```
from aide_design.play import*
from coastal import*
```

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Problem Set 3

1. You are building a small pier to be supported by piles. The water depth at the front row of piles is **h(x)= 2 m**. The beach slope is **1/30**. A wave buoy in the "deep water" measured the following wave conditions: Wave amplitude **a = 0.5 m**; wave period **T = 12 sec**.

Parameter	Value
Amplitude	0.7271 m
kh	0.238
u_{max}	1.595 m/s
w_{max}	0.3807 m/s

a. Minimum clearance needed to maintain a dry deck

Wave Energy Conservation

$$\overline{E}_0^+ \cdot c_{g0} = \overline{E}_1^+ \cdot c_{g1}$$

Using the relationship between energy and amplitude, relate the wave energy conservation to an energy normalized by density basis.

$$\overline{E}^+ = \frac{\rho g a^2}{2}$$

$$\frac{a_1}{a_0} = \sqrt{\frac{c_{g0}}{c_{g1}}}$$

In deep water conditions, phase velocity can be simplified because kh is large so the $tanh(kh)$ term approaches 1.

$$c_p = \frac{gT}{2\pi} \tanh kh$$

$$c_{p0} = \frac{gT}{2\pi}$$

In deep water conditions, n = $\frac{1}{2}$ so the group velocity is

$$c_{g0} = \frac{gT}{2\pi} \cdot \frac{1}{2}$$

Substitute the shallow group velocity with Froude's number and solve only for amplitude at the shallow regime.

$$a_1 = \frac{a_0 \cdot \sqrt{c_{g0}}}{[g \cdot h_1]^{1/4}}$$

```
amp0 = 0.5 * u.m
period = 12 * u.sec
g = 9.80665 * u.m/u.s**2
hx = 2 * u.m

amp1 = amplitude_shallow(amp0, period, hx)
print(amp1.to(u.m))
```

b. In order to confirm the shallow wave assumptions made above, $kh < \frac{\pi}{10}$ must be true.

To find k, σ must first be determined.

$$\sigma = \frac{2\pi}{T}$$

$$k = \sqrt{\frac{\sigma^2}{gh}}$$

Alternatively, the wavenumber function I have written searches for wave number based on period and height using two bounds on k.

Since $kh = 0.238 < \frac{\pi}{10}$, the shallow water assumption is valid.

```
kh = wavenumber(period, hx) * hx
print(kh)
```

c. The maximum water particle velocities are as follows:

- Maximum u @ wave crest and trough
- Maximum w @ $\eta = 0$

Horizontal Velocity

The extreme values of the horizontal velocity appear at the phase positions $(kx - \sigma t) = 0,\ \pi,\ \ldots$ (under the crest and trough positions).

$$u = a\sigma \cdot \frac{\cosh k(h+z)}{\sinh(kh)} \cos(kx - \sigma t)$$

The cosine term goes to 1.

$$u = a\sigma \cdot \frac{\cosh k(h+z)}{\sinh(kh)}$$

The vertical variation of the velocity components is best viewed by starting at the bottom where k(h + z) = 0. Here the hyperbolic terms involving z in both the u and w velocities are at their minima, 1 and 0, respectively. As we progress upward in the fluid, the magnitudes of the velocity components increase.

$$u = a\sigma \cdot \frac{1}{\sinh(kh)}$$

In shallow water, $\sinh(kh)$ goes to kh.

$$u = \frac{a\sigma}{kh}$$

Vertical Velocity

The extreme vertical velocities appear at $\pi/2$, $3\pi/2$, (where the water surface displacement is zero).

$$w = a\sigma \cdot \frac{\sinh k(h+z)}{\sinh(kh)} \sin(kx - \sigma t)$$

The sine term goes to 1.

$$w = a\sigma \cdot \frac{\sinh k(h+z)}{\sinh(kh)}$$

The z elevation is 0 where the vertical velocity is greatest. The $sinh(kh)$ terms remaining go to kh in shallow water conditions.

$$w = a\sigma \cdot \frac{\sinh(kh)}{\sinh(kh)}$$

$$w = a\sigma$$

```
sig = freq_angular(period)
uvel = amp1 * sig / kh
print(uvel)

wvel = amp1 * sig
print(wvel)
```

2. Assumptions:

- the system is at steady state and there is no dissipation from geometry
- changes so wave energy can be conserved
- linear wave theory applies

- since H is constant, group velocities can be assumed to be the same
- density of water is constant

$$\overline{E}_1^+ \cdot c_{g1} \cdot B_1 = \overline{E}_2^+ \cdot c_{g2} \cdot B_2$$

$$\overline{E}^+ = \frac{\rho g a^2}{2}$$

$$\frac{\rho g a_1^2}{2} \cdot c_{g1} \cdot B_1 = \frac{\rho g a_2^2}{2} \cdot c_{g2} \cdot B_2$$

$$a_1^2 \cdot B_1 = a_2^2 \cdot B_2$$

$$a_2 = a_1 \sqrt{\frac{B_1}{B_2}}$$

The functions I have defined are as follows:

```
@u.wraps(u.Hz, [u.s], False)
def freq_angular(T):
    sig = (2 * np.pi)/T
    return(sig)


    @u.wraps(u.m/u.s, [u.s], False)
    def celerity_phase_deep(T):
        '''Returns the phase velocity in deep water conditions'''
        cp = (g*T)/(2*np.pi)
        return(cp)


    @u.wraps(u.m/u.s, [u.s], False)
    def celerity_group_deep(T):
        '''Returns the phase velocity in deep water conditions'''
        cp = (g*T)/(4*np.pi)
        return(cp)


    def amplitude_shallow(a_deep, T, height):
        '''Returns the shallow water wave amplitude. Inputs: deep water amplitude,
        period, shallow water depth.'''
        cg0 = celerity_group_deep(T)
        a = (a_deep * np.sqrt(cg0))/((g * height)**.25)
```

```python
        return(a)


    @u.wraps(1/u.m, [u.s, u.m], False)
    def wavenumber(T, h):
        '''Returns wavelength given a period, depth of water,
        and two bounds on k. It should be noted that the bounds as a function of k
        less the angular frequency squared should be of different sign. The bounds
        are written inside this function and should be changed later.'''
        k_bound1 = 0
        k_bound2 = 10

        def _dispersion_k(k):
            return (g.magnitude * k * np.tanh(k * h) - (((2 * np.pi) / T))**2)
        root = optimize.ridder(_dispersion_k, k_bound1, k_bound2)
        return(root)
```

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