ECON 2813: Global Economics and Finance — Spring 2024
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PRACTICE FINAL

Notations: In this exam, for a generic random variable y_t, $\mathbb{E}(y_t)$ and $\text{var}(y_t)$ stand for expected value and variance of y_t, respectively. Moreover $\text{cov}(y_t, y_{t-k})$ stands for covariance between y_t and y_{t-k}.

1. Consider the following AR process :

$$y_t = 0.5 + (1 - \phi_1 - \phi_2)t + \phi_1 y_{t-1} + \phi_2 y_{t-2} + u_t, \text{ where } u_t \sim iid(0, 1).$$

   (a) Suppose that $\phi_1 = \phi_2 = 0.5$. Show that the process has a unit root.

   (b) Now suppose that $\phi_1 = \phi_2 = 0.1$. Show that the process is weakly trend stationary. Find $\mathbb{E}[y_t]$ and $\text{var}(y_t)$.

   (c) Given your answers to part 1a and 1b, discuss the difference between a weakly trend stationary process and a unit root process.

   (d) Explain the steps required to perform an Augmented Dickey-Fuller test to examine the null hypothesis that a process has a unit root vs. the alternative hypothesis that the process is trend stationary.

2. Let $y_t = 1 + 0.5y_{t-1} + u_t - 0.5u_{t-1}$ where u_t are identically independently distributed with mean zero and variance one. Let $y^f_{t+\tau|t}$ be the optimal forecast of $y_{t+\tau}$ given the information available at time t, denoted by I_t.

   (a) Show that the process for y_t is weakly stationary.

   (b) Show that $y^f_{t+1|t} = 1 + 0.5y_t - 0.5u_t$.

   (c) Let $e^f_{t+1|t} = y_{t+1} - y^f_{t+1|t}$. Show that $\text{var}\left(e^f_{t+1|t}\right) = 1$.

   (d) What is $y^f_{t+\tau|t}$ as $\tau \to \infty$.

   (e) What is $\text{var}\left(e^f_{t+\tau|t}\right)$ as $\tau \to \infty$.