361.25	
W/S (lb/f^2)	145.83		W/S_optimum	127.77			W/S (lb/f^2)	145.83		W/S_optimum	127.76	
W/S_actual	145.83		W/S_actual	127.77			W/S_actual	145.83		W/S_actual	127.77	

Use Excel Solver:
Find the altitude (H) that will result in W = L (or Delta = 0)

Use Excel Solver:
Find the Cruise Mach that will result in W = L (or Delta = 0)

Homework Assignments

HW #21 – Aircraft Design - Wing Loading (due by 11:59 pm ET on Monday)

<u>HW Help Session</u>
Monday 4:00 – 5:00 pm ET

<u>Posted on Canvas</u>
HW #21 Assignment with instructions, tips, and checklist
WINGLOAD.XLS Excel file

Questions?

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