Aircraft Performance
Equations of Motion
Maneuvering Flight

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Specific Energy

Total Energy = Potential Energy + Kinetic Energy

$$\mathbf{E = PE + KE}$$

$$\mathbf{E = mgh + \frac{1}{2}mV^2}$$

$$\mathbf{\frac{E}{W} = h + \frac{1}{2g}V^2}$$

$$\mathbf{E_s = h + \frac{1}{2g}V^2}$$

(always in ft)

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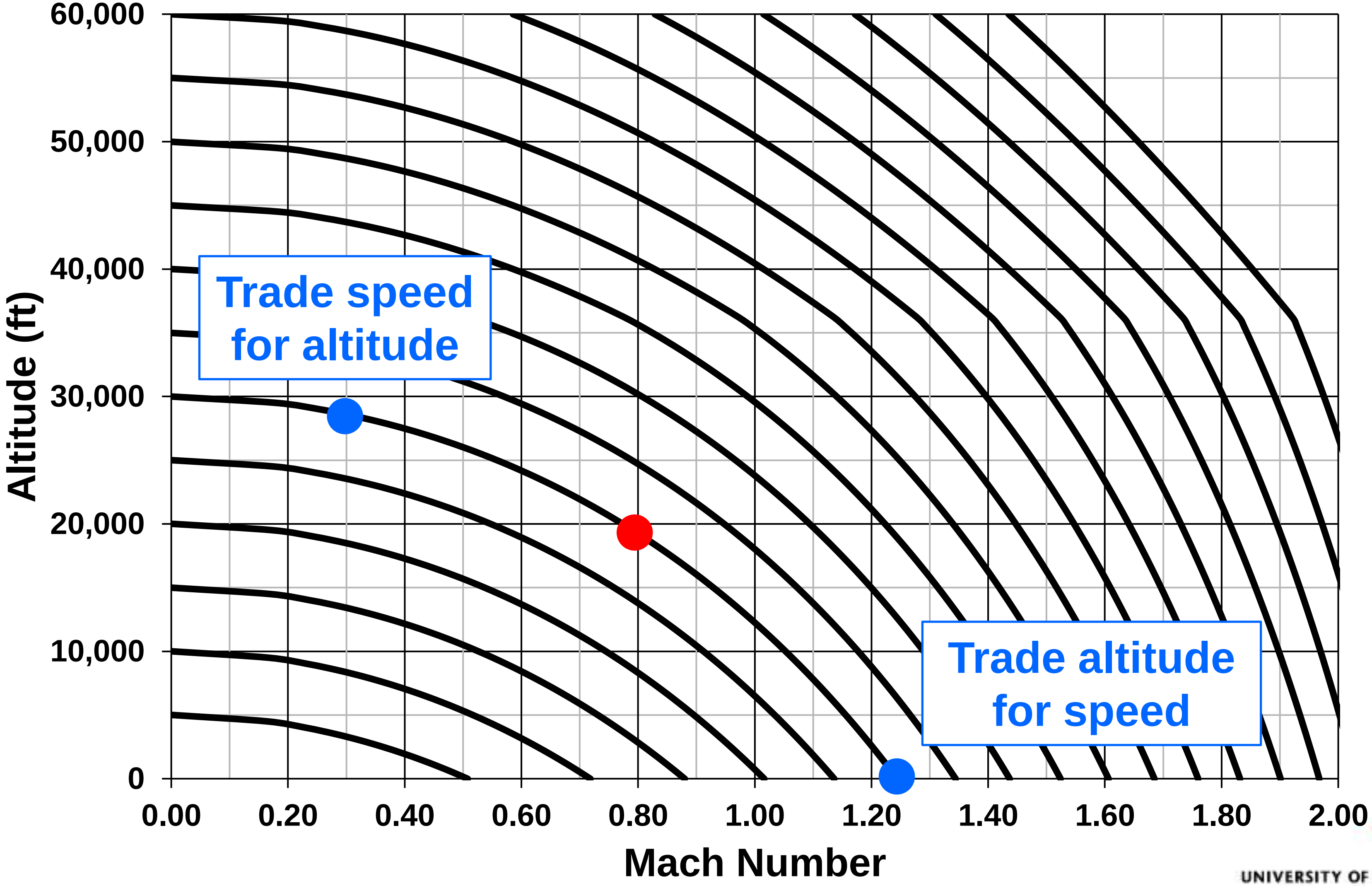
Specific Energy



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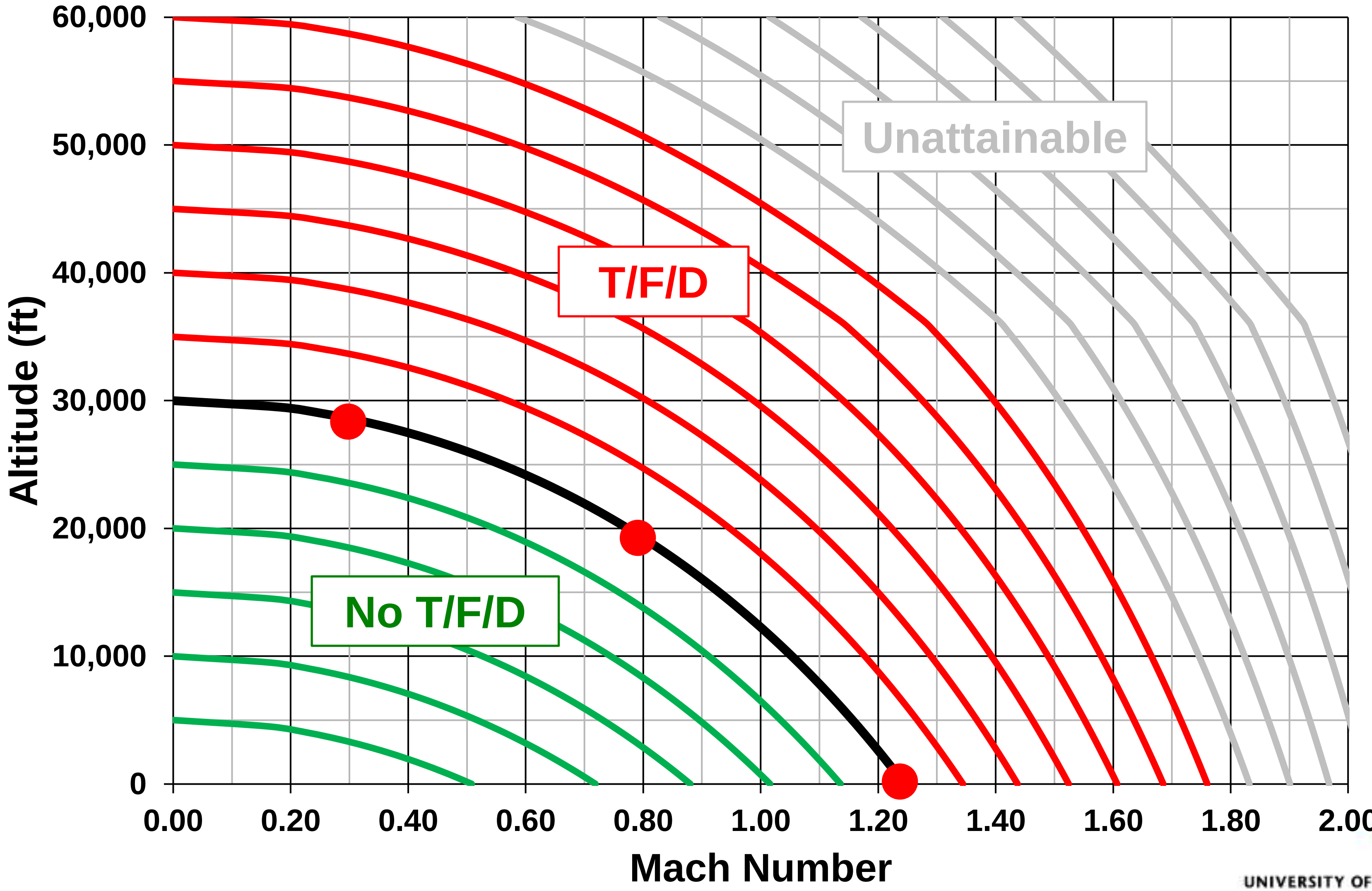
Specific Energy



Specific Energy

What is the relationship between E_s and P_s?

$$\frac{d}{dt}E_s = \frac{d}{dt}\left(h + \frac{1}{2g}V^2\right)$$

$$\boxed{\dot{E}_s = \frac{dh}{dt} + \frac{V}{g}\frac{dV}{dt} = P_s}$$

(always in ft/sec)

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Specific Excess Power

$$\frac{(T-D)\,V}{W} = \frac{dh}{dt} + \frac{V}{g}\frac{dV}{dt} = P_s$$

Specific Excess Power can help you calculate:
Absolute, Service, Cruise, and Combat Ceilings
Maximum and Minimum Velocity
Time, Fuel, and Distance to Climb or Descend
Time, Fuel, and Distance to Accel or Decel
Sustained Turn Envelope

$P_s < 0$ decelerating / descending flight

$P_s = 0$ sustained flight

$P_s > 0$ accelerating / climbing flight

Level Turning Flight

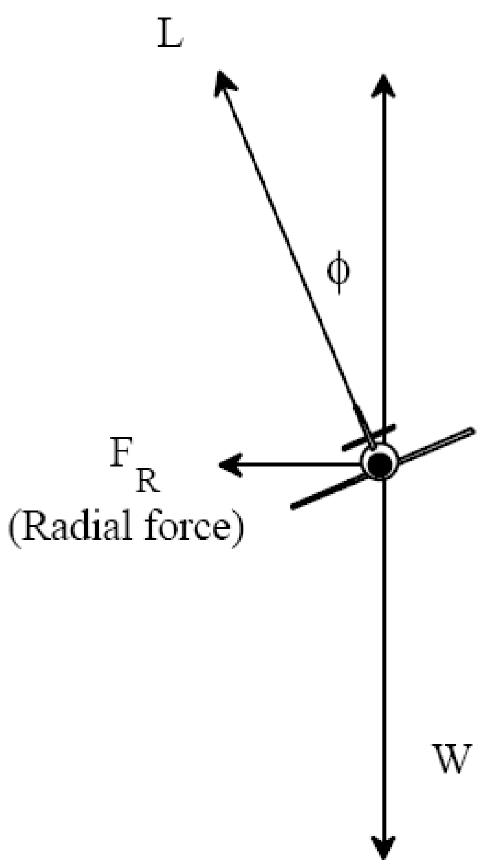

Figure 6.1
FORCES IN A STEADY LEVEL TURN

Lift acts perpendicular to the Free Stream Velocity
Lift acts perpendicular the wing surface
Drag acts parallel to the Free Stream Velocity
Weight acts vertically towards the ground
Thrust is fixed in the aircraft

Bank angle ϕ **Load factor n**

$$\Sigma F_z = L\cos\phi - W = 0$$

$$\cos\phi = \frac{W}{L} \qquad n = \frac{L}{W}$$

$$\phi = \arccos\left(\frac{1}{n}\right)$$

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Level Turning Flight

$$\Sigma\, F_R = L \sin \phi = \frac{mV^2}{R}$$

Turn Radius R

Turn Angle ψ

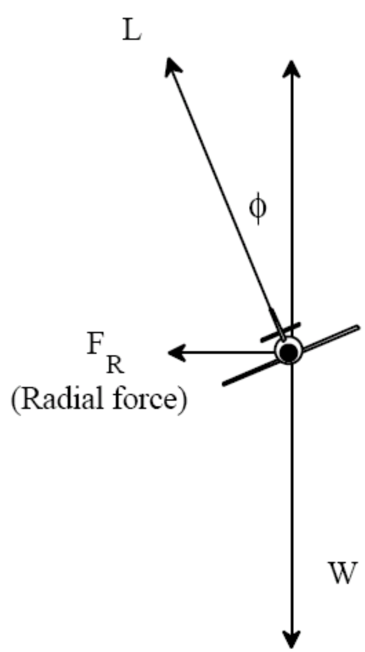
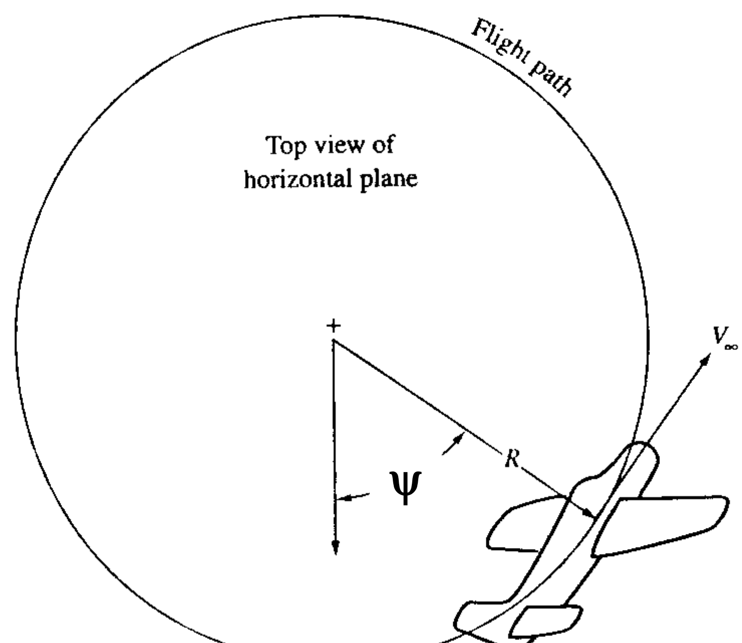


Figure 6.1
FORCES IN A STEADY LEVEL TURN



Level Turning Flight

$$\Sigma F_R = L \sin \phi = \frac{mV^2}{R}$$

Turn Radius R

Turn Angle ψ

Turn Rate ω = dψ/dt

Turn Radius: $R = \frac{mV^2}{L \sin \phi} = \frac{V^2}{g\sqrt{n^2 - 1}}$ (usually in ft)

Turn Rate: $\omega = \frac{V}{R} = \frac{g\sqrt{n^2 - 1}}{V}$ (calculated as rad/sec, but usually expressed as deg/sec)

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Level Turning Flight

Turn Radius: $$\mathbf{R = \frac{mV^2}{L\sin\phi} = \frac{V^2}{g\sqrt{n^2-1}}}$$ (tightest turn)

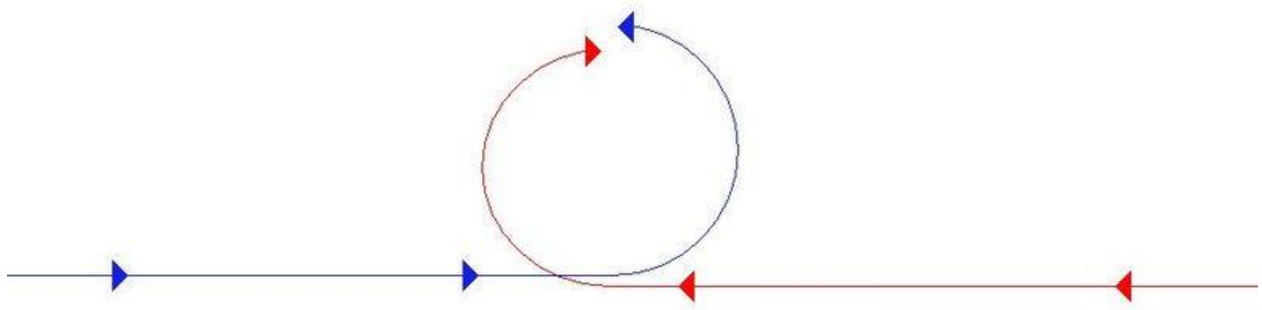
Turn Rate: $$\boldsymbol{\omega} = \mathbf{\frac{V}{R} = \frac{g\sqrt{n^2-1}}{V}}$$ (quickest turn)

To minimize turn radius for tightest turn and to maximize turn rate for quickest turn:

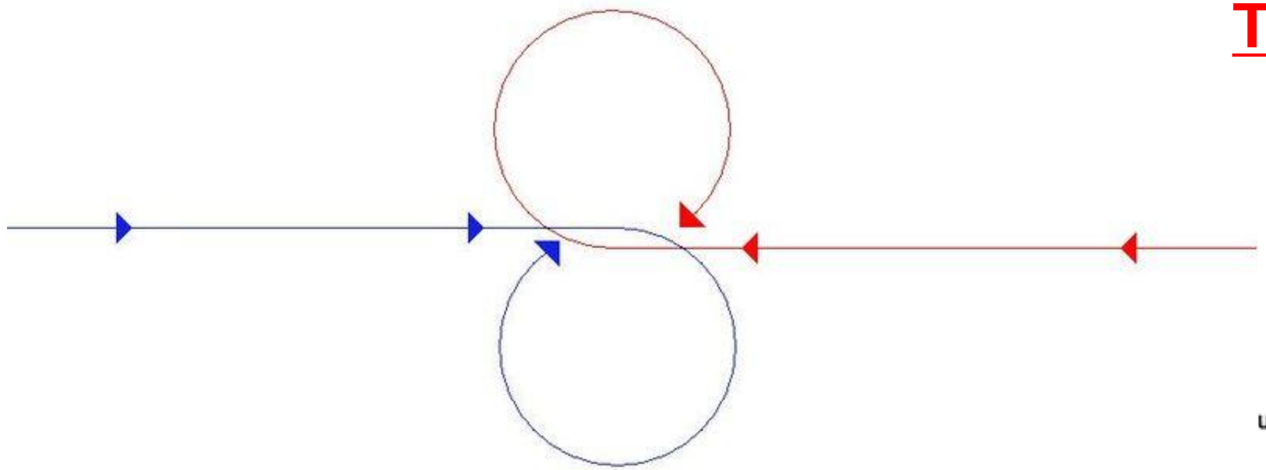
Highest possible load factor
Lowest possible velocity

Level Turning Flight

One circle flow



Two circle flow



Top Gun Video

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Level Turning Flight

Bank Angle ϕ

Load Factor n

Turn Radius R

Turn Angle ψ

Turn Rate $\omega = d\psi/dt$

Sustained Turn – turning while maintaining the same velocity and altitude $P_s = 0$

Sustained Corner Velocity – speed for highest turn rate while maintaining velocity and altitude

Instantaneous Turn – turning and not maintaining the same velocity and altitude $P_s \neq 0$

Corner Velocity – speed for highest turn rate

Sustained Turn

Sustained Turn – turning while maintaining the same velocity and altitude

$P_s = 0$

$$(n_{max})_{sust} = \sqrt{\frac{q}{K\,(W/S)}\left[\frac{T}{W} - \frac{q\,C_{D_0}}{(W/S)}\right]} = \left(\frac{T}{W}\right)\left(\frac{L}{D}\right)_{max}$$

$$V_{(n_{max})_{sust}} = \left(\frac{2}{\rho}\sqrt{\frac{K}{C_{D_0}}}\,\frac{n\,W}{S}\right)^{1/2}$$

$$(n_{max})_{C_{L_{max}}} = \frac{q\,C_{L_{max}}}{(W/S)}$$

Sustained Turn

CD0	0.0150	**Wt**	65,000	lb	**CLmax**	1.2			
K	0.08	**Alt**	20,000	ft	**(n)max**	4.5			
		rho	0.00126642						
W/S	68.42	**QMS**	680.7		**(L/D)max**	14.43			
		a	1036.85359	ft/sec	**(T/W)max**	0.2271			
TA(SL)	13850	**S**	950	sq ft	**(n)sust**	3.2773	g's		
# engines	2	**DR**	0.5328		**V (n)sust**	904.3	ft/sec		
TA(SL)	27700	14759							
T/W	0.4262	0.2271							
Vel (fps)	**q**	**n**	**CL**	**CD**	**D (lb)**	**T (lb)**	**L/D**	**(L/D)(T/W)**	**n(Clmax)**
100	6.33	0.5109	5.5211	2.4536	14759	14759	2.25	0.5109	0.1110
150	14.25	0.7634	3.6665	1.0905	14759	14759	3.36	0.7634	0.2499
200	25.33	1.0124	2.7350	0.6134	14759	14759	4.46	1.0124	0.4442
250	39.57	1.2565	2.1725	0.3926	14759	14759	5.53	1.2565	0.6941
300	56.99	1.4946	1.7945	0.2726	14759	14759	6.58	1.4946	0.9994
350	77.56	1.7252	1.5219	0.2003	14759	14759	7.60	1.7252	1.3603
400	101.31	1.9471	1.3151	0.1533	14759	14759	8.58	1.9471	1.7768
450	128.22	2.1587	1.1520	0.1212	14759	14759	9.51	2.1587	2.2487
500	158.29	2.3585	1.0195	0.0981	14759	14759	10.39	2.3585	2.7762
550	191.53	2.5448	0.9091	0.0811	14759	14759	11.21	2.5448	3.3592
600	227.94	2.7156	0.8151	0.0682	14759	14759	11.96	2.7156	3.9977
650	267.51	2.8689	0.7338	0.0581	14759	14759	12.64	2.8689	4.6918
700	310.25	3.0024	0.6621	0.0501	14759	14759	13.22	3.0024	5.4414
750	356.16	3.1134	0.5981	0.0436	14759	14759	13.71	3.1134	6.2465
800	405.23	3.1988	0.5401	0.0383	14759	14759	14.09	3.1988	7.1071
850	457.47	3.2549	0.4868	0.0340	14759	14759	14.34	3.2549	8.0232
900	512.87	3.2771	0.4372	0.0303	14759	14759	14.43	3.2771	8.9949
950	571.44	3.2597	0.3903	0.0272	14759	14759	14.36	3.2597	10.0221
1000	633.17	3.1950	0.3453	0.0245	14759	14759	14.07	3.1950	11.1048
1050	698.07	3.0724	0.3011	0.0223	14759	14759	13.53	3.0724	12.2431
1100	766.14	2.8760	0.2568	0.0203	14759	14759	12.67	2.8760	13.4369
1150	837.37	2.5790	0.2107	0.0186	14759	14759	11.36	2.5790	14.6862
1200	911.77	2.1273	0.1596	0.0170	14759	14759	9.37	2.1273	15.9910
1250	989.33	1.3554	0.0937	0.0157	14759	14759	5.97	1.3554	17.3513

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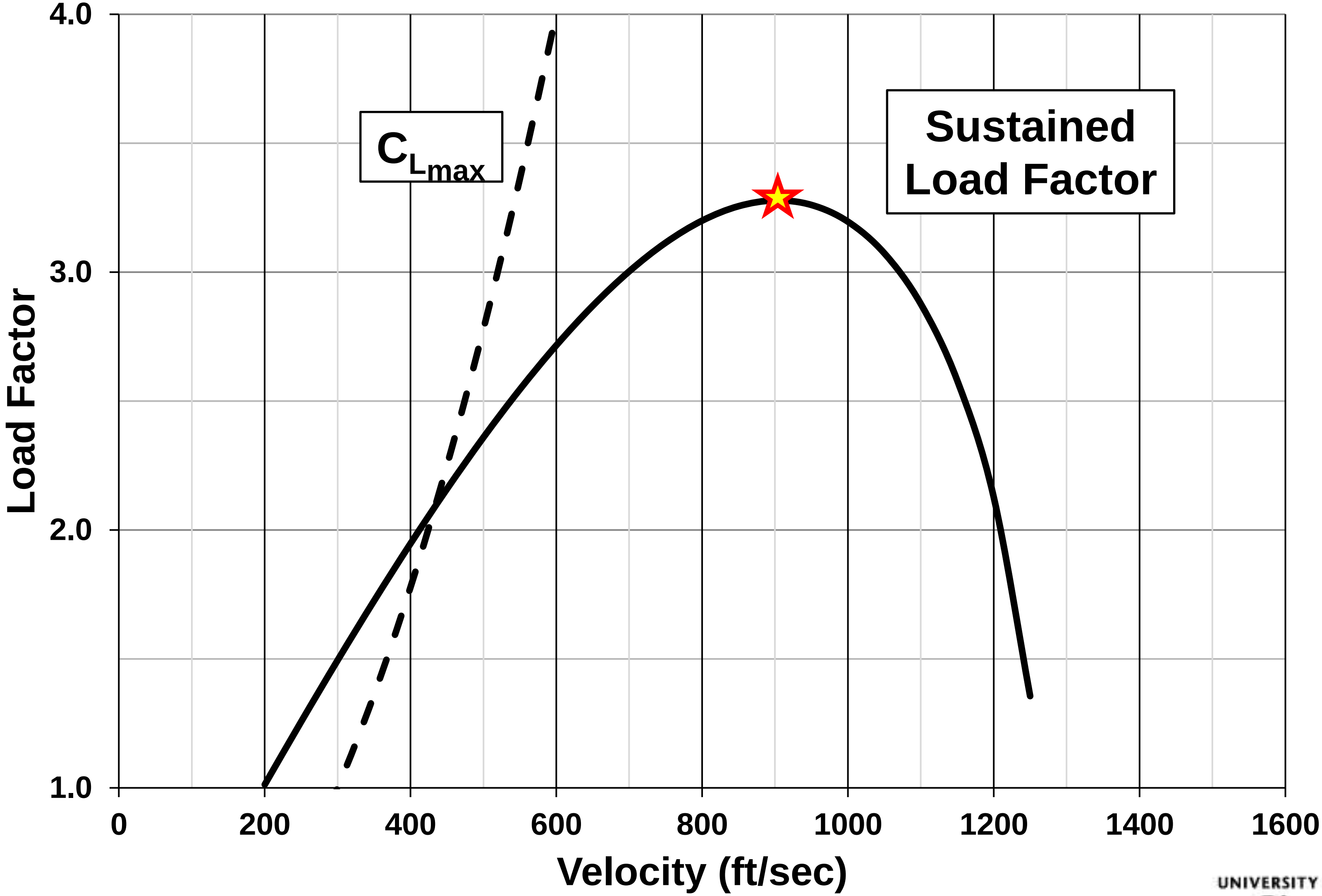
Sustained Turn



Sustained Turn

F-22 turned 360 degrees in 21 seconds

Turn Rate: $\omega = \frac{V}{R} = \frac{g\sqrt{n^2-1}}{V} = \frac{360\text{ deg}}{21\text{ sec}} = 17\text{ deg/sec}$

Turn Radius: $R = \frac{mV^2}{L\sin\phi} = \frac{V^2}{g\sqrt{n^2-1}}$

$$R = \frac{V}{\omega} = \frac{325\text{ NM/hr}}{17\text{ deg/sec}} = 1{,}850\text{ ft} \cong 0.35\text{ mile}$$

$$n = \sqrt{1 + \frac{V^2\omega^2}{g^2}} = 5.16\text{ g's} \qquad \phi = \arccos\left(\frac{1}{n}\right) = 78.8°$$

Video:

http://www.youtube.com/watch?v=n34RwIUlnAo&feature=related

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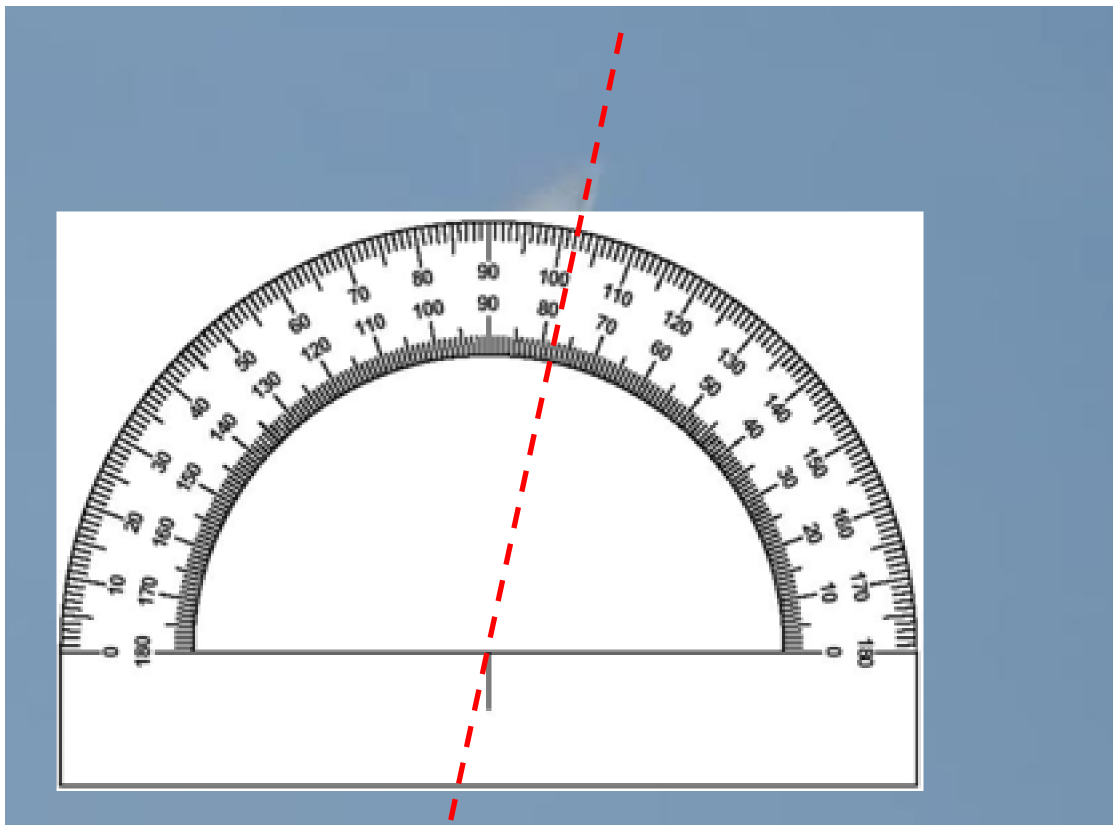
Sustained Turn

$$\mathbf{n} = \sqrt{1 + \frac{\mathbf{V^2 \omega^2}}{\mathbf{g^2}}} = \mathbf{5.16\ g's} \qquad \boldsymbol{\phi} = \mathbf{arccos}\left(\frac{\mathbf{1}}{\mathbf{n}}\right) = 78.8°$$



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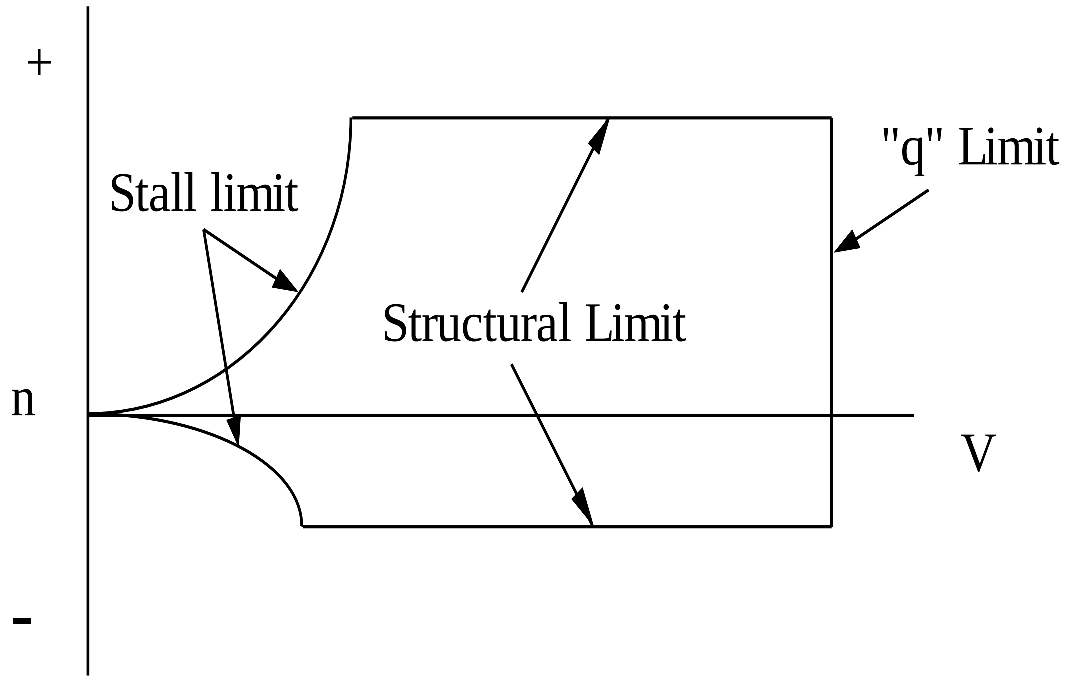
Instantaneous Turn

Instantaneous Turn – turning and not maintaining the same velocity and altitude

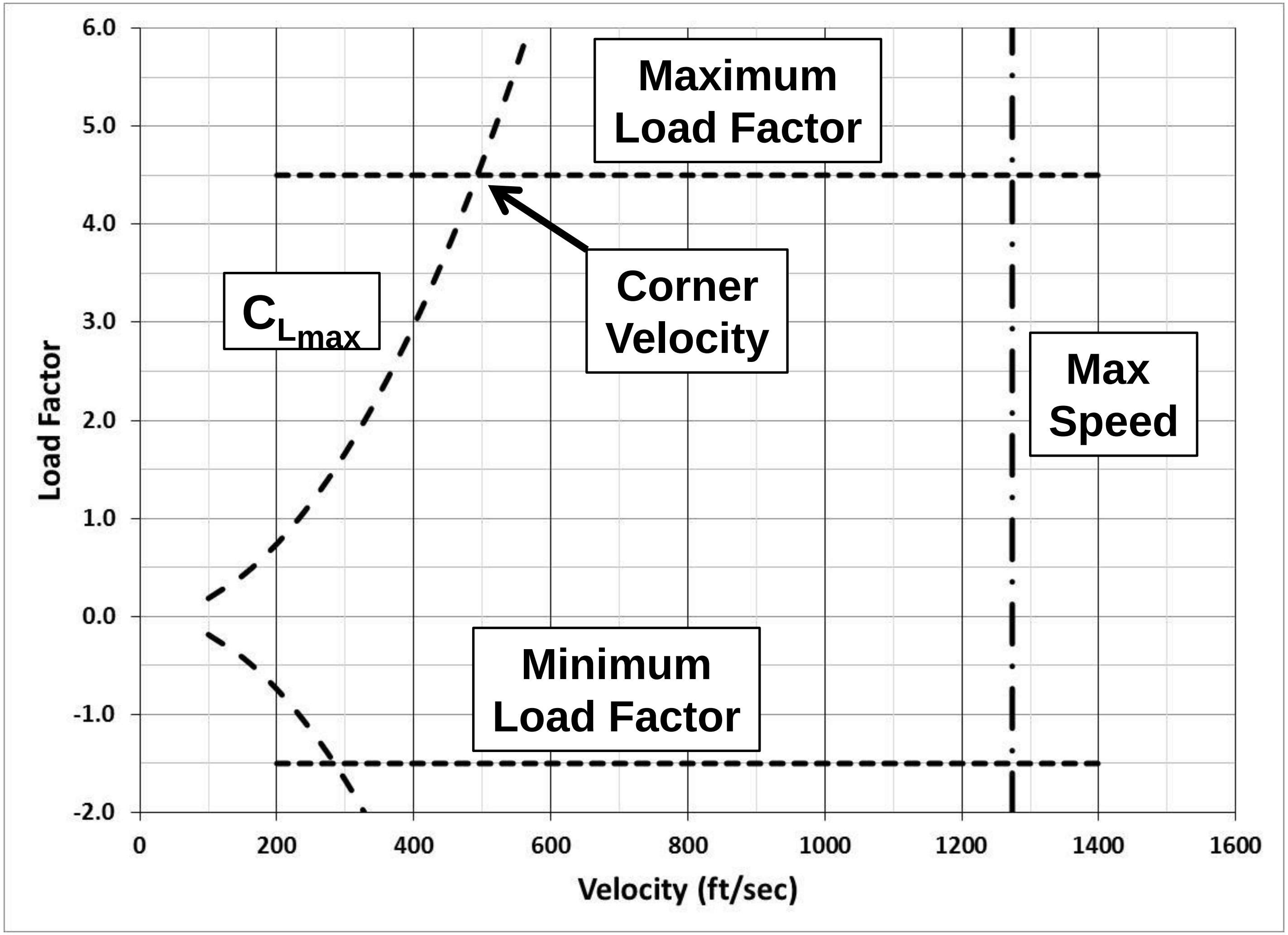
$P_s \neq 0$

V - n Diagram – Plot of Load Factor vs Velocity

- **Establishes the aircraft maneuver boundaries**
- **Includes limit load factor and maximum lift**

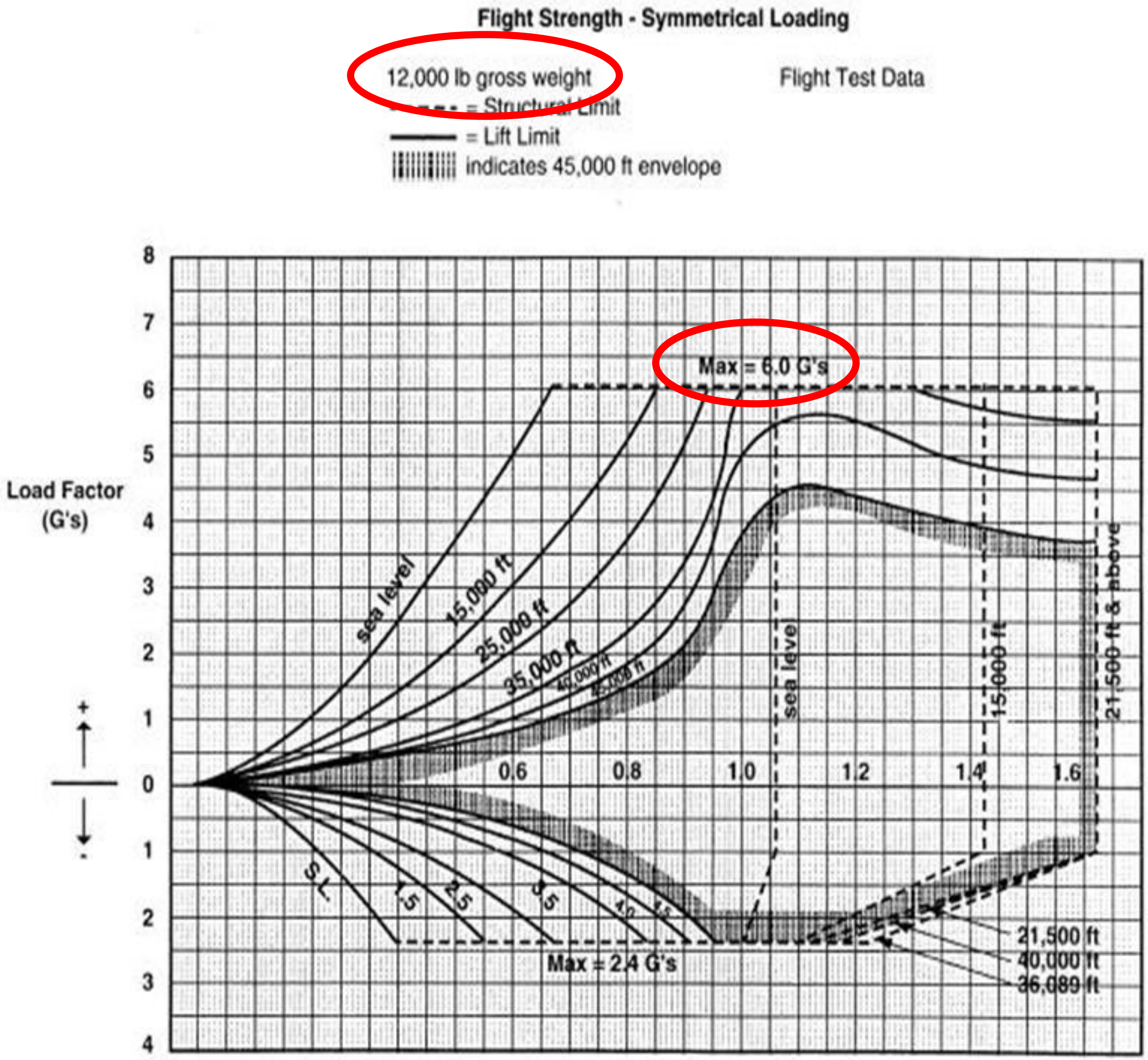


Instantaneous Turn

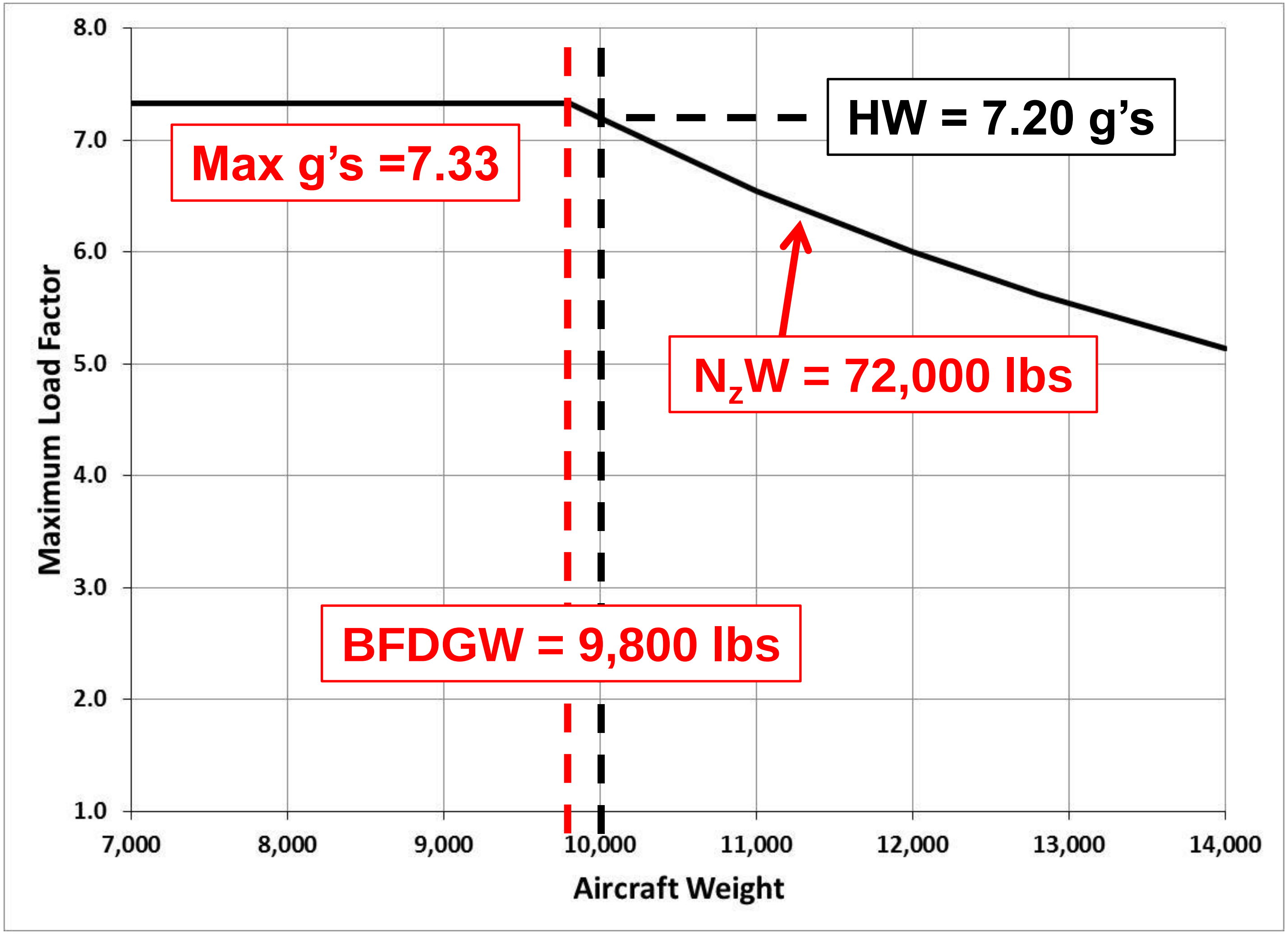


Instantaneous Turn

T-38 V-n



Instantaneous Turn - N_zW



Turning Flight Summary

Turn Radius:

$$R = \frac{mV^2}{L \sin \phi} = \frac{V^2}{g\sqrt{n^2 - 1}}$$

(usually in ft)

Turn Rate:

$$\omega = \frac{V}{R} = \frac{g\sqrt{n^2 - 1}}{V}$$

(calculated as rad/sec, but usually expressed as deg/sec)

Sustained Turn:

$$(n_{max})_{sust} = \sqrt{\frac{q}{K\,(W/S)}\left[\frac{T}{W} - \frac{q\,C_{D_0}}{(W/S)}\right]} = \left(\frac{T}{W}\right)\left(\frac{L}{D}\right)_{max}$$

$$V_{(n_{max})_{sust}} = \left(\frac{2}{\rho}\sqrt{\frac{K}{C_{D_0}}}\,\frac{n\,W}{S}\right)^{1/2}$$

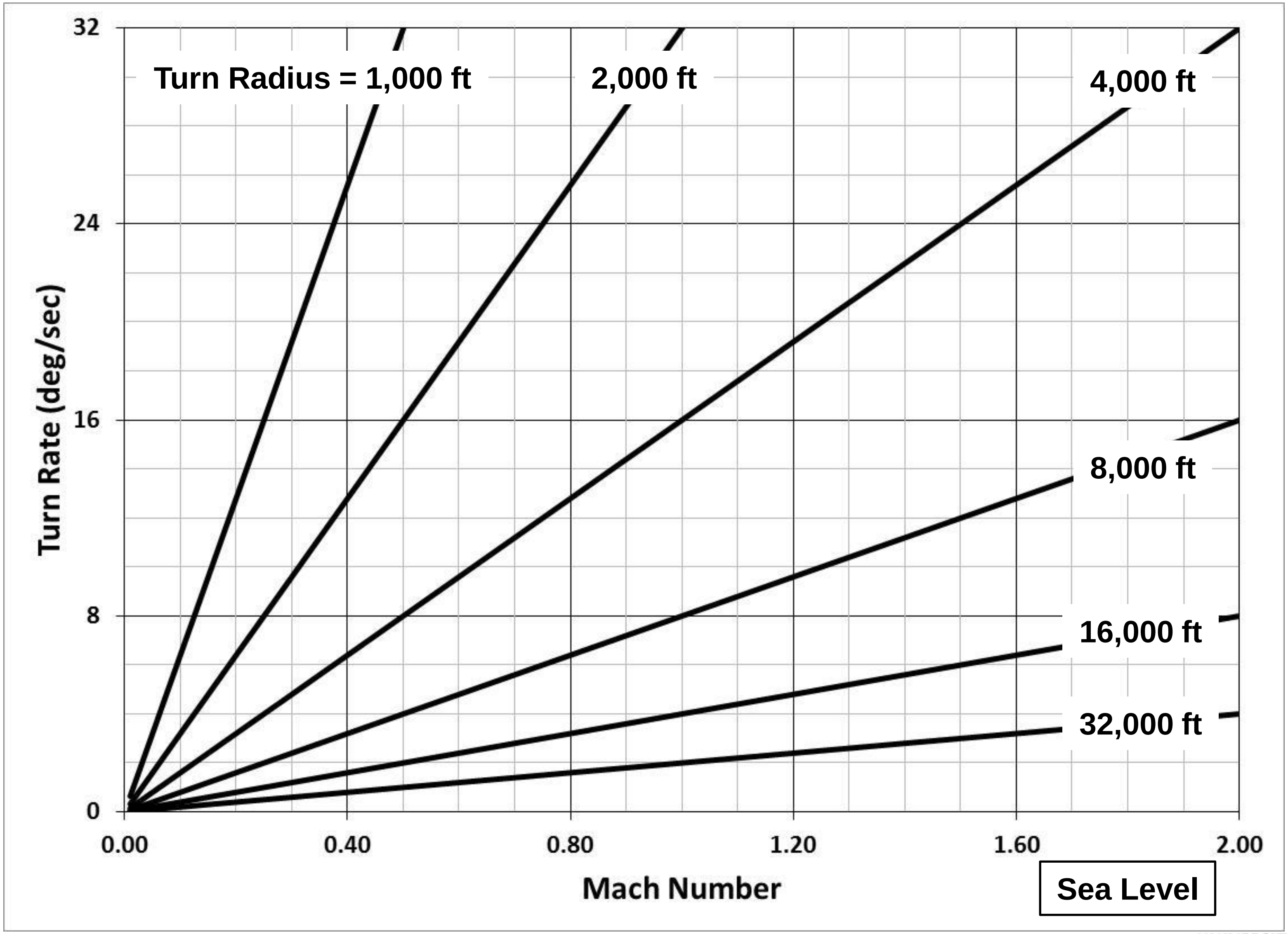
$$(n_{max})_{C_{L_{max}}} = \frac{q\,C_{L_{max}}}{(W/S)}$$

Maneuver Diagram

Plot of Turn Rate vs Velocity at a given altitude

- Establishes the aircraft maneuver boundaries
- Includes max g's, max lift, and max speed
- Includes load factor and turn radius levels
- Shows instantaneous and sustained envelopes
- Shows various P_s contours
- Used by fighter pilots for training

Maneuver Diagram



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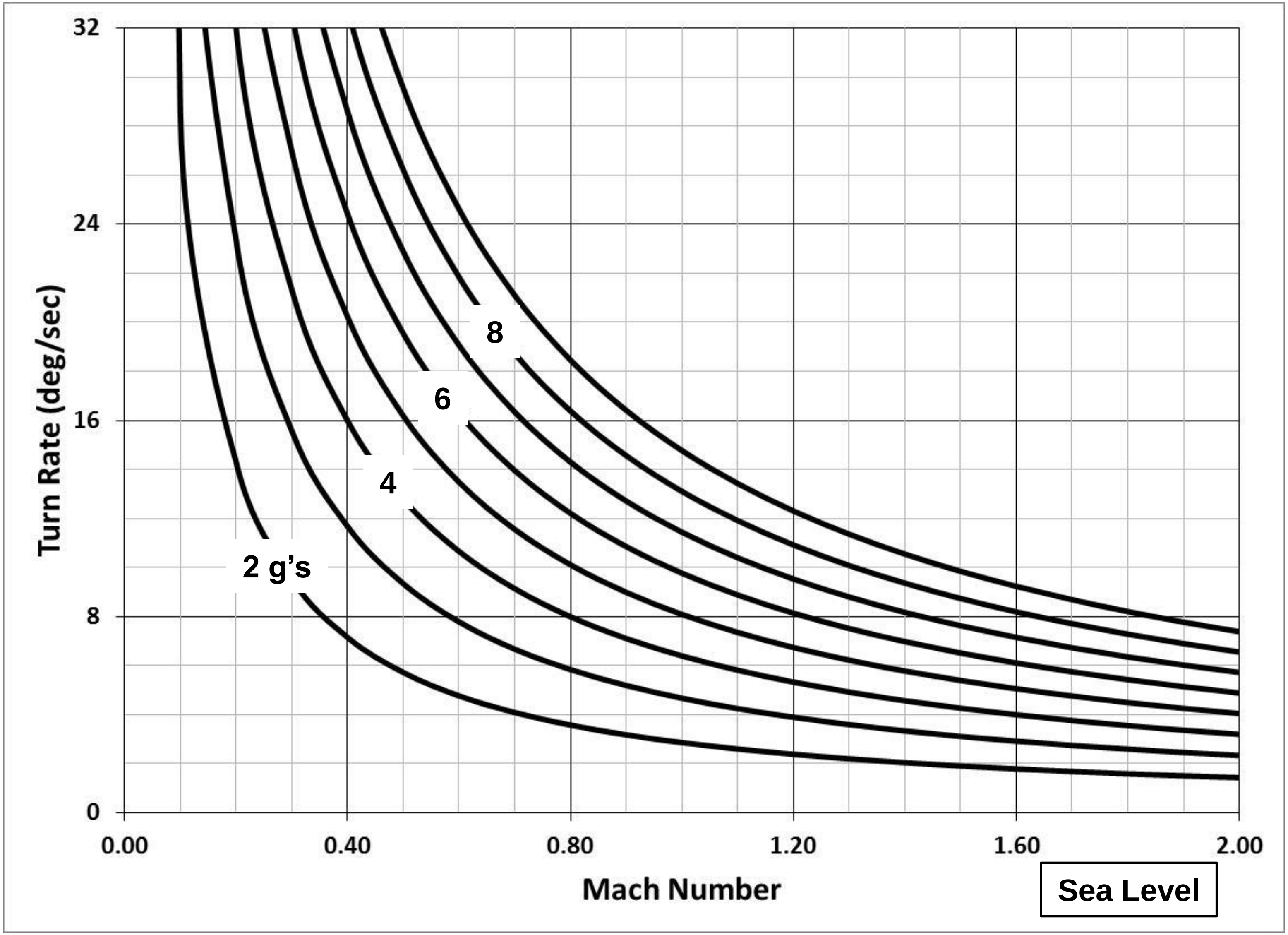
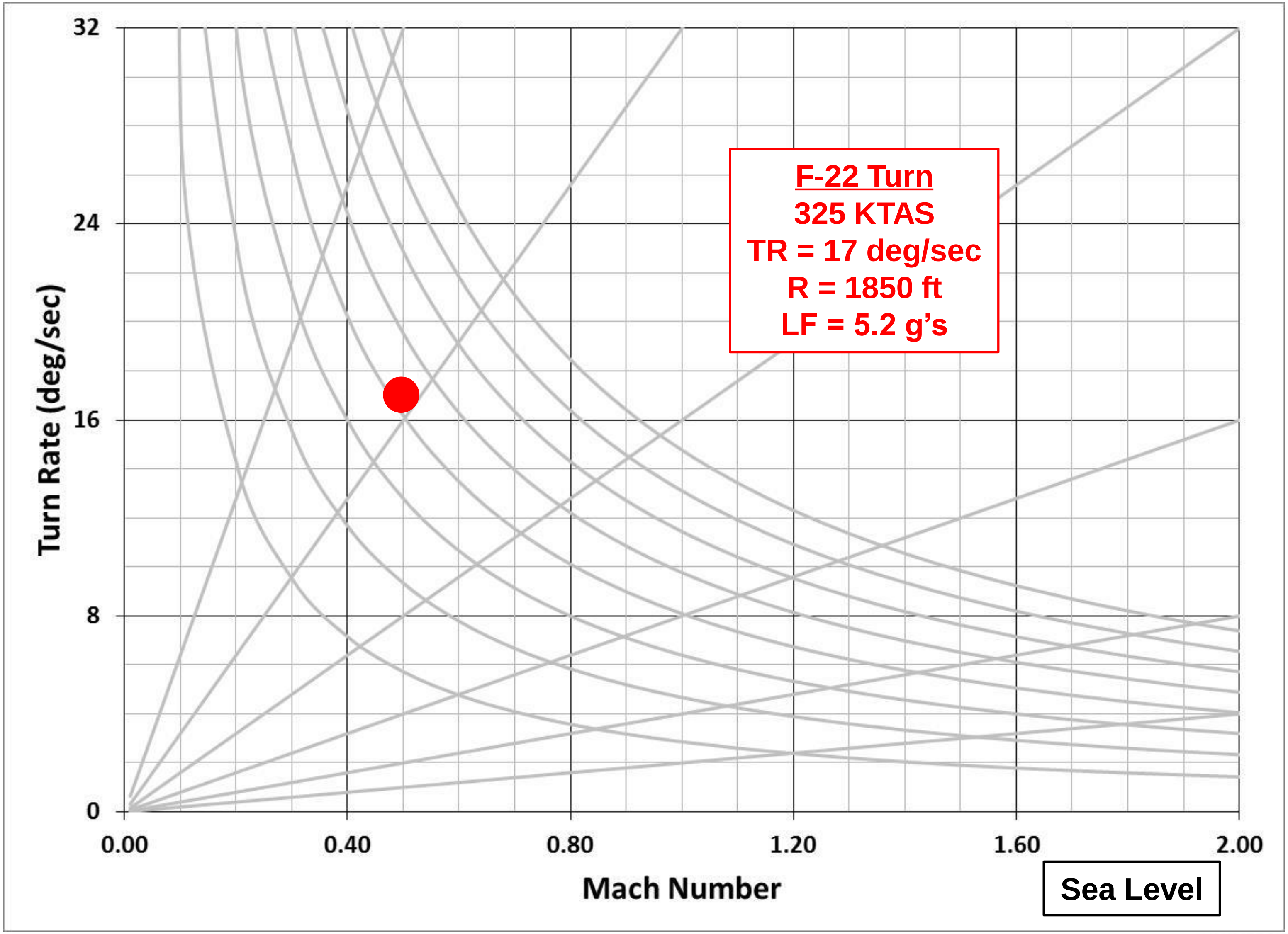
Maneuver Diagram



Maneuver Diagram



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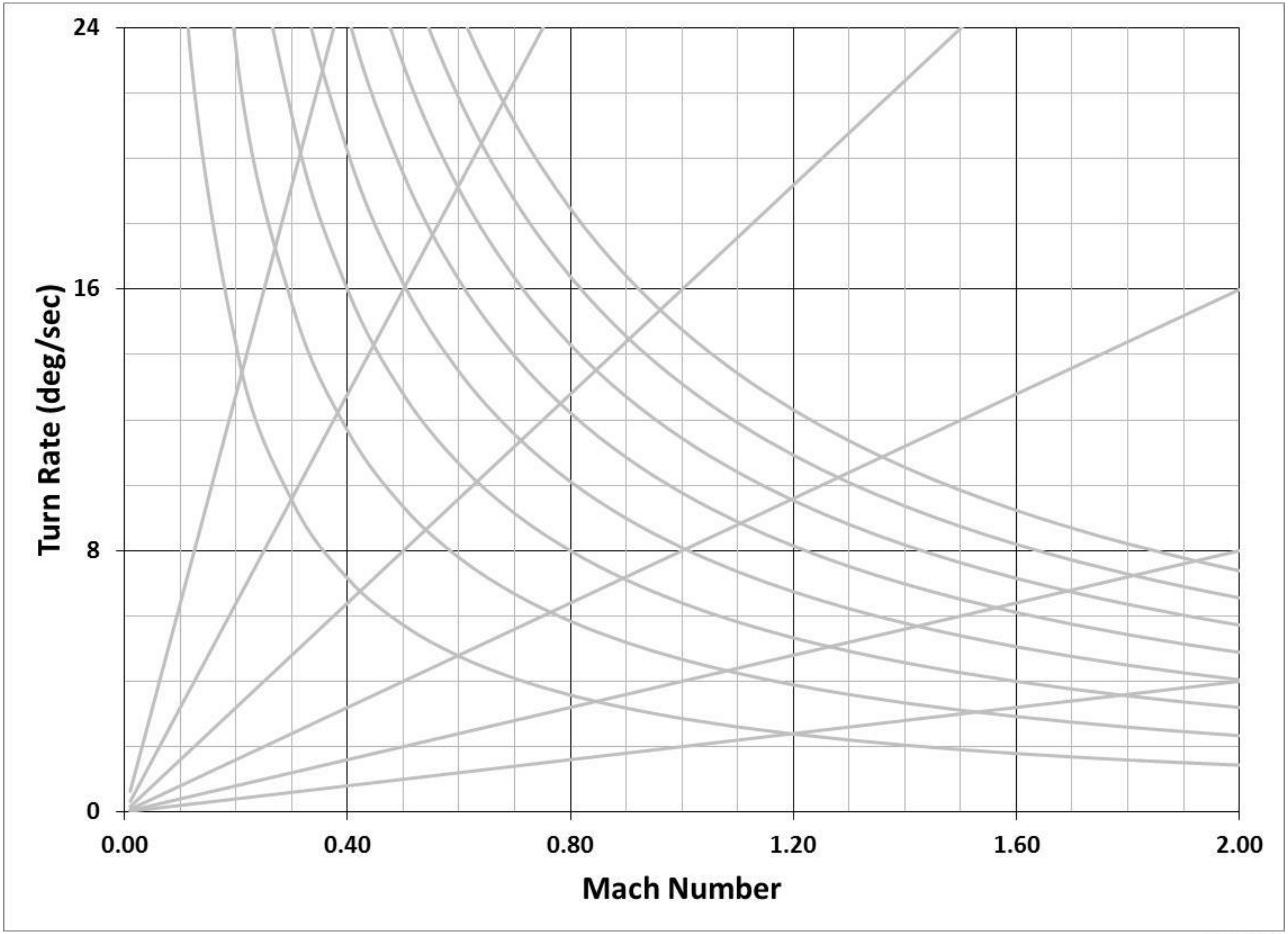
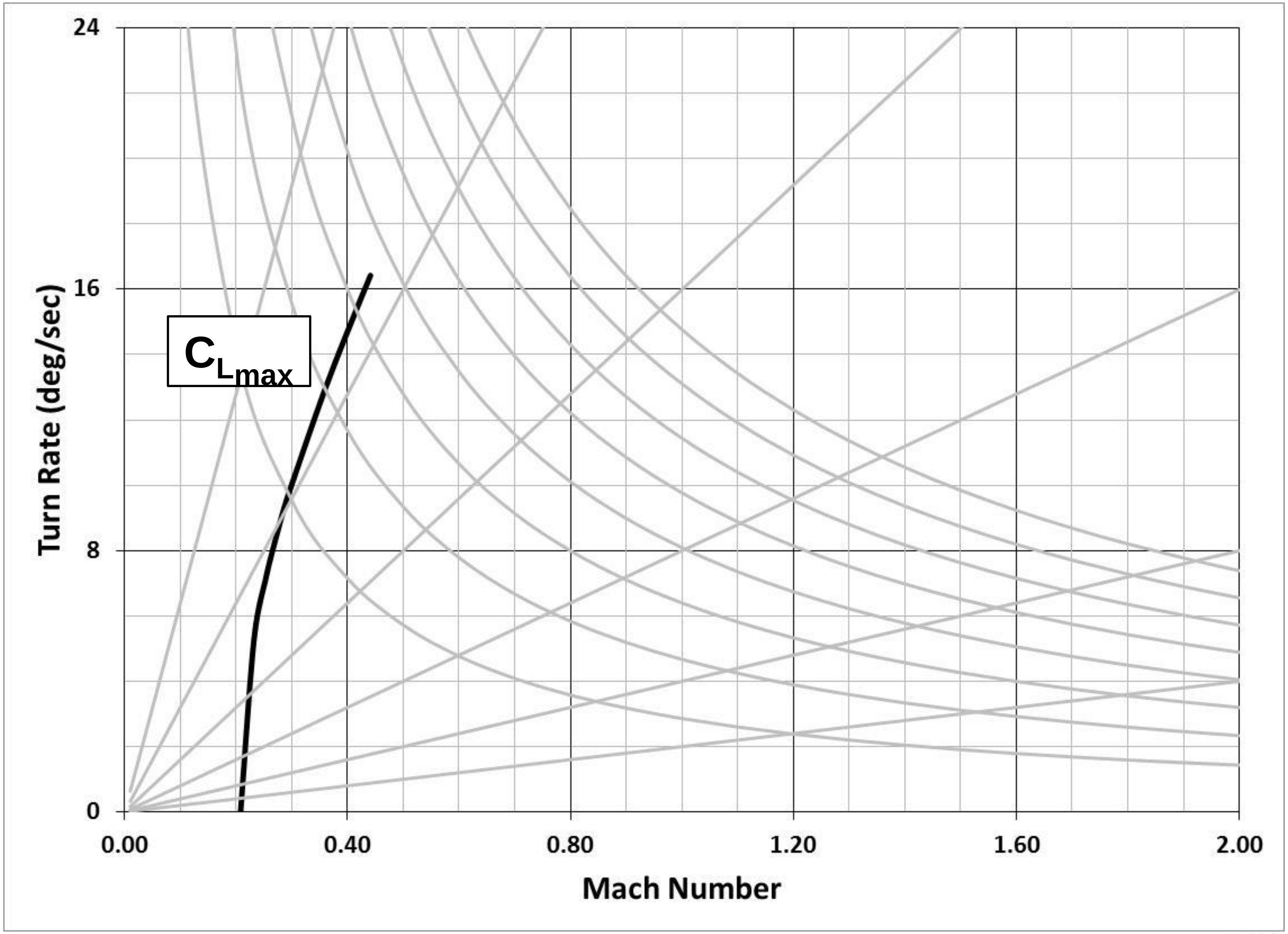
Maneuver Diagram

G-IV Example



Maneuver Diagram



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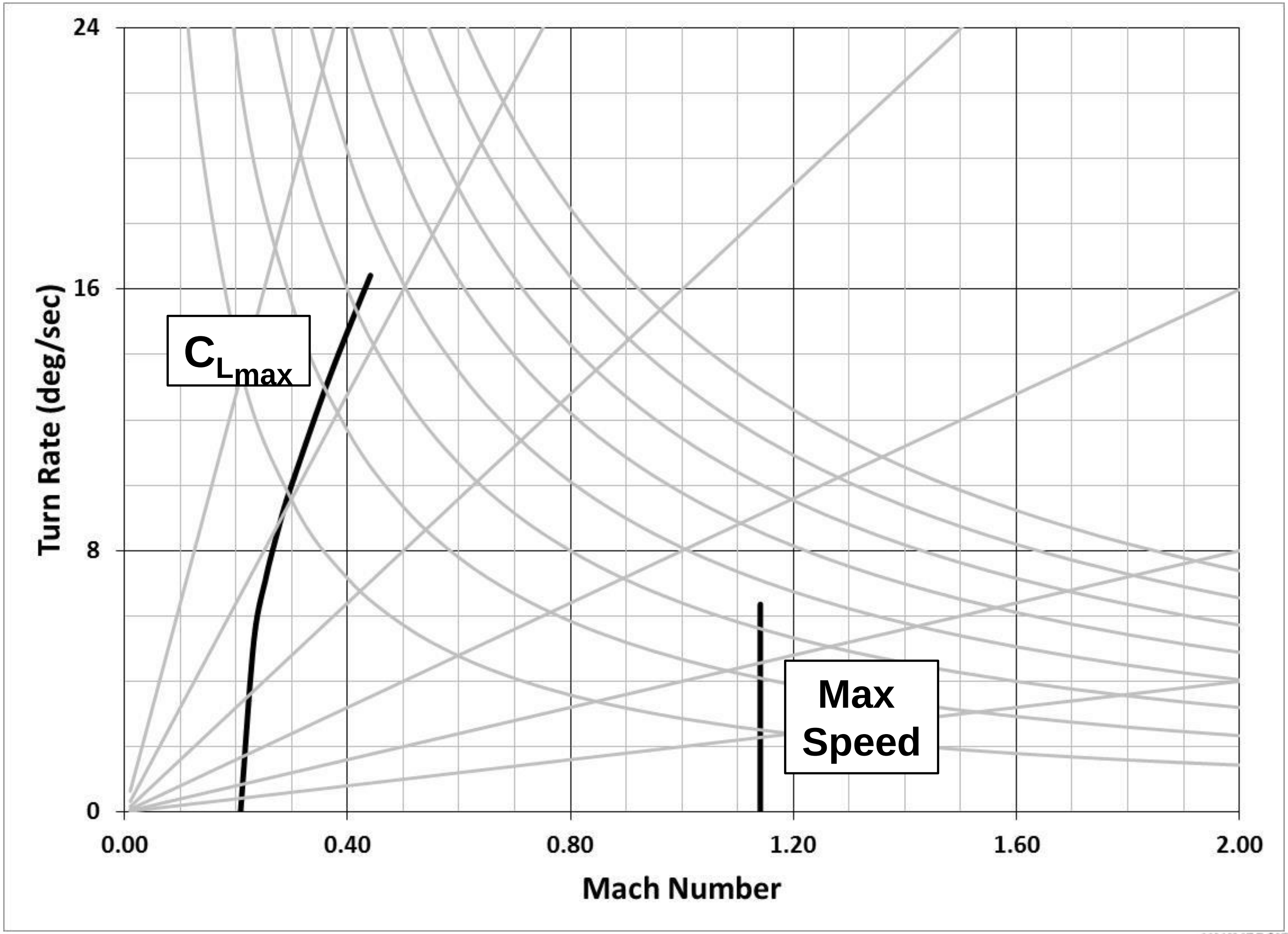
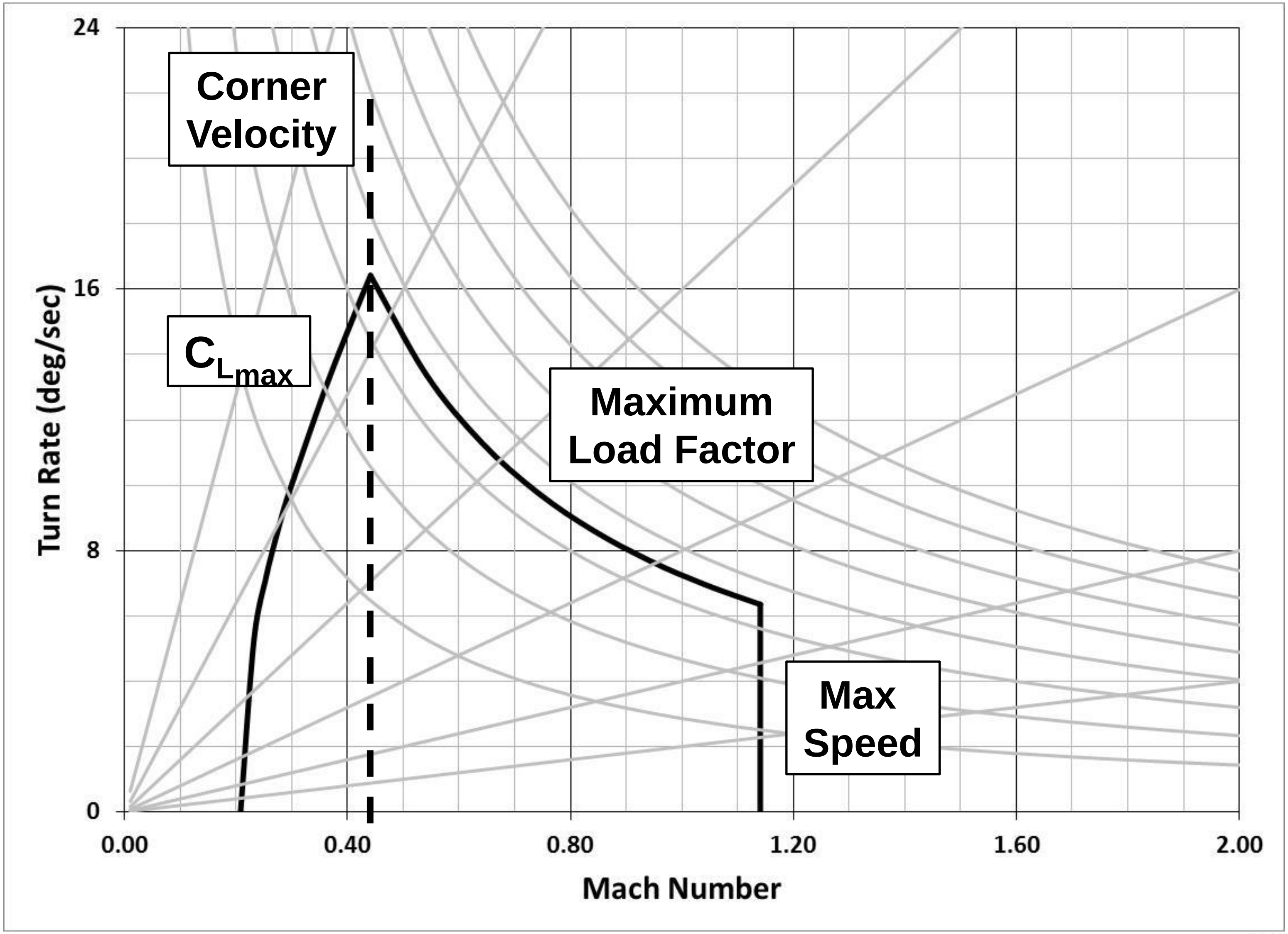
Maneuver Diagram



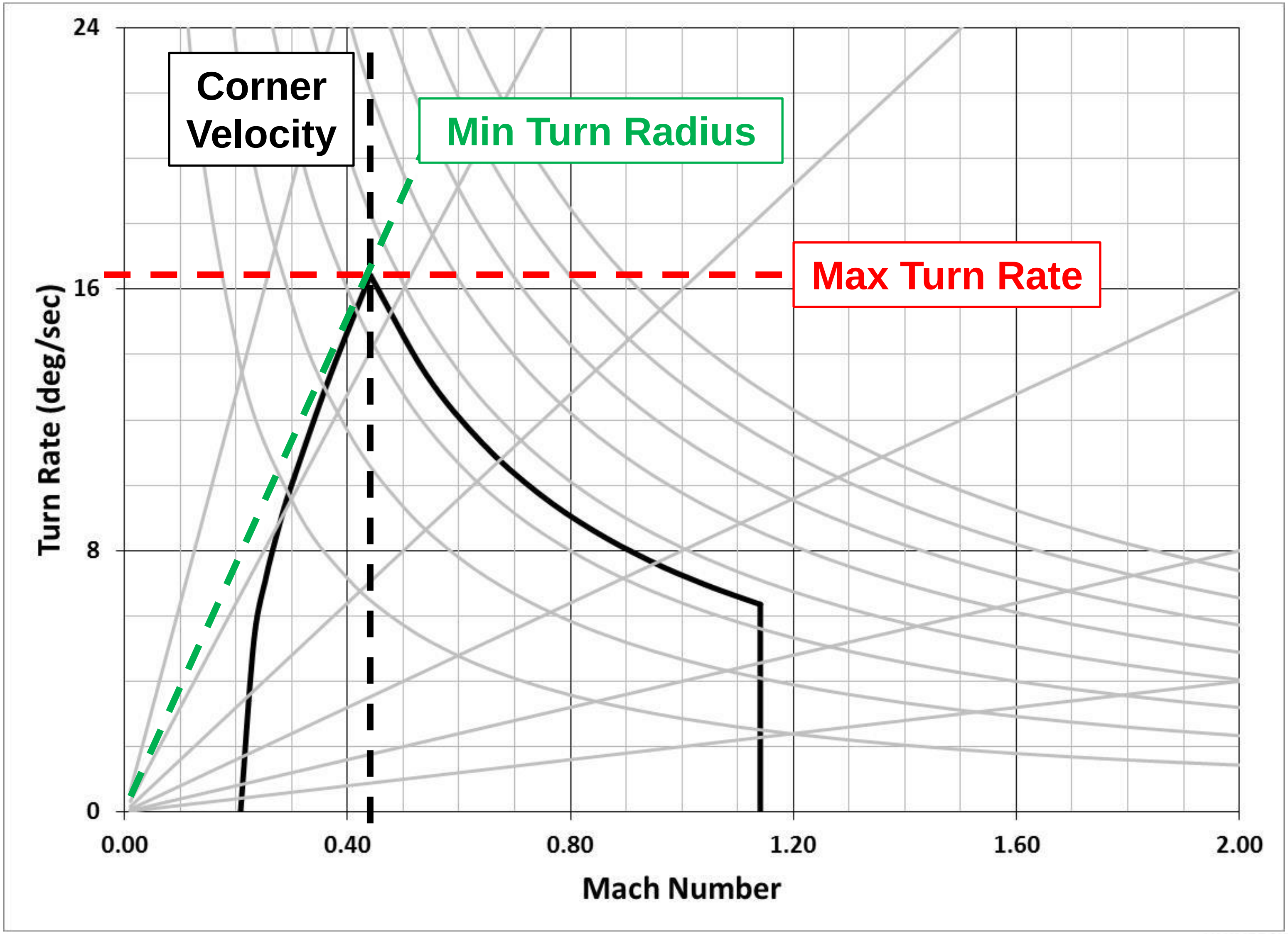
Maneuver Diagram

G-IV Example



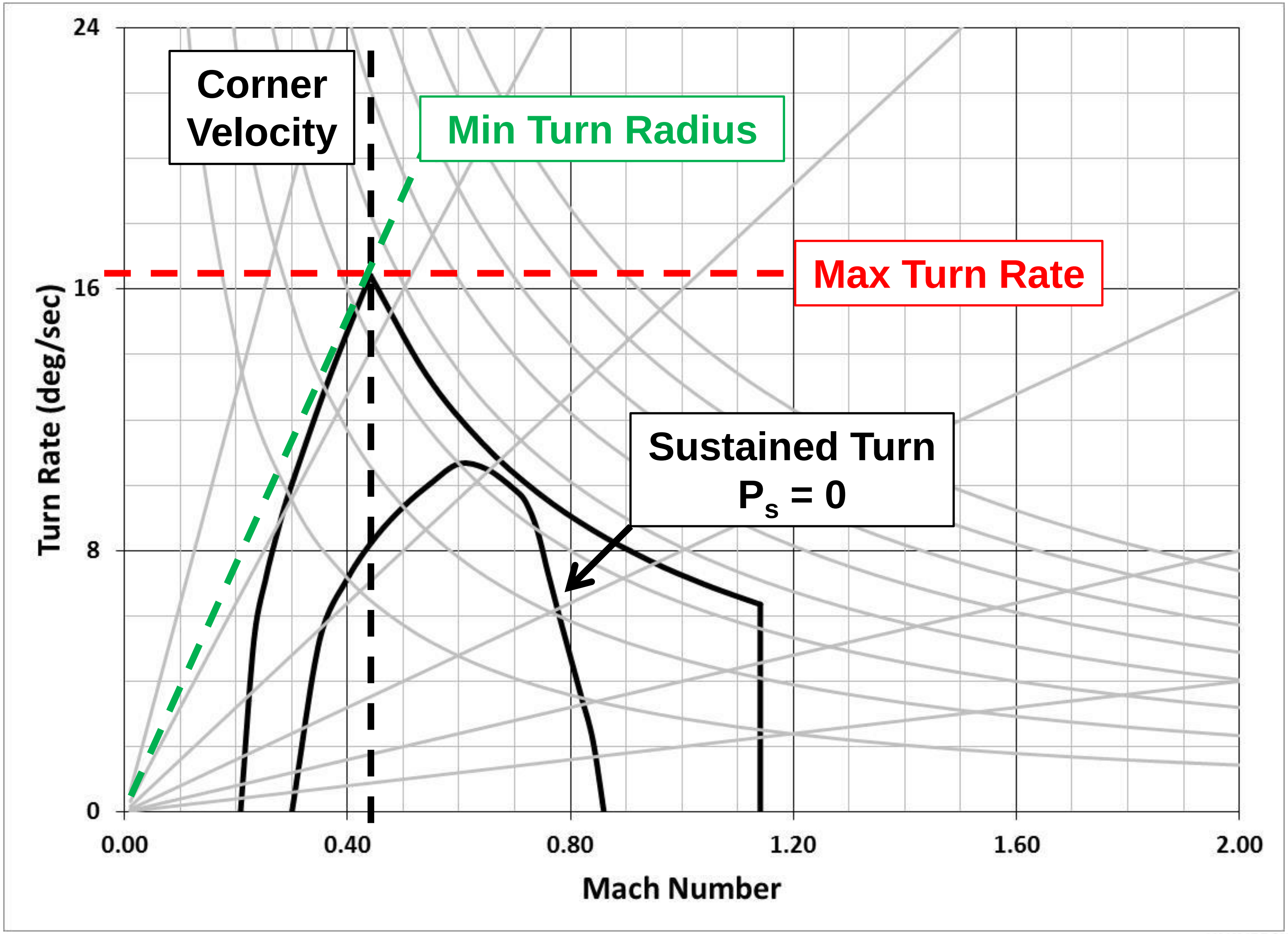
G-IV Example

Maneuver Diagram



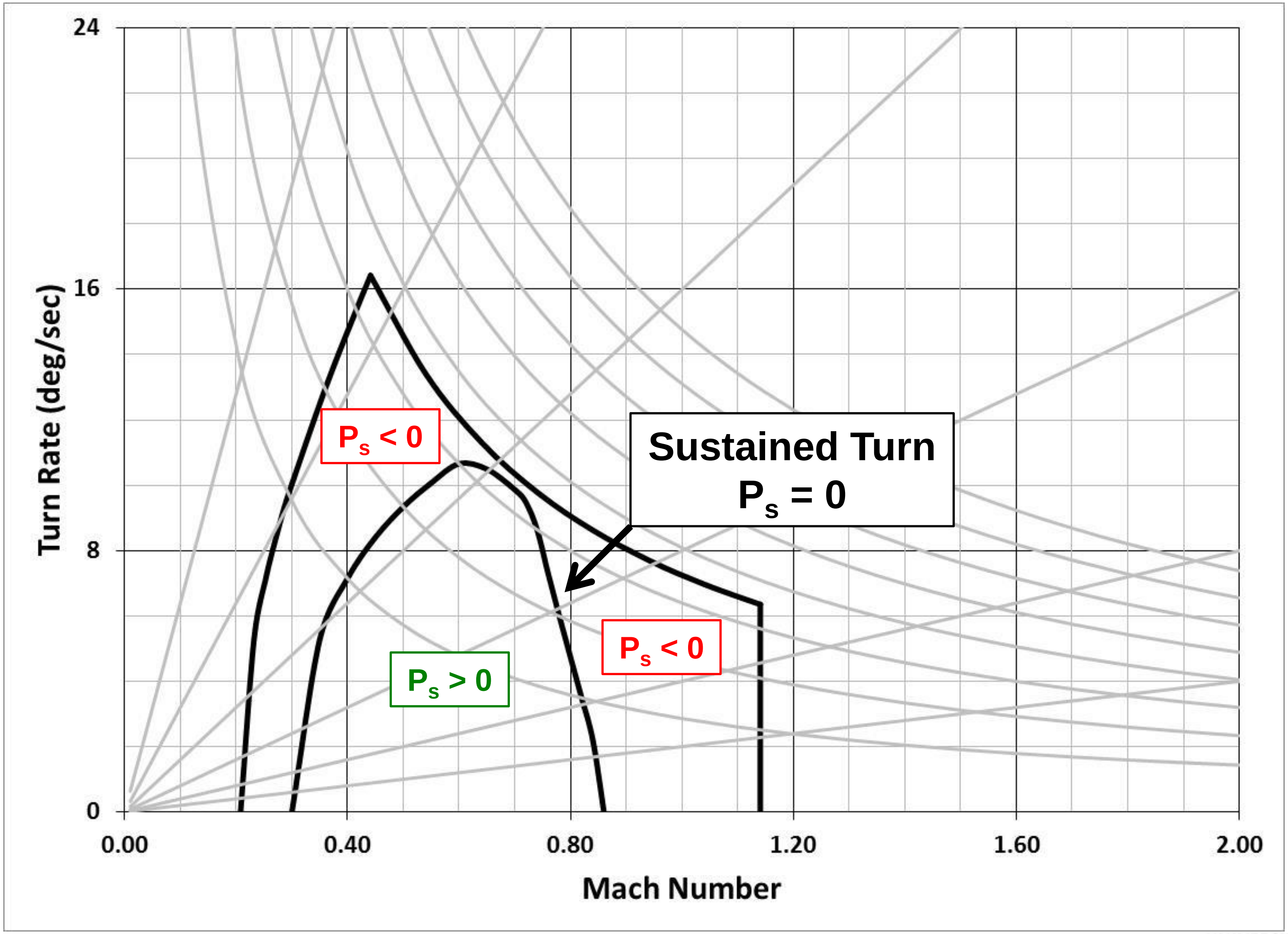
Maneuver Diagram

G-IV Example

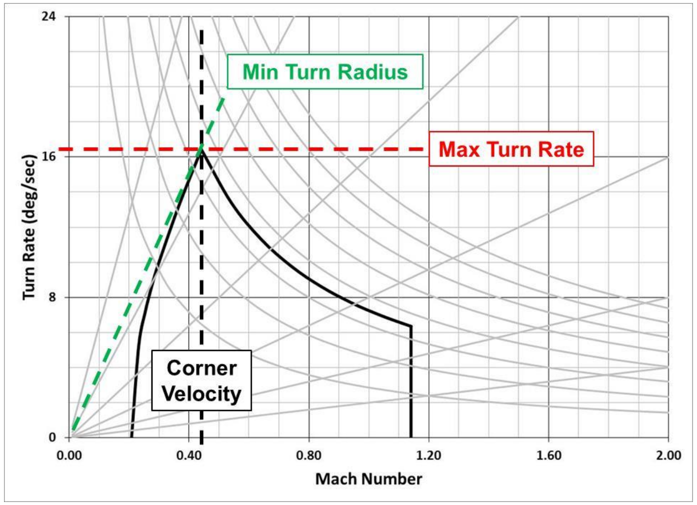


Maneuver Diagram

G-IV Example



Maneuver Calculations



Corner Velocity

$$V_{corner} = \sqrt{\frac{2\, n_{max}\, (W/S)}{\rho\, C_{L_{max}}}}$$

CD0	0.0150	**Wt**	73,000	lb	**CLmax**	1.20	
K	0.08	**Alt**	0	ft	**Vmax**	1.14	Mach
Thrust	27,700	**QMS**	1481.4	lb/sqft	**Max g**	4.5	g's
W/S	76.84	**rho**	0.00237688	slugs/ft^3			
T/W	0.3795	**a**	1116.45	ft/sec			
		S	950	sq ft			

Maximum Turn Rate & Minimum Turn Radius

Corner Velocity	492.4	ft/sec
	0.4410	Mach
Max Turn Rate	16.43	deg/sec
Min Turn Radius	1718	ft

Turn Rate

$$\omega = \frac{g\sqrt{n^2 - 1}}{V}$$

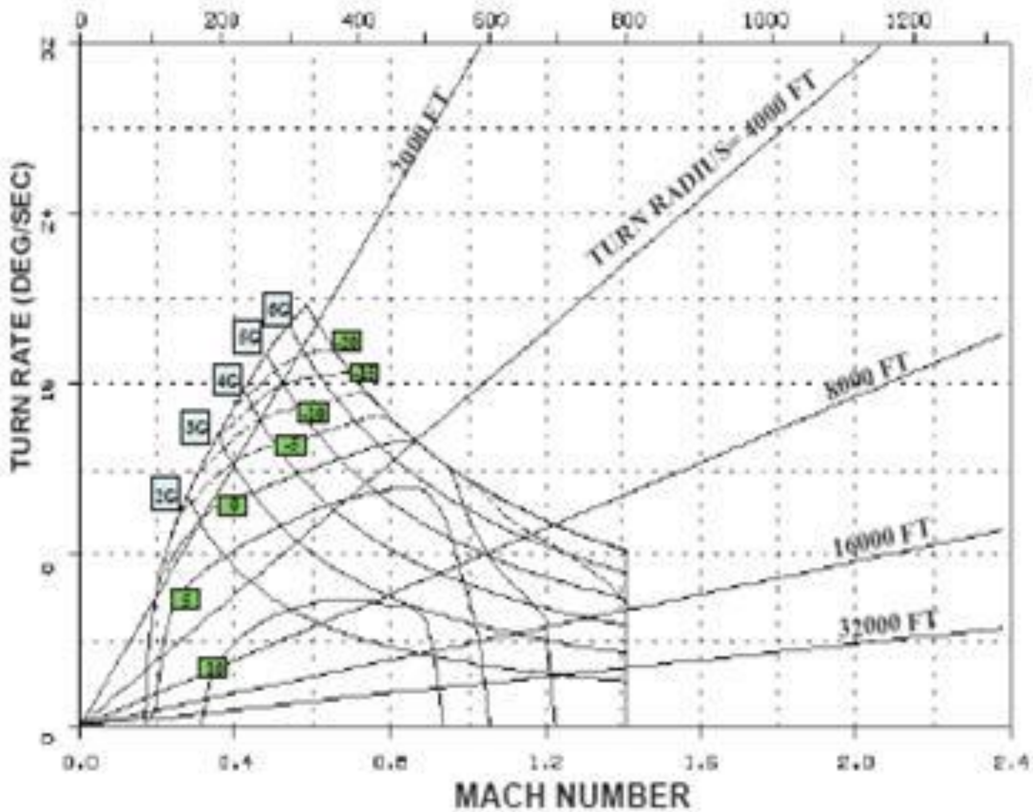
Turn Radius

$$R = \frac{V^2}{g\sqrt{n^2 - 1}}$$

F- 15A - 100

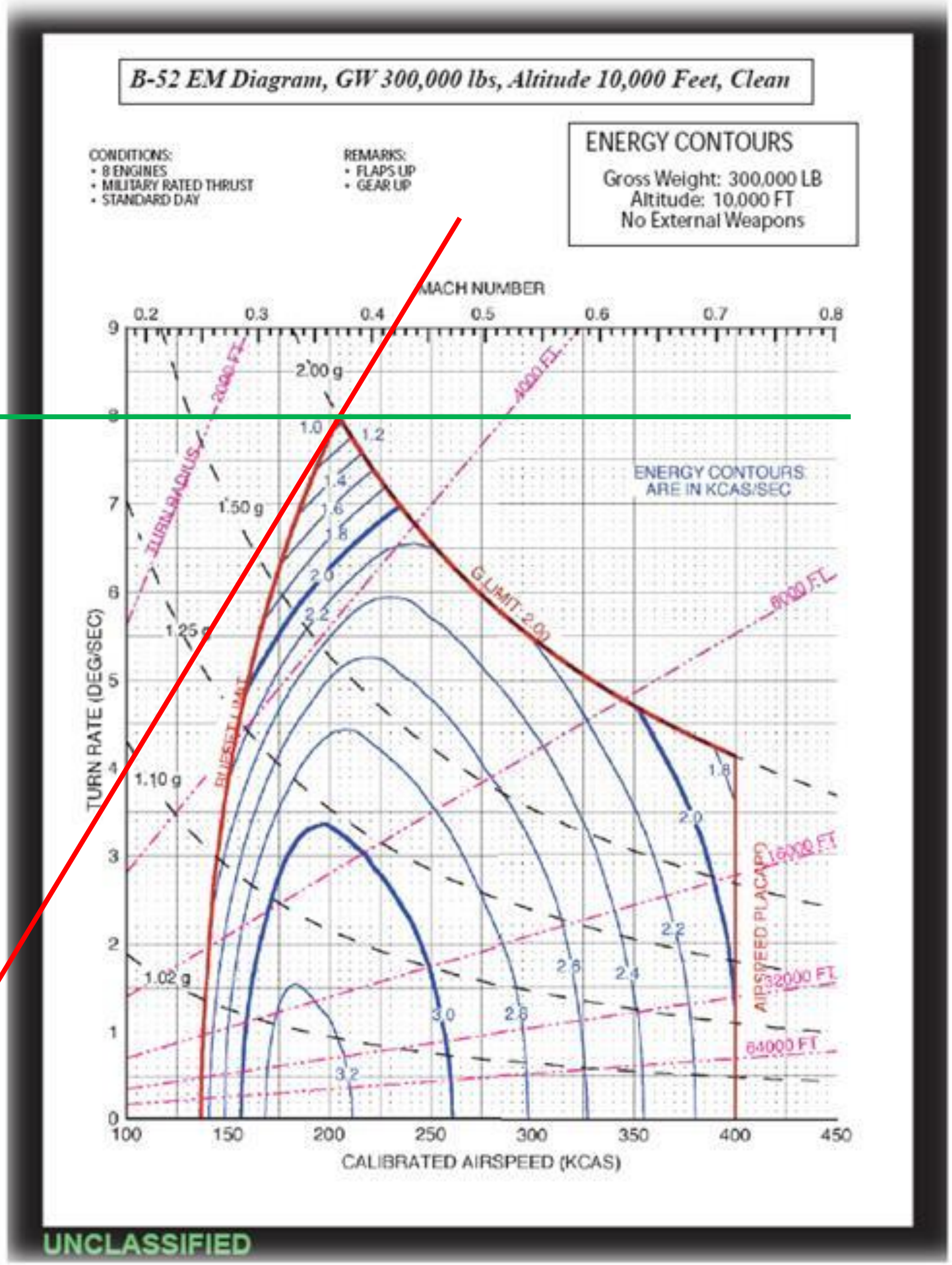
TURN RATE VS MACH

(Contours in KCAS/sec)



ALT = 10000.
WEIGHT = 40520.

CONFIGURATION
75% Internal Fuel
2 Aim-9
2 Aim-120
1 Aim-7
CL Pylon, Wing Pylons
4 LAU-128
Vmax On



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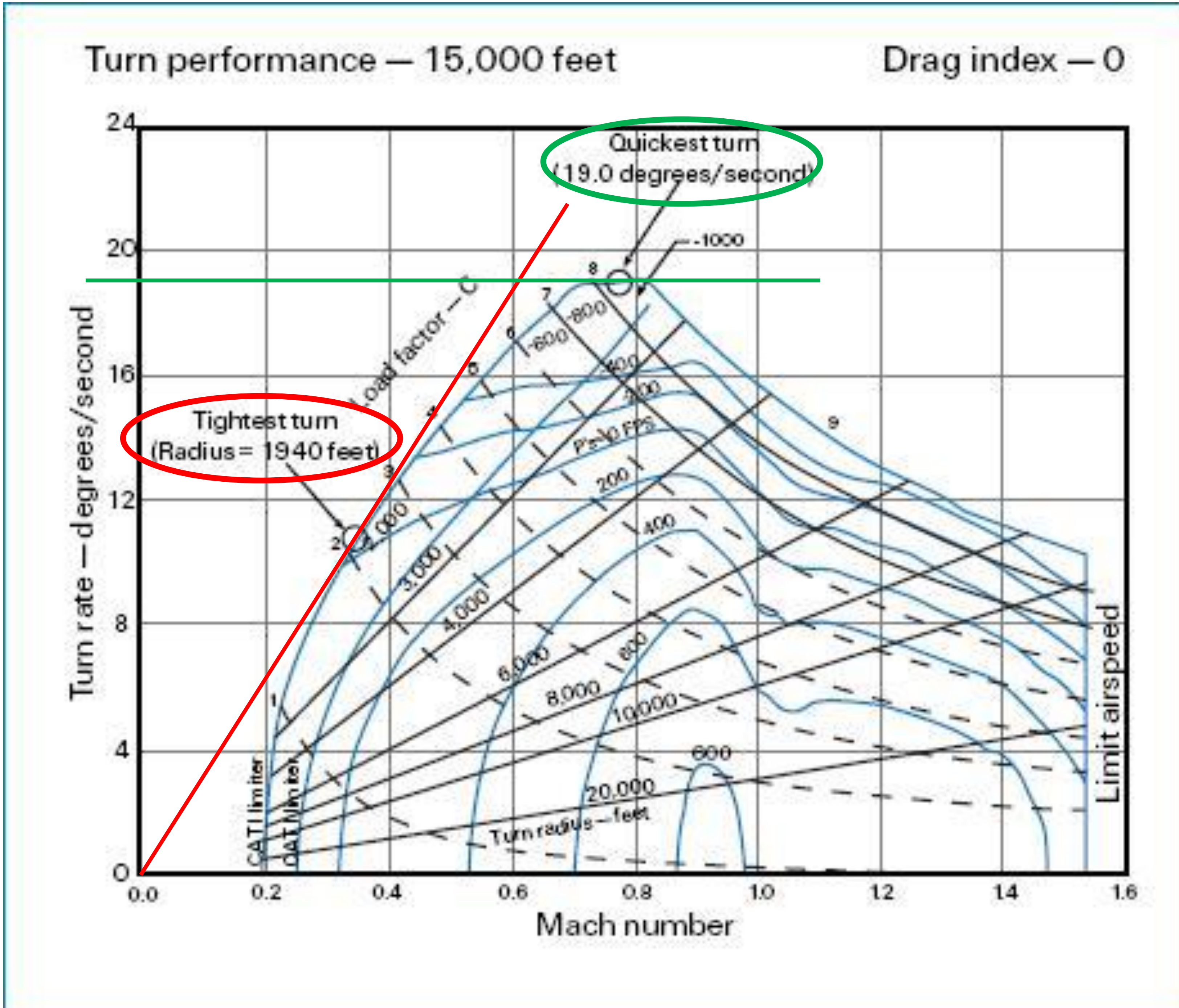

Turn performance — 15,000 feet
Drag index — 0
Quickest turn
(19.0 degrees/second)
Tightest turn
(Radius = 1940 feet)
Load factor — G
Turn rate — degrees/second
Mach number
Limit airspeed
Turn radius — feet
CATI limiter
OAT limiter
Ps=0 FPS
-1000
-800
-600
-400
-200
200
400
600
2,000
3,000
4,000
6,000
8,000
10,000
20,000
0
4
8
12
16
20
24
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6

Homework

Sustained Turn: $P_s = 0$

$$(n_{max})_{sust} = \sqrt{\frac{q}{K\,(W/S)}\left[\frac{T}{W} - \frac{q\,C_{D_0}}{(W/S)}\right]} = \left(\frac{T}{W}\right)\left(\frac{L}{D}\right)_{max}$$

$$V_{(n_{max})_{sust}} = \left(\frac{2}{\rho}\,\sqrt{\frac{K}{C_{D_0}}}\,\frac{n\,W}{S}\right)^{1/2}$$

$$\phi = \arccos\left(\frac{1}{n}\right)$$

Turn Radius:

$$R = \frac{V^2}{g\,\sqrt{n^2 - 1}}$$

Turn Rate:

$$\omega = \frac{V}{R} = \frac{g\,\sqrt{n^2 - 1}}{V}$$

Homework

Instantaneous Turn: $P_s \neq 0$

Corner Velocity: speed for highest turn rate
slowest speed for max g

$$V_{corner} = \sqrt{\frac{2\, n_{max}\, (W/S)}{\rho\, C_{L_{max}}}} \qquad \phi = \arccos\left(\frac{1}{n_{max}}\right)$$

Turn Radius: $$R = \frac{V^2}{g\sqrt{n^2 - 1}}$$

Turn Rate: $$\omega = \frac{V}{R} = \frac{g\sqrt{n^2 - 1}}{V}$$

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Homework

To calculate P_s at a given flight condition (M, h, g's):

1. **Calculate thrust** $\quad T_A = T_{SL}\left(\frac{\rho}{\rho_{SL}}\right)$
2. **Calculate drag** $\quad C_L = \frac{nW}{qS} \qquad C_D = C_{D_0} + K\,{C_L}^2 \qquad D = C_D\, q\, S$
3. **Calculate velocity (ft/sec)**

$$P_s = \frac{(T - D)\,V}{W}$$

Homework Assignment

<u>HW #14 – Maneuver</u>
(due by 11:59 pm ET on Monday, March 6)
Reading – Chapters 6.1 - 6.4

<u>HW Help Session</u>
Monday 4:00 – 5:00 pm ET

<u>Posted on Canvas</u>
HW #14 Assignment with instructions, tips, and checklist
HW #14 Template for data table in Excel

Questions?