5.

a). $y^f_{t+\tau|t} = E[y_{t+\tau} | I_t]$

$= E[1 + 0.5 y_{t+\tau-2} + u_t | I_t]$

$= 1 + 0.5 E[y_{t+\tau-2} | I_t] + E[u_t | I_t]$

$= 1 + 0.5\, y^f_{t+\tau-2|t}$

b). $e^f_{t+\tau|t} = y_{t+\tau} - y^f_{t+\tau|t}$

$= y_{t+\tau} - 1 - 0.5\, y^f_{t+\tau-2|t}$

$= 1 + 0.5 y_{t-2} + u_t - 0.5\, y^t_{t+\tau-2|t}$

$= 0.5(y_{t-2} - y^f_{t+\tau-2|t}) + u_{t+\tau}$

$= 0.5\, e^f_{t+\tau-2|t} + u_{t+\tau}$

c). $y_t = 1 + 0.5\, y_{t-2} + u_t$

As $\tau \to \infty$, $y^f_{t+\tau|t} = 1 + 0.5\, y^f_{t+\tau-2|t} = 1 + 0.5(1 + y^f_{t+\tau-4|t}) = 1 + 0.5[1 + 0.5(1 + y^f_{t+\tau-6|t})]$

$= 1 + 0.5(1 + 0.5(\cdots)) \to 1$

d). As $\tau \to \infty$, $Var(e^f_{t+\tau|t}) = \frac{1}{1-0.5^2} \cdot 1$