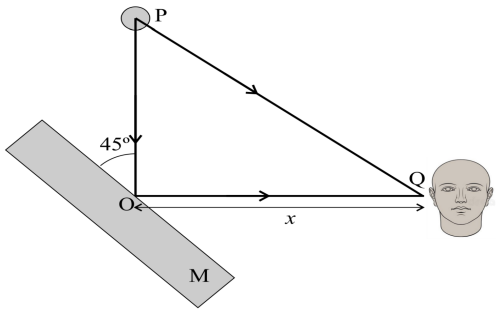
Solution by Cheng-You Ho, written 2023/07/08

Interferencia acústica (2022 OFU P4)

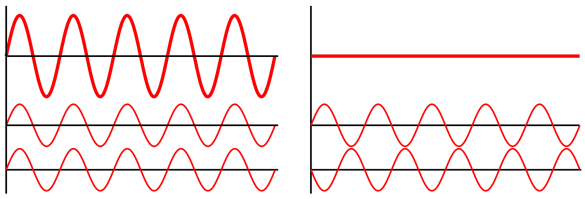
Problem A speaker **P** emits sound of frequency 330 Hz in all directions. Part of the emitted sound arrives at a point **O** at 45º on a reflecting wall **M** and is reflected in the direction of the x axis to the ear of the person. Another part of the sound comes directly to the ear from the speaker. Walking from point O on the x axis, it is heard that the sound intensity decreases until a minimum is obtained in the point Q at a distance x from O. The distance between points P and O is 3.00 m and the speed of sound in air is 340 m/s.



Part A. Calculate this distance x.

Part B. Calculate the ratio between the intensity of the sound that arrives directly and that that arrives by reflection on the wall. It is assumed that the wall does not absorb or transmit sound, nor does the floor or the surroundings.

Solution The main trick of this problem is *wave interference*. When two waves meet each other, their ampli- tudes at each position add or subtract, which is called *constructive interference* and *destructive inter- ference* (pic on the right, by Wikipedia) respectively. In the problem, the point Q is called the *first minimum*, as it is the *first* location where destructive interference occurs.



Consider two sinusoidal waves emitted from P; the first travels along PQ (length of $\sqrt{3^2 + x^2}$), and the other travels along POQ (length of $3 + x$). These two waves produce destructive interference at Q, which means the difference of their travel distance must differ by "half a wave". However, it is also OK if they differ by "1.5 waves" since the resulting pattern is identical. Hence we can write

$$\lambda = \frac{v}{f} = \frac{340}{330} \approx 1.03\ (m) \qquad (3 + x) - \sqrt{9 + x^2} = (n + \tfrac{1}{2})\lambda.$$

Q is the *first* destructive interference point, so $n = 0$. Solving the quadratic gives $x = 0.568$ m.

For Part B, no calculation is required, as the two waves must have *equal magnitude* and *opposite direction* amplitudes. Intensity is amplitude squared, hence the ratio is 1.

Answer: Part A. 0.568m, Part B. 1.