Parametric hypothesis tests

Overview

Tests for two means

- Randomization tests for two means revisited using a t-statistic
- Parametric tests and t-tests

Theories of hypothesis testing

Announcements

Ed discussions

- Use it!
  - Your class participation grade (3%) based on asking and answering questions
- But do not post full code/answers on Ed Discussion!

Get started early on homework 4, it might be a bit longer than the other homework

Next programming review will be from 4-5pm next Monday

Technical issues recording lectures the last two classes

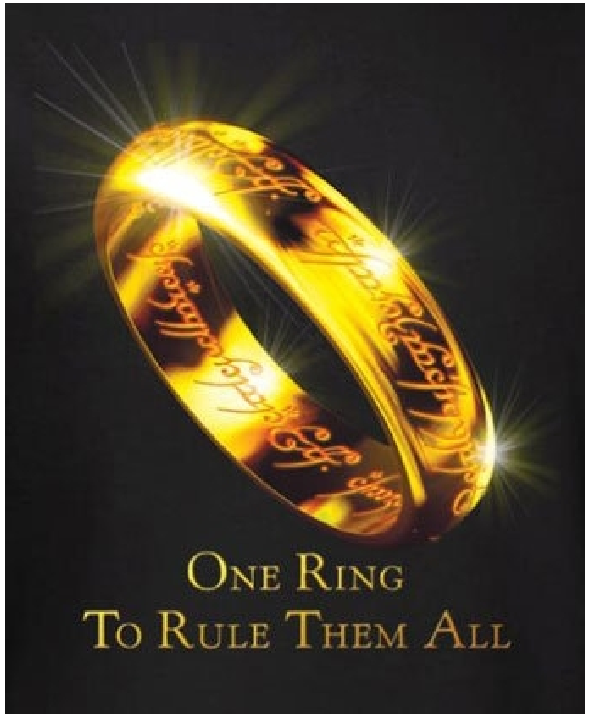
- I posted lecture material from last year and audio recordings
- Hopefully will be fixed this class!

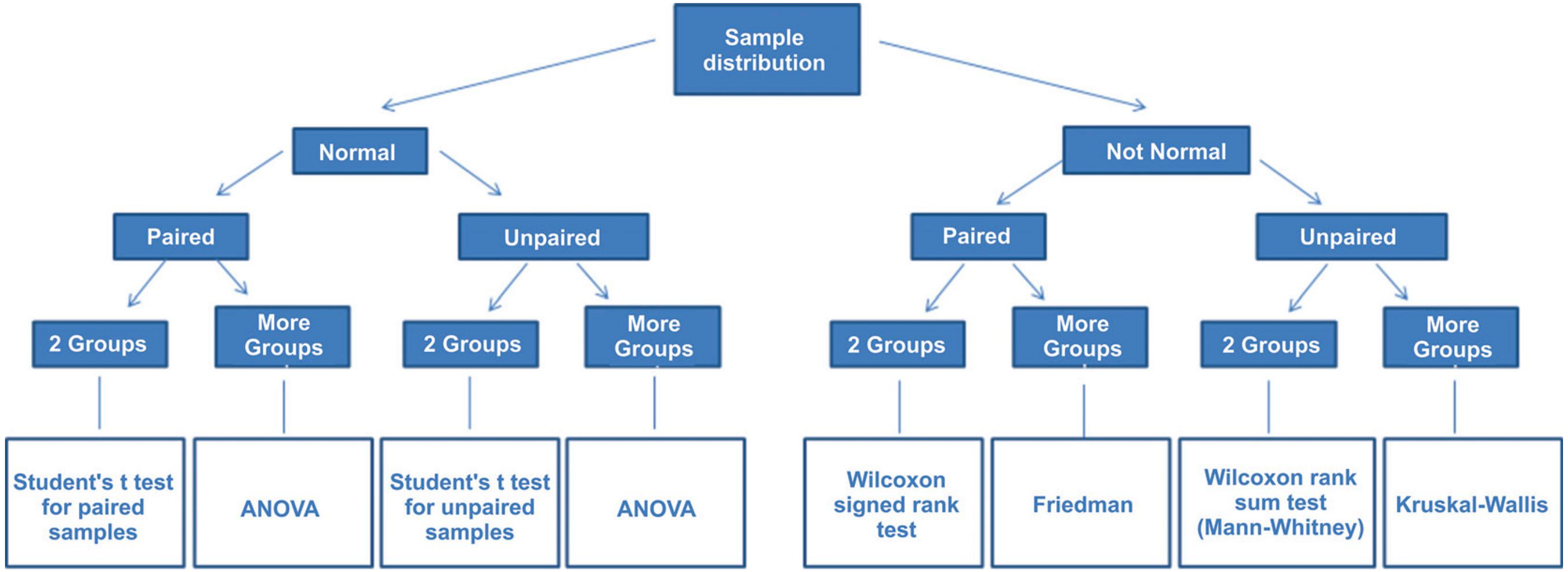
Where we are in the plan for the semester

			Analysis	R
1	Sep 2	Course overview, introduction to R, descriptive statistics		base R
2	Sep 7-9	Review of central statistical concepts and exploratory analysis using R	resampling methods	
3	Sep 14-16	Confidence Intervals and the bootstrap		
4	Sep 21-23	Review of hypothesis tests and permutation tests in R		
5	Sep 28-30	Parametric hypothesis tests and theories of hypothesis testing		

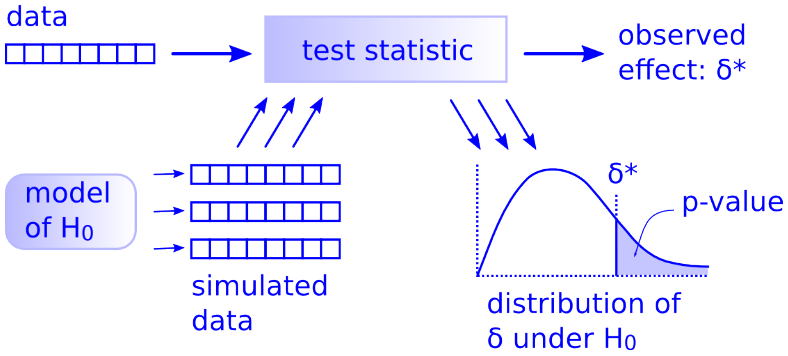
t-tests	94 respondents	80 %
confidence intervals	108 respondents	92 %
the bootstrap	18 respondents	15 %
permutation tests	18 respondents	15 %
one-way ANOVA	38 respondents	32 %

The big picture: There is only one hypothesis test!

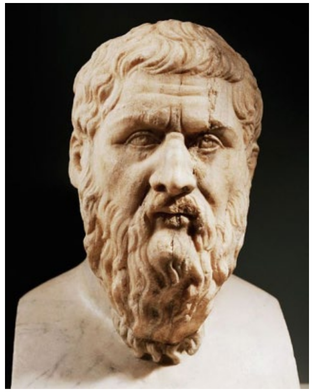




Just need to follow 5 steps!



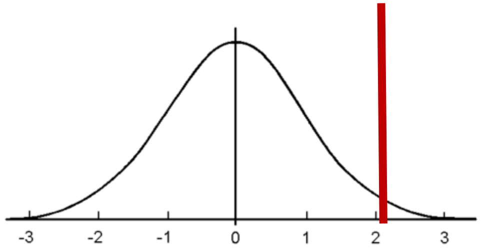
Five steps of hypothesis testing



1. State H_0 and H_A
    - Assume Gorgias (H_0) was right
    - α = .05 of the time he will be right, but we will say he is wrong

2. Calculate the actual observed statistic

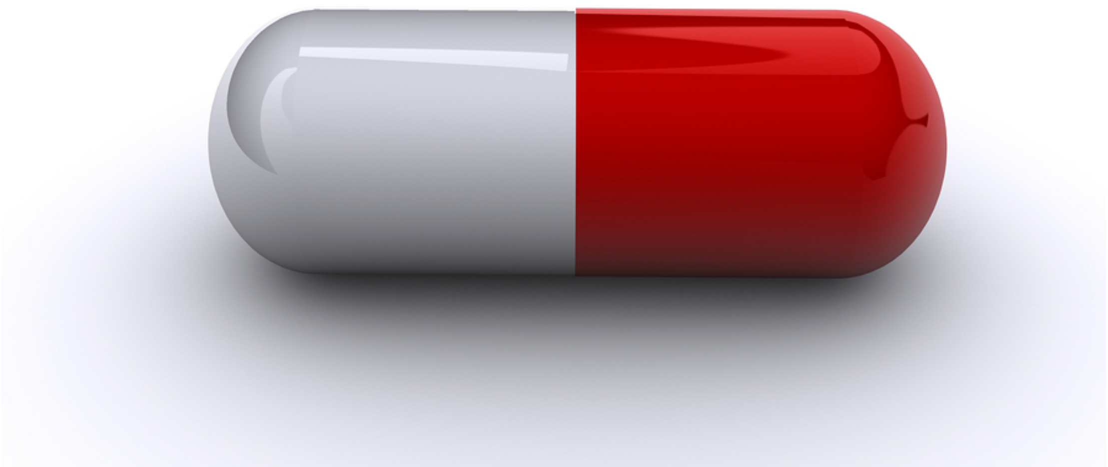
3. Create a distribution of what statistics would look like if Gorgias is right
    - Create the **null distribution** (that is consistent with H_0)



4. Get the probability we would get a statistic more than the observed statistic from the null distribution
    - p-value

5. Make a judgement
    - Assess whether the results are statistically significant

Very quick review of randomization test for two means

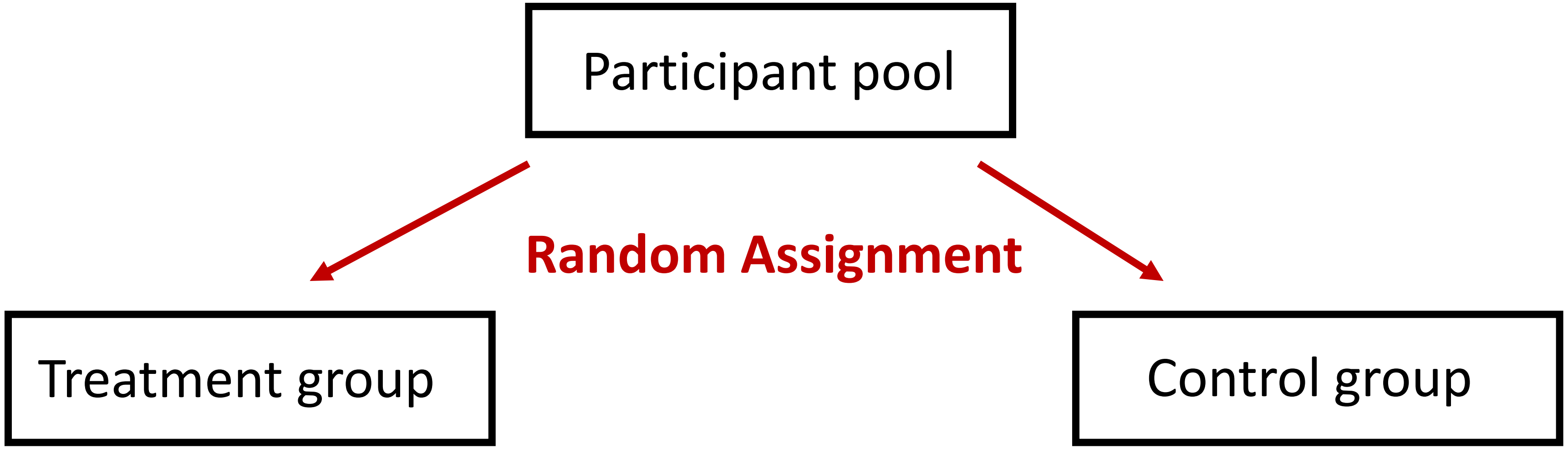


Question: Is this pill effective?

Experimental design

Take a group of participant and ***randomly assign***:

- Half to a *treatment group* where they get the pill
- Half in a *control group* where they get a fake pill (placebo)
- See if there is more improvement in the treatment group compared to the control group



Hypothesis tests for differences in two group means

1. State the null and alternative hypothesis
   - H_0: $\mu_{Treatment} = \mu_{Control}$ or $\mu_{Treatment} - \mu_{Control} = 0$
   - H_A: $\mu_{Treatment} > \mu_{Control}$ or $\mu_{Treatment} - \mu_{Control} > 0$

2. Calculate statistic of interest
   - For randomization/permutation tests we have a choice of the statistic to use

The statistic used before: $\bar{x}_{Effect} = \bar{x}_{Treatment} - \bar{x}_{Control}$

Let's try Welch's t-statistic instead: $t = \frac{\bar{x}_t - \bar{x}_c}{\sqrt{\frac{s_t^2}{n_t} + \frac{s_c^2}{n_c}}}$

Does calcium reduce blood pressure?

Treatment data (n_t = 10):

Begin	107	110	123	129	112	111	107	112	136	102
End	100	114	105	112	115	116	106	102	125	104
Decrease	**7**	**-4**	**18**	**17**	**-3**	**-5**	**1**	**10**	**11**	**-2**

Control data (n_c = 11):

Begin	123	109	112	102	98	114	119	112	110	117	130
End	124	97	113	105	95	119	114	114	121	118	133
Decrease	**-1**	**12**	**-1**	**-3**	**3**	**-5**	**5**	**2**	**-11**	**-1**	**-3**

2. What is the observed statistic of interest?
   - t = 1.604

$$t = \frac{\bar{x}_t - \bar{x}_c}{\sqrt{\frac{s_t^2}{n_t} + \frac{s_c^2}{n_c}}}$$

3. What is step 3?

3. Create the null distribution!

Treatment group

Control group

Reconstructed participant pool data under H_0

Shuffle data for random assignment consistent with H_0

Shuffled 'treatment group'

Shuffled 'control group'

One null distribution statistic: t_{shuff}

Repeat 10,000 times for null distribution

Let’s try the rest of the hypothesis test in R…

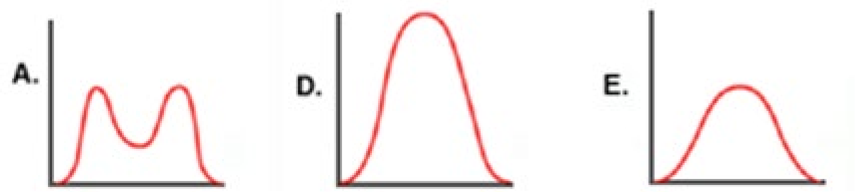
Parametric hypothesis tests

In **parametric hypothesis tests**, the null distribution is given by a density function.

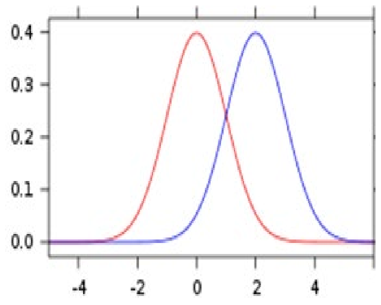
These density functions have a finite set of ***parameters*** that control the shape of these functions

- Hence the name “parametric hypothesis tests”
- Example: the normal density function has two parameters: μ and ***σ***

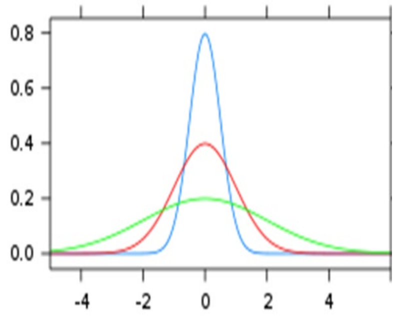
Remember density curves?



Changing μ



Changing σ

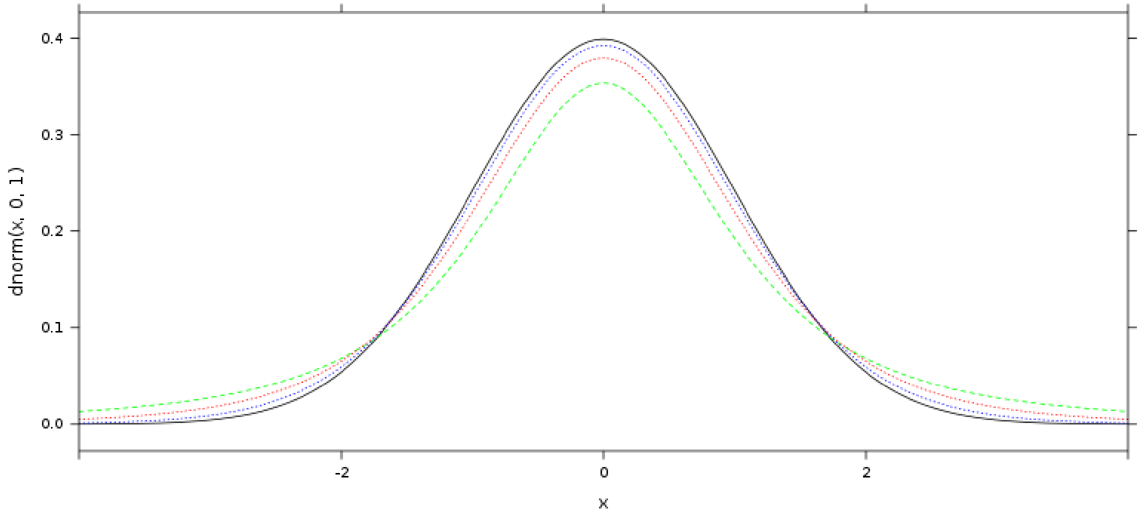


t-distributions

A commonly used density function (distribution) used for statistical inference is the t-distribution

- In R: rt(), dt(), pt() and qt()

t-distributions have one parameter called "degrees of freedom"



df = 2

df = 5

df = 15

N(0, 1)

t-distributions

When using t-distributions for statistical inference, each point in our t-distribution is a t-statistic

- i.e., we use t-distributions as null distributions for hypothesis tests and as sampling distributions when creating confidence intervals

t-statistics are a ratio of:

- The departure of an estimated value from a hypothesized parameter value
- Dived by an estimate of the standard error

θ

$$t = \frac{estimate\ \text{-}\ param_0}{\hat{SE}}$$

If the SE was known exactly the statistic would be a "z-statistic" that comes from a standard normal distribution

t-tests

t-tests are parametric hypothesis tests where the null distribution is a density function called a t-distribution.

t-tests can be used to test:

- If a mean is equal to a particular value: H_0: $\mu = 7$
- If two means are equal: H_0: $\mu_t = \mu_c$
- If a regression coefficient is equal to a particular value: H_0: $\beta = 2$
- etc.

t-tests for comparing two means

Let's examine t-tests for comparing **two means**

Step 1: what is the null hypotheses?

- H_0: $\mu_t - \mu_c = 0$

Step 2a: What is the numerator of the t-statistic?

$$t = \frac{\overbrace{estimate}^{(\bar{x}_t - \bar{x}_c)} - \overbrace{param_0}^{0}}{\hat{SE}} \leftarrow = \frac{(\bar{x}_t - \bar{x}_c) - 0}{\hat{SE}} = \frac{\bar{x}_t - \bar{x}_c}{\hat{SE}}$$

t-tests for comparing two means

Step 2b: What is the denominator of the t-statistic?

$$t = \frac{stat\ \text{-}\ param_0}{\hat{SE}}$$

Students' t-test assumes the variance in each population is the same, and uses an SE estimate of:

$$\hat{SE}_{\bar{x}_t\text{-}\bar{x}_c} = s_p \cdot \sqrt{\frac{1}{n_t} + \frac{1}{n_c}} \qquad s_p = \sqrt{\frac{\sum_i^{n_t}(x_i - \bar{x}_t)^2 + \sum_j^{n_c}(x_j - \bar{x}_c)^2}{n_t + n_c\ \text{-}\ 2}}$$

Welch's t-test does **not** assume that the variance in each population is the same and uses an estimate of:

$$\hat{SE}_{\bar{x}_t\text{-}\bar{x}_c} = \sqrt{\frac{s_t^2}{n_t} + \frac{s_c^2}{n_c}}$$

t-tests for comparing two means

Question: which statistic/test is better to use?

Students' t-test assumes the variance in each population is the same, and uses an SE estimate of:

$$t = \frac{\bar{x}_t \text{ - } \bar{x}_t}{s_p \cdot \sqrt{\frac{1}{n_t} + \frac{1}{n_c}}}$$

$$s_p = \sqrt{\frac{\sum_i^{n_t} (x_i - \bar{x}_c)^2 + \sum_j^{n_c} (x_j - \bar{x}_c)^2}{n_t + n_c \text{ - } 2}}$$

Welch's t-test does **not** assume that the variance in each population is the same and uses an estimate of:

$$\hat{SE}_{\bar{x}_t \text{-} \bar{x}_c} = \sqrt{\frac{s_t^2}{n_t} + \frac{s_c^2}{n_c}}$$

$$t = \frac{\bar{x}_t - \bar{x}_c}{\sqrt{\frac{s_t^2}{n_t} + \frac{s_c^2}{n_c}}}$$

t-tests for comparing two means

Question: which statistic/test is better to use?

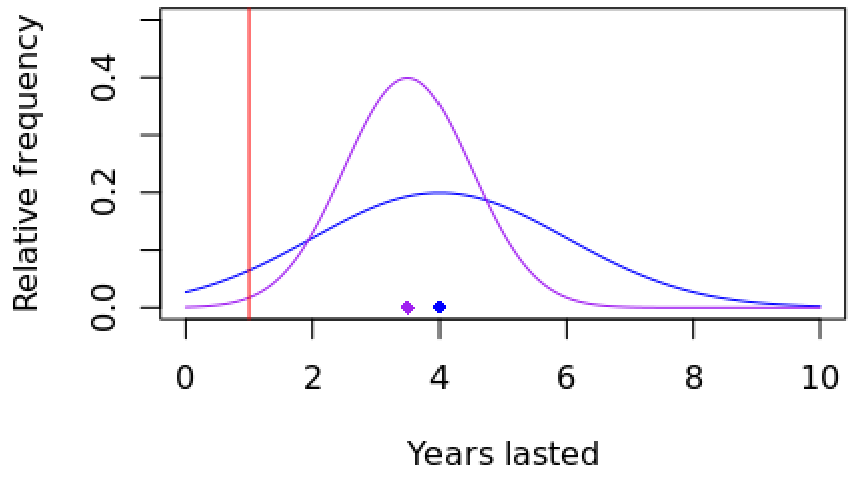
A: generally better to choose the "robust" test

- i.e., Welch's t-test is robust to unequal variances, so generally a better choice

However, we need to be careful with the decisions we make based on differences of means when there are unequal variances.

E.g., Which car battery company produces better batteries in terms of how long they last?

- Company A: μ = 4 years, σ = 2 years
- Company B: μ = 3.5 years, σ = 1 years



- Company A: 7% fail within a year
- Company B: 0.6% fail with a year

Does calcium reduce blood pressure?

Treatment data (n = 10):

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2. What is the observed statistic of interest?
    - t = 1.604

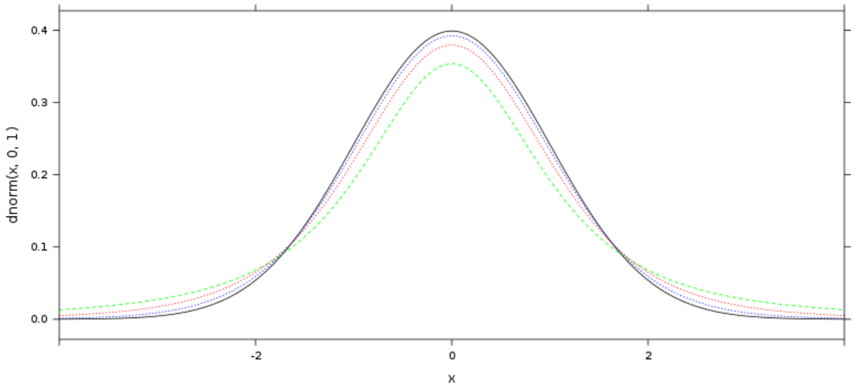
$$t = \frac{\bar{x}_t - \bar{x}_c}{\sqrt{\frac{s_t^2}{n_t} + \frac{s_c^2}{n_c}}}$$

3. What is the null distribution?
    - What additional piece of information do we need to create it?

t-tests for comparing two means

When using a t-distribution to compare two means, a conservative estimate of the degrees of freedom is the minimum of the two samples sizes, n_t and n_c, minus 1

- $df = \min(n_t, n_c) - 1$



Q: For the calcium study we had 10 people in the control group and 11 people in the treatment group so the degrees of freedom parameter is?

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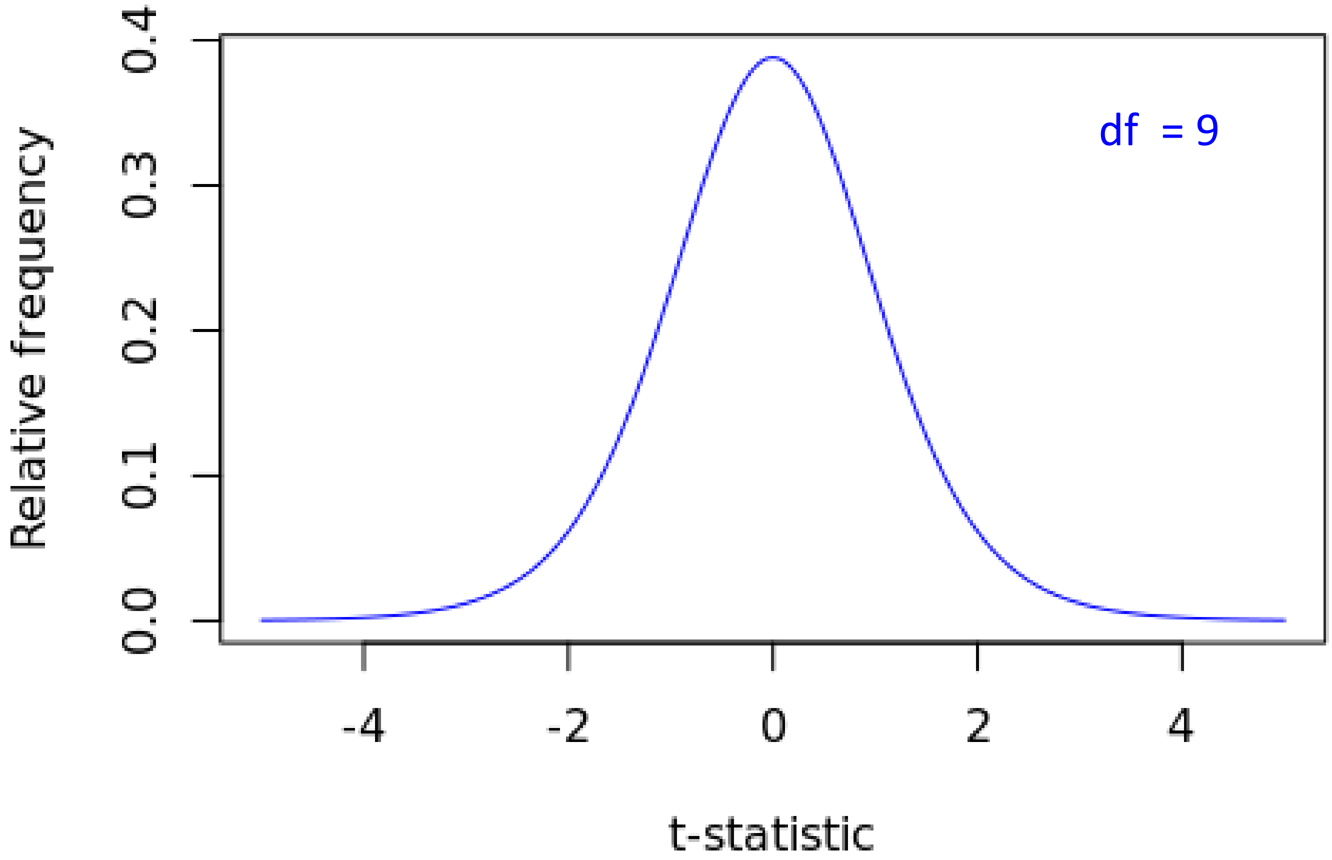
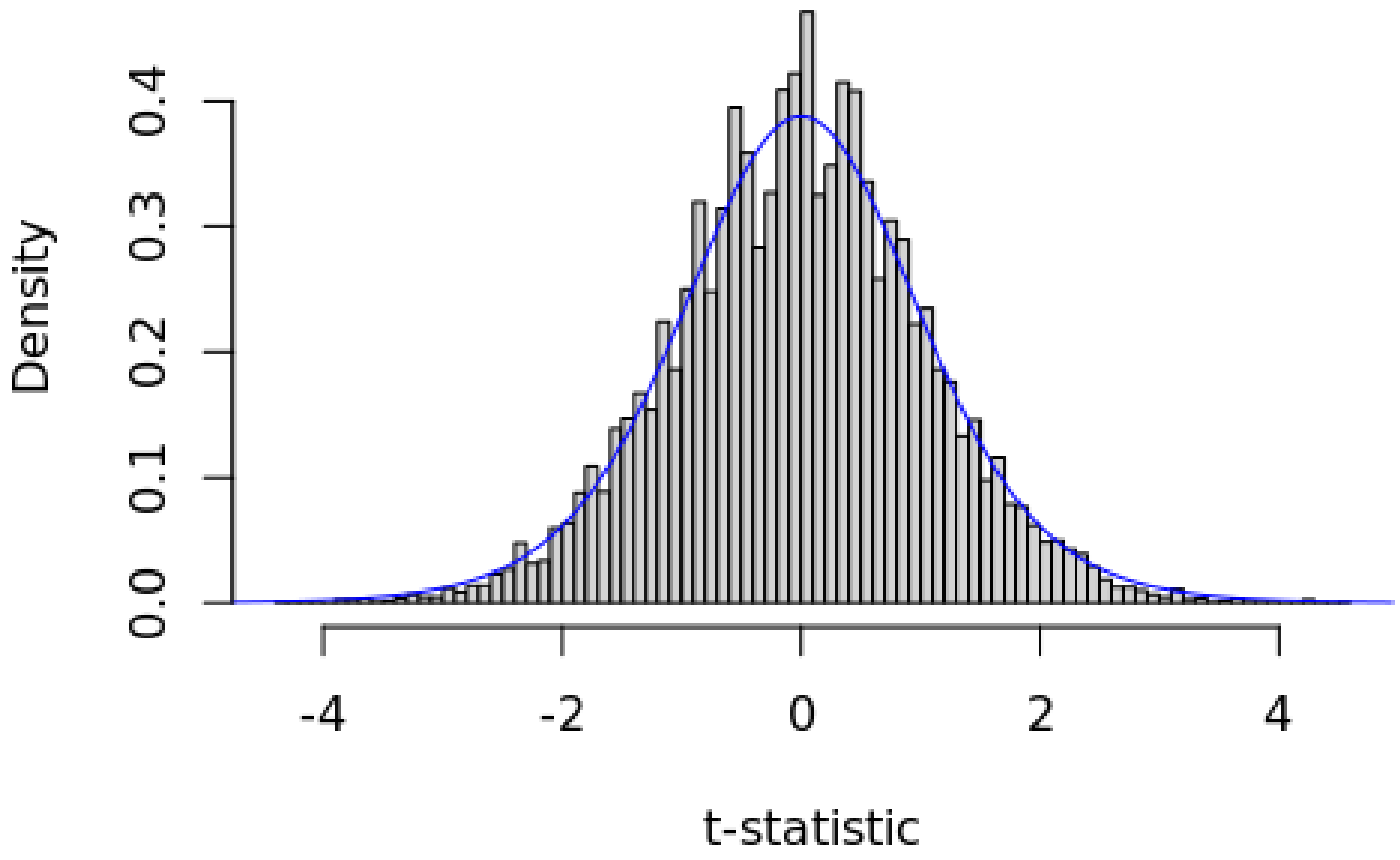
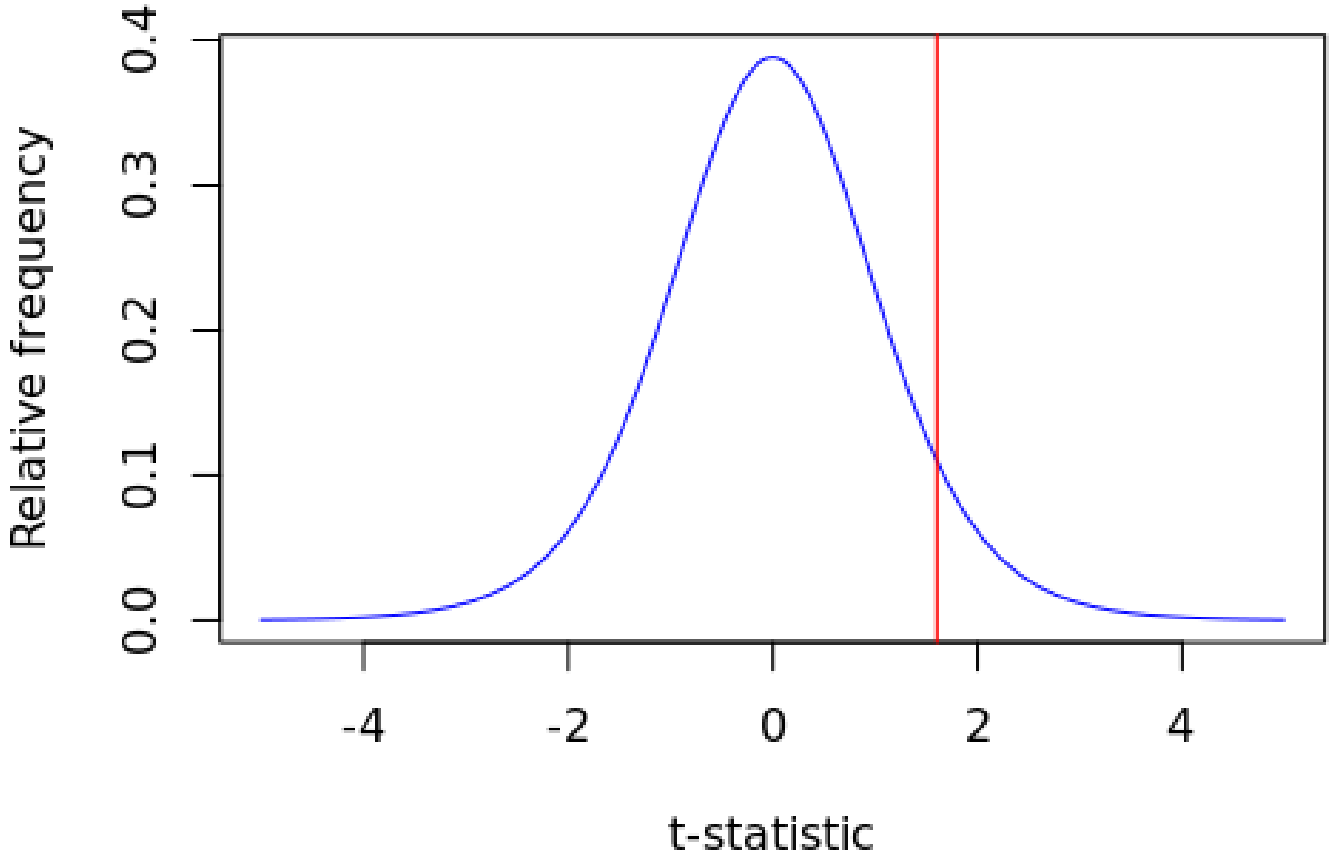
Step 3: Null t-distribution



Step 3: parametric vs. randomization distributions

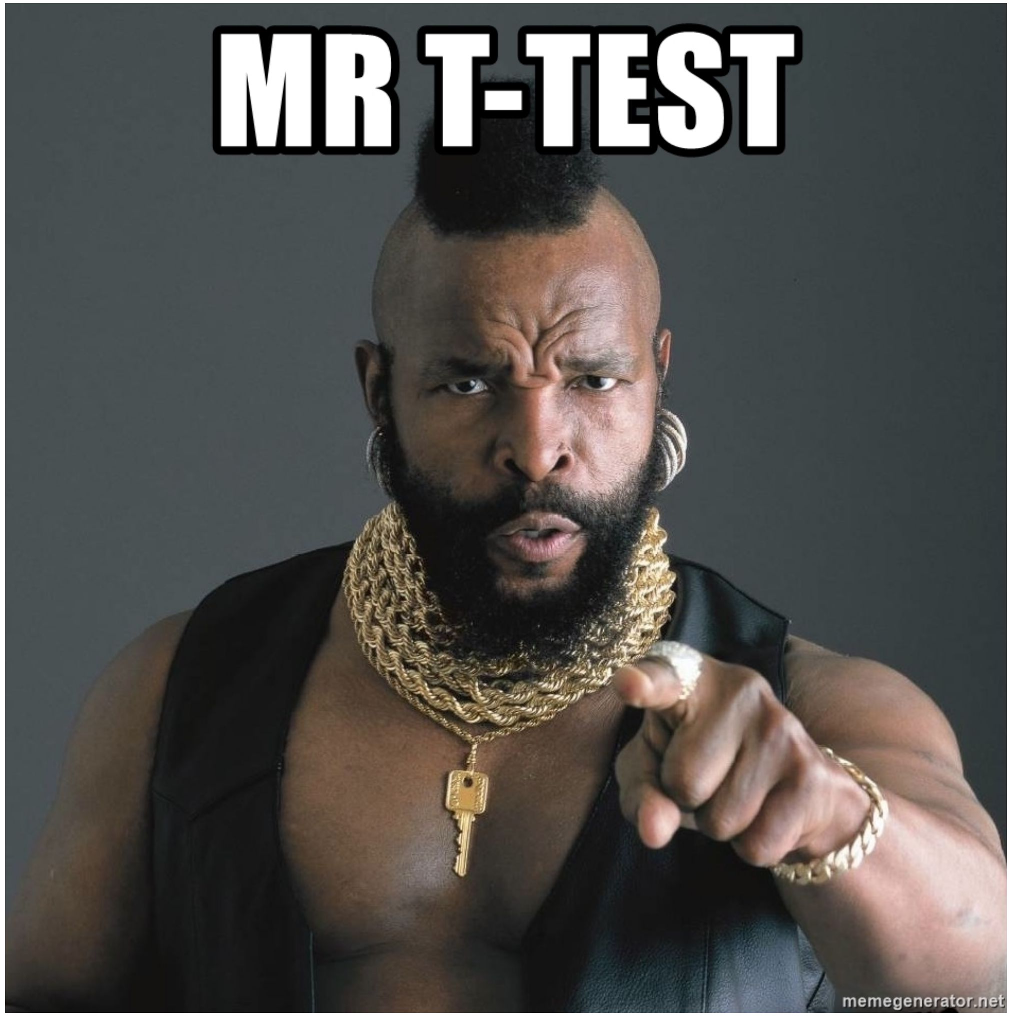


Step 4-5: p-value and conclusion



p-value = 0.072

Conclusion?



I pity the fool who doesn't want to try it in R!