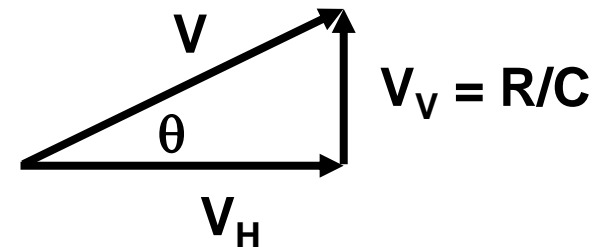
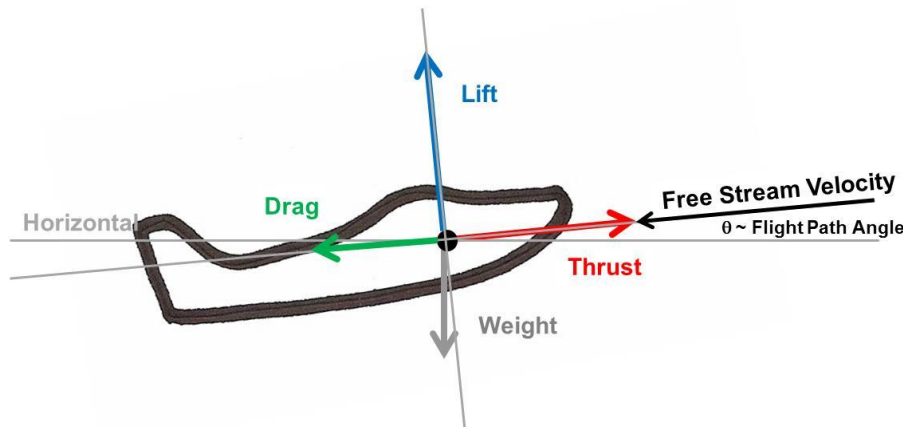


AEEM 3042 – Integrated Aircraft Engineering

Block 2b Material Review

Steady Climbing Flight



$$\sin \theta = \frac{T-D}{W} \longrightarrow V \sin \theta = \frac{(T-D) V}{W} = R/C$$

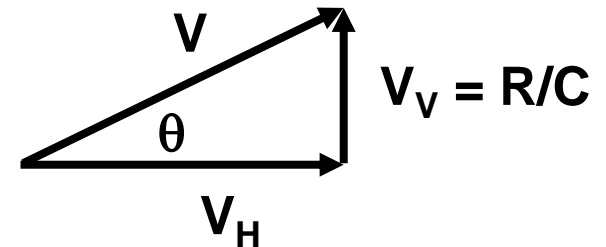
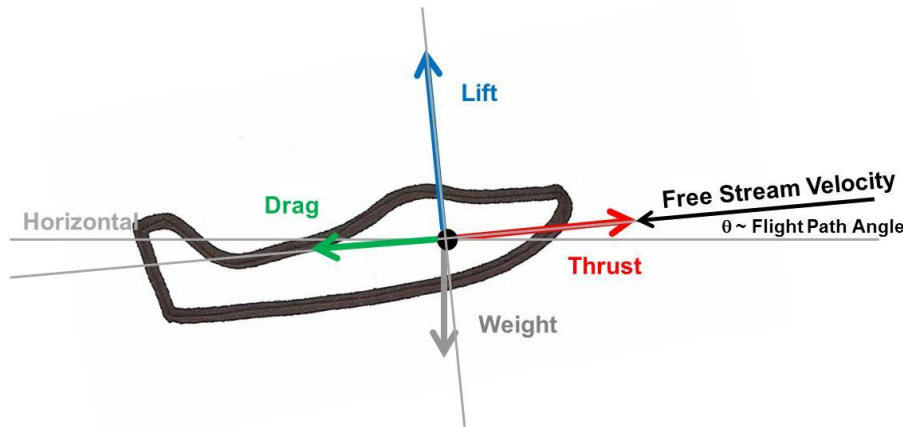
for $R/C > 0$

$$\boxed{T > D}$$

$$\cos \theta = \frac{L}{W} \longrightarrow L = W \cos \theta$$

$$\boxed{L < W}$$

Steady Climbing Flight



$$V \sin \theta = \frac{(T - D) V}{W} = R/C$$


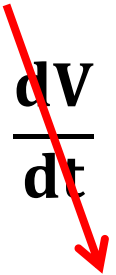
$$(T - D) V = T V - D V = \text{Excess Power}$$

$$\frac{(T - D) V}{W} = \text{Specific Excess Power} = R/C$$

Specific Excess Power

$$\frac{(T - D) V}{W} = \frac{dh}{dt} + \frac{V}{g} \frac{dV}{dt} = P_s$$

$$\frac{(T - D) V}{W} = \frac{dh}{dt} + \frac{V}{g} \frac{dV}{dt}$$

Rate
of
Climb

Climbing Flight

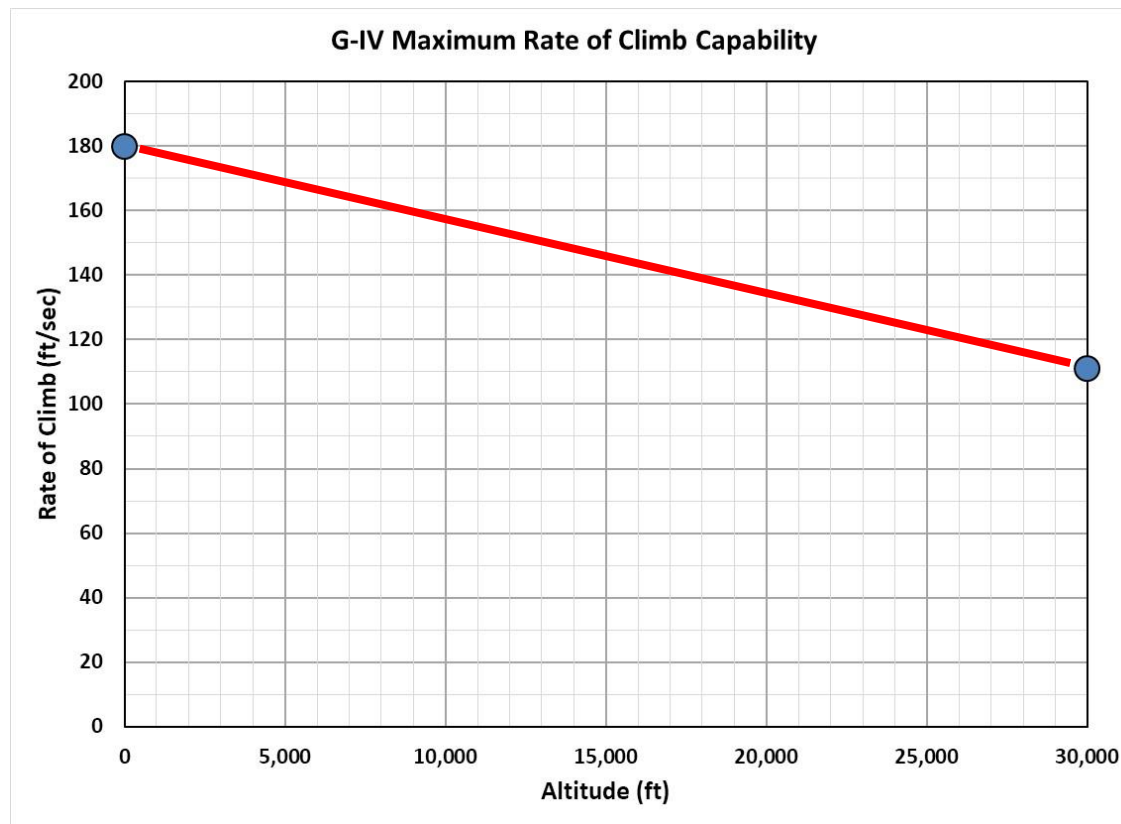
Minimum Time to Climb

Easier Method – assume linear R/C_{\max} vs Altitude

Calculate the maximum Rate of Climb at Sea Level

Calculate the maximum Rate of Climb at 30,000 ft

Calculate the average Rate of Climb



Minimum Time to Climb

Easier Method

$$t_{\min} = \frac{\Delta h}{(R/C)_{\text{avg}}}$$

At Sea Level: $(R/C)_{\max} = 179.8 \text{ ft/sec}$

At 30,000 ft: $(R/C)_{\max} = 111.0 \text{ ft/sec}$

$$t_{\min} = \frac{(30,000 - 0)}{(179.8 + 111.0)/2}$$

$$t_{\min} = 206.3 \text{ sec} = 3.44 \text{ min}$$

What about fuel and distance?

Fuel to Climb

Easier Method

$$W_{\text{fuel}} = \dot{w}_{\text{avg}} \Delta t$$

At Sea Level: $\dot{w} = 10.0028 \text{ lb/sec}$

At 30,000 ft: $\dot{w} = 5.5454 \text{ lb/sec}$

$$W_{\text{fuel}} = \frac{(10.0028 + 5.5454)}{2} 206.3$$

$$W_{\text{fuel}} = 1,603.8 \text{ lb}$$

Distance to Climb

Easier Method

$$s = V_{\text{avg}} \Delta t$$

At Sea Level: $V = 747.3 \text{ ft/sec}$

At 30,000 ft: $V = 932.2 \text{ ft/sec}$

$$s = \frac{(747.3 + 932.2)}{2} 206.3$$

$$s = 173,240 \text{ ft} = 28.51 \text{ NM}$$

Climb Summary

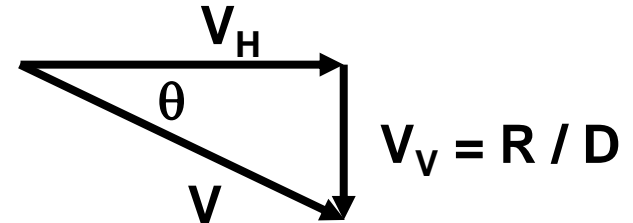
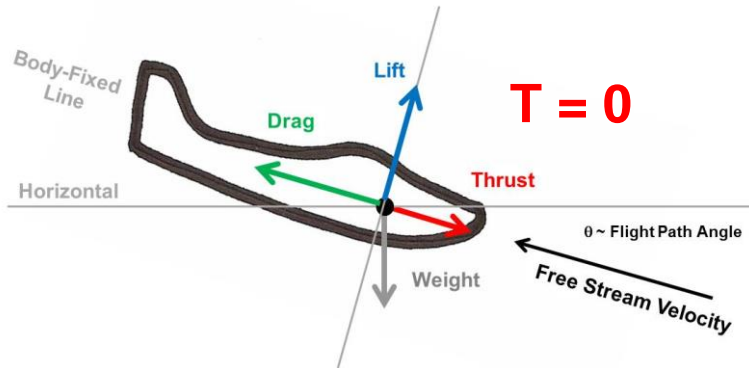
Easier Method

$$t_{\min} = \frac{\Delta h}{(R/C)_{\text{avg}}} \quad W_{\text{fuel}} = \dot{W}_{\text{avg}} \Delta t \quad s = V_{\text{avg}} \Delta t$$

Sea Level			30,000 ft		
R/C	FFR	Velocity	R/C	FFR	Velocity
(ft/sec)	(lb/sec)	(ft/sec)	(ft/sec)	(lb/sec)	(ft/sec)
179.8	10.0028	747.3	111.0	5.5454	932.2

Time	Fuel	Distance
3.44 min	1,603.8 lb	28.51 NM

Steady Gliding Flight



$$\sin \theta = \frac{D - \overset{0}{T}}{W} \longrightarrow V \sin \theta = \frac{D V}{W} = R/D$$

$$\cos \theta = \frac{L}{W} \longrightarrow L = W \cos \theta$$

$$\tan \theta = \frac{D - \overset{0}{T}}{L} = \frac{1 - (\overset{0}{T/D})}{(L/D)} = \frac{1}{(L/D)}$$

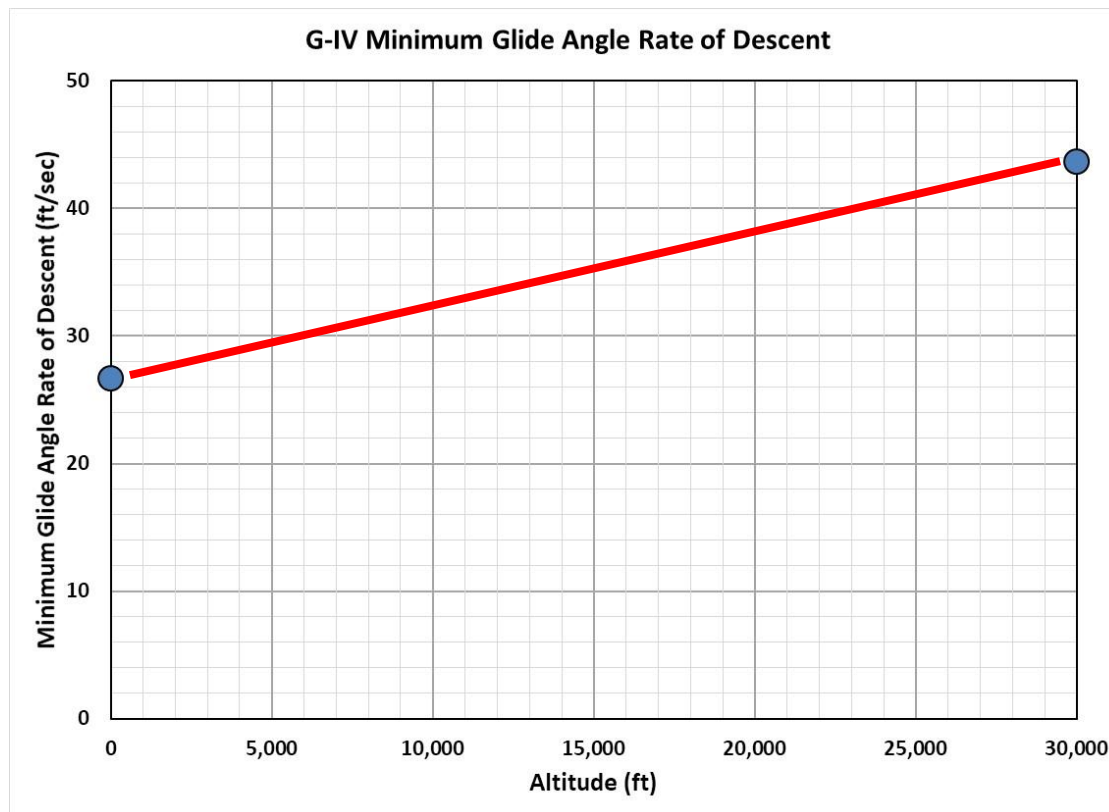
Time to Descend

Easier Method – assume linear Minimum Glide Angle vs Altitude

Calculate the Minimum Glide Angle and Velocity at Sea Level

Calculate the Minimum Glide Angle and Velocity at 30,000 ft

Calculate the average Glide Angle, average Velocity, and R/D



Time to Descend

Easier Method

$$t_{\theta_{\min}} = \frac{\Delta h}{(R/D)_{\text{avg}}}$$

At Sea Level: $\theta_{\min} = 3.96^\circ$ $V_{\theta_{\min}} = 385.1 \text{ ft/sec}$ $R/D = 26.7 \text{ ft/sec}$

At 30,000 ft: $\theta_{\min} = 3.96^\circ$ $V_{\theta_{\min}} = 631.0 \text{ ft/sec}$ $R/D = 43.6 \text{ ft/sec}$

$$t_{\theta_{\min}} = \frac{(30,000 - 0)}{(43.6 + 26.7)/2}$$

$$t_{\theta_{\min}} = 853.5 \text{ sec} = 14.22 \text{ min}$$

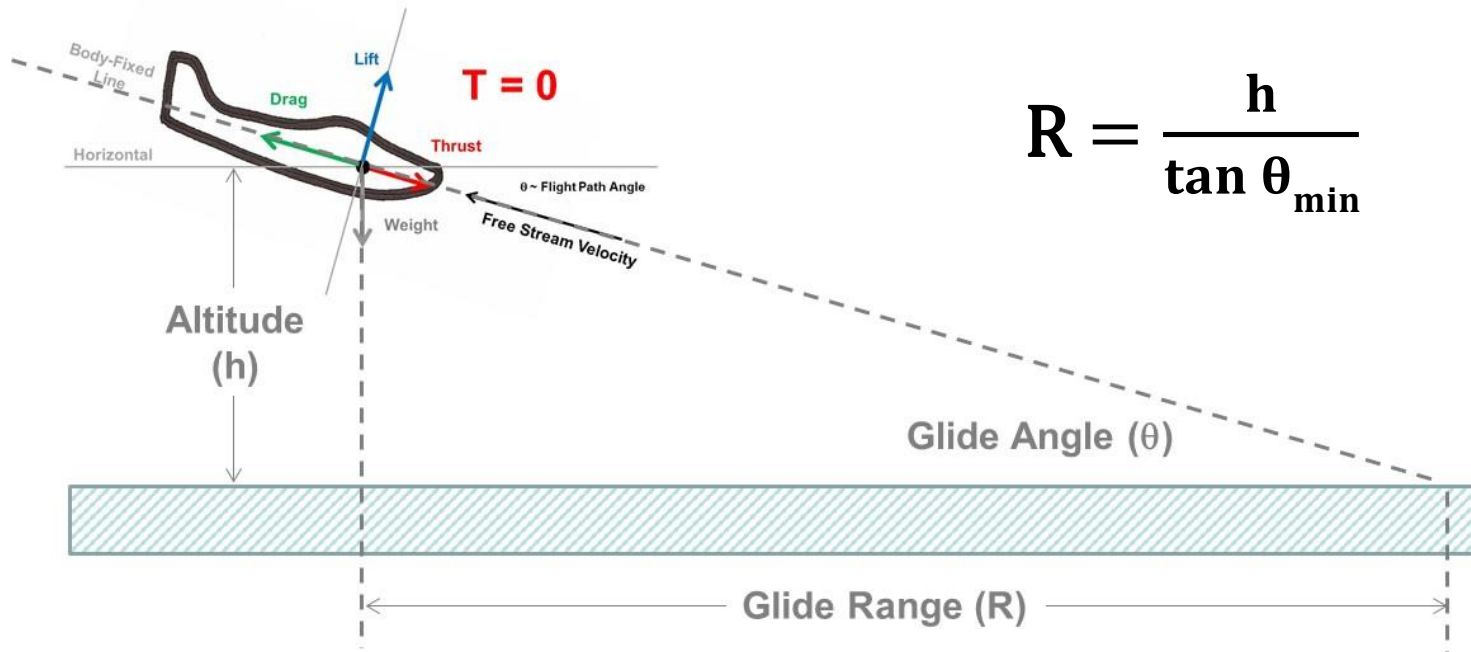
Fuel to Descend

Easier Method – assume linear Fuel Flow vs Altitude

Since this is “gliding flight” $T = 0$ and $FFR = 0$

Distance to Descend

Easier Method – assume linear Glide Angle vs Altitude



$$R = \frac{h}{\tan \theta_{\min}}$$

$$R = \frac{h}{\tan \theta_{\min}} = \frac{(30,000 - 0)}{\tan 3.96^\circ}$$

$$R = 433,013 \text{ ft} = 71.26 \text{ NM}$$

Descent Summary

Easier Method

$$t_{\theta_{\min}} = \frac{\Delta h}{(R/D)_{\text{avg}}} \quad w_{\text{fuel}} = 0 \quad R = \frac{h}{\tan \theta_{\min}}$$

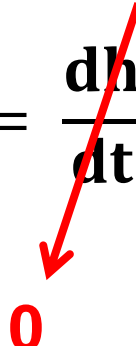

Sea Level			30,000 ft		
θ_{\min}	Velocity	R/D	θ_{\min}	Velocity	R/D
(deg)	(ft/sec)	(ft/sec)	(deg)	(ft/sec)	(ft/sec)
3.96	385.1	26.7	3.96	631.0	43.6

Time	Fuel	Distance
14.22 min	0.0 lb	71.26 NM

Specific Excess Power

$$\frac{(T - D) V}{W} = \frac{dh}{dt} + \frac{V}{g} \frac{dV}{dt} = P_s$$

$$\frac{(T - D) V}{W} = \frac{dh}{dt} + \frac{V}{g} \frac{dV}{dt}$$

Level Acceleration

**Acceleration
capability**

Time to Accelerate

Easier Method

$$t_{\text{accel}} = \frac{\Delta V}{(\dot{V})_{\text{avg}}}$$

At 20,000 ft & 518.4 ft/sec: $\dot{V} = 6.1366 \text{ ft/sec}^2$

At 20,000 ft & 829.5 ft/sec: $\dot{V} = 5.1774 \text{ ft/sec}^2$

$$t_{\text{accel}} = \frac{(829.5 - 518.4)}{(5.1774 + 6.1366)/2}$$

$$t_{\text{accel}} = 55.0 \text{ sec} = 0.92 \text{ min}$$

What about fuel and distance?

Fuel to Accelerate

Easier Method

$$W_{\text{fuel}} = \dot{W}_{\text{avg}} \Delta t$$

At V_1 (518.4 ft/sec): $\dot{w} = 6.8559$ lb/sec

At V_2 (829.5 ft/sec): $\dot{w} = 6.8559$ lb/sec

$$W_{\text{fuel}} = \frac{(6.8559 + 6.8559)}{2} 55.0$$

$$W_{\text{fuel}} = 377.1 \text{ lb}$$

Distance to Accelerate

Easier Method

$$s = V_{\text{avg}} \Delta t$$

$$s = \frac{(518.4 + 829.5)}{2} 55.0$$

$$s = 37,067 \text{ ft} = 6.10 \text{ NM}$$

Acceleration Summary

Easier Method

$$t_{\text{accel}} = \frac{\Delta V}{(\dot{V})_{\text{avg}}} \quad W_{\text{fuel}} = \dot{W}_{\text{avg}} \Delta t \quad s = V_{\text{avg}} \Delta t$$

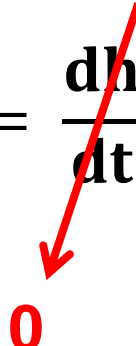

0.50 M / 20,000 ft			0.80 M / 20,000 ft		
\dot{V}	FFR	Velocity	\dot{V}	FFR	Velocity
(ft/sec ²)	(lb/sec)	(ft/sec)	(ft/sec ²)	(lb/sec)	(ft/sec)
6.1366	6.8559	518.4	5.1811	6.8559	829.5

Time	Fuel	Distance
55.0 sec	377.1 lb	6.10 NM

Specific Excess Power

$$\frac{(T - D) V}{W} = \frac{dh}{dt} + \frac{V}{g} \frac{dV}{dt} = P_s$$

$$\frac{(T - D) V}{W} = \frac{dh}{dt} + \frac{V}{g} \frac{dV}{dt}$$

Level Acceleration

**Acceleration
capability**

Time to Accelerate

Easier Method

$$t_{\text{accel}} = \frac{\Delta V}{(\dot{V})_{\text{avg}}}$$

At 20,000 ft & 518.4 ft/sec: $\dot{V} = 6.1366 \text{ ft/sec}^2$

At 20,000 ft & 829.5 ft/sec: $\dot{V} = 5.1774 \text{ ft/sec}^2$

$$t_{\text{accel}} = \frac{(829.5 - 518.4)}{(5.1774 + 6.1366)/2}$$

$$t_{\text{accel}} = 55.0 \text{ sec} = 0.92 \text{ min}$$

What about fuel and distance?

Fuel to Accelerate

Easier Method

$$W_{\text{fuel}} = \dot{W}_{\text{avg}} \Delta t$$

At V_1 (518.4 ft/sec): $\dot{w} = 6.8559$ lb/sec

At V_2 (829.5 ft/sec): $\dot{w} = 6.8559$ lb/sec

$$W_{\text{fuel}} = \frac{(6.8559 + 6.8559)}{2} 55.0$$

$$W_{\text{fuel}} = 377.1 \text{ lb}$$

Distance to Accelerate

Easier Method

$$s = V_{\text{avg}} \Delta t$$

$$s = \frac{(518.4 + 829.5)}{2} 55.0$$

$$s = 37,067 \text{ ft} = 6.10 \text{ NM}$$

Acceleration Summary

Easier Method

$$t_{\text{accel}} = \frac{\Delta V}{(\dot{V})_{\text{avg}}} \quad W_{\text{fuel}} = \dot{W}_{\text{avg}} \Delta t \quad s = V_{\text{avg}} \Delta t$$

0.50 M / 20,000 ft			0.80 M / 20,000 ft		
\dot{V}	FFR	Velocity	\dot{V}	FFR	Velocity
(ft/sec ²)	(lb/sec)	(ft/sec)	(ft/sec ²)	(lb/sec)	(ft/sec)
6.1366	6.8559	518.4	5.1811	6.8559	829.5

Time	Fuel	Distance
55.0 sec	377.1 lb	6.10 NM

Time to Decelerate

Easier Method

$$t_{\text{decel}} = \frac{\Delta V}{(\dot{V})_{\text{avg}}}$$

At 20,000 ft & 829.5 ft/sec: $\dot{V} = -3.1902 \text{ ft/sec}^2$

At 20,000 ft & 518.4 ft/sec: $\dot{V} = -2.2310 \text{ ft/sec}^2$

$$t_{\text{decel}} = \frac{(518.4 - 829.5)}{(-3.1902 - 2.2310)/2}$$

$$t_{\text{decel}} = 114.8 \text{ sec} = 1.91 \text{ min}$$

What about fuel and distance?

Fuel to Decelerate

Easier Method – assume linear Fuel Flow vs Velocity

Since this is “decelerating flight” $T = 0$ and $FFR = 0$

Distance to Decelerate

Easier Method

$$s = V_{\text{avg}} \Delta t$$

$$s = \frac{(518.4 + 829.5)}{2} 114.8$$

$$s = 77,369 \text{ ft} = 12.73 \text{ NM}$$

Deceleration Summary

Easier Method

$$t_{\text{accel}} = \frac{\Delta V}{(\dot{V})_{\text{avg}}}$$

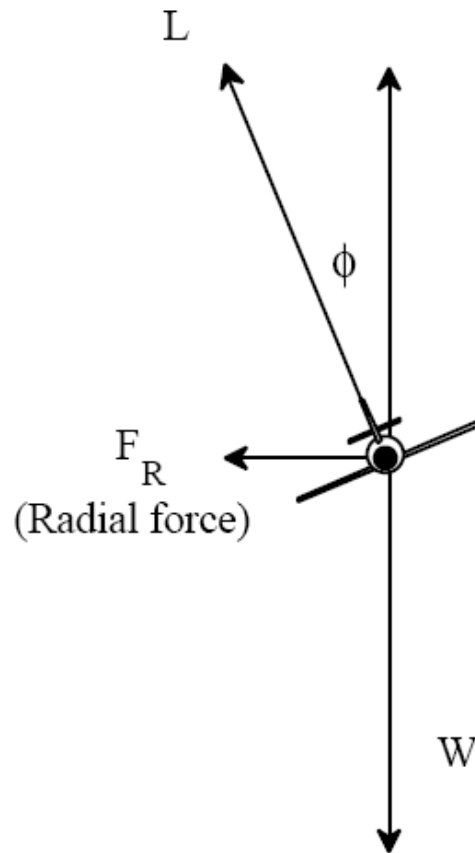
$$W_{\text{fuel}} = 0$$

$$s = V_{\text{avg}} \Delta t$$

0.80 M / 20,000 ft			0.50 M / 20,000 ft		
\dot{V}	FFR	Velocity	\dot{V}	FFR	Velocity
(ft/sec ²)	(lb/sec)	(ft/sec)	(ft/sec ²)	(lb/sec)	(ft/sec)
-3.1902	0.0	829.5	-2.2310	0.0	518.4

Time	Fuel	Distance
114.8 sec	0 lb	12.73 NM

Level Turning Flight



Lift acts perpendicular to the Free Stream Velocity

Lift acts perpendicular the wing surface

Drag acts parallel to the Free Stream Velocity

Weight acts vertically towards the ground

Thrust is fixed in the aircraft

Bank angle ϕ

Load factor n

$$\Sigma F_z = L \cos \phi - W = 0$$

$$\cos \phi = \frac{W}{L} \quad n = \frac{L}{W}$$

$$\phi = \arccos \left(\frac{1}{n} \right)$$

Figure 6.1

FORCES IN A STEADY LEVEL TURN

Level Turning Flight

Turn Radius: $R = \frac{mV^2}{L \sin \phi} = \frac{V^2}{g \sqrt{n^2 - 1}}$ (tightest turn)

Turn Rate: $\omega = \frac{V}{R} = \frac{g \sqrt{n^2 - 1}}{V}$ (quickest turn)

To minimize turn radius for tightest turn and
to maximize turn rate for quickest turn:

Highest possible load factor
Lowest possible velocity

Level Turning Flight

Bank Angle ϕ

Turn Radius R

Load Factor n

Turn Angle ψ

Turn Rate $\omega = d\psi/dt$

Sustained Turn – turning while maintaining the same velocity and altitude $P_s = 0$

Sustained Corner Velocity – speed for highest turn rate while maintaining velocity and altitude

Instantaneous Turn – turning and not maintaining the same velocity and altitude $P_s \neq 0$

Corner Velocity – speed for highest turn rate

Sustained Turn

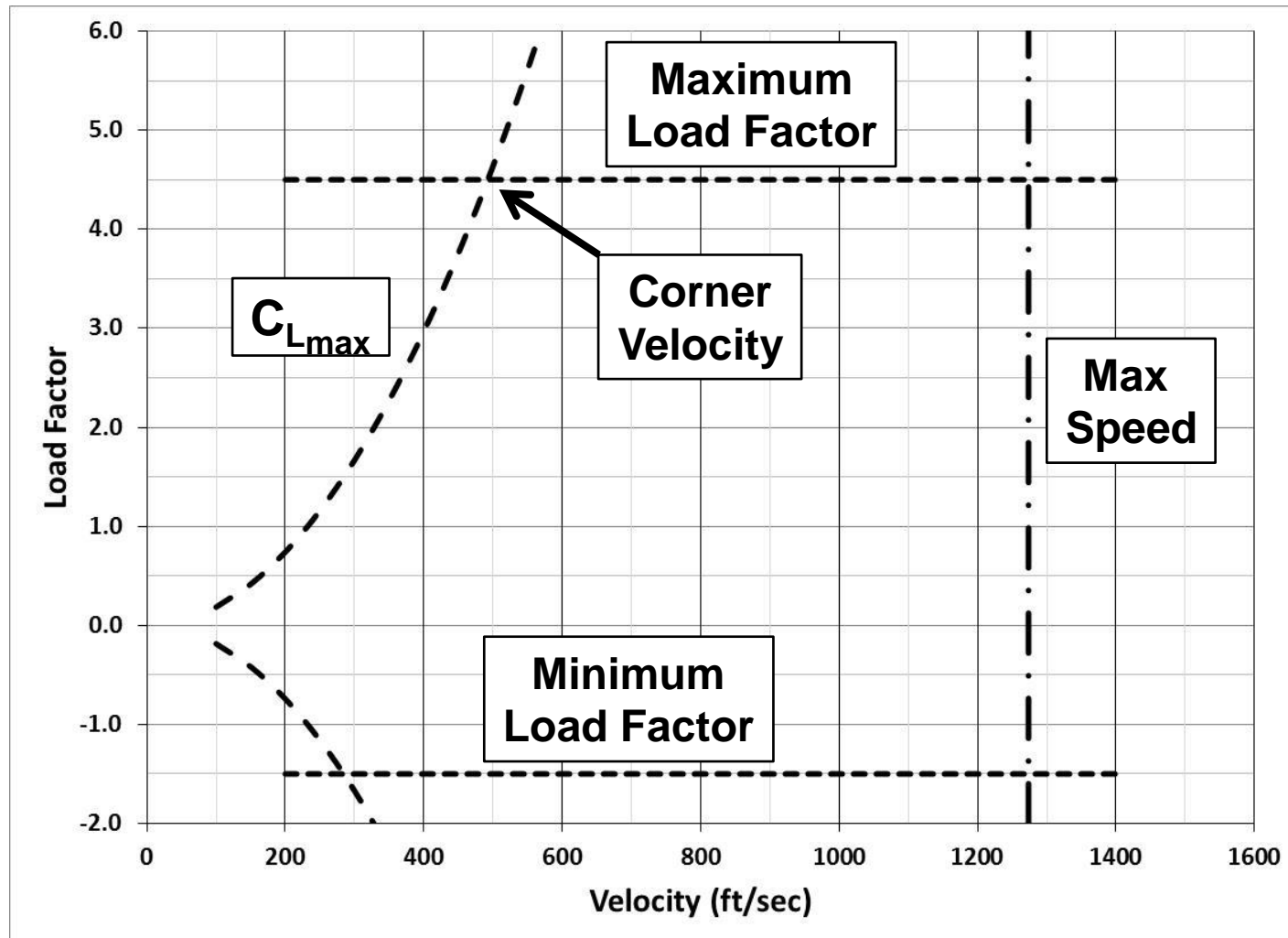
Sustained Turn – turning while maintaining the same velocity and altitude

$$P_s = 0$$

$$(n_{\max})_{\text{sust}} = \sqrt{\frac{q}{K (W/S)} \left[\frac{T}{W} - \frac{q C_{D0}}{(W/S)} \right]} = \left(\frac{T}{W} \right) \left(\frac{L}{D} \right)_{\max}$$

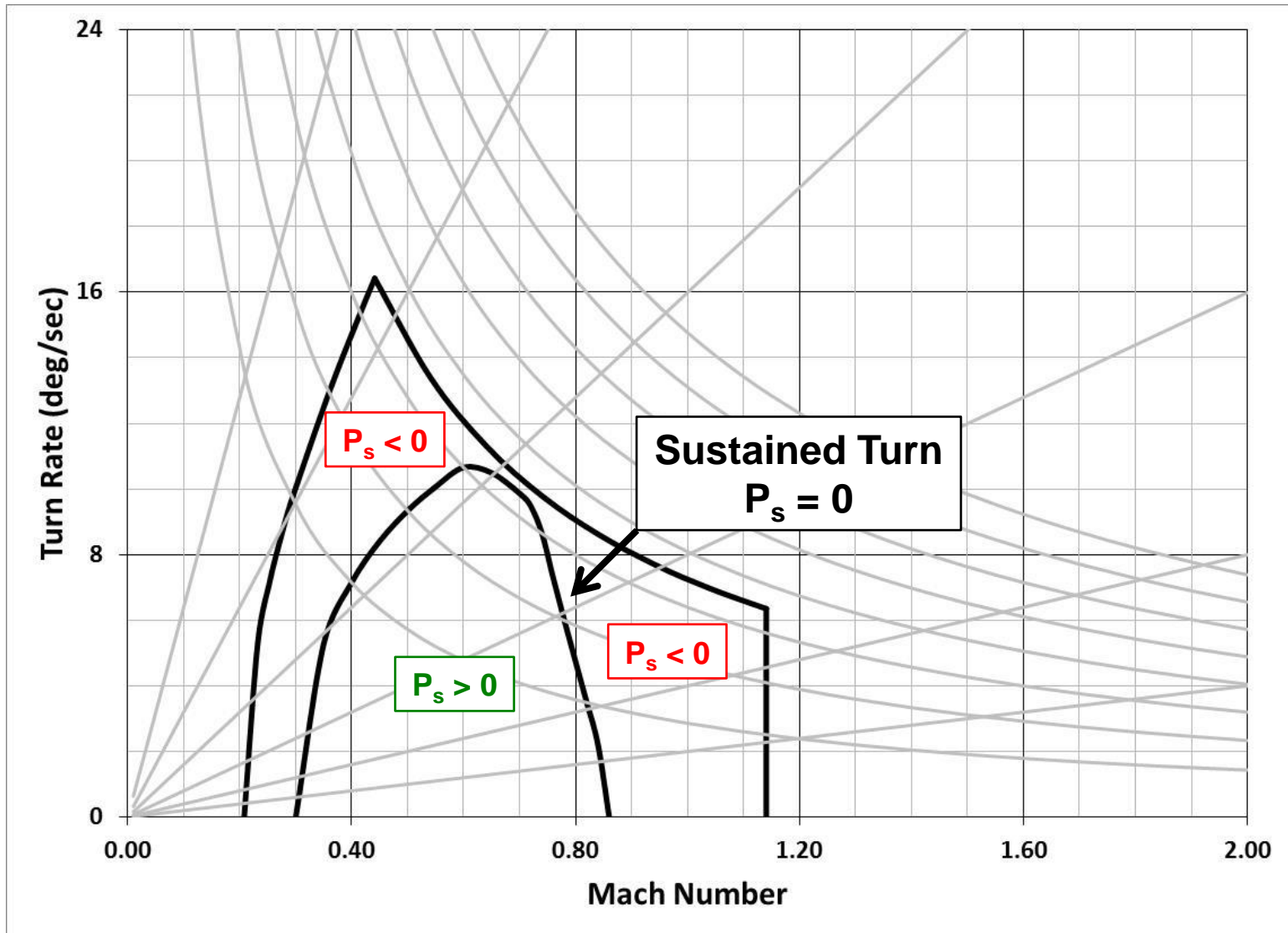
$$V_{(n_{\max})_{\text{sust}}} = \left(\frac{2}{\rho} \sqrt{\frac{K}{C_{D0}}} \frac{n W}{S} \right)^{1/2}$$

Instantaneous Turn

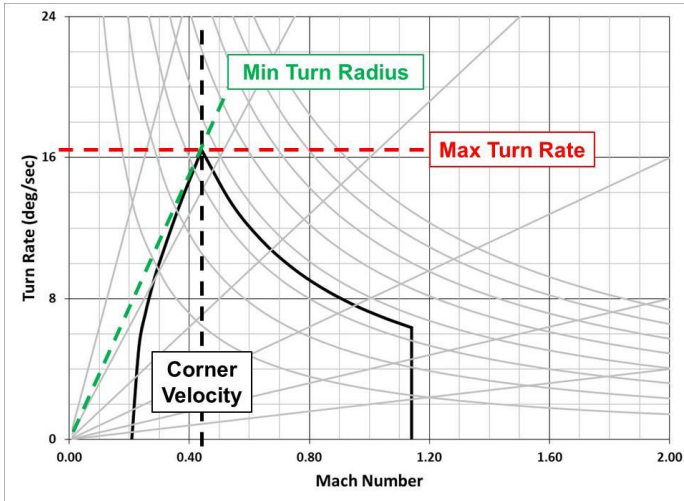


Maneuver Diagram

G-IV Example



Maneuver Calculations



Corner Velocity

$$V_{\text{corner}} = \sqrt{\frac{2 n_{\text{max}} (W/S)}{\rho C_{L_{\text{max}}}}}$$

CD0	0.0150	Wt	73,000 lb	CLmax	1.20
K	0.08	Alt	0 ft	Vmax	1.14 Mach
Thrust	27,700	QMS	1481.4 lb/sqft	Max g	4.5 g's
W/S	76.84	rho	0.00237688 slugs/ft^3		
T/W	0.3795	a	1116.45 ft/sec		
		S	950 sq ft		
Maximum Turn Rate & Minimum Turn Radius					
Corner Velocity		492.4 ft/sec			
		0.4410 Mach			
Max Turn Rate		16.43 deg/sec			
Min Turn Radius		1718 ft			

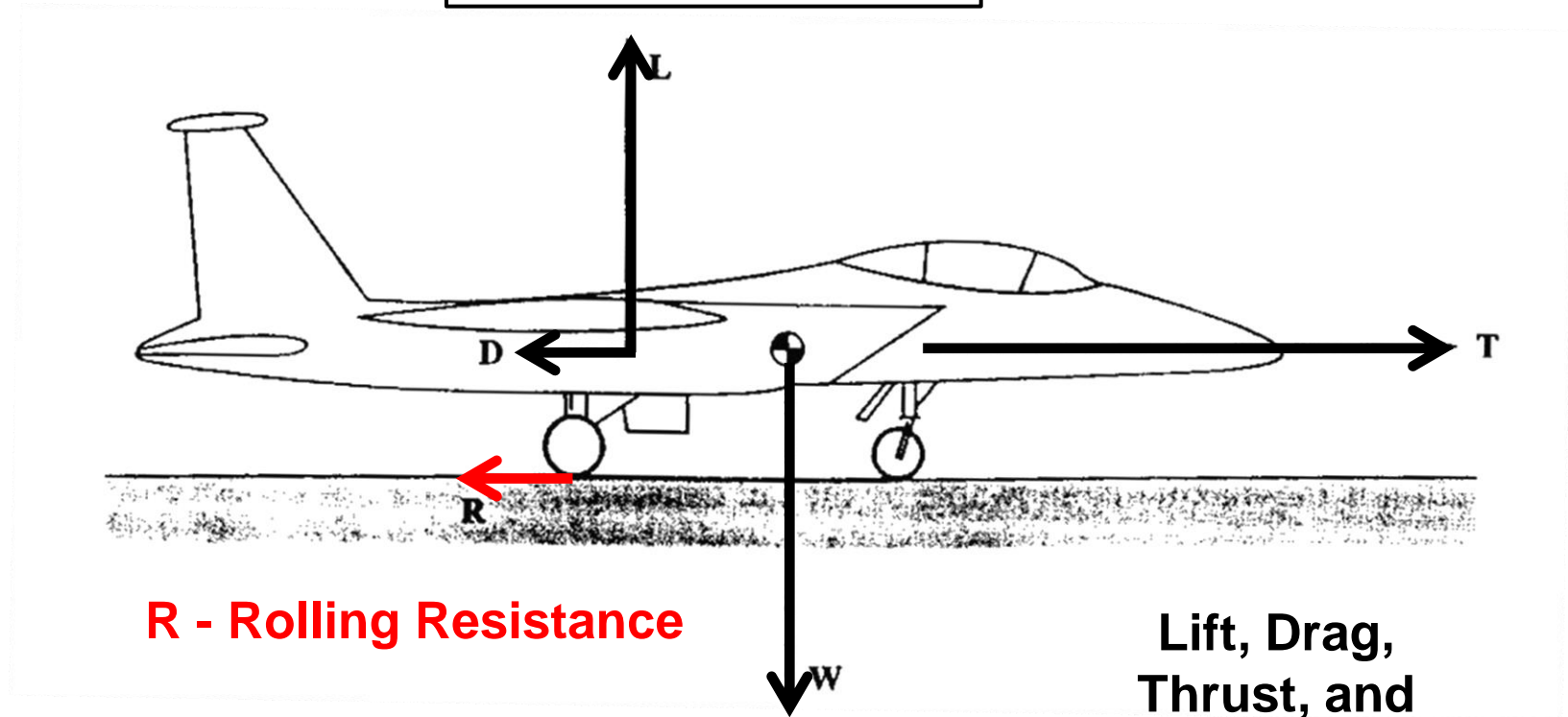
Turn Rate

$$\omega = \frac{g \sqrt{n^2 - 1}}{V}$$

Turn Radius

$$R = \frac{V^2}{g \sqrt{n^2 - 1}}$$

Takeoff



R - Rolling Resistance

Lift, Drag,
Thrust, and
Weight

$$R = \mu_r (W - L)$$

μ_r = coefficient of rolling friction

$$0.02 < \mu_r < 0.08$$

Takeoff

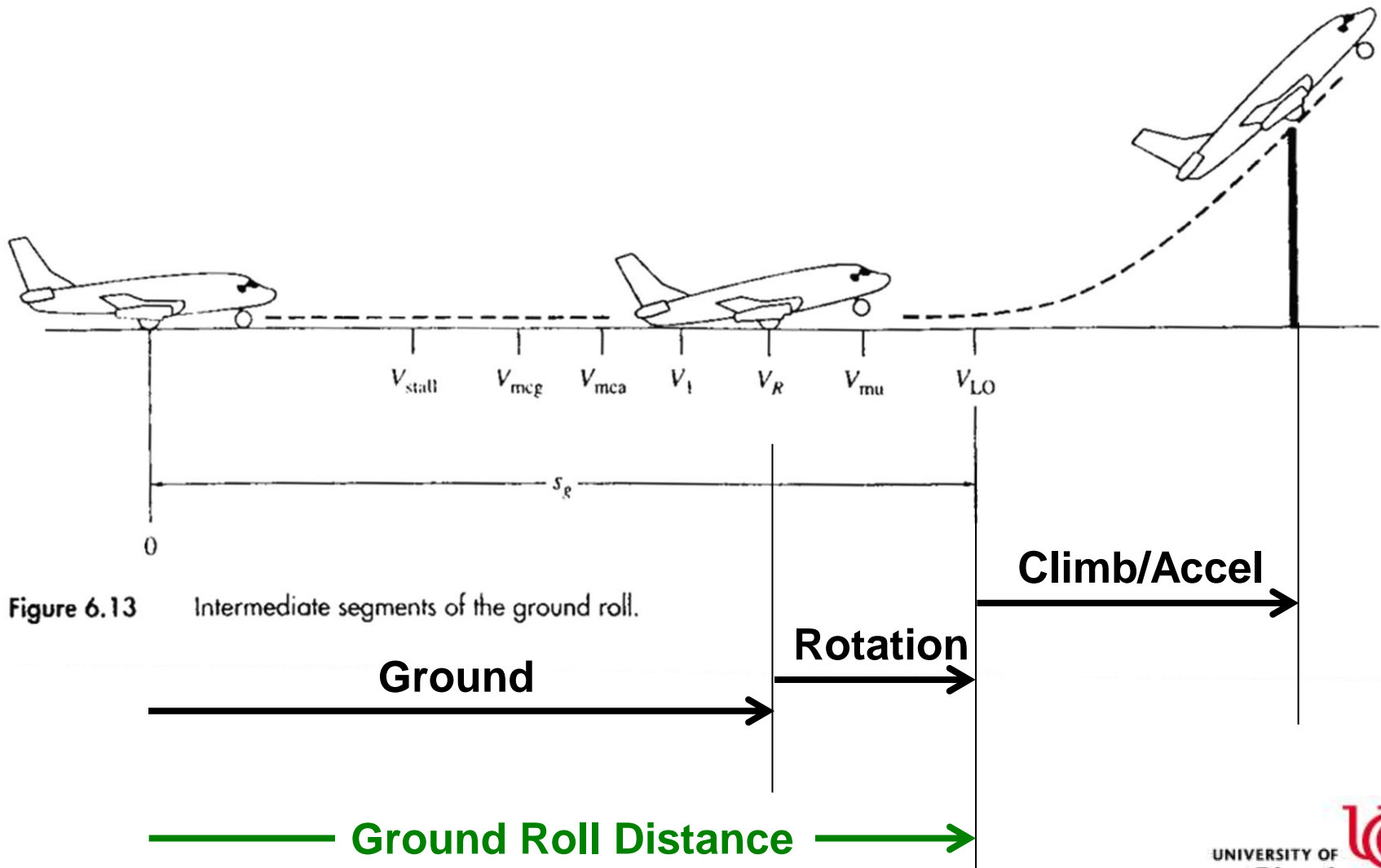
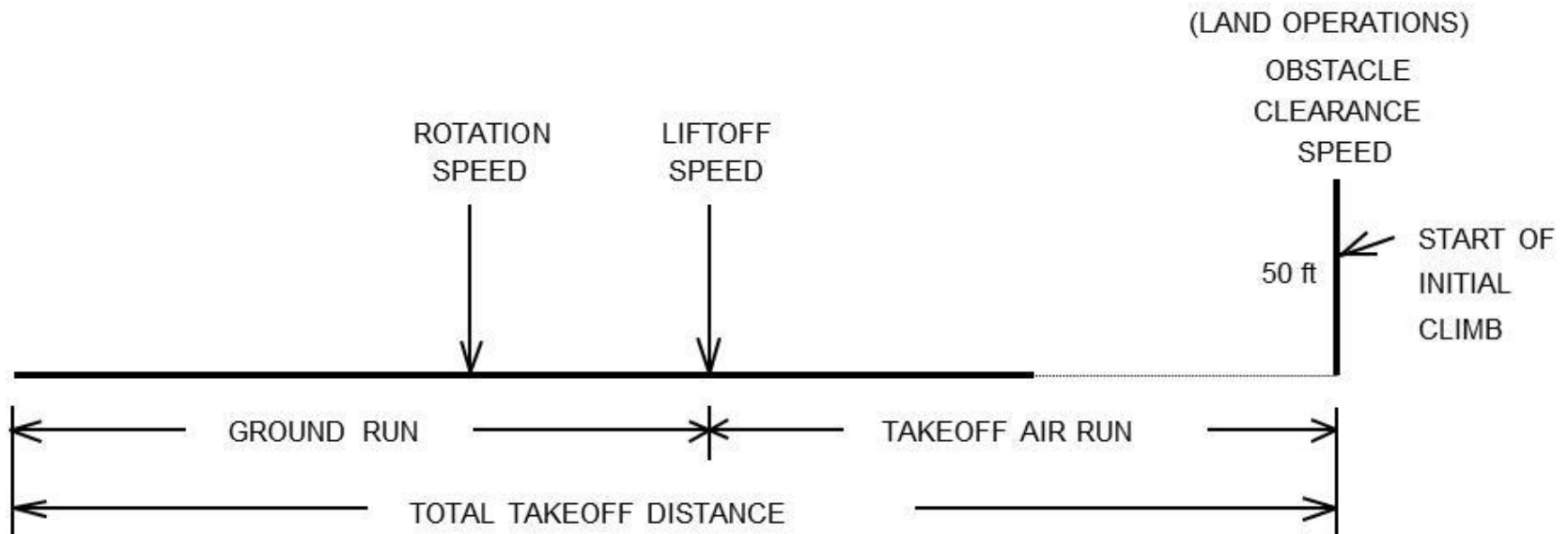


Figure 6.13 Intermediate segments of the ground roll.

Takeoff



Rotation Speed
 $> 1.05 V_{\text{stall}}$

Liftoff Speed
 $> 1.10 V_{\text{stall}}$

Obstacle Speed
 $> 1.20 V_{\text{stall}}$

$$V_{\text{stall}} = \sqrt{\frac{2W}{\rho S C_{L_{\text{max}}}}}$$

$$V_{\text{rot}} = 1.05 V_{\text{stall}}$$

Takeoff

$$s_g = \frac{V_{TO}^2}{2 a}$$

where a is the average acceleration
during the ground run

$$\Sigma F_x = T - D - R = m \frac{dV}{dt}$$

$$a = \frac{dV}{dt} = \frac{T - D - R}{W/g}$$

where T , D , and R are calculated at $0.7 V_{TO}$

Takeoff

$$a = \frac{dV}{dt} = \frac{T - D - R}{W/g}$$

where T, D, and R are calculated at $0.7 V_{TO}$

T = Thrust at $0.7 V_{TO}$ at takeoff altitude

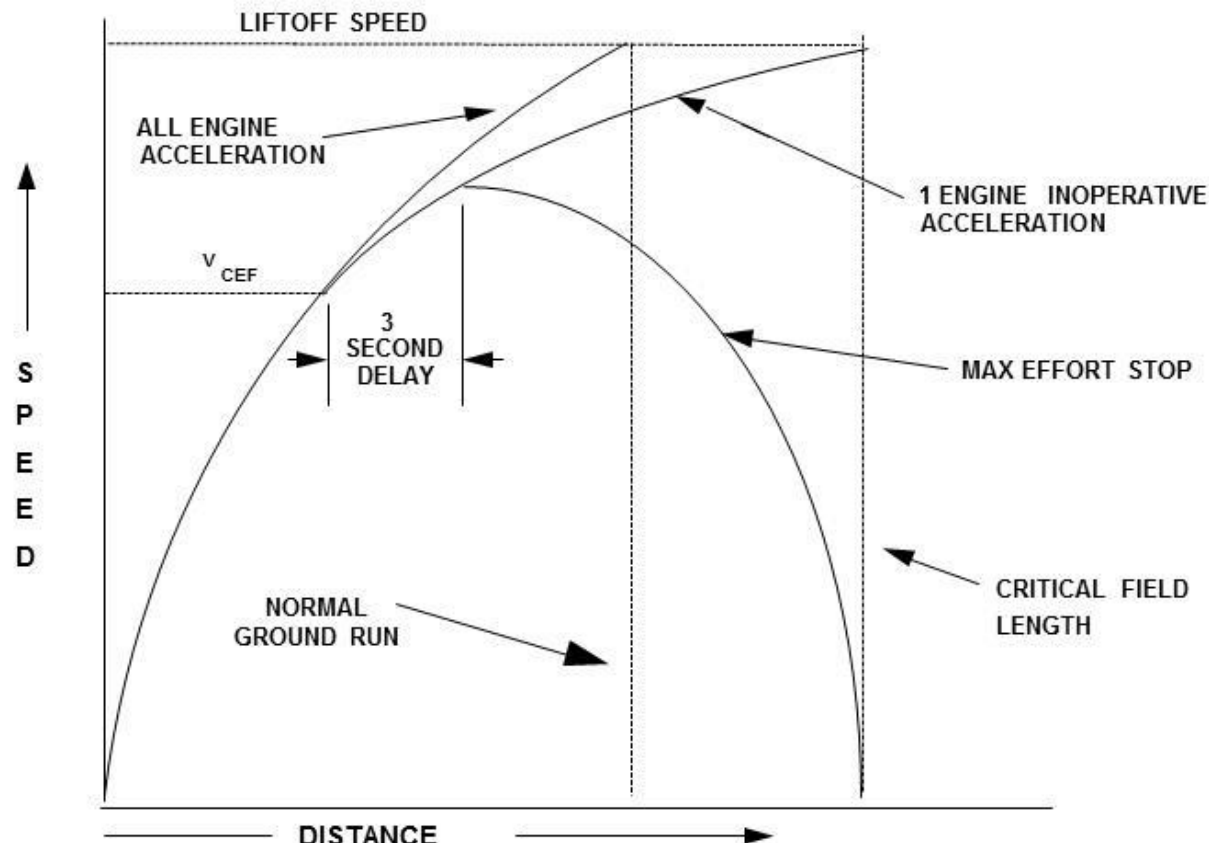
D = Drag at $0.7 V_{TO}$ at takeoff altitude
 $= (C_{D_0} + \Delta C_{D_0}) (0.5 \rho (0.7 V_{TO})^2 S)$

ΔC_{D_0} includes gear drag and flap drag

C_{D_L} and L are considered negligible

Critical Field Length

In the event of the loss of an engine during ground roll
Decision point at Critical Engine Failure Speed
Decide whether to continue takeoff or abort takeoff



Only for
multi-engine
aircraft

Landing

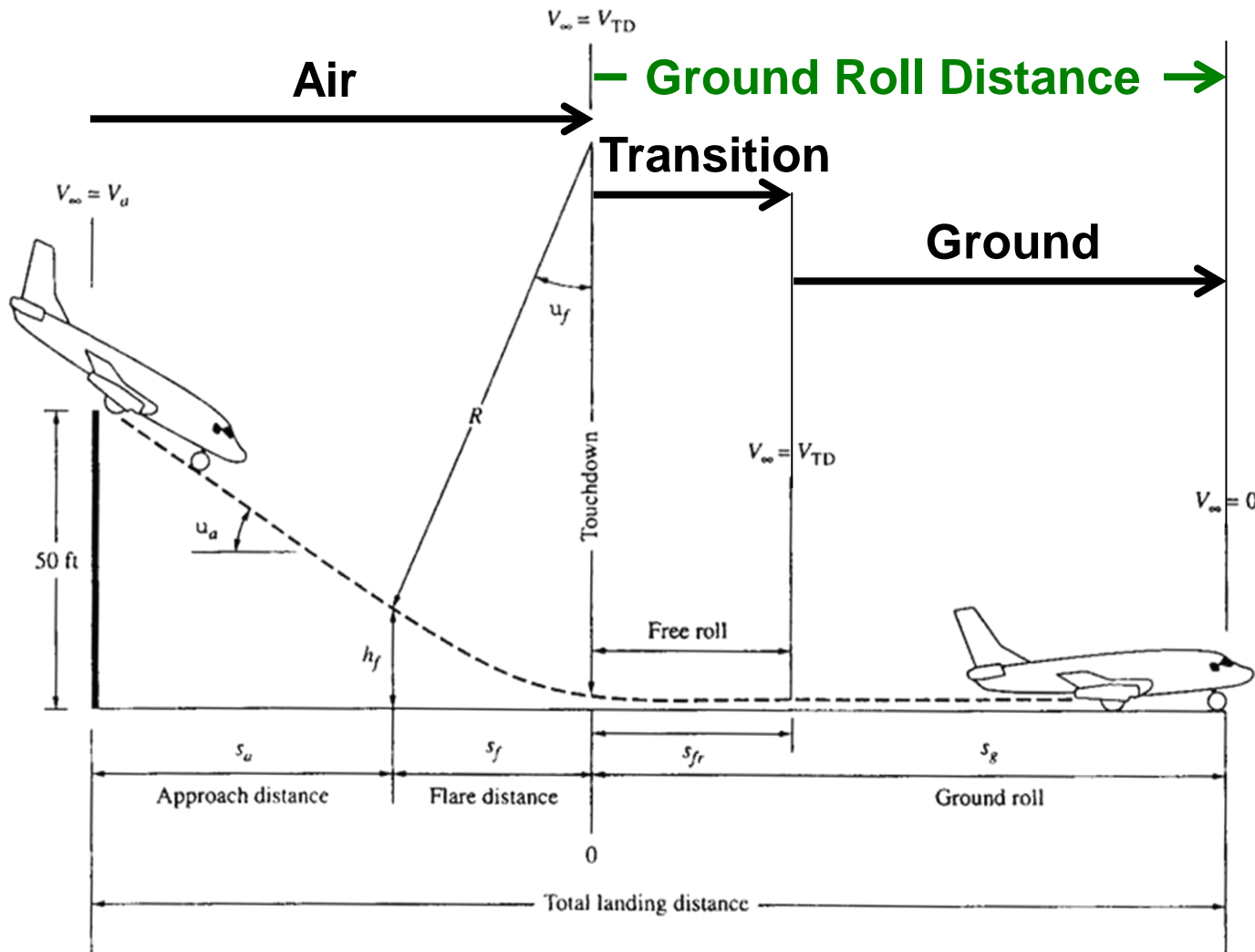


Figure 6.17 The landing path and landing distance.

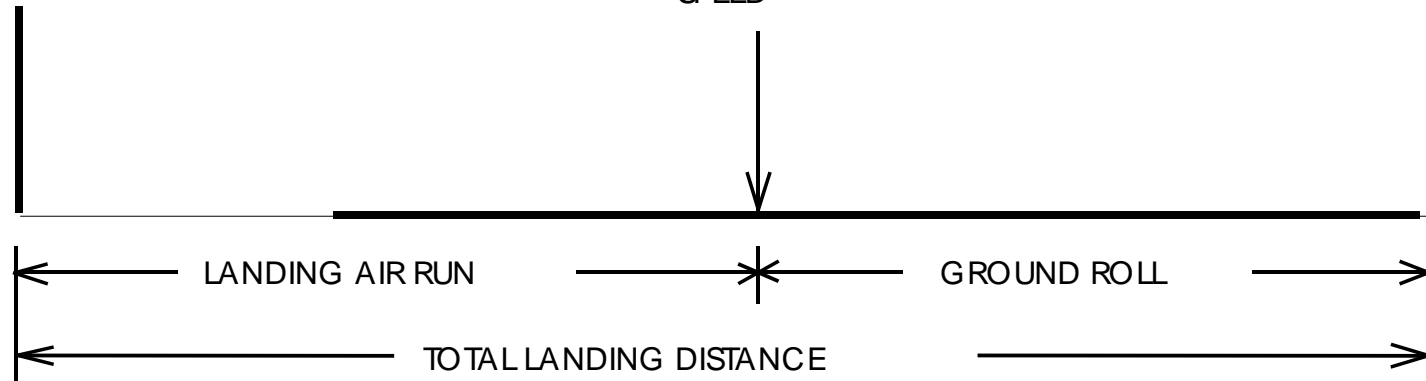
Landing

(LAND
OPERATIONS)

APPROACH
SPEED

50 ft

TOUCHDOWN
SPEED



**Approach
Speed V_{app}**
 $> 1.20 V_{stall}$

Commercial
 $> 1.30 V_{stall}$

**Touchdown
Speed V_{TD}**
 $> 1.10 V_{stall}$

Commercial
 $> 1.15 V_{stall}$

Landing – Transition

$$S_{TR} = \left[\left(\frac{V_{TD} + V_{TR}}{2} \right) - V_W \right] \Delta t_{TR}$$

Approximate by assuming 3 seconds to rotate the nose gear to the runway and deploy deceleration devices

$$S_{TR} = 3V_{TD}$$

V_{TD} – Touchdown Velocity

V_{TR} – Transition Velocity

V_W – Wind Velocity

Landing – Ground

$$s_{Brake} = - \frac{(V_{TD} - V_W)^2}{2g \left(T/W - \mu \right)} \quad 0.12 < \mu_b < 0.38$$

V_{TD} – Touchdown Velocity

V_W – Wind Velocity

For zero thrust and no wind

$$s_{brake} = \frac{V_{TD}^2}{2 g \mu}$$

$$s_g = s_{TR} + s_{brake}$$

AEEM 3042 – Aircraft Performance & Design

Course Exams

Will be done online via Canvas:

Exam #1 – Thursday, January 27 at 2:00 pm ET

Exam #2a – Tuesday, February 17 at 2:00 pm ET

Exam #2b – Thursday, March 10 at 2:00 pm ET

Start the exam between 2:00 and 2:15 pm ET

You will have **100 minutes** to complete the exam

Allowable resources include your notes, calculator, lecture slides, reference materials, spreadsheets

Academic Integrity = no help from other students!!

Questions are randomly selected

Exam will be graded automatically

Students will know their exam grade immediately after submitting

Questions during the exam should be e-mailed to the instructor

AEEM 3042 – Aircraft Performance & Design

Course Exam Tips

Be ready to take this exam!

There are 15 questions to answer (~100 points each):

- True / False
- Multiple Choice
- Matching
- Numerical values

Answers do not depend on any previous calculations, which means that there is no partial credit

You can answer the questions in any order

You can review any of the questions at any time

Do not hit the “Submit” button until you have finished

Questions?