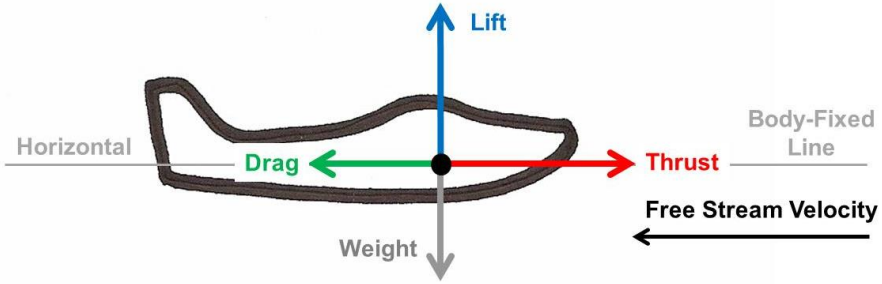
Aircraft Performance
Equations of Motion
Minimum & Maximum Velocity

Aircraft Performance



$$T = D$$

$$L = W$$

Range = How FAR can an aircraft fly?

Endurance = How LONG can an aircraft fly?

V_{max} = How FAST can an airplane fly?

V_{min} = How SLOW can an airplane fly?

Ceiling = How HIGH can an airplane fly?

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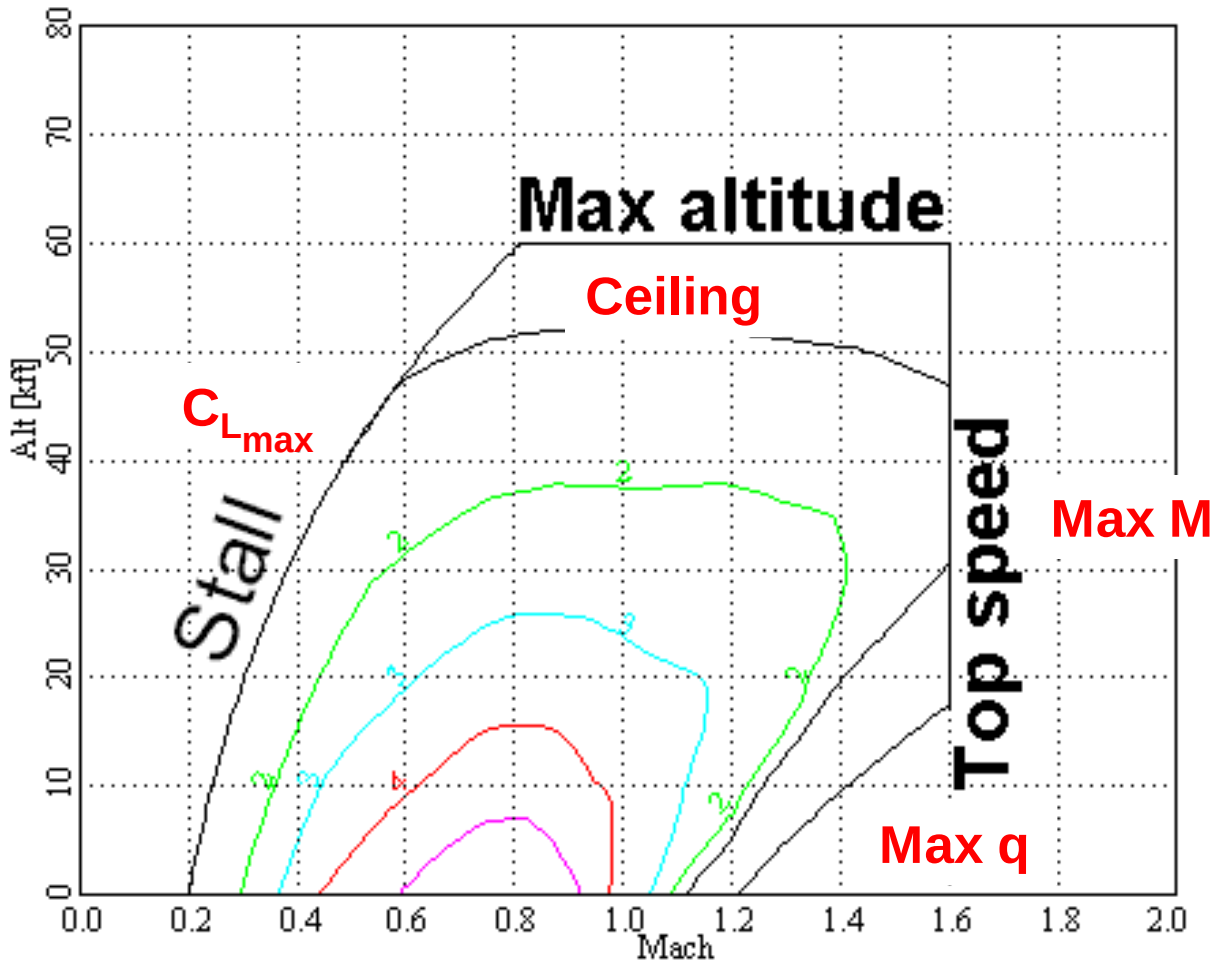
Flight Envelope

Max Speed Video



Min Speed Video

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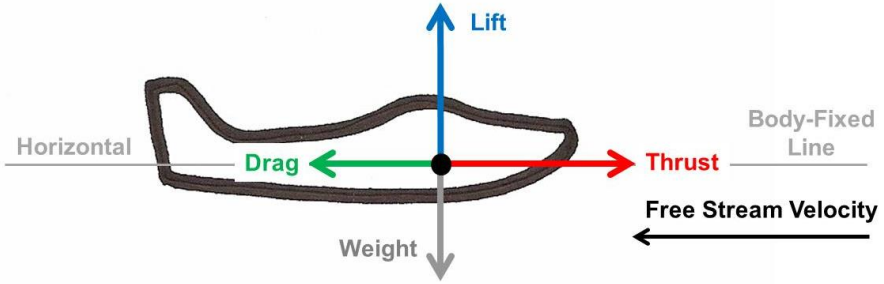
Maximum Velocity



$$T = D$$

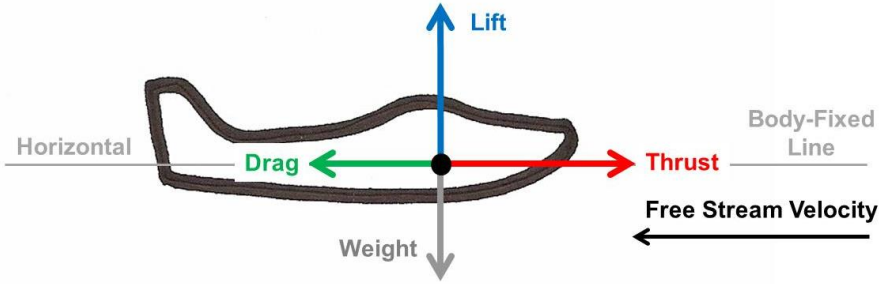
$$L = W$$

How FAST can an airplane fly in steady, level flight?

In steady, level flight, the maximum velocity of the airplane is determined by the high-speed intersection of the thrust required and the thrust available curves

$$T_{available} = D$$

Minimum Velocity



$$T = D$$

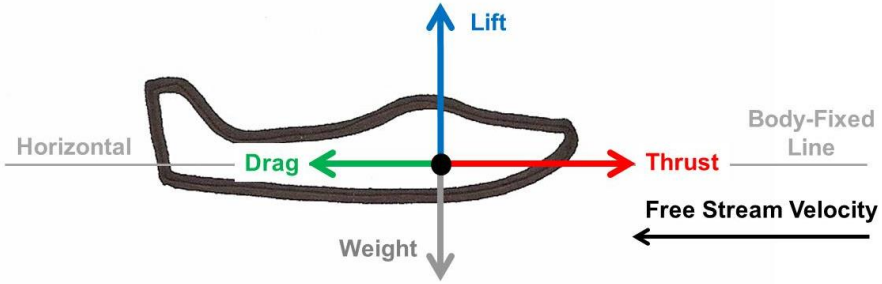
$$L = W$$

How SLOW can an airplane fly in steady, level flight?

In steady, level flight, the minimum velocity of the airplane is determined by the low-speed intersection of the thrust required and the thrust available curves

$$T_{available} = D$$

Maximum Velocity

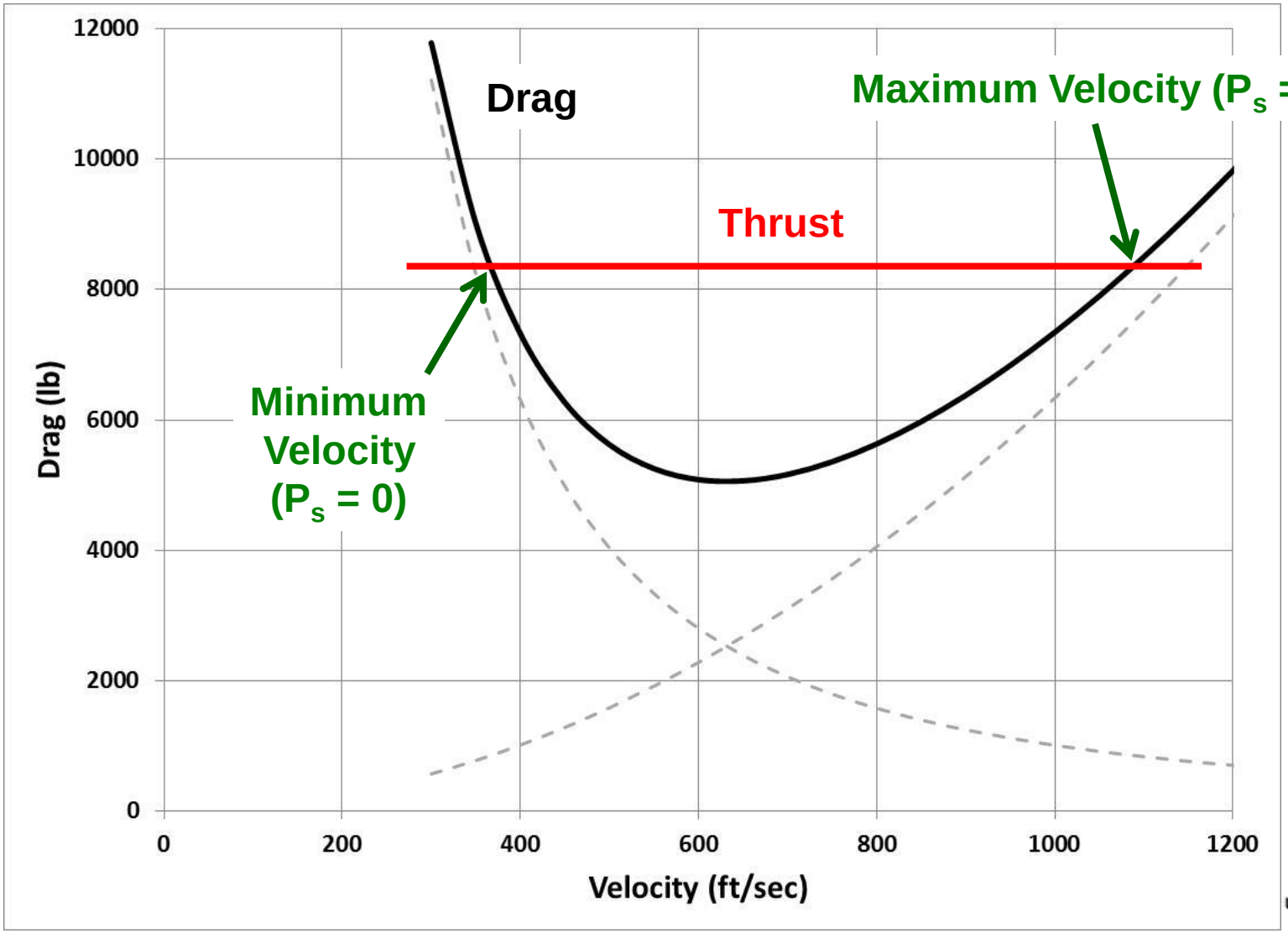


$$\frac{T}{W}$$ ~ Thrust to Weight Ratio

$$C_D = C_{D_0} + K C_L^{\ 2}$$ ~ Drag Polar

$$T_A = T_{SL}\left(\frac{\rho}{\rho_{SL}}\right)$$ ~ Thrust Available (jet)

Min & Max Velocity



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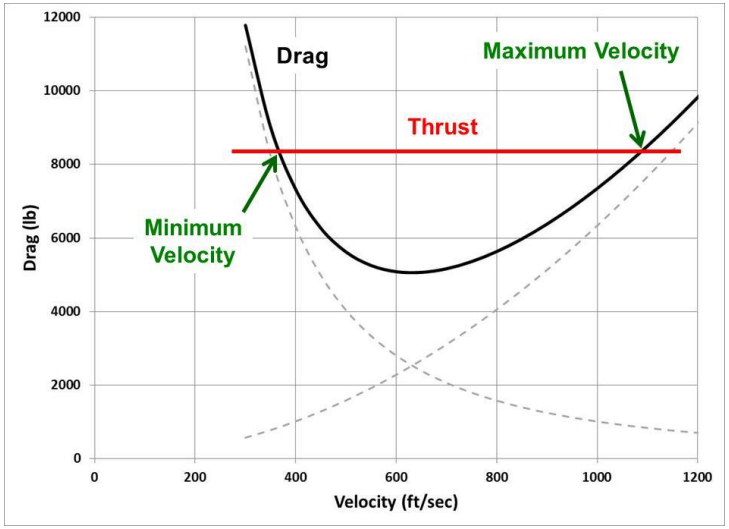
Min & Max Velocity



$$T_A = D = q\,S\,(C_{D_0} + K\,C_L{}^2)$$

$$V_{max} = \left[\frac{\left(\frac{T}{W}\right)\left(\frac{W}{S}\right) + \left(\frac{W}{S}\right)\sqrt{\left(\frac{T}{W}\right)^2 - 4C_{D_0}K}}{\rho\,C_{D_0}}\right]^{1/2}$$

$$V_{min} = \left[\frac{\left(\frac{T}{W}\right)\left(\frac{W}{S}\right) - \left(\frac{W}{S}\right)\sqrt{\left(\frac{T}{W}\right)^2 - 4C_{D_0}K}}{\rho\,C_{D_0}}\right]^{1/2}$$

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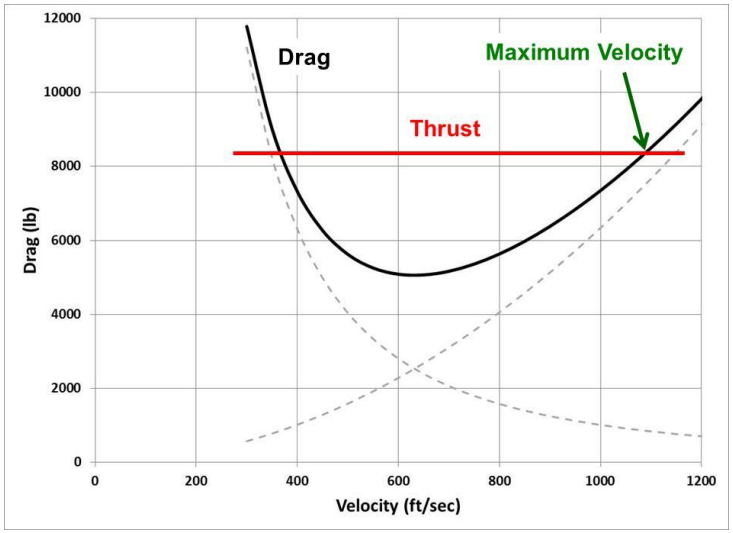
Maximum Velocity



$\frac{T}{W}$ ↑ V_{max} ↑

$\frac{W}{S}$ ↑ V_{max} ↑

C_{D_0} or K ↑ V_{max} ↓

$$V_{max} = \left[\frac{\left(\frac{T}{W}\right)\left(\frac{W}{S}\right) + \left(\frac{W}{S}\right)\sqrt{\left(\frac{T}{W}\right)^2 - 4C_{D_0}K}}{\rho\, C_{D_0}} \right]^{1/2}$$

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Thrust Required

$$C_D = C_{D_0} + K C_L^{\ 2}$$

Gulfstream IV twin-turbofan biz jet:

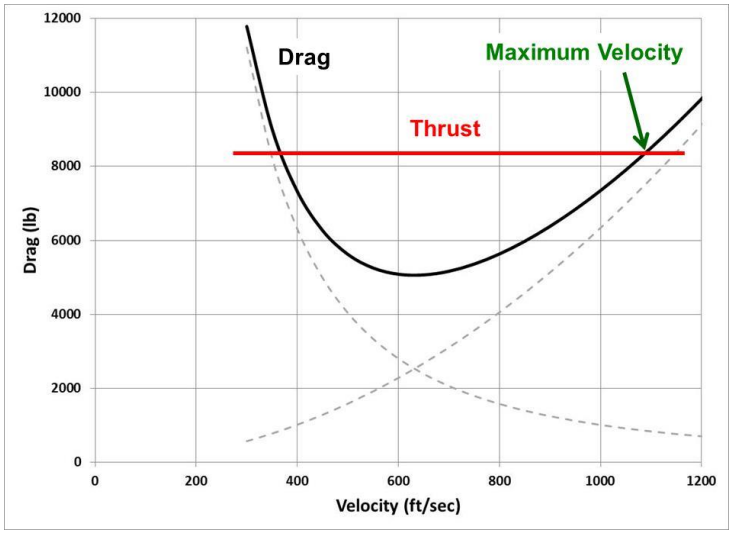
C_{D_0} = 0.0150 K = 0.08

W = 73,000 lb

h = 30,000 ft

CD0	0.0150	**Wt**	73,000	lb
K	0.08	**Alt**	30,000	ft
		QMS	439.9	
		a	994.67	ft/sec
		S	950	sq ft
Vel (fps)	**CL**	**CD**	**D (lb)**	
300	1.9203	0.3100	11785	
350	1.4108	0.1742	9015	
400	1.0801	0.1083	7322	
450	0.8535	0.0733	6267	
500	0.6913	0.0532	5621	
550	0.5713	0.0411	5253	
600	0.4801	0.0334	5085	
650	0.4091	0.0284	5066	
700	0.3527	0.0250	5164	
750	0.3072	0.0226	5358	
800	0.2700	0.0208	5632	
850	0.2392	0.0196	5975	
900	0.2134	0.0186	6378	
950	0.1915	0.0179	6837	
1000	0.1728	0.0174	7345	
1050	0.1568	0.0170	7901	
1100	0.1428	0.0166	8501	
1150	0.1307	0.0164	9142	
1200	0.1200	0.0162	9825	
1250	0.1106	0.0160	10546	
1300	0.1023	0.0158	11305	

Maximum Velocity



$$\frac{T}{W} = \frac{15,357\ \text{lb}}{73,000\ \text{lb}} = 0.2104$$

$$\frac{W}{S} = \frac{73,000\ \text{lb}}{950\ \text{ft2}} = 76.84\ \text{lb/ft}^2$$

$$C_{D_0} = 0.0150 \qquad K = 0.08$$

$$V_{max} = \left[\frac{\left(\frac{T}{W}\right)\left(\frac{W}{S}\right) + \left(\frac{W}{S}\right)\sqrt{\left(\frac{T}{W}\right)^2 - 4C_{D_0}K}}{\rho\, C_{D_0}}\right]^{1/2} = 1,535\ \text{ft/sec}$$

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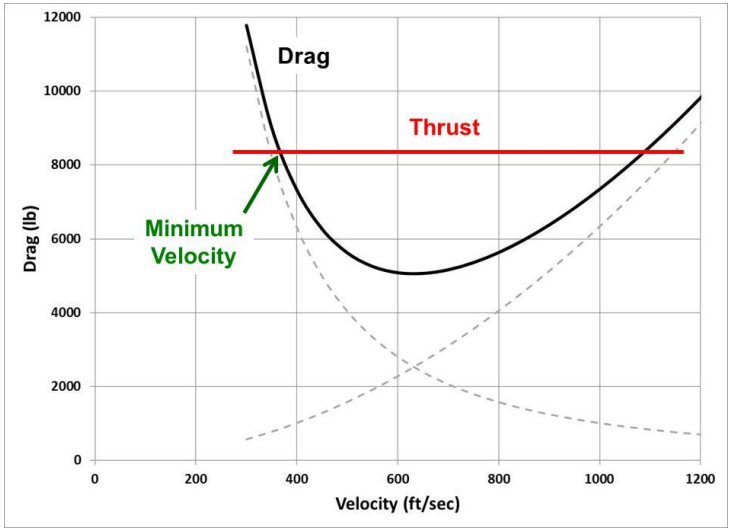
Minimum Velocity



$$\frac{T}{W} = \frac{15,357\ \text{lb}}{73,000\ \text{lb}} = 0.2104$$

$$\frac{W}{S} = \frac{73,000\ \text{lb}}{950\ \text{ft}^2} = 76.84\ \text{lb/ft}^2$$

$$C_{D_0} = 0.0150 \qquad K = 0.08$$

$$V_{min} = \left[\frac{\left(\frac{T}{W}\right)\left(\frac{W}{S}\right) - \left(\frac{W}{S}\right)\sqrt{\left(\frac{T}{W}\right)^2 - 4C_{D_0}K}}{\rho\, C_{D_0}}\right]^{1/2} = 260\ \text{ft/sec}$$

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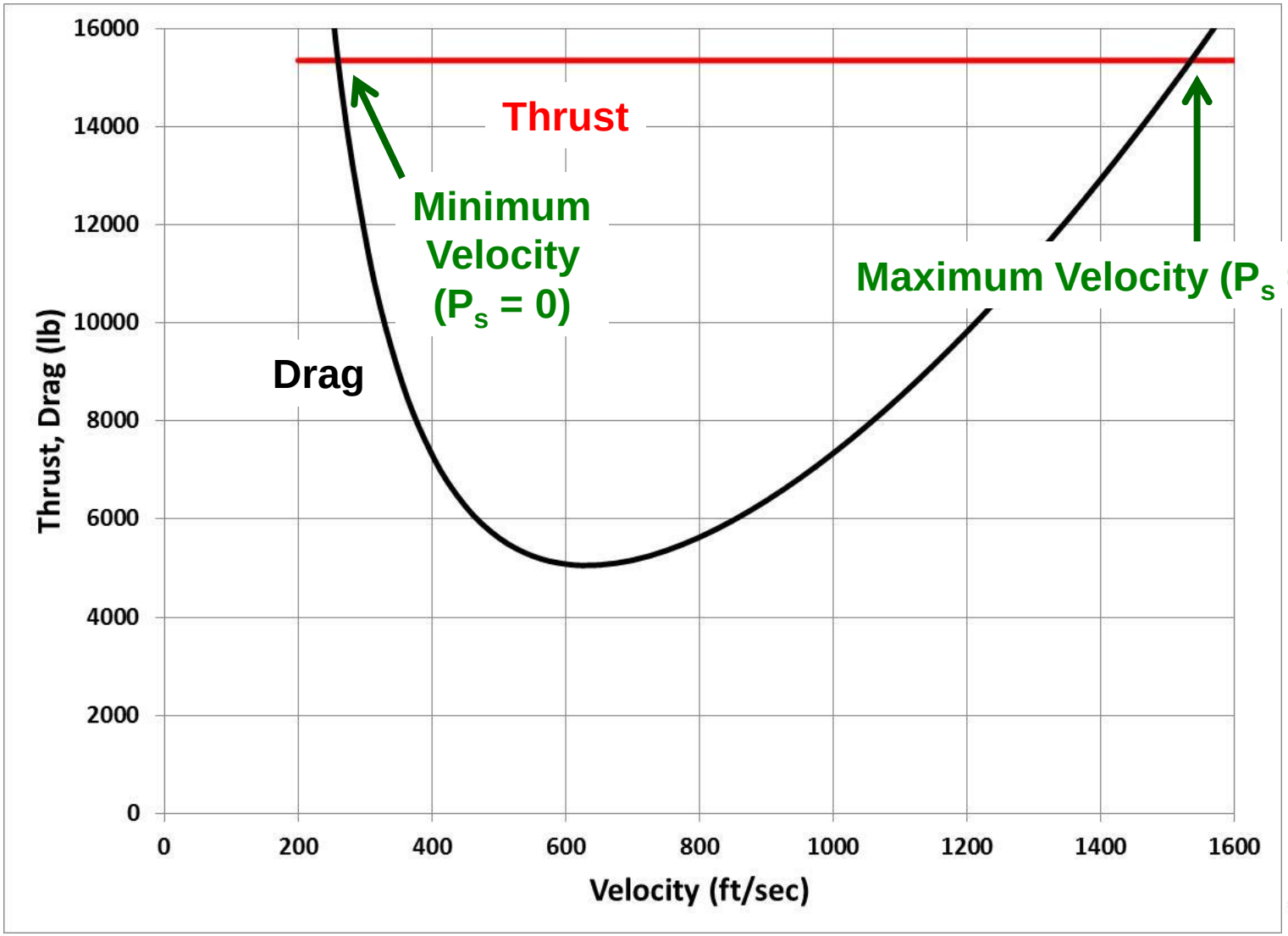
Maximum Velocity



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Minimum Velocity Constraints

We already know about Minimum Velocity where T=D, what else would be considered a minimum velocity?

1. **Velocity for maximum lift ($C_{L_{max}}$) or stall speed (V_{stall})**
2. **Buffet limitations**
3. **Stability and control constraints**

$$C_L = \frac{W}{qS} = \frac{W}{\frac{1}{2}\rho V^2 S}$$

$$C_{L_{max}} = \frac{W}{\frac{1}{2}\rho V_{stall}{}^2 S} \longrightarrow V_{stall} = \sqrt{\frac{2}{\rho}\frac{W}{S}\frac{1}{C_{L_{max}}}}$$

Maximum Velocity Constraints

We already know about Maximum Velocity where T=D, what else would be considered a maximum velocity?

1. **Velocity for maximum dynamic pressure (q_{max})**
2. **Velocity for maximum Mach number (M_{max})**
3. **Stability and control constraints**

At any altitude, an aircraft's maximum velocity is constrained to the lesser of these two speeds:

$$V_{max} = \sqrt{\frac{2}{\rho} q_{max}} \qquad \text{- or -} \qquad V_{max} = M_{max}\ a$$

Homework Assignment

HW #10 – Min & Max Velocity
(due by 11:59 pm ET on Monday)
Reading – Chapters 5.5, 5.8 - 5.9

<u>HW Help Session</u>
Monday 1:00 – 2:00 pm ET

<u>Posted on Canvas</u>
HW #10 Assignment with instructions, tips, and checklist
HW #10 Template for data table in Excel

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Homework

Plotting Charts

Weight	900	lb		QMS	1481.4	lb/ft^2				
Altitude	0	ft		a	1116.45	ft/sec				
				rho	0.00237688	slugs/ft^3				
Mach	**Vel**	**CL**	**CD0**	**CDL**	**CD**	**D**	**CL/CD**	**EF**	**CL0.5/CD**	**RF**
	(ft/sec)					**(lb)**		**(hr)**		**(NM)**
0.05	55.82	6.4291	0.0200	2.5627	2.5827	361.5	2.4893	1.91	0.9818	63.33
0.06	66.99	4.4647	0.0200	1.2359	1.2559	253.2	3.5551	2.73	1.6825	108.53
0.07	78.15	3.2802	0.0200	0.6671	0.6871	188.5	4.7740	3.67	2.6359	170.03

x axis

y axis

Add a column for Thrust

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Questions?