Hypothesis tests

Overview

Quick review of sampling distributions, confidence intervals and the bootstrap

Hypothesis tests for a single proportion

- Framework/terminology for hypothesis testing
- Hypothesis tests for a single proportion using randomization in R

Hypothesis tests for two means (if there is time)

- Randomization tests for comparing two means in R

Announcements

Homework 1 feedback has been posted

- Any regrade requests need to be made within a week
  - Official grades are on Gradescope
- Future homework:
  - Be sure to mark pages for questions on Gradescope
  - Deadlines are on the syllabus

Switching my office hours to 11:30am on Mondays and Wednesdays

Review: Creating confidence intervals

What are Confidence intervals?

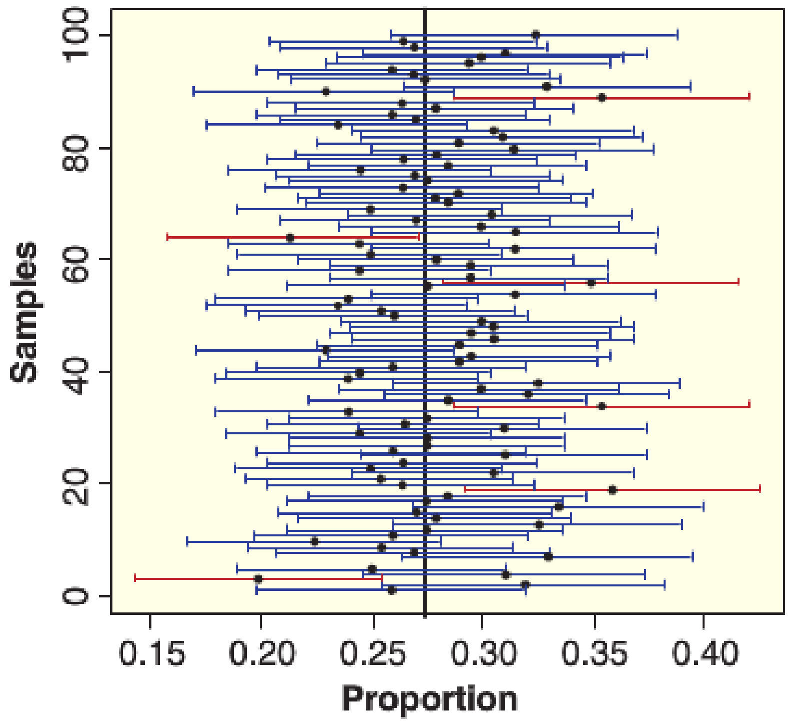
- Range of plausible values that capture the parameter a fixed % of the time

How can we create confidence intervals?

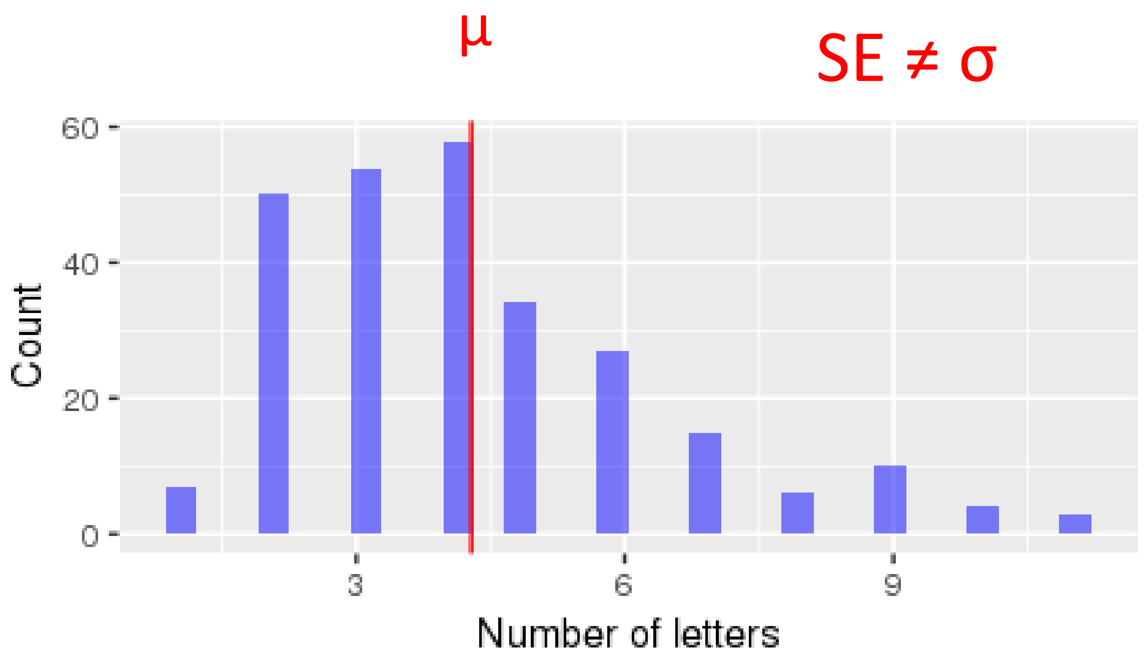
- Use the bootstrap (to estimate the SE)
- Use formulas (to estimate the SE)

μ

μ



Review of sampling distribution



SE ≠ σ

10, 3, 3, 3, 4,
3, 2, 6, 10, 5

$\bar{x} = 5$

SE ≠ s_1

2, 6, 2, 6, 6,
2, 5, 3, 2, 9

$\bar{x} = 4.3$

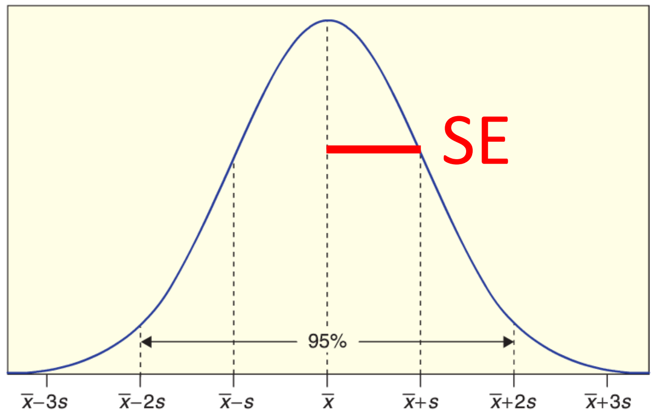
SE ≠ s_2

3, 9, 3, 4, 4,
3, 6, 6, 2, 2

$\bar{x} = 4.2$

SE ≠ s_3

The standard deviation of a sampling distribution is called the standard error (SE)

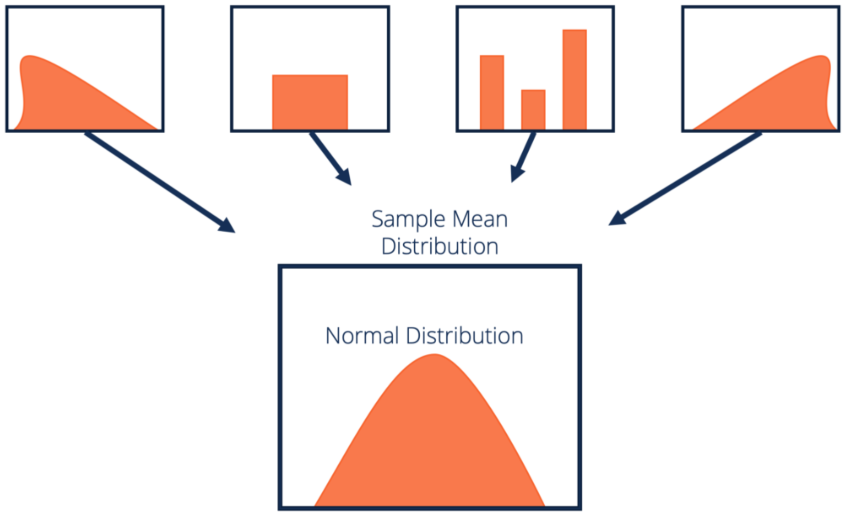


Sampling distribution!

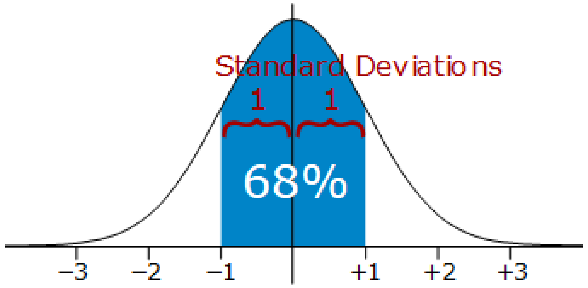
math “shortcut” for SEM = $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

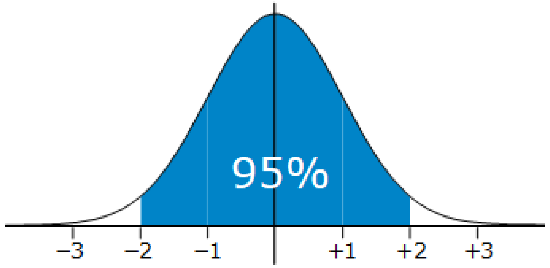
The central limit theorem

The **central limit theorem** establishes that when independent random variables are summed, the resulting statistic (sampling distribution) approaches a normal distribution as the sample size n increases.



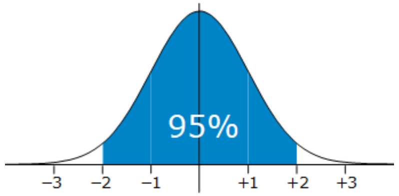
All these statistics, $\hat{p}$, $\bar{x}$, s, r, b, are sums of random data so their sampling distributions are normal





We can construct 95% confidence intervals using:

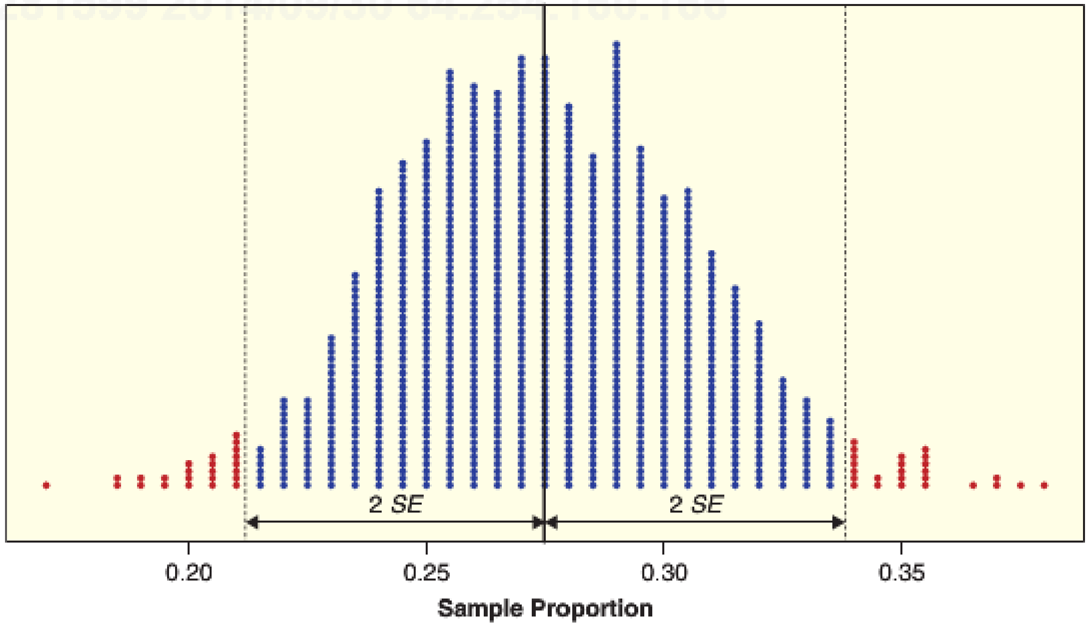
$$CI_{95} = \text{stat} \pm 2 \cdot SE$$



For example, a 95% confidence interval for the mean μ is:

$$\bar{x} \quad \pm \quad 2 \cdot SE$$

μ

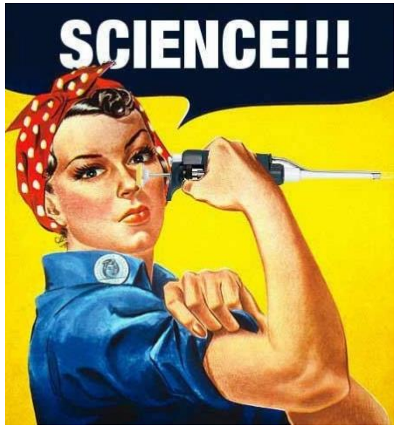


Why does this work?

2 · SE | 2 · SE

Confidence interval

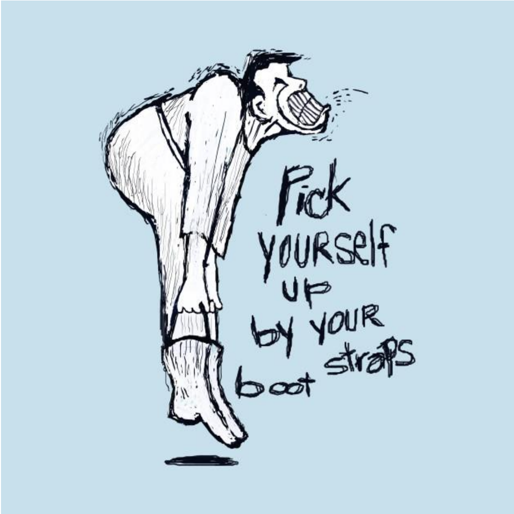
Using the bootstrap to estimate SE



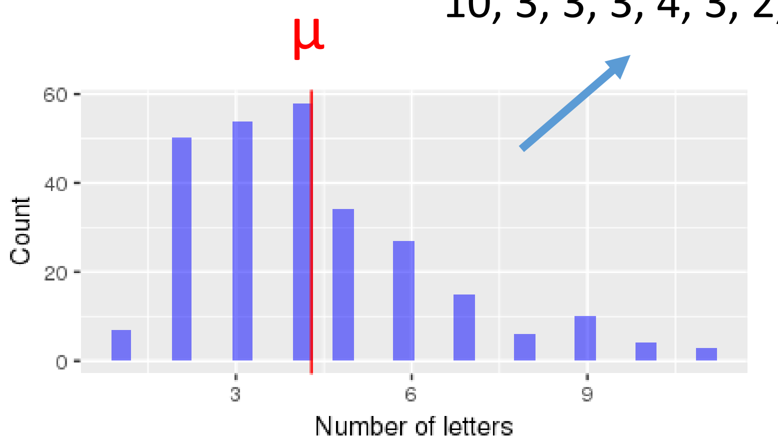
We can't calculate the sampling distribution by repeatedly sampling from the population ☹

Instead we nee do pick ourselves up from the bootstraps

1. Estimate SE with $\hat{SE}$ from a single sample
2. Then use $\bar{x} \pm 2 \cdot \hat{SE}$ to get the 95% CI



Bootstrap distribution illustration



<u>The sample (n = 10)</u>
10, 3, 3, 3, 4, 3, 2, 6, 4, 5

3, 3, 3, 5, 3,
4, 5, 2, 2, 10

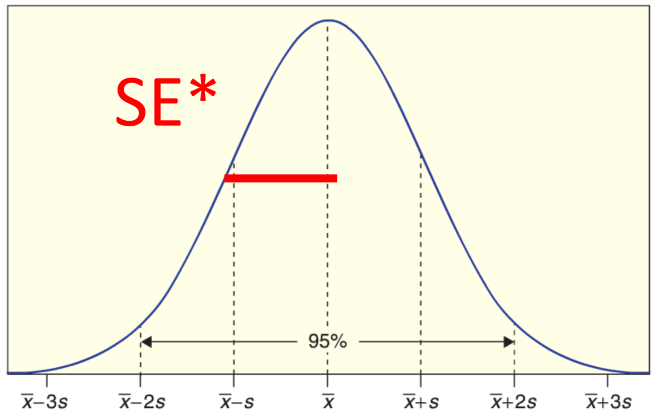
$\bar{x}^* = 4$

3, 3, 2, 3, 6,
4, 6, 5, 3, 6

$\bar{x}^* = 4.1$

5, 3, 2, 3, 3,
3, 10, 3, 4, 3

$\bar{x}^* = 3.9$



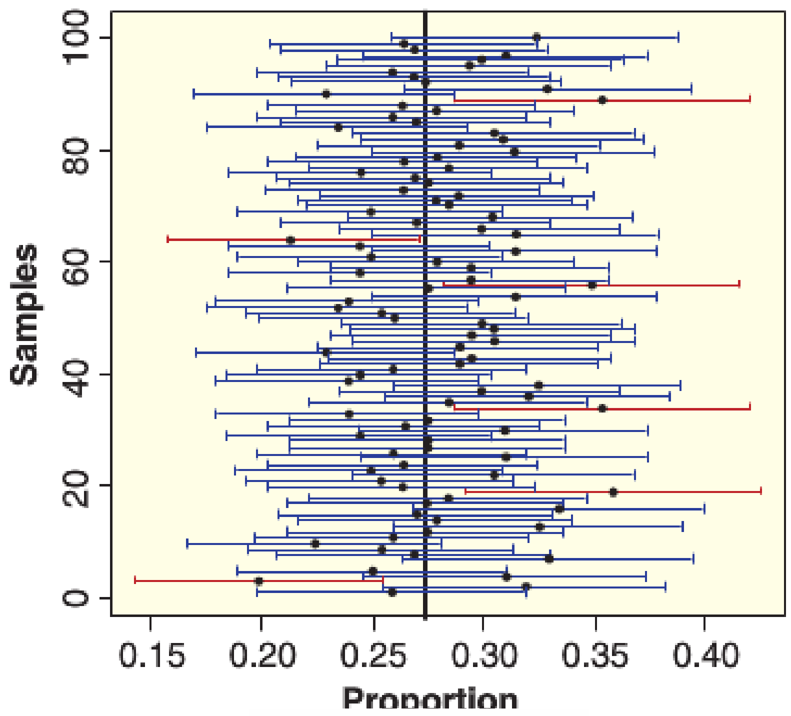
Bootstrap distribution!

Notice there is no 9's in the bootstrap samples

95% Confidence Intervals

We can estimate a 95% confidence interval using: (provided the bootstrap distribution is reasonably normal)

$$\textit{Statistic} \;\pm\; 2 \cdot SE^*$$



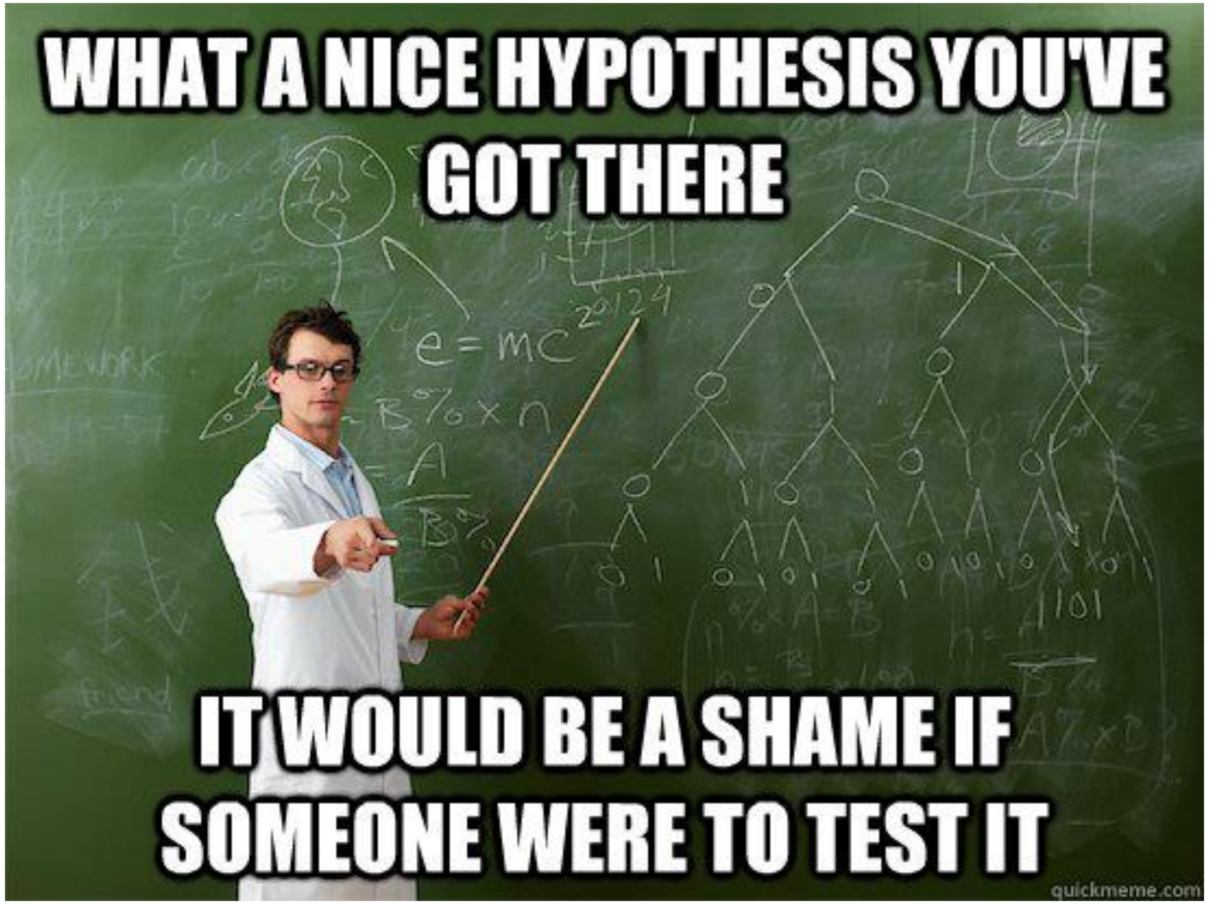
Where SE* is the standard error estimated using the bootstrap

Q: Why use the bootstrap instead of…?

$$\mathrm{CI}_{95} = \bar{x} \;\pm\; 2 \cdot \frac{s}{\sqrt{n}}$$

Questions?

Hypothesis tests



Overview

Assuming you are familiar with hypothesis tests from Intro Stats

- Particularly parametric hypothesis tests, such as the t-test

Quick review concepts of hypothesis test

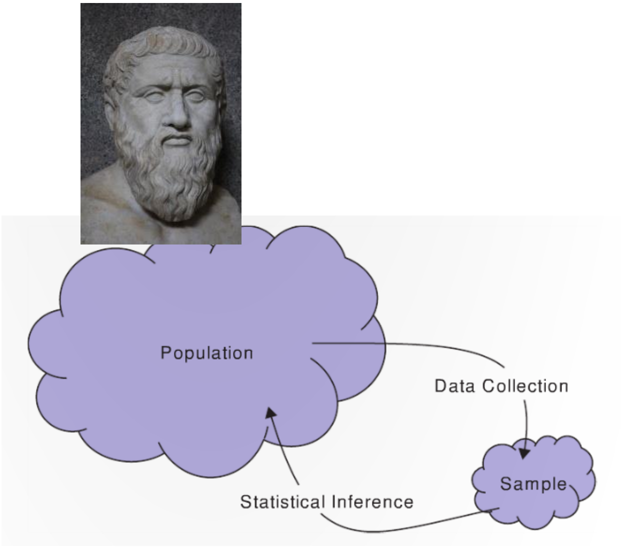
Introduce computational methods for hypothesis tests that use randomization

- These methods make fewer “assumptions” than parametric methods, so they can potentially work in more situations

t-tests	94 respondents	80 %
confidence intervals	108 respondents	92 %
the bootstrap	18 respondents	15 %
permutation tests	18 respondents	15 %
one-way ANOVA	38 respondents	32 %
multiple regression	42 respondents	36 %
logistic regression	39 respondents	33 %
sampling distributions	66 respondents	56 %
None of the above	6 respondents	5 %
No Answer	1 respondents	1 %

Statistical tests   (hypothesis test)

A **statistical test** uses data from a sample to assess a claim about a population



<u>Example 1</u>: we might make the claim that Biden's approval rating for all US citizens is 43%

How can we write this using symbols?

- $\pi = .43$

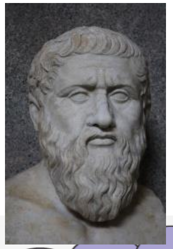
Statistical tests   (hypothesis test)

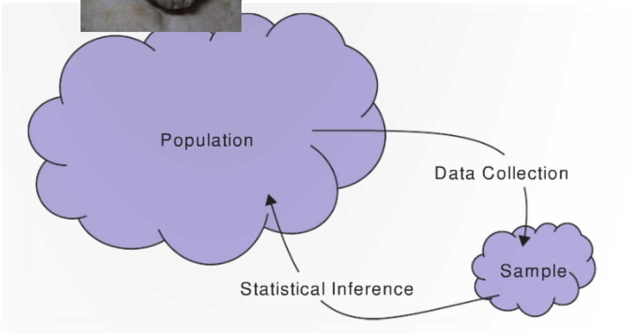
A **statistical test** uses data from a sample to assess a claim about a population

<u>Example 2</u>: we might make the claim that the average height of a baseball player is 72 inches

How can we write this using symbols?

- $\mu = 72$



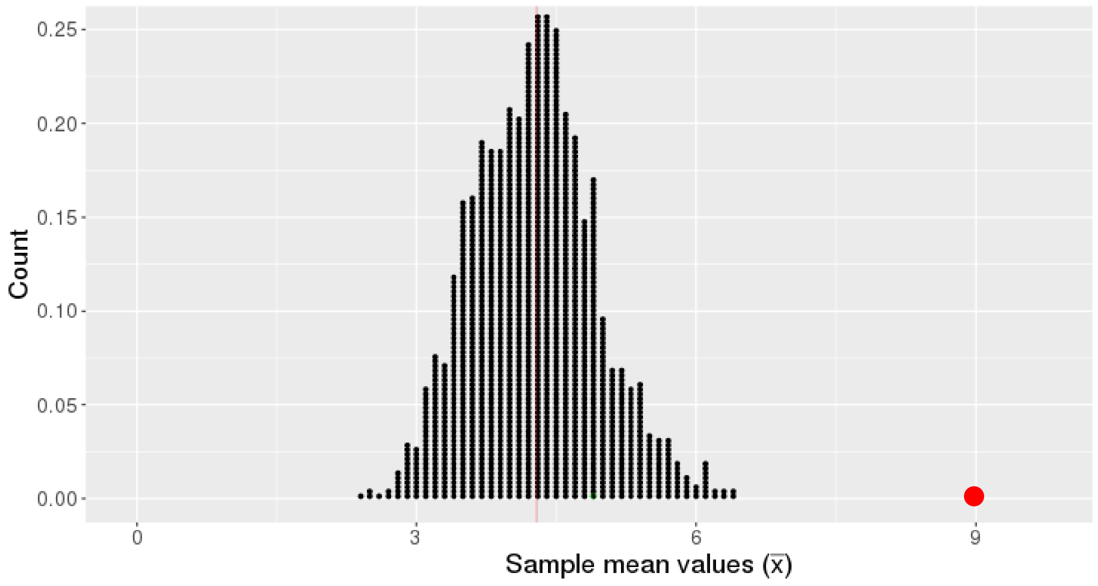


Basic hypothesis test logic

We start with a claim about a population parameter

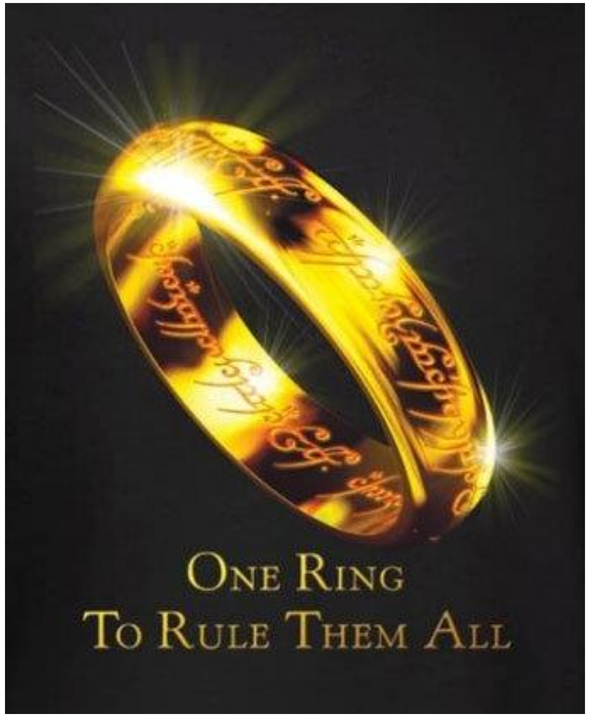
- E.g., μ = ~~4.2~~

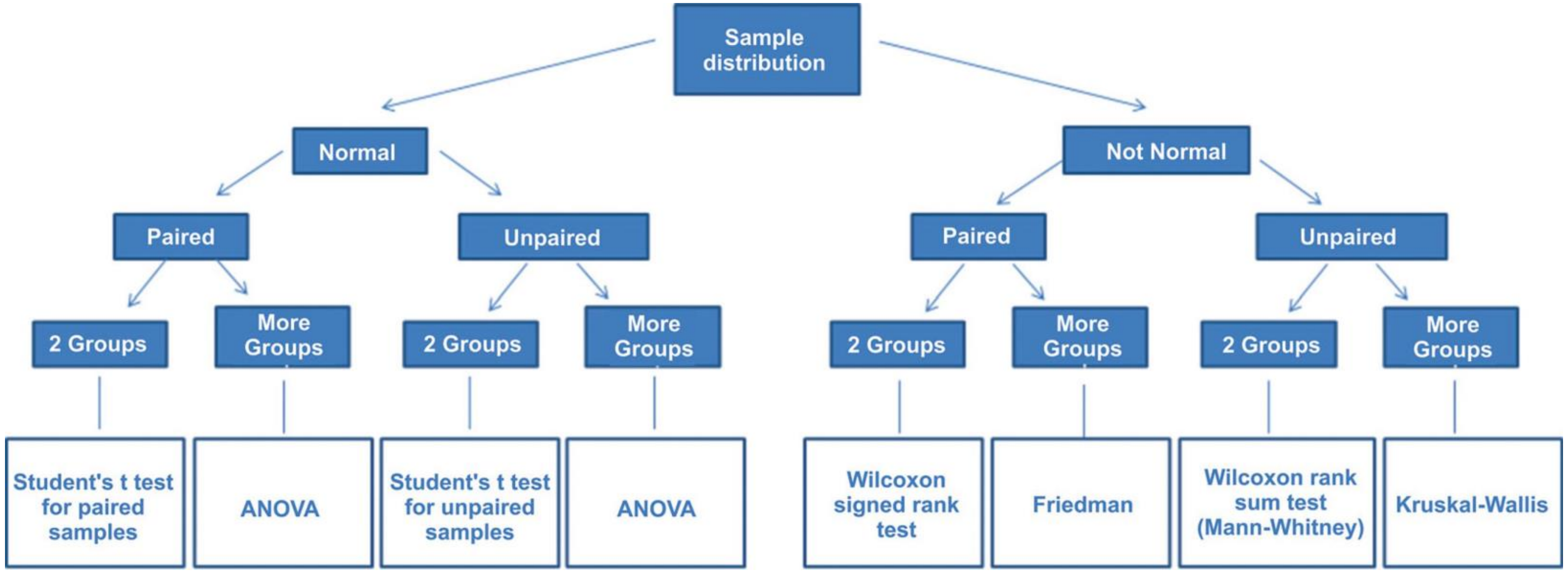
This claim implies we should get a certain distribution of statistics



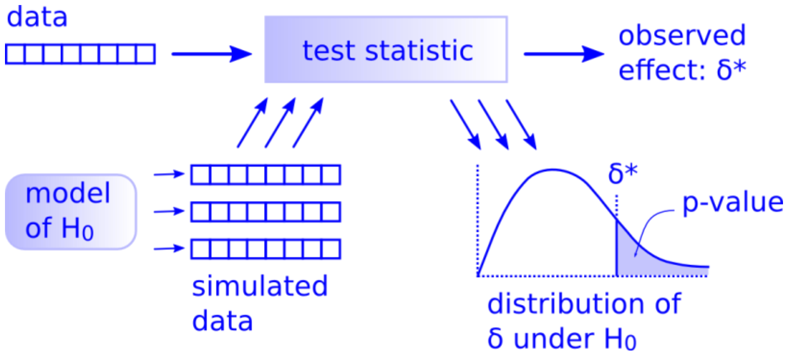
If our observed statistic is highly unlikely, we reject the claim

The big picture: There is only one hypothesis test!





Just need to follow 5 steps!



Example: Is it possible to smell whether someone has Parkinson’s disease?

Joy Milne claimed to have the ability to smell whether someone had Parkinson’s disease

To test this claim researchers gave Joy 6 shirts that had been worn by people who had Parkinson’s disease and 6 shirts by people who did not

Joy identified 11 out of the 12 shirts correctly

Questions about the experiment

1. What are the cases in this experiment?
    - Hint: draw out what the data would look like

2-3. What is the variable of interest, and is it categorical or quantitative?

4-5. What is the observed statistic - and what symbols should we use to denote it?

6. What is the population parameter we are trying to estimate, and what symbol should we use to denote it?

7. Do you think the results are due to chance?
    - i.e., do you think Joy got 11 correct answers by guessing?

8. Do you believe Joy can really smell whether someone has Parkinson's disease?

Questions about the experiment

1. What are the cases in this experiment?
    - A: Each case is trial where Joy had to smell one shirt

2-3. What is the variable of interest, and is it categorical or quantitative?

- A: The variable of interest is whether Joy correctly identified whether a shirt was worn by someone who had Parkinson’s disease.
- A: It is categorical (correct or incorrect)

4-5. What is the observed statistic - and what symbols should we use to denote it?

- A: The observed statistic is the proportion of shirts Joy correctly identified
- A: The symbols we use to denote this statistic is $\hat{p}$

Questions about the experiment

6-7. What is the population parameter we are trying to estimate, and what symbol should we use to denote it?

- A: The population parameter is how many shirts she would have correctly determined that were worn by people with/without Parkinson's disease if she had to smell infinitely many shirts.
- A: The symbol we would use to denote the population parameter is π

8. Do you think the results are due to chance?

- A: Opinions may vary. We will do more analyses to quantify this!

9. Do you believe Joy can really smell whether someone has Parkinson's disease?

- A: Opinions may vary, but it's good to have an opinion going in!

Smelling Parkinson's disease

If Joy was just guessing, what would we expect the value of the parameter to be?

$$\pi = 0.5$$

If Joy was not guessing, what would we expect the value of the parameter to be?

$$\pi > 0.5$$

Chance models

How can we assess whether 11 out of 12 correct trials ($\hat{p}$ = .916) is beyond what we would expect by chance?

If Joy was guessing, we can model his guesses as a coin flip:

Heads = correct guess

Tails = incorrect guess

We could flip 12 coins and see if we get 11 heads

Chance models

To really be sure, we should repeat flipping a coin 12 many times.

Any ideas how to do this?

Flipping coins in R

We can simulate coin flipping using the rbinom() function

```
flip_simulations <- rbinom(num_sims, size, prob)
```

num_sims: the number of simulations run

- Typically we do around 10,000 repeats

size: the number of trials on each simulation

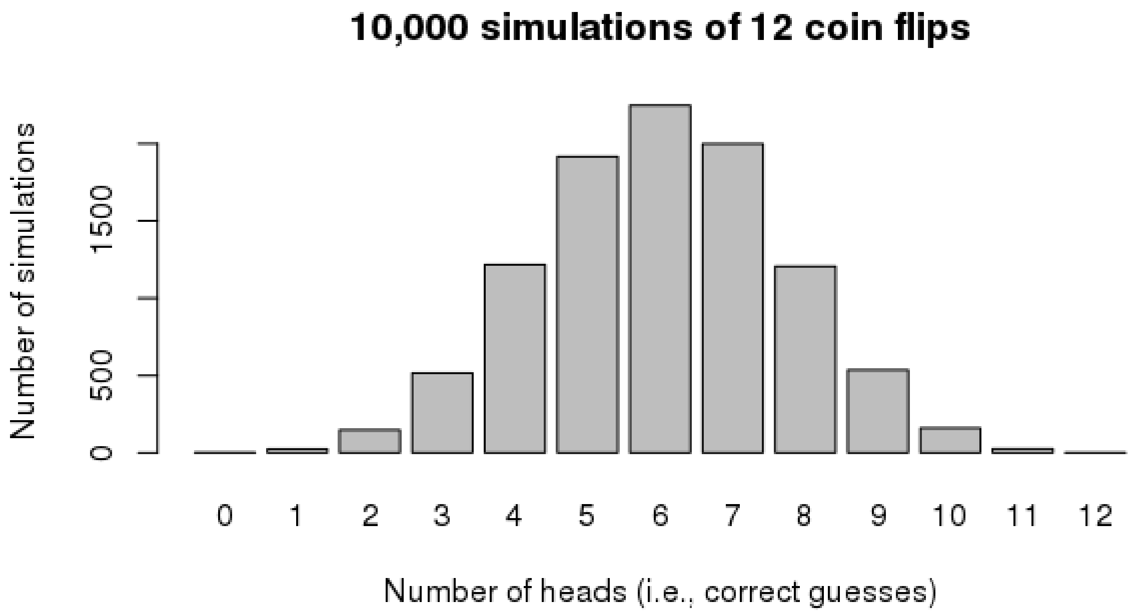
- 12 for simulating Joy's guesses

prob: the probability of success on each trial

- .5 if Joy was guessing

Simulating Flipping 12 coins 10,000

0	2
1	26
2	147
3	558
4	1269
5	1967
6	2310
7	1843
8	1142
9	537
10	162
11	33
12	4



Is it likely that Joy was guessing?

Simulating Flipping 12 coins 10,000

0	2
1	26
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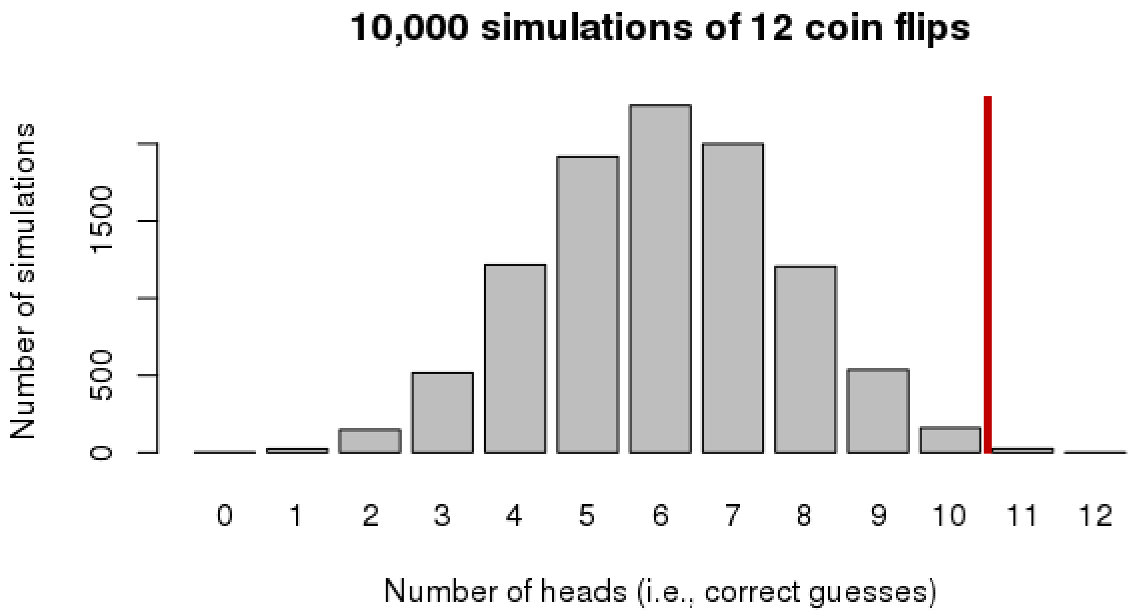


Is it likely that Joy was guessing?

Do you believe Joy can really smell whether someone has Parkinson’s disease?

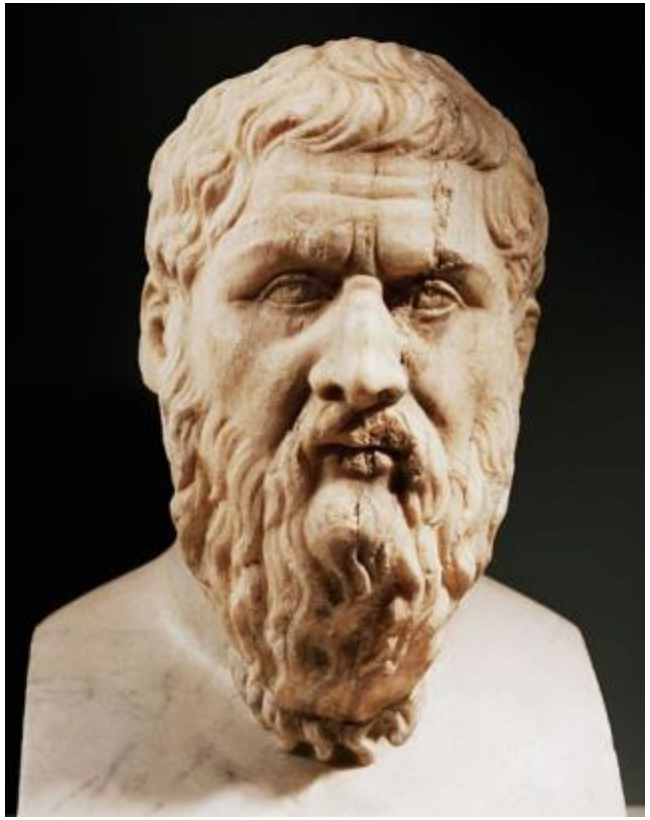
Is it possible to smell whether someone has Parkinson's disease?

We will examine this in R in a minute…

But first, let's review hypothesis testing terminology

Review of terminology and the 5 steps of hypothesis tests

Question: who is this?



A: Gorgias

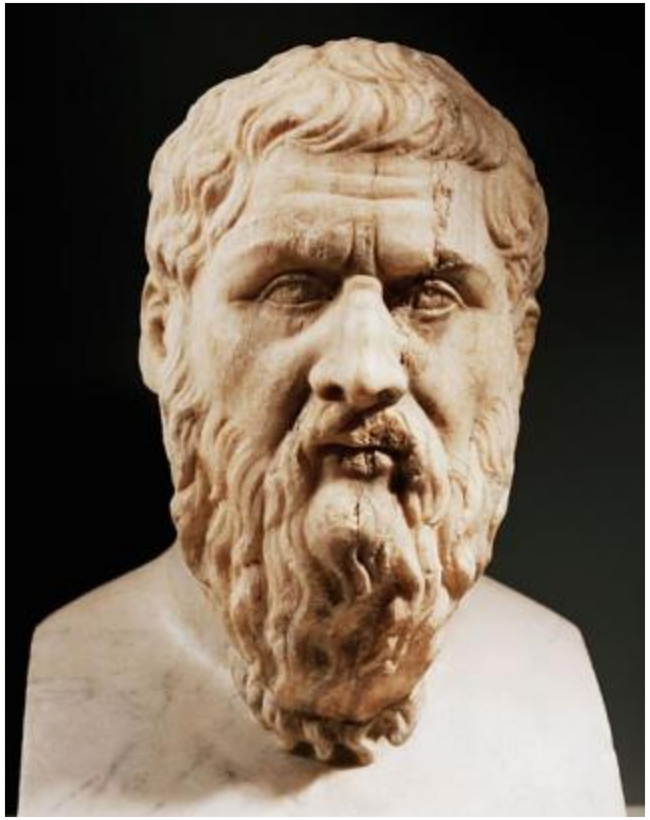
Question: Who is Gorgias?

A: a skeptic

Question: Does Gorgias believe Joy can smell Parkinson's disease?

A: No!

Question: who is this?



Gorgias believes in the ***null hypothesis***

- that Joy was guessing

How can we write the null hypothesis in symbols?

H_0: $\pi = 0.5$

We believe in the ***alternative hypothesis***

- Joy can smell Parkinson's disease

How can we write the alternative hypothesis in symbols?

H_A: $\pi > 0.5$

Question: who is this?

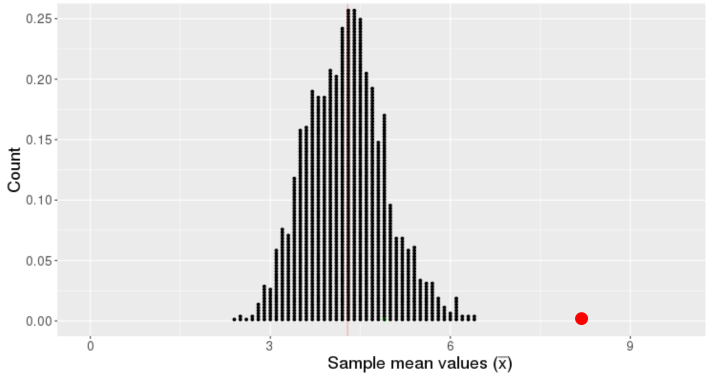
To prove Gorgias wrong, we will start by assuming he is right!

Namely, we will assume H_0 (that $\pi = 0.5$)

We will then generate a number of statistics ($\hat{p}$) that are consistent with H_0

- i.e., we will create a ***null distribution***

If our observed statistic looks very different from the statistics generated under we can reject H_0 and accept H_A

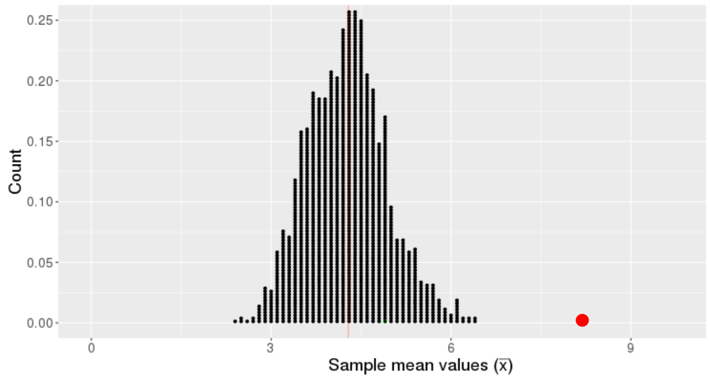


Terminology

Null Hypothesis ( ): Claim that there is no effect or no difference

Alternative Hypothesis (H_A): Claim for which we seek significant evidence

The alternative hypothesis is established <u>by observing evidence that inconsistent with the null hypothesis</u>



Review: the Joy smelling Parkinson's disease

1. What is the null hypothesis?

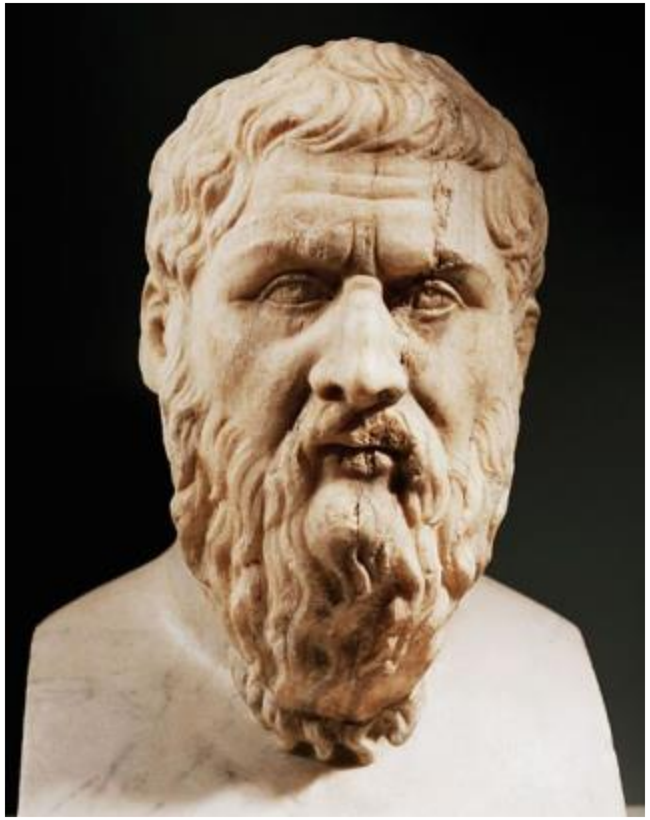
2. We can write this in terms of the population parameter as:

H_0: $\pi = 0.5$

3. What is the alternative hypothesis?

H_A: $\pi > 0.5$

Setting the rules

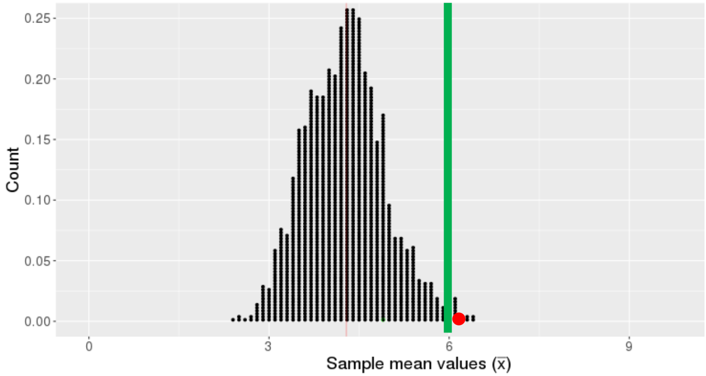


<u>Life wisdom:</u> If you are going to make a bet with a nihilist, you'd better agree to the rules first!

<u>Rules</u>

- If there is a less than 5% chance we would get a random statistic as or more extreme than the observed statistic (if H_0 is true) we will reject H_0
  - i.e., Gorgias loses the bet

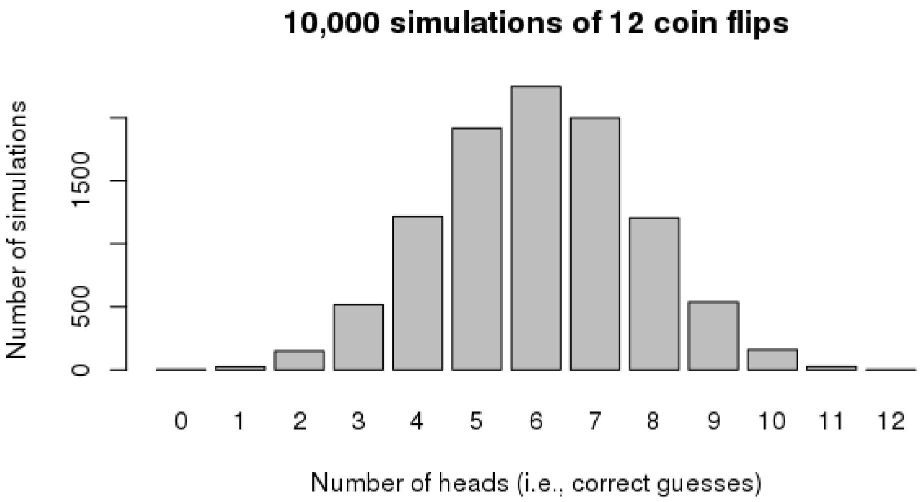
- In symbols: $\alpha = 0.05$



Null Distribution

A **null distribution** is the distribution of statistics one would expect if the null hypothesis (H_0) was true

i.e., the null distribution is the statistics one would expect to get if nothing interesting was happening



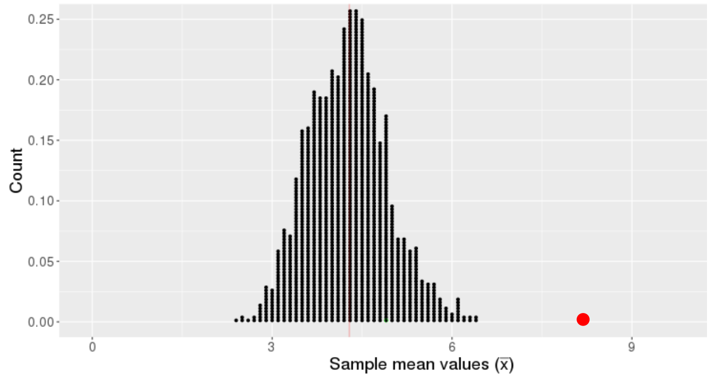
P-values

A **p-value** is the probability, of obtaining a statistic as (or more) extreme than the observed sample ***if the null hypothesis was true***

- i.e., the probability that we would get a statistic as extreme as our <u>observed statistic</u> from the <u>null distribution</u>

$$\Pr(\text{STAT} \geq \text{observed statistic} \mid H_0 = \text{True})$$

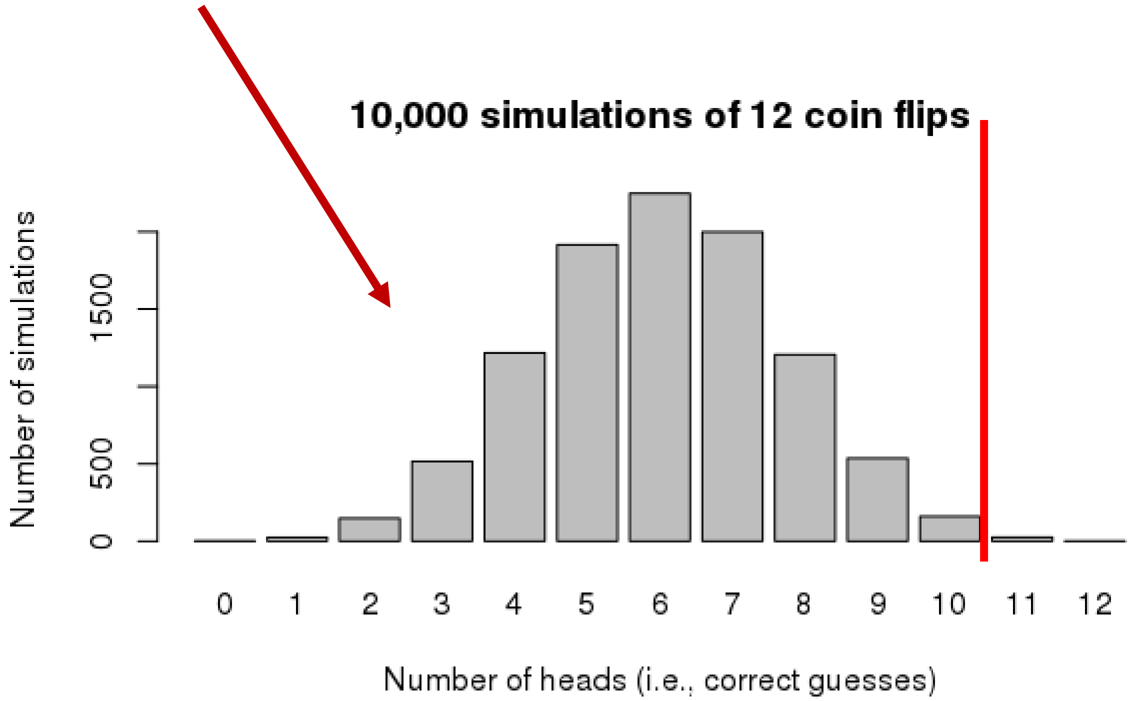
The smaller the p-value, the stronger the statistic evidence is against the null hypothesis



Joy example

0	2
1	26
2	147
3	558
4	1269
5	1967
6	2310
7	1843
8	1142
9	537
10	162
11	33
12	4

Null distribution



p-value = 37/10000 = .0037

Statistical significance

When our observed sample statistic is unlikely to co[illegible] the null distribution, people often say the results are **statistica[illegible]gnificant**

- i.e., our p-value is less than α
- i.e., Gorgias lost the bet!

'Statistically significant' results mean we have strong evidence against H_0 in favor of H_a

- The American Statistical Association rejects the phrase 'statistically significant'

5 steps for testing hypotheses

1. State the null hypothesis… and the alternative hypothesis
   - Joy was just guessing so the results are due to chance: H_0: $\pi = 0.5$
   - Joy is getting more correct results than expected by chance: H_A: $\pi > 0.5$

2. Calculate the observed statistic (and visualize the data)
   - Joy got 11 out of 12 guesses correct, or $\hat{p} = .917$

3. Create a null distribution that is consistent with the null hypothesis
   - i.e., what statistics would we expect if Joy was just guessing

4. Examine how likely the observed statistic is to come from the null distribution
   - What is the probability that the Joy would guess 11 or more correct?
   - i.e., what is the p-value

5. Make a judgement
   - If we have a small p-value, this means that $\pi = .5$ is unlikely and so $\pi > .5$
   - i.e., we could say our results are ‘statistically significant’

Is it possible to smell whether someone has Parkinson's disease?

Let's examine this in R!