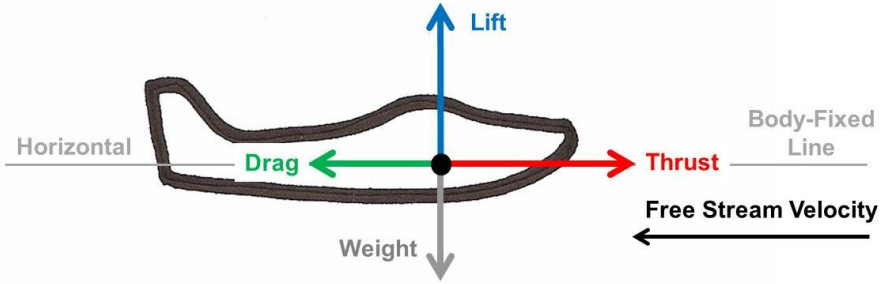
Aircraft Performance
Equations of Motion
Range & Endurance

Steady Flight



$$T = D$$

$$L = W$$

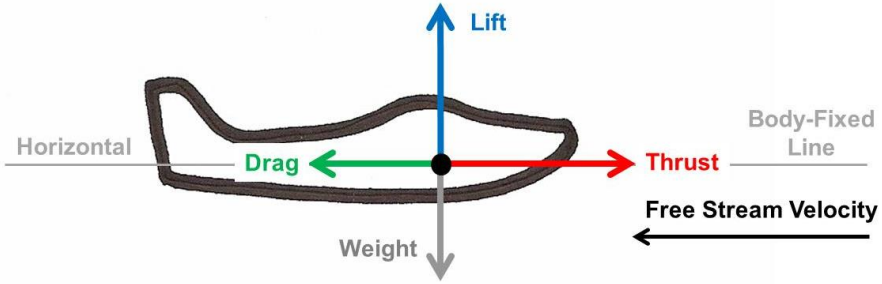
Range = How <u>FAR</u> can an aircraft fly?

Fuel Consumed per Mile Flown (lb_{fuel} / NM)

<u>Jet Aircraft</u>

$$\frac{lb_{fuel}}{NM} \propto c_t \frac{T_{req}}{V}$$

Steady Flight



$$T = D$$

$$L = W$$

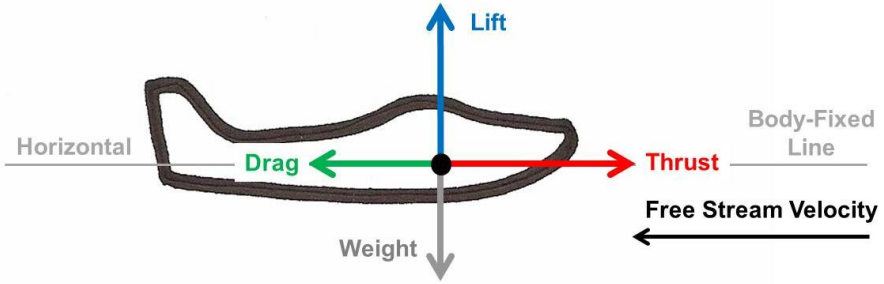
Endurance = How LONG can an aircraft fly?

Fuel Consumed per Hour (lb_{fuel} / hr)

Jet Aircraft

$$\frac{lb_{fuel}}{hr} \propto c_t \, T_{req}$$

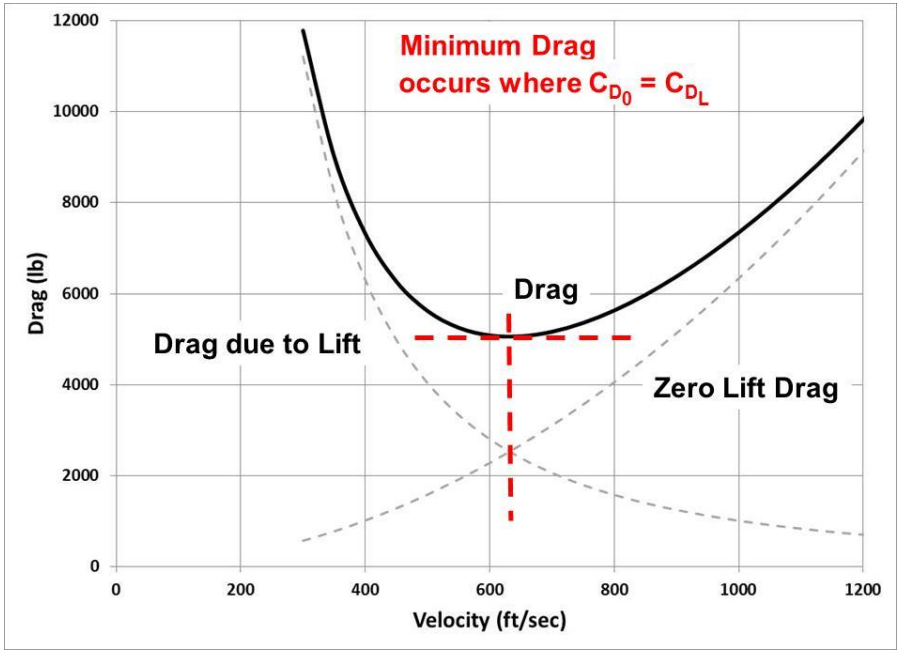
Steady Flight



$$T = D$$

$$L = W$$

Thrust Required
Minimum Drag
Velocity for D_{min}
Velocity for $(L/D)_{max}$



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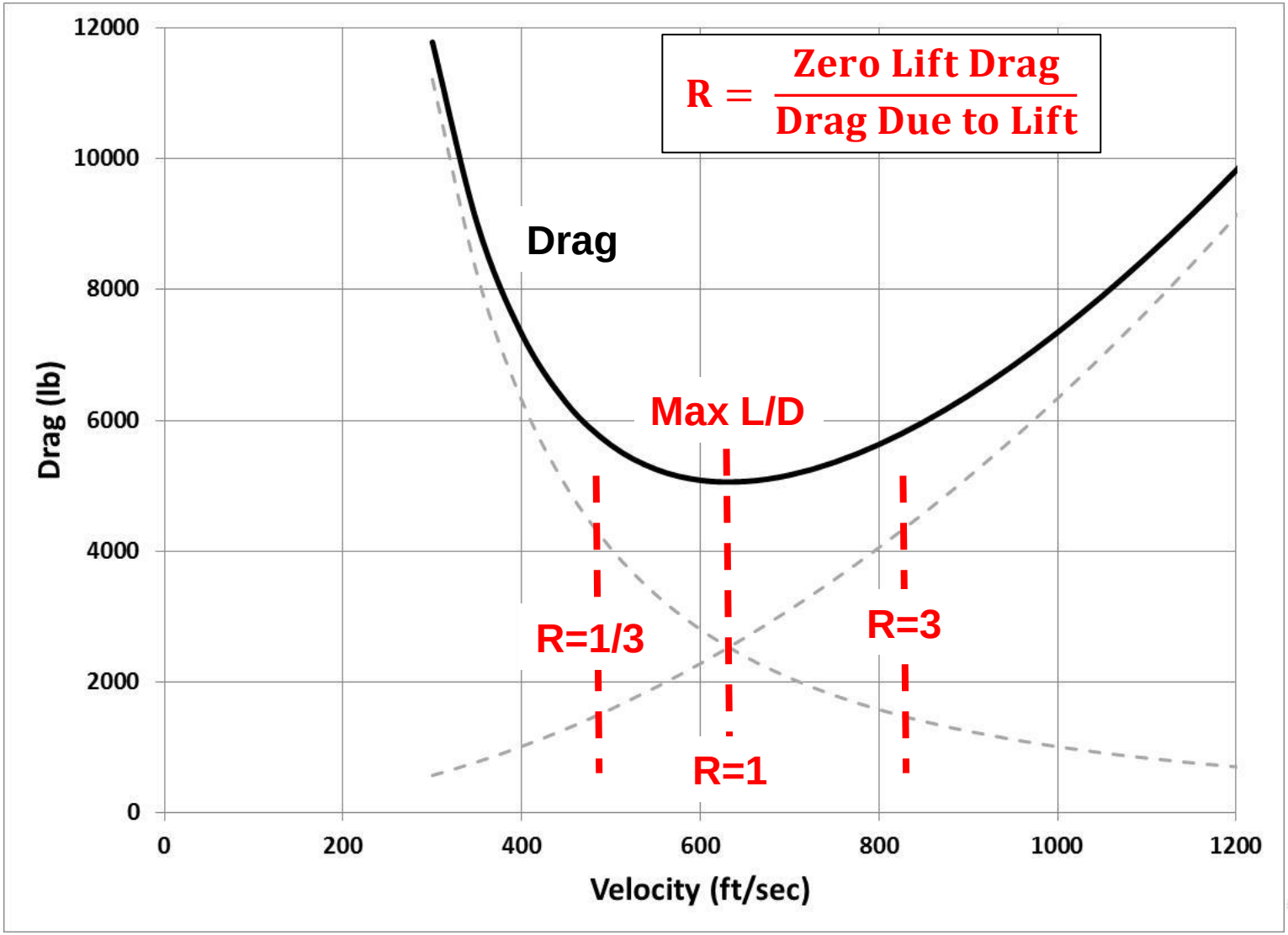
Steady Flight



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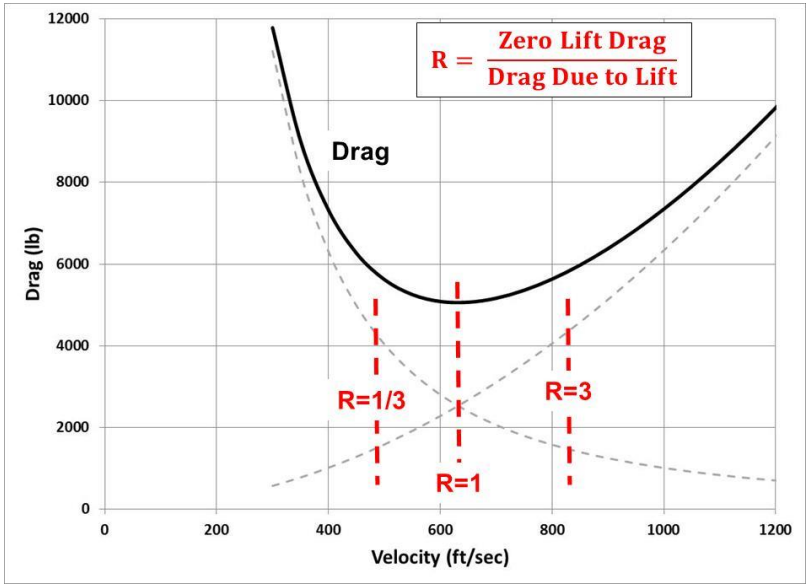
Steady Flight



$$R = \frac{\text{Zero Lift Drag}}{\text{Drag Due to Lift}}$$

$$R = 1 \qquad C_{D_0} = C_{D_L} = K\,C_L^{\,2} \qquad \text{Max } \frac{C_L}{C_D}$$

$$R = 3 \qquad C_{D_0} = 3\,C_{D_L} = 3\,K\,C_L^{\,2} \qquad \text{Max } \frac{C_L^{\,1/2}}{C_D}$$

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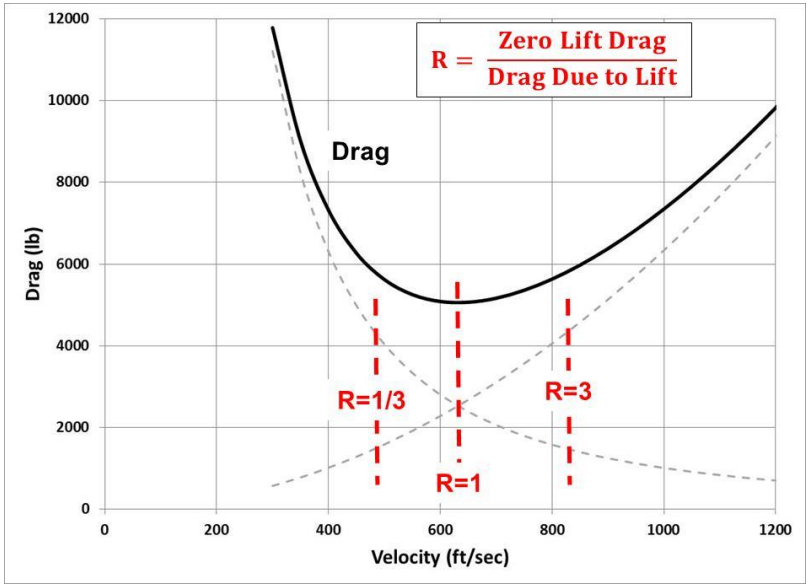
Steady Flight



$$R = \frac{\text{Zero Lift Drag}}{\text{Drag Due to Lift}}$$

$$R = 1 \qquad C_{D_0} = C_{D_L} = K\,C_L{}^2$$

$$\text{Max}\ \frac{C_L}{C_D} = \sqrt{\frac{1}{4\,C_{D_0}\,K}}$$

$$R = 3 \qquad C_{D_0} = 3\,C_{D_L} = 3\,K\,C_L{}^2$$

$$\text{Max}\ \frac{C_L{}^{1/2}}{C_D} = \frac{3}{4}\left(\frac{1}{3\,K\,C_{D_0}{}^3}\right)^{1/4}$$

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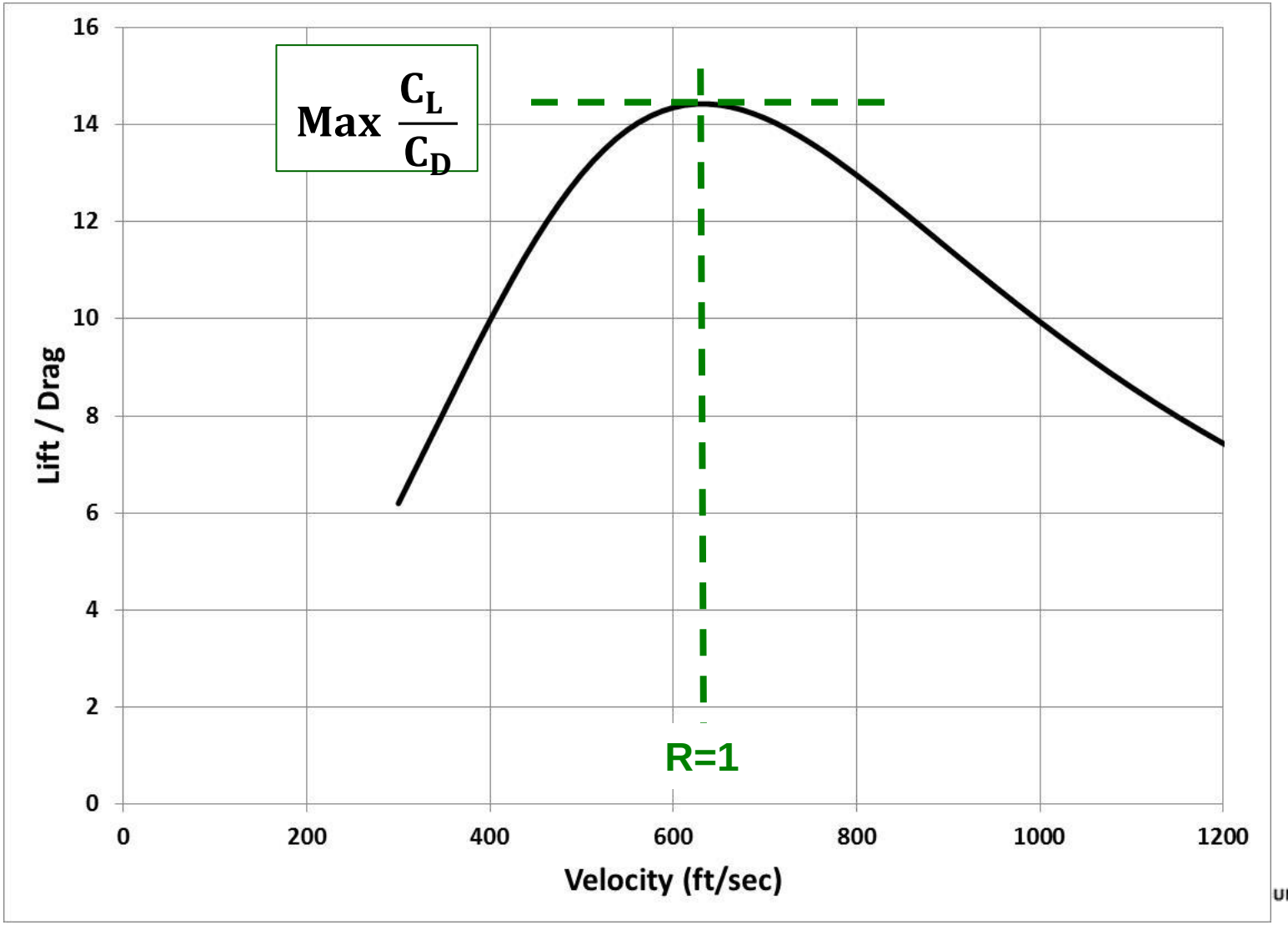
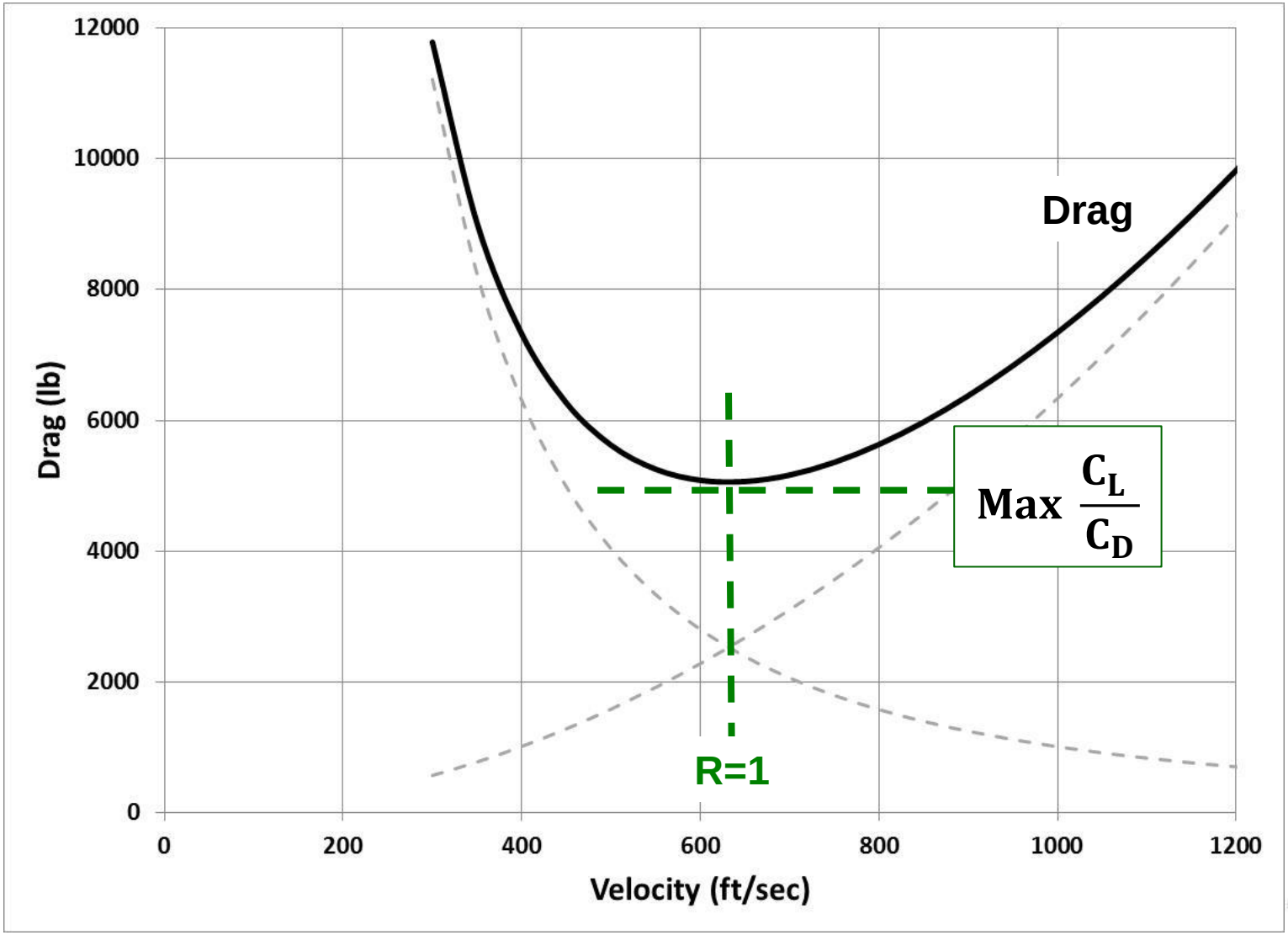
Steady Flight



Steady Flight



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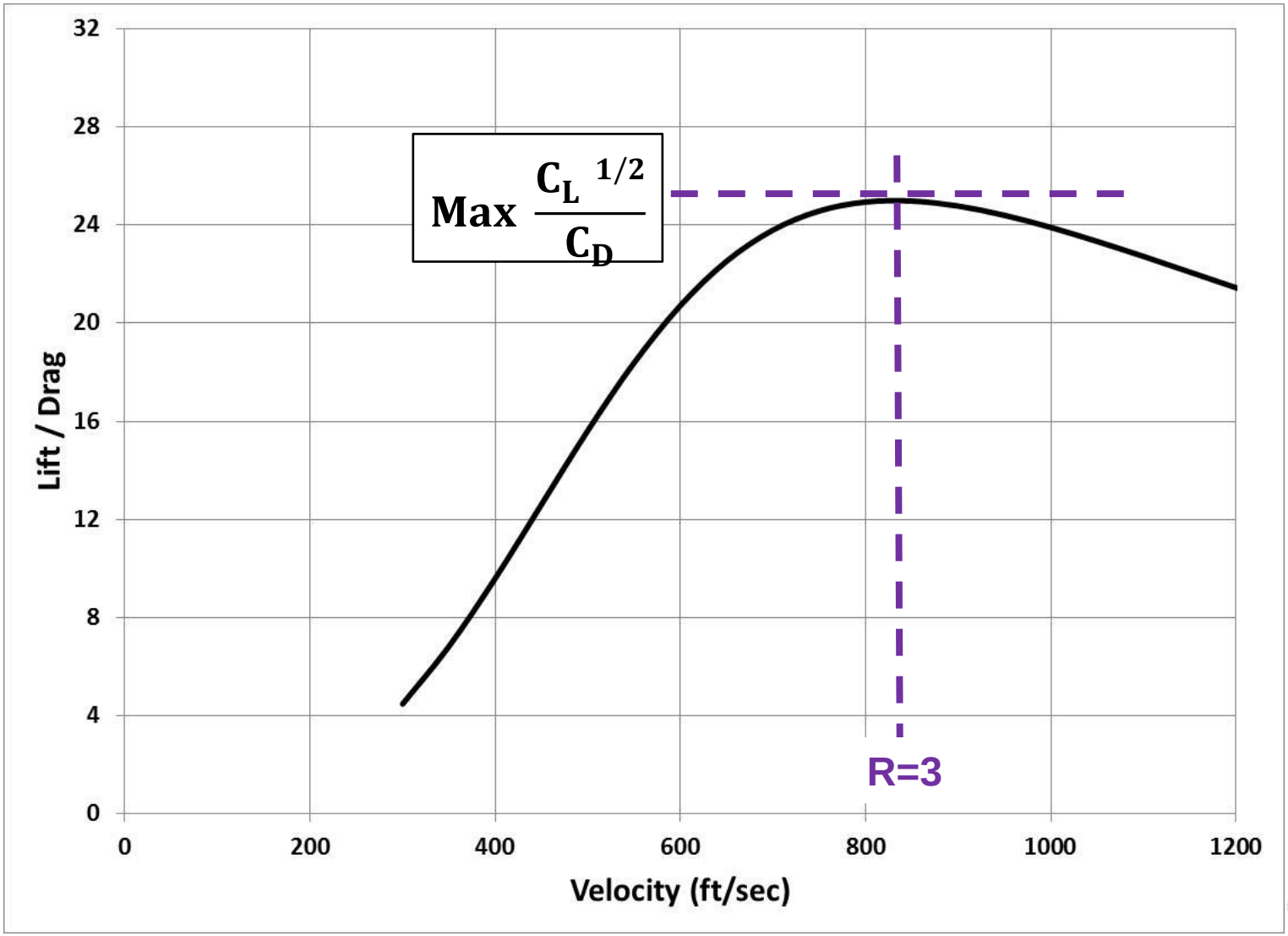
Steady Flight



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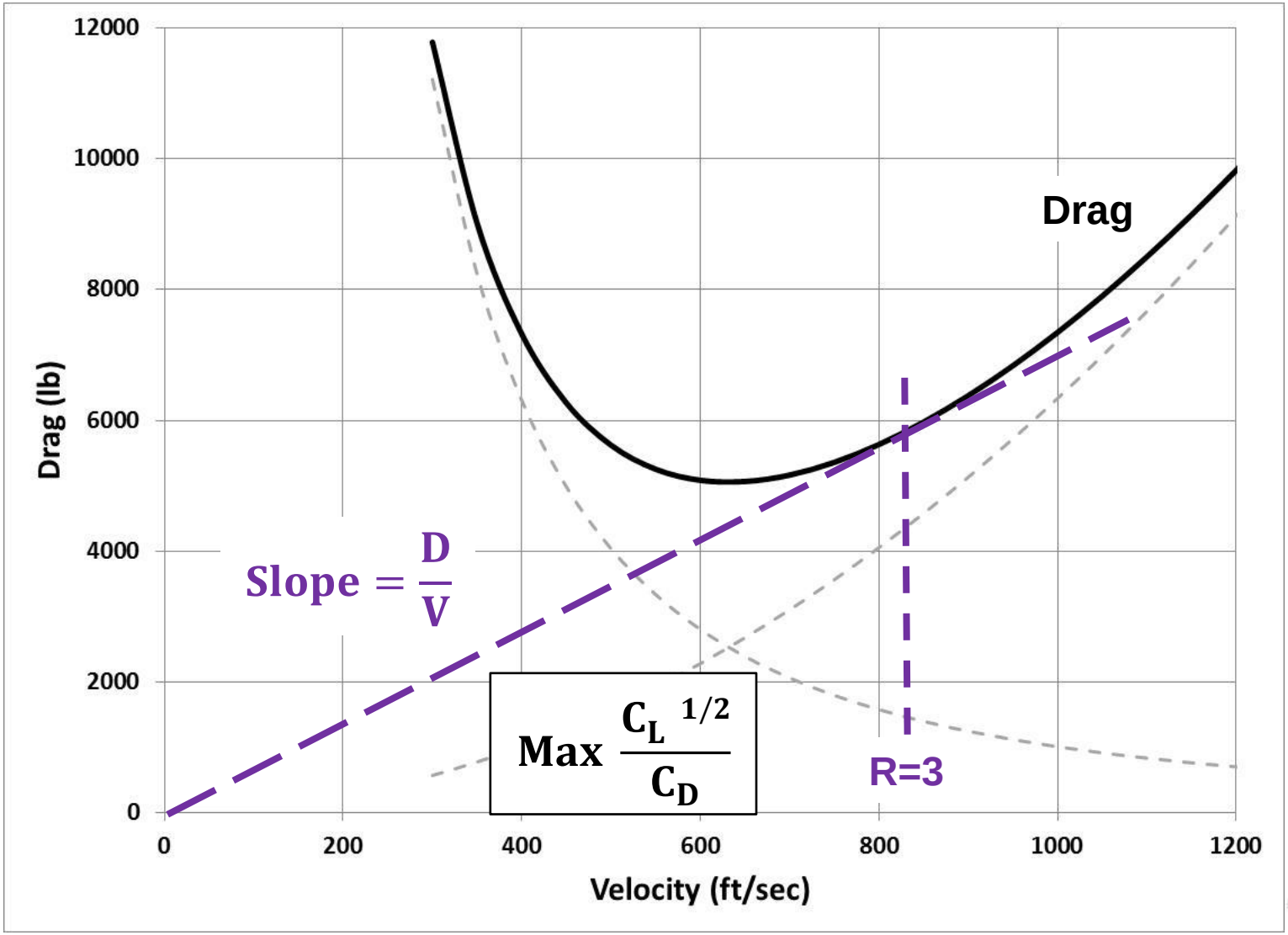
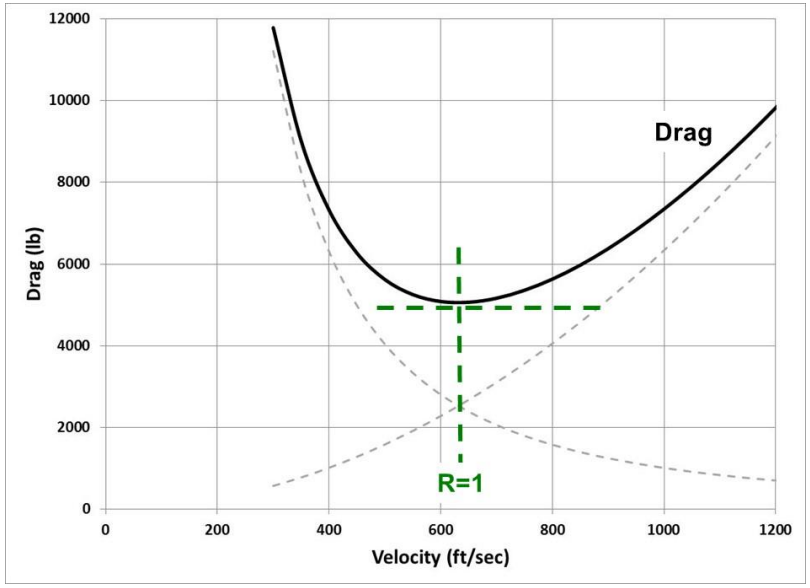
Steady Flight



Steady Flight



Thrust Required

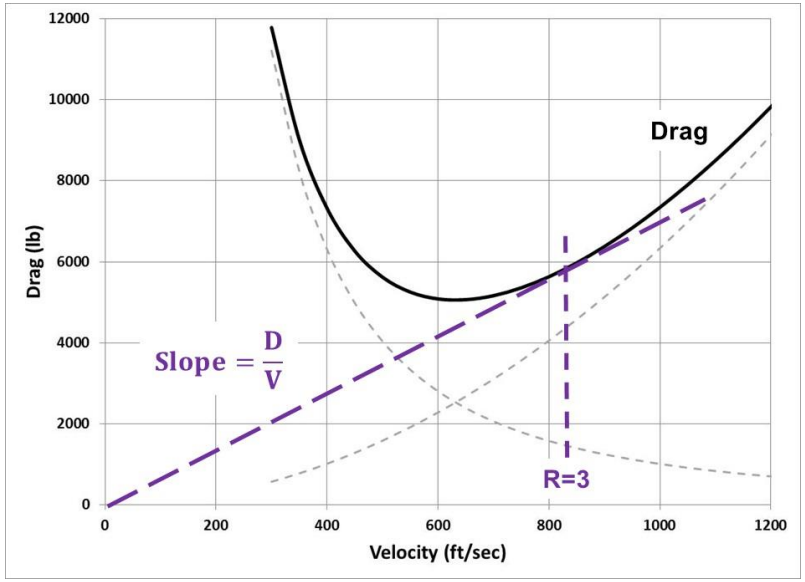
Minimum Drag

Velocity for max $\frac{C_L}{C_D}$

Jet Aircraft – Maximum Endurance

$$\frac{lb_{fuel}}{hr} \propto c_t \, T_{req} \propto c_t \, D$$

Steady Flight



Thrust Required

Minimum D / V

Velocity for max $\frac{C_L^{1/2}}{C_D}$

Jet Aircraft – Maximum Range

$$\frac{lb_{fuel}}{NM} \propto c_t \frac{T_{req}}{V}$$

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Steady Flight

		Jet Aircraft	
R = 1 / 3	$\text{Max}\ \frac{C_L^{3/2}}{C_D}$		
R = 1	$\text{Max}\ \frac{C_L}{C_D}$	Minimum D	Maximum Endurance
R = 3	$\text{Max}\ \frac{C_L^{1/2}}{C_D}$	Minimum D/V	Maximum Range

Steady Flight

$$V_{L/D_{max}} = V_{(HP/V)_{min}} = \left(\frac{2}{\rho} \sqrt{\frac{K}{C_{D_0}}} \frac{W}{S} \right)^{1/2}$$

$$V_{(D/V)_{min}} = \left(\frac{2}{\rho} \sqrt{\frac{3K}{C_{D_0}}} \frac{W}{S} \right)^{1/2} = 1.32 \; V_{L/D_{max}}$$

Steady Flight

$$V_{L/D_{max}} = V_{(HP/V)_{min}} = \left(\frac{2}{\rho} \sqrt{\frac{K}{C_{D_0}}} \frac{W}{S} \right)^{1/2}$$

$$V_{(D/V)_{min}} = \left(\frac{2}{\rho} \sqrt{\frac{3K}{C_{D_0}}} \frac{W}{S} \right)^{1/2} = 1.3161 \; V_{L/D_{max}}$$

Steady Flight

Gulfstream IV
twin-turbofan biz jet:

$C_{D_0} = 0.0150 \quad K = 0.08$

$W = 73{,}000$ lb

$h = 30{,}000$ ft

$$V_{L/D_{max}} = \left(\frac{2}{\rho} \sqrt{\frac{K}{C_{D_0}}} \frac{W}{S} \right)^{1/2} = 632 \text{ ft/sec}$$

Velocity for max endurance

$$V_{(D/V)_{min}} = 1.3161 \; V_{L/D_{max}} = 832 \text{ ft/sec}$$

Velocity for max range

Range & Endurance

Factors to consider:

Velocity / Altitude– fly at optimal conditions

Aerodynamics – maximize aero efficiency

Propulsion System – minimize fuel flow

Fuel Quantity – burn fuel efficiently

Aircraft Weights

Weight Definitions:

W = aircraft weight at any time during flight

WGTO = W_{TO} = Gross Takeoff Weight

W_{fuel} = total fuel quantity available

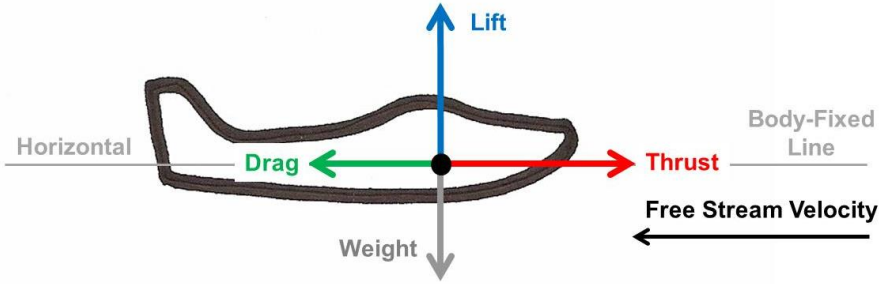
W_0 = initial weight for range calculation

W_1 = final weight for range calculation

W_f = weight of fuel remaining

$\dot{W}_f$ = fuel flow rate

Range



$$T = D$$

$$L = W$$

Range = How <u>FAR</u> can an aircraft fly?

Fuel Consumed per Mile Flown (lb_{fuel} / NM)

<u>Jet Aircraft</u>

$$\frac{lb_{fuel}}{NM} \propto c_t \frac{T_{req}}{V}$$

Range for Jet Aircraft

$$c_t = \frac{\text{Fuel Flow}}{\text{Thrust}} = -\frac{\dot{W}_f}{T} \qquad \dot{W}_f = \frac{dW}{dt} \qquad \frac{L}{D} = \frac{W}{T} \qquad V = \frac{ds}{dt}$$

$$c_t = -\frac{\dot{W}_f}{T} \longrightarrow c_t T = -\frac{dW}{dt} \longrightarrow dt = -\frac{dW}{c_t T}$$

$$V = \frac{ds}{dt} \longrightarrow ds = V\,dt \longrightarrow ds = -\frac{V\,dW}{c_t T}$$

$$ds = -\frac{V}{c_t}\frac{W}{T}\frac{dW}{W} \longrightarrow ds = -\frac{V}{c_t}\frac{L}{D}\frac{dW}{W}$$

$$R = -\frac{V}{c_t}\frac{L}{D}\int_{W_0}^{W_1}\frac{dW}{W} \longrightarrow R = \frac{V}{c_t}\frac{L}{D}\int_{W_1}^{W_0}\frac{dW}{W}$$

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Range for Jet Aircraft

$$R = \frac{V}{c_t}\frac{L}{D}\int_{W_1}^{W_0}\frac{dW}{W} \longrightarrow \boxed{R = \frac{V}{c_t}\frac{L}{D}\ln\frac{W_0}{W_1}}$$

Breguet Range Equation

$$\text{Range Factor} = \frac{V}{c_t}\frac{L}{D}$$ (in units of NM)

Maximize Range Factor = Maximize Range

- Fly at maximum V (L / D)
- Minimize c_t
- Carry a lot of fuel

Range for Jet Aircraft

$$R = \frac{V}{c_t}\frac{L}{D}\int_{W_1}^{W_0}\frac{dW}{W}$$

$$C_L = \frac{W}{1/2\,\rho V^2 S}$$

$$C_L = \frac{W}{1/2\,\rho V^2 S} \longrightarrow V = \sqrt{\frac{2\,W}{\rho\,S\,C_L}}$$

$$R = \frac{V}{c_t}\frac{L}{D}\int_{W_1}^{W_0}\frac{dW}{W} \longrightarrow R = \frac{1}{c_t}\frac{L}{D}\sqrt{\frac{2\,W}{\rho\,S\,C_L}}\int_{W_1}^{W_0}\frac{dW}{W}$$

$$R = \frac{2}{c_t}\sqrt{\frac{2}{\rho\,S}}\frac{{C_L}^{1/2}}{C_D}\left({W_0}^{1/2} - {W_1}^{1/2}\right)$$

Fly at maximum $C_L^{1/2} / C_D$

Endurance for Jet Aircraft

$$c_t = \frac{\text{Fuel Flow}}{\text{Thrust}} = -\frac{\dot{W}_f}{T} \qquad \dot{W}_f = \frac{dW}{dt} \qquad \frac{L}{D} = \frac{W}{T} \qquad V = \frac{ds}{dt}$$

$$\cdots \longrightarrow E = \frac{1}{c_t}\frac{L}{D}\int_{W_1}^{W_0}\frac{dW}{W} \longrightarrow E = \frac{1}{c_t}\frac{L}{D}\ln\frac{W_0}{W_1}$$

"Endurance Factor" **(in units of hrs)**

To Maximize Endurance:

Fly at maximum L / D or maximum C_L / C_D

Minimize c_t

Carry a lot of fuel

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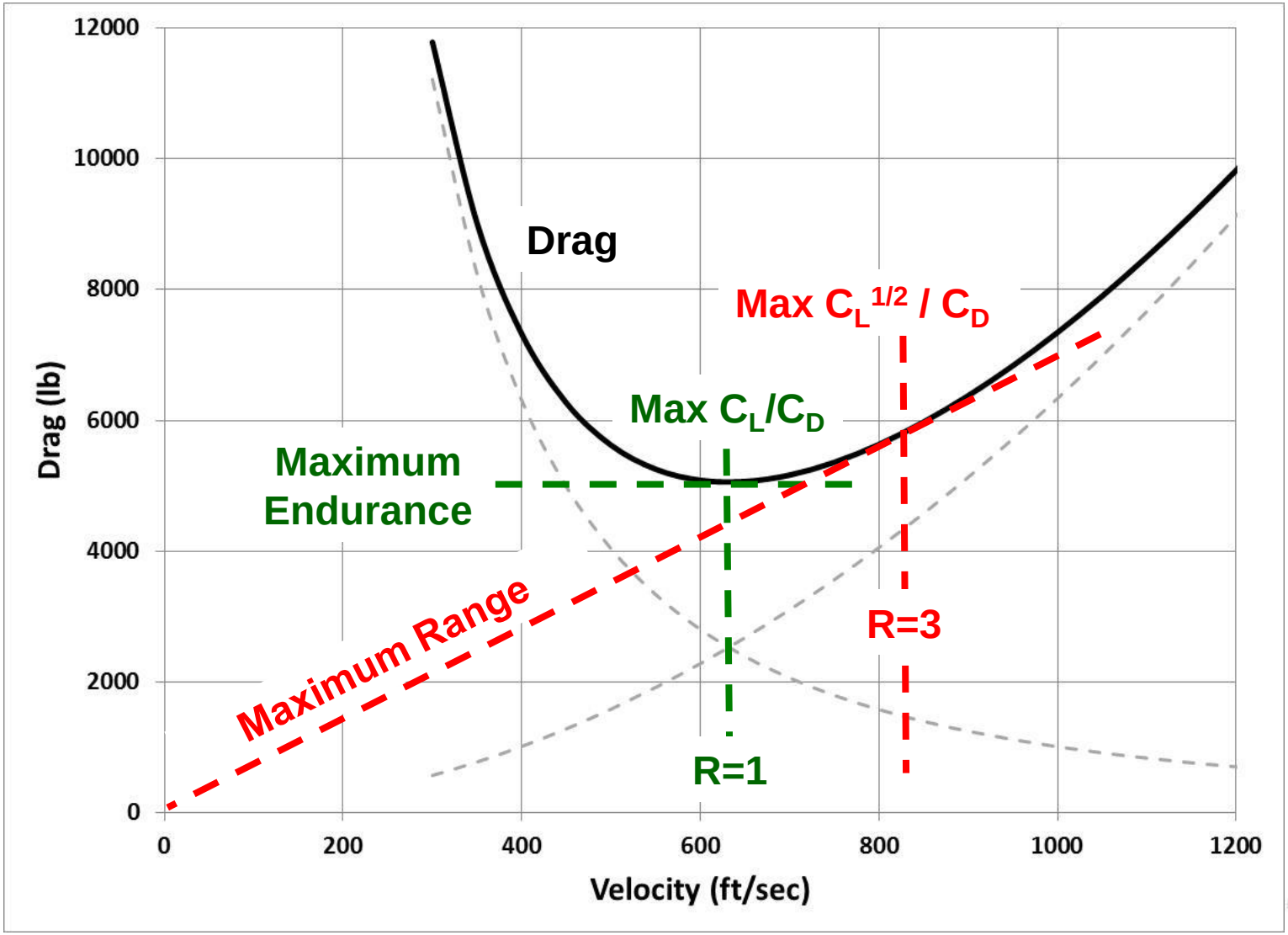
Steady Flight for Jet Aircraft



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Range & Endurance – Jet Aircraft

	Maximize	Calculate	Performance Equation
R = 1 / 3	$\text{Max}\ \frac{C_L^{3/2}}{C_D}$		
R = 1	$\text{Max}\ \frac{C_L}{C_D}$	Maximum Endurance	$E(\text{hr}) = \frac{1}{c_t}\frac{L}{D}\ln\frac{W_0}{W_1}$
R = 3	$\text{Max}\ \frac{C_L^{1/2}}{C_D}$	Maximum Range	$R(\text{NM}) = \frac{V}{c_t}\frac{L}{D}\ln\frac{W_0}{W_1}$

Range is the total distance traversed by an airplane (measured with respect to the ground)

Endurance is the amount of time that an airplane can stay in the air

Thrust Required

$$C_D = C_{D_0} + K C_L^{\ 2}$$

Gulfstream IV twin-turbofan biz jet:

C_{D_0} = 0.0150 K = 0.08

W = 73,000 lb

h = 30,000 ft

CD0	0.0150	**Wt**	73,000	lb
K	0.08	**Alt**	30,000	ft
		QMS	439.9	
		a	994.67	ft/sec
		S	950	sq ft
Vel (fps)	**CL**	**CD**	**D (lb)**	
300	1.9203	0.3100	11785	
350	1.4108	0.1742	9015	
400	1.0801	0.1083	7322	
450	0.8535	0.0733	6267	
500	0.6913	0.0532	5621	
550	0.5713	0.0411	5253	
600	0.4801	0.0334	5085	
650	0.4091	0.0284	5066	
700	0.3527	0.0250	5164	
750	0.3072	0.0226	5358	
800	0.2700	0.0208	5632	
850	0.2392	0.0196	5975	
900	0.2134	0.0186	6378	
950	0.1915	0.0179	6837	
1000	0.1728	0.0174	7345	
1050	0.1568	0.0170	7901	
1100	0.1428	0.0166	8501	
1150	0.1307	0.0164	9142	
1200	0.1200	0.0162	9825	
1250	0.1106	0.0160	10546	
1300	0.1023	0.0158	11305	

Range & Endurance

$$C_D = C_{D_0} + K C_L^{\ 2}$$

Gulfstream IV twin-turbofan biz jet:

- $C_{D_0} = 0.0150$ $K = 0.08$
- $W = 73{,}000$ lb
- $h = 20{,}000$ ft
- c (20k) = 0.720 $lb_{fuel}/hr/lb_t$

$$\text{Range Factor}(\text{NM}) = \frac{V}{c_t}\frac{L}{D}$$

$$\text{Endurance Factor}(\text{hr}) = \frac{1}{c_t}\frac{L}{D}$$

c @ altitude = c (SL) * a_{alt}/a_{SL}

Range & Endurance

$$C_D = C_{D_0} + K C_L^{\ 2}$$

Gulfstream IV
twin-turbofan biz jet:

C_{D_0} = 0.0150 K = 0.08
W = 73,000 lb
h = 20,000 ft
c (20k) = 0.720 $lb_{fuel}/hr/lb_t$

c @ altitude = c (SL) * a_{alt} / a_{SL}
G-IV: c (SL) =0.775

CD0	0.0150	**Wt**	73,000	lb		
K	0.08	**Alt**	20,000	ft		
c(SL)	0.775	**rho**	0.00126642			
c(20k)	0.720	**QMS**	680.7			
		a	1036.85	ft/sec		
		S	950	sq ft		
Vel (fps)	**CL**	**CD**	**D (lb)**	**Vel (kts)**	**RF (NM)**	**EF (hr)**
200	3.0340	0.7514	18079	118.5	664.5	5.61
250	1.9418	0.3166	11904	148.1	1261.6	8.52
300	1.3484	0.1605	8687	177.7	2074.6	11.67
350	0.9907	0.0935	6891	207.4	3051.2	14.71
400	0.7585	0.0610	5873	237.0	4091.3	17.26
450	0.5993	0.0437	5327	266.6	5074.6	19.03
500	0.4854	0.0339	5091	296.2	5900.3	19.92
550	0.4012	0.0279	5072	325.9	6513.8	19.99
600	0.3371	0.0241	5217	355.5	6909.0	19.44
650	0.2872	0.0216	5490	385.1	7113.0	18.47
700	0.2477	0.0199	5868	414.7	7166.7	17.28
750	0.2158	0.0187	6335	444.3	7111.7	16.01
800	0.1896	0.0179	6882	474.0	6983.2	14.73
850	0.1680	0.0173	7500	503.6	6808.4	13.52
900	0.1498	0.0168	8183	533.2	6606.8	12.39
950	0.1345	0.0164	8928	562.8	6391.9	11.36
1000	0.1214	0.0162	9731	592.5	6173.1	10.42
1050	0.1101	0.0160	10590	622.1	5956.0	9.57
1100	0.1003	0.0158	11503	651.7	5744.5	8.81
1150	0.0918	0.0157	12468	681.3	5540.7	8.13
1200	0.0843	0.0156	13485	710.9	5345.8	7.52
1250	0.0777	0.0155	14552	740.6	5160.3	6.97
1300	0.0718	0.0154	15668	770.2	4984.4	6.47

Maximum Endurance Calculations

Calculate V:

$$V_{L/D_{max}} = \left(\frac{2}{\rho} \sqrt{\frac{K}{C_{D_0}}} \frac{W}{S} \right)^{1/2}$$

= 529.4 ft/sec

Calculate C_L / C_D:

$$\text{Max} \frac{C_L}{C_D} = \sqrt{\frac{1}{4\, C_{D_0}\, K}}$$

= 14.43

Calculate EF:

$$EF = \frac{1}{c_t} \frac{L}{D}$$

= 20.0 hr

Calculate E:

$$E = EF \ln \frac{W_0}{W_1}$$

= 0.84 hr
burning 3,000 lb

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Maximum Range Calculations

Calculate V: $$V_{L/D_{max}} = \left(\frac{2}{\rho}\sqrt{\frac{K}{C_{D_0}}}\,\frac{W}{S}\right)^{1/2}$$ = 529.4 ft/sec

for Max R $$V_{(D/V)_{min}} = 1.3161\ V_{L/D_{max}}$$ = 696.7 ft/sec

Calculate C_L: $$C_L = \frac{W\,n}{\frac{1}{2}\,\rho\,V^2\,S} = \frac{W\,n}{(q/M^2)\,M^2\,S}$$ = 0.2500

Calculate C_D: $$C_D = C_{D_0} + K\,C_L{}^2$$ = 0.0200

Calculate RF: $$RF = \frac{V}{c_t}\,\frac{L}{D}$$ = 7167.0 NM

Calculate R: $$R = RF\ \ln\frac{W_0}{W_1}$$ = 831.9 NM burning 8,000 lb

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Homework Assignment

HW #8 – Thrust Required; Range and Endurance (due by 11:59 pm ET on Monday)
Reading – Chapters 5.1 - 5.4, 5.13 - 5.15

<u>HW Help Session</u>
Monday 1:00 – 2:00 pm ET

<u>Posted on Canvas</u>
HW #8 Assignment with instructions, tips, and checklist
HW #8 Template for data table in Excel

Homework

Plotting Charts

Weight	900	lb		QMS	1481.4	lb/ft^2				
Altitude	0	ft		a	1116.45	ft/sec				
				rho	0.00237688	slugs/ft^3				
Mach	**Vel**	**CL**	**CD0**	**CDL**	**CD**	**D**	**CL/CD**	**EF**	**CL0.5/CD**	**RF**
	(ft/sec)					**(lb)**		**(hr)**		**(NM)**
0.05	55.82	6.4291	0.0200	2.5627	2.5827	361.5	2.4893	1.91	0.9818	63.33
0.06	66.99	4.4647	0.0200	1.2359	1.2559	253.2	3.5551	2.73	1.6825	108.53
0.07	78.15	3.2802	0.0200	0.6671	0.6871	188.5	4.7740	3.67	2.6359	170.03

x axis

y axis

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Questions?