Aircraft Performance
Equations of Motion
Thrust Required

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Finite Wing

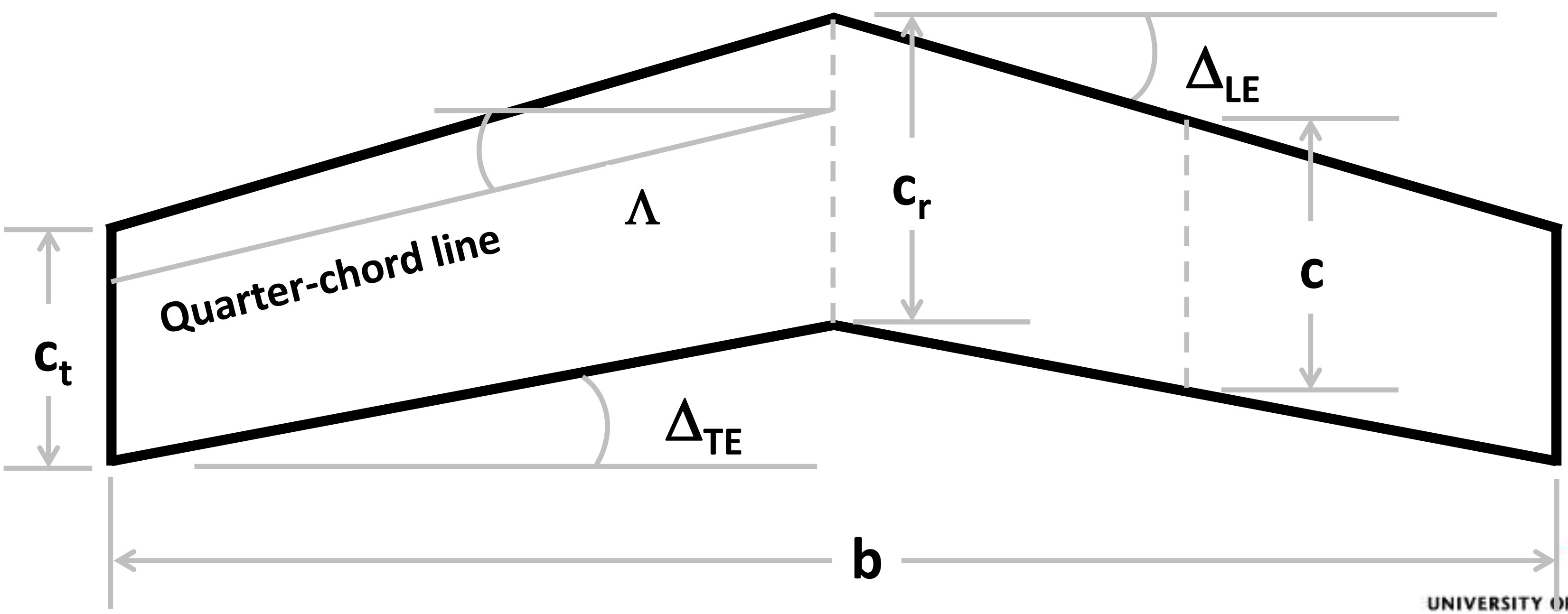
Wing Planform Characteristics

Wing Area (S)	**Leading Edge Sweep (Δ_{LE})**
Wing Span (b)	**Trailing Edge Sweep (Δ_{TE})**
Average Chord (c)	**Aspect Ratio (AR)**
Root Chord (c_r)	**Taper Ratio (λ)**
Tip Chord (c_t)	**Quarter-Chord Angle (Λ)**

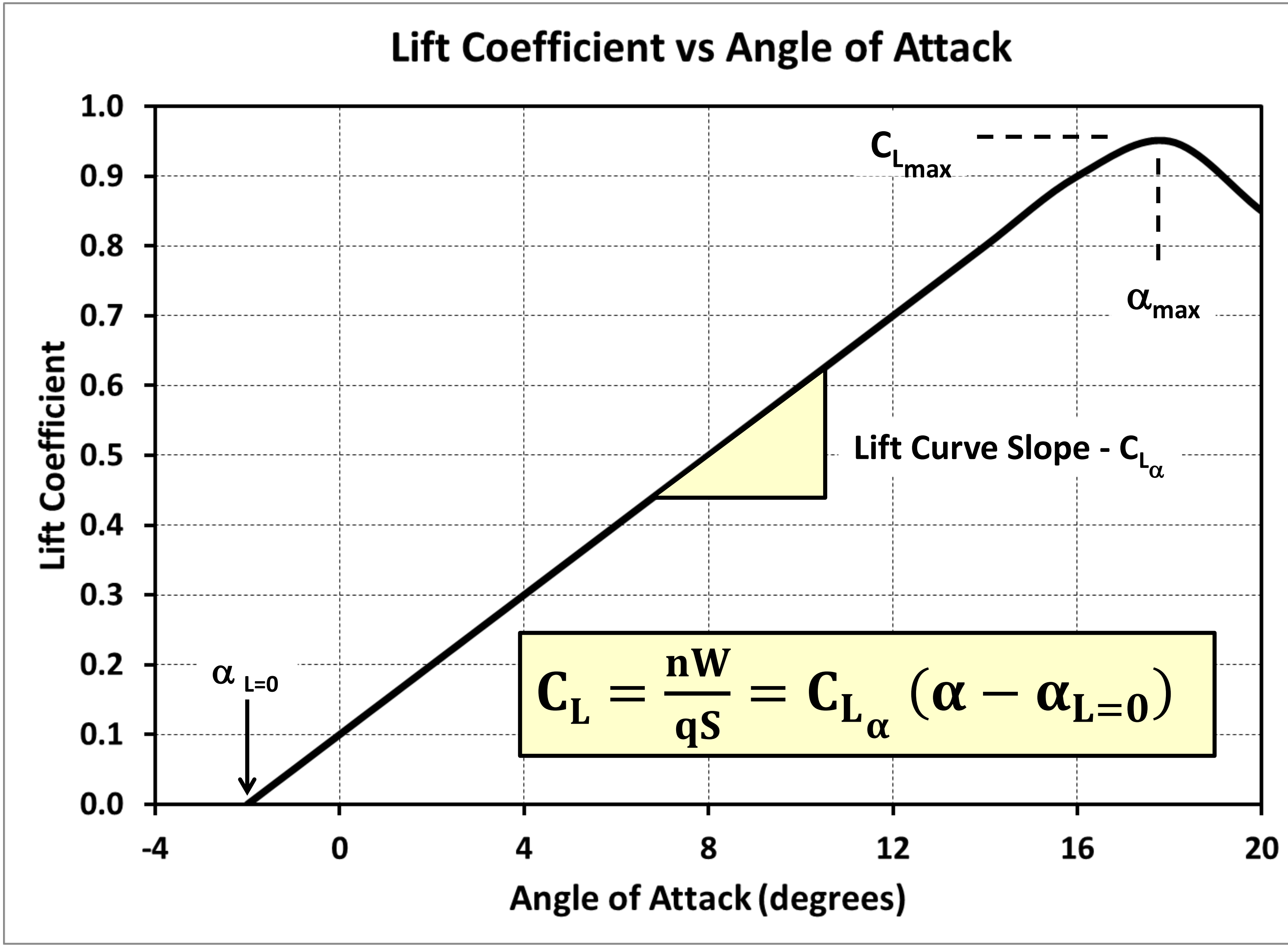
$$\mathbf{S = b\,c}$$

$$\mathbf{AR = \frac{b^2}{S} = \frac{b}{c}}$$

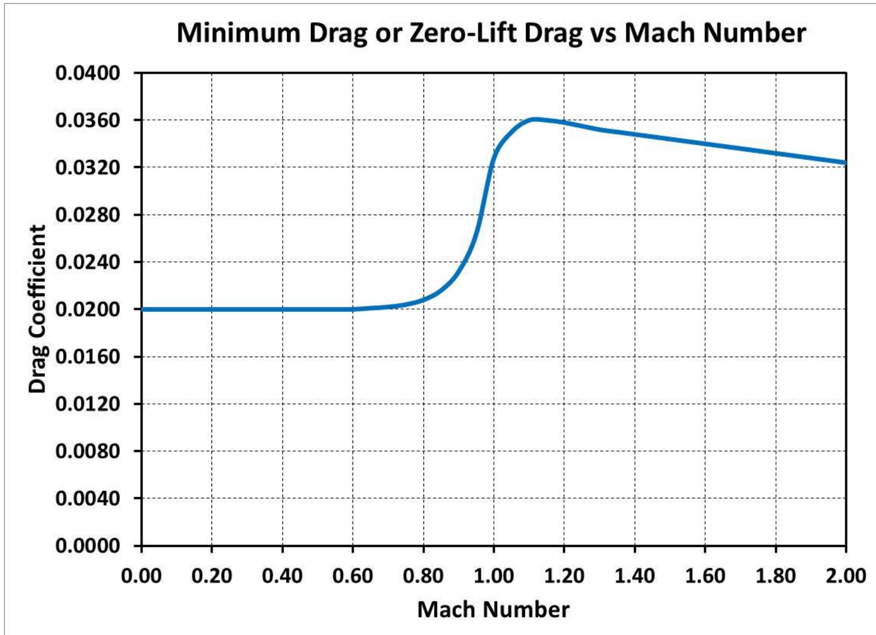
$$\boldsymbol{\lambda} = \mathbf{\frac{c_t}{c_r}}$$



Aircraft Aerodynamics



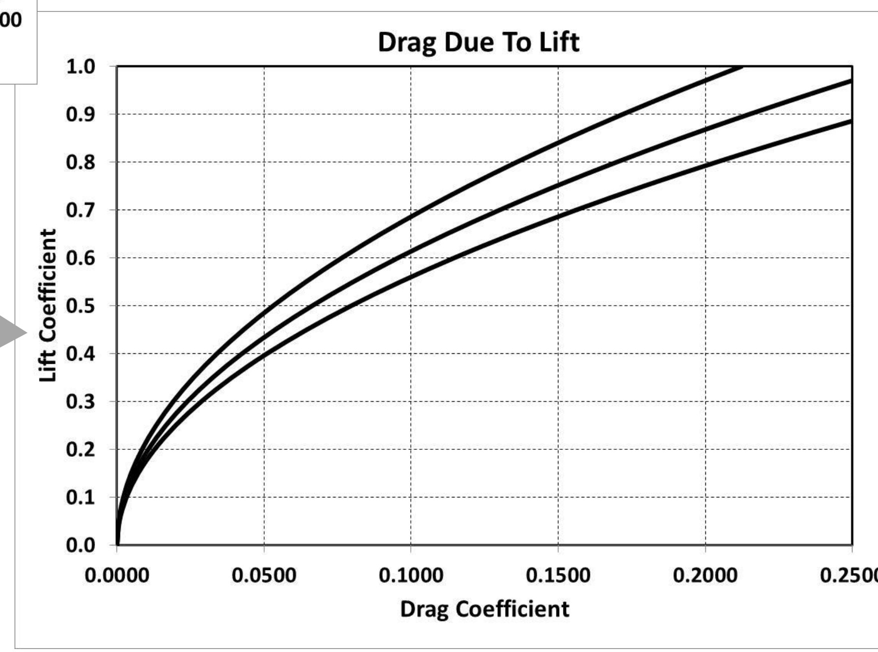
Finite Wing Drag



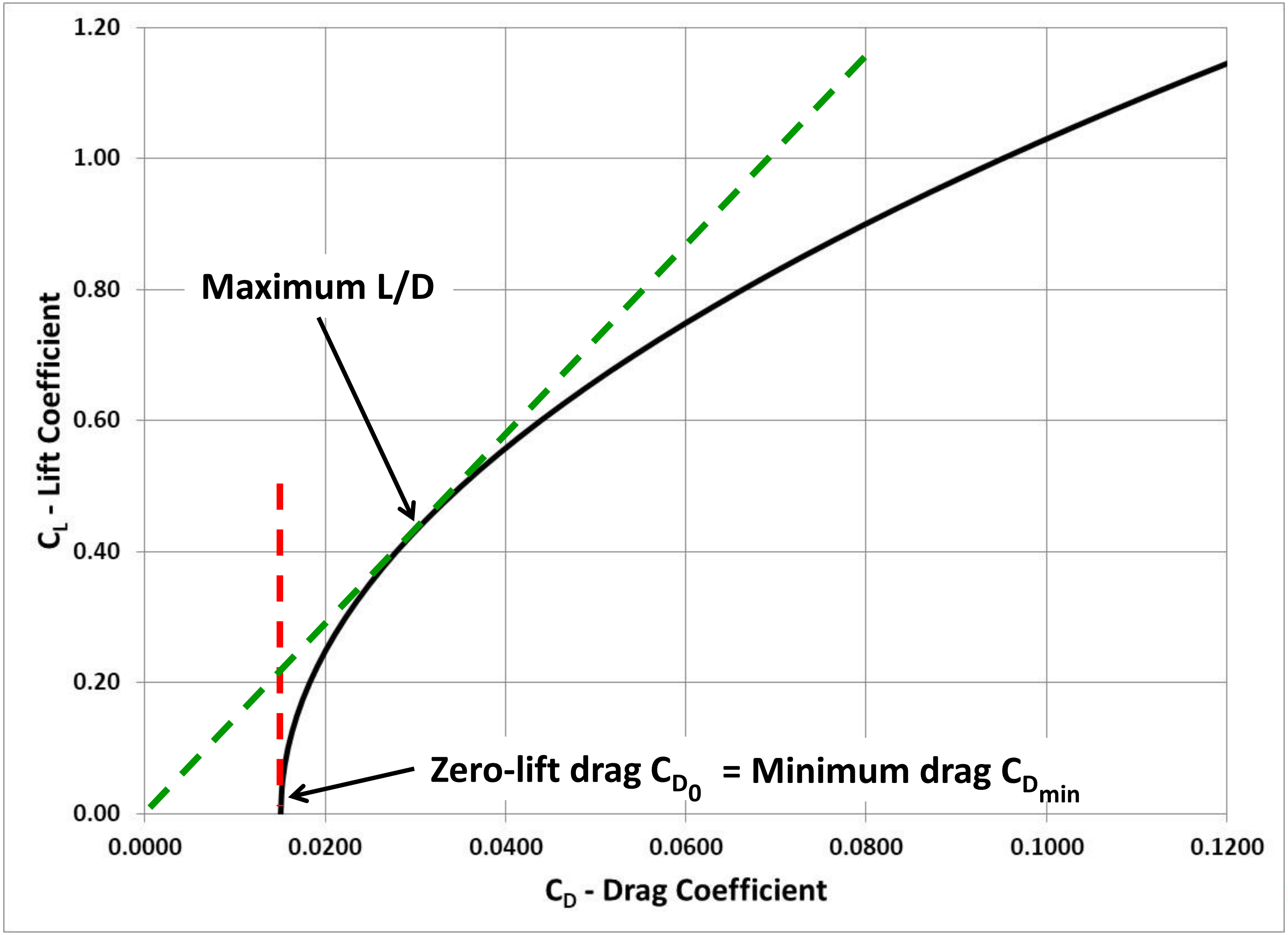
$$C_{D_0} \sim f\,(M, h)$$

$$C_{D_L} \sim f\,(C_L, M, c.g.)$$

$$C_D = C_{D_0} + K\,C_L^{\;2}$$



Aircraft Drag Polar



Aircraft Thrust

Piston engine / propeller $$\mathbf{T_A = SHP_{SL}\left(\frac{\eta_P}{V}\right)\left(\frac{\rho}{\rho_{SL}}\right)}$$

Turboprop $$\mathbf{T_A = ESHP_{SL}\left(\frac{\eta_P}{V}\right)\left(\frac{\rho}{\rho_{SL}}\right)}$$

High-bypass turbofan $$\mathbf{T_A = T_{SL}\left(\frac{0.1}{M}\right)\left(\frac{\rho}{\rho_{SL}}\right)}$$

Low-bypass turbofan & Turbojet $$\mathbf{T_A = T_{SL}\left(\frac{\rho}{\rho_{SL}}\right)}$$

Afterburner $$\mathbf{T_A = T_{SL}\left(\frac{\rho}{\rho_{SL}}\right)(1 + 0.7\,M)}$$

Aircraft Fuel Flow

Piston engine / propeller	$\mathbf{FFR = SHP\ c}$
Turboprop	$\mathbf{FFR = ESHP\ c}$
High-bypass turbofan	$\mathbf{FFR = T\ c_{t_{SL}} \left(\frac{a}{a_{SL}}\right)}$
Low-bypass turbofan & Turbojet	$\mathbf{FFR = T\ c_{t_{SL}} \left(\frac{a}{a_{SL}}\right)}$
Afterburner	$\mathbf{FFR = T\ c_{t_{SL}} \left(\frac{a}{a_{SL}}\right)}$

$$\mathbf{c\left(\frac{lb}{HP\ hr}\right)}$$

$$\mathbf{c_t\left(\frac{lb}{lb_t\ hr}\right)}$$

Aircraft Weights

Basic Mission Takeoff Gross Weight =
OW + Mission Payload + Mission Fuel

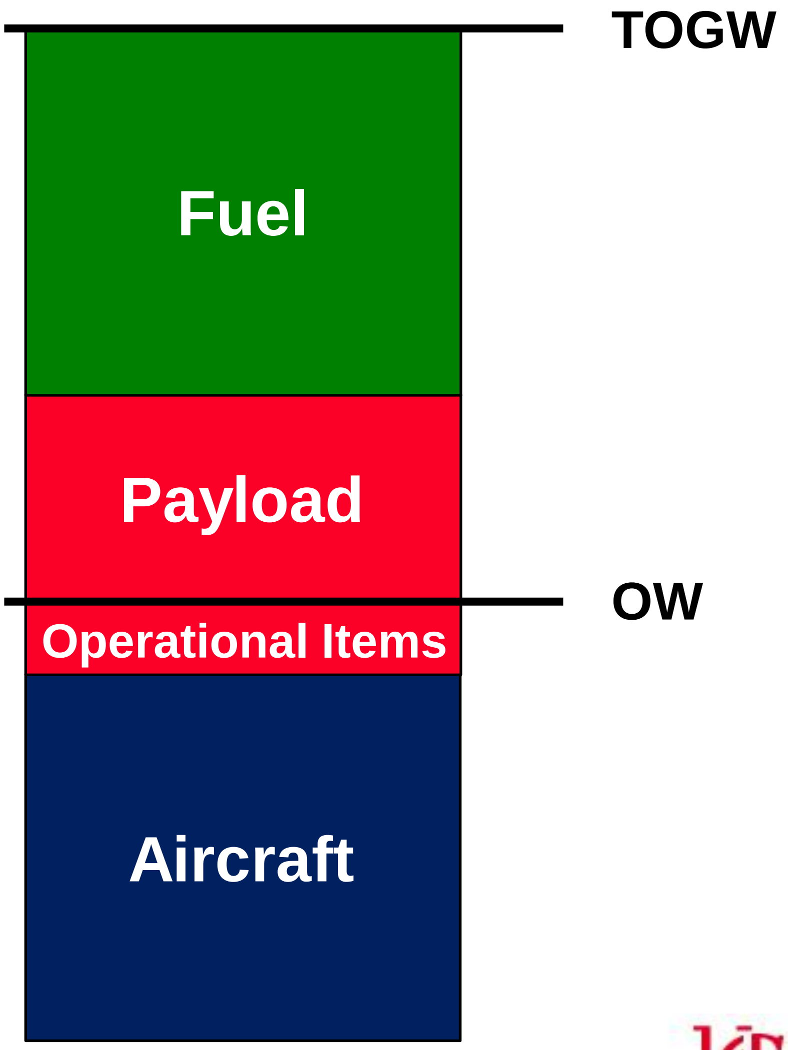
Maximum Fuel – full capacity
Mission Fuel – specific mission capability

Maximum Payload – full capacity loadout
Mission Payload – specific mission loadout

Maximum Takeoff Gross Weight =
OW + Maximum Payload + Maximum Fuel

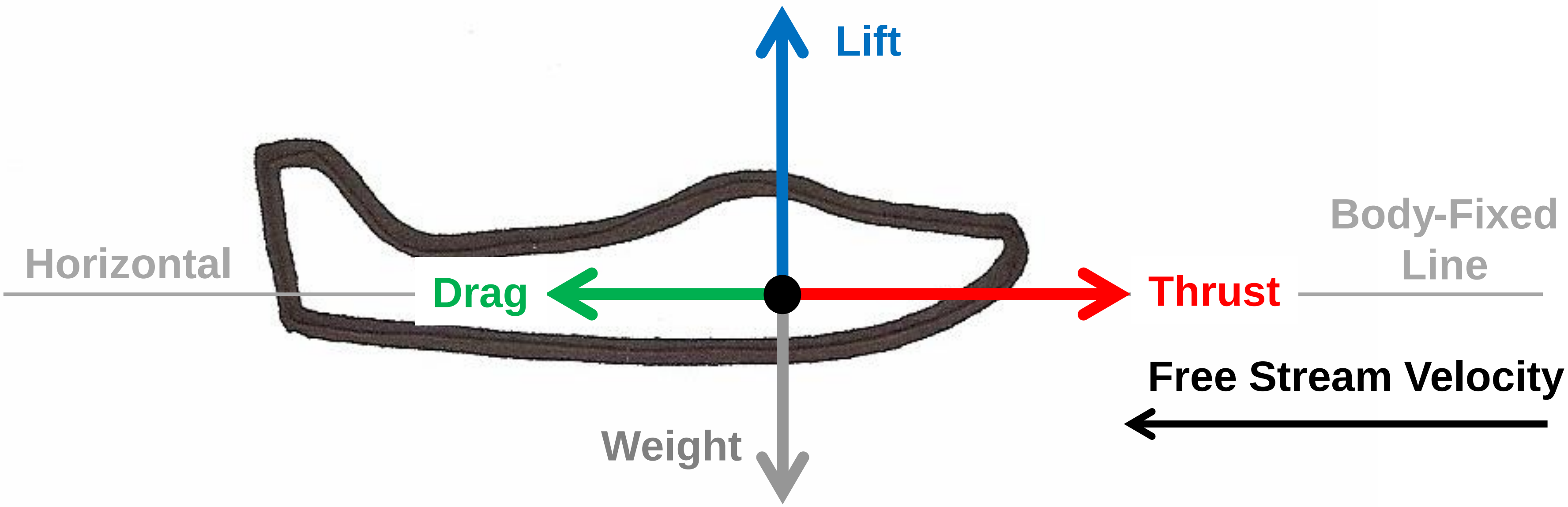
- or -

Maximum Takeoff Gross Weight
could be set by other factors
(landing gear limit, c.g. limits, etc)



Simple Free Body Diagram

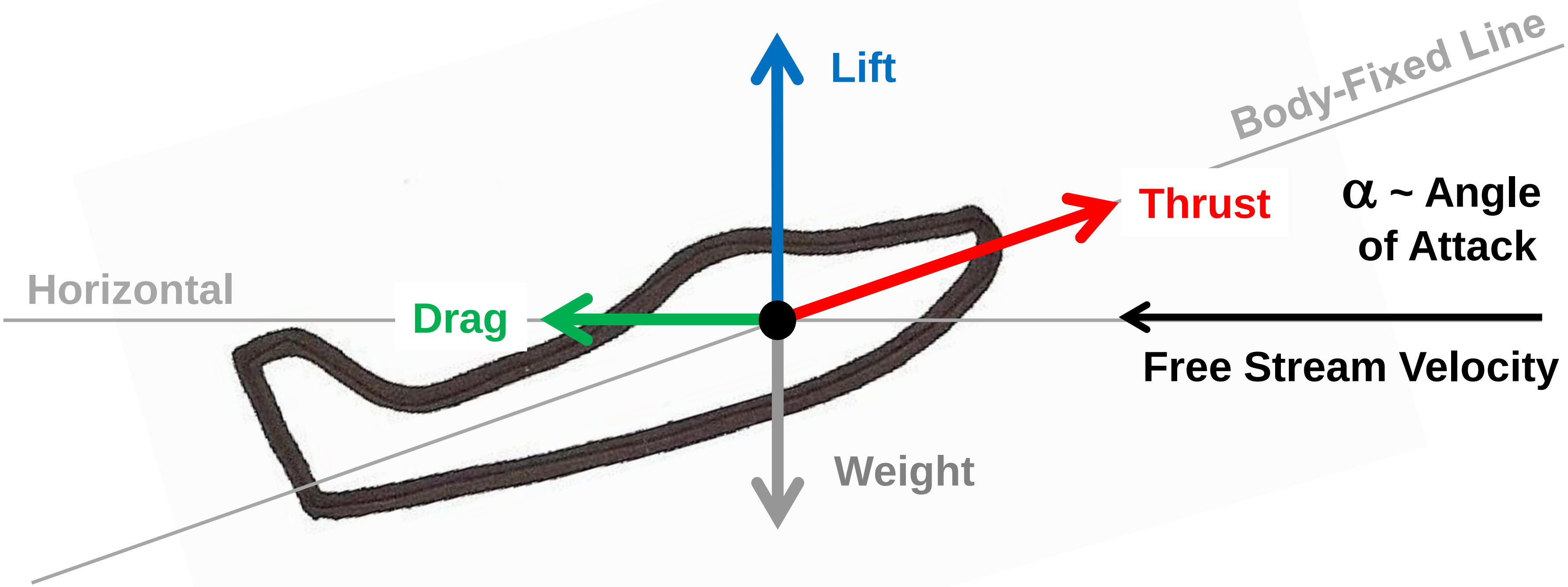
Straight and Level Flight



Lift acts perpendicular to the Free Stream Velocity
Drag acts parallel to the Free Stream Velocity
Weight acts vertically towards the ground
Thrust is fixed in the aircraft

Stability Axis System

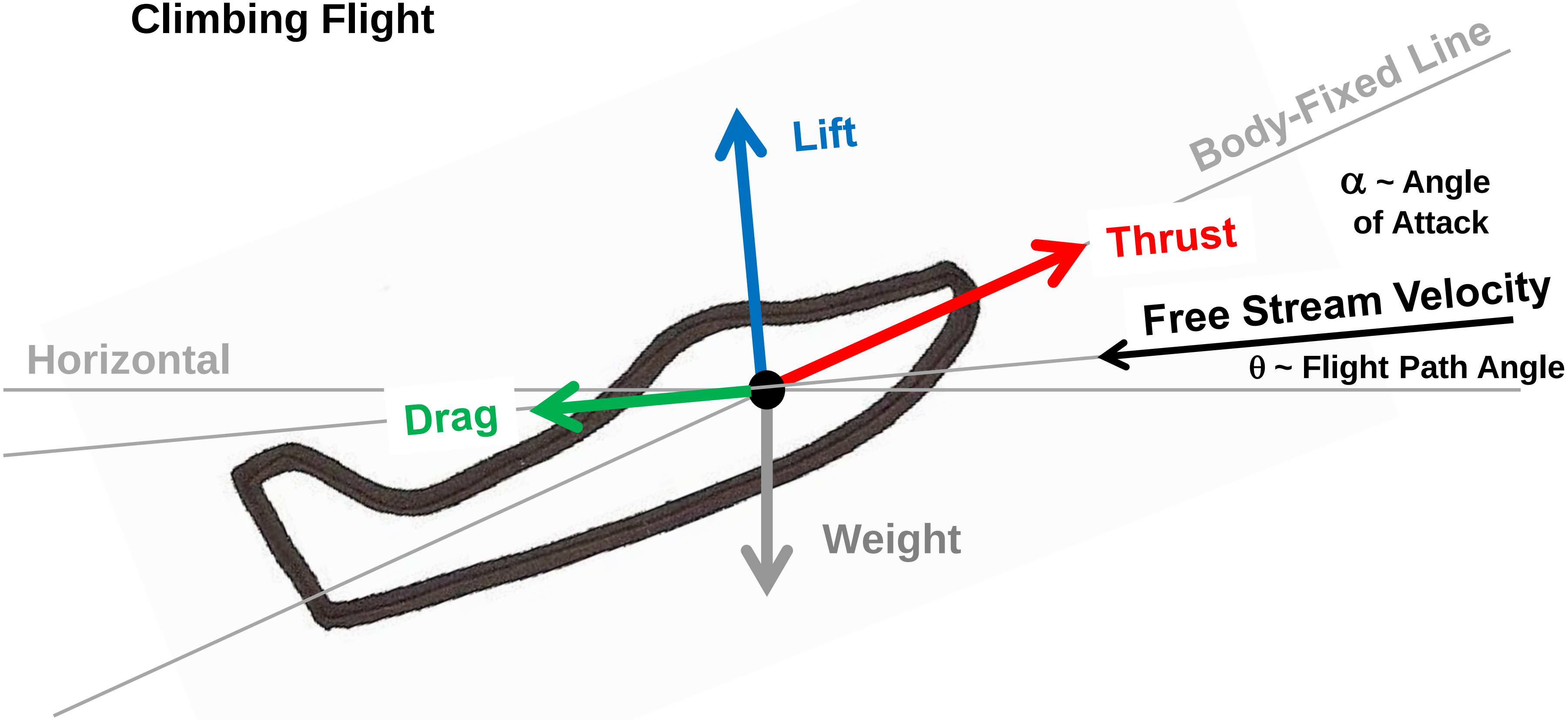
Straight and Level Flight



Lift acts perpendicular to the Free Stream Velocity
Drag acts parallel to the Free Stream Velocity
Weight acts vertically towards the ground
Thrust is fixed in the aircraft

Stability Axis System

Climbing Flight

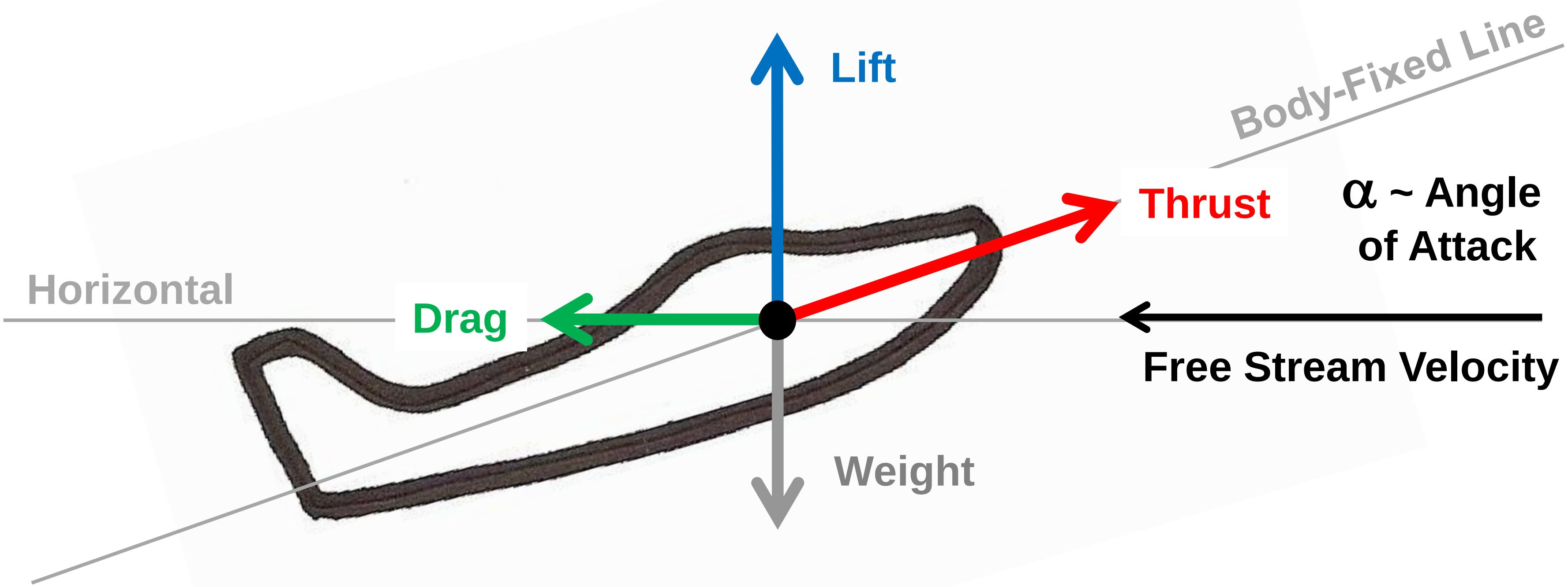


Lift acts perpendicular to the Free Stream Velocity
Drag acts parallel to the Free Stream Velocity
Weight acts vertically towards the ground
Thrust is fixed in the aircraft

Aircraft Performance
Equations of Motion
for Steady Flight

Stability Axis System

Straight and Level Flight



Lift acts perpendicular to the Free Stream Velocity
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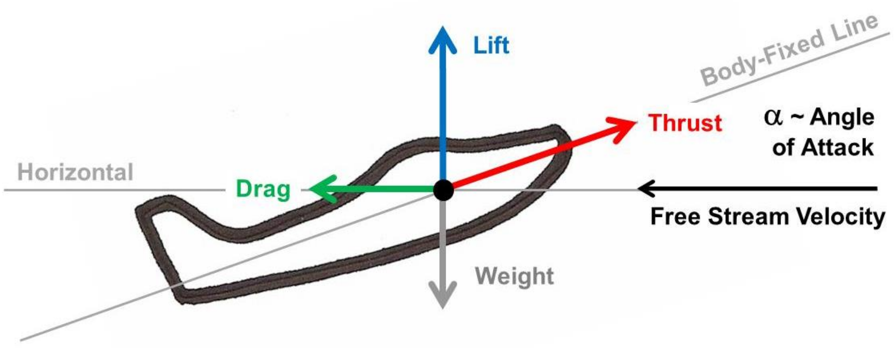
Straight and Level Flight

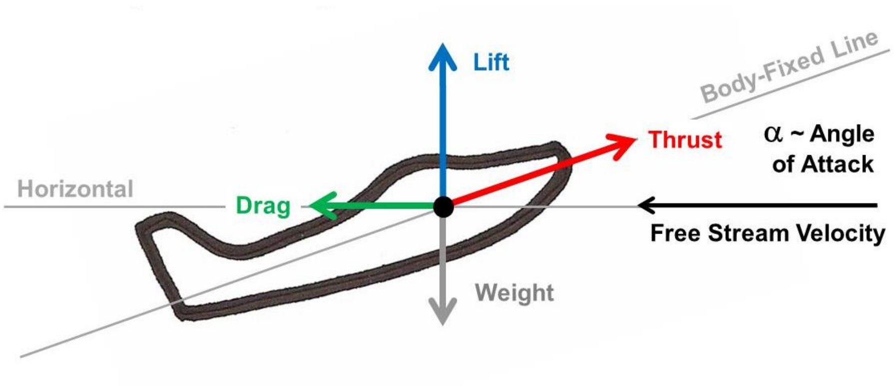


$$\Sigma F_x = T\cos\alpha - D = \frac{d(mV)}{dt} = \dot{m}V + m\dot{V}$$

$$\Sigma F_z = L + T\sin\alpha - W = 0$$

Straight and Level = Constant Altitude = Not Climbing or Descending

Steady Flight



$$\Sigma F_x = T\cos\alpha - D = \frac{d(mV)}{dt} = \dot{m}V + \cancel{m\dot{V}}^{\,0}$$

$$\Sigma F_z = L + T\sin\alpha - W = 0$$

Steady = Constant Speed = Not Accelerating or Decelerating

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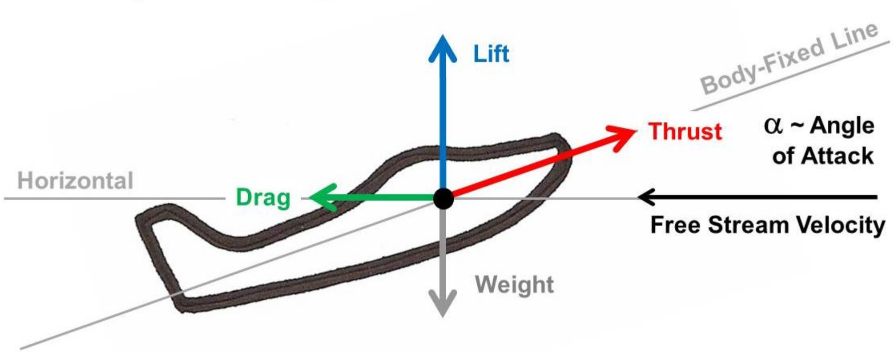
Steady Flight

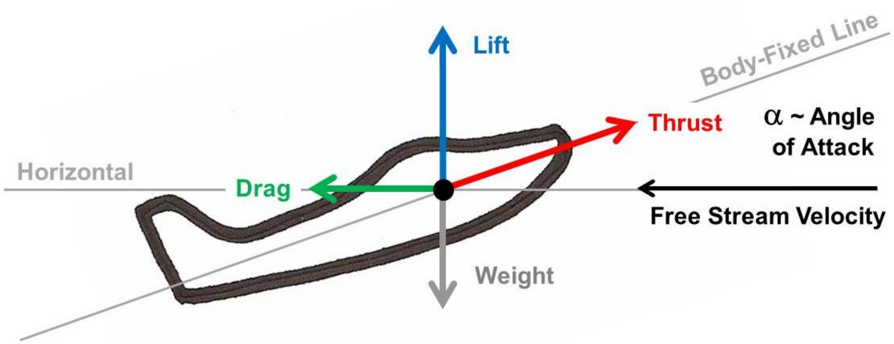


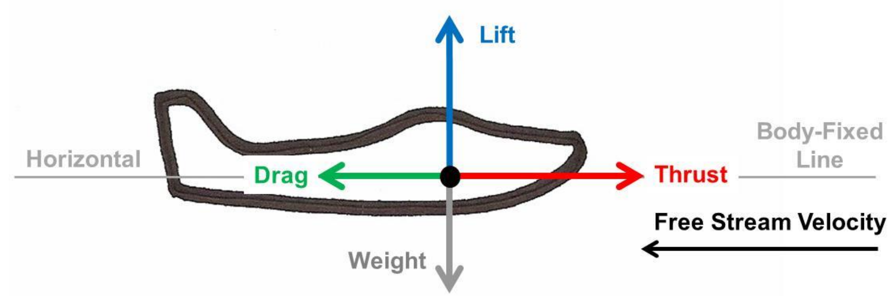
$$\Sigma F_x = T\cos\alpha - D = \frac{d(mV)}{dt} = \cancel{\dot{m}}^{\,0}\, V + m\,\cancel{\dot{V}}^{\,0}$$

$$\Sigma F_z = L + T\sin\alpha - W = 0$$

Steady = Constant Weight = "Point" Calculations

Steady Flight



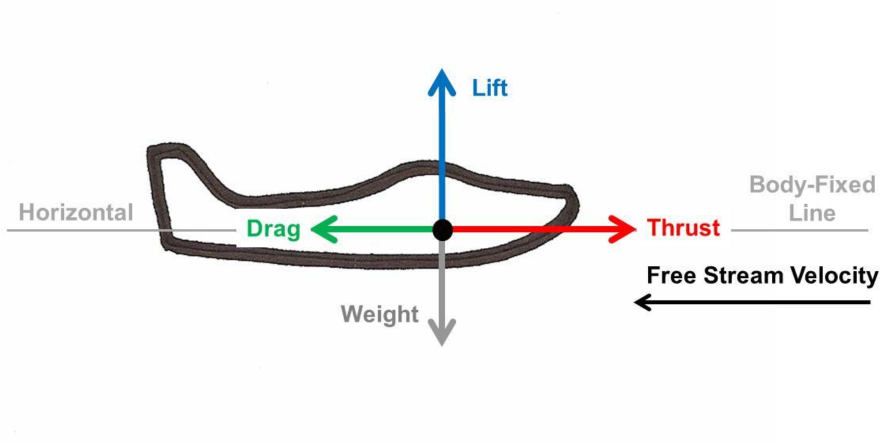


$$\Sigma\, F_x = T \overset{1}{\cos\alpha} - D = \frac{d(mV)}{dt} = 0 \qquad \boxed{T = D}$$

$$\Sigma\, F_z = L + T \overset{0}{\sin\alpha} - W = 0 \qquad \boxed{L = W}$$

Small Angle Approximation Assumption for Angle of Attack (α)

Steady Flight



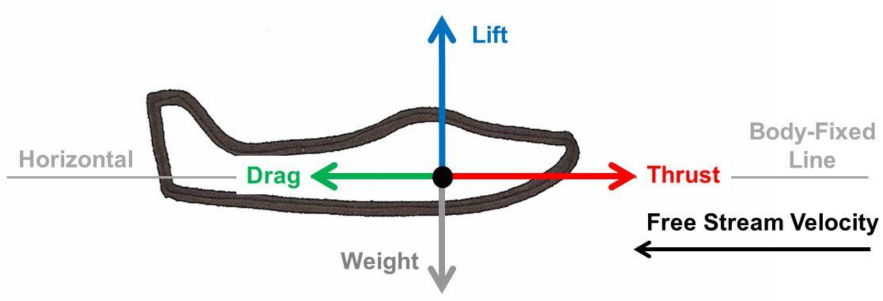
$$\boxed{T = D} \qquad \boxed{L = W}$$

An aircraft is flying at a given flight condition (velocity and altitude at the aircraft weight)

We already know that Lift is determined by Weight

How much Thrust is required to keep the aircraft at that flight condition?

Thrust Required



$\boxed{T = D}$ $\boxed{L = W}$

The amount of Thrust Required is dependent on:

- **Velocity**
- **Altitude**
- **Weight**
- **Aerodynamics (Aspect Ratio, Wing Sweep, etc)**

Example 5.1

Thrust Required

$$C_D = C_{D_0} + K\,C_L^{\;2}$$

Gulfstream IV twin-turbofan biz jet:

$C_{D_0} = 0.0150$ $K = 0.08$

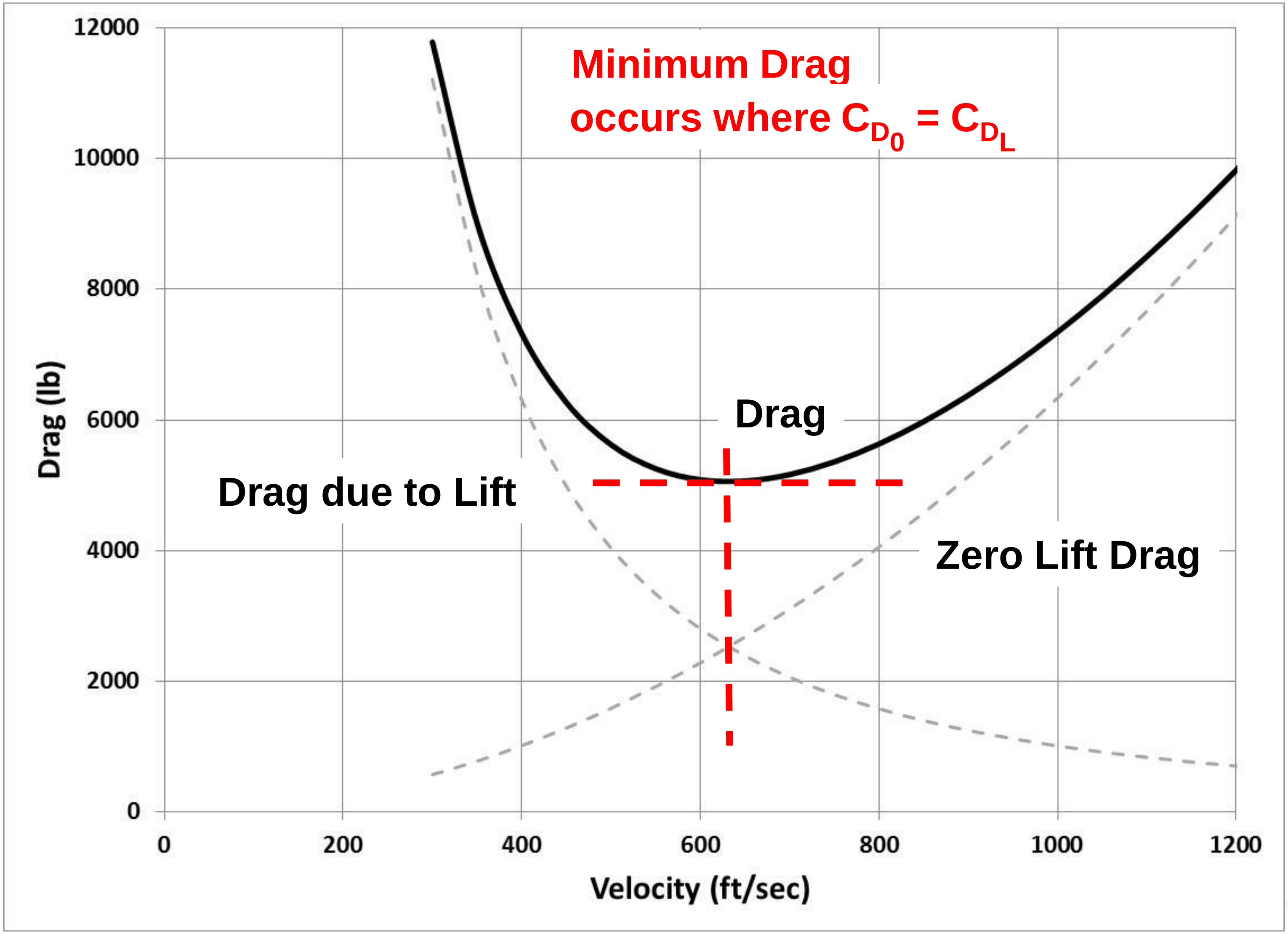
$W = 73{,}000$ lb

$h = 30{,}000$ ft

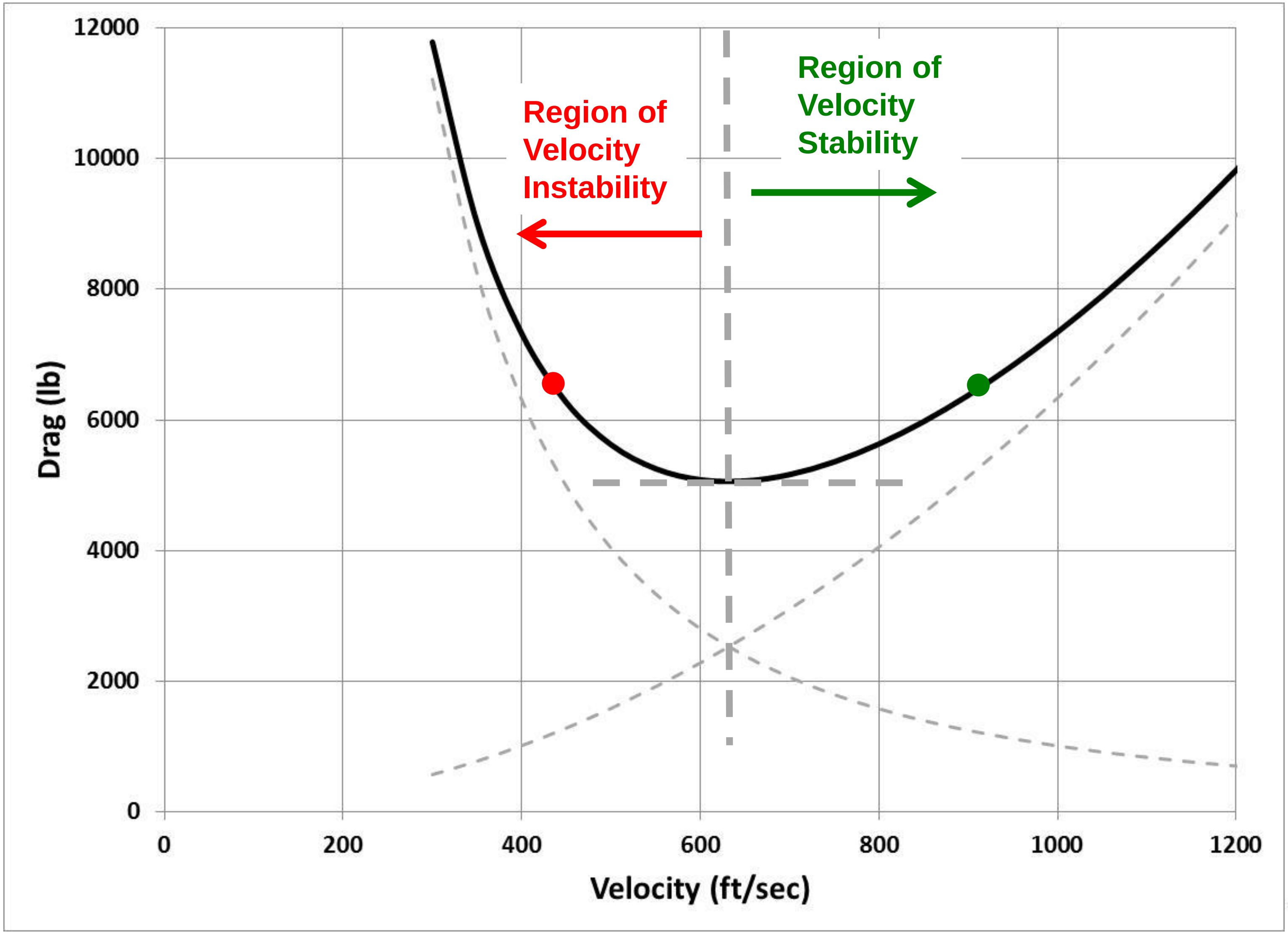
CD0	0.0150	**Wt**	73,000	lb
K	0.08	**Alt**	30,000	ft
		QMS	439.9	
		a	994.67	ft/sec
		S	950	sq ft
Vel (fps)	**CL**	**CD**	**D (lb)**	
300	1.9203	0.3100	11785	
350	1.4108	0.1742	9015	
400	1.0801	0.1083	7322	
450	0.8535	0.0733	6267	
500	0.6913	0.0532	5621	
550	0.5713	0.0411	5253	
600	0.4801	0.0334	5085	
650	0.4091	0.0284	5066	
700	0.3527	0.0250	5164	
750	0.3072	0.0226	5358	
800	0.2700	0.0208	5632	
850	0.2392	0.0196	5975	
900	0.2134	0.0186	6378	
950	0.1915	0.0179	6837	
1000	0.1728	0.0174	7345	
1050	0.1568	0.0170	7901	
1100	0.1428	0.0166	8501	
1150	0.1307	0.0164	9142	
1200	0.1200	0.0162	9825	
1250	0.1106	0.0160	10546	
1300	0.1023	0.0158	11305	

Example 5.1

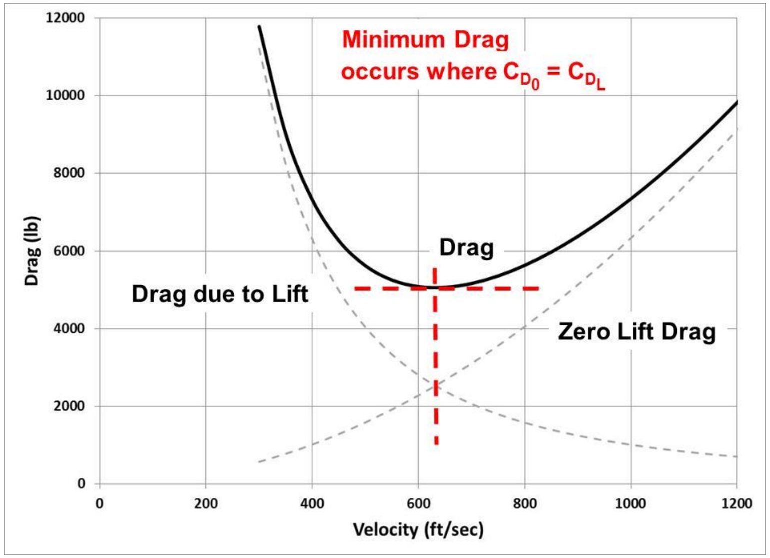
Thrust Required



Thrust Required



Thrust Required



Minimum Drag occurs where the Drag due to Lift and Zero Lift Drag curves intersect

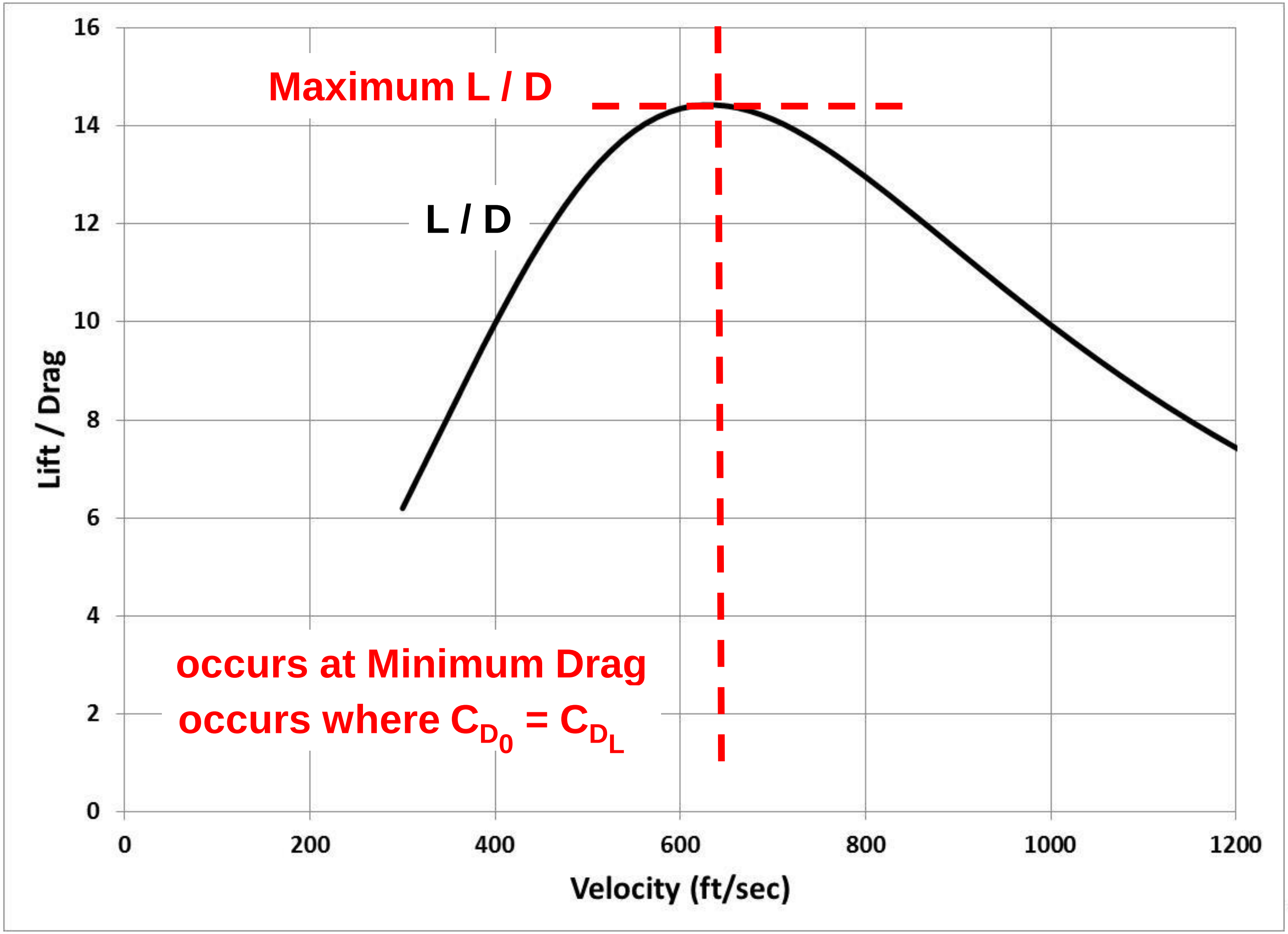
$$C_{D_0} = K\,C_L^{\;2}$$

Minimum Drag also occurs where L/D is maximum

L/D is one of the most important parameters affecting aircraft performance

Example 5.1

Lift to Drag Ratio



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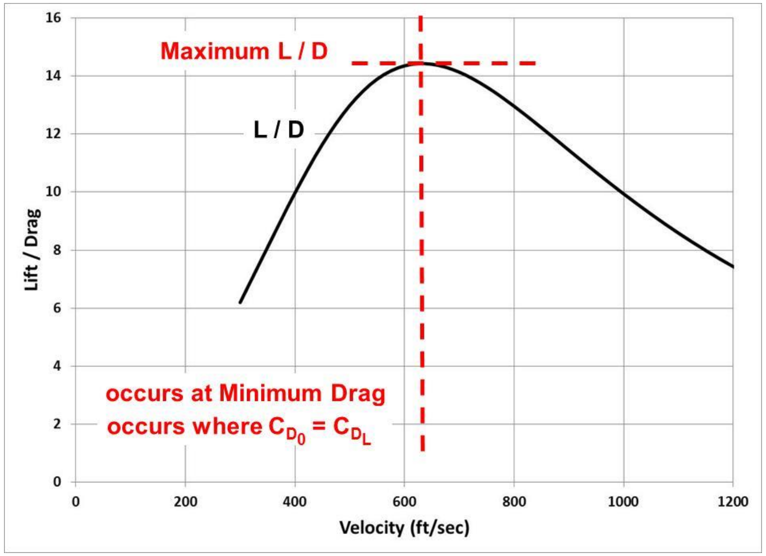
Lift to Drag Ratio



Maximum L / D occurs where the Drag due to Lift and Zero Lift Drag curves intersect

$$C_{D_0} = K\,C_L^{\,2}$$

$$\left(\frac{L}{D}\right)_{max} = \left(\frac{C_L}{C_D}\right)_{max} = \sqrt{\frac{1}{4\,C_{D_0}\,K}}$$

$$V_{L/D_{max}} = \left(\frac{2}{\rho}\sqrt{\frac{K}{C_{D_0}}}\,\frac{W}{S}\right)^{1/2}$$

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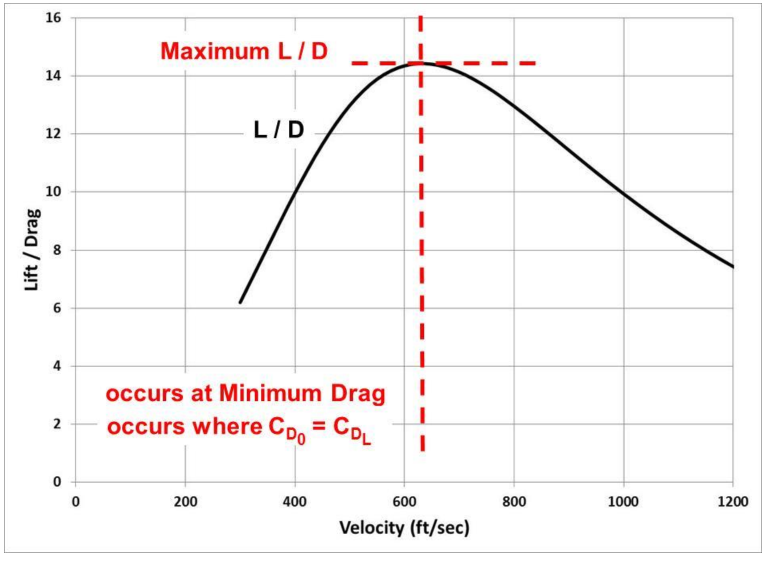
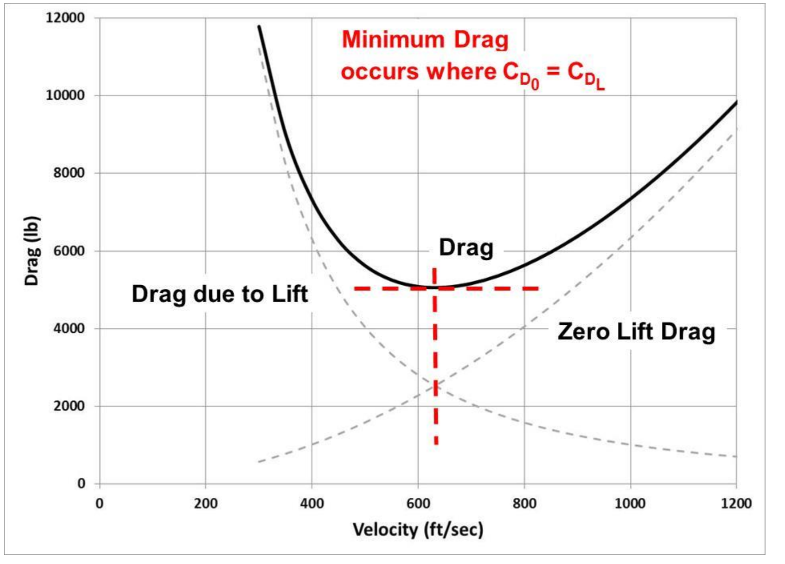
Lift to Drag Ratio





$$\left(\frac{L}{D}\right)_{max} = \left(\frac{C_L}{C_D}\right)_{max} = \sqrt{\frac{1}{4\ C_{D_0}\ K}}$$

$$\left(\frac{D}{W}\right)_{min} = \left(\frac{T_{req}}{W}\right)_{min} = \sqrt{4\ C_{D_0} K}$$

$$V_{L/D_{max}} = \left(\frac{2}{\rho}\sqrt{\frac{K}{C_{D_0}}}\ \frac{W}{S}\right)^{1/2}$$

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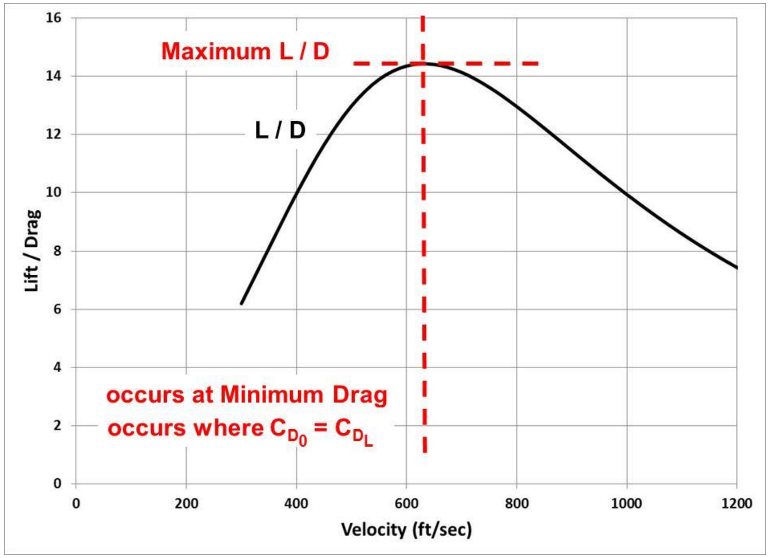
Example 5.1

Lift to Drag Ratio



Gulfstream IV twin-turbofan biz jet:

C_{D_0} = 0.0150 K = 0.08

W = 73,000 lb

h = 30,000 ft

$$\left(\frac{L}{D}\right)_{max} = \left(\frac{C_L}{C_D}\right)_{max} = \sqrt{\frac{1}{4\,C_{D_0}\,K}} = 14.4$$

$$V_{L/D_{max}} = \left(\frac{2}{\rho}\sqrt{\frac{K}{C_{D_0}}}\,\frac{W}{S}\right)^{1/2} = 632 \text{ ft/sec}$$

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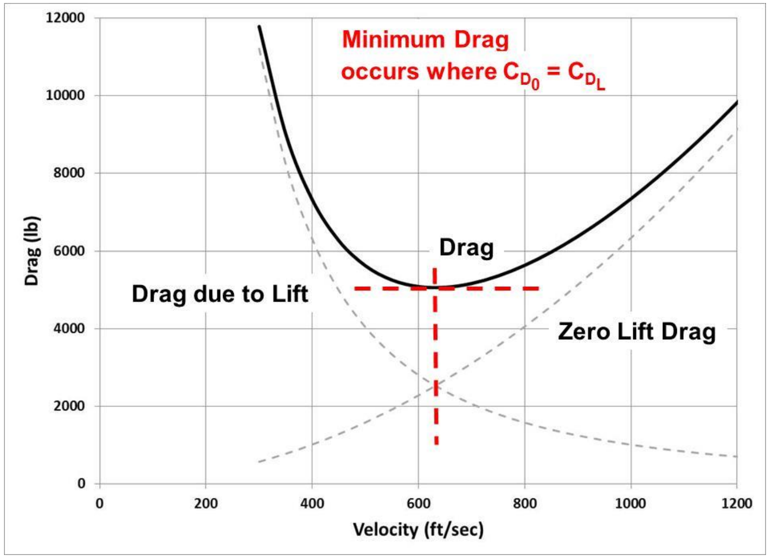
Thrust Required



Gulfstream IV twin-turbofan biz jet:

$C_{D_0} = 0.0150$ $K = 0.08$

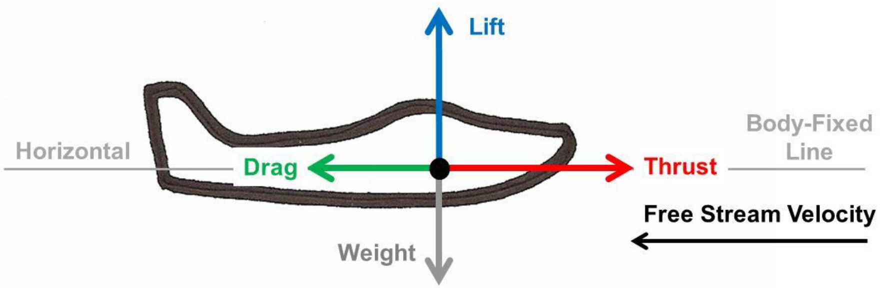
$W = 73{,}000$ lb

$h = 30{,}000$ ft

$$\left(\frac{D}{W}\right)_{min} = \left(\frac{T_{req}}{W}\right)_{min} = \sqrt{4\, C_{D_0} K}$$

$$D_{min} = \left(T_{req}\right)_{min} = W\sqrt{4\, C_{D_0} K} \quad = 5{,}058 \text{ lb}$$

Steady Flight Summary



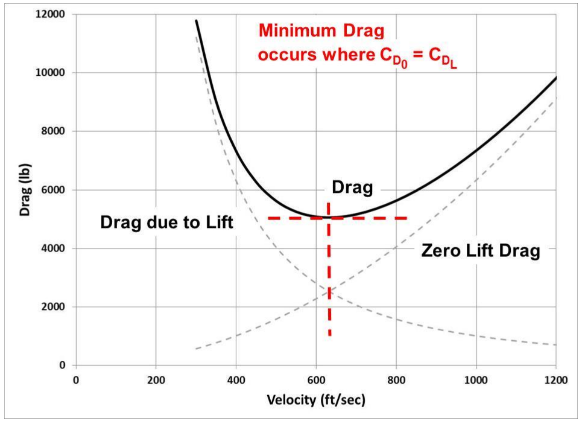
$$\boxed{T = D} \qquad \boxed{L = W}$$

Minimum Drag & Max L / D occur where the Drag due to Lift and Zero Lift Drag curves intersect

$$V_{L/D_{max}} = \left(\frac{2}{\rho}\sqrt{\frac{K}{C_{D_0}}}\frac{W}{S}\right)^{1/2}$$

$$\left(\frac{L}{D}\right)_{max} = \left(\frac{C_L}{C_D}\right)_{max} = \sqrt{\frac{1}{4\,C_{D_0}\,K}}$$

$$\left(\frac{D}{W}\right)_{min} = \left(\frac{T_{req}}{W}\right)_{min} = \sqrt{4\,C_{D_0}K}$$



$$C_{D_0} = K\,C_L^{\,2}$$

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Altitude Effects

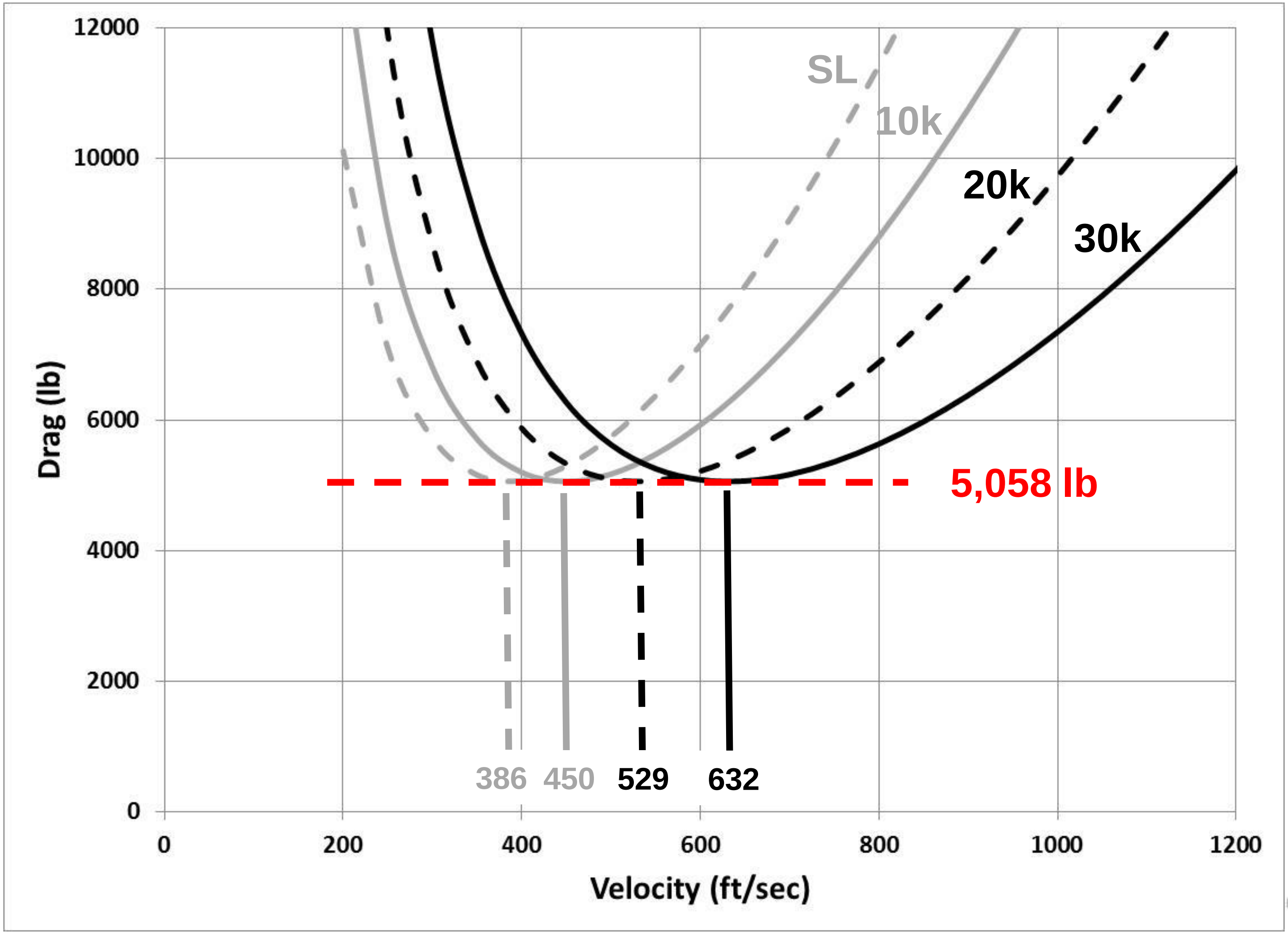
$$V_{L/D_{max}} = \left(\frac{2}{\rho}\sqrt{\frac{K}{C_{D_0}}}\,\frac{W}{S}\right)^{1/2}$$

$$\left(\frac{L}{D}\right)_{max} = \left(\frac{C_L}{C_D}\right)_{max} = \sqrt{\frac{1}{4\,C_{D_0}\,K}}$$

$$\left(\frac{D}{W}\right)_{min} = \left(\frac{T_{req}}{W}\right)_{min} = \sqrt{4\,C_{D_0}K}$$

Hint: if a velocity equation is a function of $\sqrt{\rho}$ then it is probably correlated to dynamic pressure and a direct function of KEAS

Altitude Effects



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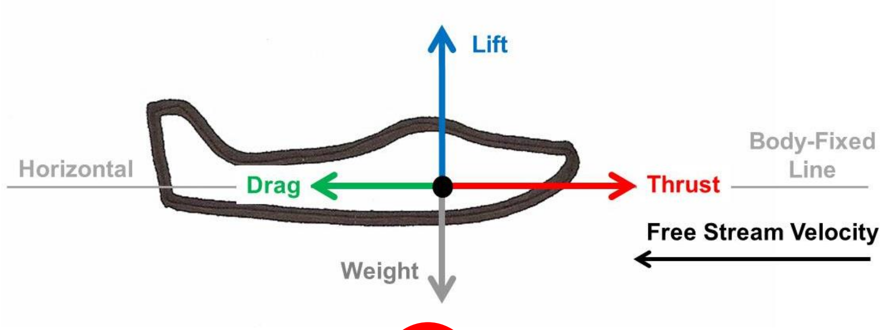
Steady Flight



$$\boxed{T = D}$$

$$\boxed{L = W}$$

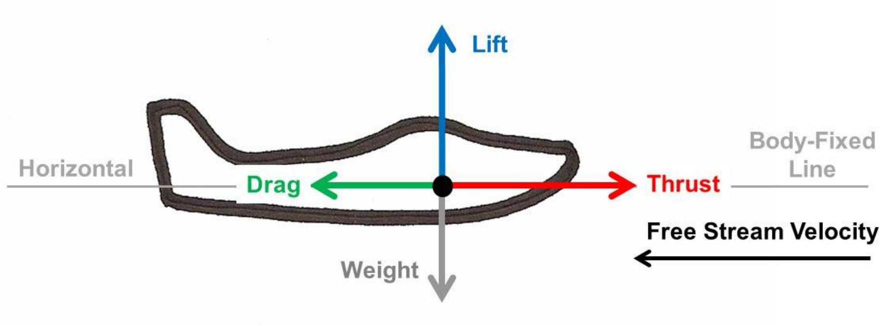
$$\left(\frac{D}{W}\right)_{min} = \left(\frac{T_{req}}{W}\right)_{min} = \sqrt{4\, C_{D_0} K}$$

$$C_D = C_{D_0} + K\, C_L^{\;2}$$

$$V_{L/D\,max} = \left(\frac{2}{\rho}\sqrt{\frac{K}{C_{D_0}}}\,\frac{W}{S}\right)^{1/2}$$

$$\left(\frac{L}{D}\right)_{max} = \left(\frac{C_L}{C_D}\right)_{max} = \sqrt{\frac{1}{4\, C_{D_0}\, K}}$$

Fundamental Parameters



$$\frac{T}{W}$$ ~ Thrust to Weight Ratio

$$\frac{W}{S}$$ ~ Wing Loading (lb/ft^2)

$$\frac{L}{D}$$ ~ Lift to Drag Ratio

$$C_D = C_{D_0} + K\,C_L^{\;2}$$ ~ Drag Polar

Questions?