PROBLEM SET 4

Due: Tuesday February. 19th

1. We are interested in forecasting quarterly GNP growth by AR model with the number of lags selected by AIC. Use observations from 1960Q1 to 1989Q4, i.e. the first 120 observations, as your initial training set to estimate the model.

   (a) Compute one to eight quarters ahead forecasts of GNP growth between 1990Q1 to 2019Q4 using expanding windows.

   (b) Plot one and eight quarters ahead forecasts of GNP growth v.s. the actual GNP growth. You can see that the eight quarters ahead forecasts of GNP growth is closer to the flat line, compared to the one quarter ahead forecasts. Explain why?

   (c) Compute root mean square forecast errors for one and eight quarters ahead.

2. Repeat question 1 for an AR model with the number of lags selected by BIC.

3. Repeat questions 1 and 2, but use rolling windows rather than expanding window.

4. Consider the average forecasts of GNP growth computed by expanding window and rolling window models with the number of lags selected with AIC as additional forecasts of GNP growth.

   (a) Compute root mean square forecast errors for one and eight quarters ahead.

   (b) Compare root mean square forecast errors of the average model with that of expanding window and rolling window models.

      i. Do we have any of these three models to perform worse than the others in terms of forecasting errors over all the 8-steps ahead forecasts?

      ii. Is there any step ahead forecasts over which the average model perform the best or the worst?

5. Consider the following weakly stationary autoregressive (AR) process:

   (1) $y_t = 1 + 0.5y_{t-2} + u_t$

   where u_t are identically independently distributed with mean zero and variance one. Let $y^f_{t+\tau|t}$ be the optimal forecast of $y_{t+\tau}$ given the information available at time t, denoted by I_t.

   (a) Show that $y^f_{t+\tau|t} = 1 + 0.5y^f_{t+\tau-2|t}$.

   (b) Show that $e^f_{t+\tau|t} = 0.5e^f_{t+\tau-2|t} + u_{t+\tau}$, where $e^f_{t+\tau|t}$ is the corresponding optimal forecast error of $y_{t+\tau}$ given the information available at time t.

   (c) Given equation (1), what is the value of $y^f_{t+\tau|t}$ as $\tau \to \infty$.

   (d) Given equation (1), what is the variance of $e^f_{t+\tau|t}$ as $\tau \to \infty$.