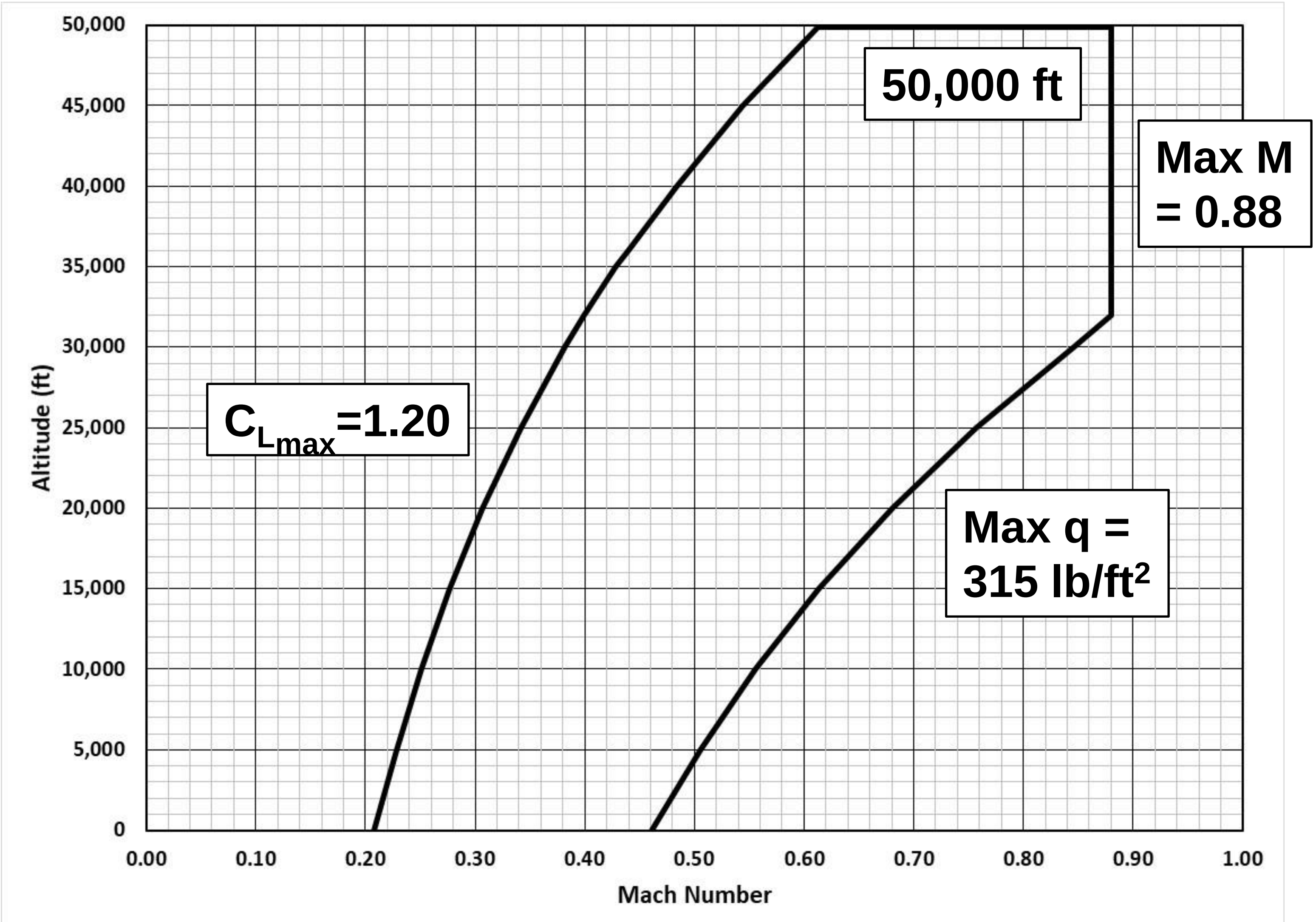
Aircraft Performance
Flight Envelope

UNIVERSITY OF Cincinnati

Flight Envelope



Stall Speed

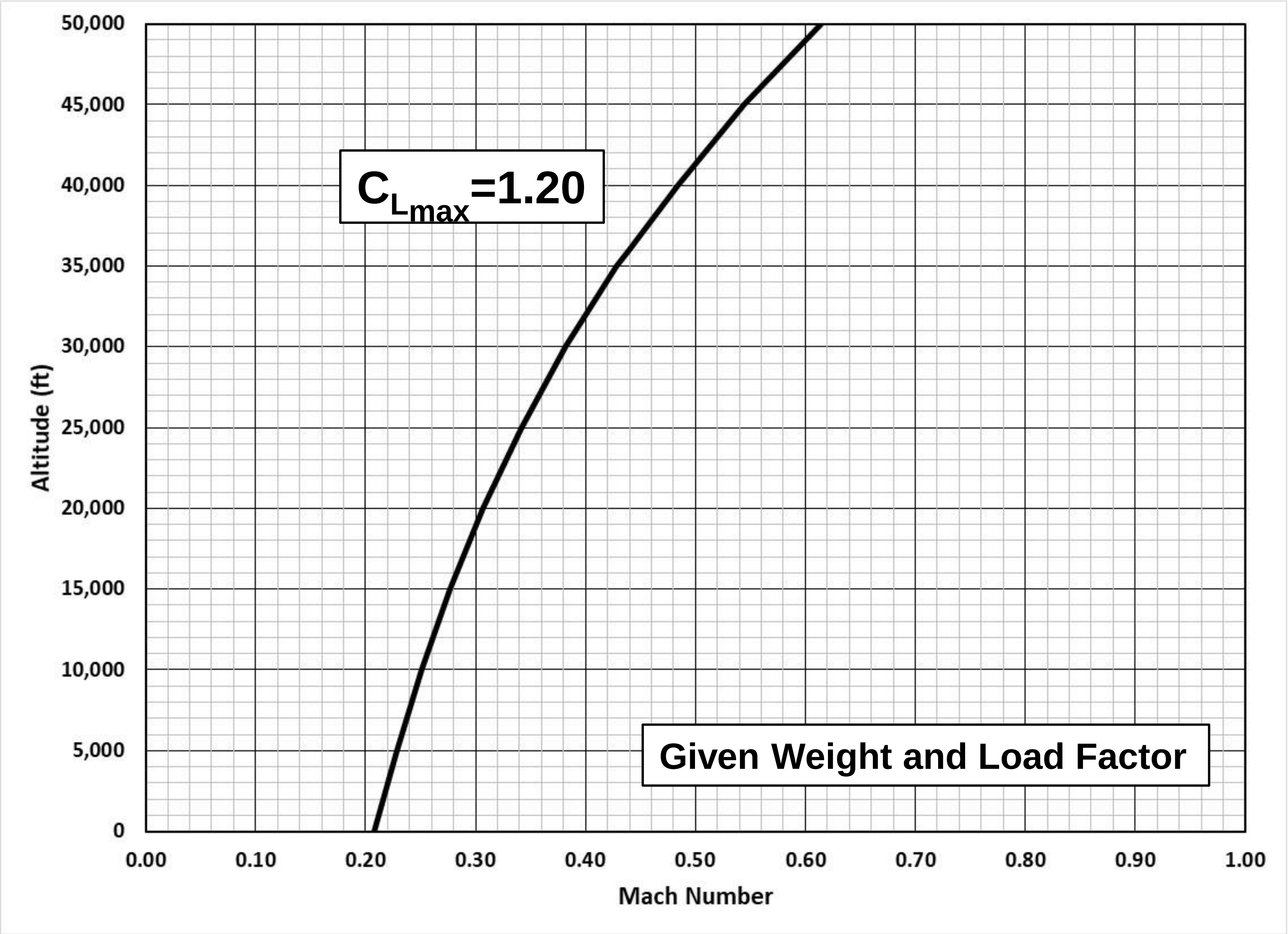
$$C_L = \frac{n\,W}{q\,S} = \frac{n\,W}{\frac{1}{2}\,\rho\,V^2\,S}$$

$$C_{L_{max}} = \frac{n\,W}{\frac{1}{2}\,\rho\,{V_{stall}}^2\,S} \longrightarrow V_{stall} = \sqrt{\frac{2}{\rho}\,\frac{W}{S}\,\frac{n}{C_{L_{max}}}}$$

$$\longrightarrow M_{stall} = V_{stall}\,/\,a$$

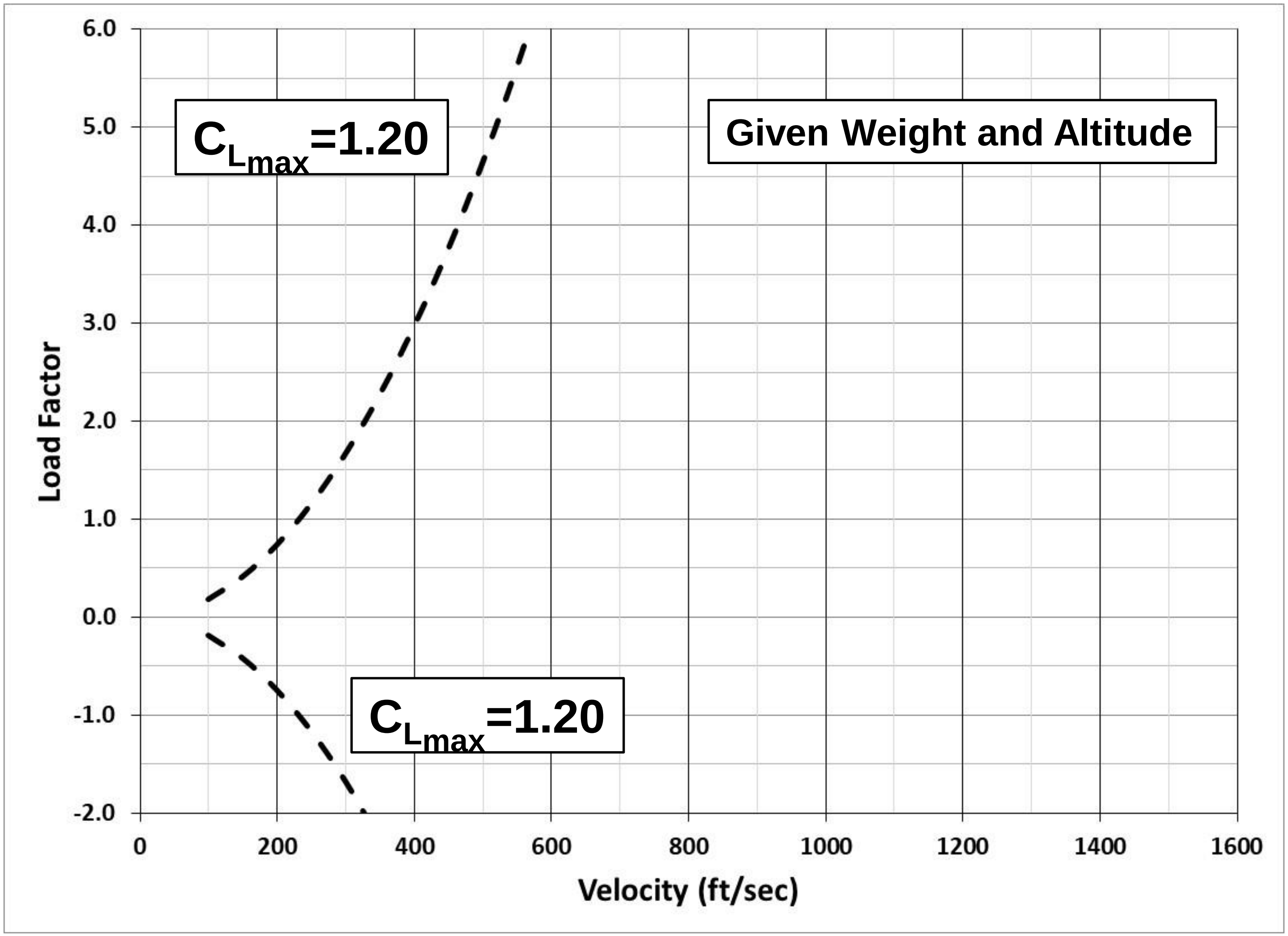
G-IV Example

Stall Speed – Flight Envelope



UNIVERSITY OF Cincinnati

Stall Speed – V-n Diagram



Limit Speed

Maximum Mach Number (Max M = X.YY)

Maximum dynamic pressure (q_{max} = X.Y lb/ft^2)

Maximum KEAS (Max KEAS = X.Y NM/hr or kts)

At any altitude, an aircraft's maximum velocity is constrained to the least of these speeds

$$V_{maxq} = \sqrt{\frac{2}{\rho} q_{max}} \quad \text{- or -} \quad V_{maxM} = M_{max} \, a$$

University of Cincinnati

Limit Speed

Maximum Mach Number (Max M = X.YY)

Maximum dynamic pressure (q_{max} = X.Y lb/ft^2)

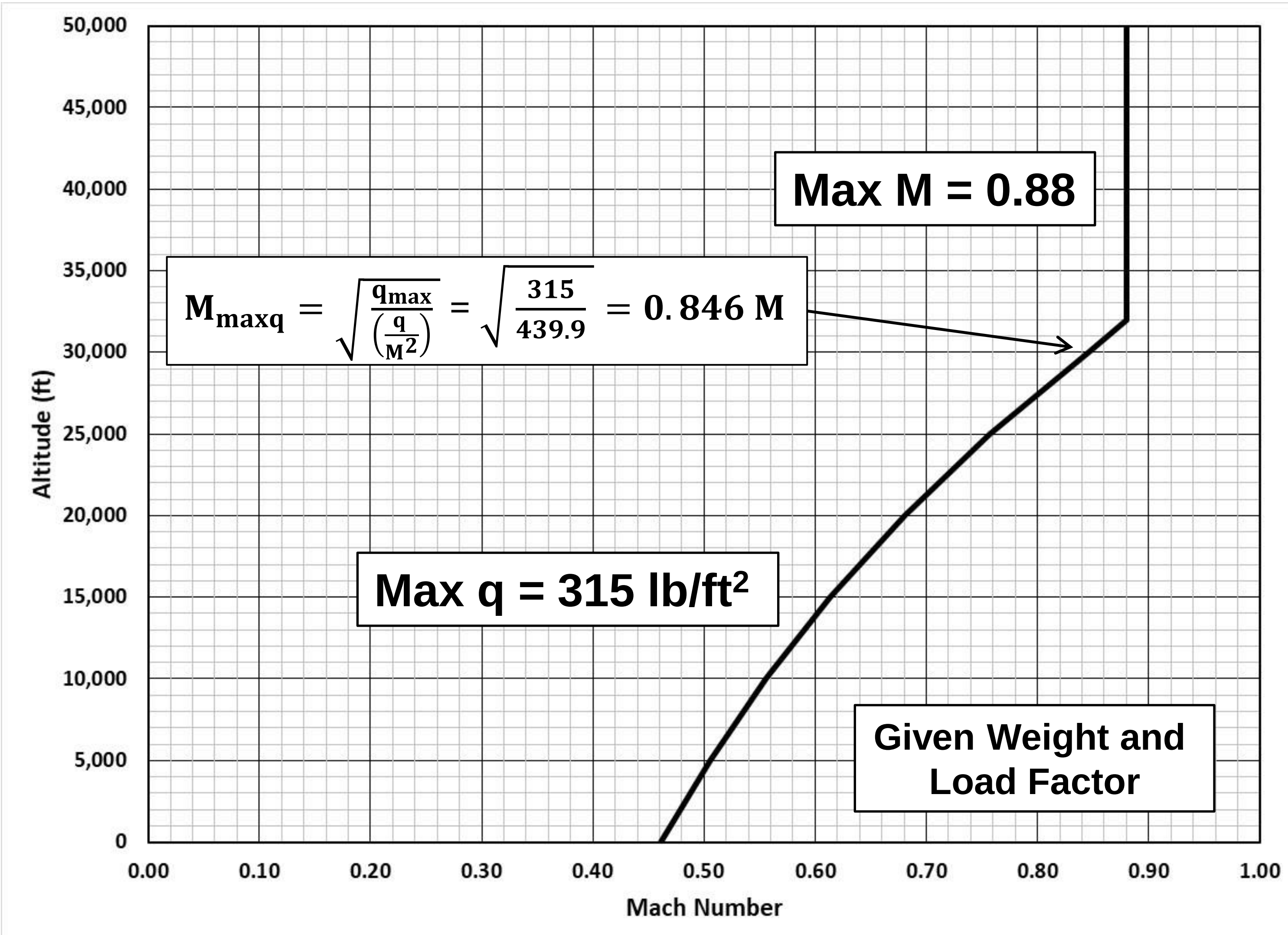
Maximum KEAS (Max KEAS = X.Y NM/hr or kts)

$$q_{max} = \left(\frac{q}{M^2}\right)_{SL}\left(\frac{\text{Max KEAS}}{a_{SL}}\right)^2 \longrightarrow V_{maxq} = \sqrt{\frac{2}{\rho}\,q_{max}}$$

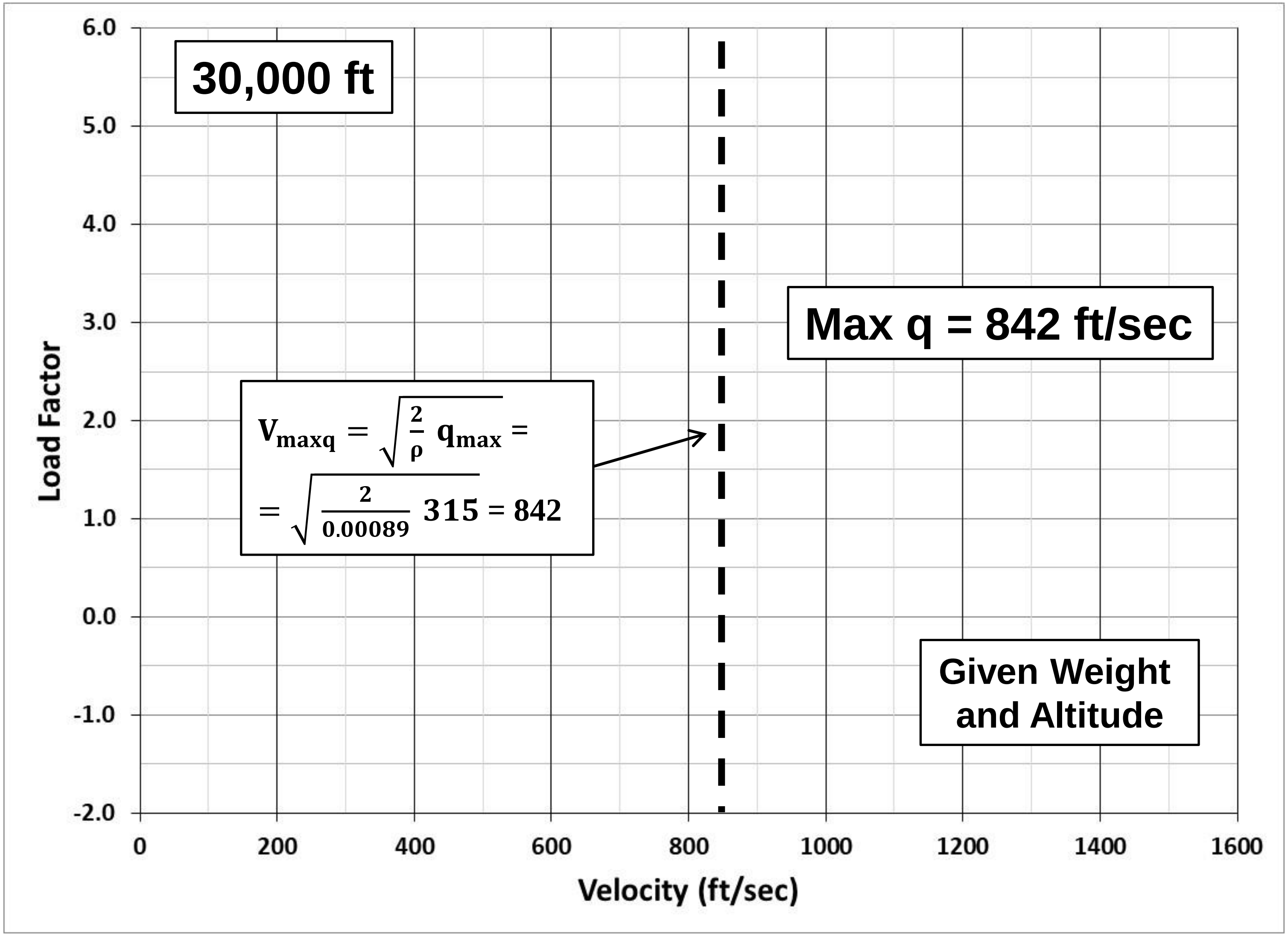
- or -

$$M_{maxq} = \sqrt{\frac{q_{max}}{\left(\frac{q}{M^2}\right)}}$$

Limit Speed – Flight Envelope



Limit Speed – V-n Diagram



G-IV Example

Limit Speed – Flight Envelope

Max KEAS = 305 NM / hr

Max q = 315 lb/ft²

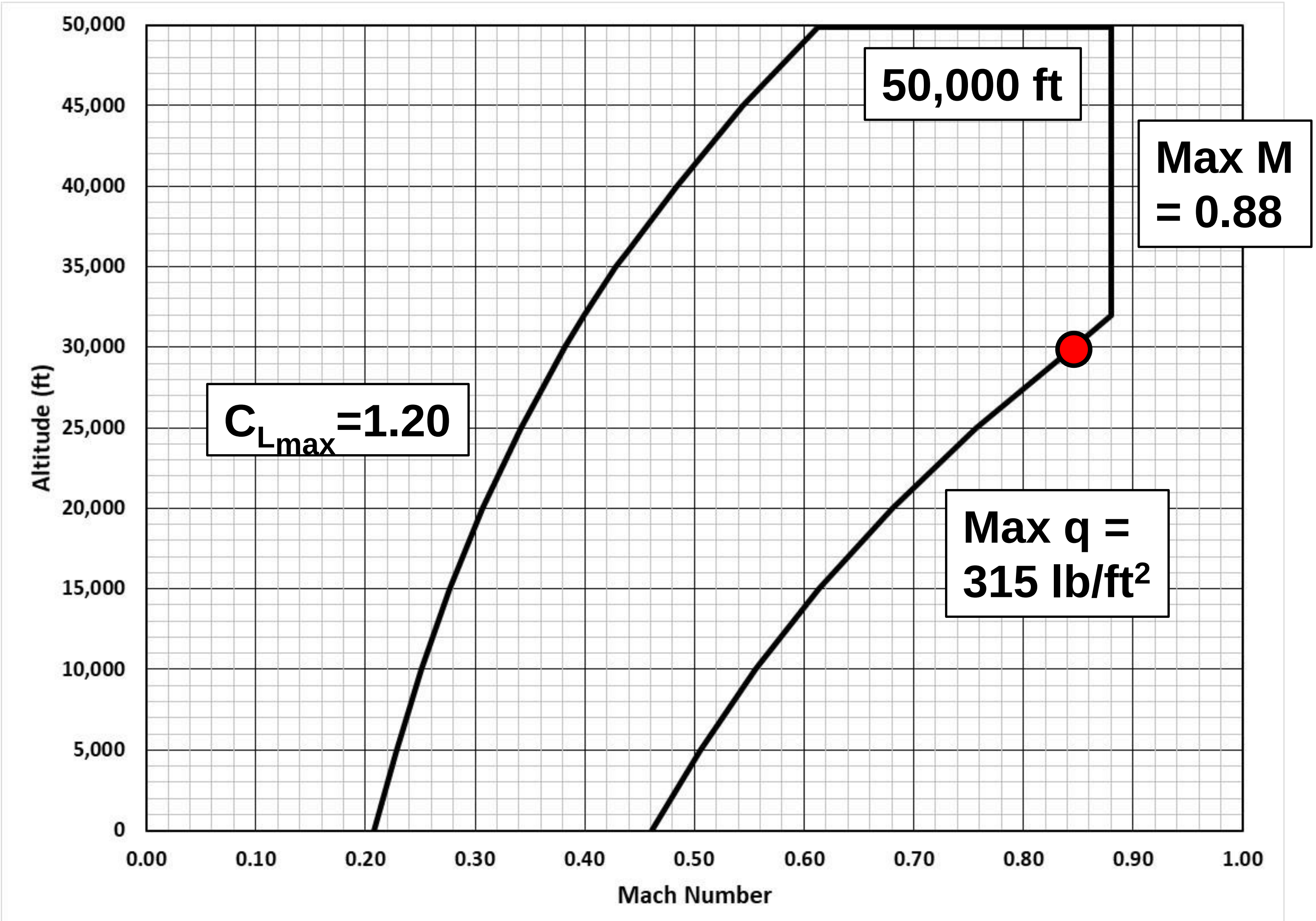
Max M = 0.88

$$\text{Max q} = \left(\frac{q}{M^2}\right)_{SL}\left(\frac{\text{Max KEAS}}{a_{SL}}\right)^2$$

Limit Mach = MIN(M_{maxq}, M_{max})

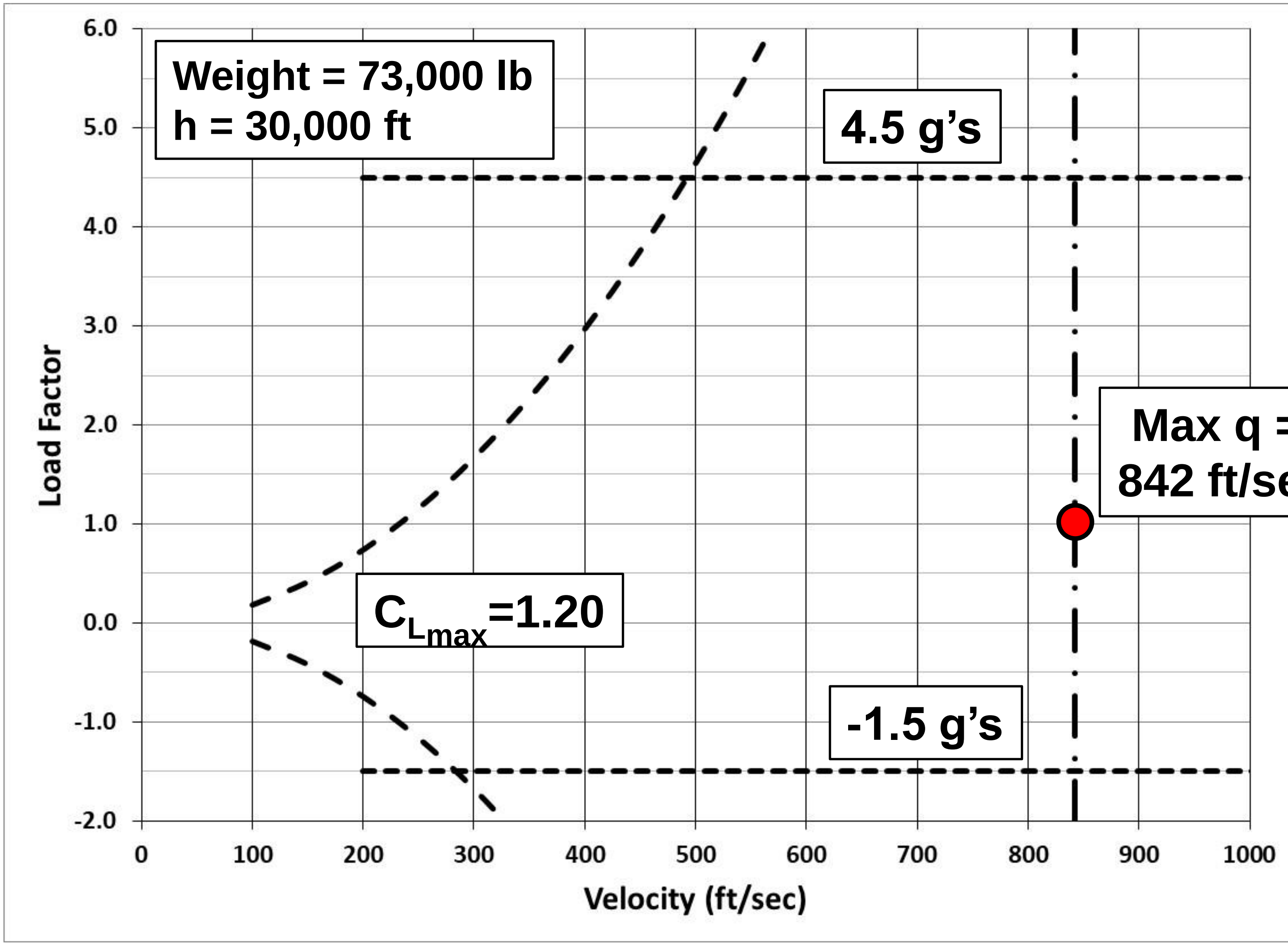
Altitude	rho	a	QMS	g's	CLmax	Vel	Mach	Max q	Max Mach	Max Mach	Limit Mach
(ft)		(ft/sec)				(ft/sec)		(lb/ft^2)	(max q)		
0	0.00237688	1116.45	1481.4	1.00	1.20	232.1	0.2079	315.0	0.4611	0.88	0.4611
5,000	0.00204808	1097.09	1232.6	1.00	1.20	250.1	0.2279	315.0	0.5055	0.88	0.5055
10,000	0.00175527	1077.39	1018.7	1.00	1.20	270.1	0.2507	315.0	0.5560	0.88	0.5560
15,000	0.00149561	1057.31	836.0	1.00	1.20	292.6	0.2768	315.0	0.6138	0.88	0.6138
20,000	0.00126642	1036.85	680.7	1.00	1.20	318.0	0.3067	315.0	0.6802	0.88	0.6802
25,000	0.00106511	1015.98	549.7	1.00	1.20	346.8	0.3413	315.0	0.7570	0.88	0.7570
30,000	0.00088926	994.67	439.9	1.00	1.20	379.5	0.3815	315.0	0.8462	0.88	0.8462
32,000	0.00082551	986.02	401.3	1.00	1.20	393.9	0.3995	315.0	0.8859	0.88	0.8800
35,000	0.00073652	972.89	348.6	1.00	1.20	417.0	0.4286	315.0	0.9506	0.88	0.8800
36,000	0.00070856	968.48	332.3	1.00	1.20	425.1	0.4390	315.0	0.9736	0.88	0.8800
40,000	0.00058512	968.08	274.2	1.00	1.20	467.8	0.4833	315.0	1.0718	0.88	0.8800
45,000	0.00046012	968.08	215.6	1.00	1.20	527.6	0.5450	315.0	1.2087	0.88	0.8800
50,000	0.00036183	968.08	169.5	1.00	1.20	594.9	0.6146	315.0	1.3630	0.88	0.8800

G-IV Flight Envelope



University of Cincinnati

G-IV V-n Diagram



Aircraft Performance
Equations of Motion
Range & Endurance

UNIVERSITY OF Cincinnati

Range & Endurance

$$C_D = C_{D_0} + K\,C_L^{\,2}$$

Gulfstream IV
twin-turbofan biz jet:

C_{D_0} = 0.0150 K = 0.08
W = 73,000 lb
h = 20,000 ft
c (20k) = 0.720 lb_{fuel}/hr/lb_t

$$\text{Range Factor(NM)} = \frac{V}{c_t}\frac{L}{D}$$

$$\text{Endurance Factor(hr)} = \frac{1}{c_t}\frac{L}{D}$$

c @ altitude = c (SL) * a_{alt}/a_{SL}

Maximum Endurance Calculations

Calculate V:

$$V_{L/D_{max}} = \left(\frac{2}{\rho}\sqrt{\frac{K}{C_{D_0}}}\frac{W}{S}\right)^{1/2}$$

= 529.4 ft/sec

Calculate C_L / C_D:

$$\text{Max}\ \frac{C_L}{C_D} = \sqrt{\frac{1}{4\ C_{D_0}\ K}}$$

= 14.43

Calculate EF:

$$EF = \frac{1}{c_t}\frac{L}{D}$$

= 20.0 hr

Calculate E:

$$E = EF\ \ln\frac{W_0}{W_1}$$

= 0.84 hr
burning 3,000 lb

UNIVERSITY OF Cincinnati

Maximum Range Calculations

Calculate V: $$V_{L/D_{max}} = \left(\frac{2}{\rho} \sqrt{\frac{K}{C_{D_0}}} \frac{W}{S} \right)^{1/2}$$ = 529.4 ft/sec

for Max R $$V_{(D/V)_{min}} = 1.3161 \ V_{L/D_{max}}$$ = 696.7 ft/sec

Calculate C_L: $$C_L = \frac{W\,n}{\frac{1}{2}\,\rho\,V^2\,S} = \frac{W\,n}{(q/M^2)\,M^2\,S}$$ = 0.2500

Calculate C_D: $$C_D = C_{D_0} + K\,C_L{}^2$$ = 0.0200

Calculate RF: $$RF = \frac{V}{c_t}\,\frac{L}{D}$$ = 7167.0 NM

Calculate R: $$R = RF\ \ln\frac{W_0}{W_1}$$ = 831.9 NM burning 8,000 lb

University of Cincinnati

Questions?

UNIVERSITY OF Cincinnati