Elastic collision

In an elastic collision, both momentum and kinetic energy are conserved. Here's how we can determine the final velocities.

Step-by-Step Solution

1. **Define the variables:**
   - m_1 and m_2 = masses of car 1 and car 2.
   - u_1 and u_2 = initial velocities of car 1 and car 2.
   - v_1 and v_2 = final velocities of car 1 and car 2.

2. **Conservation of momentum**
   The total momentum before the collision is equal to the total momentum after the collision.
   $$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

3. **Conservation of Kinetic energy**
   The total kinetic energy before the collision is equal to the total kinetic energy after the collision.
   $$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

Solving the equations

You have two equations with two unknowns (v1 and v2):

1. $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

2. $\frac{1}{2} m_1u_1^2+\frac{1}{2} m_2u_2^2 = \frac{1}{2} m_1v_1^2 + \frac{1}{2} m_2v_2^2$

Rearranging and solving

For simplicity, we can use the following methods to solve for v_1 and v_2:

Relative velocity approach:

In an elastic collision, the relative velocity of approach before the collision is equal to the relative velocity of separation after the collision (but with the opposite sign):

$$u_1-u_2 = -(v_1-v_2)$$

Rewriting it, we get:

$$u_1-u_2 = v_2-v_1$$

So,

$$v_2-v_1 = u_1 - u_2$$

Now we have two equations,

1. $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$
2. $v_2-v_1 = u_1 - u_2$

Solving the equations:

From the second equation, we can express v_2 in terms of v_1:

$$v_2 = v_1 + (u_1-u_2)$$

Substitute v_2 in the first equation:

$$m_1u_1+m_2u_2 = m_1v_1+m_2(v_1+(u_1-u_2))$$

Simplify the equation:

$$m_1u_1+m_2u_2 = m_1v_1+m_2v_1+m_2(u_1-u_2))$$

$$m_1u_1+m_2u_2 = (m_1+m_2)\ v_1+m_2u_1-m_2u_2))$$

$$m_1u_1+m_2u_2 - m_2u_1+m_2u_2= (m_1+m_2)\ v_1$$

$$(m_1- m_2)\ u_1+2m_2u_2= (m_1+m_2)\ v_1$$

$$v_1 = \frac{(m1 - m2)\ u1+2m2u2}{(m1+m2)}$$

using $v_2 = v_1 + (u_1-u_2)$:

$$v_2 = \frac{(m1 - m2)\ u1+2m2u2}{(m1+m2)} + (u_1-u_2)$$

$$v_2 = \frac{(m1 - m2)\ u1+2m2u2+(u1-u2)(m1+m2)}{(m1+m2)}$$

$$v2 = \frac{m1u1 - m2u1+2m2u2+u1m1+u1m2-u2m1-u2m2}{(m1+m2)}$$

$$v2 = \frac{2m1u1 - m1u2 - m2u2 + 2m2u2}{(m1+m2)}$$

$$v2 = \frac{2m1u1 - m1u2 + m2u2}{(m1+m2)}$$

$$v2 = \frac{2m1u1 + u2(m2 - m1)}{(m1+m2)}$$

$$v1 = \frac{2m2u2 + u1(m1 - m2)}{(m1+m2)}$$

These are the final velocities of the two objects after an elastic collision, expressed in terms of their initial velocities and masses.

Inelastic collision

In an inelastic collision, momentum is conserved, but kinetic energy is not conserved. Typically, in a completely inelastic collision, the two objects stick together after the collision. Let's derive the final velocity for this scenario, where the objects stick together.

Given:

- m_1 and u_1 for the mass and initial velocity of object 1
- m_2 and u_2 for the mass and initial velocity of object 2

After the collision, they stick together and move with a common final velocity v.

Step 1: Conservation of momentum

The total momentum before the collision is equal to the total momentum after the collision. This can be written as:

$$m_1u_1 + m_2u_2 = (m_1+m_2)\, v \qquad (1)$$

Step 2: Solving for the final velocity

From equation (1), we can solve for v:

$$v = \frac{\mathrm{m1u1 + m2u2}}{(\mathrm{m1+m2})}$$

This equation represents the final velocity v of the combined mass after a completely inelastic collision.

1. **Conservation of momentum:**
   - The law of conservation of momentum states that the total momentum of a system remains constant if no external forces act on it. In the case of a collision, this means that the momentum before the collision must equal the momentum after the collision.
   - The momentum before the collision is the sum of the momenta of the two objects: $m_1u_1 + m_2u_2$.
   - After the collision, since the objects stick together, they move with a common velocity v, and the combined mass is $m_1 + m_2$.

2. **Inelastic collision:**
   - In an inelastic collision, the objects do not retain their individual kinetic energies. Some of the kinetic energy is converted into other form of energy, such as heat, sound or deformation energy.
   - Because kinetic energy is not conserved, we do not use the conservation of kinetic energy equation in this scenario.
   - Instead, we rely solely on the conservation of momentum to determine the final velocity.