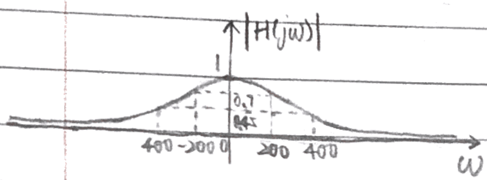
Prelab-6

1. a). i) $a=200$, $H(j\omega)=\frac{200}{200+j\omega}$

$$|H(j\omega)| = \frac{200}{\sqrt{40000+\omega^2}} \longrightarrow \omega=0,\ |H(j\omega)|_{max}=1$$

$$|\omega|\rightarrow\infty,\ |H(j\omega)|\longrightarrow 0$$



ii). This is an Low-Pass Filter

iii) $H(j\omega)=\frac{200}{200+j\omega} = \frac{200}{1\cdot(j\omega)+200} \longrightarrow a=[1, 200]$

$b=[200]$

b). $|H(j\omega_c)| = \frac{H(0)}{\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{200}{\sqrt{80000}} \longrightarrow 40000+\omega_c^2 = 80000$

$\omega_c = \sqrt{40000}$

$\omega_c = 200$

$\because |\omega| < \omega_c = 200$, the signal can pass

$\because x_1 = \cos(100t)$, $\omega = 100 < 200$

$\therefore x_1(t)$ is in the pass-band of the filter

$$y_1(t) = |H(j\omega)|_{\omega=100} \cos(100t + \angle H(j\omega)_{\omega=100})$$

$$|H(j\omega)|_{\omega=100} = \frac{200}{\sqrt{50000}} = 0.8944$$

$$\angle H(j\omega)|_{\omega=100} = (\tan^{-1}(0) - \tan^{-1}(\tfrac{\omega}{200}))_{\omega=100} = -\tan^{-1}(\tfrac{1}{2}) = -26.57^\circ$$

$$\therefore y_1(t) = 0.8944\cos(100t - 26.57^\circ)$$

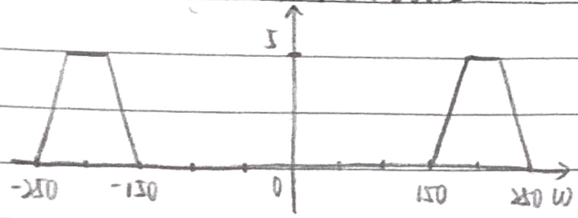
c). length should be $0.5s \times 4000Hz + 1 = 2001$

2) a) $y(t) = x(t)c(t)$

$Y(j\omega) = \frac{1}{2\pi} X(j\omega) * C(j\omega)$

$Y(j\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) \cdot \cancel{\pi}(\delta(\alpha-(\omega-200)) + \delta(\alpha-(\omega+200)))\,d\omega$

$= \frac{1}{2}\left[X(\omega+200) + X(\omega-200)\right]$



carrier frequency is 200 rad/s

b). Without aliasing: $2\omega_{max} = 2 \times 50 = 100$ rad/s

3). a). $z(t) = y(t)\cos(200t) = x(t)\cos^2(200t)$

$= x(t)\left(\frac{1}{2}(1+\cos(400t))\right)$

$= \frac{1}{2}x(t) + \frac{1}{2}x(t)\cos(400t)$

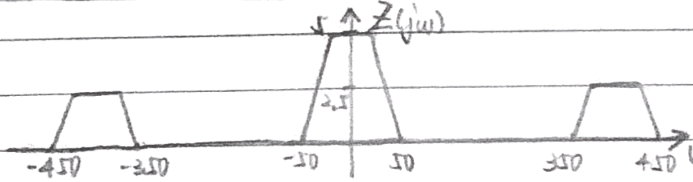
$Z(j\omega) = \frac{1}{2}X(j\omega) + \frac{1}{2}\left[\frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\cdot\cancel{\pi}(\delta(\alpha-(\omega-200)) + \delta(\alpha-(\omega+200)))\,d\omega\right]$

$= \frac{1}{2}X(j\omega) + \frac{1}{2}\left(\frac{1}{2}\left[X(\omega+400) + X(\omega-400)\right]\right)$

$= \frac{1}{2}X(j\omega) + \frac{1}{4}\left[X(\omega+400) + X(\omega-400)\right]$

$\because$ similarly, $Z(j\omega) = \frac{1}{2\pi} Y(j\omega) * C(j\omega)$ (like in 2.a), $c(t) = \cos(200t)$

$\therefore \boxed{Z(j\omega) = \frac{1}{2}\left[Y(\omega+200) + Y(\omega-200)\right]}$



b). From original equation: $240(j\omega)^4 X_r(j\omega) + 3\times10^4 (j\omega)^3 X_r(j\omega) + 2.2\times10^6 (j\omega)^2 X_r(j\omega) + 10^8 (j\omega) X_r(j\omega)$
$+ 2\times10^9 X(j\omega) = 2\times10^9 Z(j\omega)$

i). $\therefore H(j\omega) = \frac{X_r(j\omega)}{Z(j\omega)} = \frac{2\times10^9}{240(j\omega)^4 + 3\times10^4(j\omega)^3 + 2.2\times10^6(j\omega)^2 + 10^8(j\omega) + 2\times10^9}$

ii). $a = [240, 30000, 2.2\times10^6, 10^8, 2\times10^9]$, $b = [2\times10^9]$

iii). DC gain $= H(0) = \frac{2\times10^9}{2\times10^9} = 1$