PROBLEM SET 2

Due: Tuesday January 30th

1. Which of the following autoregressive processes are weakly stationary?

   (a) $y_t = 3 - 0.3y_{t-1} + 0.04y_{t-2} + \varepsilon_t$,

   (b) $y_t = 5 + 3.1y_{t-1} - 0.03y_{t-2} + \varepsilon_t$,

   (c) $y_t = 2.5y_{t-1} - 2y_{t-2} + 0.5y_{t-3} + \varepsilon_t$,

   where $\varepsilon_t \sim \text{iid}(0, \sigma^2)$.

2. Consider the following weakly stationary AR(3) process,

   $$y_t = 0.5 + 0.6y_{t-3} + v_t,$$

   where $v_t \sim \text{iid}(0, 1)$.

   (a) Show that $\mathbb{E}(y_t) = 1.25$.

   (b) Let $\gamma(k) = \text{cov}(y_t, y_{t-k})$. show $\gamma(0) = \frac{1}{0.64} = 1.5625$.

   (c) Show that $\gamma(1) = 0.6\gamma(2)$.

   (d) Show that $\gamma(2) = 0.6\gamma(1)$.

   (e) Use the results from parts (c) and (d) to show that $\gamma(1) = \gamma(2) = 0$.

   (f) Show that $\gamma(3) = 0.6\gamma(0)$.

   (g) We can show that in general $\gamma(k) = 0.6\gamma(k-3)$ for all $k = 3, 4, 5, \cdots$. Use this result and the result of parts (e) and (f) to show

   $$\rho(k) = \begin{cases} 0.6^{k/3} & \text{for } k = 3, 6, 9, \cdots, \\ 0 & \textit{otherwise}, \end{cases}$$

   where

   $$\rho(k) = \frac{\gamma(k)}{\gamma(0)}.$$

3. Consider the following time-series process for y_t, $t = 1, 2, \cdots,$

$$y_t = 2(u_t - \varepsilon_t) + 0.5u_{t-1} \text{ for } t = 1, 2, \cdots .$$

Suppose

$$\mathbb{E}(u_t) = \mathbb{E}(\varepsilon_t) = 0 \text{ and } \mathrm{var}(u_t) = \mathrm{var}(\varepsilon_t) = 1 \text{ for all } t.$$

Moreover, assume that

$$\mathrm{cov}(u_t, \varepsilon_t) = -0.5 \text{ for all } t, \text{ and } \mathrm{cov}(u_t, \varepsilon_{t'}) = 0 \text{ for all } t \neq t'.$$

(a) Show that $\mathbb{E}(y_t) = 0$.

(b) Show that $\mathrm{var}(y_t) = 12.25$.

(c) Show that $\mathrm{cov}(y_t, y_{t-1}) = 1.5$.

(d) Show that $\mathrm{cov}(y_t, y_{t-2}) = 0$.

(e) Given the result of part (d), what do you expect about $\mathrm{cov}(y_t, y_{t-k})$ for $k = 3, 4, \cdots$.

(f) Given the result of parts (a)-(e), can you conclude that the process is weakly stationary? Explain why?