YEAR 2024

Star Cluster Evolution
– Revealed by GAIA

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Acknowledgment

This research was partially supported by the National Taiwan Science Education Center.

This work has used data from the European Space Agency (ESA) mission Gaia, processed by the Gaia Data Processing and Analysis Consortium. I'm very grateful for the open data release.

I thank Mr. Huang from National Chiayi Senior High School, who provided valuable insight for this research.

I'm also thankful to Professor Wen-Hsien Li for assistance on the manuscript, research poster, and project presentation.

And, finally, I thank my family for their encouragement and unwavering support for me.

ABSTRACT

In this work, the membership ratio of RR Lyrae (RRL) variable stars in 20 globular clusters were calculated. In addition to cluster age and metallicity, which were previously known as the two main factors of star cluster evolution, this work reveals the significance of the cluster radius. Globular clusters are dense, low-metallicity star clusters. These objects are home to most RRL variables, which are important in determining the evolution of star clusters. Although considerable research has been devoted to discussing their period and luminosities, the membership ratio of RRLs within globular clusters remains an area of investigation. Using the GAIA Data Release 3 database, the apparent dimension of the clusters and an estimated number of observed stars in the clusters were calculated. Then, the membership ratio of RRLs in 20 globular clusters was analyzed. Variables combining the effect of the three key parameters were defined and used in a regression analysis. It is concluded that three factors play an essential role in deciding the ratio of RRLs to the number of stars in the home cluster, which are the cluster age, cluster radius, and metallicity, listed in order of descending correlation. Age and metallicity are negatively correlated with the membership ratio of RRLs, while radius is positively correlated to it. By discussing the clusters M54 and M10, the formation of RRLs is believed to be correlated with the home galaxy and the cluster formation history. Additionally, the negative correlation of metallicity with the membership ratio signify that RRLs are more likely to form in metal-poor environments. Given the positive correlation between the membership ratio and cluster radius, it is hypothesized that the formation of RRLs is correlated with tidal forces and properties of the home galaxy.

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List of Abbreviations and Acronyms

Abbreviation or Acronym	Full Form
GAIA	Global Astrometric Interferometer for Astrophysics
STD	Standard Deviation
RRL	RR Lyrae
RRLs	RR Lyrae Variable Stars
GC	Globular Cluster
CMD	Color-Magnitude Diagram
HRD	Hertzberg-Russell Diagram
DEC	Declination
RA	Right Ascension

List of Symbols

Physical Parameter	Symbol	Unit
Cluster age	$\boldsymbol{A}$	Gyr (10^9 years)
Cluster metallicity [Fe/H]	$\boldsymbol{M}$	dex
Cluster half-light radius	$\boldsymbol{r}$	Light years (ly)
Membership ratio of RRLs	$\boldsymbol{R}$	Parts per million (ppm)

I. INTRODUCTION

1. Relationship to Other Studies

In this research, I calculated the membership ratio of RR Lyrae variable stars (RRLs) in 20 globular clusters. It is found that three factors play an essential role in deciding the ratio of RRL variables to the number of stars in the home cluster, which are the cluster age, cluster radius, and metallicity, listed in order of descending correlation.

RRL variable stars (RRLs) often appear in globular clusters, and they have played a key role in determining the evolution of star clusters. Past research has mainly focused on the classification of RRLs based on brightness curves and pulsation periods, and it is further known that there is a correlation between metallicity and mass [1], [2]. This study sheds light on the importance of the number of RRLs in globular clusters, further highlighting the significance of this type of variable star in a statistical context. By knowing the relationship between cluster parameters and the membership ratio of RRLs in globular clusters, future research can not only determine cluster parameters based on the number of RRLs in the cluster but also investigate further the mechanism behind how these parameters influence the formation of RRLs.

Furthermore, age and metallicity were known as the primary and secondary factors of cluster evolution in previous research [1]. This work reveals that radius is even more essential than metallicity in analyzing the membership ratio of RR Lyrae stars.

2. Problem

Due to the limited accuracy of past apparatus, the number of stars that could be observed is limited, leaving statistical analyses of variable stars in star clusters an open area of research. In

June 2022, the European Space Agency released its third data release of the GAIA mission, which boasts photometric data for over 1.8 billion stars [3]. Using the newly released GAIA database, it is possible to calculate the membership of RRLs in globular clusters.

Interestingly, after searching for the RRLs in a few globular clusters, I found that the number of RRLs in different globular clusters can vary a lot. While some clusters host hundreds of RRLs, others may only have less than ten. Remarkably, the total number of stars in most clusters is on the same order of magnitude, so the difference in the number of RRLs isn't simply due to the fact that clusters with more stars have proportionally more RRLs. Instead, it may be related to some properties of the home cluster.

3. Hypothesis

The membership ratio of RRLs is defined as the number of RRLs in a globular cluster divided by the total number of stars in the cluster, represented with the symbol **R** hereafter. I hypothesize that the membership ratio of RRLs in globular clusters is correlated with some cluster parameters, including, though possibly not all of, age, radius, metallicity, total mass, right ascension, and declination.

4. Objectives

- Using GAIA Data Release 3, analyze the membership ratio **R** of RRLs in GCs.
- Determine which cluster parameters are most correlated with **R**.
- Propose explanations regarding the cause of such a result.

II. BIBLIOGRAPHIC REFERENCES

Star clusters are objects composed of large numbers of stars. They can be classified into globular clusters and open clusters, where the former is more densely packed, more easily found near the Milky Way halo, and have a lower metallicity [4]. Since globular clusters are formed at the early universe, investigating them can reveal a lot about the evolution of the universe [4], [5]. It has been pointed out that the formation of star clusters is mostly influenced by the age of the cluster, with metallicity being the secondary factor [1]. However, due to the highly complex chemical evolutions, in addition to processes such as nucleosynthesis and mass-loss [6], globular clusters remain an open area of investigation.

Variable stars are stars whose brightness changes over time [7]. RR Lyrae variable stars (RRLs) often appear in globular clusters. They play a key role in determining the evolution of star clusters. One of their most notable roles is serving as standard candles: since RRLs have a clear period-luminosity relationship, the amount of light they give off can be determined by measuring their period, and the distance from Earth to the cluster where the RRL is located can be determined [8]. The instability strip on the Hertzsprung-Russell diagram (HRD) is located in a diagonal region, running from the upper right to the lower left. It covers stars with intermediate temperatures, typically from around 5,000 K to 7,500 K, and spans a range of luminosities. Stars found in this region are in the later stages of stellar evolution, and their position on the HRD corresponds to their pulsating nature, with hotter, more luminous stars like Cepheids near the top and fainter, cooler stars like RRL near the bottom [9]. RRLs typically have a mass of around 0.5 solar masses [10].

III. METHODOLOGY

Start of the study: July 6^{th}, 2023

End of the study: October 10^{th}, 2024

1. Selection of Target Clusters

We selected 20 globular clusters, 18 of which have a designation in the Messier catalog with data collected from various sources [11], [12], [13], [14]. Two other clusters, namely Omega Centauri (NGC 5139) and NGC 6441, are selected due to their contrasting characteristics: Omega Centauri is the largest known cluster in the Milky Way, while NGC 6441 is a small cluster. The age, dimension, radius, metallicity, right ascension, and declination for all the GCs in this research were given from the references.

2. Determining the Total Number of Stars

For every target cluster, a search around the cluster was first conducted to find the apparent dimension of the cluster with a novel method I proposed. The region around the cluster was divided into a 3 by 3 square grid, where the center of the cluster lies in the middle of the center cell. The number of stars in each grid, retrieved with Python from the GAIA database, was calculated. By subtracting the average number of stars in the surrounding 8 cells from the number of stars in the center cell, an estimate of the total number of stars in the cluster can be made, effectively filtering out foreground and background stars. This approach assumes that foreground and background stars are homogeneously distributed throughout space. **(Figure 1)**.

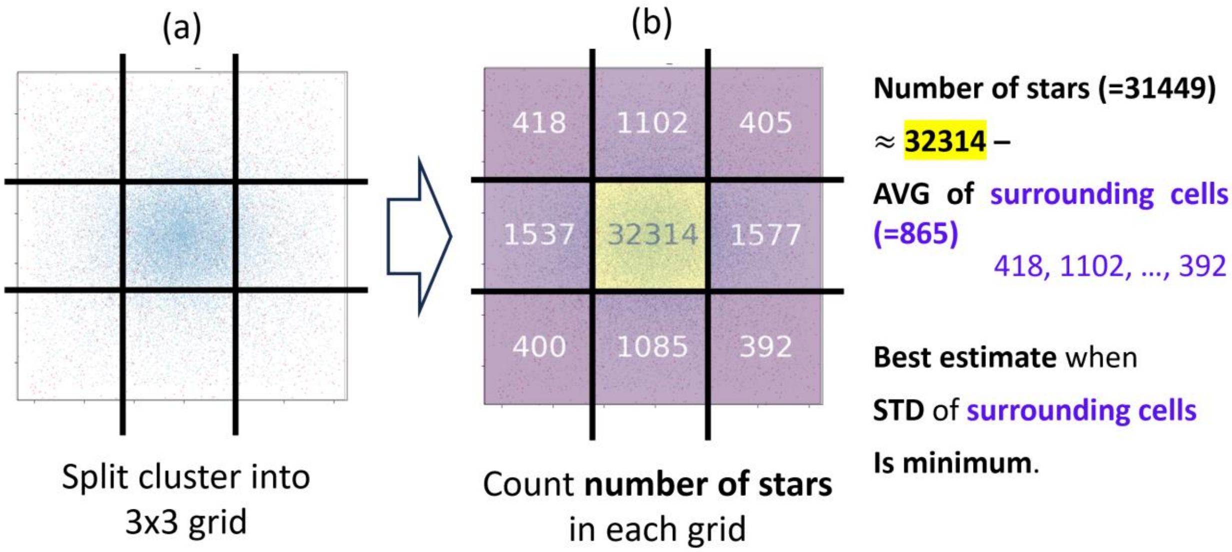
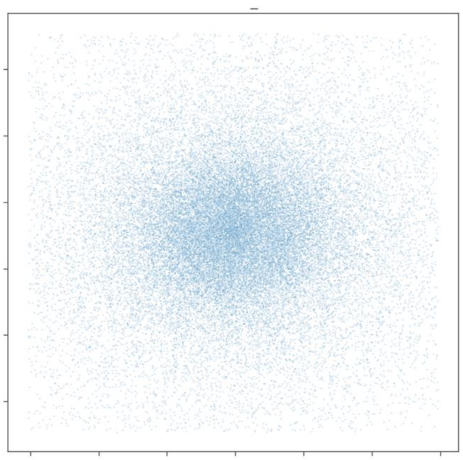


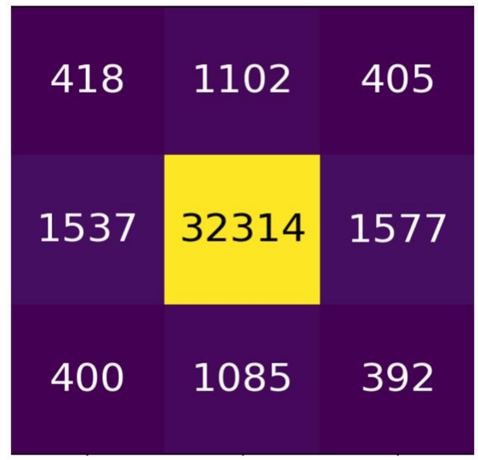
Figure 1. Visual description of estimating the number of stars in GCs.
(a) The GC was split into a 3 by 3 grid. (b) Then, the number of stars in each grid was counted. The number of stars in the GC was estimated by subtracting the average number of stars in the surrounding 8 cells from the number of stars in the center cell.

The side length of the 3 by 3 grid was then continuously increased while calculating the standard deviation of the number of stars in the bordering 8 cells. When increasing the side length of the 3 by 3 grid, there will be a critical length, above which the grid at the center can completely contain the globular cluster. Since it is assumed that in the space where globular clusters are not present, stars are distributed evenly, the number of stars in the bordering 8 cells should be roughly even. In other words, the standard deviation of the number of stars in these 8 cells is small. Therefore, at the side length which yields the lowest standard deviation, it could be said that the center cell has covered the globular cluster entirely, so such side length is taken to calculate the dimension of the globular cluster **(Figure 2)**. The step of increasing the side length was 0.25 arcminutes. Note that determining the apparent dimensions of globular clusters was done simultaneously with removing non-cluster stars and calculating the number of stars in the cluster. This is possible because non-cluster stars are assumed to be evenly distributed in space.

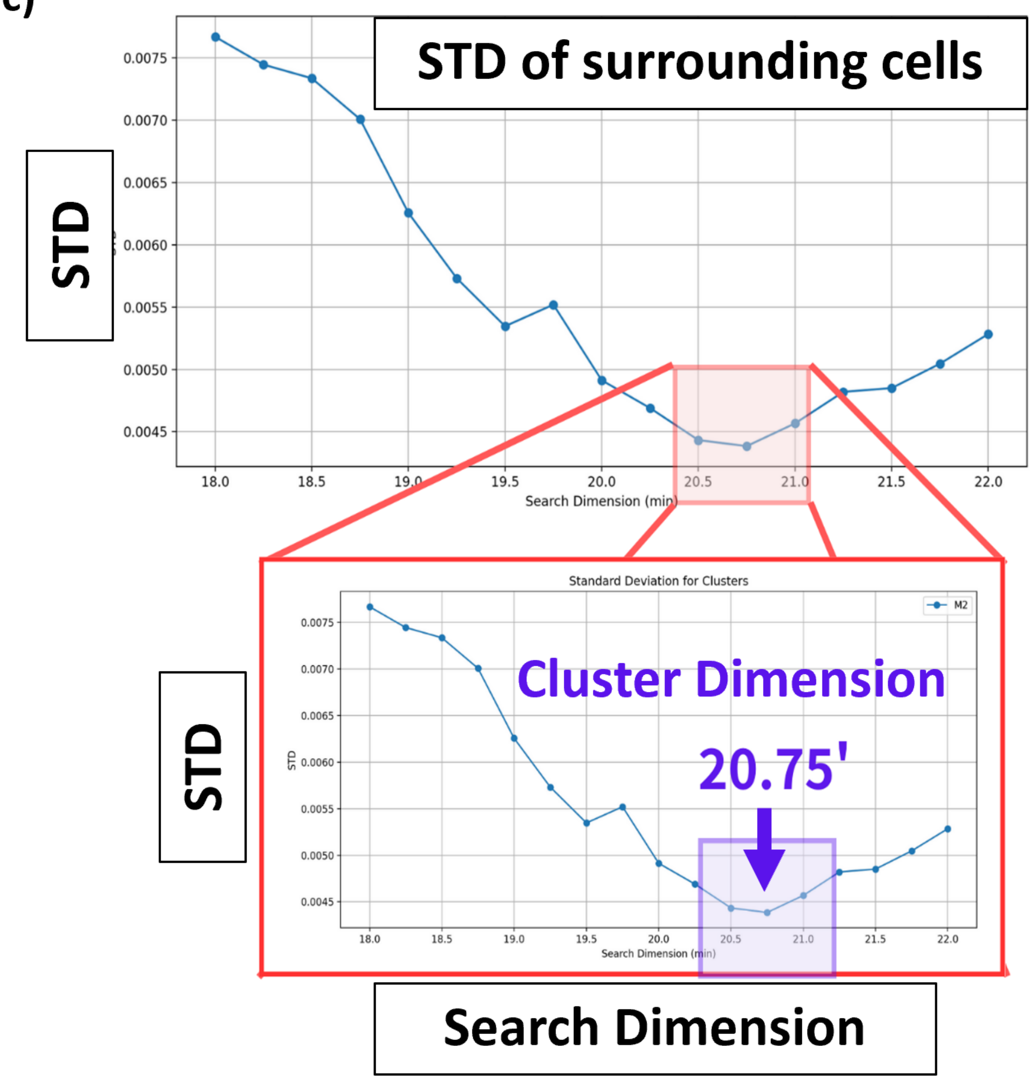
a)



b)


418
1102
405
1537
32314
1577
400
1085
392

c)


STD of surrounding cells
STD
0.0075
0.0070
0.0065
0.0060
0.0055
0.0050
0.0045
18.0
18.5
19.0
19.5
20.0
20.5
21.0
21.5
22.0
Search Dimension (min)
Standard Deviation for Clusters
M2
STD
Cluster Dimension
20.75'
Search Dimension

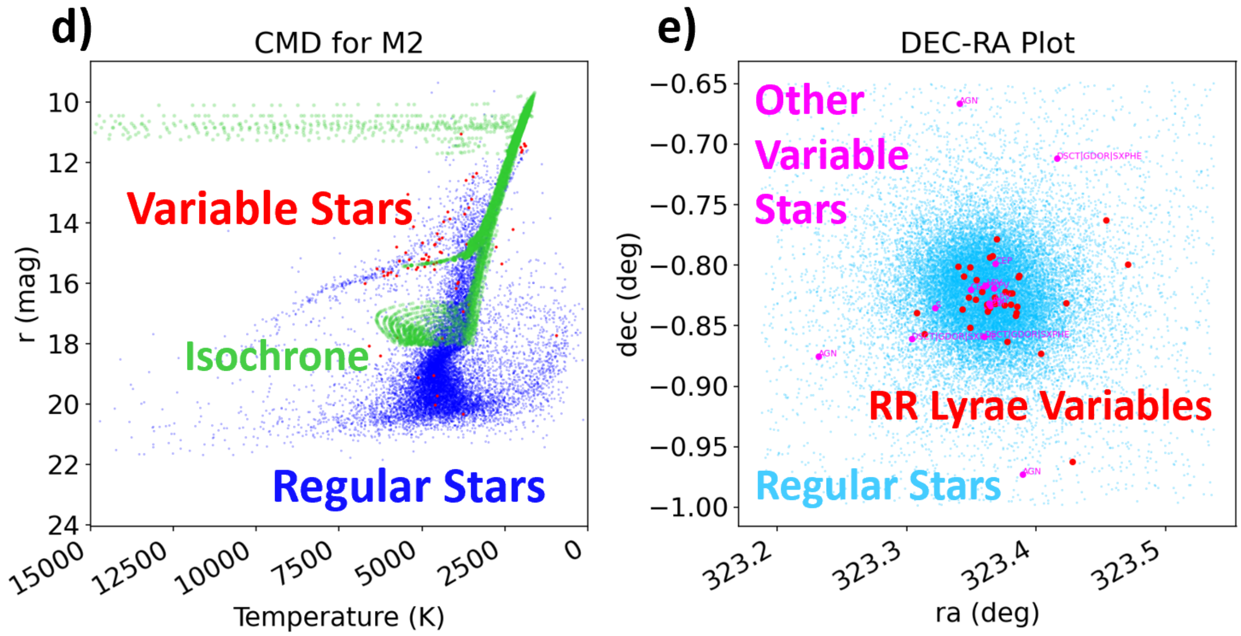


Figure 2. Methodology of finding R.

All subplots in this figure use data from the cluster M2. (a) The plot of declination vs. right ascension of cluster M2 depicts what the cluster would look like to a human eye, with the center of the cluster significantly denser. (b) The plot labeled (a) was divided into a 3 by 3 grid and the number of stars in each grid was counted with Python, with the center cell having significantly more stars. (c) The dimension of the 3 by 3 grid that yields the lowest standard deviation of the number of stars in the bordering 8 cells is determined with Python. (d) From the GAIA database, the color-magnitude diagram was plotted, with the blue dots representing normal stars, red stars representing variable stars and the green dots being the fitted isochrone curve. (e) RRLs, colored in red, and other types of variable stars, colored in purple, are shown on the plot of declination vs. right ascension with GAIA data.

3. Removal of Non-Member RRLs

The difference in magnitude of the visual blue and red bands given by the GAIA database was recorded as (b-r). A color-magnitude diagram (CMD) is a plot that shows the magnitude of stars on one axis and their temperature on the other. Isochrones are lines on a CMD representing stars of the same age but different masses. Since the age of stars in a globular cluster should be almost the same, stars belonging to the cluster should all lie on the same isochrone line. The isochrone curve from the "isochrone.mist" library was plotted onto the color-magnitude diagram (CMD) of a given globular cluster, where the x-axis is the effective temperature determined with the equation.

$$\mathrm{T_{eff}} = 10^{3.939-0.395\times(\mathrm{b}-\mathrm{r})}\ (\mathrm{K})$$

and the y-axis is the visual magnitude of the red band [15]. The isochrone curve was manually fitted to the CMD, and stars that deviated from the isochrone were removed. After filtering out stars that do not belong to the target cluster, **R** could be calculated.

4. Definition of Variables for Statistical Analysis

To find a relationship between **R** and cluster parameters, some variables combining various parameters were defined. To combine the effects of age and metallicity, the variable α was defined in exponent form as:

$$10^{\alpha} = A^{a} + (10^{M})^{b}$$

which can be rewritten as

$$\alpha = a_{\alpha} \cdot \log A + b_{\alpha} \cdot M$$

Where $\log X = \log_{10} X$. a_{α} and b_{α} are coefficients whose values have yet to be determined. The subscript α (and β below) are used to distinguish the variable α from β defined below.

To consider the effect of the cluster radius, the variable β is defined in exponent form as:

$$10^{\beta} = A^{a} + (10^{M})^{b} + r^{c}$$

which can be rewritten as

$$\beta = a_{\beta} \cdot \log A + b_{\beta} \cdot M + c_{\beta} \cdot \log r$$

The optimal coefficients $a_\alpha, b_\alpha, a_\beta, b_\beta, c_\beta$ that give the highest correlation coefficient between the two variables and the membership ratio **R** were determined with Python, and the highest correlation coefficient were calculated, along with the corresponding p-value. The p-value is calculated based on the t-distribution:

$$t = \frac{c\sqrt{N-2}}{\sqrt{1-c^2}}$$

where c is the correlation coefficient and $N = 20$.

There are two reasons why age (A) and radius (r) were taken as logarithms, while metallicity (M) wasn't. First, the metallicity value defined in this research is the chemical abundance ratio of the logarithm of the ratio of a cluster's iron abundance compared to that of the Sun. Since a logarithm has already been taken, the [Fe/H] value is directly used to define α and β above. Moreover, metallicity values can range from negative (for globular clusters), to close to zero or positive (for some metal-rich open clusters), so generally speaking, taking the logarithm of it isn't reasonable. The variables α and β enable one to determine the relative significance of each cluster parameter based on the parameters' coefficients.

Finding optimal coefficients for α and β was done with Python with 2 and 3 nested loops respectively to find the coefficients to 4 significant decimal places. Python programs were run on Google Colaboratory.

IV. RESULTS AND DISCUSSION

1. Discussion of Potential Data Bias

We selected 20 globular clusters mainly from the Messier catalog. These clusters are selected because they have enough past research determining their cluster parameters, such as age, radius, and metallicity since clusters in the Messier catalog are generally brighter and easier to observe. However, selecting these clusters may have a potential bias as they are generally brighter than the clusters not selected in this research.

2. Cluster Dimension Estimation with GAIA

Since instruments with a better resolution would be able to see more stars, the dimension determined from the GAIA database would be larger than that determined with a telescope of lower resolution. As a result, most of the dimensions determined in this research are larger than those from the references, shedding light on GAIA's ability to see stars up to a magnitude of 20.7 [3].

3. Comparison of Individual Cluster Parameters

The value of **R** of the 20 globular clusters selected ranges from 123 to 7544 **(Figure 3).** Comparing **R** to various cluster parameters, such as age, metallicity, radius, and mass, the relationship between such parameters and the membership ratio of RRLs can be deduced **(Table 1).** Plots of **R** with age, metallicity, radius, mass, right ascension, and declination show that **R** is negatively correlated with age and metallicity, but is positively correlated with radius, and shows no significant relationship with mass, right ascension, and declination **(Figure 4, Table 2)**.

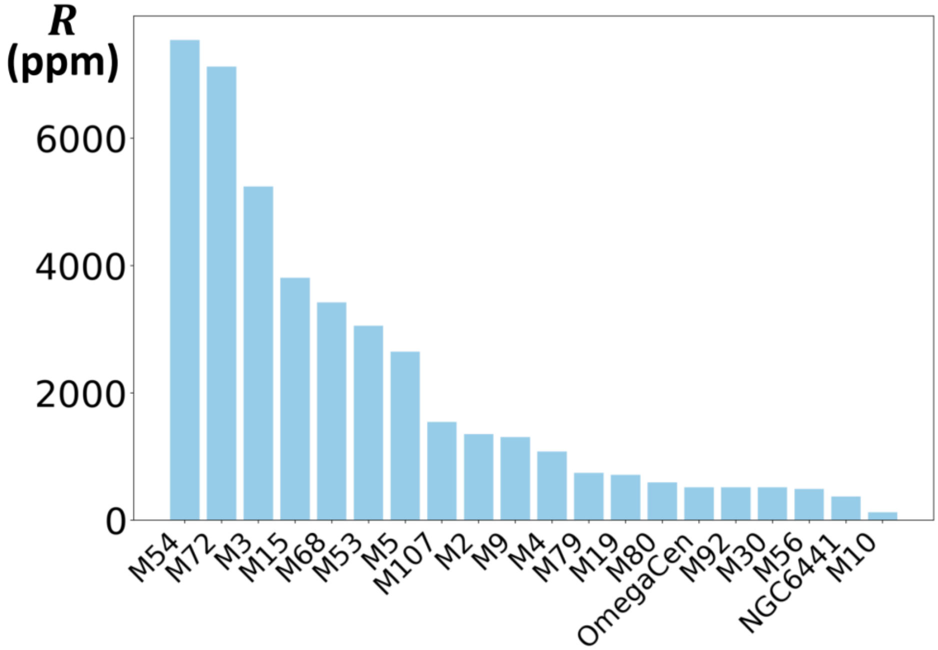


Figure 3. Value of R of 20 globular clusters.

The bar graph lists the part per million membership ratio of RRLs in decreasing order. After a query to the GAIA database retrieves the position of individual stars, foreground, and background stars were filtered, and **R** could be calculated.

Table 1. Cluster parameters and R of the 20 researched globular clusters.

Cluster parameters were collected from the Messier catalog and references. The total number of stars and the number of RRLs were determined from the GAIA database. R, the part per million membership ratio of RRLs in each globular cluster, was calculated using Excel.

Globular Cluster	Age (Gyr)	[Fe/H]	Radius (Light Years)	Total Number of Stars	Number of RRLs	*R*
M2	12.50	-1.65	87.3	22912	31	1353
M3	11.39	-1.34	90.0	37189	195	5243
M4	12.20	-1.07	36.0	28710	31	1080
M5	10.62	-1.12	81.0	36259	96	2648

M9	12.00	-1.77	45.0	12240	16	1307
M10	11.39	-1.25	41.6	32551	4	123
M15	12.00	-2.37	88.0	28638	109	3806
M19	11.90	-1.53	70.0	13991	10	715
M30	13.00	-2.27	32.0	15555	8	514
M53	12.67	-1.86	113.0	17026	52	3054
M54	13.00	-1.49	153.0	6230	47	7544
M56	13.70	-2.00	42.0	10200	5	490
M68	11.20	-2.23	53.0	12559	43	3424
M72	9.50	-1.48	39.0	5613	40	7126
M79	11.70	-1.55	65.0	10722	8	746
M80	13.50	-1.47	47.0	11815	7	592
M92	14.20	-2.32	109.0	23668	15	634
M107	13.95	-0.95	30.0	11653	18	1545
OmegaCen	11.52	-1.35	86.0	224722	116	516
NGC6441	13.00	-0.53	4.8	2690	1	372

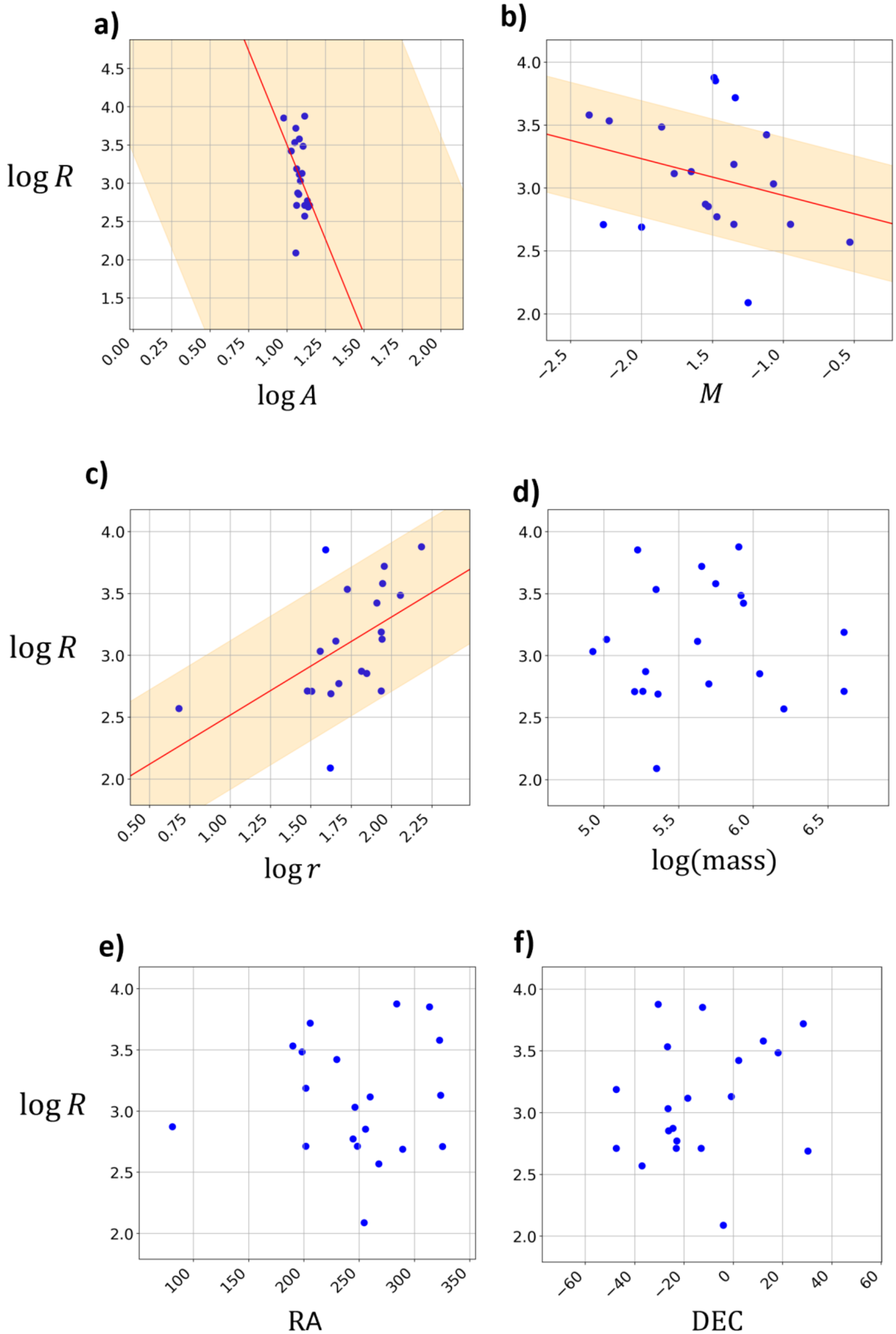

a)
b)
c)
d)
e)
f)
log R
log A
M
log r
log(mass)
RA
DEC

Figure 4. Plots of R versus various cluster parameters.

The 20 blue dots shown in each subplot represent a globular cluster. (a) $\log R$ vs. the logarithm of cluster age shows a negative correlation with a correlation coefficient of -0.411. (b) $\log R$ vs. cluster metallicity shows a negative correlation with a correlation coefficient of -0.280. (c) $\log R$ vs. the logarithm of cluster radius shows a positive correlation with a correlation coefficient of 0.520. (d) $\log R$ vs. the logarithm of cluster mass with a correlation coefficient of 0.035. (e) $\log R$ vs. cluster right ascension with a correlation coefficient of 0.049. (f) $\log R$ vs. cluster declination with a correlation coefficient of 0.222. Plots (d), (e), and (f) do not present significant correlation between $\log R$ and the respective cluster parameters. For the three key parameters with significant trends, the best-fit line shown in red and the orange 95% confidence interval are shown in subplots (a), (b), and (c).

Table 2. Regression results between cluster parameters and $\log R$.

The correlation coefficients and p-values between the logarithm of the membership ratio $\log R$ and the transformed parameters $\log A$, $\log r$, M, log(mass), right ascension, and declination are shown. The transformed parameters may be the logarithm of the cluster parameter or simply the cluster parameter itself. The three key parameters age, radius, and metallicity exhibit stronger correlations with $\log R$ compared to the other three cluster parameters.

Cluster Parameter	Transformed Parameter	Correlation Coefficient	P-Value
Age (Gyr)	$\log A$	-0.411	0.0718
Radius (Light years)	$\log r$	0.520	0.0188
Metallicity [Fe/H] (dex)	M	-0.280	0.2318
Mass (Solar mass)	log(mass)	0.035	0.8835
Right Ascension	RA	0.049	0.8375
Declination	DEC	0.222	0.3469

4. Regression Results of α and β Variables

Using Python to calculate the value of α for each of the 20 clusters with a given a_α and b_α, the optimal a_α and b_α that yield the highest correlation coefficient with $\log R$ is determined to be $a_\alpha = -6.26$ and $b_\alpha = -0.37$. The correlation coefficient between α and

$\log R$, using the optimal parameters, is -0.501, with a p-value of 0.0244. The values of $a_\beta, b_\beta, c_\beta$ that give the highest correlation coefficient between β and $\log R$ are -7.11, 1.11, and -0.21, with a correlation coefficient of 0.613 and a p-value of 0.0040. Such a p-value means that if age, metallicity, and radius do not play a role in **R**, there is only a 0.4% probability that I'll get a correlation coefficient of 0.613 or higher. Thus, the low p-value strengthens that the result has statistical significance. The lines of best fit and the uncertainties of the slopes and intercepts are listed in **Table 3**. Most of the 20 data points lie in the 95% confidence interval **(Figure 5)**.

Table 3. Regression results between $\boldsymbol{\alpha, \beta}$ and $\log \boldsymbol{R}$.
Optimal coefficients, correlation coefficients, and p-values of the regression between α, β and $\log R$. The small p-values indicate a statistically significant association between α, β and $\log R$.

Quantity	a	b	c	Correlation Coefficient	P-value
$\boldsymbol{\alpha}$	-6.26	-0.37		0.501	0.0244
$\boldsymbol{\beta}$	-7.11	-0.21	1.11	0.613	0.0040

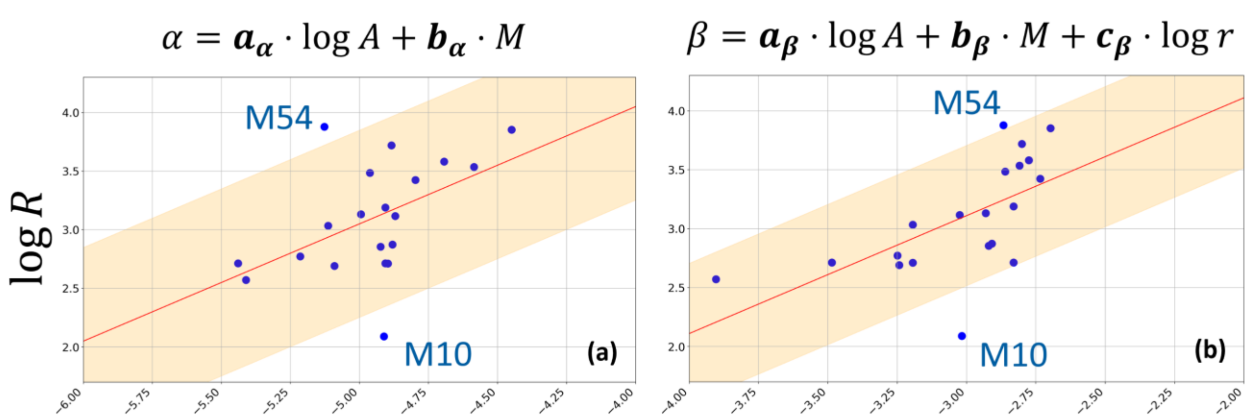


Figure 5. Plots of $\log \boldsymbol{R}$ versus $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$.
The 20 blue dots shown each represent a globular cluster. The red line is the regression line with its equation shown on top of each subfigure, and the orange-shaded region is the 95% confidence

interval. (a) $\log R$ vs. α (p=0.0244). (b) $\log R$ vs. β (p=0.0040). The slopes of the best-fit lines are normalized to 1. Most clusters fall inside the 95% confidence interval.

The dots in the plot of $\log R$ vs. β are more concentrated than in the plot of $\log R$ vs. α. Also, the area of the 95% confidence interval in the plot of $\log R$ vs. β is 25% smaller. Thus, the correlation between $\log R$ and β is greater than that between $\log R$ and α, which means radius is an essential factor in understanding star cluster evolution. Notably, clusters M54 and M10 are outside of the 95% confidence interval, and have high and low **R** respectively.

5. Relative Significance of Cluster Parameters

Plots of **R** vs. cluster parameters show the positive or negative correlations of **R** with cluster age, metallicity, and radius, although the trend is not very strong. Therefore, analyses using the variables α and β are required. It is shown in the plots that the linear fit is better than simply plotting **R** vs. individual cluster parameters **(Figure 4, Figure 5)**. The p-value 0.0040 suggests that the cluster parameters do influence **R**.

The variable β is positively correlated with **R**, and the coefficients of metallicity and age are negative, while the coefficient of radius is positive. This confirms that metallicity and age are negatively correlated with **R** while the radius is positively correlated, agreeing with the result obtained from **Figure 4**. From the coefficients of various parameters in β, it is possible to take the absolute value of the product of the coefficient in β and the median value of a cluster parameter to compare the relative significance of age, metallicity, and radius **(Table 4)**. It is found that the importance of these three factors, listed in decreasing order, are age, radius, and metallicity.

Table 4. Estimation of relative significance of cluster age, radius and metallicity.
The absolute value of the coefficient of each parameter in β was multiplied by the median value of the 20 globular clusters. The product was normalized so that the most significant parameter, which is age, has a relative significance of 1. The calculation shown here strengthens the fact that age, radius, and metallicity are three significant factors influencing **R** listed in decreasing order of correlation. The calculation was made with Excel.

Parameter	Value Range	Median Value	Coefficient in $\boldsymbol{\beta}$	Relative Significance
$\log A$	0.978~1.152	1.08	-7.11	$\equiv$ 1
$\log R$	0.681~2.185	1.77	1.11	≈0.26
M	-2.37~-0.95	-1.51	-0.21	≈0.04

6. Interpretations of the Results

It is found that the clusters M54 and M10, have a high and low **R**, respectively **(Figure 3, Figure 5)**. Since M54 belongs to the Sagittarius Dwarf Galaxy while other clusters are in the Milky Way, I propose that the membership ratio of RRLs may be dependent on the properties of the home galaxy. Moreover, the lack of RRLs in M10 is in agreement with past research, which suggests that this may have to do with variations in the Red Giant Branch evolution of M10 [16], [17].

As stated previously, age and metallicity are the two primary factors governing the evolution of star clusters [1]. The expected timescale for RR Lyrae star evolution is around 1.2 to 8 billion years [18]. Therefore, RRLs in older globular clusters may have left the instability strip and moved further into stellar evolution, explaining why **R** is lower at larger ages. The negative correlation of metallicity with **R** could signify that RRLs are more likely to form in metal-poor environments. From past research, I found that the instability strip, which is where the RRLs are located on the Hertzberg-Russell Diagram, is narrower when the metallicity is higher [19], and this is a possible explanation of the result.

In addition to cluster age and metallicity, this work reveals that the radius of the cluster is an essential factor in understanding star cluster evolution. To explain why there are likely to be more RRLs (higher **R**) when the radius of the cluster is larger, I propose that the formation of RRLs could be related to tidal forces, which are proportional to the radius of the cluster **(Figure 6)**. This hypothesis is in agreement with past research suggesting that the formation of RRLs is correlated with mass loss [16].

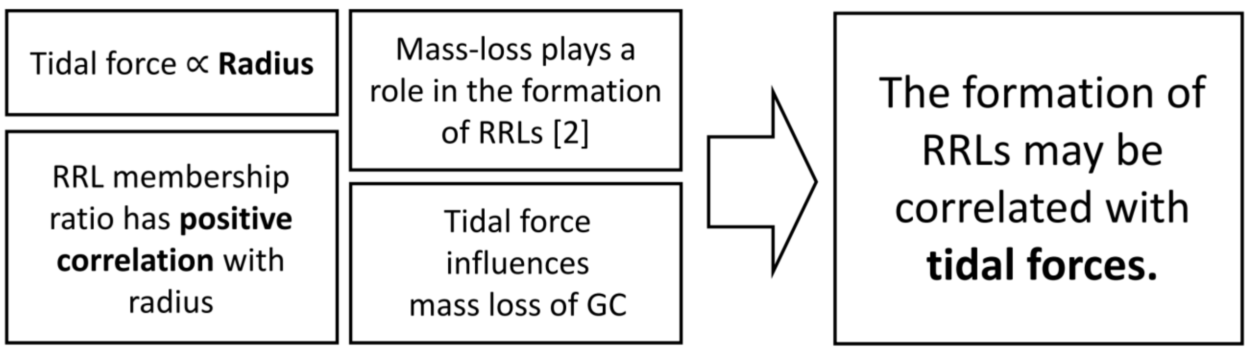


Figure 6. Hypotheses of the relationship between the formation of RRLs and tidal forces.

V. CONCLUSION

1. Novel Method of Determining Cluster Dimension

In this research, I used a novel method to determine the dimension of the globuar cluster with the GAIA database. The region surrounding the cluster was divided into a 3x3 grid, and the standard deviation of the bordering eight cells was calculated. This approach is made possible by the extensive number of stars recorded in the GAIA database. The method was also used to determine the number of stars in a globular cluster visible to the GAIA spacecraft. After removing non-member RRLs, the membership ratio of RRLs, defined as the number of RRLs in a globular cluster divided by the total number of stars in the cluster, was calculated for 20 globular clusters. Potential data bias is discussed.

2. Relationship Between R and Cluster Parameters

The regression results between the membership ratio of RRLs (**R**) and the six cluster parameters investigated in this study (cluster age, radius, metallicity, mass, right ascension, and declination) show that age, radius, and metallicity are more strongly correlated with **R** compared to the other three cluster parameters. Variables α and β were defined to combine the effects of age, radius, and metallicity. The optimal coefficients for each transformed cluster parameter were determined so that α and β exhibit the greatest correlation coefficient and lowest p-value with **R**. By quantitatively comparing the coefficients in β, the relative significance of the three key cluster parameters is found, which, listed in order of descending correlation, are age, radius and metallicity.

3. Interpretation of the Results

Because **R** is very weakly correlated with right ascension and declination, the formation of RRLs may not be significantly influenced by their spatial distribution in the sky. Since **R** is lower at larger ages, RRLs in older globular clusters may have left the instability strip. Additionally, the negative correlation of metallicity with **R** could signify that RRLs are more likely to form in metal-poor regions.

This work reveals that radius is even more essential than metallicity in understanding star cluster evolution. Given the positive correlation between **R** and cluster radius, I propose that the formation of RRLs could be related to tidal forces. By investigating the clusters M54 and M10, I propose that the membership ratio of RRLs may be dependent on the properties of the home galaxy and variations in the Red Giant Branch evolution of the cluster. However, the exact formation mechanism of RRLs requires further research.

4. Future Prospects

- Similar analyses could be conducted on open clusters, given that member stars can be properly distinguished from background and foreground stars.
- Future research can apply the method used in this work to consider the effect from radius to other star clusters.

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