Solution for Problem Set 1

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1. Refer to Lecture Note 2, Pages 3-4
2. Refer to Lecture Note 2, Pages 4-5.
3.

The following lines call the required packages and source the required local functions for this problem set

```
library("quantmod")
library("fBasics")
source("r_functions/model_selection_function.R")
source("r_functions/ols_function.R")
source("r_functions/t_test_function.R")
```

Now, we start with answering the questions.

(a) We can use the getSymbols command to fetch the data from Fred as

```
getSymbols(Symbols ="GPDIC1",src = "FRED") # Download Quarterly data for Domestic Investment
```

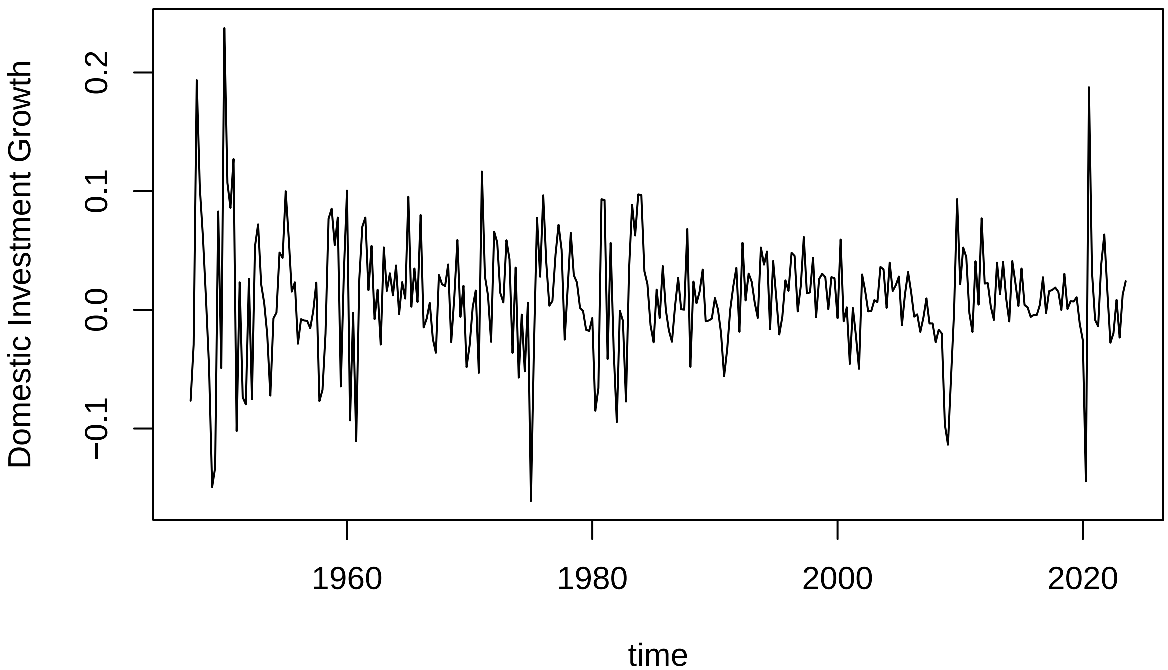
```
## [1] "GPDIC1"
```

```
keep_data <- seq(from = as.Date("1947-01-01"), to = as.Date("2024-01-01"), by = "quarter")
ID_all = as.matrix(GPDIC1[,1])
ID = as.matrix(ID_all[as.Date(rownames(ID_all)) %in% keep_data,])
```

(b)

```
GID <- diff(ID)/ID[1:(dim(ID)[1]-1),]
colnames(GID) = "Domestic Investment Growth"
n_obs = dim(GID)[1]
GID_date = as.Date(row.names(GID))

plot(x = GID_date, y = GID,xlab='time',ylab='Domestic Investment Growth',type='l',col="black")
```

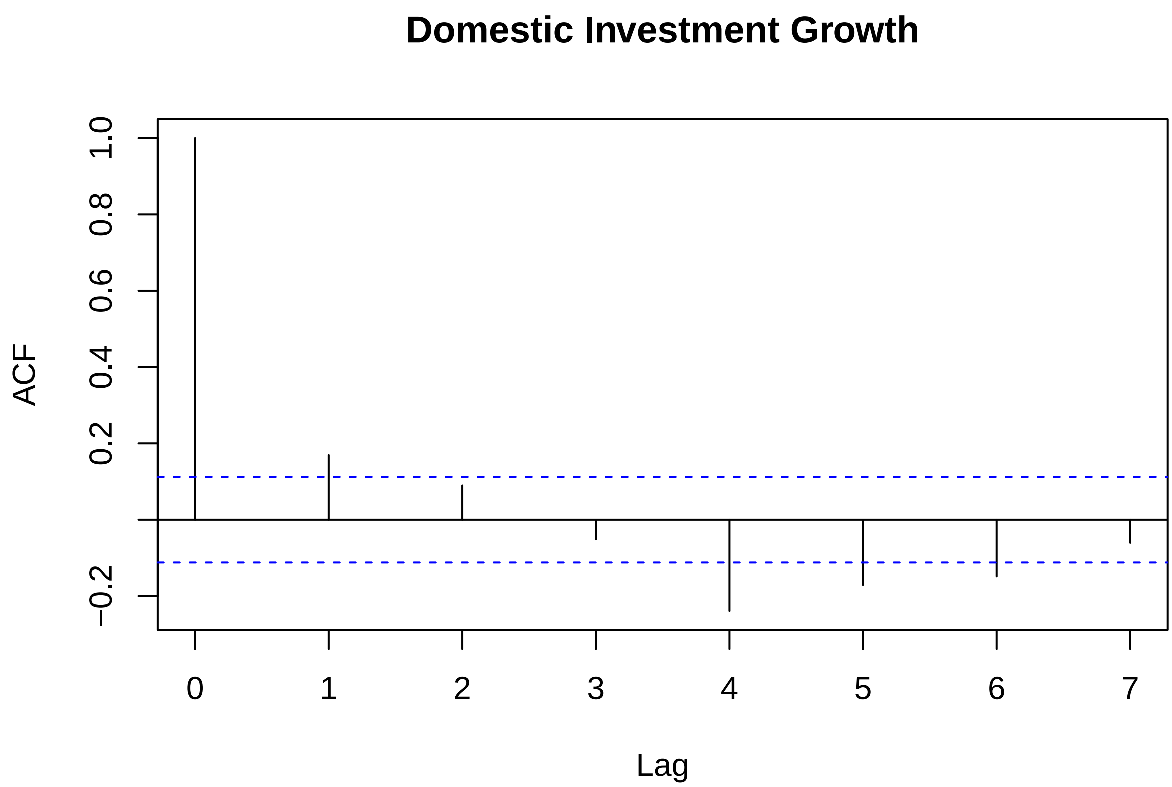



```
basicStats(GID)
```

```
##                 Domestic.Investment.Growth
## nobs                         306.000000
## NAs                            0.000000
## Minimum                       -0.161079
## Maximum                        0.237403
## 1. Quartile                   -0.009684
## 3. Quartile                    0.034516
## Mean                           0.010692
## Median                         0.009620
## Sum                            3.271648
## SE Mean                        0.002799
## LCL Mean                       0.005183
## UCL Mean                       0.016200
## Variance                       0.002398
## Stdev                          0.048966
## Skewness                       0.133155
## Kurtosis                       2.950689
```

(c)

```
acf(GID,lag=round(n_obs^(1/3))) # command to obtain sample ACF of the data
```



Yes, since the computed sample autocorrelation of order 1,4 and 5 are significantly different from zero.

(d)

```
Box.test(GID, lag = round(n_obs^(1/3)), type = "Ljung-Box") # applying Ljung and Box (1978) joint test
```

```
##
##  Box-Ljung test
##
## data:  GID
## X-squared = 47.288, df = 7, p-value = 4.905e-08
```

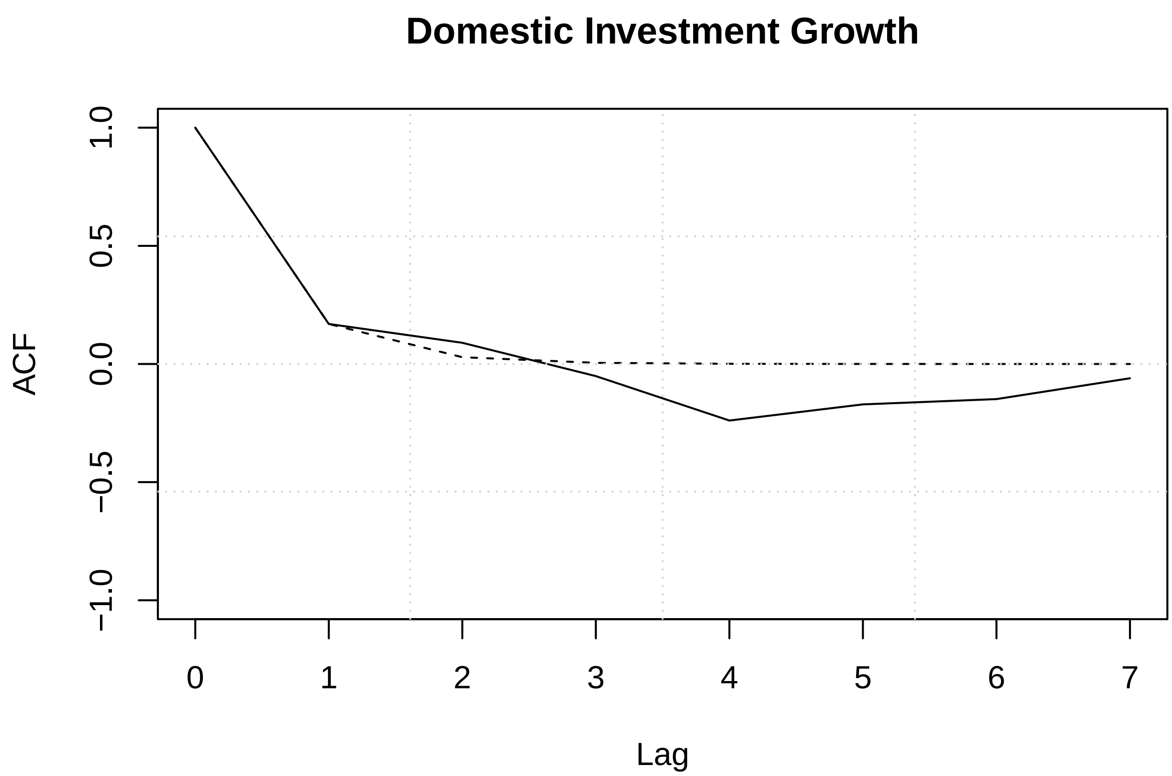
As expected the Ljung and Box (1978) test reject the null that there exists no serial correlation.

(e)

```
num_lags = 1
lags_GID = matrix(NA,nrow = n_obs, ncol = num_lags)
for (i in 1:num_lags) {
  lags_GID[(i+1):n_obs,i] = as.matrix(GID[1:(n_obs-i),1])
}
intercept = matrix(1,n_obs)
X = cbind(intercept,lags_GID)
y = GID
reg_result = ols(X[(num_lags+1):n_obs,],as.matrix(y[(num_lags+1):n_obs,1]))
beta_hat = reg_result$beta_hat

ar_coeff <- as.numeric(beta_hat[2:(num_lags+1)])
ma_coeff <- 0
```

```r
ACF = acf(GID,lag=round(n_obs^(1/3)),plot = FALSE) # command to obtain sample ACF of the data
TACF <- ARMAacf(ar_coeff, ma_coeff, lag.max = round(n_obs^(1/3))) # command to obtain theoretical ACF
plot(c(0:round(n_obs^(1/3))),ACF$acf,type='l',xlab='Lag',
     ylab='ACF',ylim=c(-1,1),main = "Domestic Investment Growth")
lines(0:round(n_obs^(1/3)),TACF,lty=2)
grid(nx = 4, ny = 4)
```



As can be seen in the above figure, the empirical autocorrelation function of Part 3 is not close to the selected theoretical autocorrelation function. This suggests that we should probably consider more number of lags, $p > 1$, for the AR(p) model to analyse the Domestic Investment Growth.