

Linear Models

Yangyang Chen

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To gain a better understanding of the factors that predict recovery time from COVID-19 illness, a study was designed to combine three existing cohort studies that have been tracking participants for several years. The study collects recovery information through questionnaires and medical records, and leverages existing data on personal characteristics prior to the pandemic.

In this project, we predict the recovery time based on important risk factors. The training data is in “training_df”, and the test data is in “training_df”. The response is in the column “Time to recovery (tt_recovery_time)”, and other variables can be used as predictors. The variable definitions can be found in “dictionary.txt”.

First, we import the data and adjust the variable type. As we want to compare different models afterwards, we use caret.

```
library(caret)

## Loading required package: ggplot2
## Loading required package: lattice
library(glmnet)

## Loading required package: Matrix
## Loaded glmnet 4.1-8
library(tidymodels)

## -- Attaching packages ----- tidymodels 1.1.1 --
## v broom          1.0.5      v rsample          1.2.0
## v dials          1.2.0      v tibble          3.2.1
## v dplyr          1.1.4      v tidyr           1.3.1
## v infer          1.0.5      v tune            1.1.2
## v modeldata      1.2.0      v workflows       1.1.3
## v parsnip        1.1.1      v workflowsets    1.0.1
## v purrr          1.0.2      v yardstick       1.2.0
## v recipes        1.0.8

## -- Conflicts ----- tidymodels_conflicts() --
## x purrr::discard()      masks scales::discard()
## x tidyr::expand()       masks Matrix::expand()
## x dplyr::filter()       masks stats::filter()
## x dplyr::lag()          masks stats::lag()
## x purrr::lift()         masks caret::lift()
## x tidyr::pack()         masks Matrix::pack()
## x yardstick::precision() masks caret::precision()
## x yardstick::recall()   masks caret::recall()
## x yardstick::sensitivity() masks caret::sensitivity()
```

```
## x yardstick::specificity() masks caret::specificity()
## x recipes::step()          masks stats::step()
## x tidyr::unpack()          masks Matrix::unpack()
## x recipes::update()        masks Matrix::update(), stats::update()
## * Search for functions across packages at https://www.tidymodels.org/find/
```

```
set.seed(2024)
load("~/Desktop/Data Science II/Group_Project/recovery.RData")
dat = dat |> janitor::clean_names()
```

```
# data splitting
data.split = initial_split(dat, prop = 0.8)
training_data = training(data.split)
testing_data = testing(data.split)
```

```
# check missing values
training_data |> is.na() |> sum()
```

```
## [1] 0
```

```
testing_data |> is.na() |> sum()
```

```
## [1] 0
```

```
# summary of training and testing data
training_data |> summary()
```

```
##      id      age      gender      race      smoking
## Min.   : 1.0   Min.   :42.00   Min.   :0.0000   1:1578   0:1444
## 1st Qu.:741.8   1st Qu.:57.00   1st Qu.:0.0000   2: 128   1: 700
## Median :1484.5   Median :60.00   Median :0.0000   3: 489   2: 256
## Mean   :1490.8   Mean    :60.28   Mean    :0.4863   4: 205
## 3rd Qu.:2230.5   3rd Qu.:63.00   3rd Qu.:1.0000
## Max.   :3000.0   Max.    :79.00   Max.    :1.0000
##      height      weight      bmi      hypertension
## Min.   :147.8   Min.   : 55.90   Min.   :18.80   Min.   :0.0000
## 1st Qu.:166.0   1st Qu.: 75.30   1st Qu.:25.90   1st Qu.:0.0000
## Median :169.9   Median : 79.75   Median :27.70   Median :0.0000
## Mean   :169.9   Mean    : 79.99   Mean    :27.79   Mean    :0.4958
## 3rd Qu.:173.9   3rd Qu.: 84.90   3rd Qu.:29.60   3rd Qu.:1.0000
## Max.   :188.6   Max.    :103.70   Max.    :38.90   Max.    :1.0000
##      diabetes      sbp      ldl      vaccine
## Min.   :0.00   Min.   :105.0   Min.   : 28.0   Min.   :0.0000
## 1st Qu.:0.00   1st Qu.:125.8   1st Qu.: 97.0   1st Qu.:0.0000
## Median :0.00   Median :130.0   Median :110.0   Median :1.0000
## Mean   :0.15   Mean    :130.4   Mean    :110.7   Mean    :0.5933
## 3rd Qu.:0.00   3rd Qu.:136.0   3rd Qu.:125.0   3rd Qu.:1.0000
## Max.   :1.00   Max.    :156.0   Max.    :178.0   Max.    :1.0000
##      severity      study      recovery_time
## Min.   :0.0000   Length:2400   Min.    : 2.00
## 1st Qu.:0.0000   Class :character   1st Qu.: 31.00
## Median :0.0000   Mode  :character   Median : 39.00
## Mean    :0.1087                      Mean    : 42.02
## 3rd Qu.:0.0000                      3rd Qu.: 49.00
## Max.    :1.0000                      Max.    :365.00
```

```
testing_data |> summary()
```

```
##           id           age           gender           race           smoking
## Min.      : 4.0    Min.    :45.00    Min.      :0.0000    1:389    0:378
## 1st Qu.: 801.5    1st Qu.:57.00    1st Qu.:0.0000    2: 30    1:159
## Median :1538.0    Median :60.00    Median :0.0000    3:115    2: 63
## Mean     :1539.3    Mean     :59.86    Mean      :0.4817    4: 66
## 3rd Qu.:2312.2    3rd Qu.:63.00    3rd Qu.:1.0000
## Max.     :2997.0    Max.      :72.00    Max.      :1.0000
##           height           weight           bmi           hypertension
## Min.      :148.1    Min.      :60.20    Min.      :19.20    Min.      :0.0000
## 1st Qu.:166.2    1st Qu.:75.00    1st Qu.:25.50    1st Qu.:0.0000
## Median :169.9    Median :79.80    Median :27.55    Median :1.0000
## Mean     :170.1    Mean      :79.85    Mean      :27.67    Mean      :0.5033
## 3rd Qu.:174.0    3rd Qu.:84.50    3rd Qu.:29.50    3rd Qu.:1.0000
## Max.     :188.5    Max.      :98.60    Max.      :35.90    Max.      :1.0000
##           diabetes           sbp           ldl           vaccine
## Min.      :0.0000    Min.      :108.0    Min.      : 50.0    Min.      :0.0000
## 1st Qu.:0.0000    1st Qu.:125.0    1st Qu.: 96.0    1st Qu.:0.0000
## Median :0.0000    Median :131.0    Median :109.5    Median :1.0000
## Mean     :0.1717    Mean      :130.7    Mean      :109.6    Mean      :0.6067
## 3rd Qu.:0.0000    3rd Qu.:136.0    3rd Qu.:123.0    3rd Qu.:1.0000
## Max.     :1.0000    Max.      :155.0    Max.      :162.0    Max.      :1.0000
##           severity           study           recovery_time
## Min.      :0.0    Length:600    Min.      : 2.00
## 1st Qu.:0.0    Class :character    1st Qu.: 30.00
## Median :0.0    Mode  :character    Median : 40.00
## Mean     :0.1                                     Mean      : 42.76
## 3rd Qu.:0.0                                     3rd Qu.: 49.00
## Max.     :1.0                                     Max.      :330.00
```

```
# training data
train.x = model.matrix(recovery_time ~ ., training_data)[, -1]
train.y = training_data$recovery_time

# test data
test.x = model.matrix(recovery_time ~ ., testing_data)[, -1]
test.y = testing_data$recovery_time

# cross validation
ctrl = trainControl(method = "cv", number = 10)
ctrl_1SE = trainControl(method = "cv", number = 10,
                         selectionFunction = "oneSE")
```

There is no missing data in both datasets. The training dataset has 2400 observation and 16 variables, and the test dataset has 600 samples and 16 variables.

Lasso Regression Model

```
set.seed(2024)
lasso.fit = training_data |>
  train(recovery_time ~ ., data = _, method = "glmnet",
        tuneGrid = expand.grid(
```

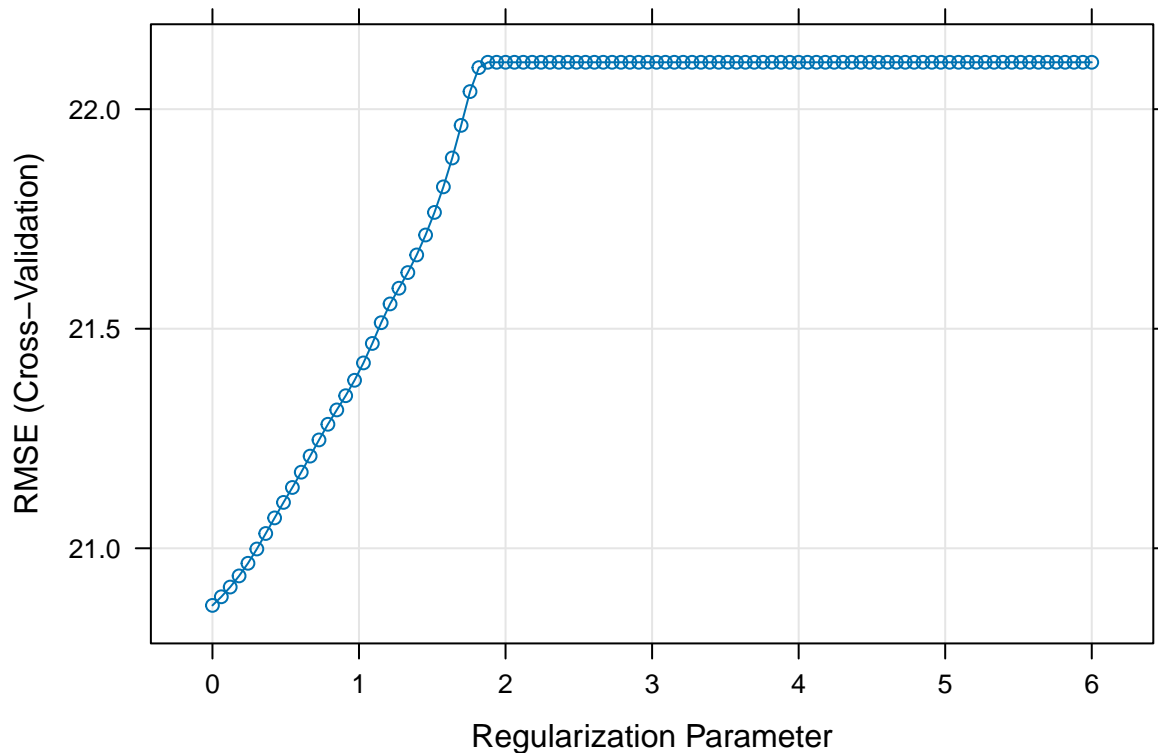
```

    alpha = 1,
    lambda = exp(seq(6, 0, length = 100))),
    trControl = ctrl,
    preProcess = c("center", "scale"))

## Warning in nominalTrainWorkflow(x = x, y = y, wts = weights, info = trainInfo,
## : There were missing values in resampled performance measures.

plot(lasso.fit, xTrans = log)

```



The flattening in the curve occurs because the lasso regression model has reached a stable solution where no more coefficients become zero as the regularization parameter increases. At this point, further increasing the regularization parameter does not change the set of non-zero coefficients or their values, resulting in a flat region where the model's performance remains constant. This behavior indicates that the model has achieved the optimal level of sparsity, with the regularization parameter value at the start of the flat region corresponding to the desired sparse solution.

```

lasso.pred = predict(lasso.fit, newdata = testing_data)
mse.lasso = mean((test.y - lasso.pred) ^ 2)

```

The selected tuning parameter is 1, and the test error (MSE) is 608.0764. Now, we apply the 1SE rule and refit the model.

```

lasso_1SE.fit = training_data |>
  train(recovery_time ~ .,
    data = _,
    method = "glmnet",
    tuneGrid = expand.grid(
      alpha = 1,
      lambda = exp(seq(6, 0, length = 100))),
    trControl = ctrl_1SE)

```

```
## Warning in nominalTrainWorkflow(x = x, y = y, wts = weights, info = trainInfo,
## : There were missing values in resampled performance measures.
```

```
lasso.coef_1SE = coef(lasso_1SE.fit$finalModel,
                      s = lasso_1SE.fit$bestTune$lambda)
```

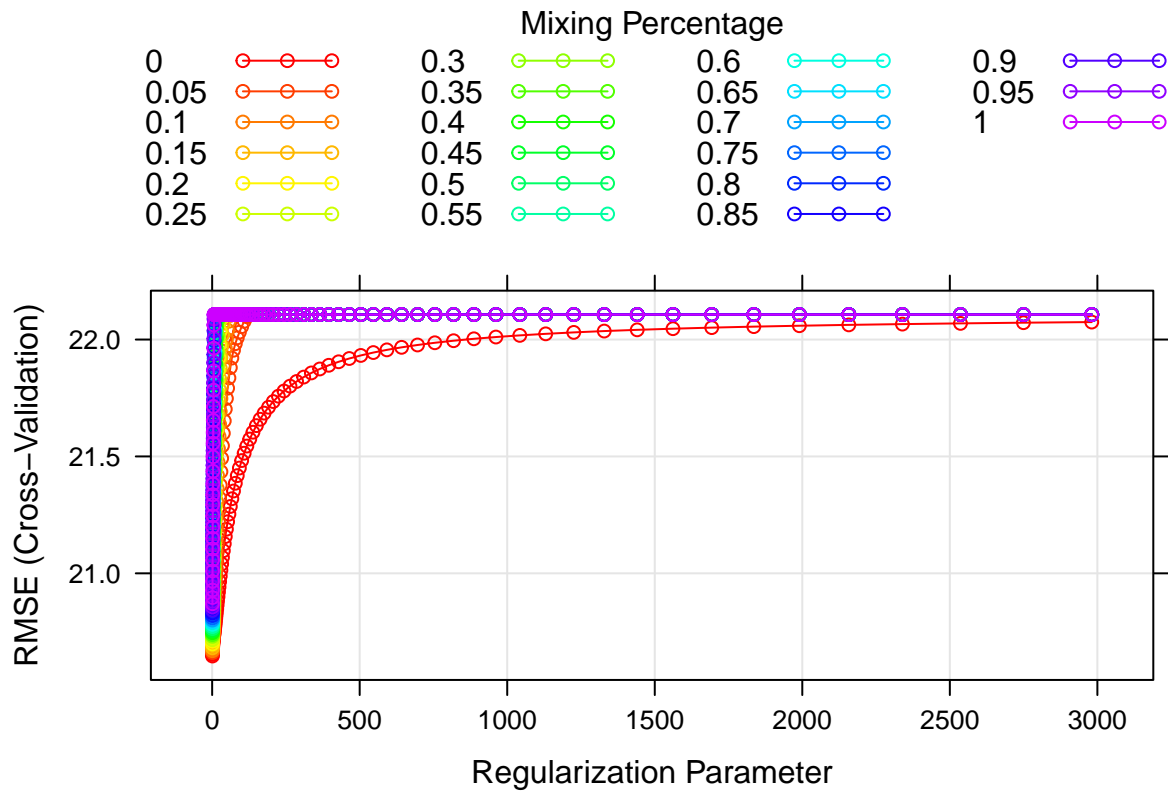
There exists problem in our model. When the 1SE rule is applied, the selected tuning parameter is $\alpha = 1$, $\lambda = 403.43$, and 0 predictors are included in the model.

Elastic Net Model

```
set.seed(2024)
enet.fit = training_data |>
  train(recovery_time ~ ., data = _, method = "glmnet",
        tuneGrid = expand.grid(
          alpha = seq(0, 1, length = 21),
          lambda = exp(seq(8, 0, length = 100))),
        preprocess = c("center", "scale"),
        trControl = ctrl)
```

```
## Warning in nominalTrainWorkflow(x = x, y = y, wts = weights, info = trainInfo,
## : There were missing values in resampled performance measures.
```

```
myCol = rainbow(25)
myPar =
  list(superpose.symbol = list(col = myCol),
        superpose.line = list(col = myCol))
plot(enet.fit, par.settings = myPar)
```



The vertical line pattern in the graph is a characteristic behavior of the elastic net model. It occurs because the

elastic net combines lasso and ridge regularization, which can lead to sparse solutions with some coefficients becoming exactly zero. As the regularization parameter increases, the model transitions between different sparse solutions, causing sudden drops or jumps in the cross-validation error curve.

```
enet.pred = predict(enet.fit, newdata = testing_data)
mse.enet = mean((test.y - enet.pred) ^ 2)
```

The selected tuning parameter is $\alpha = 0$, $\lambda = 1$, and the test error (MSE) is 589.642. Now, we try to apply the 1SE rule and refit the model.

```
set.seed(2024)
enet_1SE.fit = training_data |>
  train(recovery_time ~ .,
        data = _,
        method = "glmnet",
        tuneGrid = expand.grid(
          alpha = seq(0, 1, length = 21),
          lambda = exp(seq(8, 0, length = 100))),
        preProcess = c("center", "scale"),
        trControl = ctrl_1SE)
```

```
## Warning in nominalTrainWorkflow(x = x, y = y, wts = weights, info = trainInfo,
## : There were missing values in resampled performance measures.
```

```
enet_1SE.pred = predict(enet_1SE.fit, newdata = testing_data)
mse.enet_1SE = mean((test.y - enet_1SE.pred) ^ 2)
```

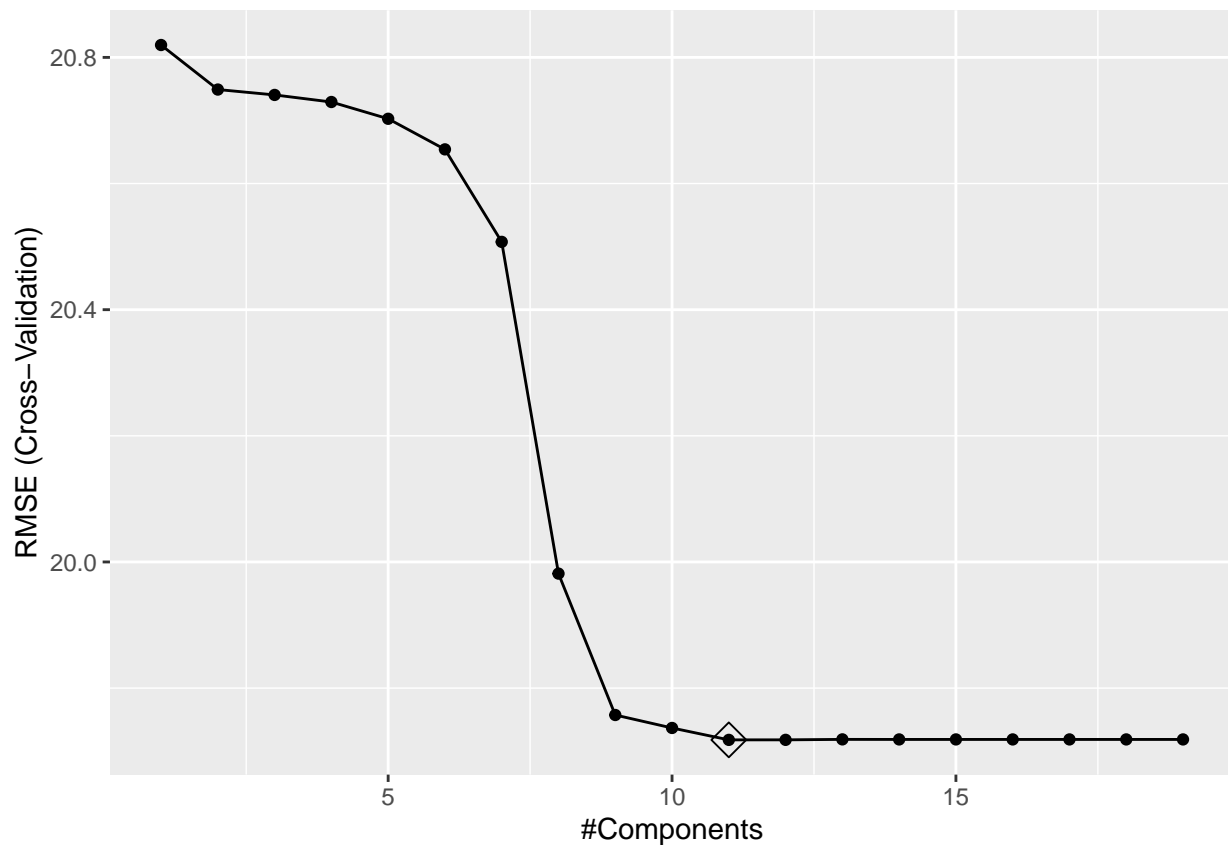
When the 1SE rule is applied, the selected tuning parameter is $\alpha = 0$, $\lambda = 191.0481$, and the test error (MSE) is 654.9077.

Partial Least Square

```
set.seed(2024)

pls.fit = train(train.x, train.y, method = "pls",
  tuneGrid = data.frame(ncomp = 1:19),
  trControl = ctrl, preProcess = c("center", "scale"))

ggplot(pls.fit, highlight = T)
```



```
pls.pred = predict(pls.fit, newdata = test.x)
mse.pls = mean((test.y - pls.pred) ^ 2)
```

As illustrated in the plot, 11 components are included in my model, and the test error (MSE) is 472.1037.

Linear Model Comparison

Here, we compare the CV results of different models and choose the model with the smallest median RMSE.

```
resamp =
  resamples(list(lasso = lasso.fit,
                 lasso_1SE = lasso_1SE.fit,
                enet =enet.fit,
                 enet_1SE =enet_1SE.fit,
                 pls = pls.fit))
summary(resamp)
```

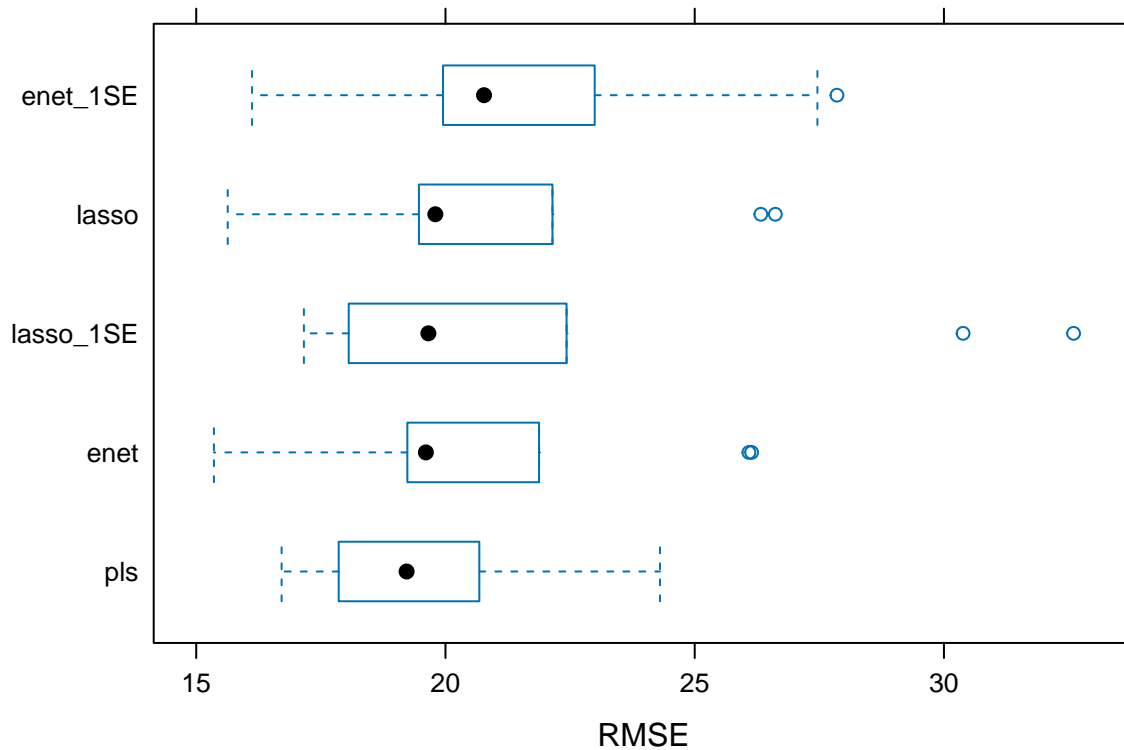
```
##
## Call:
## summary.resamples(object = resamp)
##
## Models: lasso, lasso_1SE, enet, enet_1SE, pls
## Number of resamples: 10
##
## MAE
```

	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.	NA's
lasso	11.70820	12.80710	12.91981	12.98865	13.31698	14.11550	0
lasso_1SE	12.02208	12.59751	13.18138	13.49821	13.78426	16.11071	0

```
## enet      11.61245 12.92027 13.01641 13.09761 13.50941 14.31622    0
## enet_1SE  11.89810 12.94601 13.07245 13.24746 13.72694 14.26906    0
## pls      11.73828 12.57180 13.04293 12.97226 13.36581 14.03871    0
##
## RMSE
##           Min.   1st Qu.   Median     Mean  3rd Qu.     Max. NA's
## lasso      15.63217 19.47182 19.79685 20.86990 22.00463 26.61706    0
## lasso_1SE  17.16031 18.12459 19.65723 21.78256 22.37995 32.60110    0
## enet       15.35438 19.26202 19.60704 20.64619 21.75758 26.14188    0
## enet_1SE   16.12041 20.02620 20.77401 21.71048 22.90480 27.85372    0
## pls       16.71277 17.90735 19.22022 19.71794 20.53556 24.30313    0
##
## Rsquared
##           Min.   1st Qu.   Median     Mean  3rd Qu.     Max. NA's
## lasso      0.07871429 0.09239047 0.1087820 0.1188363 0.1385637 0.1964512    0
## lasso_1SE   NA         NA         NA         NaN         NA         NA    10
## enet       0.09101487 0.10983939 0.1291095 0.1318636 0.1408881 0.1941670    0
## enet_1SE   0.07217084 0.09373823 0.1192631 0.1199881 0.1304252 0.1780023    0
## pls       0.06761502 0.20238751 0.2266513 0.2120997 0.2458746 0.2837441    0
```

Using bw-plot to compare their RMSE.

```
bwplot(resamp, metric = "RMSE")
```



Hence, we selected partial least square model as it has smallest RMSE.