PROBLEM SET 1

Due: Tuesday January 23

1. Let $\{x_t \text{ for } t = \cdots, -2, -1, 0, 1, 2, ...\}$ be a time series process.
   (a) When we say x_t is weakly stationary?
   (b) Suppose $x_t = \sum_{j=0}^{4} \alpha_j \varepsilon_{t-j}$, where $\varepsilon_{t'} \sim \text{iid}(0, \sigma_\varepsilon^2)$ for all t'. Show that x_t is weakly stationary.
   (c) Now suppose that $x_t = \sum_{j=0}^{n} \alpha_j \varepsilon_{t-j}$, where $\varepsilon_{t'} \sim \text{iid}(0, \sigma_\varepsilon^2)$ for all t' and n is a positive constant. Show that x_t is weakly stationary.
   (d) Now suppose that $x_t = \sum_{j=0}^{\infty} \alpha_j \varepsilon_{t-j}$, where $\varepsilon_{t'} \sim \text{iid}(0, \sigma_\varepsilon^2)$ for all t'. Show that x_t is weakly stationary if $\sum_{j=0}^{\infty} \alpha_j^2 \leq c < \infty$, where c is positive constant.

2. Consider the following AR(1) process,

$$y_t = 0.5 y_{t-1} + \varepsilon_t, \text{ where } \varepsilon_t \sim \text{iid}(0, 1).$$

   (a) Find coefficients $\{\alpha_j\}$ in the representation $y_t = \sum_{j=0}^{\infty} \alpha_j \varepsilon_{t-j}$. Use the result in part (d) of question 1 to conclude y_t is weakly stationary.
   (b) Find the mean of y_t.
   (c) Find the variance of y_t.
   (d) Find the auto-covariance function of y_t.

3. In this question we are going to analyze the time series process for real investment growth in the U.S.
   (a) Compile quarterly data for the U.S. real gross private domestic investment (DI) from 1947Q1 to 2023Q4.
   (b) Compute growth in quarterly DI (GDI), provide its summary statistics and plot the data.
   (c) Compute and plot empirical autocorrelation function. Given the plot, do you expect any time-series correlation among the observations? Explain why?

(d) Set the maximum number of lags to the integer closest to the number of observations to the power one-third. Perform a test for joint autocorrelation in GDI and report your result. Does your finding consistent with that of Part 3c? Explain why?

(e) Consider an AR(1) model and compute the theoretical autocorrelation function. Compare your findings with that of Part 3c.