

2019

Time : $1\frac{1}{2}$ hours

Full Marks : 40

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Answer from both the Sections as directed.

Section – A

Answer any four questions of the following :

$$6 \times 4 = 24$$

1. (a) State order-completeness property of \mathbb{R} .
- (b) Find the infimum and supremum of the following sets.

(i) $\{1, 3, 5, 7, 9\}$

(ii) $\left\{\frac{1}{n} : n \in \mathbb{N}\right\}$

2. State and prove Cauchy's general principle of convergence.
3. Define convergent sequence. Prove that a monotonically increasing sequence which is bounded above is convergent.
4. State and prove Ratio test.
5. Prove that every absolutely convergent series is convergent but the converse need not be true.
6. State and prove Leibnitz's test.

Section – B

Answer any four questions of the following :

4×4 = 16

7. Show that the sequence $\left\{1 + \frac{1}{n}\right\}$ converges to the limit 1.
8. Show that the sequence $a_n = \frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{n+n}$ is convergent.

9. Test the convergency of the series :

$$\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \dots \text{to } \infty$$

10. Show that the series $1 - \frac{1}{2} + \frac{1}{2^2} - \frac{1}{2^3} + \frac{1}{2^4} - \dots$ is conditionally convergent.

11. Give examples of countable and uncountable sets.

12. Give examples of monotonic increasing and monotonic decreasing sequences.

