AEEM 3042 – Integrated Aircraft Engineering

Block 2a
Material
Review

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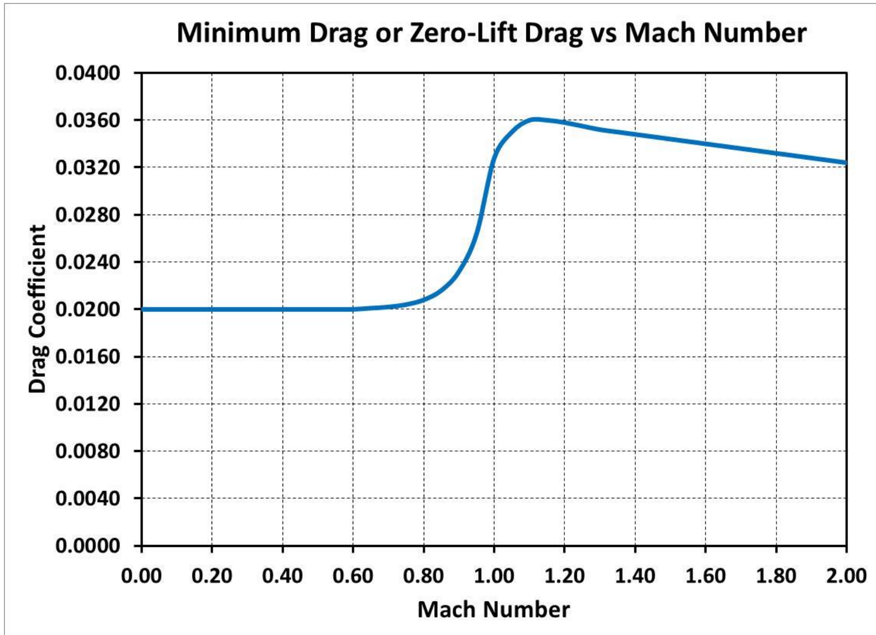
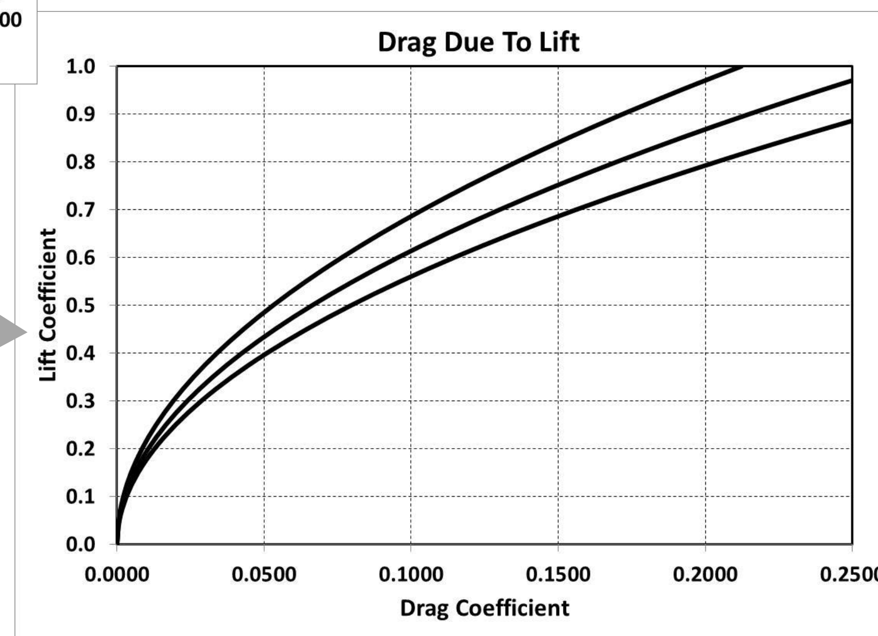
Finite Wing Drag



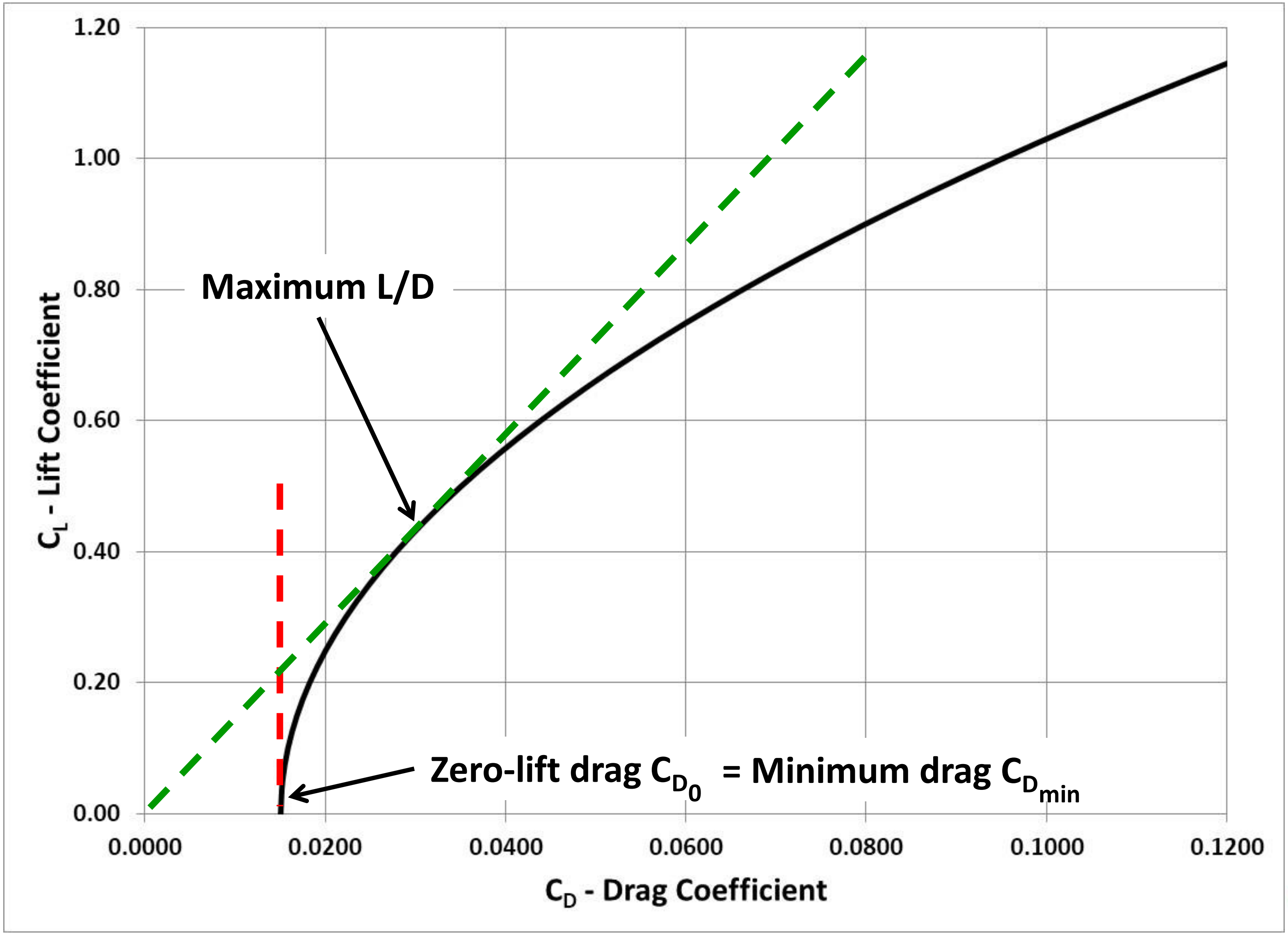
$$C_{D_0} \sim f\ (M, h)$$

$$C_{D_L} \sim f\ (C_L, M, c.g.)$$



$$C_D = C_{D_0} + K\, C_L^{\ 2}$$

Aircraft Drag Polar



Aircraft Thrust

Piston engine / propeller $$\mathbf{T_A = SHP_{SL}\left(\frac{\eta_P}{V}\right)\left(\frac{\rho}{\rho_{SL}}\right)}$$

Turboprop $$\mathbf{T_A = ESHP_{SL}\left(\frac{\eta_P}{V}\right)\left(\frac{\rho}{\rho_{SL}}\right)}$$

High-bypass turbofan $$\mathbf{T_A = T_{SL}\left(\frac{0.1}{M}\right)\left(\frac{\rho}{\rho_{SL}}\right)}$$

Low-bypass turbofan & Turbojet $$\mathbf{T_A = T_{SL}\left(\frac{\rho}{\rho_{SL}}\right)}$$

Afterburner $$\mathbf{T_A = T_{SL}\left(\frac{\rho}{\rho_{SL}}\right)(1 + 0.7\,M)}$$

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Aircraft Fuel Flow

Piston engine / propeller	$\mathbf{FFR = SHP\ c}$
Turboprop	$\mathbf{FFR = ESHP\ c}$
High-bypass turbofan	$\mathbf{FFR = T\ c_{t_{SL}} \left(\frac{a}{a_{SL}}\right)}$
Low-bypass turbofan & Turbojet	$\mathbf{FFR = T\ c_{t_{SL}} \left(\frac{a}{a_{SL}}\right)}$
Afterburner	$\mathbf{FFR = T\ c_{t_{SL}} \left(\frac{a}{a_{SL}}\right)}$

$\mathbf{c\left(\frac{lb}{HP\ hr}\right)}$

$\mathbf{c_t\left(\frac{lb}{lb_t\ hr}\right)}$

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Aircraft Weights

Basic Mission Takeoff Gross Weight =
OW + Mission Payload + Mission Fuel

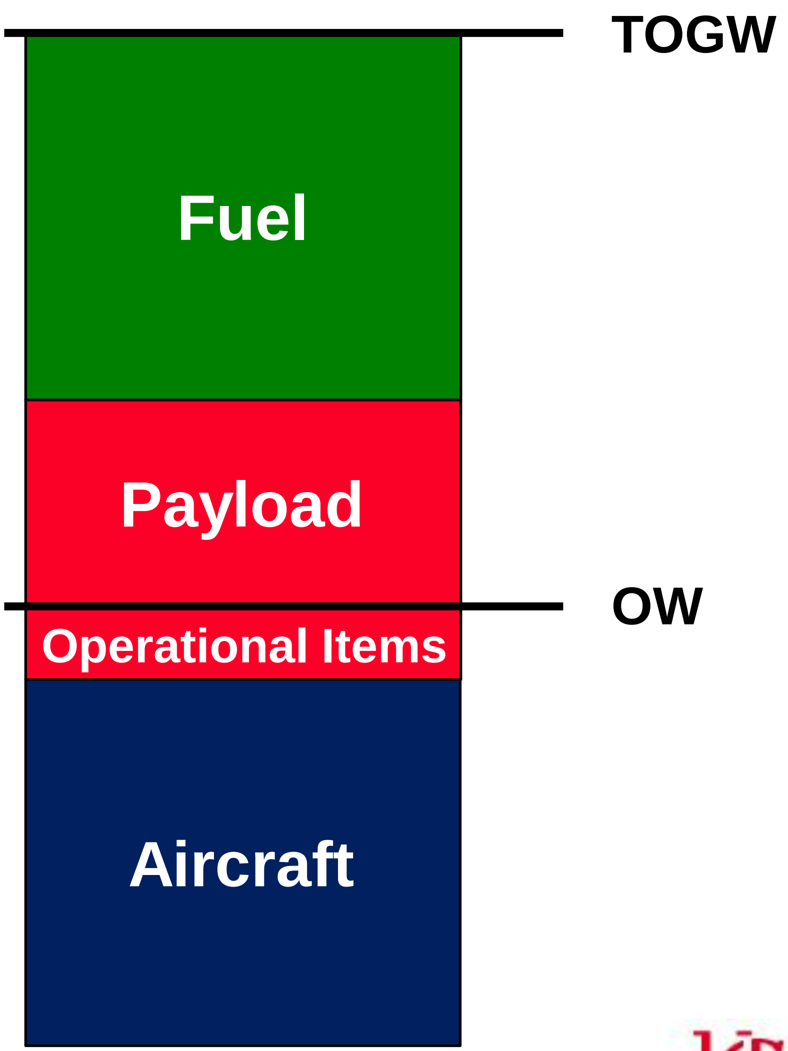
Maximum Fuel – full capacity
Mission Fuel – specific mission capability

Maximum Payload – full capacity loadout
Mission Payload – specific mission loadout

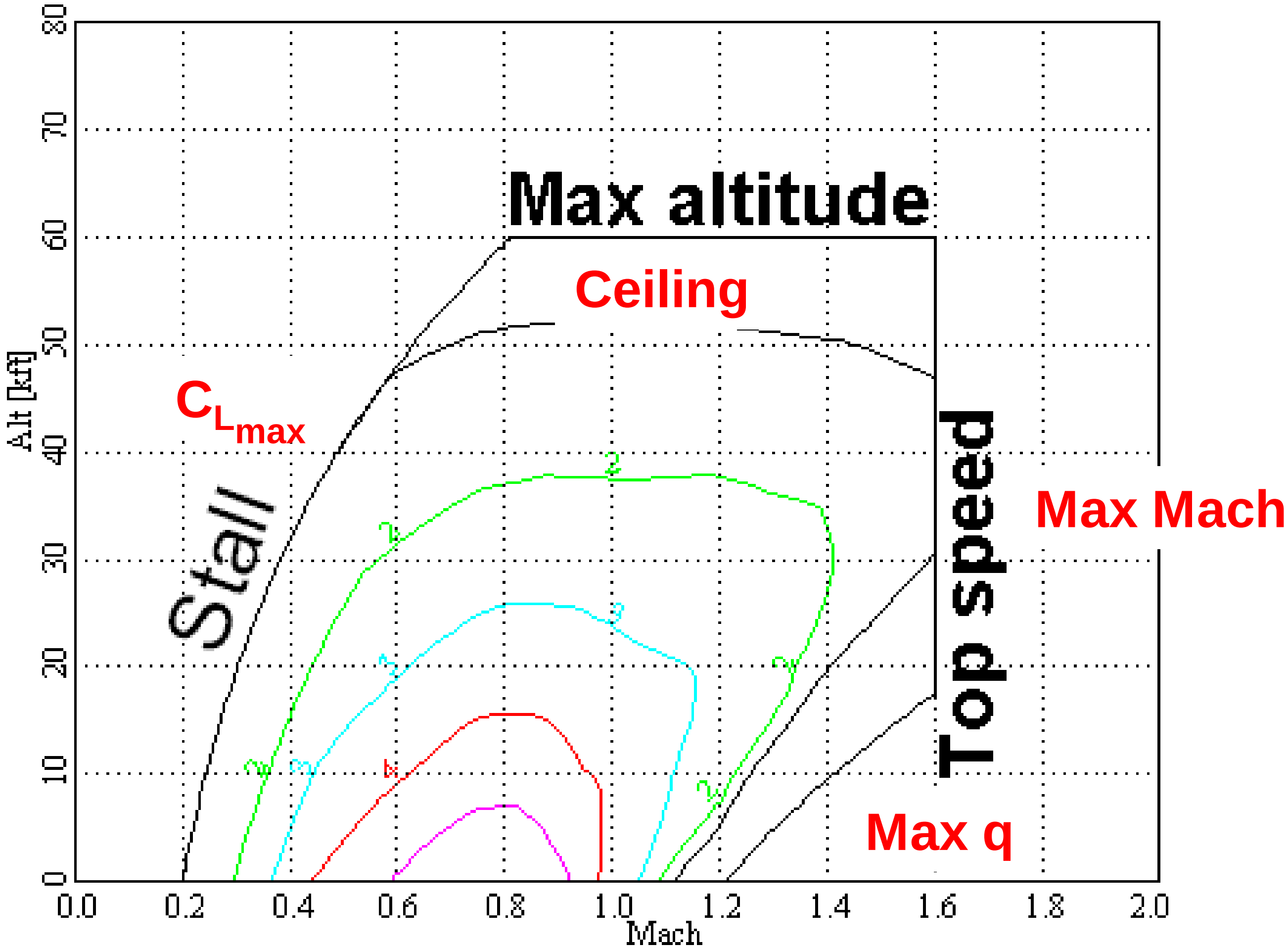
Maximum Takeoff Gross Weight =
OW + Maximum Payload + Maximum Fuel

- or -

Maximum Takeoff Gross Weight
could be set by other factors
(landing gear limit, c.g. limits, etc)



Flight Envelope



Stall Speed

$$C_L = \frac{n\,W}{q\,S} = \frac{n\,W}{\frac{1}{2}\,\rho\,V^2\,S}$$

$$C_{L_{max}} = \frac{n\,W}{\frac{1}{2}\,\rho\,{V_{stall}}^2\,S} \longrightarrow V_{stall} = \sqrt{\frac{2}{\rho}\,\frac{W}{S}\,\frac{n}{C_{L_{max}}}}$$

Limit Speed

Maximum Mach Number

Maximum dynamic pressure (max q = q_{max})
Maximum KEAS

At any altitude, an aircraft's maximum velocity is constrained to the least of these speeds

$$V_{max} = \sqrt{\frac{2}{\rho}\, q_{max}} \qquad \text{- or -} \qquad V_{max} = M_{max}\; a$$

$$q_{max} = \left(\frac{q}{M^2}\right)_{SL} \left(\frac{\text{Max KEAS}}{a_{SL}}\right)^2$$

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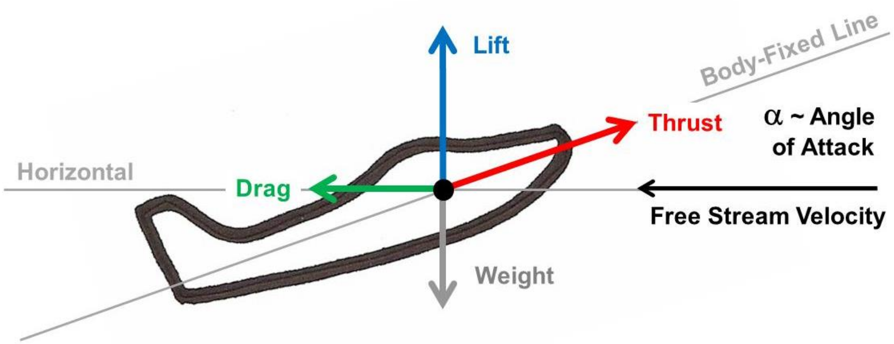
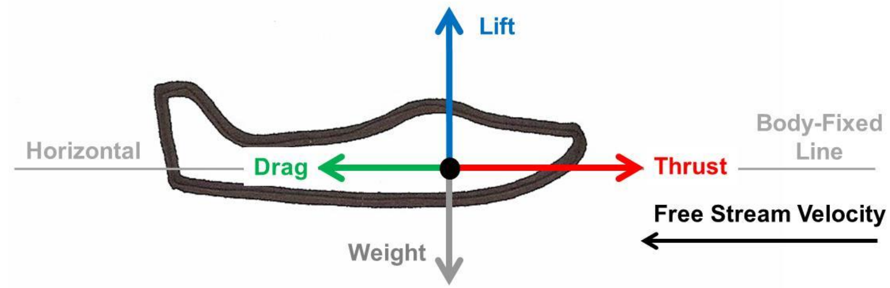
Steady Flight





$$\Sigma F_x = T\,\cancelto{1}{\cos\alpha} - D = \frac{d(mV)}{dt} = 0$$

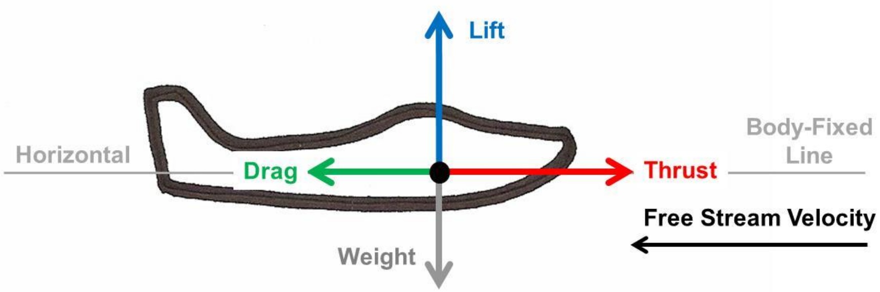
$$\boxed{T = D}$$

$$\Sigma F_z = L + T\,\cancelto{0}{\sin\alpha} - W = 0$$

$$\boxed{L = W}$$

Small Angle Approximation Assumption for Angle of Attack (α)

Thrust Required



$$\boxed{T = D}$$

$$\boxed{L = W}$$

The amount of Thrust Required is dependent on:

- **Velocity**
- **Altitude**
- **Weight**
- **Aerodynamics (Aspect Ratio, Wing Sweep, etc)**

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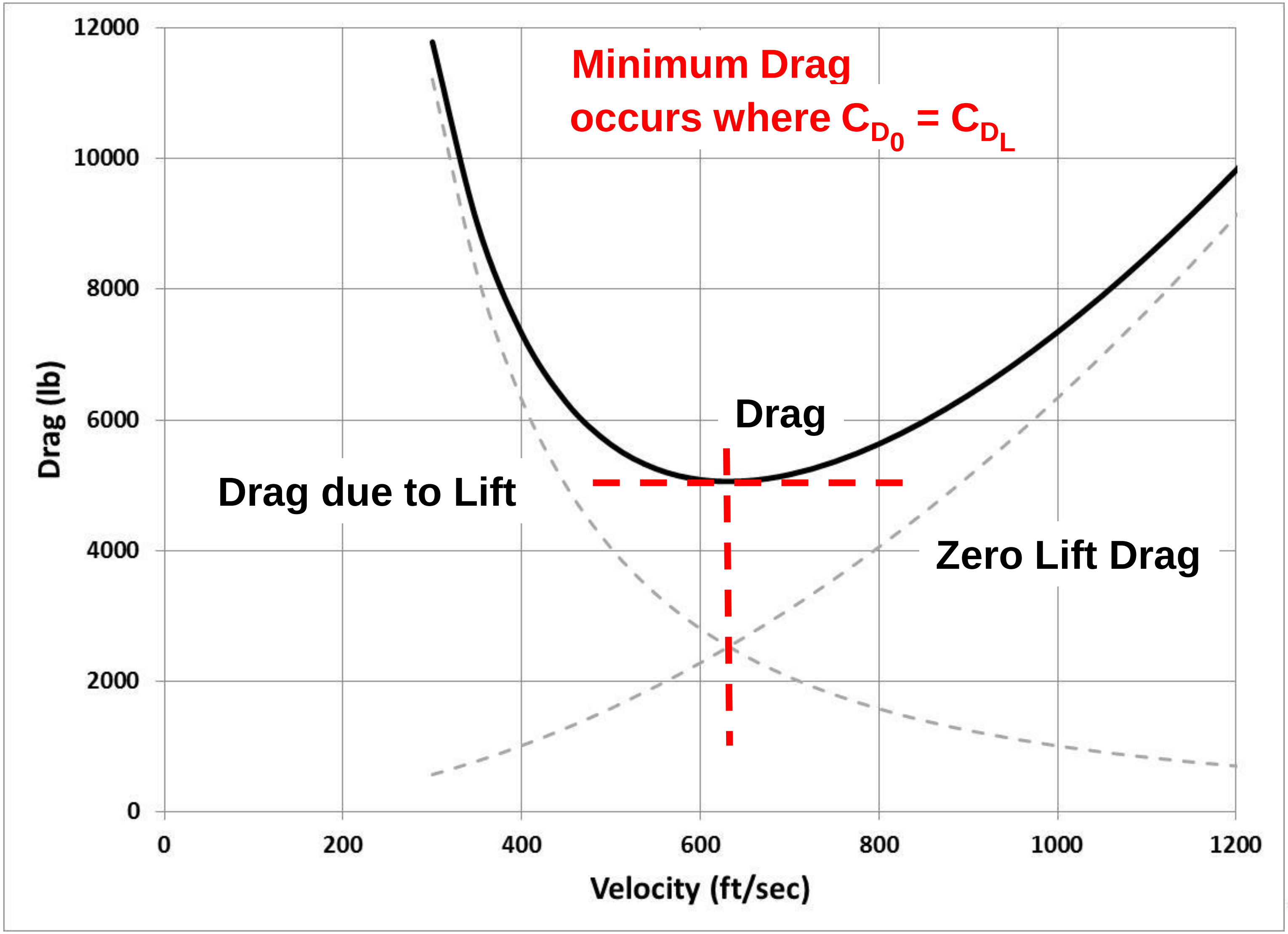
Example 5.1

Thrust Required



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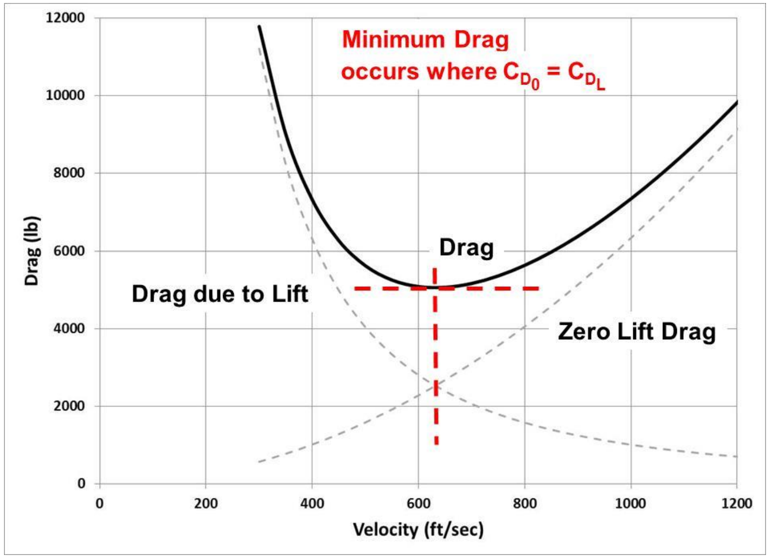
Thrust Required



Minimum Drag occurs where the Drag due to Lift and Zero Lift Drag curves intersect

$$C_{D_0} = K\, C_L^{\ 2}$$

Minimum Drag also occurs where L/D is maximum

L/D is one of the most important parameters affecting aircraft performance

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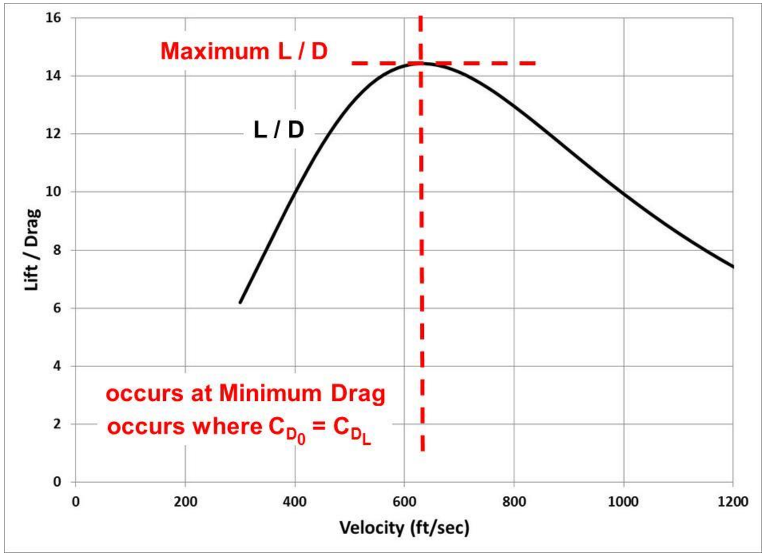
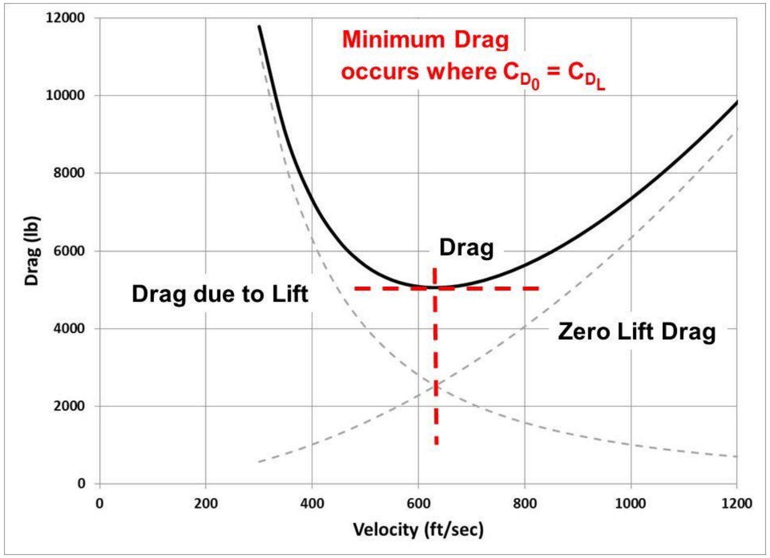
Lift to Drag Ratio





$$\left(\frac{L}{D}\right)_{max} = \left(\frac{C_L}{C_D}\right)_{max} = \sqrt{\frac{1}{4\ C_{D_0}\ K}}$$

$$\left(\frac{D}{W}\right)_{min} = \left(\frac{T_{req}}{W}\right)_{min} = \sqrt{4\ C_{D_0} K}$$

$$V_{L/D_{max}} = \left(\frac{2}{\rho}\sqrt{\frac{K}{C_{D_0}}}\ \frac{W}{S}\right)^{1/2}$$

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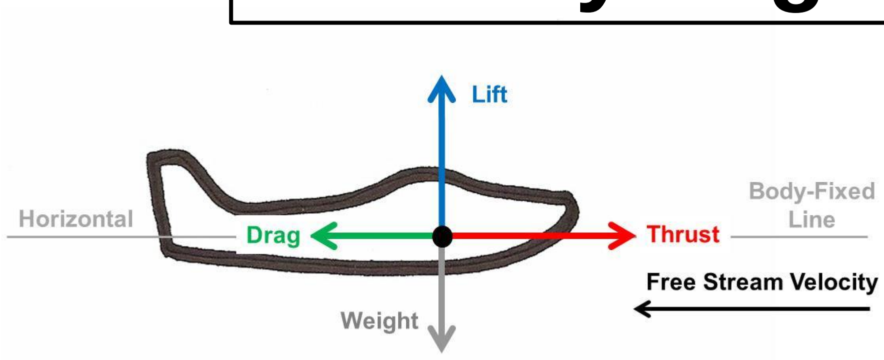
Steady Flight Summary



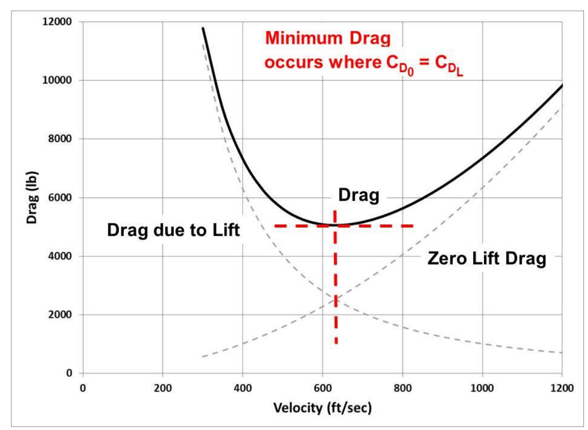
$$\boxed{T = D} \qquad \boxed{L = W}$$

$$V_{L/D_{max}} = \left(\frac{2}{\rho} \sqrt{\frac{K}{C_{D_0}}} \frac{W}{S} \right)^{1/2}$$

$$\left(\frac{L}{D}\right)_{max} = \left(\frac{C_L}{C_D}\right)_{max} = \sqrt{\frac{1}{4\, C_{D_0}\, K}}$$

$$\left(\frac{D}{W}\right)_{min} = \left(\frac{T_{req}}{W}\right)_{min} = \sqrt{4\, C_{D_0} K}$$

Minimum Drag & Max L / D occur where the Drag due to Lift and Zero Lift Drag curves intersect



$$C_{D_0} = K\, C_L^{\,2}$$

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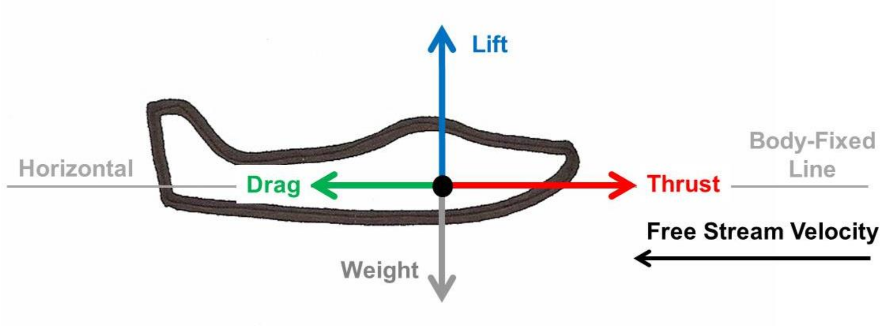
Steady Flight



$$\boxed{T = D}$$ $$\boxed{L = W}$$

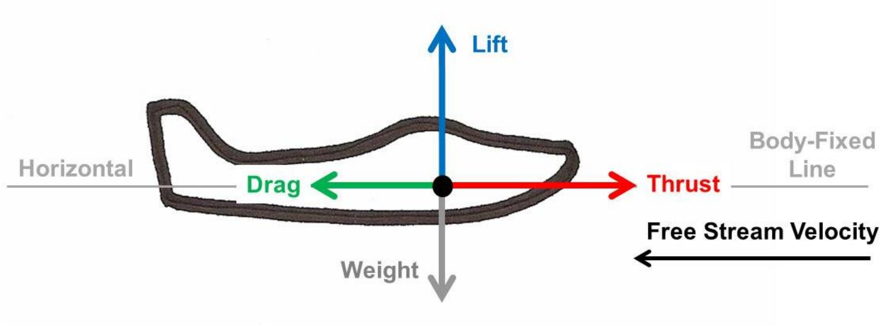
Range = How <u>FAR</u> can an aircraft fly?

Fuel Consumed per Mile Flown (lb_{fuel} / NM)

<u>Jet Aircraft</u>

$$\frac{lb_{fuel}}{NM} \propto c_t \frac{T_{req}}{V}$$

Steady Flight



$$\boxed{T = D} \qquad \boxed{L = W}$$

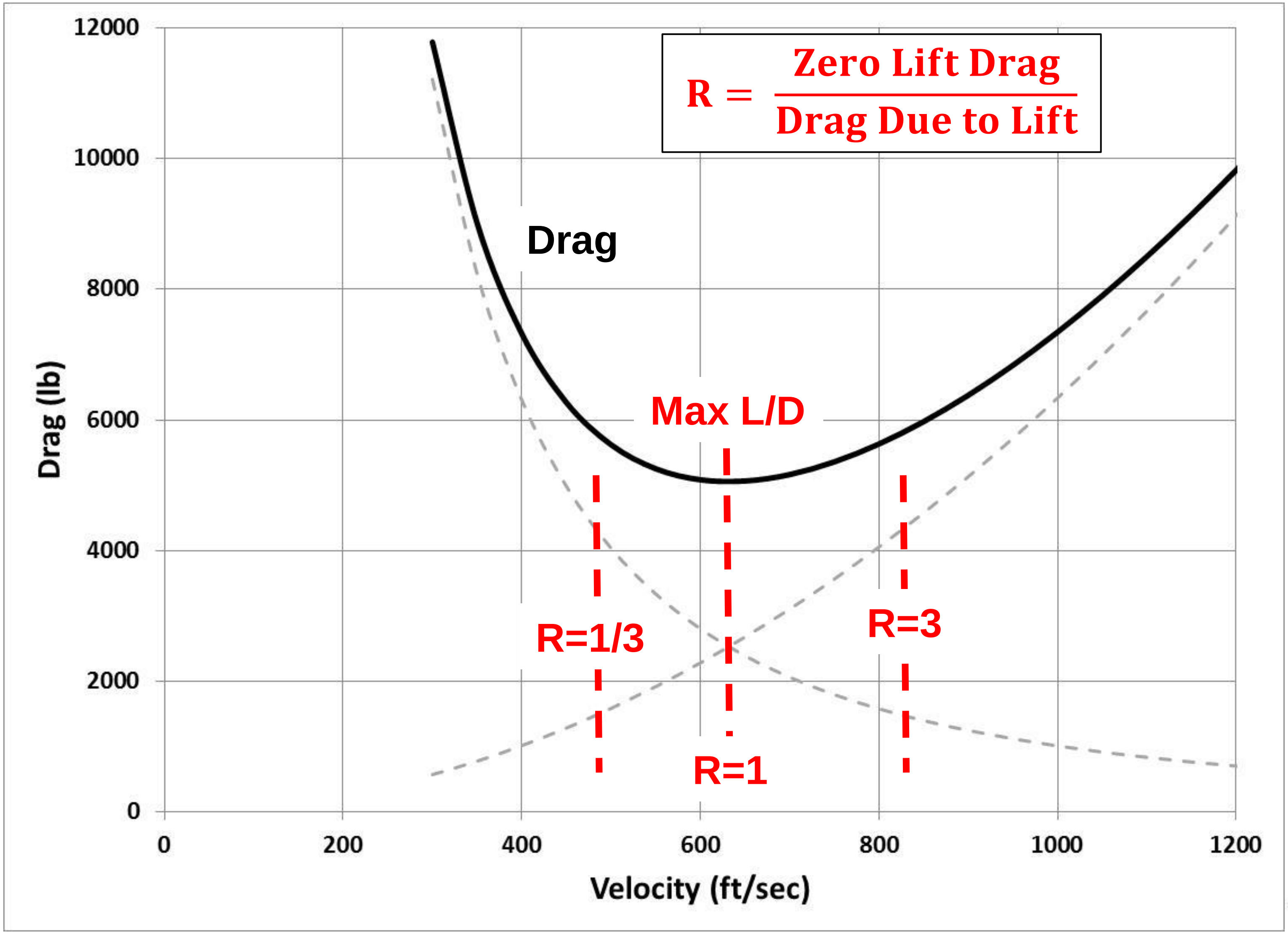
Endurance = How <u>LONG</u> can an aircraft fly?

Fuel Consumed per Hour (lb$_{fuel}$ / hr)

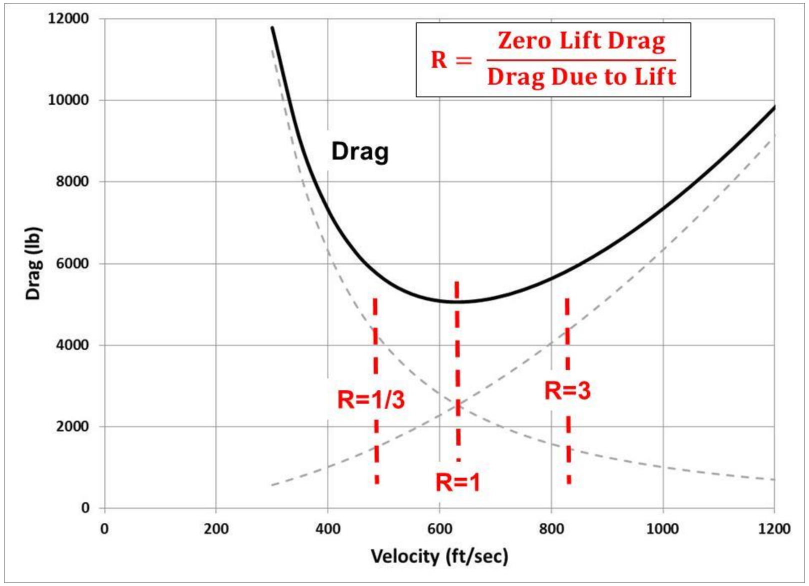
<u>Jet Aircraft</u>

$$\frac{\mathrm{lb_{fuel}}}{\mathrm{hr}} \propto c_t \, T_{req}$$

Steady Flight



Steady Flight



$$R = \frac{\text{Zero Lift Drag}}{\text{Drag Due to Lift}}$$

$$R = 1 \qquad C_{D_0} = C_{D_L} = K\,C_L^{\ 2}$$

$$\text{Max}\ \frac{C_L}{C_D} = \sqrt{\frac{1}{4\,C_{D_0}\,K}}$$

$$R = \frac{1}{3} \qquad C_{D_0} = \frac{1}{3}\,C_{D_L} = \frac{1}{3}\,K\,C_L^{\ 2}$$

$$\text{Max}\ \frac{C_L^{\ 3/2}}{C_D} = \frac{1}{4}\left(\frac{3}{K\,C_{D_0}^{\ 1/3}}\right)^{3/4}$$

$$R = 3 \qquad C_{D_0} = 3\,C_{D_L} = 3\,K\,C_L^{\ 2}$$

$$\text{Max}\ \frac{C_L^{\ 1/2}}{C_D} = \frac{3}{4}\left(\frac{1}{3\,K\,C_{D_0}^{\ 3}}\right)^{1/4}$$

Steady Flight

		Jet Aircraft	
R = 1 / 3	Max $\frac{C_L^{3/2}}{C_D}$		
R = 1	Max $\frac{C_L}{C_D}$	Minimum D	Maximum Endurance
R = 3	Max $\frac{C_L^{1/2}}{C_D}$	Minimum D/V	Maximum Range

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Steady Flight

$$\mathbf{V_{L/D_{max}} = V_{(HP/V)_{min}} = \left(\frac{2}{\rho}\sqrt{\frac{K}{C_{D_0}}}\,\frac{W}{S}\right)^{1/2}}$$

$$\mathbf{V_{(D/V)_{min}} = \left(\frac{2}{\rho}\sqrt{\frac{3\,K}{C_{D_0}}}\,\frac{W}{S}\right)^{1/2} = 1.3161\ V_{L/D_{max}}}$$

Range for Jet Aircraft

$$R = \frac{V}{c_t}\frac{L}{D}\int_{W_1}^{W_0}\frac{dW}{W} \longrightarrow \boxed{R = \frac{V}{c_t}\frac{L}{D}\ln\frac{W_0}{W_1}}$$

Breguet Range Equation

$$\text{Range Factor} = \frac{V}{c_t}\frac{L}{D}$$

Maximize Range Factor = Maximize Range

Fly at maximum V (L / D)

Minimize c_t

Carry a lot of fuel

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Endurance for Jet Aircraft

$$\boxed{c_t = \frac{\text{Fuel Flow}}{\text{Thrust}} = -\frac{\dot{W}_f}{T}} \qquad \boxed{\dot{W}_f = \frac{dW}{dt}} \qquad \boxed{\frac{L}{D} = \frac{W}{T}} \qquad \boxed{V = \frac{ds}{dt}}$$

$$\cdots \longrightarrow E = \frac{1}{c_t}\frac{L}{D}\int_{W_1}^{W_0}\frac{dW}{W} \longrightarrow \boxed{E = \frac{1}{c_t}\frac{L}{D}\ln\frac{W_0}{W_1}}$$

"Endurance Factor"

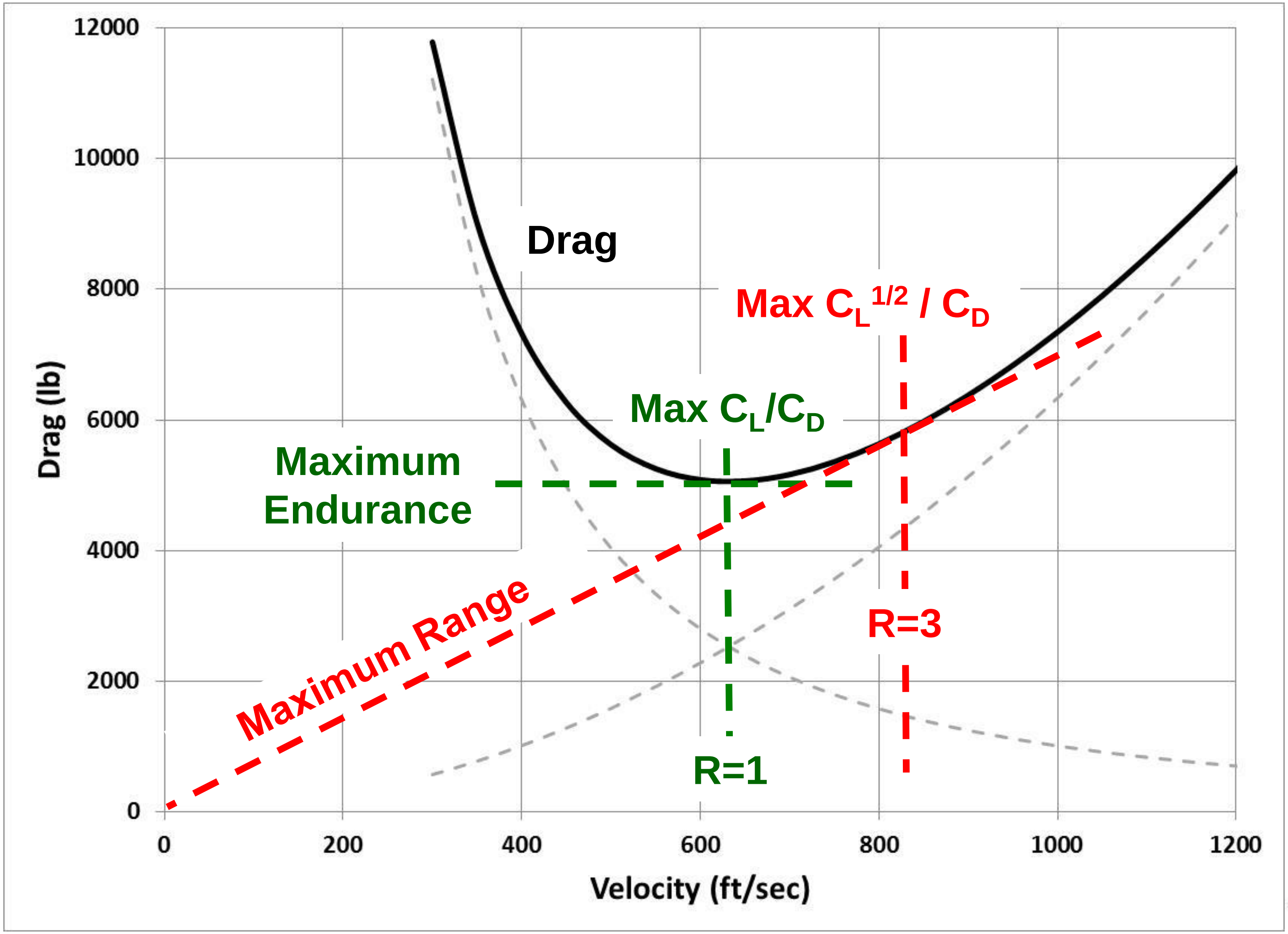
To Maximize Endurance:

Fly at maximum L / D

Minimize c_t

Carry a lot of fuel

Steady Flight for Jet Aircraft



Range & Endurance – Jet Aircraft

	Maximize	Calculate	Performance Equation
R = 1 / 3	Max $\frac{C_L^{3/2}}{C_D}$		
R = 1	Max $\frac{C_L}{C_D}$	Maximum Endurance	$E(hr) = \frac{1}{c_t} \frac{L}{D} \ln \frac{W_0}{W_1}$
R = 3	Max $\frac{C_L^{1/2}}{C_D}$	Maximum Range	$R(NM) = \frac{V}{c_t} \frac{L}{D} \ln \frac{W_0}{W_1}$

Range is the total distance traversed by an airplane (measured with respect to the ground)

Endurance is the amount of time that an airplane can stay in the air

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BD-5J Range & Endurance

Hand calculation checks to make sure this data makes sense

SEA LEVEL					
	AIRCRAFT & ATMOSPHERIC DATA				
Weight	900	lb	QMS	1481.4	lb/ft^2
Altitude	0	ft	a	1116.5	ft/sec
			rho	0.00237688	slugs/ft^3
	ENDURANCE CALCULATIONS				
			EF	Endurance	
Max (CL/CD)	14.1990		10.9223	1.2865	hr
Velocity	187.81	ft/sec		77.2	min
	0.168	Mach		143.1	NM
	RANGE CALCULATIONS				
			RF	Range	
MAX (CL^0.5/CD)	21.4739		1385.19	163.2	NM
Velocity	247.18	ft/sec		1.1141	hr
	0.221	Mach		66.8	min

10,000 FT					
	AIRCRAFT & ATMOSPHERIC DATA				
Weight	900	lb	QMS	1018.7	lb/ft^2
Altitude	10,000	ft	a	1077.4	ft/sec
			rho	0.00175527	slugs/ft^3
	ENDURANCE CALCULATIONS				
			EF	Endurance	
Max (CL/CD)	14.1990		11.3184	1.3331	hr
Velocity	218.55	ft/sec		80.0	min
	0.203	Mach		172.6	NM
	RANGE CALCULATIONS				
			RF	Range	
MAX (CL^0.5/CD)	21.4739		1670.36	196.7	NM
Velocity	287.64	ft/sec		1.1545	hr
	0.267	Mach		69.3	min

For the same fuel weight: Endurance Time > Range Time

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HW#8

BD-5J Range & Endurance

Hand calculation checks to make sure this data makes sense

SEA LEVEL					
	AIRCRAFT & ATMOSPHERIC DATA				
Weight	900	lb	QMS	1481.4	lb/ft^2
Altitude	0	ft	a	1116.5	ft/sec
			rho	0.00237688	slugs/ft^3
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	0.203	Mach		172.6	NM
	RANGE CALCULATIONS				
			RF	Range	
MAX (CL^0.5/CD)	21.4739		1670.36	196.7	NM
Velocity	287.64	ft/sec		1.1545	hr
	0.267	Mach		69.3	min

For the same fuel weight: Max Range > Endurance Range

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HW#8

BD-5J Range & Endurance

Hand calculation checks to make sure this data makes sense

SEA LEVEL					
	AIRCRAFT & ATMOSPHERIC DATA				
Weight	900	lb	**QMS**	1481.4	lb/ft^2
Altitude	0	ft	**a**	1116.5	ft/sec
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	ENDURANCE CALCULATIONS				
			EF	**Endurance**	
Max (CL/CD)	14.1990		10.9223	1.2865	hr
Velocity	187.81	ft/sec		77.2	min
	0.168	Mach		143.1	NM
	RANGE CALCULATIONS				
			RF	**Range**	
MAX (CL^0.5/CD)	21.4739		1385.19	163.2	NM
Velocity	247.18	ft/sec		1.1141	hr
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10,000 FT					
	AIRCRAFT & ATMOSPHERIC DATA				
Weight	900	lb	**QMS**	1018.7	lb/ft^2
Altitude	10,000	ft	**a**	1077.4	ft/sec
			rho	0.00175527	slugs/ft^3
	ENDURANCE CALCULATIONS				
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Max (CL/CD)	14.1990		11.3184	1.3331	hr
Velocity	218.55	ft/sec		80.0	min
	0.203	Mach		172.6	NM
	RANGE CALCULATIONS				
			RF	**Range**	
MAX (CL^0.5/CD)	21.4739		1670.36	196.7	NM
Velocity	287.64	ft/sec		1.1545	hr
	0.267	Mach		69.3	min

Velocity X Time = Range

$$(247.18)(3600/6076.4) \times 1.1141 = 163.2$$

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Specific Energy

Total Energy = Potential Energy + Kinetic Energy

$$\mathbf{E = PE + KE}$$

$$\mathbf{E = mgh + \frac{1}{2}mV^2}$$

$$\mathbf{\frac{E}{W} = h + \frac{1}{2g}V^2}$$

$$\mathbf{E_s = h + \frac{1}{2g}V^2}$$

(always in ft)

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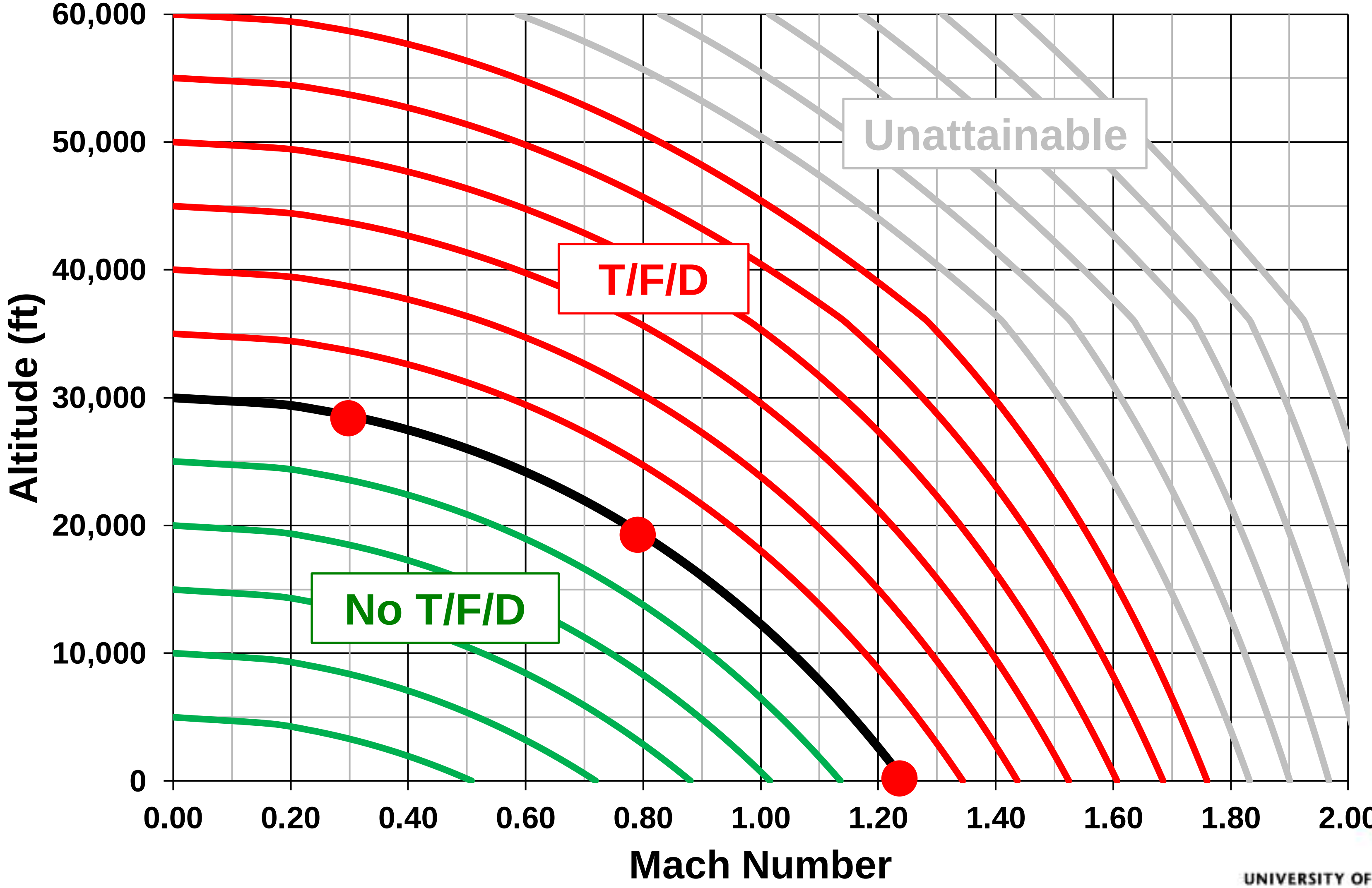
Specific Energy



Specific Energy

$$E_s = h + \frac{1}{2g} V^2$$

Flight Condition		
Alt	0	ft
Mach	0.75	
Atmospheric Data		
a	1116.45	ft/sec
Specific Energy Calculations		
h	0	ft
V	837.34	ft/sec
E_s	10,896	ft

Flight Condition		
Alt	10,000	ft
Mach	0.75	
Atmospheric Data		
a	1077.39	ft/sec
Specific Energy Calculations		
h	10,000	ft
V	808.04	ft/sec
E_s	20,147	ft

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Specific Energy

What is the relationship between E_s and P_s?

$$\frac{d}{dt}E_s = \frac{d}{dt}\left(h + \frac{1}{2g}V^2\right)$$

$$\boxed{\dot{E}_s = \frac{dh}{dt} + \frac{V}{g}\frac{dV}{dt} = P_s}$$

(always in ft/sec)

Specific Excess Power

$$\boxed{\frac{(T-D)\,V}{W} = \frac{dh}{dt} + \frac{V}{g}\frac{dV}{dt} = P_s}$$

$P_s > 0$ accelerating / climbing flight $T > D$

$P_s = 0$ sustained flight $T = D$

$P_s < 0$ decelerating / descending flight $T < D$

Specific Excess Power

$$\frac{(T - D)\,V}{W} = \frac{dh}{dt} + \frac{V}{g}\frac{dV}{dt} = P_s$$

Flight Condition			Aircraft Data		
Wt	60,000	lb	CD0	0.0150	
Alt	0	ft	K	0.08	
Mach	0.75		Thrust	27,700	lb
g's	1.0		S	950	sq ft
Atmospheric Data			**Performance Data**		
QMS	1481.4	lb/sqft	CL	0.0758	
a	1116.45	ft/sec	CD	0.0155	
			T	27,700	lb
			D	12,238	lb
			V	837.34	ft/sec
			W	60,000	lb
			P_s	215.78	ft/sec

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Specific Excess Power

$$\frac{(T-D)\,V}{W} = \frac{dh}{dt} + \frac{V}{g}\frac{dV}{dt} = P_s$$

Flight Condition			Aircraft Data		
Wt	60,000	lb	**CD0**	0.0150	
Alt	0	ft	**K**	0.08	
Mach	0.75		**Thrust**	27,700	lb
g's	3.5		**S**	950	sq ft
Atmospheric Data			**Performance Data**		
QMS	1481.4	lb/sqft	**CL**	0.2653	
a	1116.45	ft/sec	**CD**	0.0206	
			T	27,700	lb
			D	16,331	lb
			V	837.34	ft/sec
			W	60,000	lb
			P_s	158.66	ft/sec

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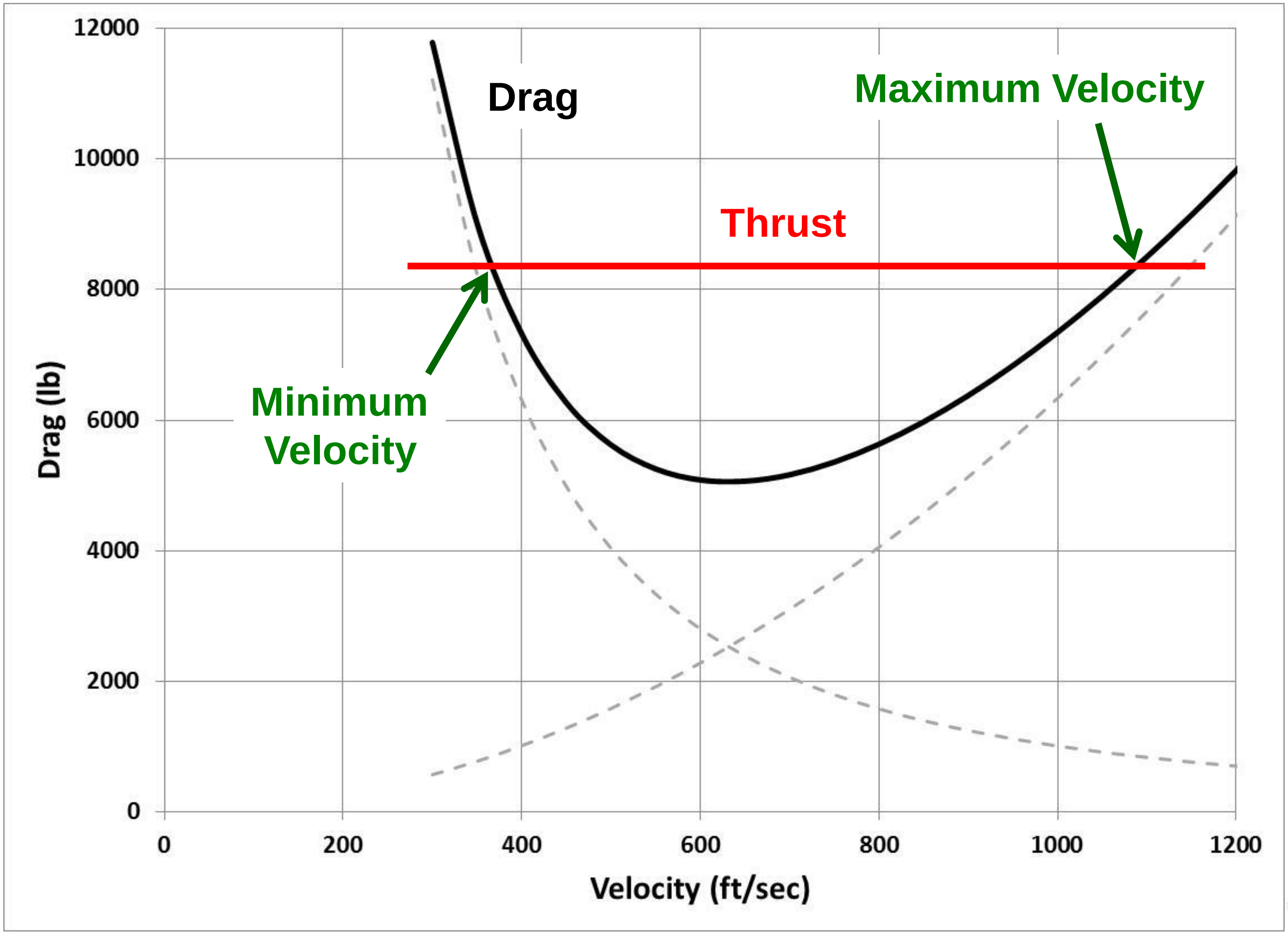
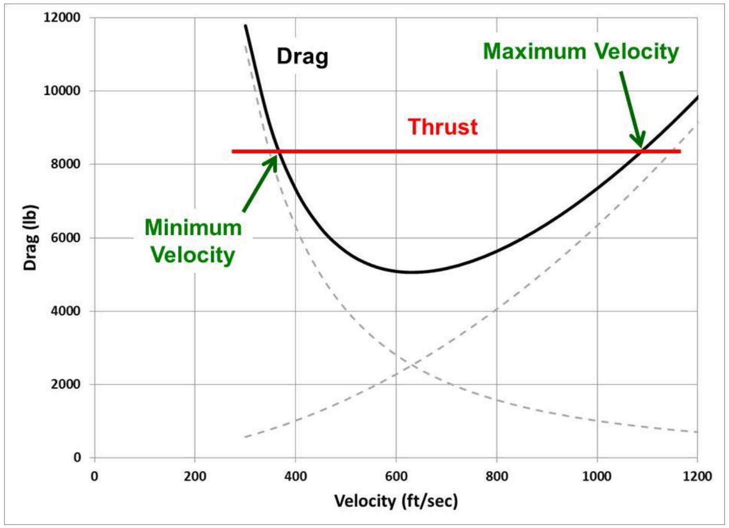
Min & Max Velocity



Min & Max Velocity



$$T_A = D = q\,S\,(C_{D_0} + K\,C_L{}^2)$$

$$V_{max} = \left[\frac{\left(\frac{T}{W}\right)\left(\frac{W}{S}\right) + \left(\frac{W}{S}\right)\sqrt{\left(\frac{T}{W}\right)^2 - 4C_{D_0}K}}{\rho\,C_{D_0}}\right]^{1/2}$$

$$V_{min} = \left[\frac{\left(\frac{T}{W}\right)\left(\frac{W}{S}\right) - \left(\frac{W}{S}\right)\sqrt{\left(\frac{T}{W}\right)^2 - 4C_{D_0}K}}{\rho\,C_{D_0}}\right]^{1/2}$$

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Minimum Velocity Constraints

We already know about Minimum Velocity where T=D, what else would be considered a minimum velocity?

1. **Velocity for maximum lift ($C_{L_{max}}$) or stall speed (V_{stall})**
2. Buffet limitations
3. Stability and control constraints

$$C_L = \frac{W}{qS} = \frac{W}{\frac{1}{2}\,\rho\,V^2\,S}$$

$$C_{L_{max}} = \frac{W}{\frac{1}{2}\,\rho\,{V_{stall}}^2\,S} \longrightarrow V_{stall} = \sqrt{\frac{2}{\rho}\,\frac{W}{S}\,\frac{1}{C_{L_{max}}}}$$

Maximum Velocity Constraints

We already know about Maximum Velocity where T=D, what else would be considered a maximum velocity?

1. **Velocity for maximum dynamic pressure (q_{max})**
2. **Velocity for maximum Mach number (M_{max})**
3. Stability and control constraints

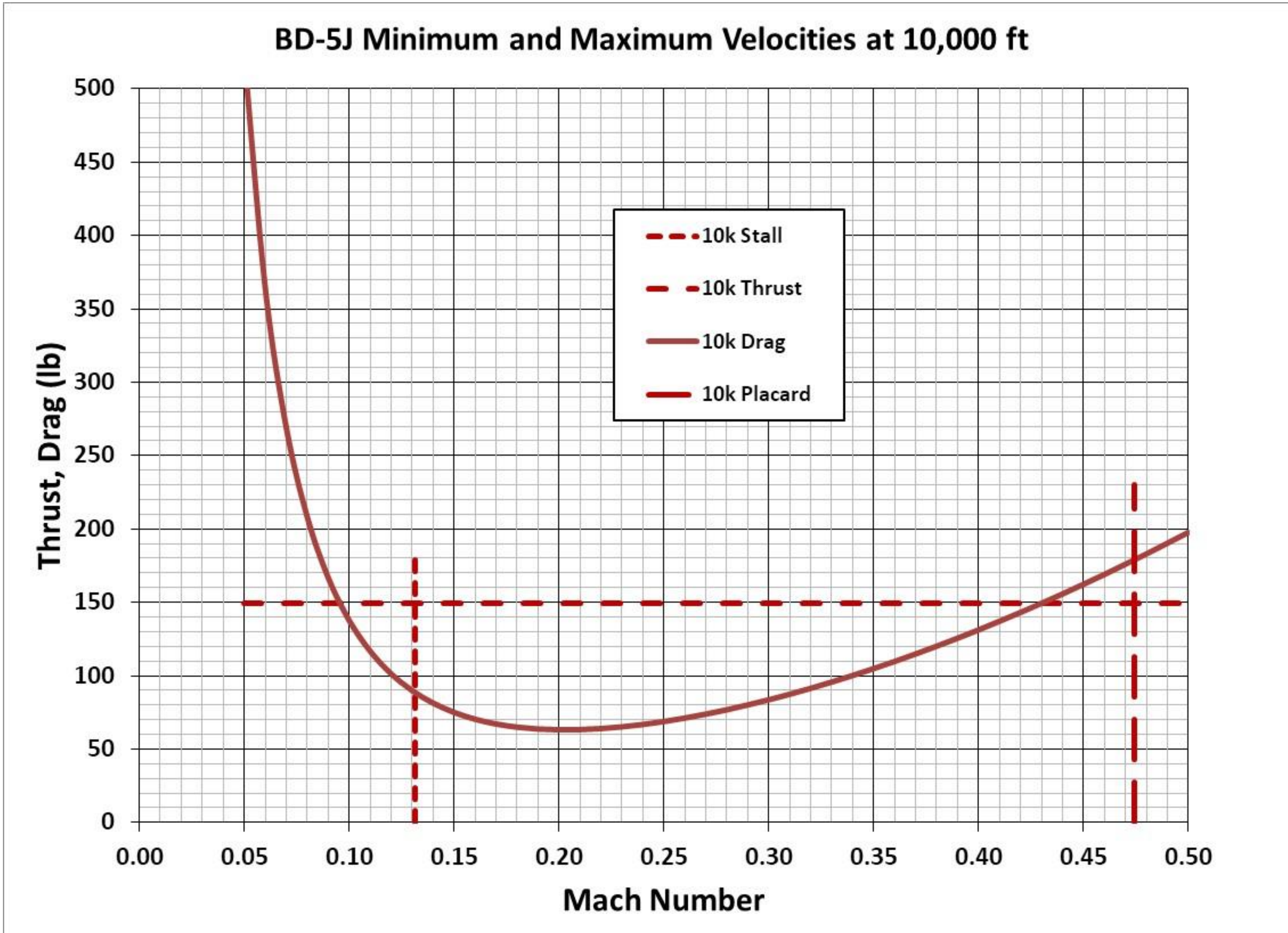
At any altitude, an aircraft's maximum velocity is constrained to the lesser of these two speeds:

$$\mathbf{V_{max} = \sqrt{\frac{2}{\rho}\, q_{max}}}$$

- or -

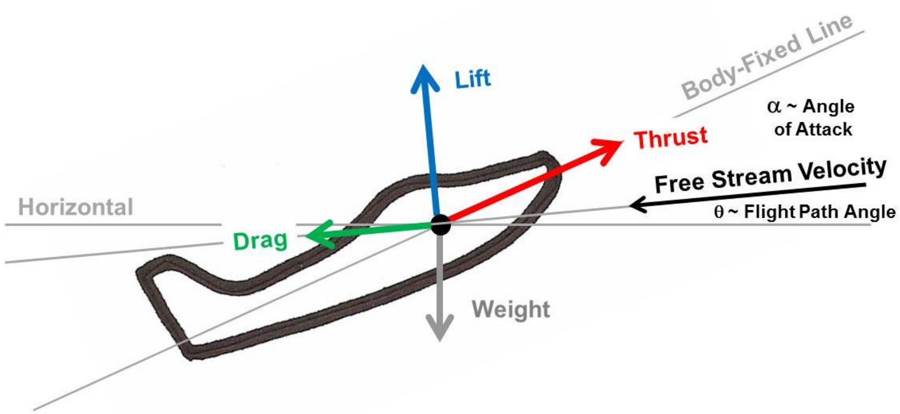
$$\mathbf{V_{max} = M_{max}\ a}$$

HW #10



BD-5J Minimum and Maximum Velocities at 10,000 ft
500
450
400
350
300
250
200
150
100
50
0
Thrust, Drag (lb)
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.50
Mach Number
10k Stall
10k Thrust
10k Drag
10k Placard

Steady Climbing Flight



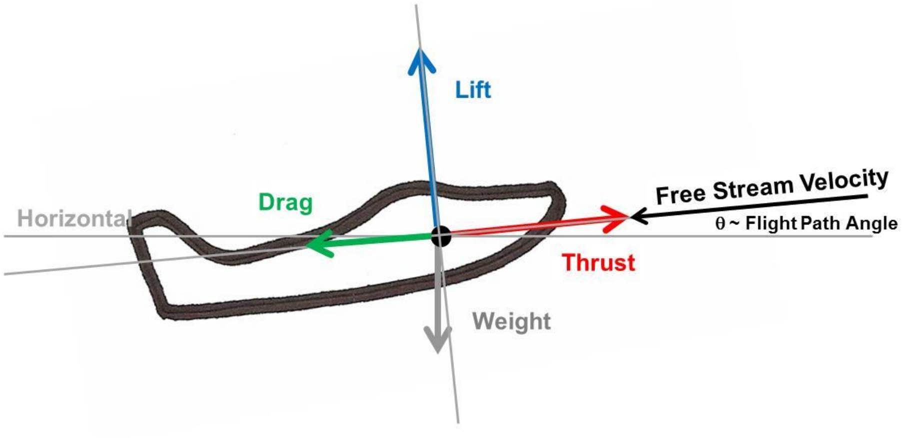
$$\Sigma F_x = T\cos\alpha - D - W\sin\theta = \frac{d(mV)}{dt} = \cancelto{0}{\dot{m}}V + m\cancelto{0}{\dot{V}}$$

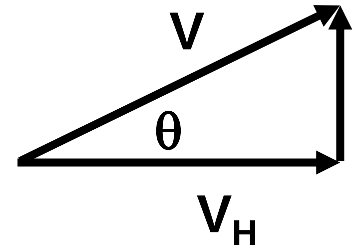
$$\Sigma F_z = L + T\sin\alpha - W\cos\theta \; = 0$$

Steady Climb = Not Accelerating

Steady Climb = Constant Weight

Steady Climbing Flight





$$R/C = V \sin\theta = \frac{(T - D)\,V}{W}$$

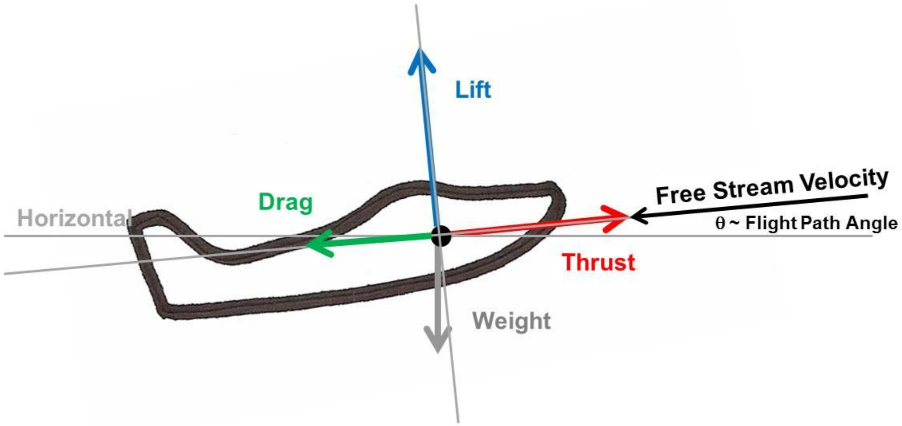
$$R/C = V\left[\left(\frac{T}{W}\right) - \frac{q\,C_{D_0}}{(W/S)} - \left(\frac{W}{S}\right)\frac{K}{q}\right]$$

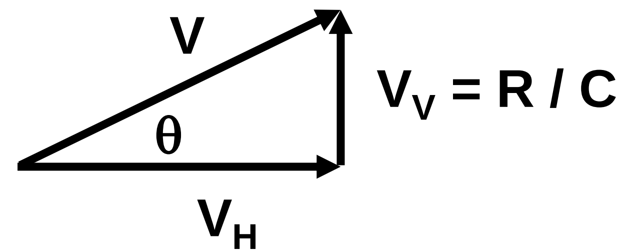
Fundamental Parameters

Speed and Altitude

Drag Polar Characteristics

Steady Climbing Flight





Maximum Climb Angle (jet aircraft)

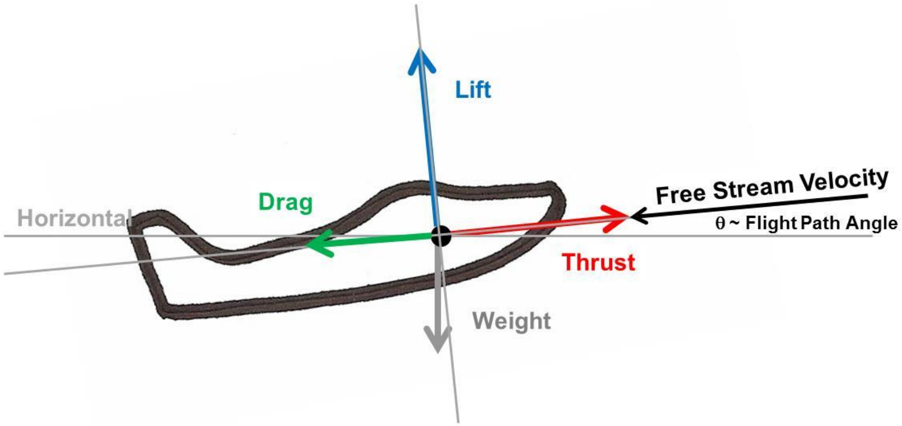
$$\sin\theta_{max} = \left(\frac{T}{W}\right) - \frac{1}{(L/D)_{max}} = \left(\frac{T}{W}\right) - \sqrt{4\,C_{D_0}K}$$

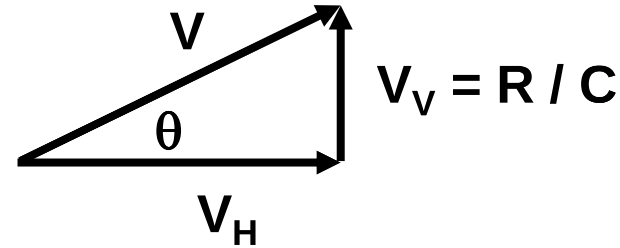
Dominated by thrust term

$$V_{\theta_{max}} = \sqrt{\frac{2}{\rho}\left(\frac{K}{C_{D_0}}\right)^{1/2}\left(\frac{W}{S}\right)\cos\theta_{max}}$$

Dominated by aero terms

Maximum Rate of Climb



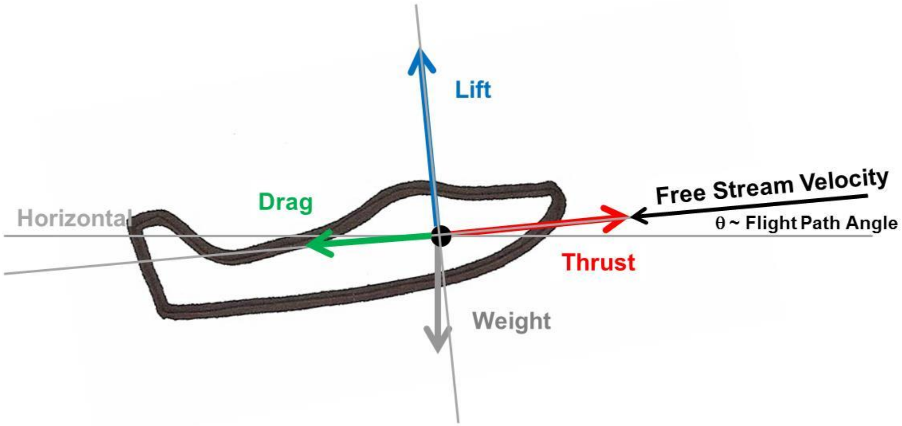


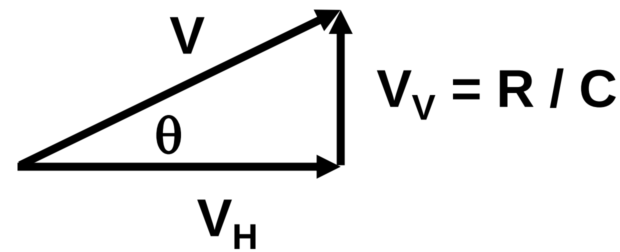
<u>Maximum Rate of Climb</u> (jet aircraft)

$$(R/C)_{max} = \left[\left(\frac{W}{S}\right)\frac{Z}{3\,\rho\,C_{D_0}}\right]^{1/2}\left(\frac{T}{W}\right)^{3/2}\left[1-\frac{Z}{6}-\frac{3}{2\,Z\,(T/W)^2\,(L/D)^2_{max}}\right]$$

$$\text{where}\quad Z = 1+\sqrt{1+\frac{3}{(T/W)^2\,(L/D)^2_{max}}}$$

Maximum Rate of Climb





Maximum Rate of Climb (jet aircraft)

$$V_{(R/C)_{max}} = \left[\left(\frac{T}{W}\right) \left(\frac{W}{S}\right) \frac{Z}{3\,\rho\, C_{D_0}} \right]^{1/2}$$

$$\text{where} \quad Z = 1 + \sqrt{1 + \frac{3}{\left(T/W\right)^2 \left(L/D\right)^2_{max}}}$$

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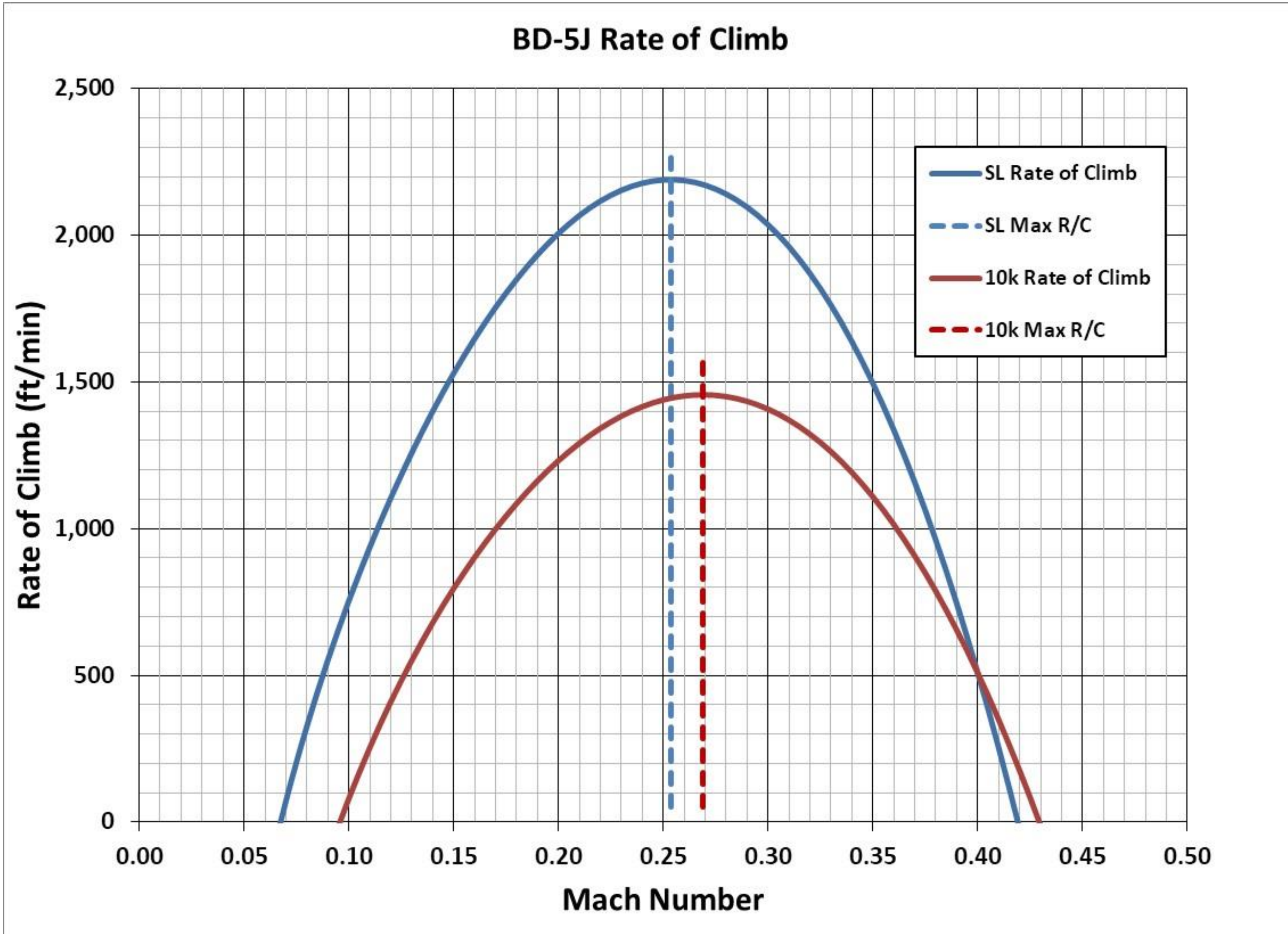
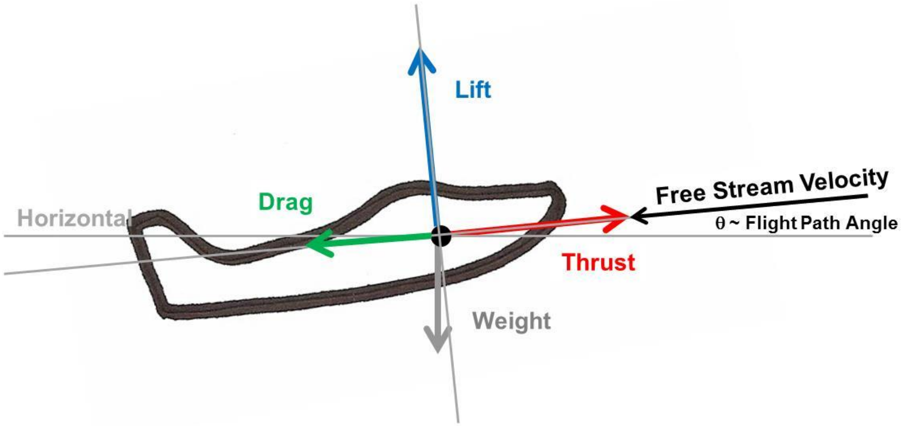
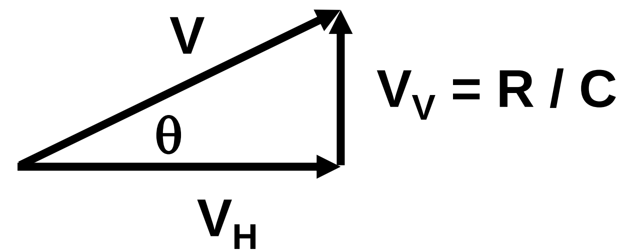
HW #11


BD-5J Rate of Climb
Rate of Climb (ft/min)
2,500
2,000
1,500
1,000
500
0
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.50
Mach Number
SL Rate of Climb
SL Max R/C
10k Rate of Climb
10k Max R/C

Time to Climb





$$\frac{dh}{dt} = R/C \longrightarrow dt = \frac{dh}{R/C} \longrightarrow t = \int_{h_1}^{h_2} \frac{dh}{R/C}$$

$$\boxed{t_{min} = \int_{h_1}^{h_2} \frac{dh}{(R/C)_{max}}}$$

Time to Climb

<u>Alternative Method in Textbook</u> – assume linear R/C_{max} vs Altitude

Calculate the maximum Rate of Climb at Sea Level
Calculate the maximum Rate of Climb at 30,000 ft
Derive the linear relationship $R/C_{max} = a + b\,\Delta h$

$$t_{min} = \int_{h_1}^{h_2} \frac{dh}{(R/C)_{max}} = \int_{h_1}^{h_2} \frac{dh}{a + b\,\Delta h}$$

$$t_{min} = \frac{1}{b}\left[\ln(a + b\,\Delta h) - \ln a\right]$$

Time to Climb

Easier Method

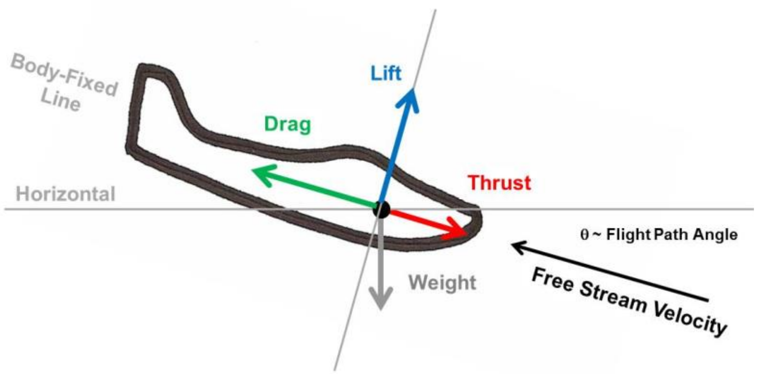
$$\mathbf{t_{min} = \frac{\Delta\, h}{(R/C)_{avg}}}$$

At Sea Level: $(R/C)_{max}$ = 179.8 ft/sec
At 30,000 ft: $(R/C)_{max}$ = 111.0 ft/sec

$$\mathbf{t_{min} = \frac{(30,000\ - 0)}{(179.8 + 111.0)/2}}$$

$$\boxed{\mathbf{t_{min} = 206.3\ sec = 3.44\ min}}$$

Steady Descending Flight



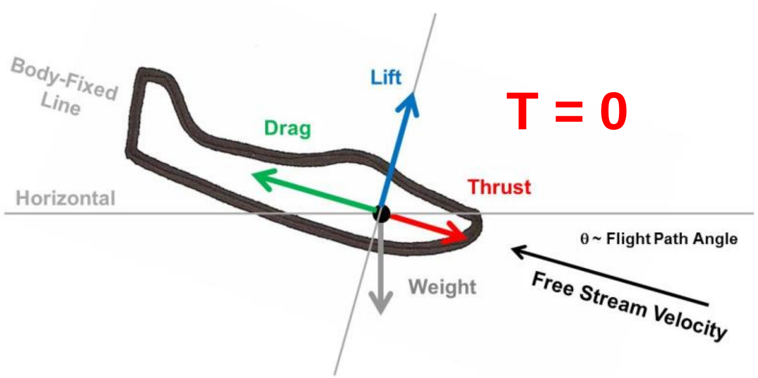
$$\Sigma F_x = T - D + W \sin\theta = \frac{d(mV)}{dt} = \cancelto{0}{\dot{m}}\, V + m\, \cancelto{0}{\dot{V}}$$

$$\Sigma F_z = L - W \cos\theta \quad = 0$$

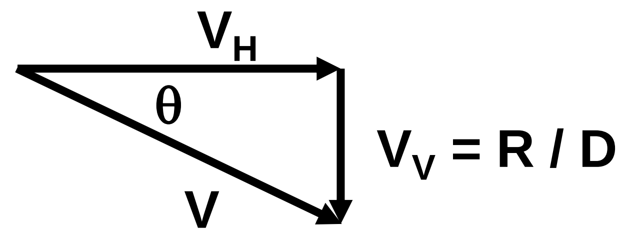
Steady Descent = Not Accelerating

Steady Descent = Constant Weight

Steady Gliding Flight



T = 0



$$\tan\theta_{min} = \frac{1}{(L/D)_{max}}$$

Minimum descent angle
= Maximum glide range

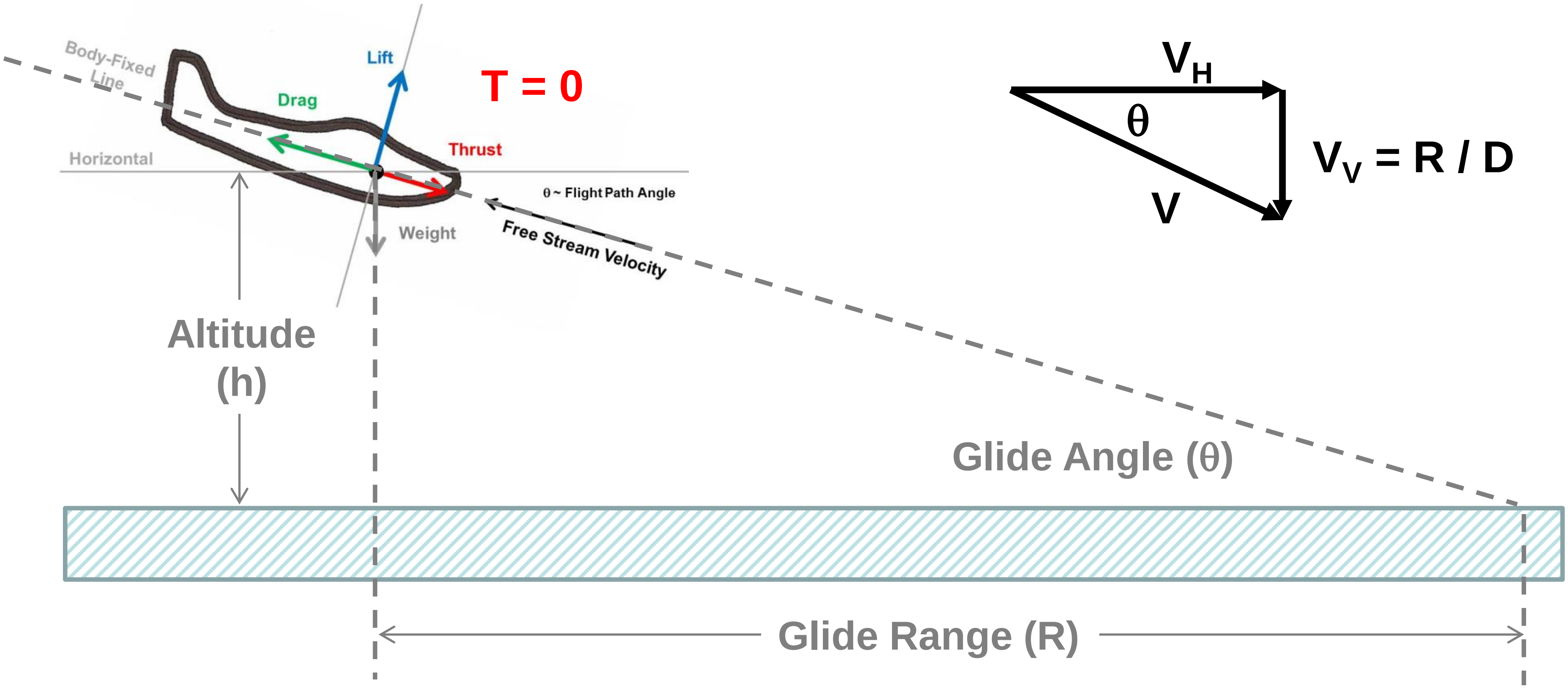
$$V_{\theta_{min}} = \sqrt{\frac{2}{\rho}\left(\frac{K}{C_{D_0}}\right)^{1/2}\left(\frac{W}{S}\right)\cos\theta}$$

$$V_{(L/D)_{max}} = \sqrt{\frac{2}{\rho}\left(\frac{K}{C_{D_0}}\right)^{1/2}\left(\frac{W}{S}\right)}$$

$V_{\theta_{min}} \approx V_{(L/D)_{max}}$

Maximum endurance for jet aircraft R=1

Steady Gliding Flight



$$\tan\theta_{min} = \frac{1}{\left(L/D\right)_{max}} = \frac{h}{R}$$

Steady Flight

$$\boxed{R = \frac{\text{Zero Lift Drag}}{\text{Drag Due to Lift}}}$$

Max Endurance **Max Glide Range**

$$\boxed{R = 1 \qquad C_{D_0} = C_{D_L} = K\,C_L^{\ 2}} \qquad \text{Max}\ \frac{C_L}{C_D} \qquad V_{L/D_{max}} = \left(\frac{2}{\rho}\sqrt{\frac{K}{C_{D_0}}}\,\frac{W}{S}\right)^{1/2}$$

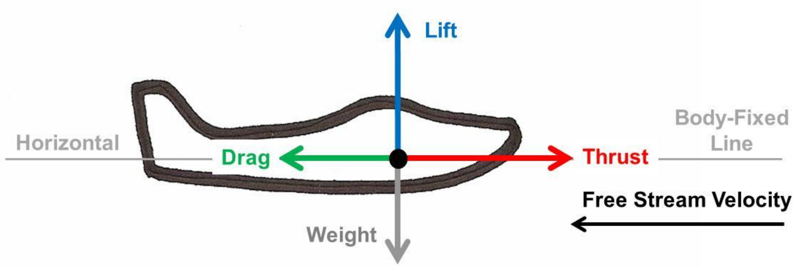
Max Range

$$\boxed{R = 3 \qquad C_{D_0} = 3\,C_{D_L} = 3\,K\,C_L^{\ 2}} \qquad \text{Max}\ \frac{C_L^{\ 1/2}}{C_D} \qquad V = 1.3161\ V_{L/D_{max}}$$

Min Sink Rate

$$\boxed{R = \frac{1}{3} \qquad C_{D_0} = \frac{1}{3}\,C_{D_L} = \frac{1}{3}\,K\,C_L^{\ 2}} \qquad \text{Max}\ \frac{C_L^{\ 3/2}}{C_D} \qquad V = 0.7598\ V_{L/D_{max}}$$

Aircraft Flight Ceiling



How high can an airplane fly in steady, level flight?

Upper altitude limit = ceiling

Ceiling	R / C Capability
Absolute	0 ft / min
Service	100 ft / min
Cruise	300 ft / min
Combat	500 ft / min

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Estimating Absolute Ceiling

Calculate the maximum Rate of Climb at Sea Level

Calculate the maximum Rate of Climb at another altitude

Derive the linear relationship $R/C_{max} = a + b\,h$
and solve for h where $R/C_{max} = 0$

At Sea Level: $(R/C)_{max} = 4{,}600$ ft/min
At 30,000 ft: $(R/C)_{max} = 1{,}600$ ft/min

$a = 4{,}600$ $b = -0.1$

$(R/C)_{max} = 4{,}600 - 0.1\,h$ $h = [4{,}600 - (R/C)_{max}] / 0.1$

When $(R/C)_{max} = 0$, then $h = 46{,}000$ ft

Course Exams

Will be done online via Canvas:

- Exam #1 – Thursday, January 27 at 2:00 pm ET
- **Exam #2a – Tuesday, February 17 at 2:00 pm ET**
- Exam #2b – Thursday, March 10 at 2:00 pm ET

Start the exam between 2:00 and 2:15 pm ET

You will have **90 minutes** to complete the exam

Allowable resources include your notes, calculator, lecture slides, reference materials, spreadsheets

Academic Integrity = no help from other students!!

Questions are randomly selected

Exam will be graded automatically

Students will know their exam grade immediately

Questions during the exam should be e-mailed to the instructor

Course Exam Tips

Be ready to take this exam!

There are 15 questions to answer (~100 points each):

- True / False
- Multiple Choice
- Matching
- Numerical values

Answers do not depend on any previous calculations, which means that there is no partial credit

You can answer the questions in any order

You can review any of the questions at any time

Do not hit the "Submit" button until you have finished

Questions?