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Advanced Bayesian Learning

Computer Lab 4 - Bayesian Model Inference

The labs are the only examination, so you should do the labs **individually**.
You can use any programming language you prefer, but do **submit the code**.
Submit a readable report in **PDF** (no Word documents!) or a **JuPyteR notebook**

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1. **Robust regression modeling**. Consider the student-t regression model:

$$y_i = \boldsymbol{x}_i^T \beta + \varepsilon_i, \qquad \varepsilon_i \overset{\text{iid}}{\sim} t_\nu(0, \sigma^2),$$

where $t_\nu(\mu, \sigma^2)$ is the student-t distribution with ν degrees of freedom, location μ and scale parameter σ^2 such that the variance is $\sigma^2\nu/(\nu - 2)$ whenever $\nu > 2$. Assume that ν is known, and use the prior $\beta \sim N(0, \tau^2 I_q)$ and non-informative prior for the scale: $p(\sigma) \propto 1/\sigma$. The file `StudentTRegression.R` contains the function `GibbsTReg` that implements a regression extension of the Gibbs sampler on Page 294 in the Bayesian Data Analysis book to simulate from the posterior $p(\beta, \sigma^2 | \boldsymbol{y}, \boldsymbol{X})$ for a given ν. The same file also contains the function `PredTReg` to simulate from the posterior predictive distribution.

   (a) Consider the dataset `regression.csv` (loaded from the code `StudentTRegression.R`). Let M_ν denote the above regression model with ν degrees of freedom, and let $\tau = 1$. Compare the models M_2, M_5, M_{10} and M_{30} using the following Bayesian model inference criteria:

      i. Posterior model probabilities using the BIC approximation of the marginal likelihood. Assume uniform prior over the set of models.

      ii. Posterior model probabilities using the Laplace approximation of the marginal likelihood. Assume uniform prior over the set of models.

      iii. WAIC

      iv. Bayesian leave-one-out cross-validation (no need to do code up importance sampling or the Pareto smoothed version since the data set is relatively small).

   (b) Repeat 1a) using $\tau = 10$ and $\tau = 100$.

Good luck! May the Bayes be with you.