Aircraft Performance
Equations of Motion
Energy Concepts

Specific Energy

Total Energy = Potential Energy + Kinetic Energy

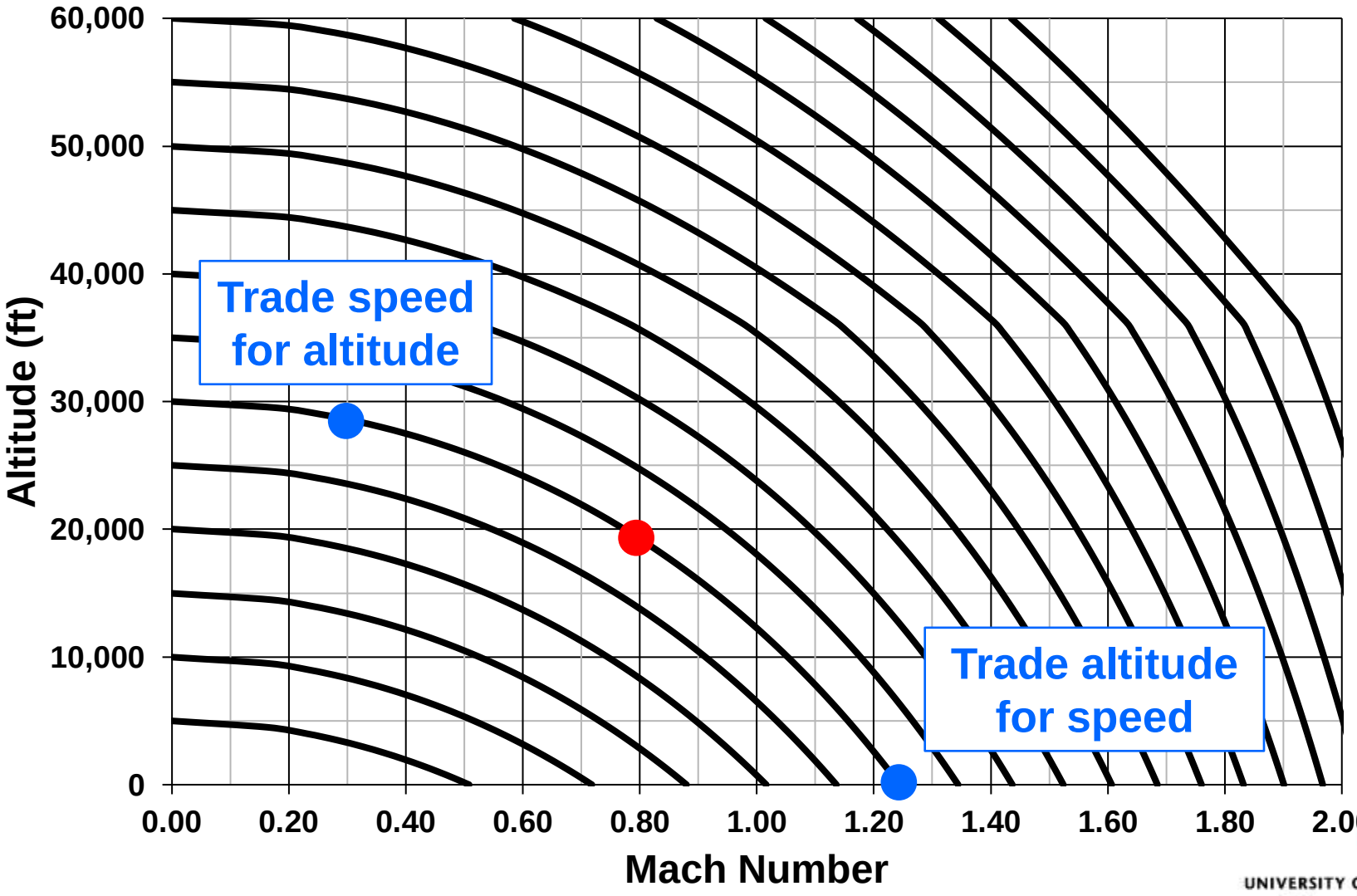
$$E = PE + KE$$

$$E = mgh + \frac{1}{2}mV^2$$

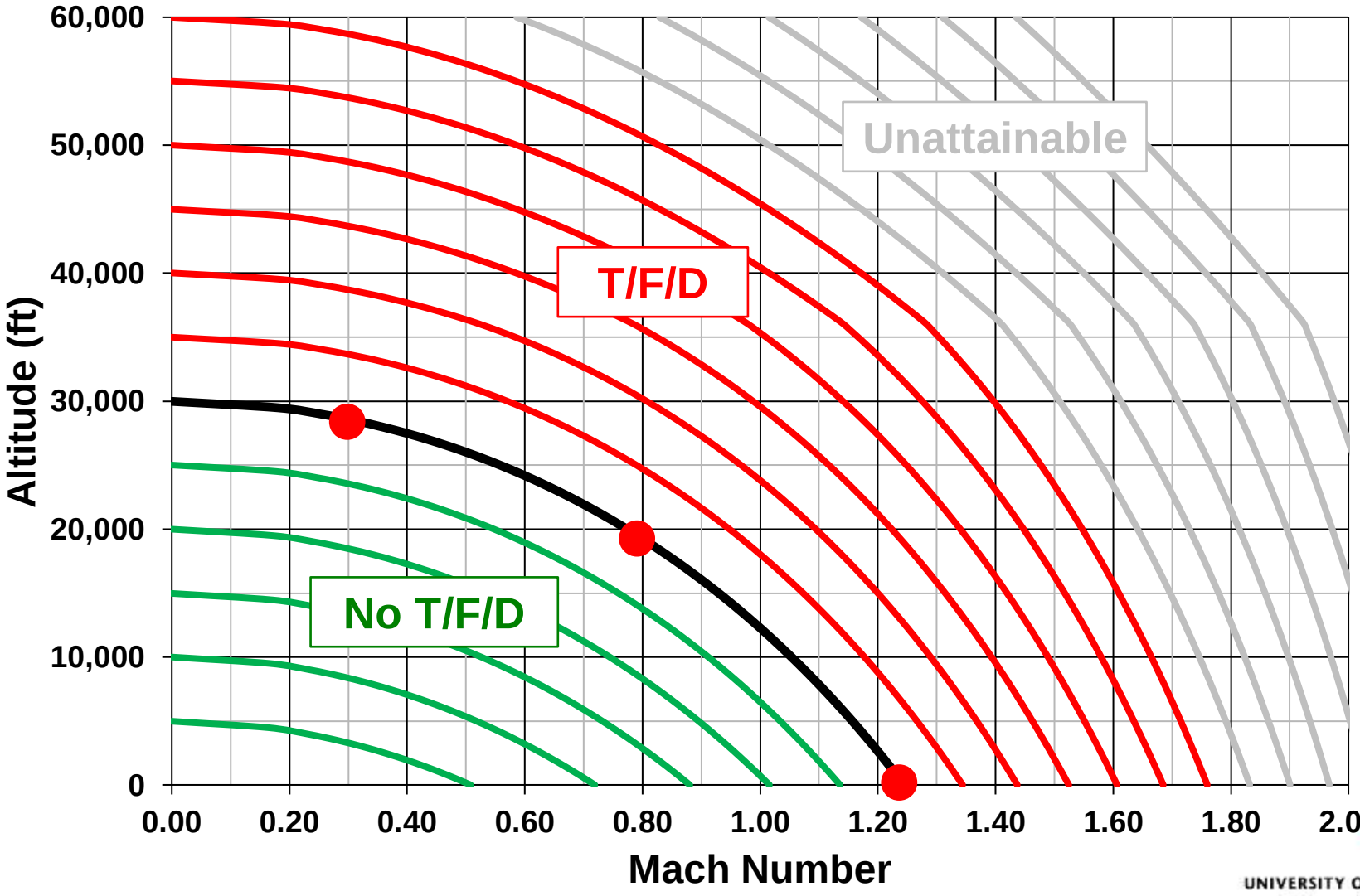
$$\frac{E}{W} = h + \frac{1}{2g}V^2$$

$$E_s = h + \frac{1}{2g}V^2 \qquad \text{(always in ft)}$$

Specific Energy



Specific Energy



Specific Energy

$$E_s = h + \frac{1}{2g}V^2$$

Flight Condition		
Alt	0	ft
Mach	0.75	
Atmospheric Data		
a	1116.45	ft/sec
Specific Energy Calculations		
h	0	ft
V	837.34	ft/sec
E_s	10,896	ft

Flight Condition		
Alt	10,000	ft
Mach	0.75	
Atmospheric Data		
a	1077.39	ft/sec
Specific Energy Calculations		
h	10,000	ft
V	808.04	ft/sec
E_s	20,147	ft

F-15 “Streak Eagle”

Stripped down F-15A – 1800 lbs lighter ⇒ T/W = 1.4

Set time-to-climb record to 30,000 m on 2/1/1975 by USAF test pilot Major Roger Smith

McDonnell Douglas test pilot “Pete” Garrison developed the flight profile

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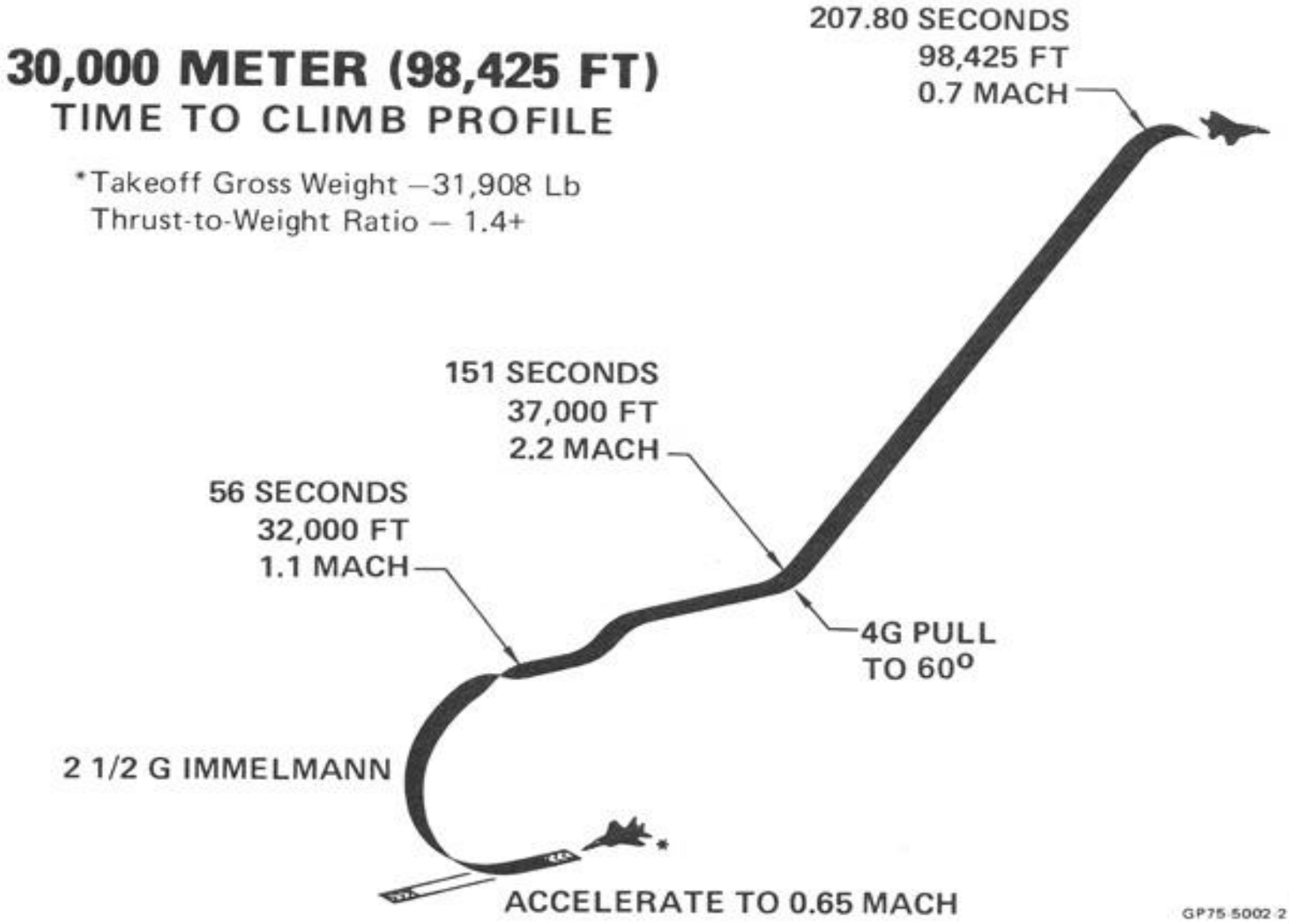
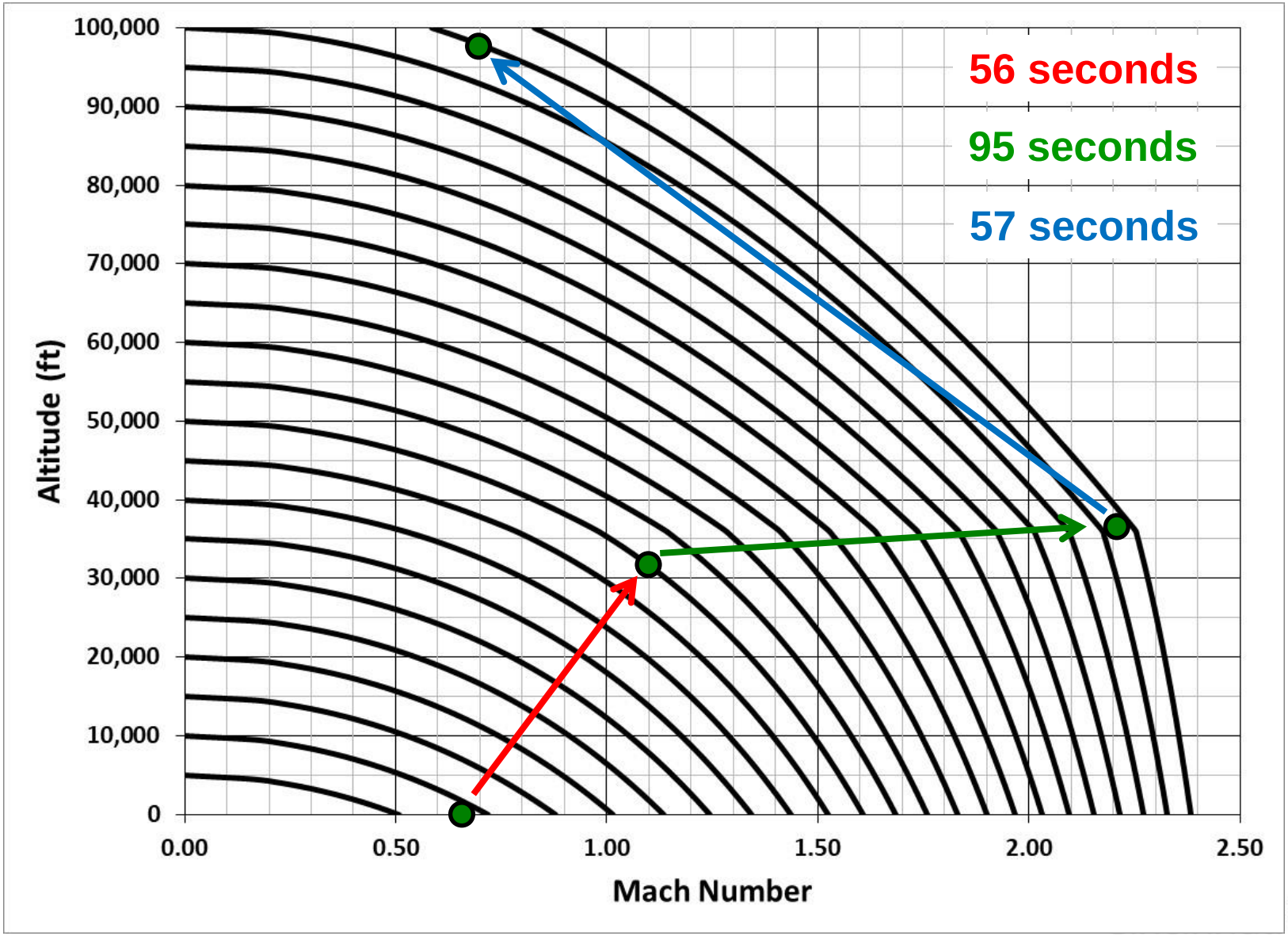
F-15 Minimum Time to Climb



F-15 Minimum Time to Climb



Specific Energy

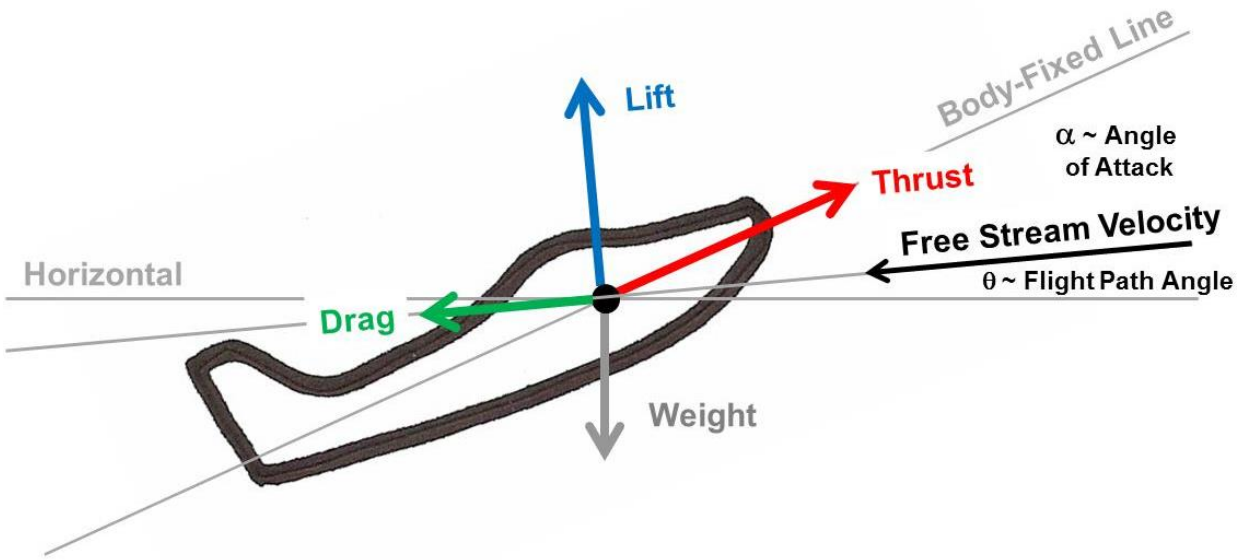
What is the relationship between E_s and P_s?

$$\frac{d}{dt}E_s = \frac{d}{dt}\left(h + \frac{1}{2g}V^2\right)$$

$$\dot{E}_s = \frac{dh}{dt} + \frac{V}{g}\frac{dV}{dt} = P_s$$

(always in ft/sec)

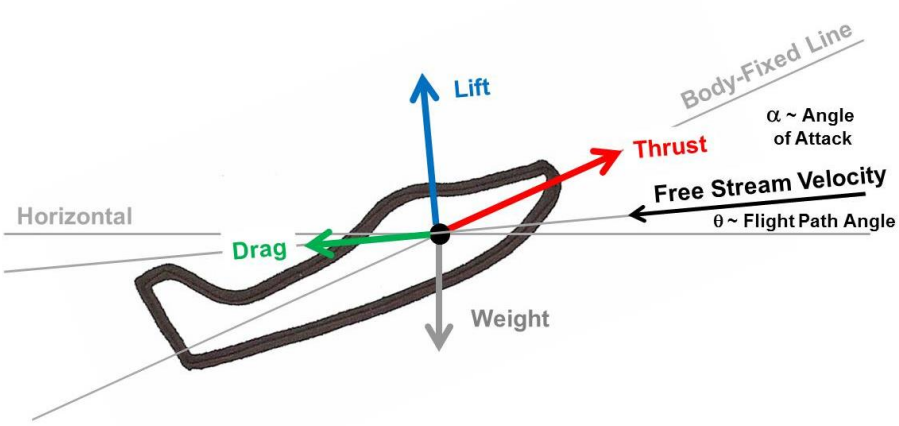
Climbing Flight

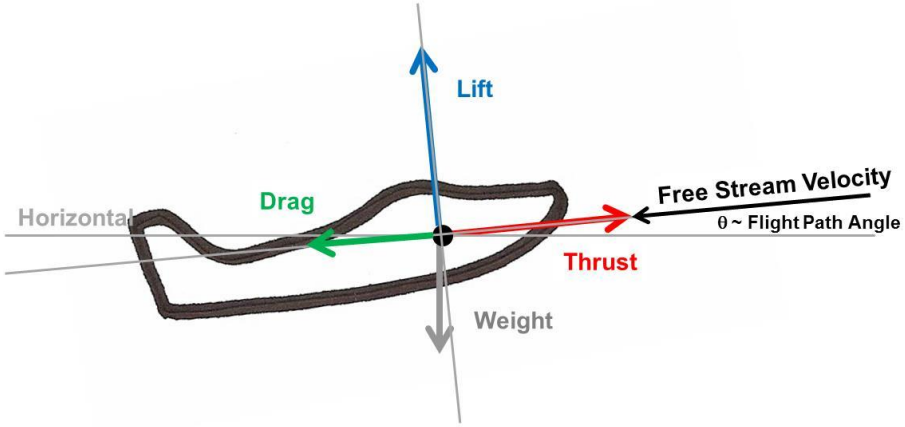


$$\Sigma F_x = T\cos\alpha - D - W\sin\theta = \frac{d(mV)}{dt} = \cancel{\dot{m}}^{\,0}V + m\dot{V}$$

Steady Climb = Constant Weight

Climbing Flight





$$\Sigma F_x = T \cos\alpha - D - W \sin\theta = m\dot{V}$$

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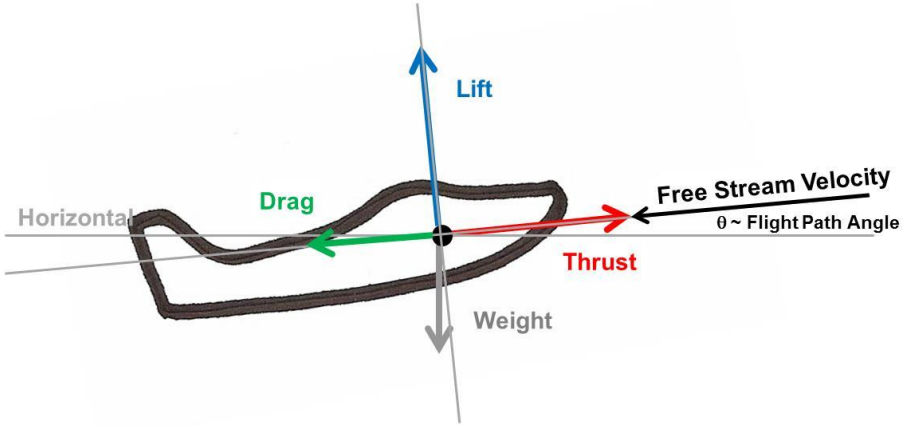
Small Angle Approximation Assumption for Angle of Attack (α)

Climbing Flight

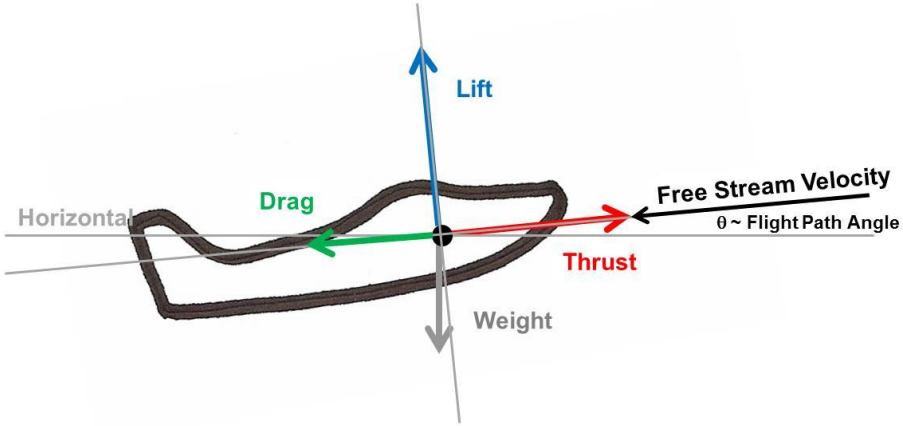


$$\Sigma F_x = T - D - W \sin\theta = m\dot{V}$$

$$T - D - W \sin\theta = \frac{W}{g}\frac{dV}{dt}$$

$$T - D = W \sin\theta + \frac{W}{g}\frac{dV}{dt}$$

Climbing Flight

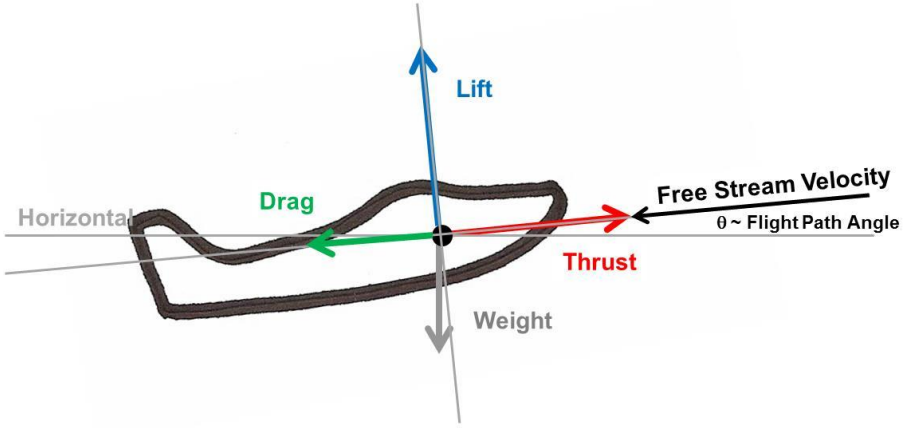


$$T - D = W \sin\theta + \frac{W}{g}\frac{dV}{dt}$$

$$T - D = W\left(\sin\theta + \frac{1}{g}\frac{dV}{dt}\right)$$

$$\frac{(T-D)\,V}{W} = V \sin\theta + \frac{V}{g}\frac{dV}{dt}$$

Climbing Flight



$$\frac{(T-D)\,V}{W} = V \sin\theta + \frac{V}{g}\frac{dV}{dt}$$

$$\text{where} \quad V \sin\theta = \frac{dh}{dt}$$

$$\boxed{\frac{(T-D)\,V}{W} = \frac{dh}{dt} + \frac{V}{g}\frac{dV}{dt} = P_s}$$

Specific Excess Power

$$\frac{(T - D)\,V}{W} = \frac{dh}{dt} + \frac{V}{g}\frac{dV}{dt} = P_s$$

Specific Excess Power can help you calculate:

- Maximum and Minimum Velocity (next lecture)
- Absolute, Service, Cruise, and Combat Ceilings (next class)
- Time, Fuel, and Distance to Climb or Descend
- Time, Fuel, and Distance to Accel or Decel
- Sustained Turn Envelope

Specific Excess Power

$$\frac{(T - D)\,V}{W} = \frac{dh}{dt} + \frac{V}{g}\frac{dV}{dt} = P_s$$

$P_s > 0$ accelerating / climbing flight $T > D$

$P_s = 0$ sustained flight $T = D$

$P_s < 0$ decelerating / descending flight $T < D$

Specific Excess Power Calculations

$$P_s = \frac{(T - D)\,V}{W}$$

Need four values: T, D, V, and W

T : Calculate T (lb) using the characteristic equation

$$T_A = T_{SL}\left(\frac{\rho}{\rho_{SL}}\right) \quad \text{- or -} \quad T_A = T_{SL}\left(\frac{\rho}{\rho_{SL}}\right)(1 + 0.7\,M)$$

V : Calculate V (ft/sec) for the provided flight condition

W : The aircraft weight W (lb) is usually provided

Specific Excess Power Calculations

$$P_s = \frac{(T - D)\,V}{W}$$

Need four values: T, D, V, and W

D : Calculate C_L $\quad C_L = \dfrac{W\,n}{\frac{1}{2}\,\rho\,V^2\,S} = \dfrac{W\,n}{(q/M^2)\,M^2\,S}$

Calculate C_D using the parabolic drag polar

$$C_D = C_{D_0} + K\,C_L^2$$

Calculate D (lb) $\quad D = C_D\,q\,S$

Specific Excess Power Calculations

$$P_s = \frac{(T - D)\,V}{W}$$

Flight Condition		
Wt	60,000	lb
Alt	0	ft
Mach	0.75	
g's	1.0	

Atmospheric Data		
QMS	1481.4	lb/sqft
a	1116.45	ft/sec

Aircraft Data		
CD0	0.0150	
K	0.08	
Thrust	27,700	lb
S	950	sq ft

Performance Data		
CL	0.0758	
CD	0.0155	
T	27,700	lb
D	12,238	lb
V	837.34	ft/sec
W	60,000	lb
P_s	215.78	ft/sec

Specific Excess Power Calculations

$$P_s = \frac{(T - D)\,V}{W}$$

Flight Condition		
Wt	60,000	lb
Alt	0	ft
Mach	0.75	
g's	3.5	

Atmospheric Data		
QMS	1481.4	lb/sqft
a	1116.45	ft/sec

Aircraft Data		
CD0	0.0150	
K	0.08	
Thrust	27,700	lb
S	950	sq ft

Performance Data		
CL	0.2653	
CD	0.0206	
T	27,700	lb
D	16,331	lb
V	837.34	ft/sec
W	60,000	lb
P_s	158.66	ft/sec

Homework Assignment

HW #9 – Specific Energy; Specific Excess Power (due by 11:59 pm ET on Monday)
Reading – Chapter 6.6

<u>HW Help Session</u>
Monday 1:00 – 2:00 pm ET

<u>Posted on Canvas</u>
HW #9 Assignment with instructions, tips, and checklist
HW #9 Template for data table in Excel

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Questions?