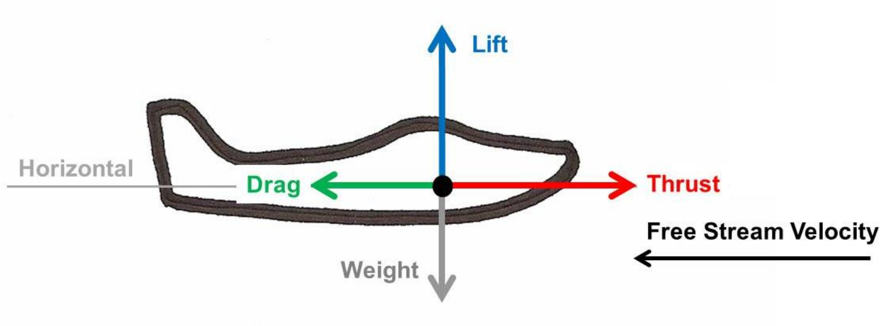
Aircraft Performance
Equations of Motion
Climb & Descent
Ceilings

Aircraft Performance



$\boxed{T = D}$ $\boxed{L = W}$

Range = How <u>FAR</u> can an aircraft fly?

Endurance = How <u>LONG</u> can an aircraft fly?

V_{max} = How <u>FAST</u> can an airplane fly?

V_{min} = How <u>SLOW</u> can an airplane fly?

Ceiling = How <u>HIGH</u> can an airplane fly?

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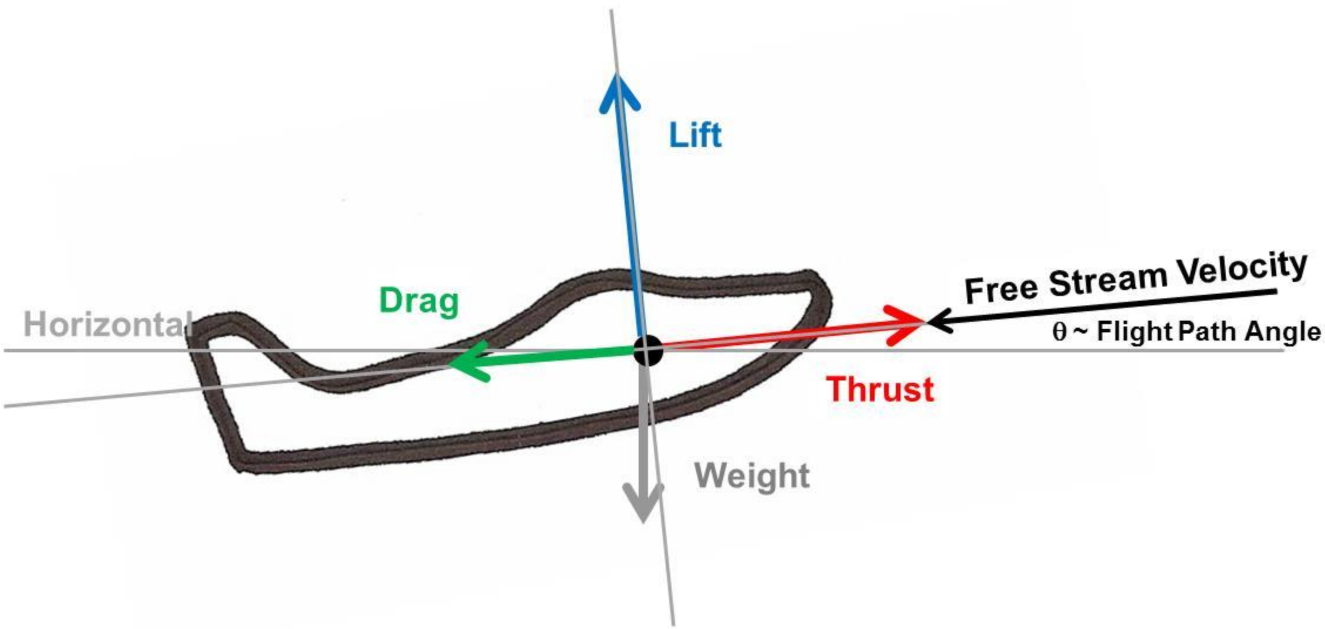
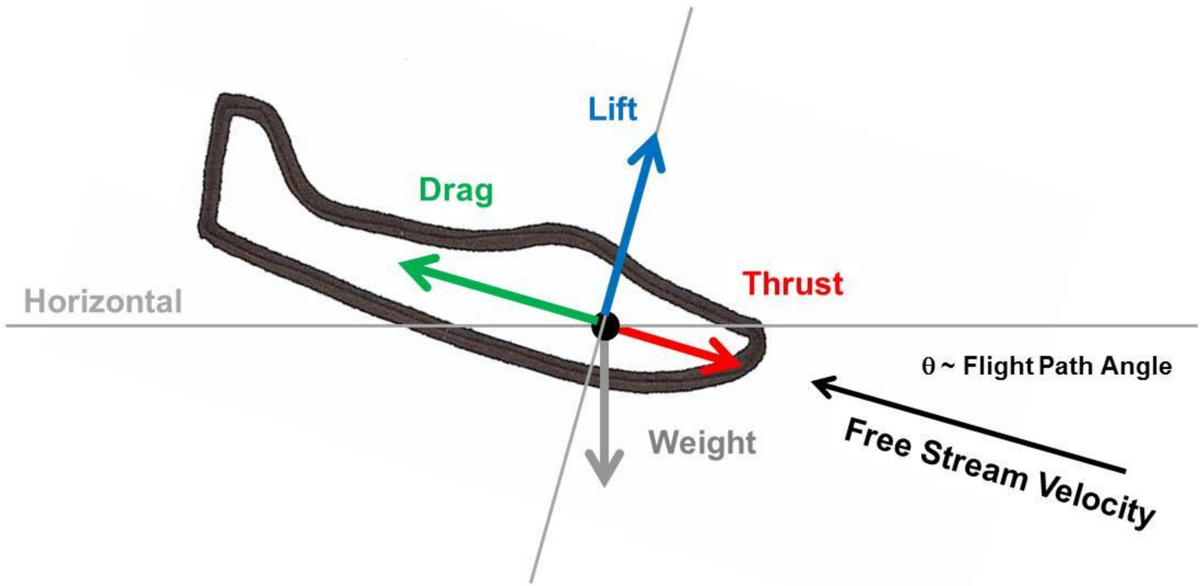
Aircraft Performance



Climb = How long does it take an aircraft to get to a higher altitude? How fast can it climb?

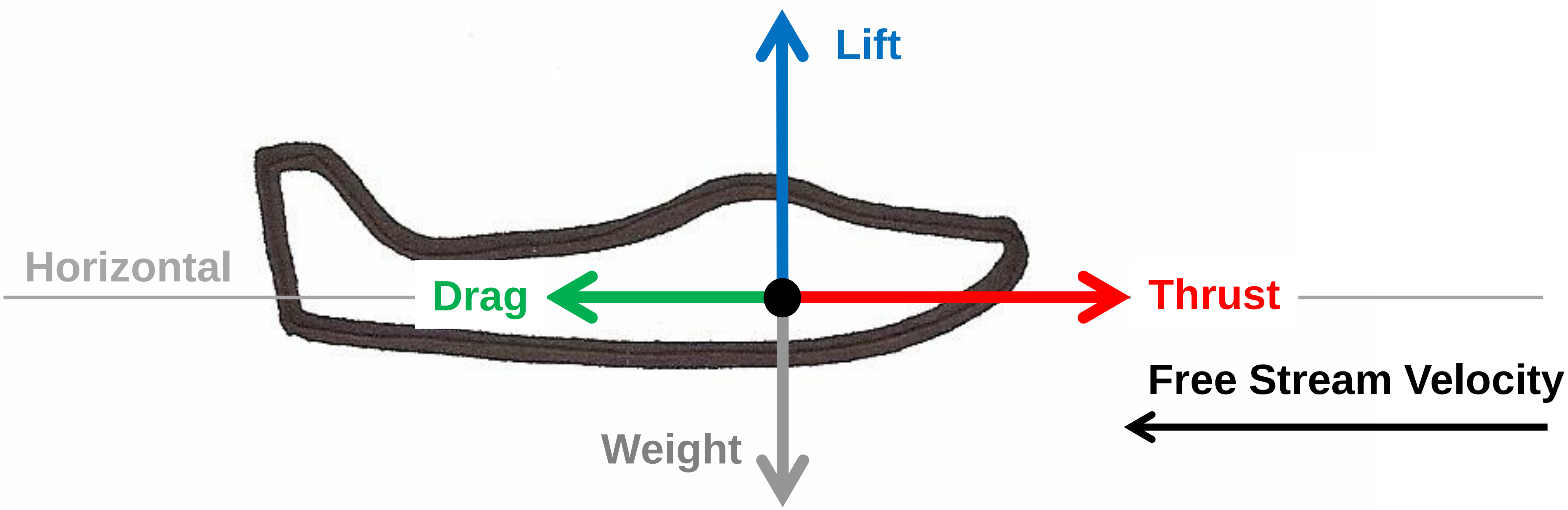
Aircraft Performance



Descent = How long does it take an aircraft to get to a lower altitude? How slow can it descend?

Simple Free Body Diagram

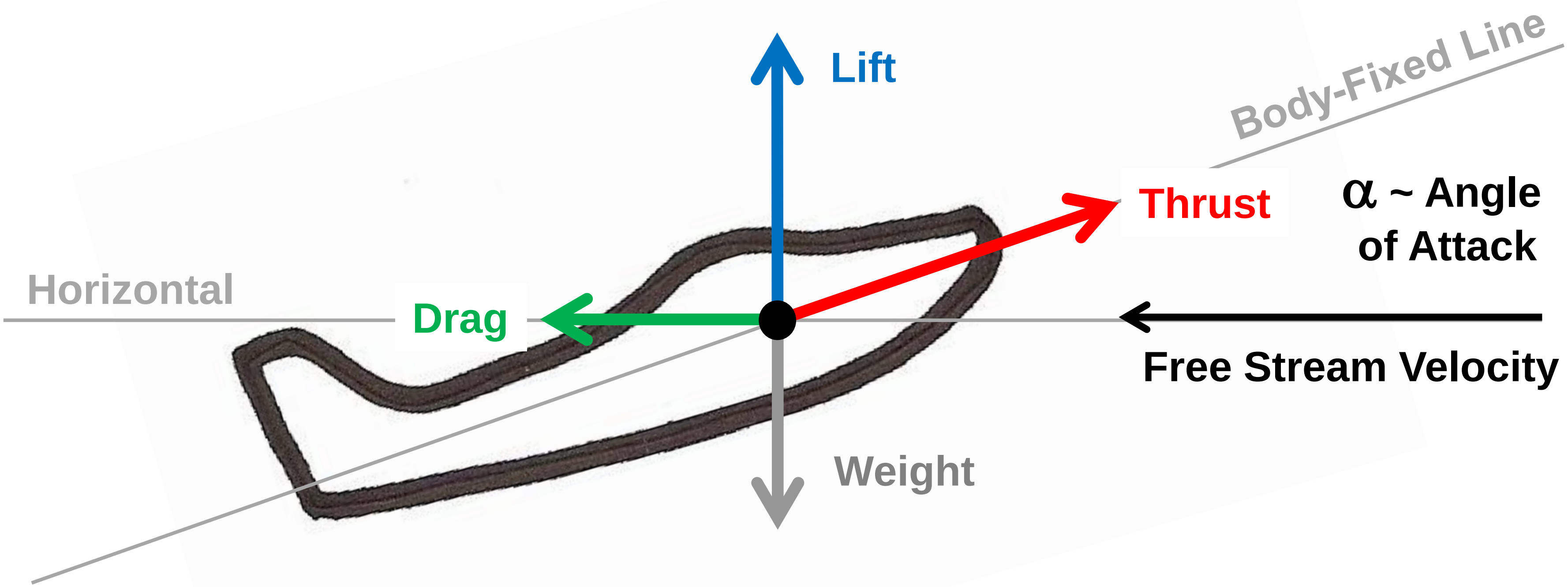
Straight and Level Flight



Lift acts perpendicular to the Free Stream Velocity
Drag acts parallel to the Free Stream Velocity
Weight acts vertically towards the ground
Thrust is fixed in the aircraft

Stability Axis System

Straight and Level Flight



Lift acts perpendicular to the Free Stream Velocity
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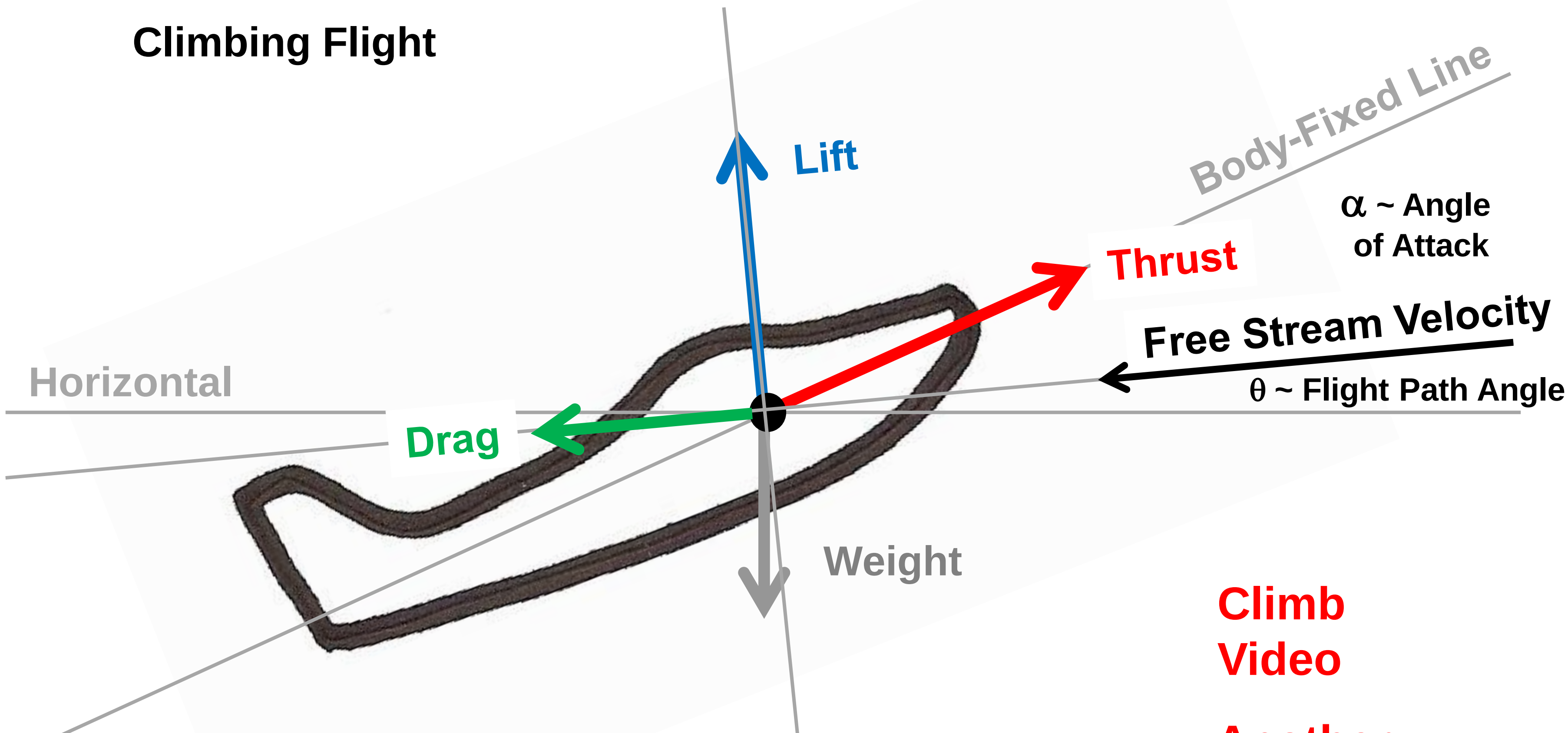
Stability Axis System



Climb Video

Another Climb Video

Lift acts perpendicular to the Free Stream Velocity
Drag acts parallel to the Free Stream Velocity
Weight acts vertically towards the ground
Thrust is fixed in the aircraft

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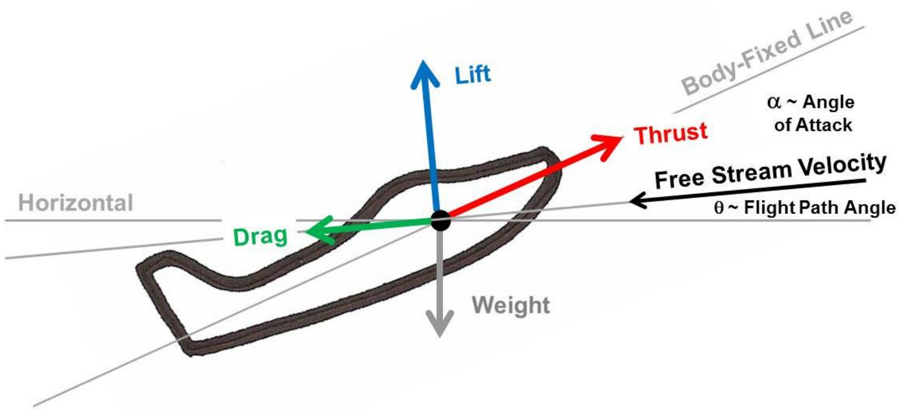
Steady Climbing Flight



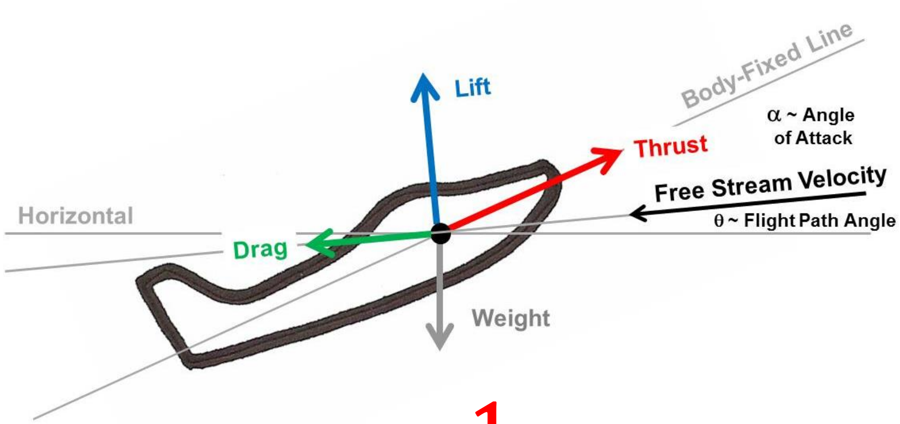
$$\Sigma F_x = T\cos\alpha - D - W\sin\theta = \frac{d(mV)}{dt} = \cancelto{0}{\dot{m}}V + m\cancelto{0}{\dot{V}}$$

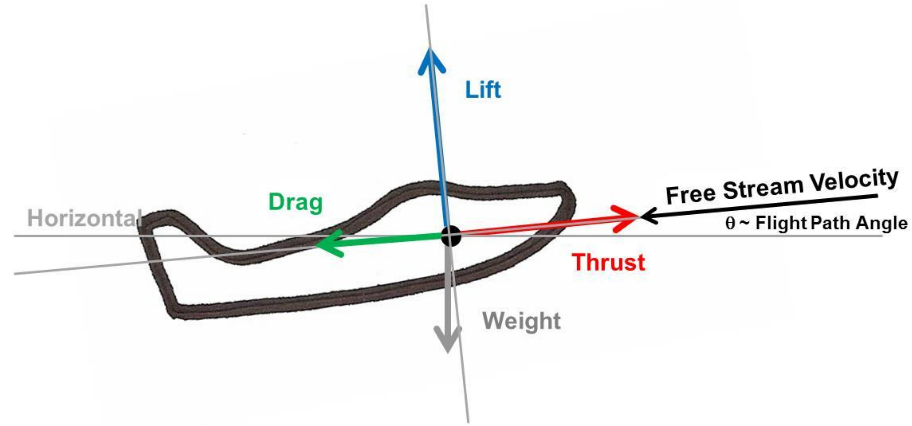
$$\Sigma F_z = L + T\sin\alpha - W\cos\theta = 0$$

Steady Climb = Not Accelerating

Steady Climb = Constant Weight

Steady Climbing Flight



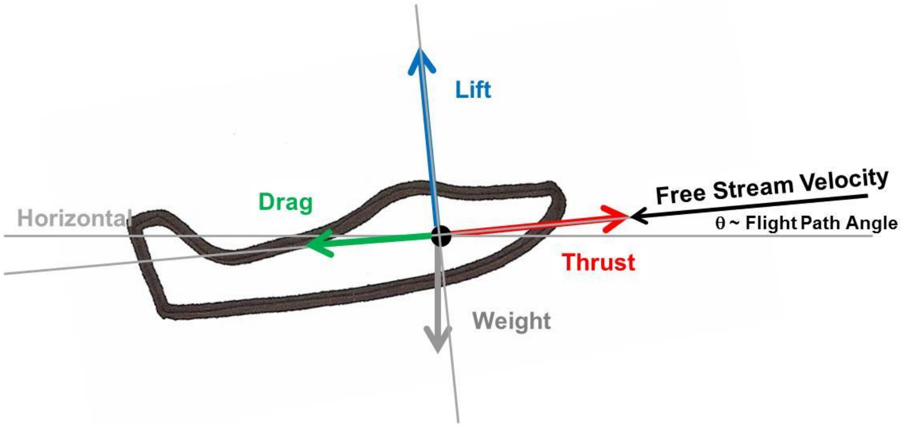


$$\Sigma\, F_x = T \overset{1}{\cos\alpha} - D - W \sin\theta = 0$$

$$\Sigma\, F_z = L + T \overset{0}{\sin\alpha} - W \cos\theta = 0$$

Small Angle Approximation Assumption for Angle of Attack (α)

Steady Climbing Flight



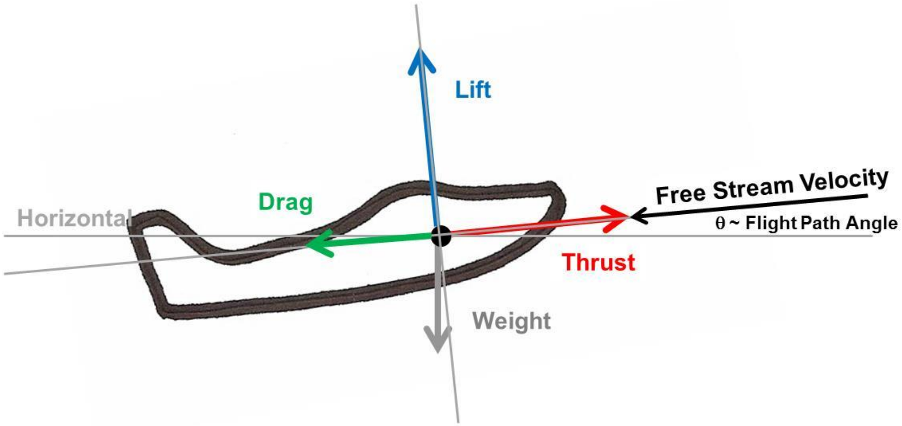
$$\Sigma F_x = T - D - W \sin\theta = 0$$

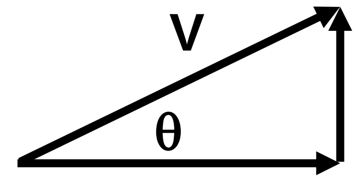
$$\boxed{\sin\theta = \frac{T-D}{W}}$$

$$\Sigma F_z = L - W \cos\theta = 0$$

$$\boxed{\cos\theta = \frac{L}{W}}$$

Steady Climbing Flight

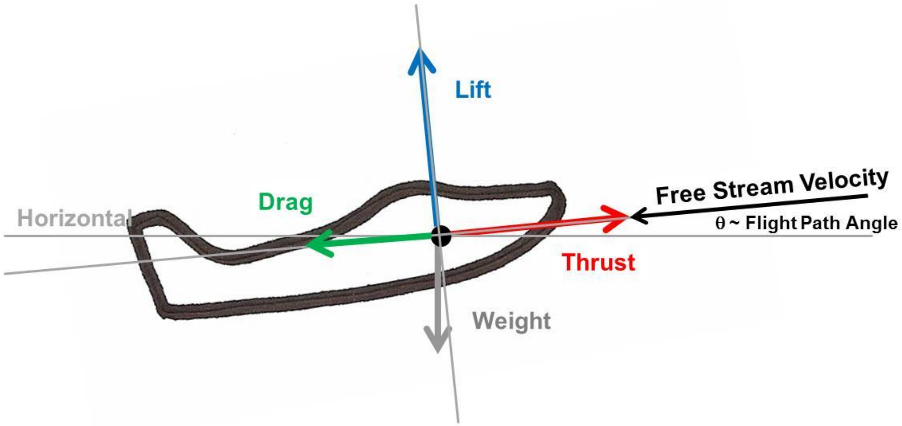


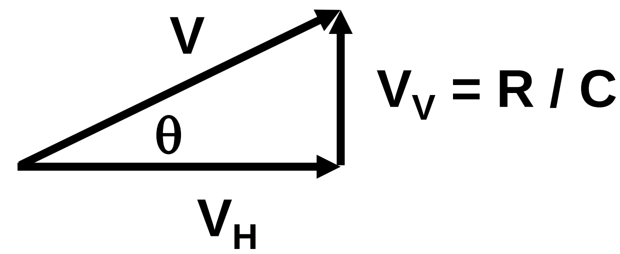


$$\sin\theta = \frac{T-D}{W} \longrightarrow V\sin\theta = \frac{(T-D)\,V}{W} = R/C$$

$$\cos\theta = \frac{L}{W} \longrightarrow L = W\cos\theta$$

Steady Climbing Flight



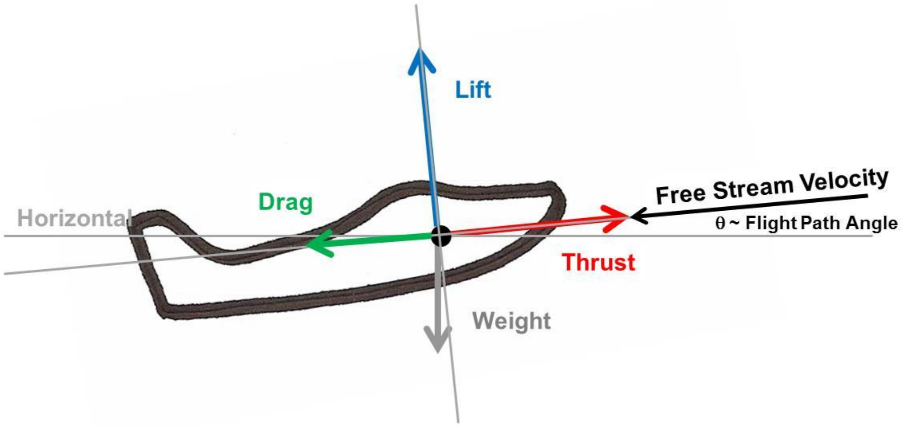


$$V \sin\theta = \frac{(T - D)\,V}{W} = R/C$$

$$(T - D)\,V = T\,V - D\,V = \text{Excess Power}$$

$$\frac{(T - D)\,V}{W} = \text{Specific Excess Power} = R/C$$

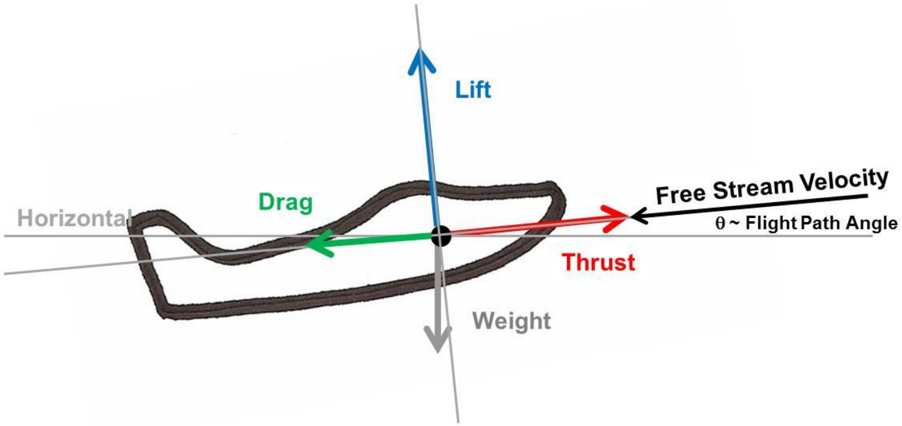
Aircraft Performance

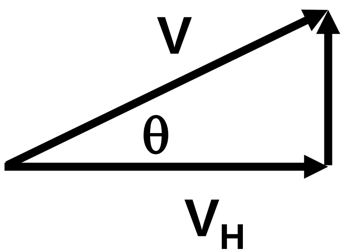


$\boxed{T > D}$ $\boxed{L < W}$

Climb = How long does it take an aircraft to get to a higher altitude? How fast can it climb?

Steady Climbing Flight





$$R/C = V \sin\theta = \frac{(T - D)\,V}{W}$$

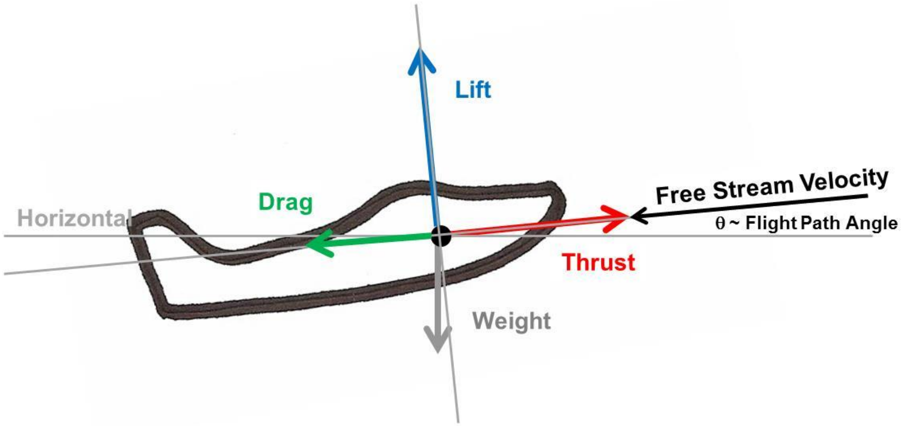
$$R/C = V\left[\left(\frac{T}{W}\right) - \frac{q\,C_{D_0}}{(W/S)} - \left(\frac{W}{S}\right)\frac{K}{q}\right]$$

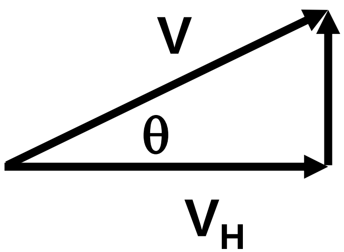
Fundamental Parameters

Speed and Altitude

Drag Polar Characteristics

Steady Climbing Flight

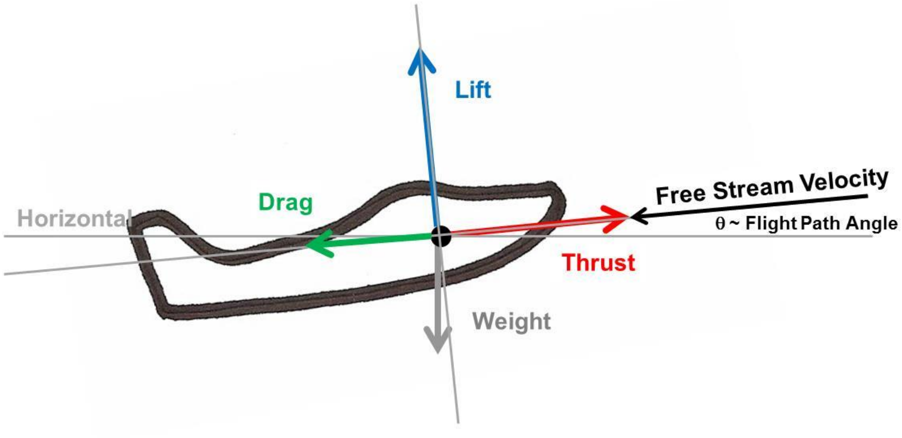


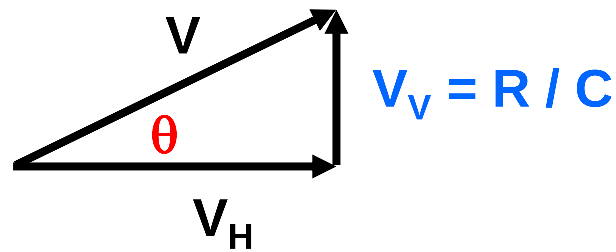


2 cases to examine: **Maximum Climb Angle**

Maximum Rate of Climb

Steady Climbing Flight

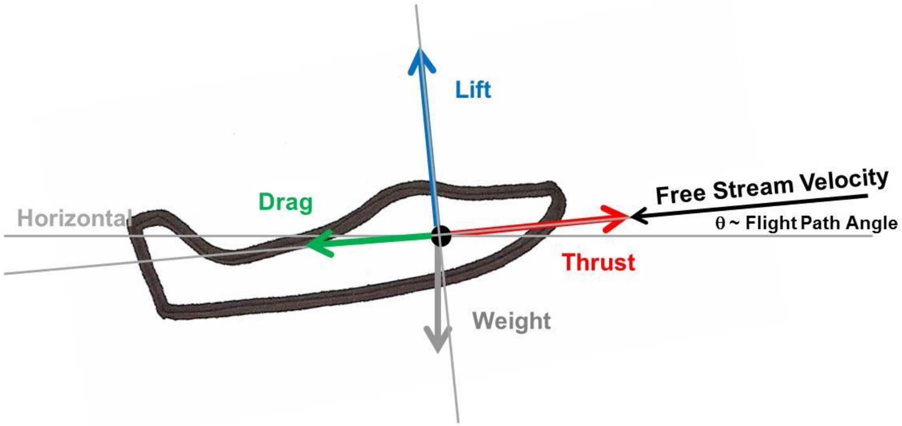


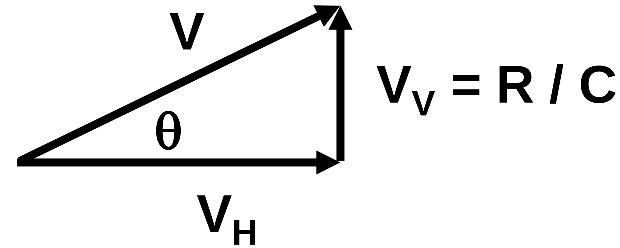


2 cases to examine: **Maximum Climb Angle**

Maximum Rate of Climb

Maximum Climb Angle





Maximum Climb Angle (jet aircraft)

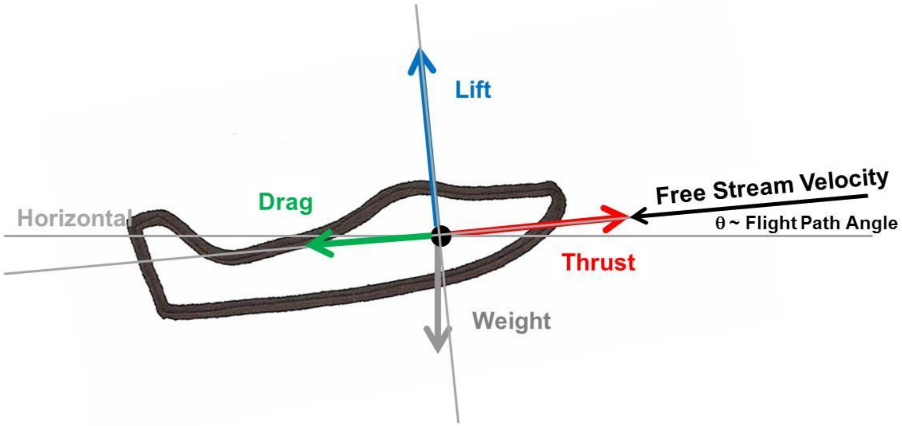
$$\sin\theta_{max} = \left(\frac{T}{W}\right) - \frac{1}{(L/D)_{max}} = \left(\frac{T}{W}\right) - \sqrt{4\,C_{D_0}K}$$

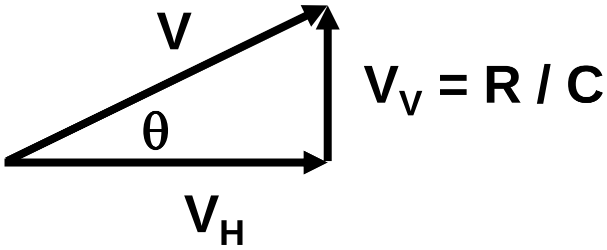
Dominated by thrust term

$$V_{\theta_{max}} = \sqrt{\frac{2}{\rho}\left(\frac{K}{C_{D_0}}\right)^{1/2}\left(\frac{W}{S}\right)\cos\theta_{max}}$$

Dominated by aero terms

Maximum Climb Angle





<u>Maximum Climb Angle</u> (jet aircraft)

$$V_{\theta_{max}} = \sqrt{\frac{2}{\rho}\left(\frac{K}{C_{D_0}}\right)^{1/2}\left(\frac{W}{S}\right)\cos\theta_{max}}$$

377 ft/sec @ SL

<u>Maximum Endurance</u> (jet aircraft R=1)

$$V_{(L/D)_{max}} = \sqrt{\frac{2}{\rho}\left(\frac{K}{C_{D_0}}\right)^{1/2}\left(\frac{W}{S}\right)}$$

386 ft/sec @ SL

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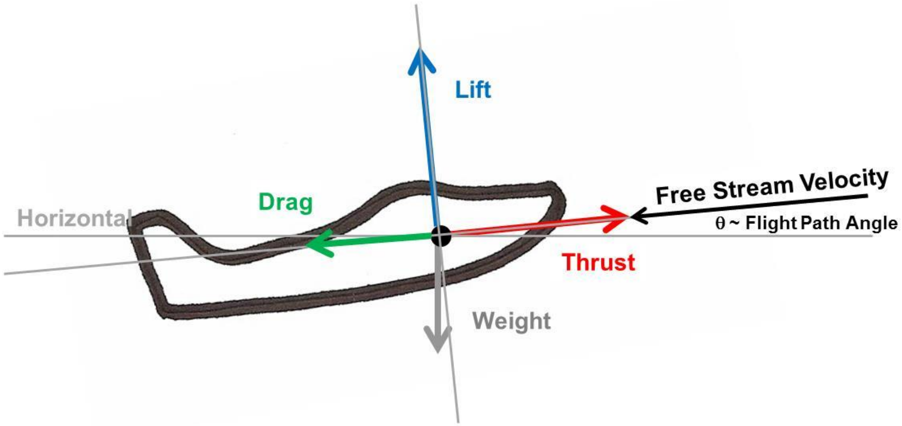
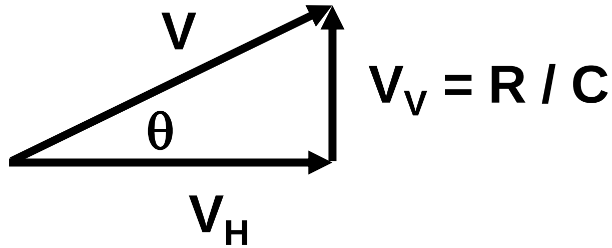
Maximum Rate of Climb



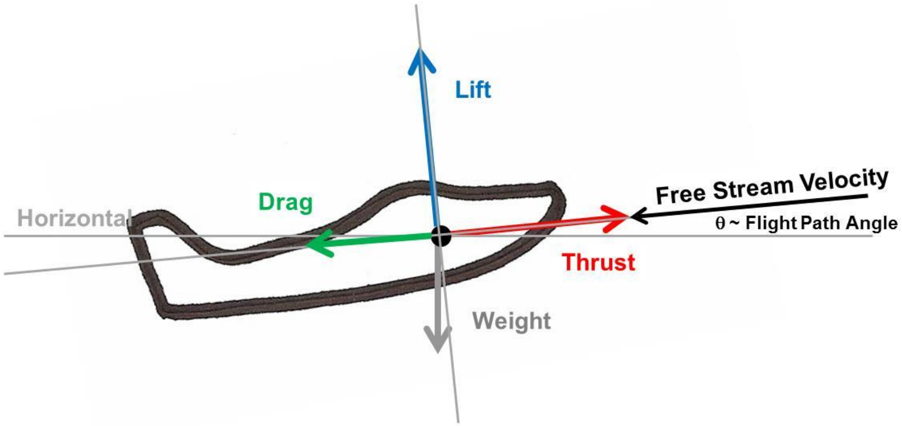


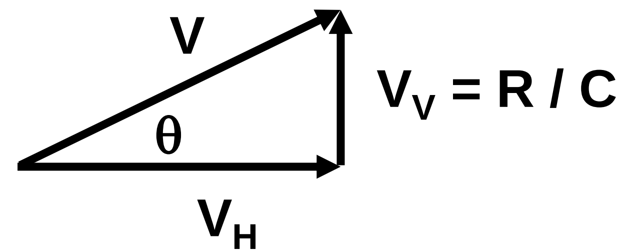
Maximum Rate of Climb (jet aircraft)

$$(R/C)_{max} = \left[\left(\frac{W}{S}\right)\frac{Z}{3\,\rho\,C_{D_0}}\right]^{1/2}\left(\frac{T}{W}\right)^{3/2}\left[1-\frac{Z}{6}-\frac{3}{2\,Z\,(T/W)^2\,(L/D)^2_{max}}\right]$$

$$\text{where}\quad Z = 1+\sqrt{1+\frac{3}{(T/W)^2\,(L/D)^2_{max}}}$$

Maximum Rate of Climb





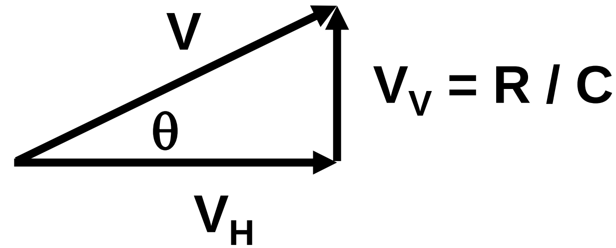
<u>Maximum Rate of Climb</u> (jet aircraft)

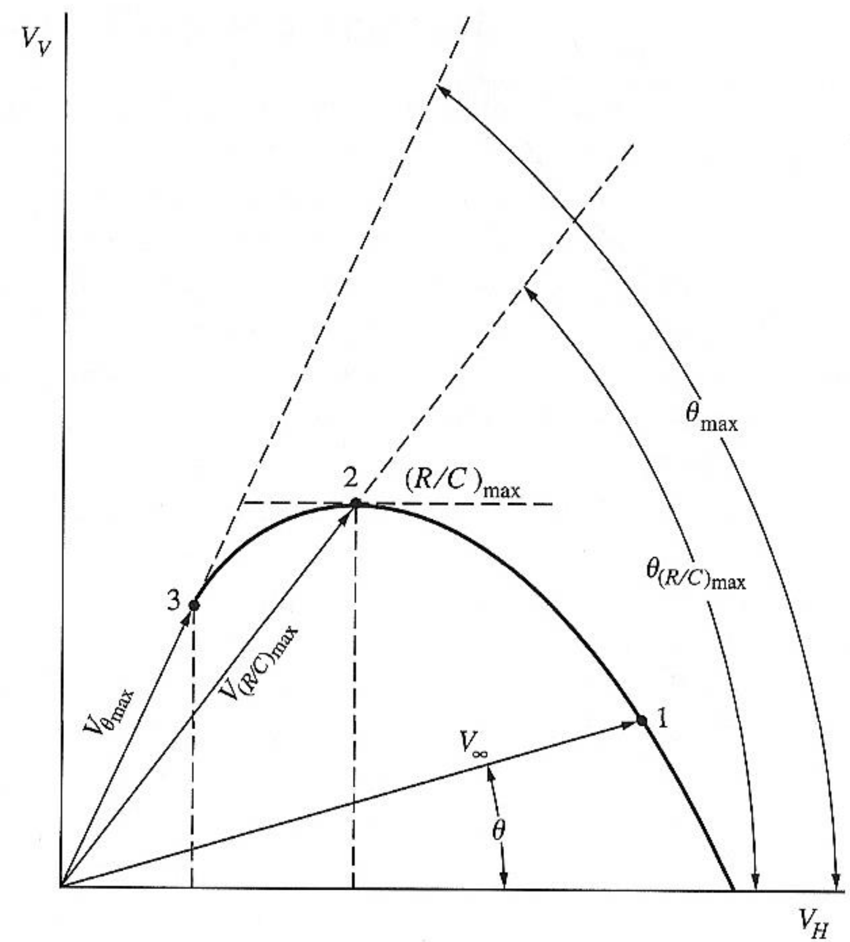
$$V_{(R/C)_{max}} = \left[\left(\frac{T}{W}\right) \left(\frac{W}{S}\right) \frac{Z}{3\,\rho\, C_{D_0}} \right]^{1/2}$$

$$\text{where} \quad Z = 1 + \sqrt{1 + \frac{3}{(T/W)^2 \, (L/D)^2_{max}}}$$

Steady Climbing Flight

Hodograph: Velocity Diagram





Vel (fps)	Theta	VH	VV
200	13.94	194.1	48.2
250	16.39	239.8	70.5
300	17.52	286.1	90.3
350	17.99	332.9	108.1
400	18.06	380.3	124.0
450	17.87	428.3	138.1
500	17.50	476.8	150.4
550	16.99	526.0	160.7
600	16.35	575.7	168.9
650	15.62	626.0	175.0
700	14.79	676.8	178.7
750	13.87	728.1	179.8
800	12.88	779.9	178.3
850	11.81	832.0	174.0
900	10.67	884.4	166.6
950	9.46	937.1	156.1
1000	8.18	989.8	142.3
1050	6.83	1042.5	124.9
1100	5.42	1095.1	103.9
1150	3.94	1147.3	79.0
1200	2.39	1199.0	50.1
1250	0.78	1249.9	17.1
1300	-0.90	1299.8	-20.4

Example 5.13

Maximum Rate of Climb

$$R/C = V \sin\theta = \frac{(T - D)\,V}{W}$$

Gulfstream IV twin-turbofan biz jet:

$C_{D_0} = 0.0150$ $K = 0.08$

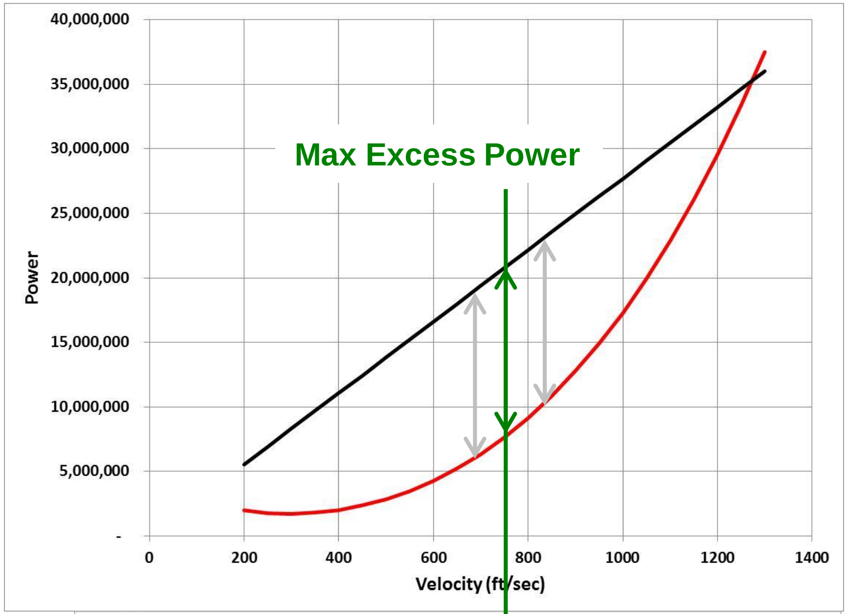
$W = 73{,}000$ lb

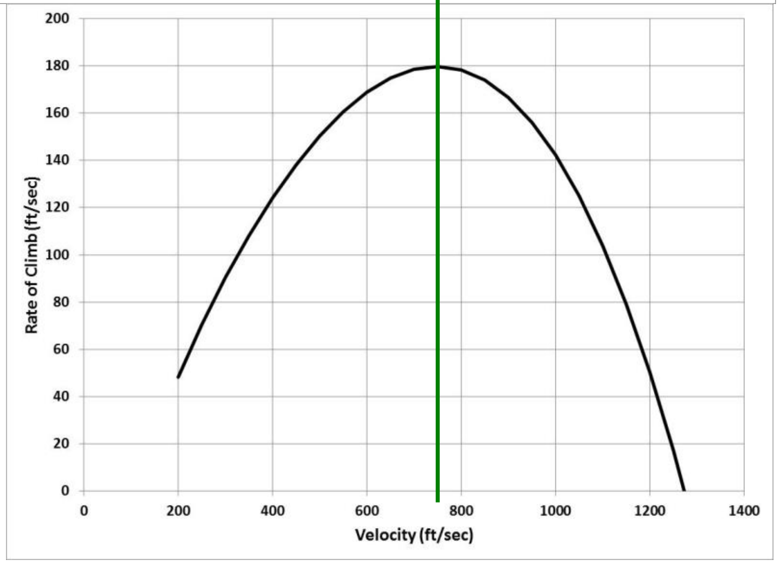
h = sea level

CD0	0.0150	**Wt**	73,000	lb			
K	0.08	**Alt**	0	ft			
		rho	0.00237688				
		QMS	1481.4				
		a	1116.45	ft/sec			
		S	950	sq ft			
					Power		
Vel (fps)	**CL**	**CD**	**D (lb)**	**T (lb)**	**TV**	**DV**	**R/C**
200	1.6164	0.2240	10117	27700	5,540,000	2,023,428	48.2
250	1.0345	0.1006	7100	27700	6,925,000	1,774,976	70.5
300	0.7184	0.0563	5720	27700	8,310,000	1,715,897	90.3
350	0.5278	0.0373	5157	27700	9,695,000	1,804,951	108.1
400	0.4041	0.0281	5070	27700	11,080,000	2,027,869	124.0
450	0.3193	0.0232	5294	27700	12,465,000	2,382,370	138.1
500	0.2586	0.0204	5744	27700	13,850,000	2,872,166	150.4
550	0.2137	0.0187	6371	27700	15,235,000	3,504,237	160.7
600	0.1796	0.0176	7146	27700	16,620,000	4,287,471	168.9
650	0.1530	0.0169	8049	27700	18,005,000	5,231,930	175.0
700	0.1319	0.0164	9069	27700	19,390,000	6,348,430	178.7
750	0.1149	0.0161	10198	27700	20,775,000	7,648,290	179.8
800	0.1010	0.0158	11429	27700	22,160,000	9,143,173	178.3
850	0.0895	0.0156	12759	27700	23,545,000	10,844,990	174.0
900	0.0798	0.0155	14184	27700	24,930,000	12,765,824	166.6
950	0.0716	0.0154	15703	27700	26,315,000	14,917,891	156.1
1000	0.0647	0.0153	17314	27700	27,700,000	17,313,502	142.3
1050	0.0586	0.0153	19014	27700	29,085,000	19,965,046	124.9
1100	0.0534	0.0152	20805	27700	30,470,000	22,884,964	103.9
1150	0.0489	0.0152	22683	27700	31,855,000	26,085,746	79.0
1200	0.0449	0.0152	24650	27700	33,240,000	29,579,917	50.1
1250	0.0414	0.0151	26704	27700	34,625,000	33,380,028	17.1
1300	0.0383	0.0151	28845	27700	36,010,000	37,498,656	(20.4)

Maximum Rate of Climb

Example 5.13





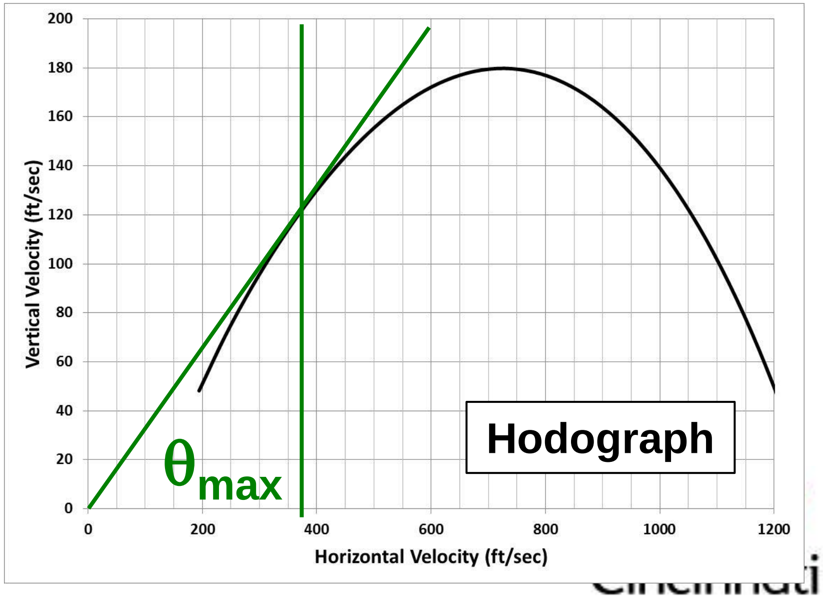
From the graphs:

$V_{(R/C)_{max}} = 750$ ft/sec

$(R/C)_{max} = 180$ ft/sec

$V_{\theta_{max}} = 375$ ft/sec

$\theta_{max} = \arctan(125/375) = 18.4^{\circ}$



Example 5.13

Maximum Rate of Climb

Maximum Climb Angle

$$\sin\theta_{max} = \left(\frac{T}{W}\right) - \sqrt{4\, C_{D_0} K}$$

$$V_{\theta_{max}} = \sqrt{\frac{2}{\rho}\left(\frac{K}{C_{D_0}}\right)^{1/2}\left(\frac{W}{S}\right)\cos\theta_{max}}$$

Maximum Rate of Climb

$$V_{(R/C)_{max}} = \left[\left(\frac{T}{W}\right)\left(\frac{W}{S}\right)\frac{Z}{3\,\rho\, C_{D_0}}\right]^{1/2}$$

CD0	0.0150	**Wt**	73,000	lb	
K	0.08	**Alt**	0	ft	
T	27,700	**rho**	0.00237688		
Z	2.0488	**QMS**	1481.4		
		a	1116.45	ft/sec	
		S	950	sq ft	
T/W	0.3795				
W/S	76.84				
L/D max	14.43				
Max Climb Angle					
	Max Theta	18.07	degrees		**18.4**
	Velocity	376.8	ft/sec		**375**
Max Rate of Climb					
	Max R/C	179.8	ft/sec		**180**
	Velocity	747.3	ft/sec		**750**

$$(R/C)_{max} = \left[\left(\frac{W}{S}\right)\frac{Z}{3\,\rho\, C_{D_0}}\right]^{1/2}\left(\frac{T}{W}\right)^{3/2}\left[1 - \frac{Z}{6} - \frac{3}{2\,Z\,(T/W)^2\,(L/D)^2_{max}}\right]$$

$$\text{where}\quad Z = 1 + \sqrt{1 + \frac{3}{(T/W)^2\,(L/D)^2_{max}}}$$

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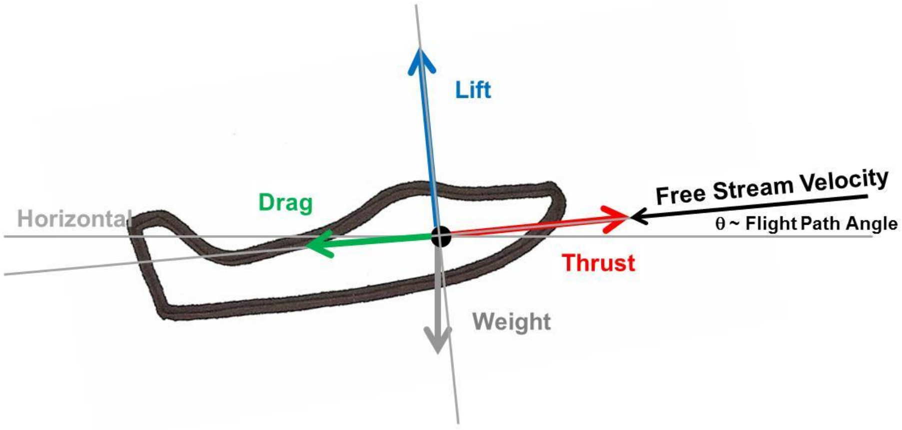
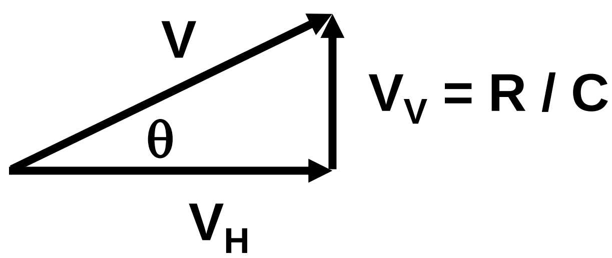
Time to Climb





$$\frac{dh}{dt} = R/C \longrightarrow dt = \frac{dh}{R/C} \longrightarrow t = \int_{h_1}^{h_2} \frac{dh}{R/C}$$

$$\boxed{t_{min} = \int_{h_1}^{h_2} \frac{dh}{(R/C)_{max}}}$$

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Time to Climb

Example 5.17

Calculate the maximum Rate of Climb at many altitudes

Divide Δh by $(R/C)_{max}$ at each altitude

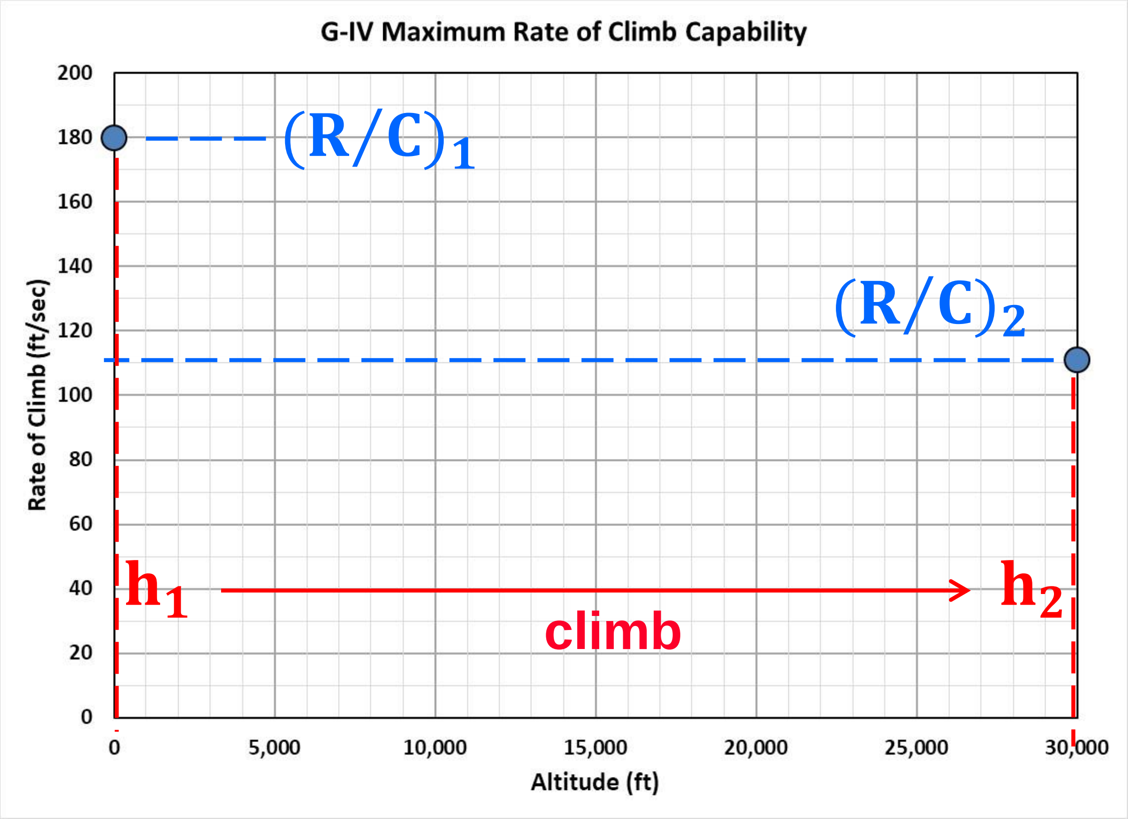
Sum up all of the increments to get Time to Climb

Altitude	DR	rho	Thrust	T/W	W/S	Z	max R/C	dh/(R/C)	
0	1.0000	0.00237688	27,700	0.3795	76.84	2.0488	179.8		
2,000	0.9428	0.00224086	26,738	0.3663	76.84	2.0523	175.1	11.3	
4,000	0.8881	0.00211087	25,796	0.3534	76.84	2.0561	170.5	11.6	
6,000	0.8359	0.00198673	24,875	0.3408	76.84	2.0602	165.8	11.9	
8,000	0.7860	0.00186826	23,974	0.3284	76.84	2.0647	161.2	12.2	
10,000	0.7385	0.00175527	23,093	0.3163	76.84	2.0695	156.6	12.6	
12,000	0.6932	0.00164758	22,232	0.3046	76.84	2.0748	152.0	13.0	
14,000	0.6500	0.00154502	21,391	0.2930	76.84	2.0806	147.4	13.4	
16,000	0.6090	0.00144742	20,570	0.2818	76.84	2.0869	142.9	13.8	
18,000	0.5699	0.00135461	19,768	0.2708	76.84	2.0938	138.3	14.2	
20,000	0.5328	0.00126642	18,985	0.2601	76.84	2.1013	133.8	14.7	
22,000	0.4976	0.00118269	18,222	0.2496	76.84	2.1096	129.2	15.2	
24,000	0.4642	0.00110326	17,478	0.2394	76.84	2.1186	124.7	15.8	
26,000	0.4325	0.00102798	16,752	0.2295	76.84	2.1285	120.1	16.3	
28,000	0.4025	0.00095670	16,045	0.2198	76.84	2.1393	115.5	17.0	
30,000	0.3741	0.00088926	15,356	0.2104	76.84	2.1513	111.0	17.7	
								Time to Climb	
								210.5	**seconds**
								3.51	**minutes**

Time to Climb

Alternative Method in Textbook – assume linear R/C$_{max}$ vs Altitude

$$t_{min} = \int_{h_1}^{h_2} \frac{dh}{(R/C)_{max}}$$



Time to Climb

<u>Alternative Method in Textbook</u> – assume linear R/C$_{max}$ vs Altitude

Calculate the maximum Rate of Climb at Sea Level
Calculate the maximum Rate of Climb at 30,000 ft
Derive the linear relationship R/C$_{max}$ = a + b Δh

$$\mathbf{t_{min}} = \int_{h_1}^{h_2} \frac{dh}{(R/C)_{max}} = \int_{h_1}^{h_2} \frac{dh}{a + b\,\Delta h}$$

$$\mathbf{t_{min}} = \frac{1}{b}\left[\ln(a + b\,\Delta h) - \ln a\right]$$

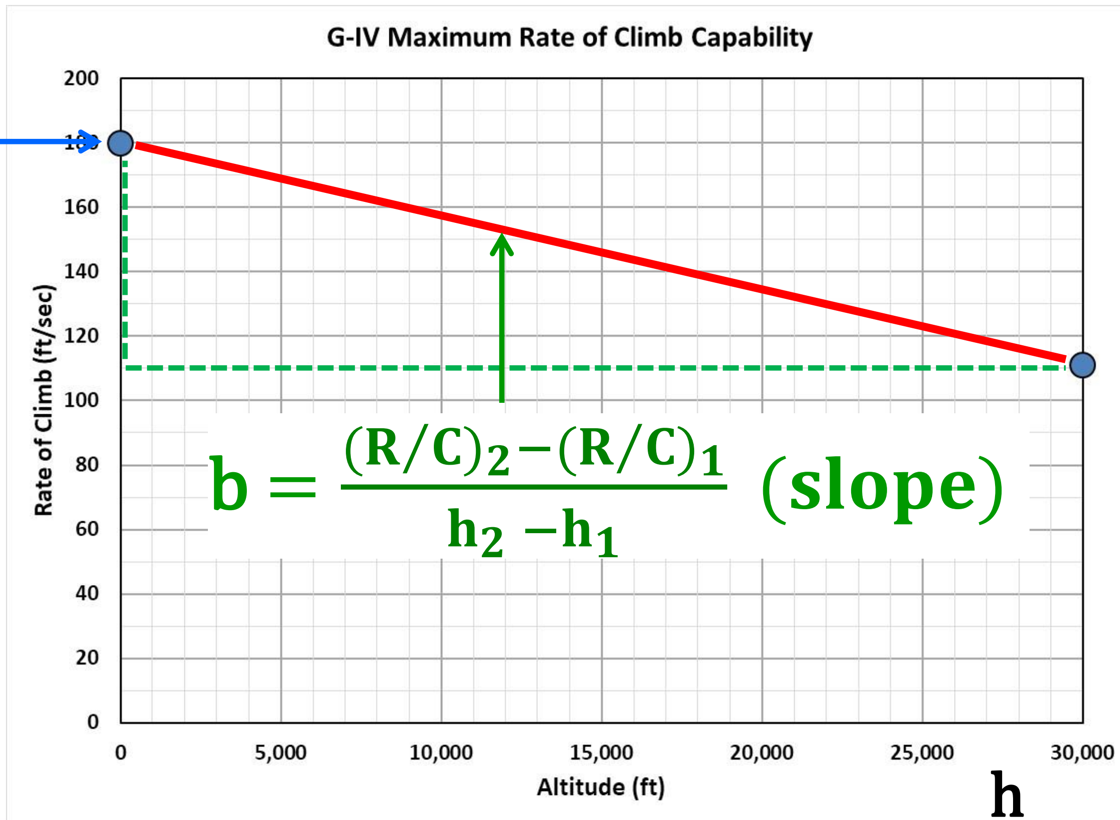
Time to Climb

<u>Alternative Method in Textbook</u> – assume linear R/C_{max} vs Altitude

Calculate the maximum Rate of Climb at Sea Level
Calculate the maximum Rate of Climb at 30,000 ft
Derive the linear relationship $R/C_{max} = a + b\,\Delta h$

$a = (R/C)_1$

R/C



Example 5.17

Time to Climb

Alternative Method in Textbook

$$\mathbf{t_{min}} = \frac{\mathbf{1}}{\mathbf{b}}\left[\mathbf{ln(a + b\ \Delta h) - ln\ a}\right]$$

since $(R/C)_{max} = a + b\ \Delta h = a + b\ (h_2 - h_1)$

$$\mathbf{t_{min}} = \frac{\mathbf{1}}{\mathbf{b}}\left[\mathbf{ln(R/C)_2 - ln(R/C)_1}\right]$$

Time to Climb

Alternative Method in Textbook

$$\mathbf{t_{min}} = \frac{\mathbf{1}}{\mathbf{b}}\left[\mathbf{ln}(\mathbf{R/C})_{\mathbf{2}} - \mathbf{ln}(\mathbf{R/C})_{\mathbf{1}}\right]$$

Solve for b

At Sea Level: $(R/C)_{max}$ **= 179.8 ft/sec**
At 30,000 ft: $(R/C)_{max}$ **= 111.0 sec**

a = 179.8 **b = (111.0 - 179.8) / (30,000 – 0) = -0.0022933**

$(R/C)_{max}$ **= 179.8 – 0.0022933 Δh**

$$\mathbf{t_{min}} = \frac{\mathbf{1}}{\mathbf{-0.0022933}}\left[\mathbf{ln}(\mathbf{111.0}) - \mathbf{ln}(\mathbf{179.8})\right]$$

$$\boxed{\mathbf{t_{min} = 210.3\ sec = 3.51\ min}}$$

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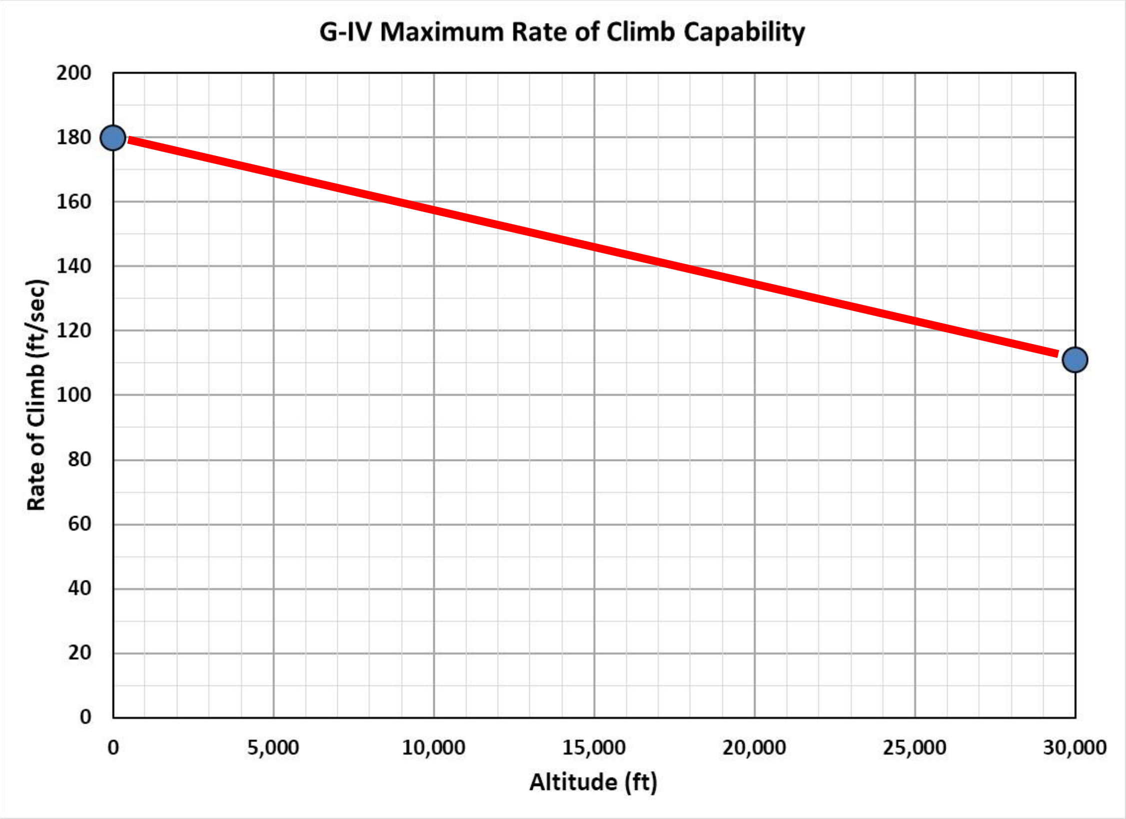
Time to Climb

<u>Easier Method</u> – assume linear R/C_{max} vs Altitude

Calculate the maximum Rate of Climb at Sea Level
Calculate the maximum Rate of Climb at 30,000 ft
Calculate the average Rate of Climb



Time to Climb

Easier Method

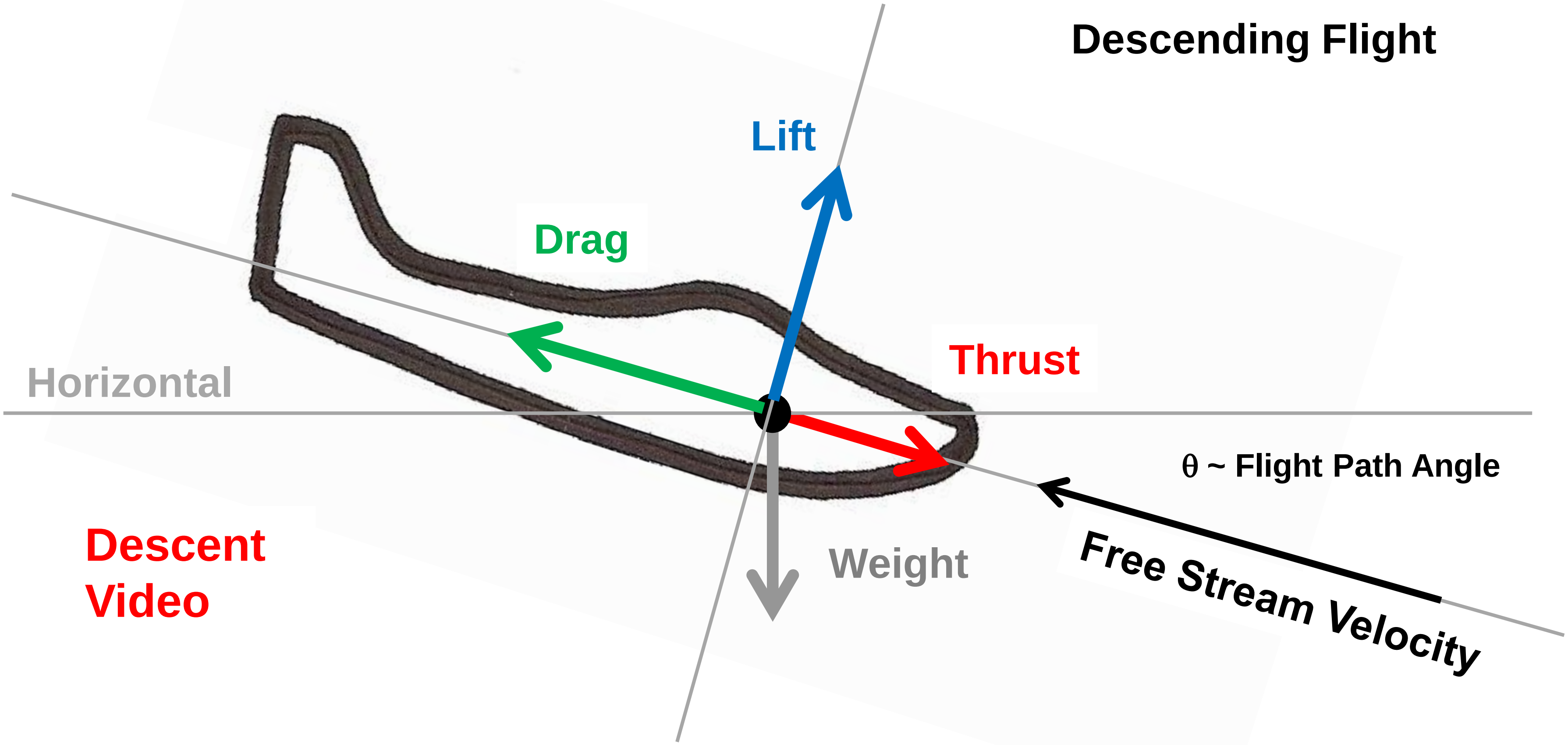
$$t_{min} = \frac{\Delta h}{(R/C)_{avg}}$$

At Sea Level: $(R/C)_{max}$ = 179.8 ft/sec
At 30,000 ft: $(R/C)_{max}$ = 111.0 ft/sec

$$t_{min} = \frac{(30,000 - 0)}{(179.8 + 111.0)/2}$$

$$\boxed{t_{min} = 206.3 \text{ sec} = 3.44 \text{ min}}$$

Stability Axis System



Descent Video

Lift acts perpendicular to the Free Stream Velocity
Drag acts parallel to the Free Stream Velocity
Weight acts vertically towards the ground
Thrust is fixed in the aircraft

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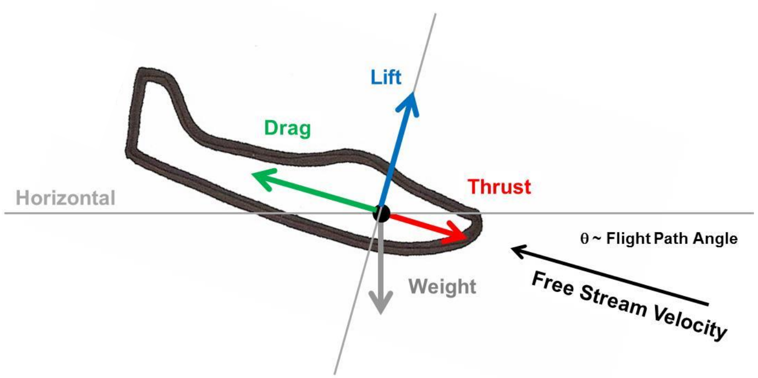
Steady Descending Flight



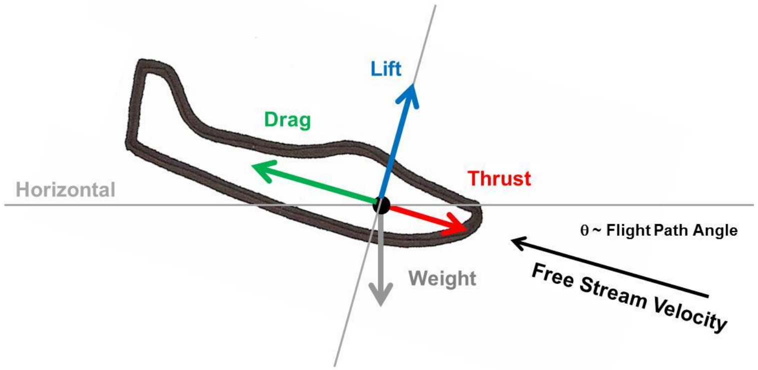
$$\Sigma F_x = T - D + W \sin\theta = \frac{d(mV)}{dt} = \cancelto{0}{\dot{m}}\, V + m\, \cancelto{0}{\dot{V}}$$

$$\Sigma F_z = L - W \cos\theta \quad = 0$$

Steady Descent = Not Accelerating

Steady Descent = Constant Weight

Steady Descending Flight



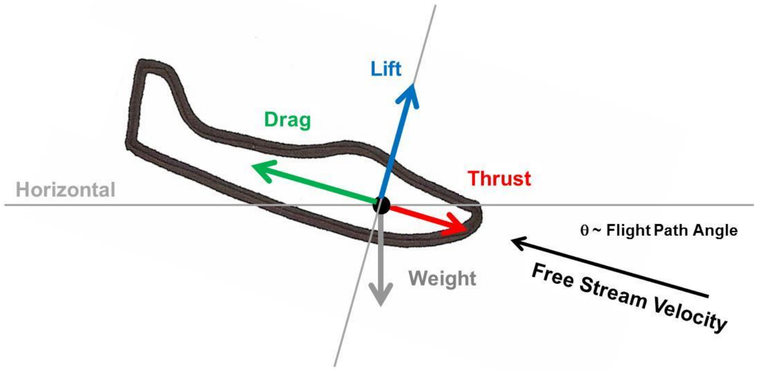
$$\Sigma F_x = T - D + W \sin\theta = 0$$

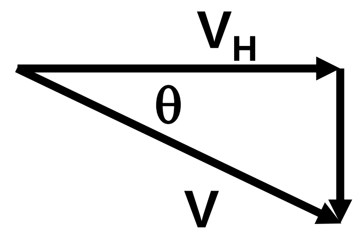
$$\boxed{\sin\theta = \frac{D-T}{W}}$$

$$\Sigma F_z = L - W \cos\theta = 0$$

$$\boxed{\cos\theta = \frac{L}{W}}$$

Steady Descending Flight



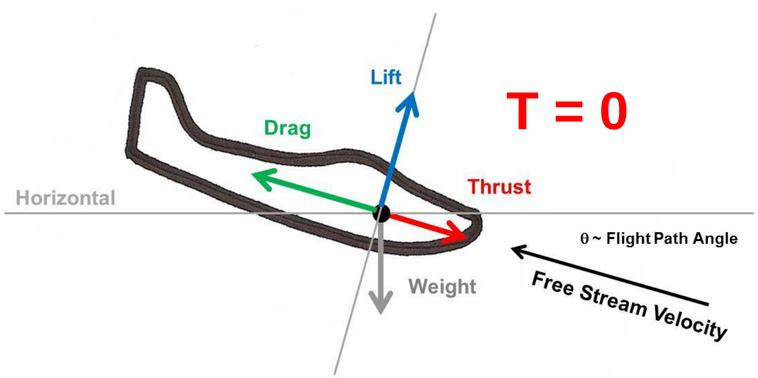


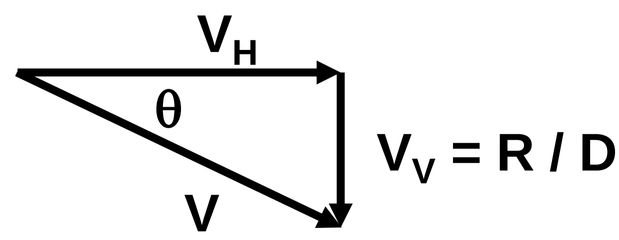
$$\sin\theta = \frac{D - T}{W} \longrightarrow V\sin\theta = \frac{(D - T)\,V}{W} = R/D$$

$$\cos\theta = \frac{L}{W} \longrightarrow L = W\cos\theta$$

$$\tan\theta = \frac{D - T}{L} = \frac{1 - \left(T/D\right)}{\left(L/D\right)}$$

Steady Gliding Flight



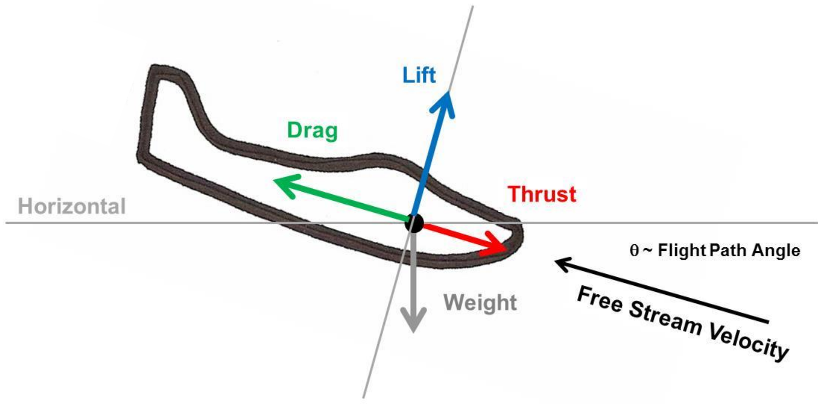


$$\sin\theta = \frac{D - \cancelto{0}{T}}{W} \longrightarrow V\sin\theta = \frac{D\,V}{W} = R/D$$

$$\cos\theta = \frac{L}{W} \longrightarrow L = W\cos\theta$$

$$\tan\theta = \frac{D - \cancelto{0}{T}}{L} = \frac{1 - \left(\cancelto{0}{T}/_D\right)}{\left(L/_D\right)} = \frac{1}{\left(L/_D\right)}$$

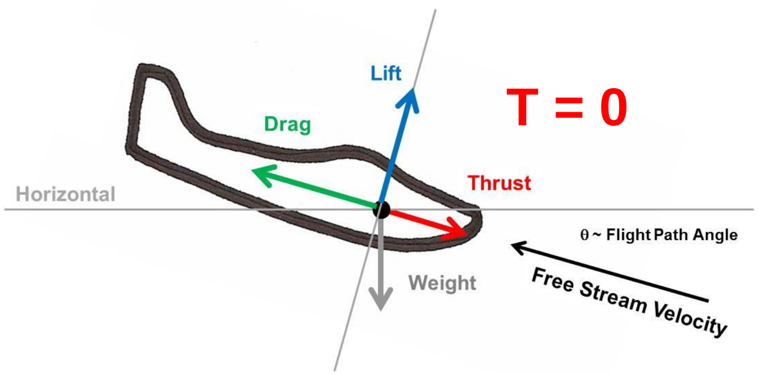
Aircraft Performance



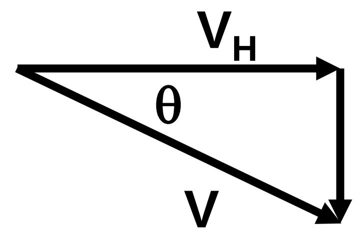
$\boxed{T < D}$ $\boxed{L < W}$

Descent = How long does it take an aircraft to get to a lower altitude? How slow can it descend?

Minimum Glide Angle



T = 0



$$\tan\theta_{min} = \frac{1}{(L/D)_{max}}$$

Minimum descent angle = Maximum glide range

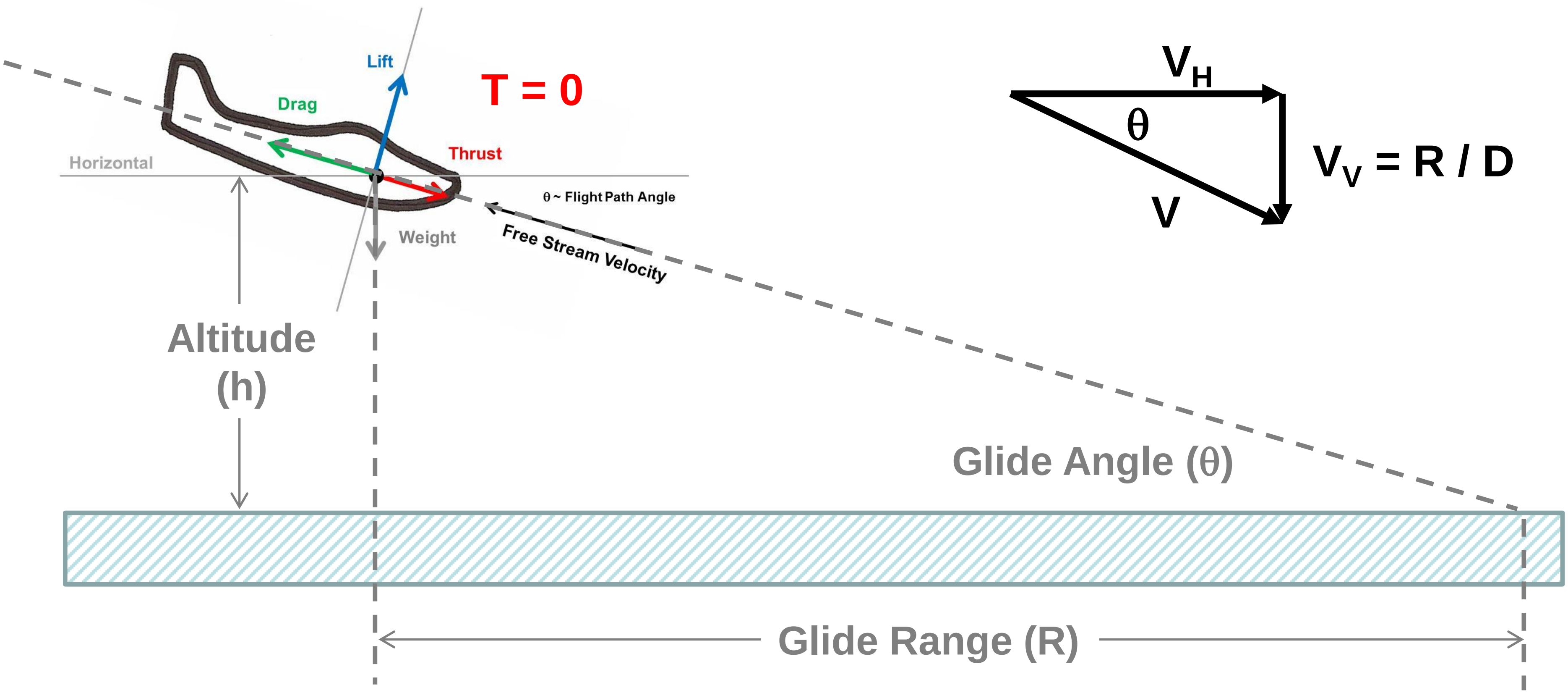
$$V_{\theta_{min}} = \sqrt{\frac{2}{\rho}\left(\frac{K}{C_{D_0}}\right)^{1/2}\left(\frac{W}{S}\right)\cos\theta}$$

$$V_{(L/D)_{max}} = \sqrt{\frac{2}{\rho}\left(\frac{K}{C_{D_0}}\right)^{1/2}\left(\frac{W}{S}\right)}$$

$V_{\theta_{min}} \approx V_{(L/D)_{max}}$

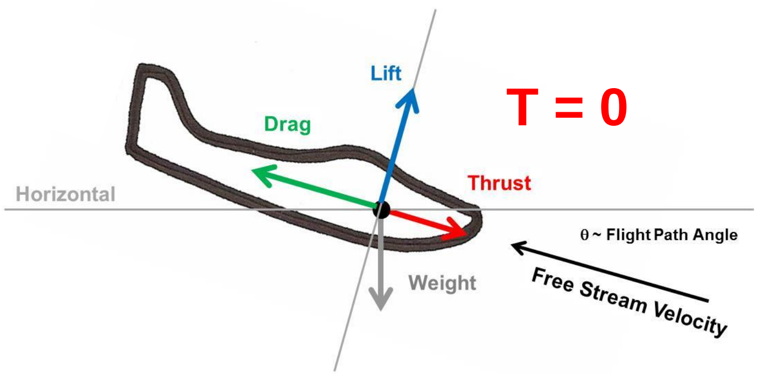
Maximum endurance for jet aircraft R=1

Minimum Glide Angle

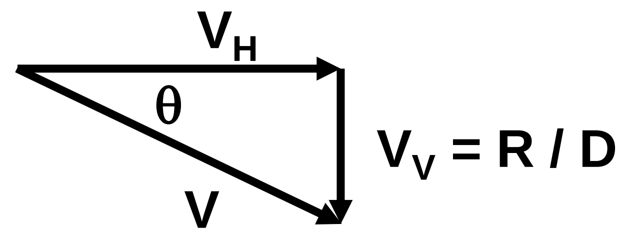


$$\tan\theta_{min} = \frac{1}{\left(L/D\right)_{max}} = \frac{h}{R}$$

Minimum Sink Rate



T = 0



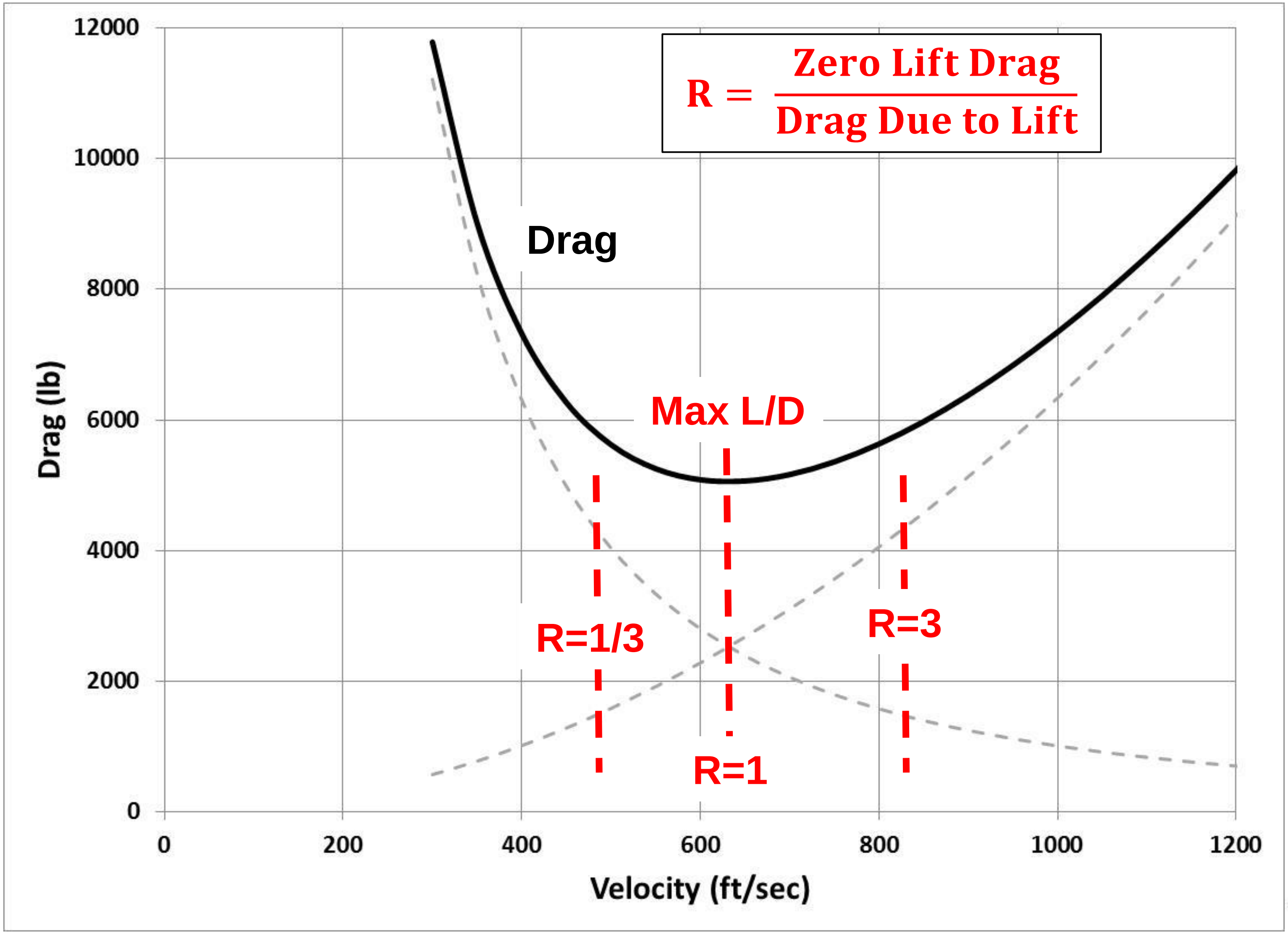
Rate of Descent (R/D) = Sink Rate (V_V)

$$V_{V_{min}} = \sqrt{\frac{2}{\rho\left(C_L^{\ 3/2}/C_D\right)^2}\left(\frac{W}{S}\right)}$$

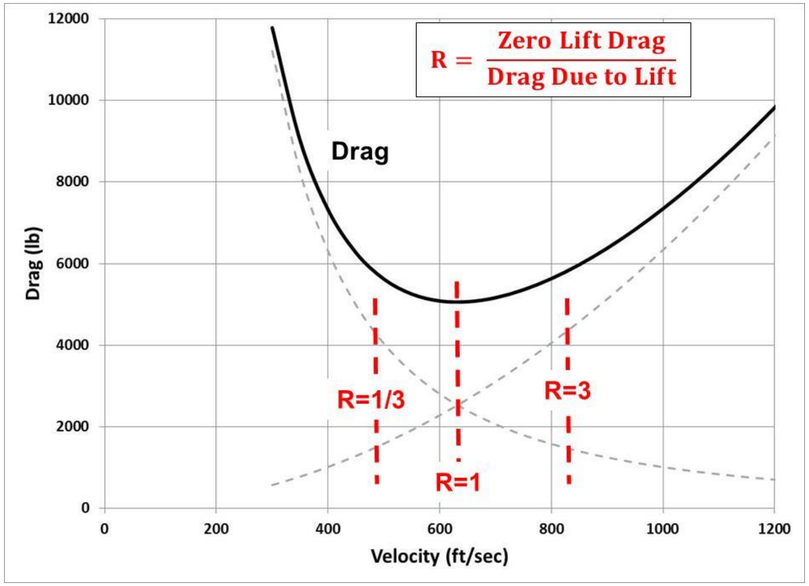
Minimum Sink Rate occurs when $C_L^{3/2} / C_D$ is a maximum

$$V_{(VV)_{min}} = \left(\frac{2}{\rho}\sqrt{\frac{K}{3\,C_{D_0}}}\,\frac{W}{S}\right)^{1/2} = 0.7598\ V_{L/D_{max}}$$

Steady Flight



Steady Flight



$$R = \frac{\text{Zero Lift Drag}}{\text{Drag Due to Lift}}$$

$$\boxed{R = 1 \qquad C_{D_0} = C_{D_L} = K\,C_L^{\ 2}} \qquad \text{Max } \frac{C_L}{C_D}$$

Max Endurance

Max Glide Range

$$\boxed{R = 3 \qquad C_{D_0} = 3\,C_{D_L} = 3\,K\,C_L^{\ 2}} \qquad \text{Max } \frac{C_L^{\ 1/2}}{C_D}$$

Max Range

$$\boxed{R = \frac{1}{3} \qquad C_{D_0} = \frac{1}{3}\,C_{D_L} = \frac{1}{3}\,K\,C_L^{\ 2}} \qquad \text{Max } \frac{C_L^{\ 3/2}}{C_D}$$

Min Sink Rate

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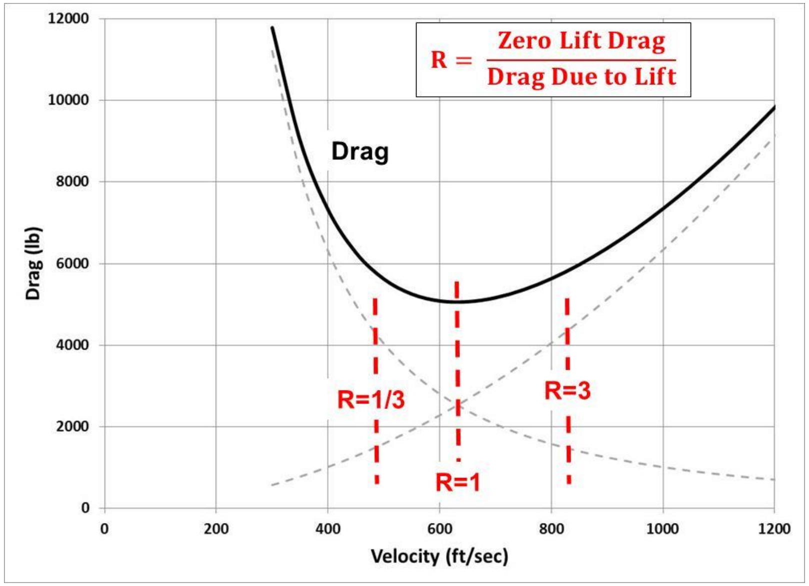
Steady Flight



$$\boxed{R = \frac{\text{Zero Lift Drag}}{\text{Drag Due to Lift}}}$$

$$\boxed{R = 1 \qquad C_{D_0} = C_{D_L} = K\, C_L^{\ 2}}$$

$$\text{Max}\ \frac{C_L}{C_D} = \sqrt{\frac{1}{4\, C_{D_0}\, K}}$$

$$\boxed{R = 3 \qquad C_{D_0} = 3\, C_{D_L} = 3\, K\, C_L^{\ 2}}$$

$$\text{Max}\ \frac{C_L^{\ 1/2}}{C_D} = \frac{3}{4}\left(\frac{1}{3\, K\, C_{D_0}^{\ 3}}\right)^{1/4}$$

$$\boxed{R = \frac{1}{3} \qquad C_{D_0} = \frac{1}{3}\, C_{D_L} = \frac{1}{3}\, K\, C_L^{\ 2}}$$

$$\text{Max}\ \frac{C_L^{\ 3/2}}{C_D} = \frac{1}{4}\left(\frac{3}{K\, C_{D_0}^{\ 1/3}}\right)^{3/4}$$

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Steady Flight

$$\boxed{R = \frac{\text{Zero Lift Drag}}{\text{Drag Due to Lift}}}$$

Max Endurance **Max Glide Range**

$$\boxed{R = 1 \qquad C_{D_0} = C_{D_L} = K\,C_L^{\ 2}}$$

$$\text{Max } \frac{C_L}{C_D}$$

$$V_{L/D_{max}} = \left(\frac{2}{\rho}\sqrt{\frac{K}{C_{D_0}}}\,\frac{W}{S}\right)^{1/2}$$

Max Range

$$\boxed{R = 3 \qquad C_{D_0} = 3\,C_{D_L} = 3\,K\,C_L^{\ 2}}$$

$$\text{Max } \frac{C_L^{\ 1/2}}{C_D}$$

$$V = 1.3161\ V_{L/D_{max}}$$

Min Sink Rate

$$\boxed{R = \frac{1}{3} \qquad C_{D_0} = \frac{1}{3}\,C_{D_L} = \frac{1}{3}\,K\,C_L^{\ 2}}$$

$$\text{Max } \frac{C_L^{\ 3/2}}{C_D}$$

$$V = 0.7598\ V_{L/D_{max}}$$

G-IV Descent

$$R/D = V \sin\theta = \frac{(D - T)\,V}{W}$$

Gulfstream IV twin-turbofan biz jet:

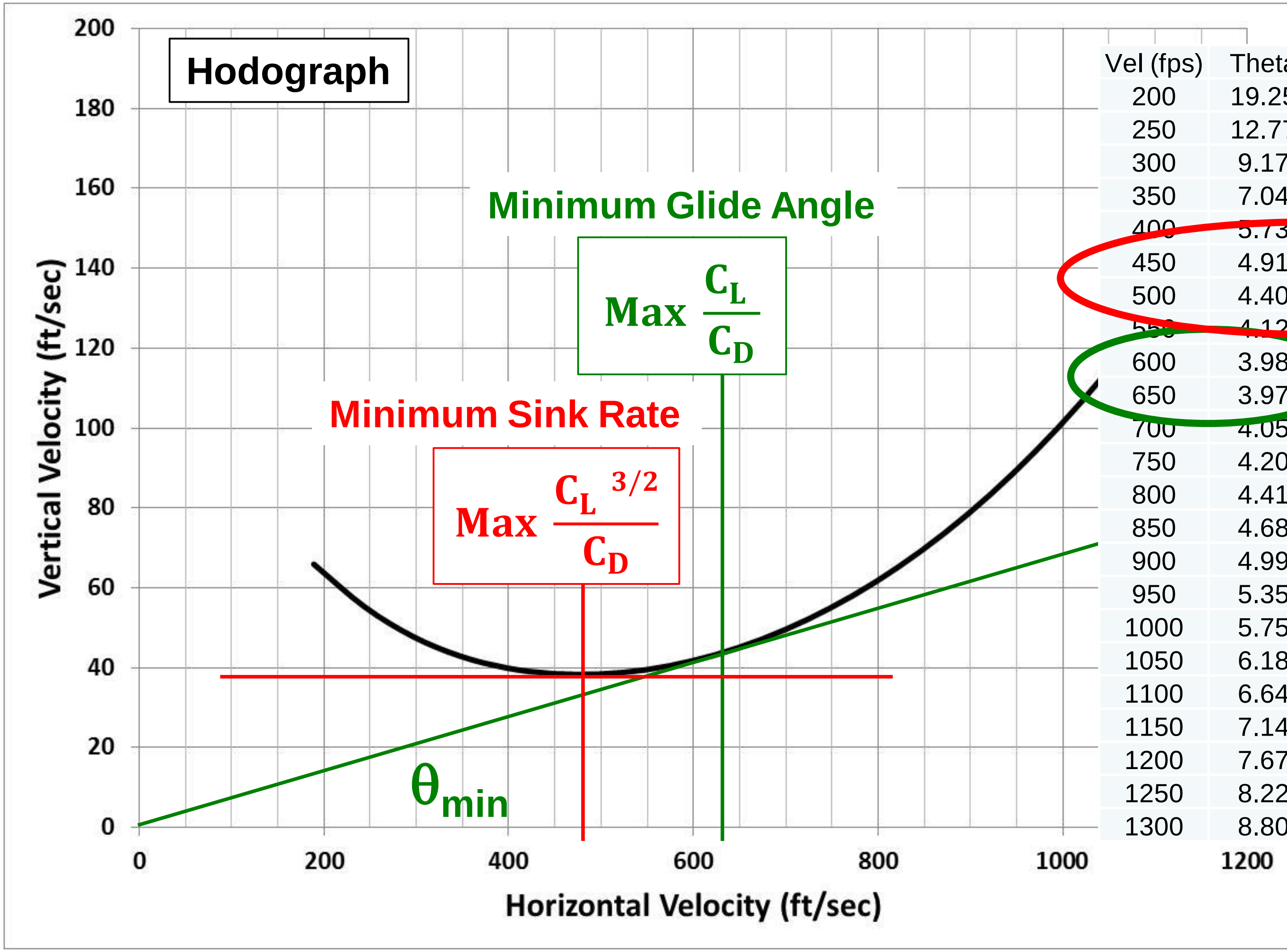
$C_{D_0} = 0.0150$ $K = 0.08$

$W = 73{,}000$ lb

$h = 30{,}000$ ft

CD0	0.0150	**Wt**	73,000	lb
K	0.08	**Alt**	30,000	ft
T	0	**rho**	0.00088926	
		QMS	439.9	
		a	994.67	ft/sec
		S	950	sq ft
T/W	0.0000			
W/S	76.84			
L/D max	14.43			
CL^3/2/CD	10.83			
Min Glide Angle = Max Glide Range				
	Min Theta	3.96	degrees	
	Velocity	631.0	ft/sec	
	Distance	433,013	ft	
		71.3	NM	
	Sink Rate	43.6	ft/sec	
Min Sink Rate				
	Min sink rate	38.4	ft/sec	
	Velocity	479.4	ft/sec	
	L/D	12.5		
	Theta	4.57	degrees	
	Distance	375,000	ft	
		61.7	NM	

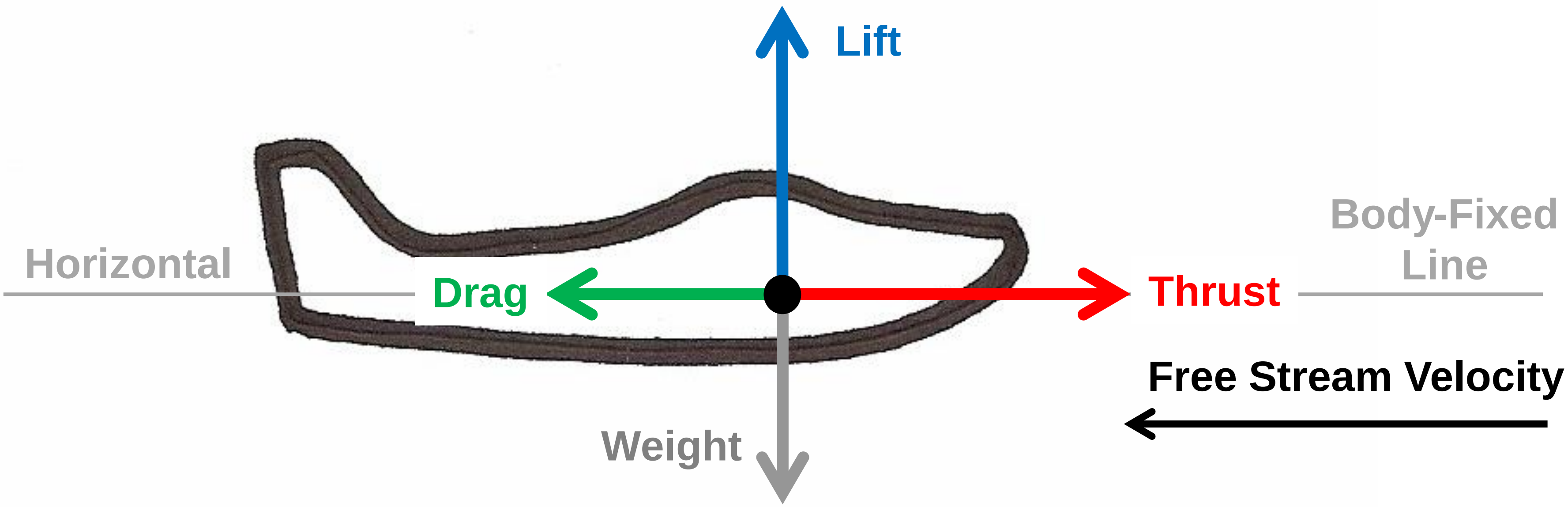
G-IV Descent



Vel (fps)	Theta	VH	VV
200	19.25	188.8	65.9
250	12.77	243.8	55.3
300	9.17	296.2	47.8
350	7.04	347.4	42.9
400	5.73	398.0	39.9
450	4.91	448.4	38.5
500	4.40	498.5	38.4
550	4.12	548.6	39.5
600	3.98	598.5	41.7
650	3.97	648.4	45.0
700	4.05	698.3	49.4
750	4.20	748.0	54.9
800	4.41	797.6	61.5
850	4.68	847.2	69.3
900	4.99	896.6	78.3
950	5.35	945.9	88.6
1000	5.75	995.0	100.1
1050	6.18	1043.9	113.0
1100	6.64	1092.6	127.2
1150	7.14	1141.1	142.9
1200	7.67	1189.3	160.1
1250	8.22	1237.2	178.7
1300	8.80	1284.7	198.9

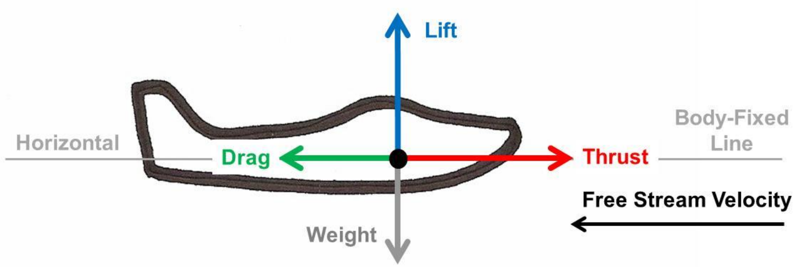
Simple Free Body Diagram

Straight and Level Flight



Lift acts perpendicular to the Free Stream Velocity
Drag acts parallel to the Free Stream Velocity
Weight acts vertically towards the ground
Thrust is fixed in the aircraft

Aircraft Flight Ceiling



How HIGH can an airplane fly in steady, level flight?

Upper altitude limit = ceiling

Ceiling	R / C Capability	P_s Capability
Absolute	0 ft / min	0 ft / sec
Service	100 ft / min	1.67 ft / sec
Cruise	300 ft / min	5.00 ft / sec
Combat	500 ft / min	8.33 ft / sec

Estimating Absolute Ceiling

Calculate the maximum Rate of Climb at Sea Level

Calculate the maximum Rate of Climb at another altitude

Derive the linear relationship $R/C_{max} = a + b\, h$
and solve for h where $R/C_{max} = 0$

At Sea Level: $(R/C)_{max} = 4{,}600$ ft/min
At 30,000 ft: $(R/C)_{max} = 1{,}600$ ft/min

$a = 4{,}600$ $b = -0.1$

$(R/C)_{max} = 4{,}600 - 0.1\, h$ $h = [4{,}600 - (R/C)_{max}] / 0.1$

When $(R/C)_{max} = 0$, then $h = 46{,}000$ ft

Estimating Service Ceiling

Calculate the maximum Rate of Climb at Sea Level

Calculate the maximum Rate of Climb at another altitude

Derive the linear relationship $R/C_{max} = a + b\ h$
and solve for h where $R/C_{max} = 100$ ft/min

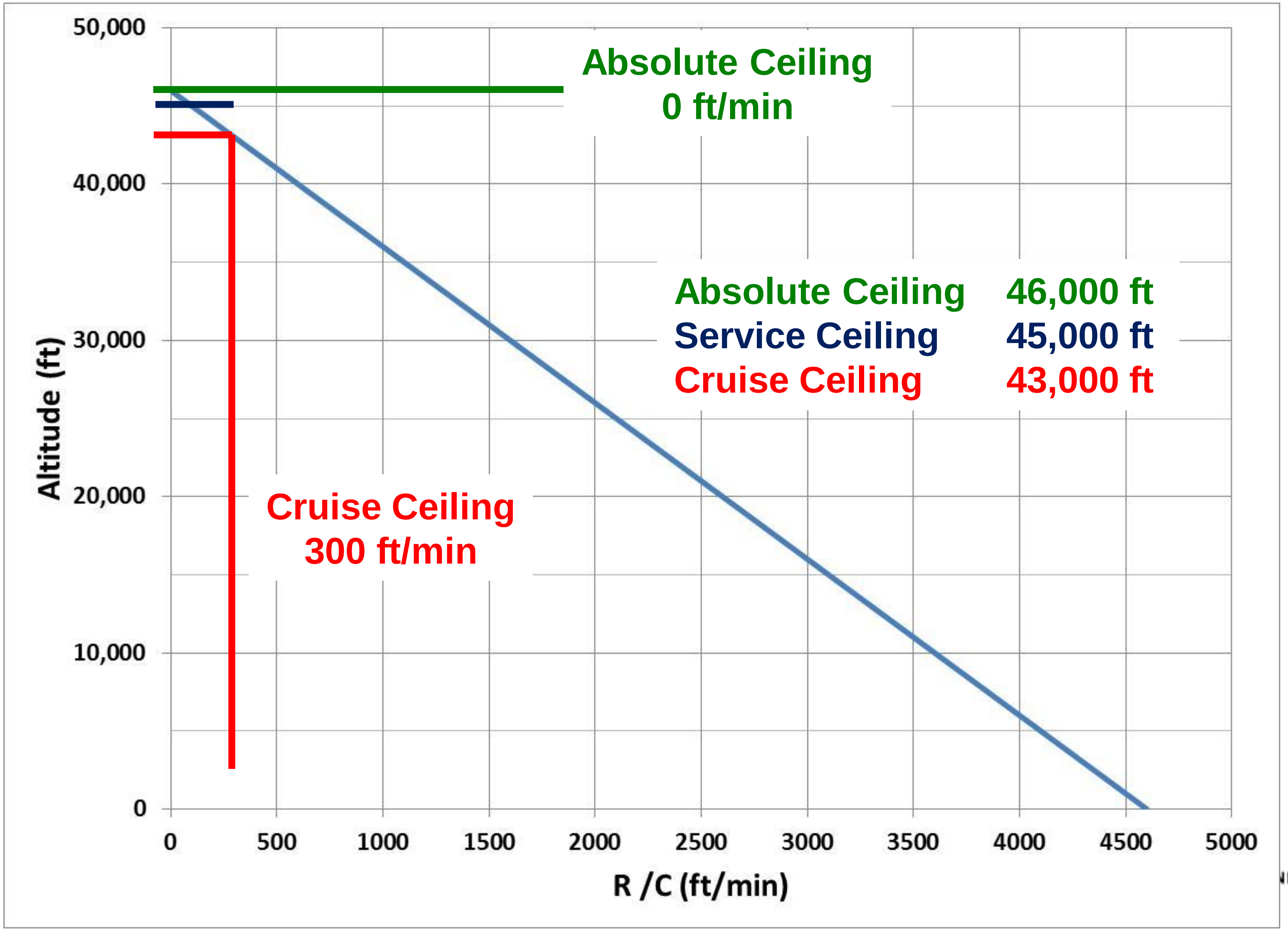
At Sea Level: $(R/C)_{max} = 4{,}600$ ft/min
At 30,000 ft: $(R/C)_{max} = 1{,}600$ ft/min

$a = 4{,}600$ $b = -0.1$

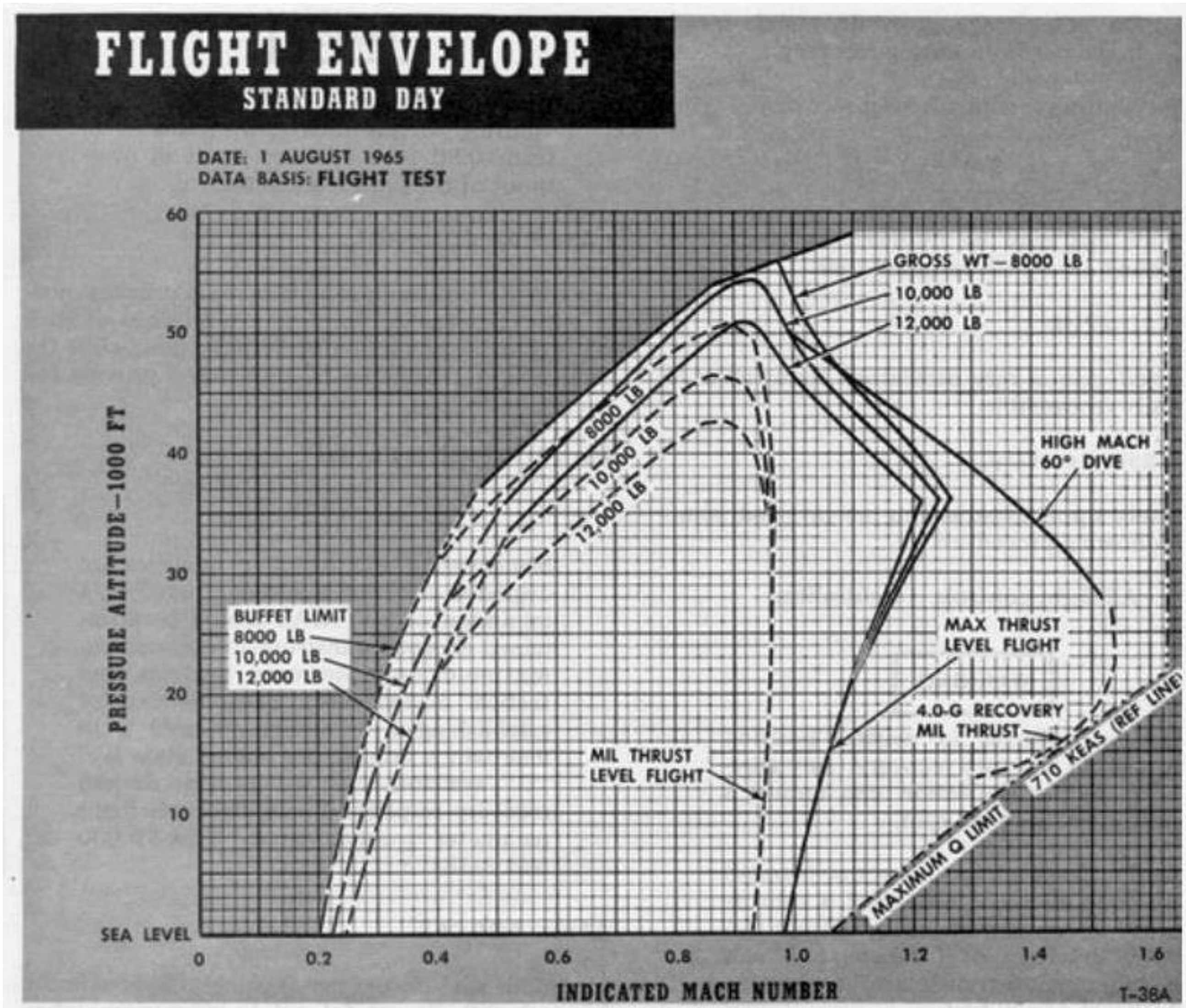
$(R/C)_{max} = 4{,}600 - 0.1\ h$ $h = [4{,}600 - (R/C)_{max}] / 0.1$

When $(R/C)_{max} = 100$ ft/min, then $h = 45{,}000$ ft

Aircraft Flight Ceiling



T-38 Flight Envelope



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Homework Assignment

HW #11 – Rate of Climb; Rate of Descent; Ceilings (due by 11:59 pm ET on Monday)
Reading – Chapters 5.10 - 5.12

<u>HW Help Session</u>
Monday 1:00 – 2:00 pm ET

<u>Posted on Canvas</u>
HW #11 Assignment with instructions, tips, and checklist
HW #11 Template for data table in Excel

Homework

Plotting Charts

Weight	900	lb		QMS	1481.4	lb/ft^2			
Altitude	0	ft		a	1116.45	ft/sec			
				rho	0.00237688	slugs/ft^3			
				DR	1.0000				
Mach	**Vel**	**CL**	**CD0**	**CDL**	**CD**	**D**	**Thrust**	**R/C**	**Climb Angle**
	(ft/sec)					**(lb)**	**(lb)**	**(ft/min)**	**(deg)**
0.05	55.82	6.4291	0.0200	2.5627	2.5827	362	202.0	-593.75	-10.21
0.06	66.99	4.4647	0.0200	1.2359	1.2559	253	202.0	-228.47	-3.26
0.07	78.15	3.2802	0.0200	0.6671	0.6871	189	202.0	70.23	0.86

x axis

y axis

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Questions?