Aircraft Performance
Equations of Motion
Takeoff and Landing

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Takeoff

The aircraft takeoff maneuver can be divided up into three distinct phases:

Ground – all landing gear on the ground
– brake release to start of rotation

Rotation – pilot inputs an elevator deflection that lifts the nose gear off the runway
– ends when the main gear lifts off the runway

Climb / Acceleration – transitions to target airspeed
– flaps are retracted
– landing gear is retracted

McDonnell-Douglas MD-80 Takeoff Video

Boeing 757 Crosswind Takeoff Video

Boeing 787 Takeoff Video

F-35C Catapult Video

Takeoff

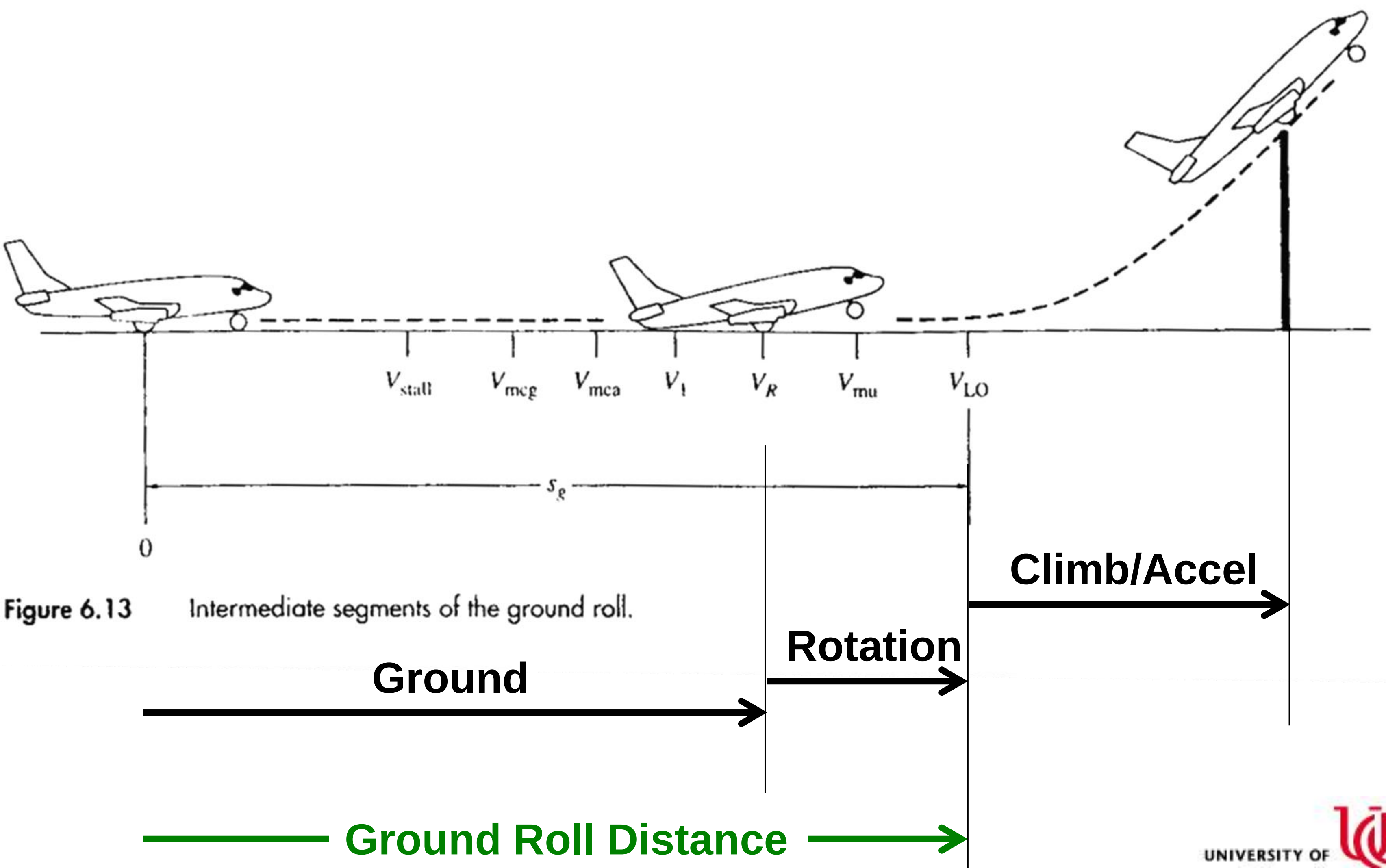
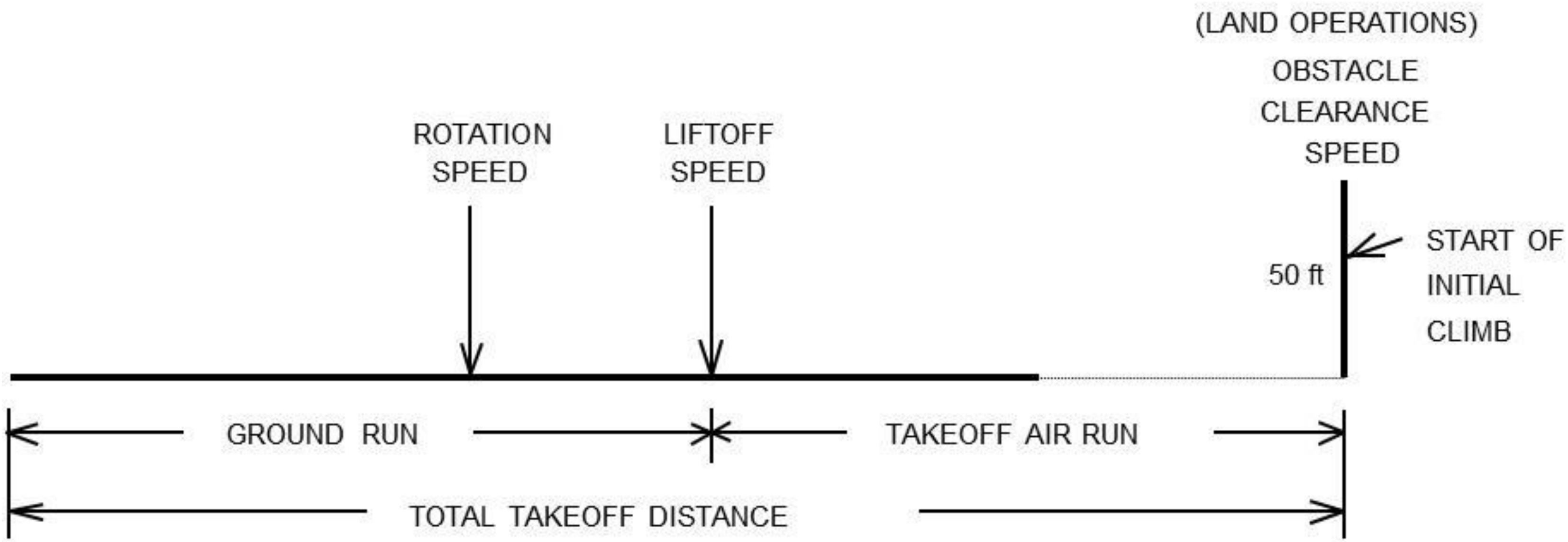


Figure 6.13 Intermediate segments of the ground roll.

Takeoff Speeds



Rotation Speed $> 1.05\ V_{stall}$

Liftoff Speed $> 1.10\ V_{stall}$

Obstacle Speed $> 1.20\ V_{stall}$

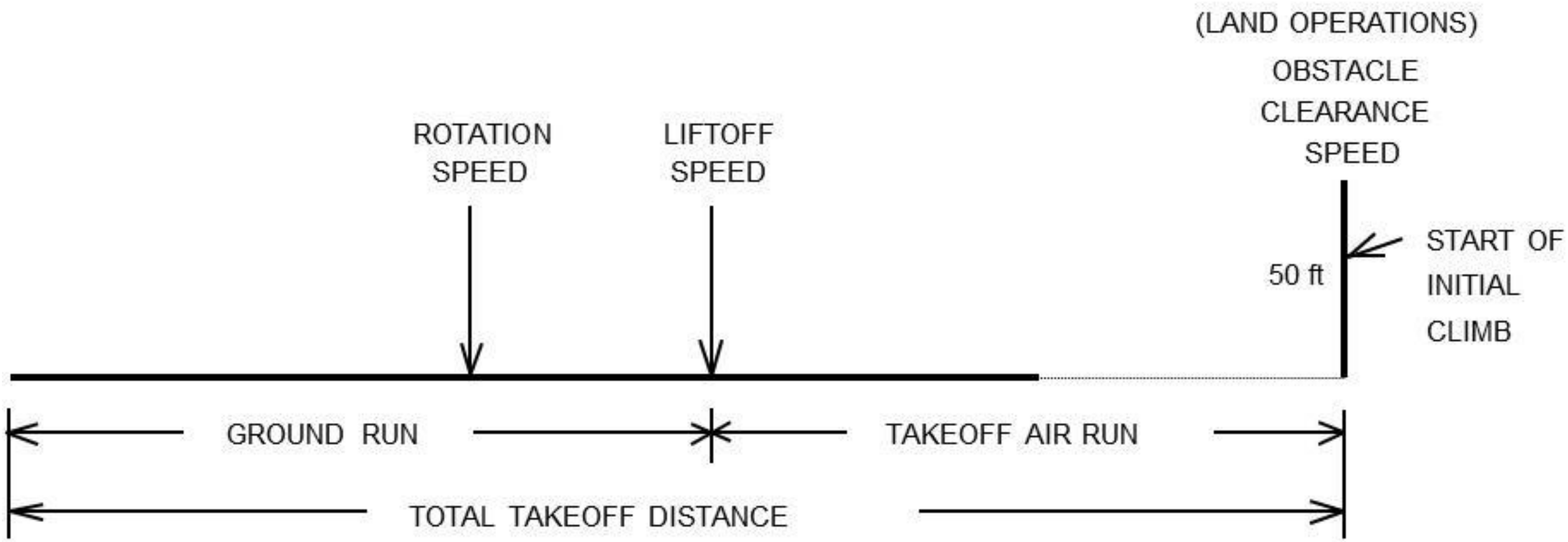
$$V_{stall} = \sqrt{\frac{2\,W}{\rho\,S\,C_{L_{max}}}}$$

$$V_{rot} = 1.05\ V_{stall}$$

Stall Speed is dominated by W/S

Stall Speed ↓
Wing Area ↑

Takeoff



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$$V_{rot} = 1.05\ V_{stall}$$

Stall Speed is dominated by $C_{L_{max}}$

Stall Speed ↓

$C_{L_{max}}$ ↑

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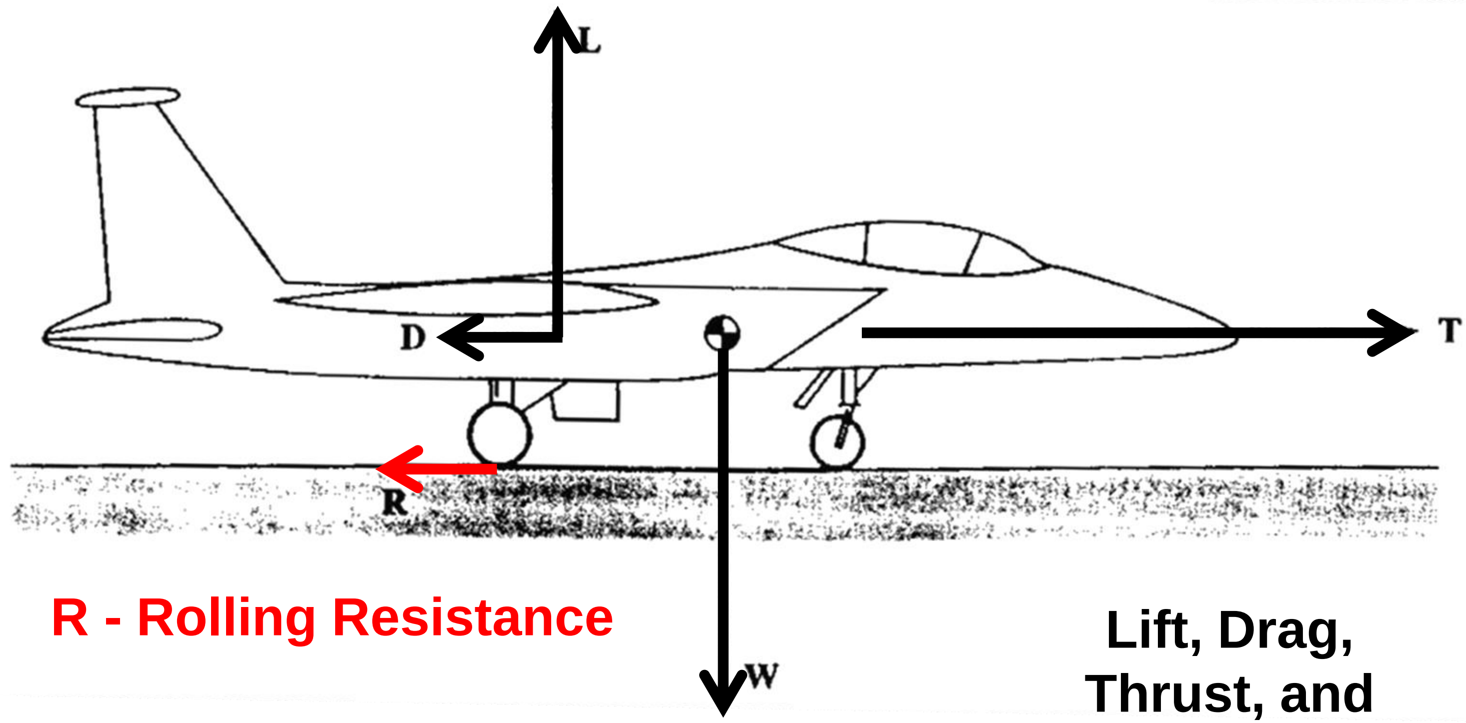
Takeoff



R - Rolling Resistance

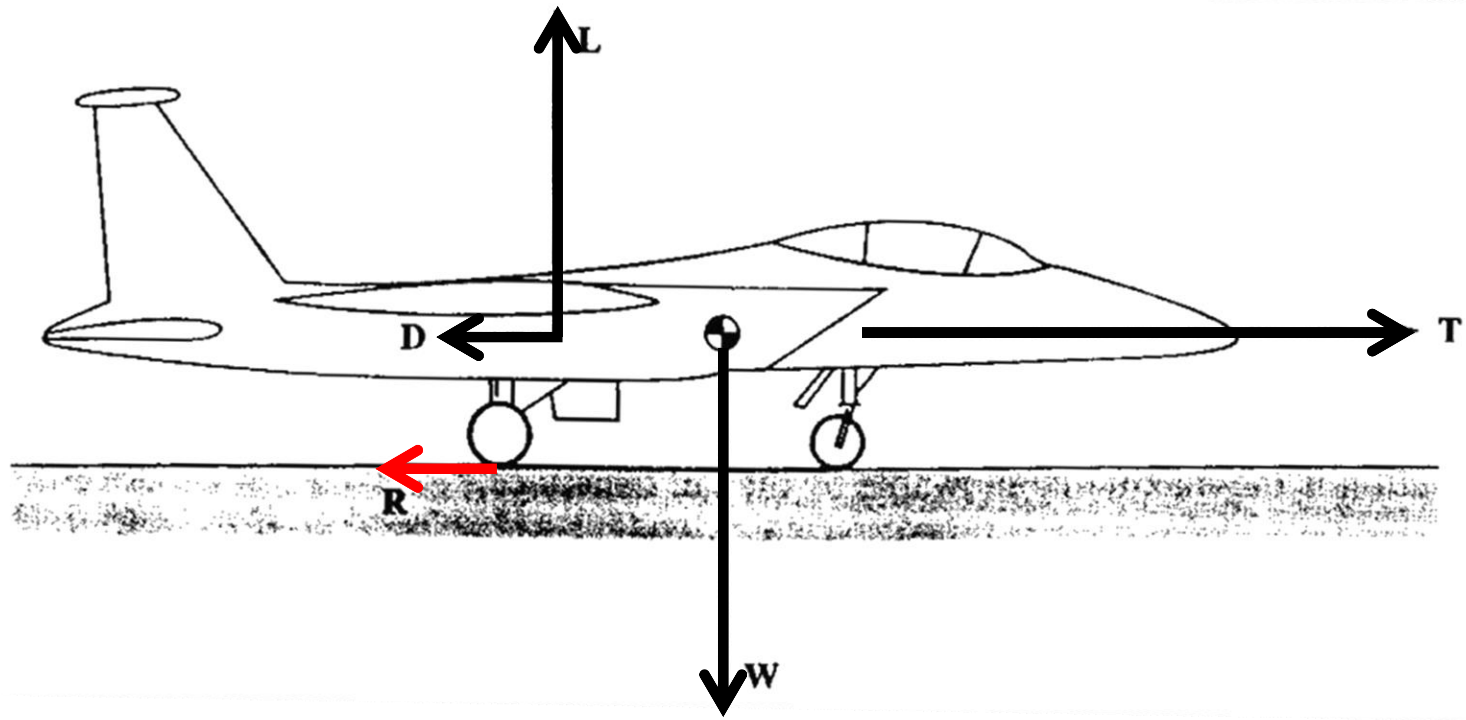
Lift, Drag, Thrust, and Weight

$R = \mu_r (W - L)$

μ_r = coefficient of rolling friction

$0.02 < \mu_r < 0.08$

Takeoff



$$\Sigma F_x = T - D - R = \frac{d(mV)}{dt} = \overset{0}{\cancel{\dot{m}}}\,V + m\,\dot{V}$$

Takeoff

$$\Sigma F_x = T - D - R = m\frac{dV}{dt}$$

$$ds = \frac{ds}{dt}dt = V\,dt = V\,\frac{dt}{dV}\,dV = \frac{V\,dV}{a}$$

$$\int_0^{TO} ds = \int_0^{V_{TO}} \frac{V\,dV}{a}$$

$$\boxed{s_g = \frac{{V_{TO}}^2}{2\,a}}$$

where a is the average acceleration during the ground run

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Takeoff

$$\boxed{s_g = \frac{V_{TO}{}^2}{2\,a}}$$

where a is the average acceleration during the ground run

$$\Sigma\,F_x = T - D - R = m\frac{dV}{dt}$$

$$a = \frac{dV}{dt} = \frac{T - D - R}{W/g}$$

where T, D, and R are calculated at 0.7 V_{TO}

Takeoff capability is dominated by T/W

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Takeoff

$$\mathbf{a} = \frac{\mathbf{dV}}{\mathbf{dt}} = \frac{\mathbf{T} - \mathbf{D} - \mathbf{R}}{\mathbf{W/g}}$$

where T, D, and R are calculated at 0.7 V_{TO}

T = Thrust at 0.7 V_{TO} at takeoff altitude

D = Drag at 0.7 V_{TO} at takeoff altitude
= $(C_{D_0} + \Delta C_{D_0})\,(0.5\,\rho\,(0.7\,V_{TO})^2\,S)$

ΔC_{D_0} includes gear drag and flap drag

C_{D_L} and L are considered negligible

Takeoff

T-38 Characteristics:

C_{D_0} = 0.0158

$C_{L_{max}}$ (with flaps) = 1.20

ΔC_{D_0} = 0.0240

T (mil) at SLS = 1900 lb

W = 10,000 lb

μ_r = 0.025

Sea Level Ground Roll

Calculate V_{TO}

Calculate T @ 0.7 V_{TO}

Calculate D @ 0.7 V_{TO}

Calculate a

Calculate ground roll

$$a = \frac{dV}{dt} = \frac{T - D - R}{W/g}$$

where T and D are calculated at 0.7 V_{TO}

$$V_{TO} = 1.1\sqrt{\frac{2\,(W/S)}{\rho\, C_{L_{max}}}}$$

$$= 223.4\ \text{ft/sec}$$

$$0.7\,V_{TO} = 156.4\ \text{ft/sec} = 0.1400\ \text{Mach}$$

$$T = 2\,(\text{Tmil} + 400M) = 3912\ \text{lb}$$

$$D = (0.0158 + 0.0240)\,q\,S = 196\ \text{lb}$$

$$a = (3912 - 196 - 250)\,\frac{32.174}{10,000}$$

$$= 11.15\ \text{ft/sec}^2$$

$$s_g = \frac{{V_{TO}}^2}{2\,a} = 2,238\ \text{ft}$$

Other Factors

Headwinds, Tailwinds, Crosswinds
Headwinds reduce ground roll, Tailwinds increase ground roll
Crosswinds increase the various controllability speeds

Runway Gradient – Not all runways are flat
Going up grade increases ground roll
Going down grade decreases ground roll

Engine Spool-Up/Spool-Down Times
Engines don't produce instantaneous thrust

In-Ground Effect Aerodynamics
Changes the induced drag characteristics
Dependent on the height and span of the wing

Tail Bump Angle

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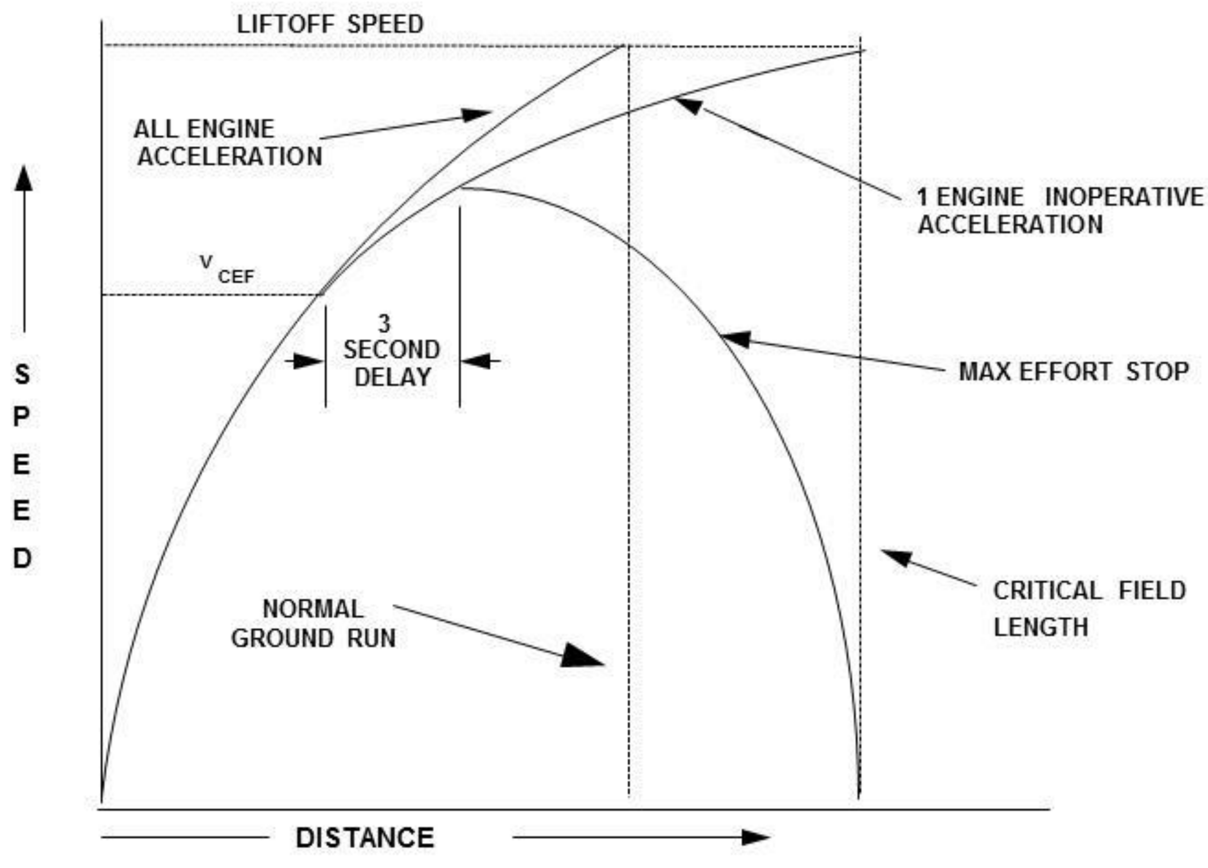
Critical Field Length

In the event of the loss of an engine during ground roll
Decision point at Critical Engine Failure Speed
Decide whether to continue takeoff or abort takeoff



Only for multi-engine aircraft

Landing

The aircraft landing maneuver can be divided up into three distinct phases:

Air – flight at 50 ft to main gear touchdown
– includes flare

Transition – begins with main gear on the runway
– ends with nose gear on the runway

Ground – all landing gear on the runway
– deceleration devices are deployed
– wheel braking is used
– ends when the aircraft completely stops

Boeing 757 Landing Video
Boeing 747 Crosswind Landing Video
F-35C Landing Video
F-35B Landing Video

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Landing

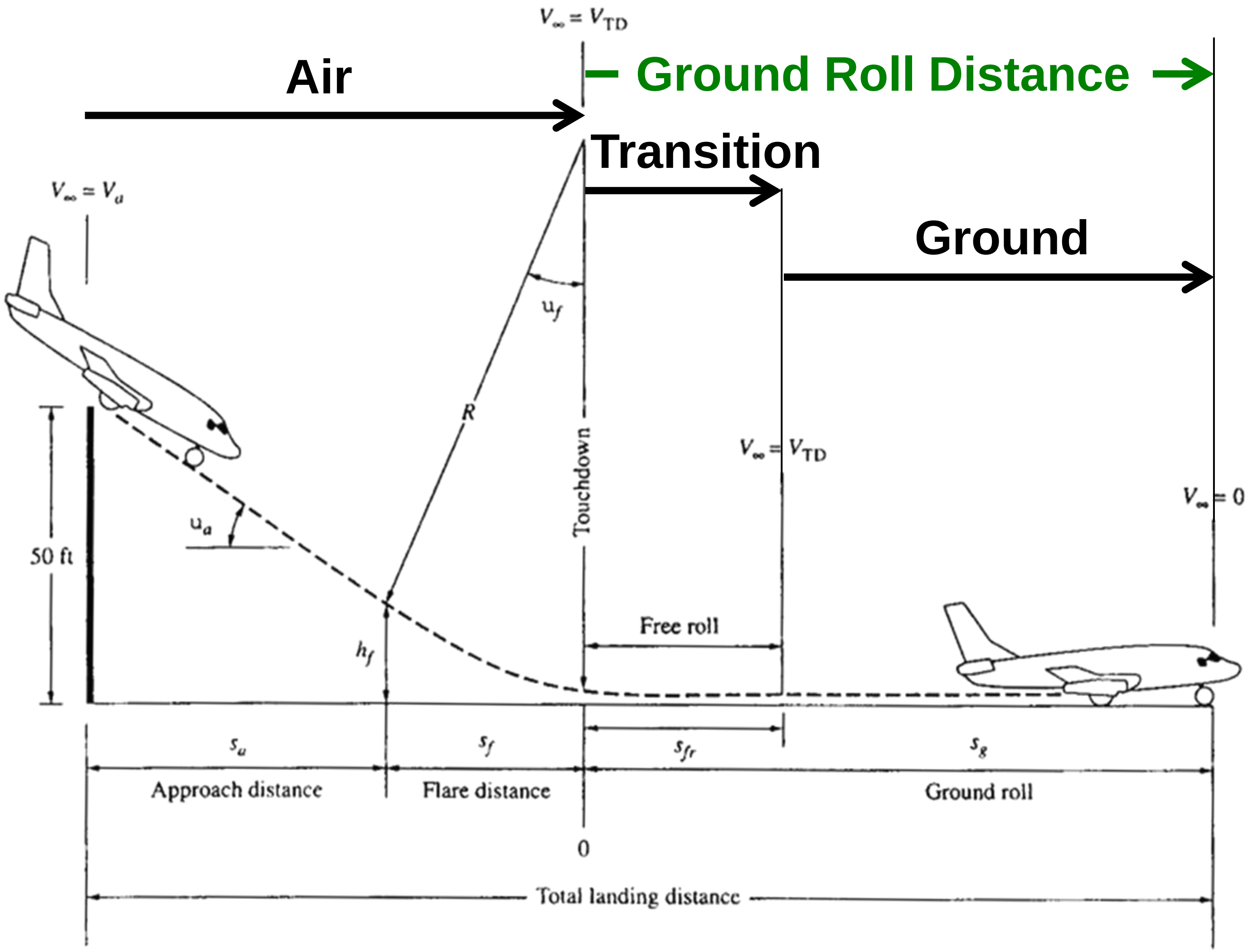
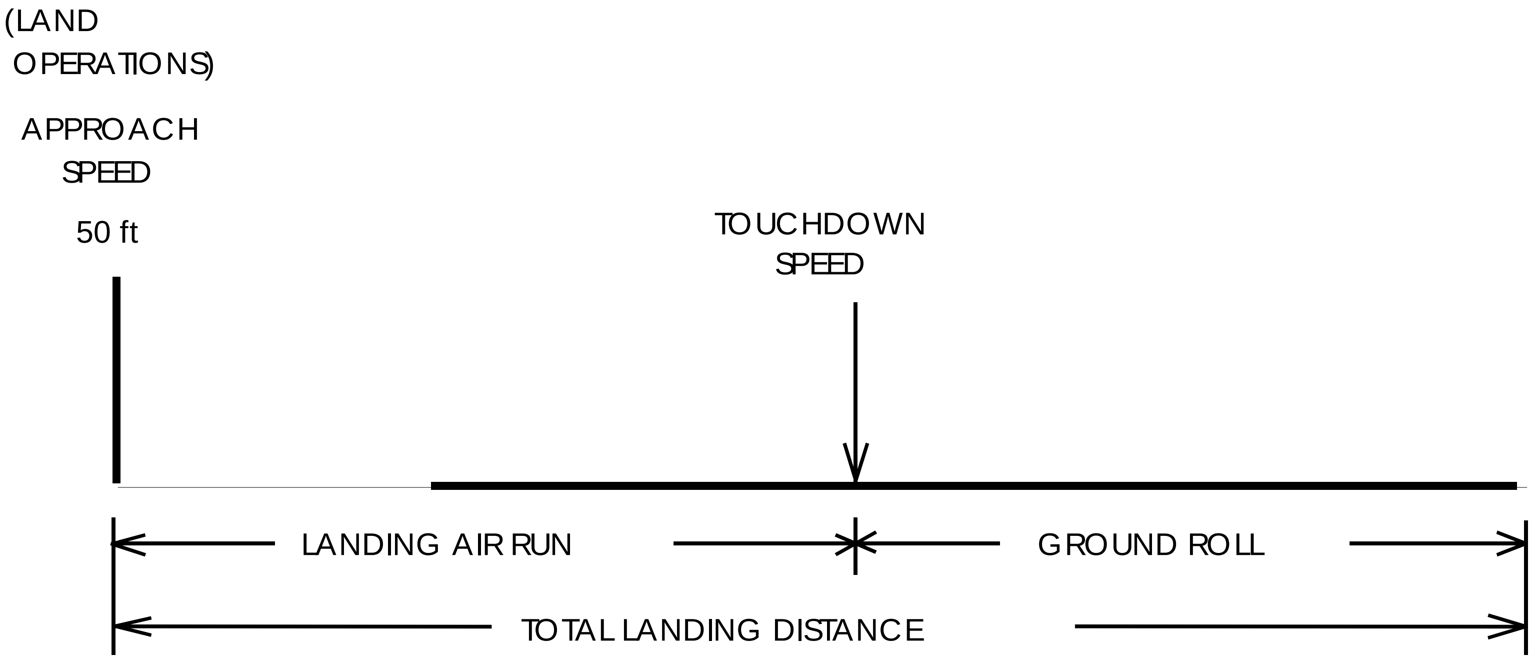


Figure 6.17 The landing path and landing distance.

Landing



Approach Speed V_{app} > 1.20 V_{stall}

Commercial > 1.30 V_{stall}

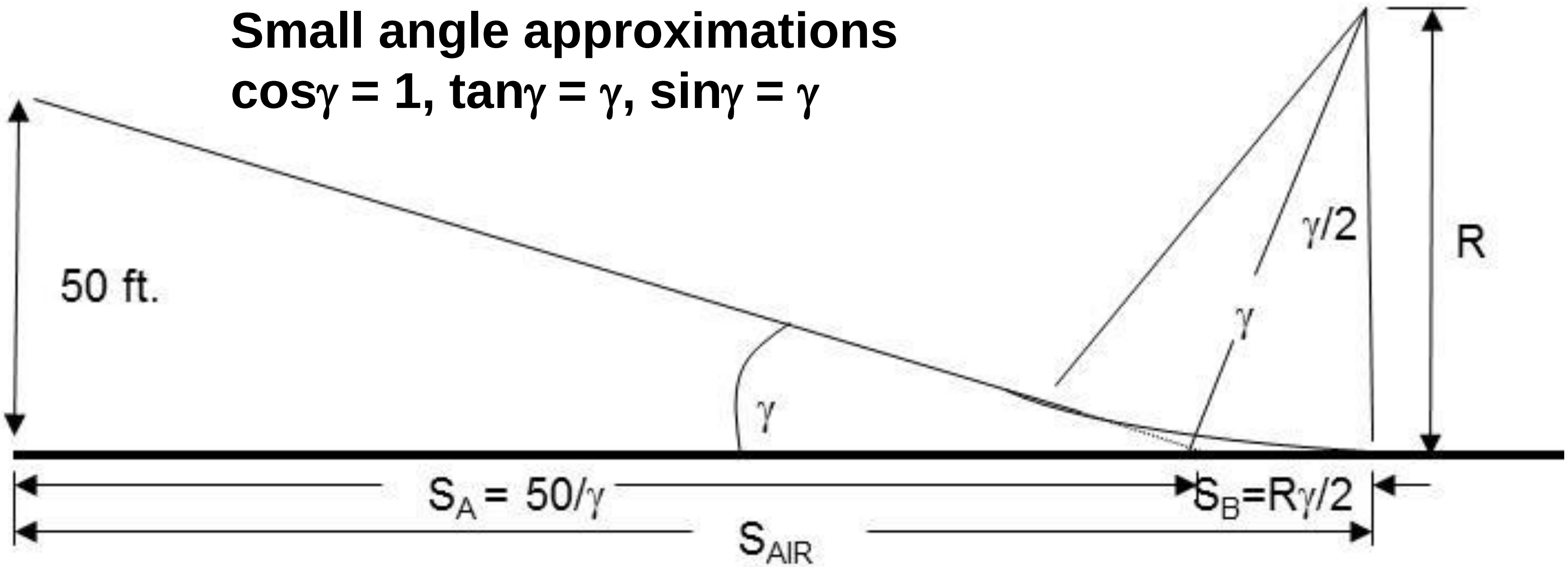
Touchdown Speed V_{TD} > 1.10 V_{stall}

Commercial > 1.15 V_{stall}

Landing – Air Segment

Air – flight at 50 ft to main gear touchdown
– includes flare

Small angle approximations
$\cos\gamma = 1$, $\tan\gamma = \gamma$, $\sin\gamma = \gamma$



$$S_A = {}^{50}/_{\gamma} = \frac{50}{\left({}^{C_D}/_{C_L}\right) - \left({}^{T}/_{W}\right)}$$

$$S_B = R\left({}^{\gamma}/_{2}\right)$$

$$S_{AIR} = \left({}^{50}/_{\gamma}\right) + R\left({}^{\gamma}/_{2}\right)$$

Landing – Transition

$$S_{TR} = \left[\left(\frac{V_{TD} + V_{TR}}{2}\right) - V_W\right] \Delta t_{TR}$$

Approximate by assuming 3 seconds to rotate the nose gear to the runway and deploy deceleration devices

$$S_{TR} = 3V_{TD}$$

V_{TD} – Touchdown Velocity
V_{TR} – Transition Velocity
V_W – Wind Velocity

Landing – Ground

$$S_{Brake} = -\frac{(V_{TD} - V_W)^2}{2g\left({}^{T}\!/_{W} - \mu\right)}$$

$0.12 < \mu_b < 0.38$

V_{TD} – Touchdown Velocity
V_W – Wind Velocity

For zero thrust and no wind

$$\mathbf{s_{brake}} = \frac{\mathbf{V_{TD}}^{2}}{\mathbf{2\ g\ \mu}}$$

$$\mathbf{s_g} = \mathbf{s_{TR}} + \mathbf{s_{brake}}$$

Landing – Ground

For zero thrust and no wind

$$S_{TR} = 3V_{TD}$$

$$s_{brake} = \frac{V_{TD}^{\ 2}}{2\,g\,\mu}$$

$$s_g = s_{TR} + s_{brake}$$

Touchdown Speed V_{TD} > 1.10 V_{stall}

Commercial > 1.15 V_{stall}

$$V_{stall} = \sqrt{\frac{2\,W}{\rho\,S\,C_{L_{max}}}}$$

Stall Speed is dominated by W/S

Stall Speed ↓
Wing Area ↑

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Landing – Ground

For zero thrust and no wind

$$\boxed{S_{TR} = 3V_{TD}}$$

$$\boxed{s_{brake} = \frac{{V_{TD}}^2}{2\,g\,\mu}}$$

$$\boxed{s_g = s_{TR} + s_{brake}}$$

Touchdown Speed V_{TD} > 1.10 V_{stall}

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$$V_{stall} = \sqrt{\frac{2\,W}{\rho\,S\,C_{L_{max}}}}$$

Stall Speed ↓

$C_{L_{max}}$ ↑

Stall Speed is dominated by $C_{L_{max}}$

Homework Assignment

<u>HW #15 – Takeoff and Landing</u>
(due by 11:59 pm ET on Monday)
Reading – Chapters 6.7 and 6.8

<u>Quiz #5 – Maneuver; Takeoff and Landing</u>
(due by 11:59 pm ET on Monday)

<u>HW Help Session</u>
Monday 4:00 – 5:00 pm ET

<u>Posted on Canvas</u>
HW #15 Assignment with instructions, tips, and checklist
HW #15 Template for data table in Excel

Questions?