

Software Implementation

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#-----#
# Linear Growth mixture modeling Based on Conditional Medians
#-----#
# model {
  for (i in 1:N) {
    # ----- class membership -----
    mem[i] ~ dcat(lambda[1:2])
    for(t in 1:Time) {
      # ----- growth curve -----
      V[i,t] ~ dexp(pre_sigma)
      y[i,t] ~ dnorm(muy[i,t], pre_sig2[i,t])
      muy[i,t] <- LS[i,1]+(t-1)*LS[i,2] + zeta*V[i,t]
      pre_sig2[i,t]<- 1/sig2_y[i,t]
      sig2_y[i,t] <- V[i,t]*xi*xi/pre_sigma
    }
    LS[i,1:2] ~ dmnorm(muLS[mem[i],1:2], Inv_cov[1:2,1:2])
  }

  zeta <- (1-2*tau)/(tau*(1-tau))
  xi <- sqrt(2/(tau*(1-tau)))

#-----#
# priors
#-----#
# -- sigma --
pre_sigma ~ dgamma(0.1,0.1)
sigma <- 1/pre_sigma

# -- muLS --
# Intercept
muLS[1,1] ~ dnorm(0, 0.01) I(muLS[2,1],)
muLS[2,1] ~ dnorm(0, 0.01)
# Slope
muLS[1,2] ~ dnorm(0, 0.1)
muLS[2,2] ~ dnorm(0, 0.1)

# -- Inv_cov --
Inv_cov[1:2,1:2]~dwish(R[1:2,1:2], 3)

```

```
Cov_b <- inverse(Inv_cov[1:2,1:2])  
R[1,1]<-1  
R[2,2]<-1  
R[2,1]<-R[1,2]  
R[1,2]<-0  
  
# -- lambda --  
lambda[1:2] ~ ddirich(alpha[1:2])  
}
```

Supplementary Results

Table 1

Convergence rates for the Median GMM and Mean GMM when N=300

	Unbalanced mixing proportions				Balanced mixing proportions			
	Median GMM		Mean GMM		Median GMM		Mean GMM	
	MD1	MD2	MD1	MD2	MD1	MD2	MD1	MD2
D1	1.00	1.00	0.99	1.00	0.99	1.00	0.99	1.00
D2	0.99	1.00	0.92	0.99	0.99	0.99	0.99	1.00
D3	0.99	1.00	0.68	0.97	1.00	1.00	0.99	0.99
D4	1.00	1.00	0.94	1.00	1.00	1.00	0.99	1.00

Note. Median GMM: Growth mixture modeling based on conditional medians; Mean GMM: Traditional growth mixture modeling based on conditional means. The total number of replications was 500.

Table 2

Convergence rates for the Median GMM and Mean GMM when N=1000

	Unbalanced mixing proportions				Balanced mixing proportions			
	Median GMM		Mean GMM		Median GMM		Mean GMM	
	MD1	MD2	MD1	MD2	MD1	MD2	MD1	MD2
D1	0.99	1.00	0.99	1.00	0.99	1.00	0.98	1.00
D2	0.99	1.00	0.46	1.00	0.98	1.00	0.97	1.00
D3	0.99	1.00	0.20	0.99	0.99	1.00	0.72	1.00
D4	0.99	1.00	0.62	1.00	0.99	1.00	0.98	1.00

Note. Median GMM: Growth mixture modeling based on conditional medians; Mean GMM: Traditional growth mixture modeling based on conditional means. The total number of replications was 500.

Table 3

Mixing proportion and membership recovery when N=300 and unbalanced mixing proportions

	Mixing proportion				Membership			
	MD1		MD2		MD1		MD2	
	Median	Mean	Median	Mean	Median	Mean	Median	Mean
	GMM	GMM	GMM	GMM	GMM	GMM	GMM	GMM
D1	0.00 (0.02)	0.00 (0.02)	0.00 (0.02)	0.00 (0.03)	0.76 (0.05)	0.77 (0.04)	0.96 (0.01)	0.96 (0.01)
D2	0.00 (0.02)	-0.06 (0.09)	0.00 (0.03)	0.01 (0.03)	0.77 (0.04)	0.76 (0.04)	0.95 (0.01)	0.94 (0.02)
D3	0.00 (0.02)	-0.07 (0.10)	0.00 (0.03)	0.01 (0.03)	0.78 (0.03)	0.75 (0.04)	0.95 (0.01)	0.94 (0.01)
D4	0.00 (0.02)	-0.06 (0.10)	0.00 (0.02)	0.01 (0.02)	0.79 (0.04)	0.76 (0.05)	0.96 (0.01)	0.96 (0.01)

Note. Numbers in the mixing proportion column show bias and MSE (in the parenthesis) of π_1 .

Numbers in the membership column show average membership recovery and its standard deviation (in the parenthesis).

Table 4

Mixing proportion and membership recovery when N=1000 and unbalanced mixing proportions

	Mixing proportion				Membership			
	MD1		MD2		MD1		MD2	
	Median	Mean	Median	Mean	Median	Mean	Median	Mean
	GMM	GMM	GMM	GMM	GMM	GMM	GMM	GMM
D1	0.01 (0.03)	0.00 (0.03)	0.00 (0.02)	0.00 (0.02)	0.78 (0.04)	0.79 (0.03)	0.96 (0.01)	0.96 (0.01)
D2	0.00 (0.03)	-0.15 (0.18)	0.00 (0.02)	0.01 (0.02)	0.78 (0.03)	0.74 (0.04)	0.96 (0.01)	0.95 (0.01)
D3	0.00 (0.03)	-0.18 (0.19)	0.00 (0.02)	0.01 (0.02)	0.79 (0.02)	0.72 (0.03)	0.96 (0.01)	0.94 (0.01)
D4	0.00 (0.03)	-0.16 (0.19)	0.00 (0.02)	0.00 (0.02)	0.80 (0.03)	0.74 (0.05)	0.96 (0.01)	0.96 (0.01)

Note. Numbers in the mixing proportion column show bias and MSE (in the parenthesis) of π_1 .

Numbers in the membership column show average membership recovery and its standard deviation (in the parenthesis).

Table 5

Mixing proportion and membership recovery when N=300 and balanced mixing proportions

	Mixing proportion				Membership			
	MD1		MD2		MD1		MD2	
	Median	Mean	Median	Mean	Median	Mean	Median	Mean
	GMM	GMM	GMM	GMM	GMM	GMM	GMM	GMM
D1	0.00 (0.02)	0.00 (0.02)	0.00 (0.03)	0.00 (0.03)	0.74 (0.03)	0.74 (0.05)	0.95 (0.01)	0.95 (0.01)
D2	0.00 (0.02)	-0.01 (0.03)	0.00 (0.03)	0.00 (0.03)	0.72 (0.06)	0.74 (0.03)	0.95 (0.01)	0.94 (0.01)
D3	0.00 (0.02)	-0.02 (0.04)	0.00 (0.03)	0.01 (0.03)	0.74 (0.04)	0.73 (0.03)	0.95 (0.01)	0.93 (0.01)
D4	0.00 (0.02)	-0.01 (0.03)	0.00 (0.03)	0.00 (0.03)	0.74 (0.03)	0.76 (0.03)	0.96 (0.01)	0.95 (0.01)

Note. Numbers in the mixing proportion column show bias and MSE (in the parenthesis) of π_1 .

Numbers in the membership column show average membership recovery and its standard deviation (in the parenthesis).

Table 6

Mixing proportion and membership recovery when N=500 and balanced mixing proportions

	Mixing proportion				Membership			
	MD1		MD2		MD1		MD2	
	Median	Mean	Median	Mean	Median	Mean	Median	Mean
	GMM	GMM	GMM	GMM	GMM	GMM	GMM	GMM
D1	0.00 (0.03)	0.00 (0.03)	0.00 (0.02)	0.00 (0.02)	0.72 (0.06)	0.74 (0.04)	0.95 (0.01)	0.95 (0.01)
D2	0.00 (0.02)	-0.02 (0.03)	0.00 (0.02)	0.00 (0.02)	0.74 (0.04)	0.74 (0.02)	0.95 (0.01)	0.94 (0.01)
D3	0.00 (0.03)	-0.03 (0.03)	0.00 (0.02)	0.01 (0.02)	0.75 (0.03)	0.73 (0.03)	0.95 (0.01)	0.93 (0.01)
D4	0.00 (0.02)	-0.02 (0.03)	0.00 (0.02)	0.00 (0.02)	0.76 (0.04)	0.76 (0.02)	0.96 (0.01)	0.95 (0.01)

Note. Numbers in the mixing proportion column show bias and MSE (in the parenthesis) of π_1 .

Numbers in the membership column show average membership recovery and its standard deviation (in the parenthesis).

Table 7

Mixing proportion and membership recovery when N=1000 and balanced mixing proportions

	Mixing proportion				Membership			
	MD1		MD2		MD1		MD2	
	Median	Mean	Median	Mean	Median	Mean	Median	Mean
	GMM	GMM	GMM	GMM	GMM	GMM	GMM	GMM
D1	0.00 (0.03)	0.01 (0.03)	0.00 (0.02)	0.00 (0.02)	0.72 (0.07)	0.75 (0.04)	0.95 (0.01)	0.95 (0.01)
D2	0.00 (0.03)	-0.03 (0.05)	0.00 (0.02)	0.01 (0.02)	0.74 (0.04)	0.74 (0.03)	0.95 (0.01)	0.94 (0.01)
D3	0.00 (0.03)	-0.06 (0.10)	0.00 (0.02)	0.01 (0.02)	0.75 (0.02)	0.72 (0.045)	0.95 (0.01)	0.94 (0.01)
D4	0.00 (0.03)	-0.03 (0.05)	0.00 (0.02)	0.00 (0.02)	0.77 (0.03)	0.76 (0.03)	0.96 (0.01)	0.95 (0.01)

Note. Numbers in the mixing proportion column show bias and MSE (in the parenthesis) of π_1 .

Numbers in the membership column show average membership recovery and its standard deviation (in the parenthesis).

Table 8

Descriptive Statistics for the Empirical Data

	Time 1	Time 2	Time 3	Time 4
Minimum	3.20	1.75	1.50	0.75
Q1	9.13	9.00	9.50	9.54
Median	11.00	11.25	11.86	11.75
Mean	13.29	13.34	13.98	14.18
Q3	14.75	15.33	16.00	16.25
Max	141.50	62.92	83.33	97.50
SD	7.75	7.04	7.62	8.16
Skewness	5.36	2.42	2.97	3.20

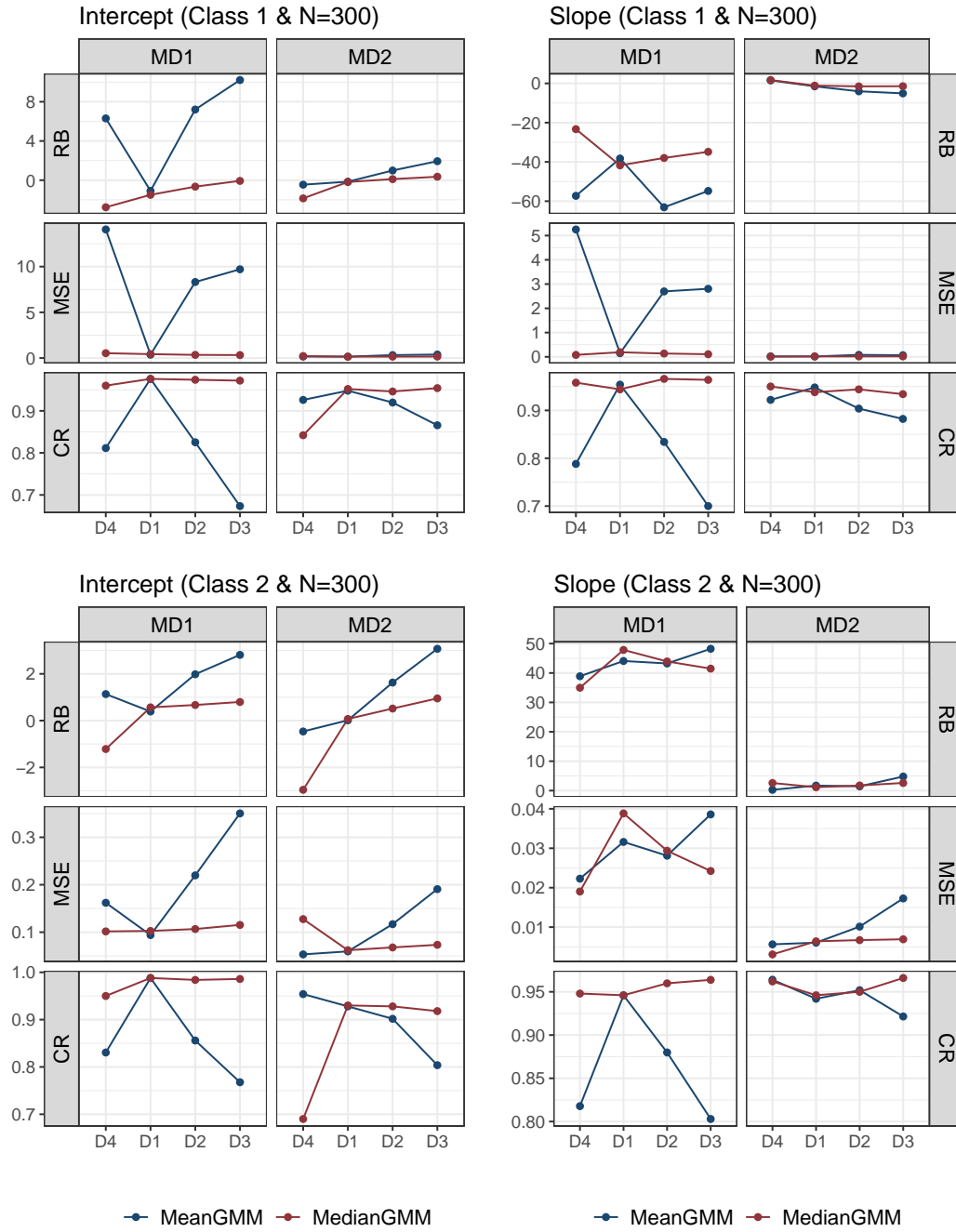


Figure 1. Estimation results for the intercept and slope parameters when $N=300$ and mixing proportions were unbalanced. RB represents relative bias, and CR represents coverage rate.

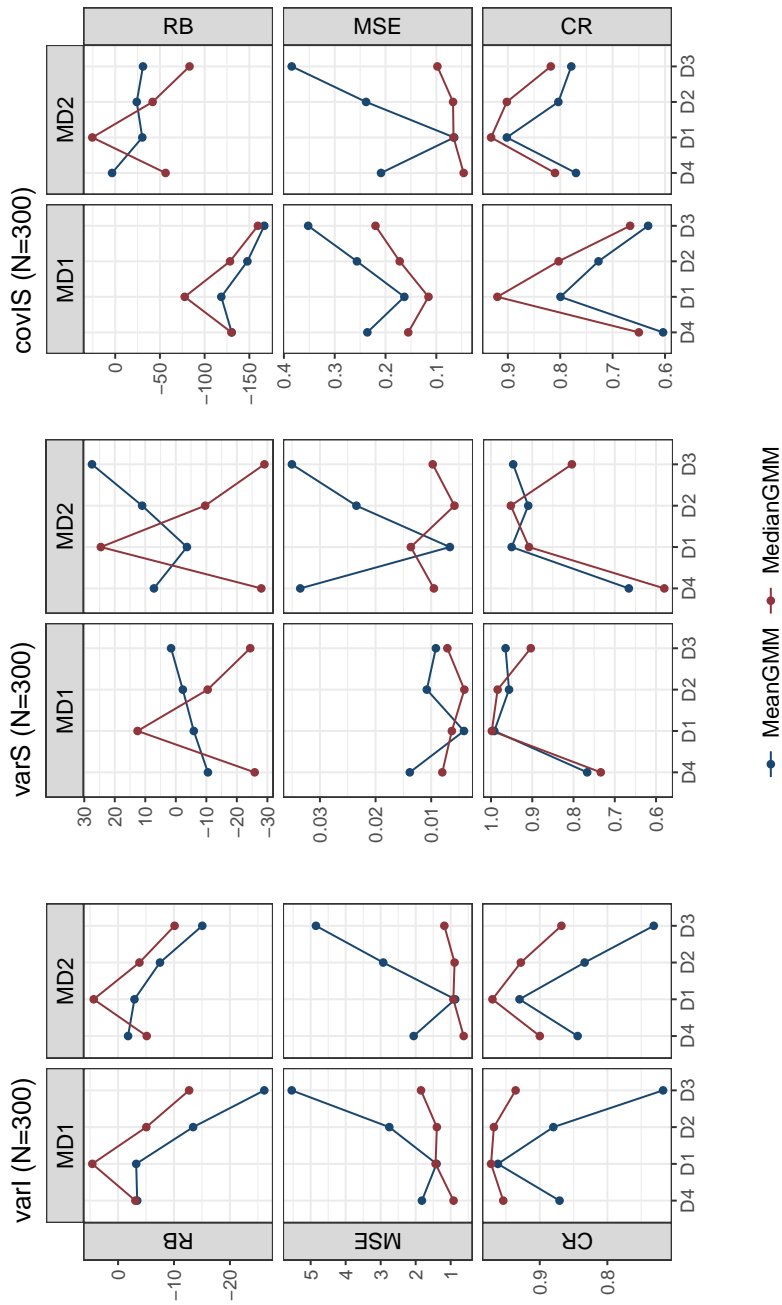


Figure 2. Estimation results for the parameters in Ψ when $N=300$ and mixing proportions were unbalanced. RB represents relative bias, and CR represents coverage rate. varI shows results for intercept variance estimates, varS shows results for slope variance estimates, and covIS shows results for intercept-slope covariance estimates.

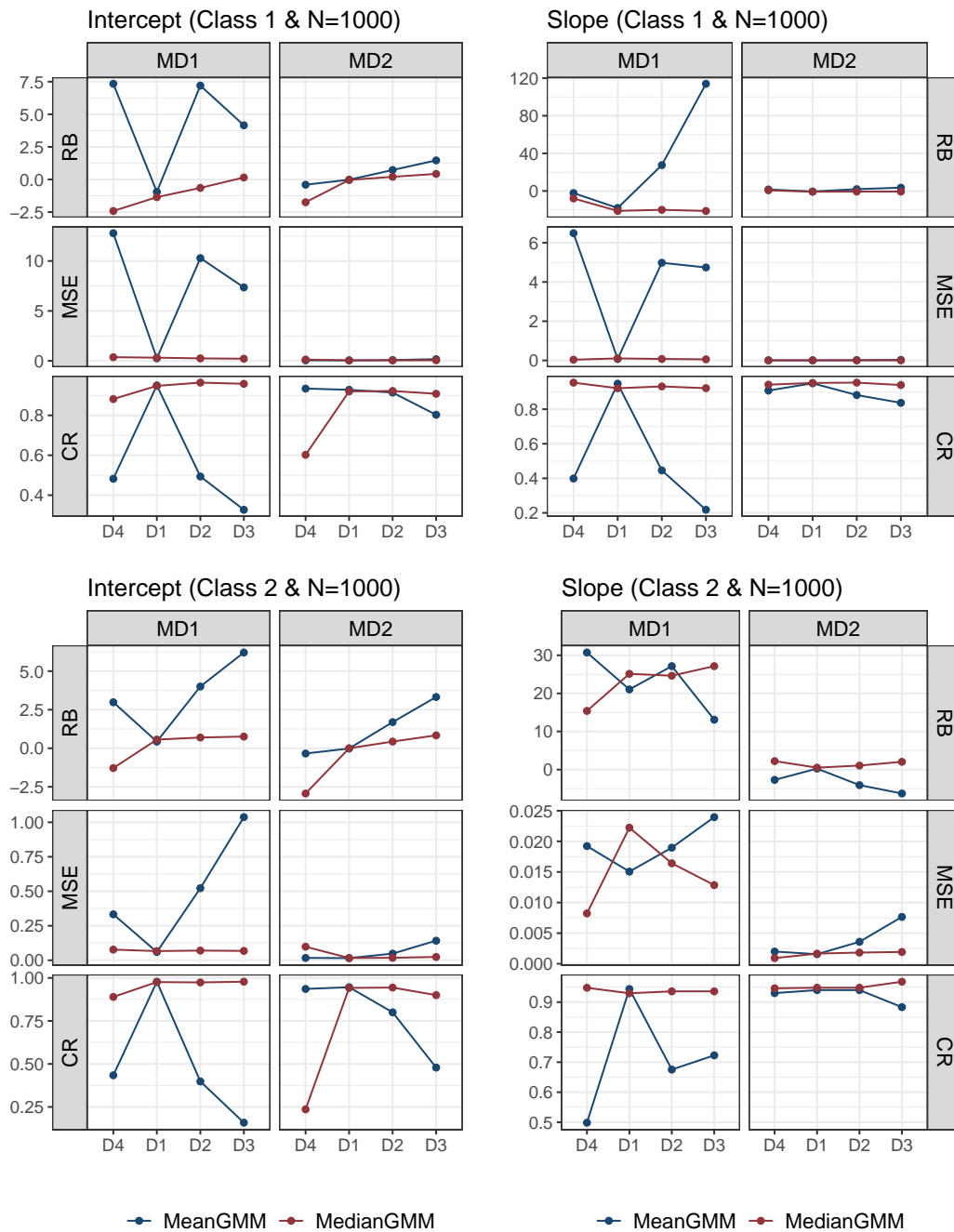


Figure 3. Estimation results for the intercept and slope parameters when $N=1000$ and mixing proportions were unbalanced. RB represents relative bias, and CR represents coverage rate.

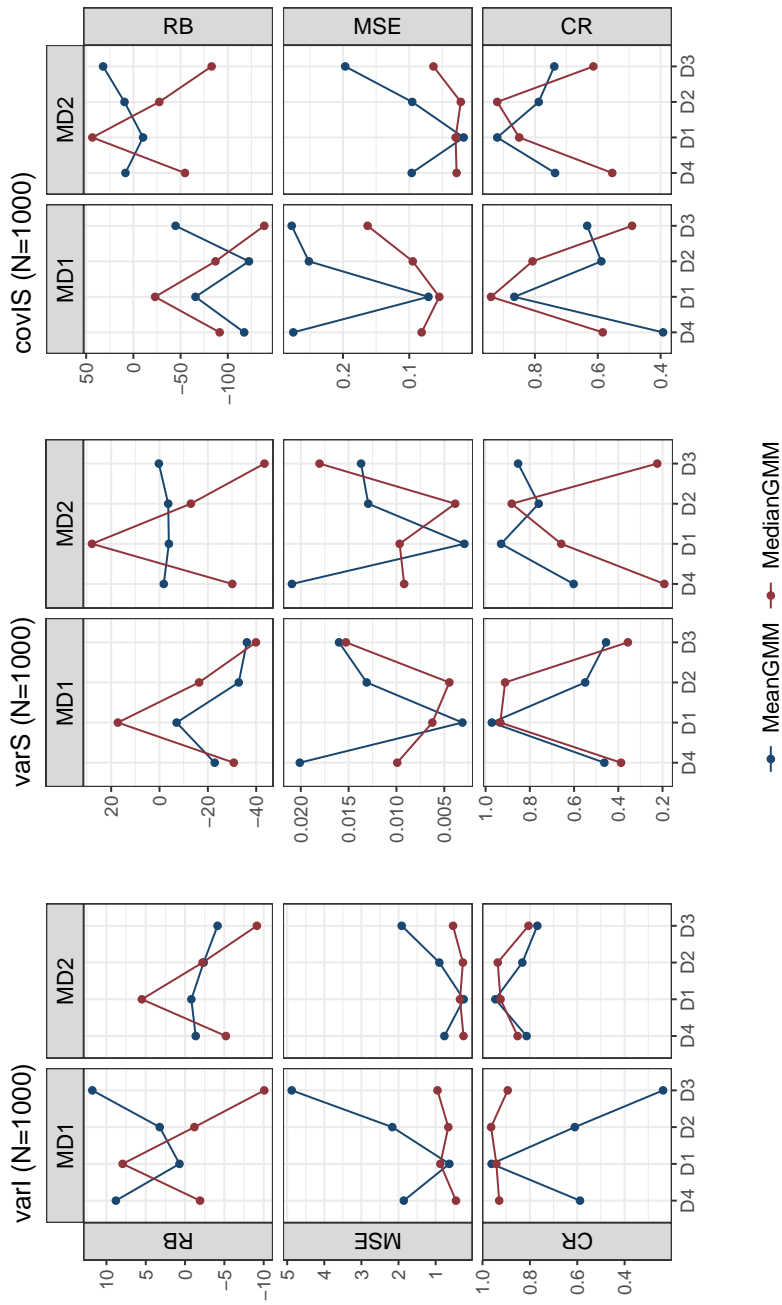


Figure 4. Estimation results for the parameters in Ψ when $N=1000$ and mixing proportions were unbalanced. RB represents relative bias, and CR represents coverage rate. varI shows results for intercept variance estimates, varS shows results for slope variance estimates, and covIS shows results for intercept-slope covariance estimates.

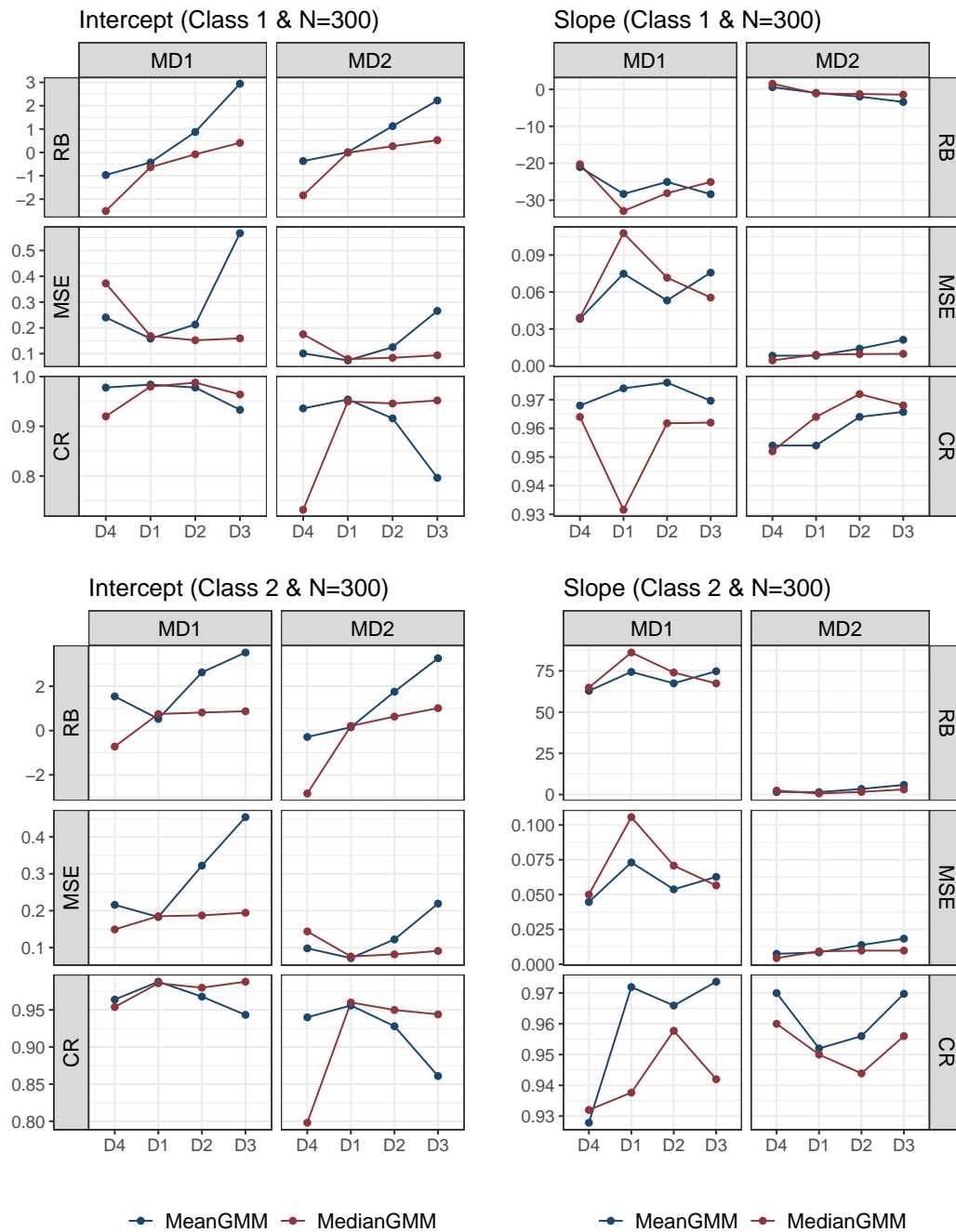


Figure 5. Estimation results for the intercept and slope parameters when $N=300$ and mixing proportions were balanced. RB represents relative bias, and CR represents coverage rate.

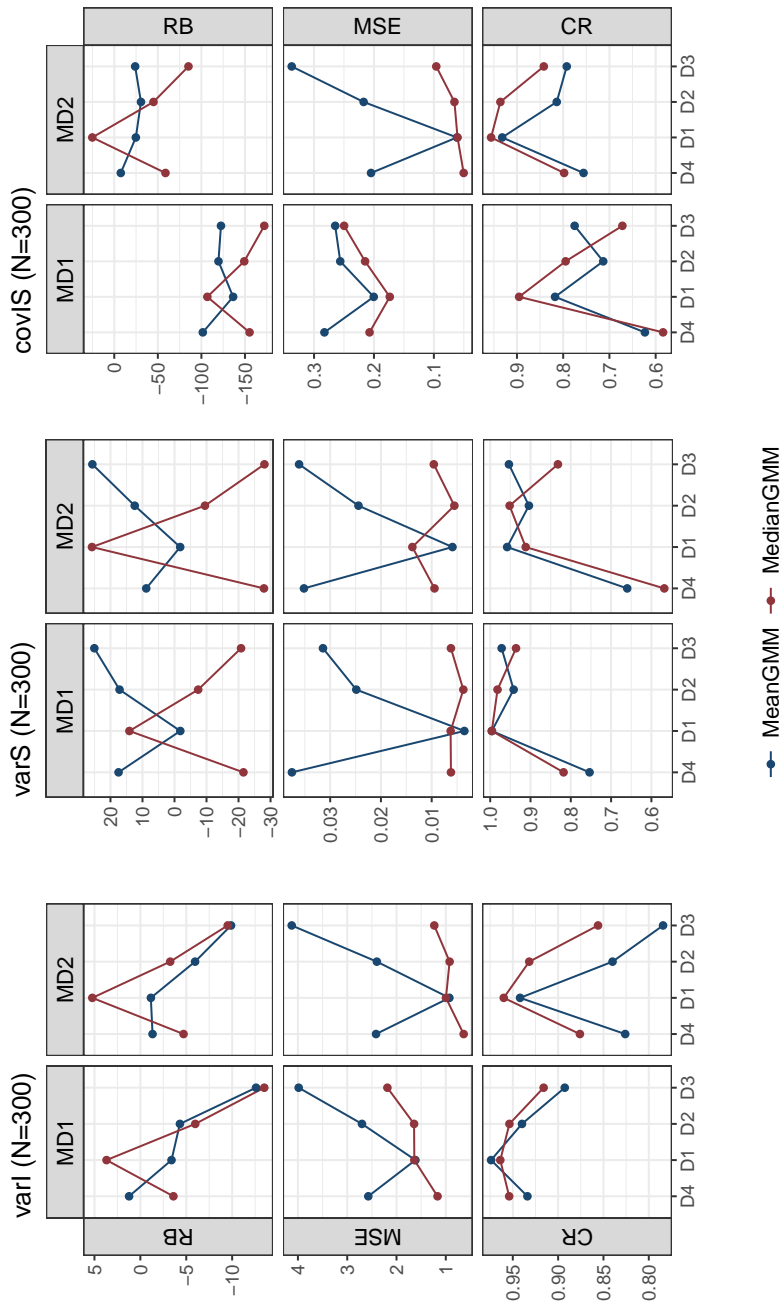


Figure 6. Estimation results for the parameters in Ψ when $N=300$ and mixing proportions were balanced. RB represents relative bias, and CR represents coverage rate. varI shows results for intercept variance estimates, varS shows results for slope variance estimates, and covIS shows results for intercept-slope covariance estimates.

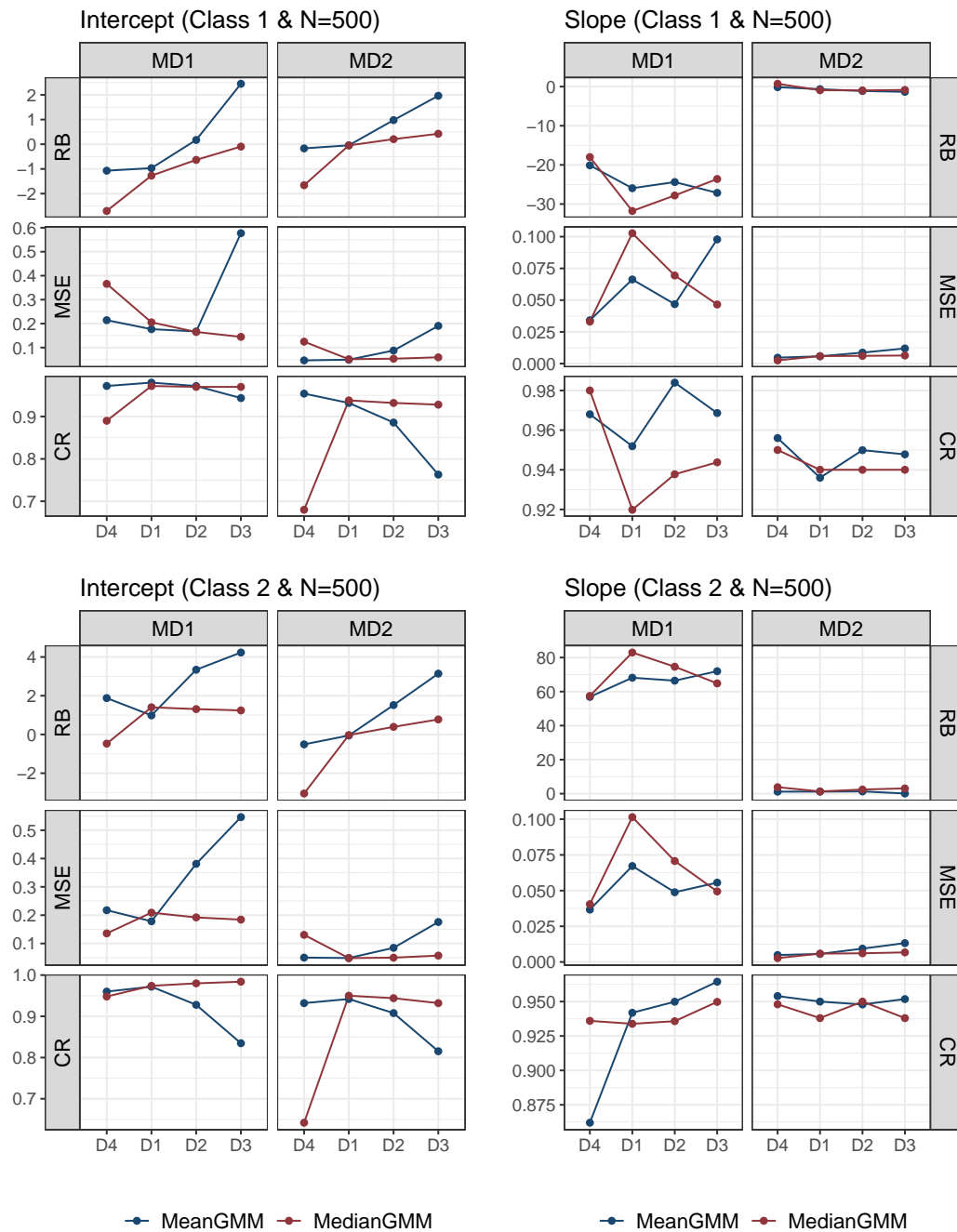


Figure 7. Estimation results for the intercept and slope parameters when $N=500$ and mixing proportions were balanced. RB represents relative bias, and CR represents coverage rate.

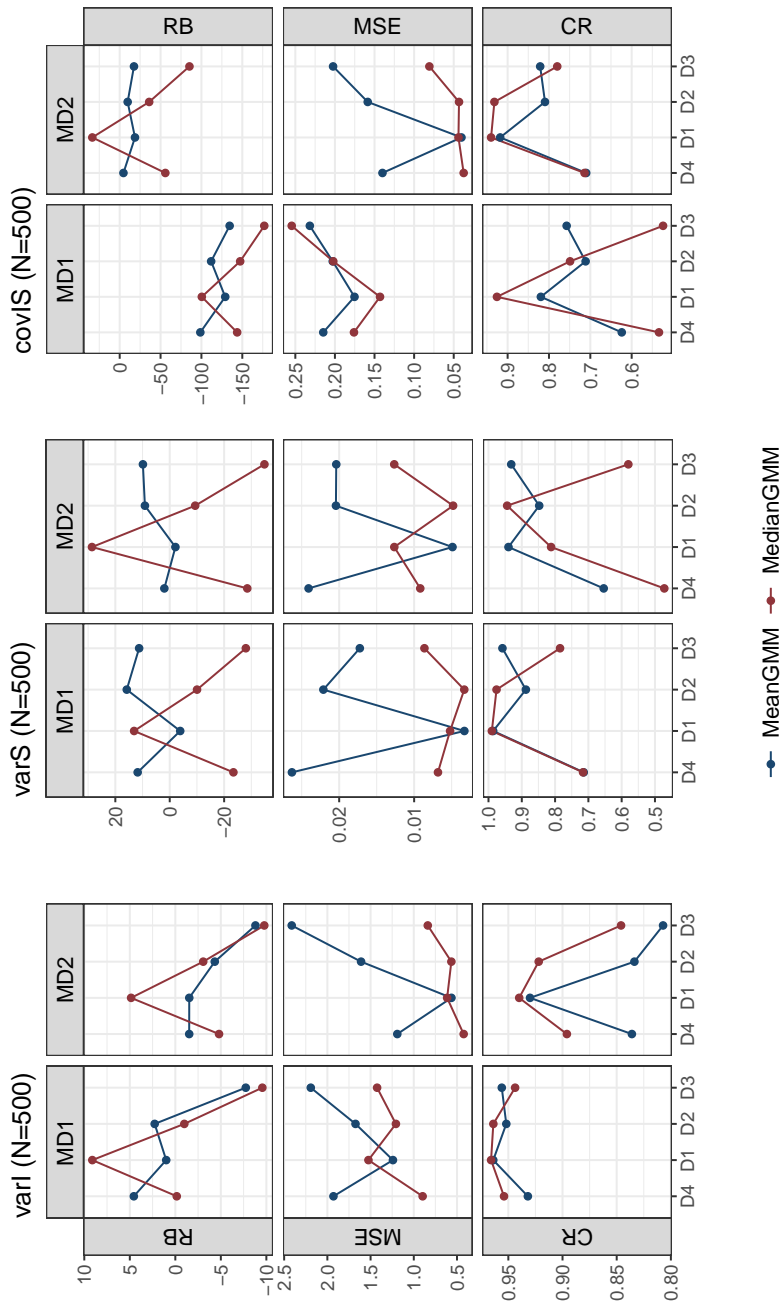


Figure 8. Estimation results for the parameters in Ψ when $N=500$ and mixing proportions were balanced. RB represents relative bias, and CR represents coverage rate. varI shows results for intercept variance estimates, varS shows results for slope variance estimates, and covIS shows results for intercept-slope covariance estimates.

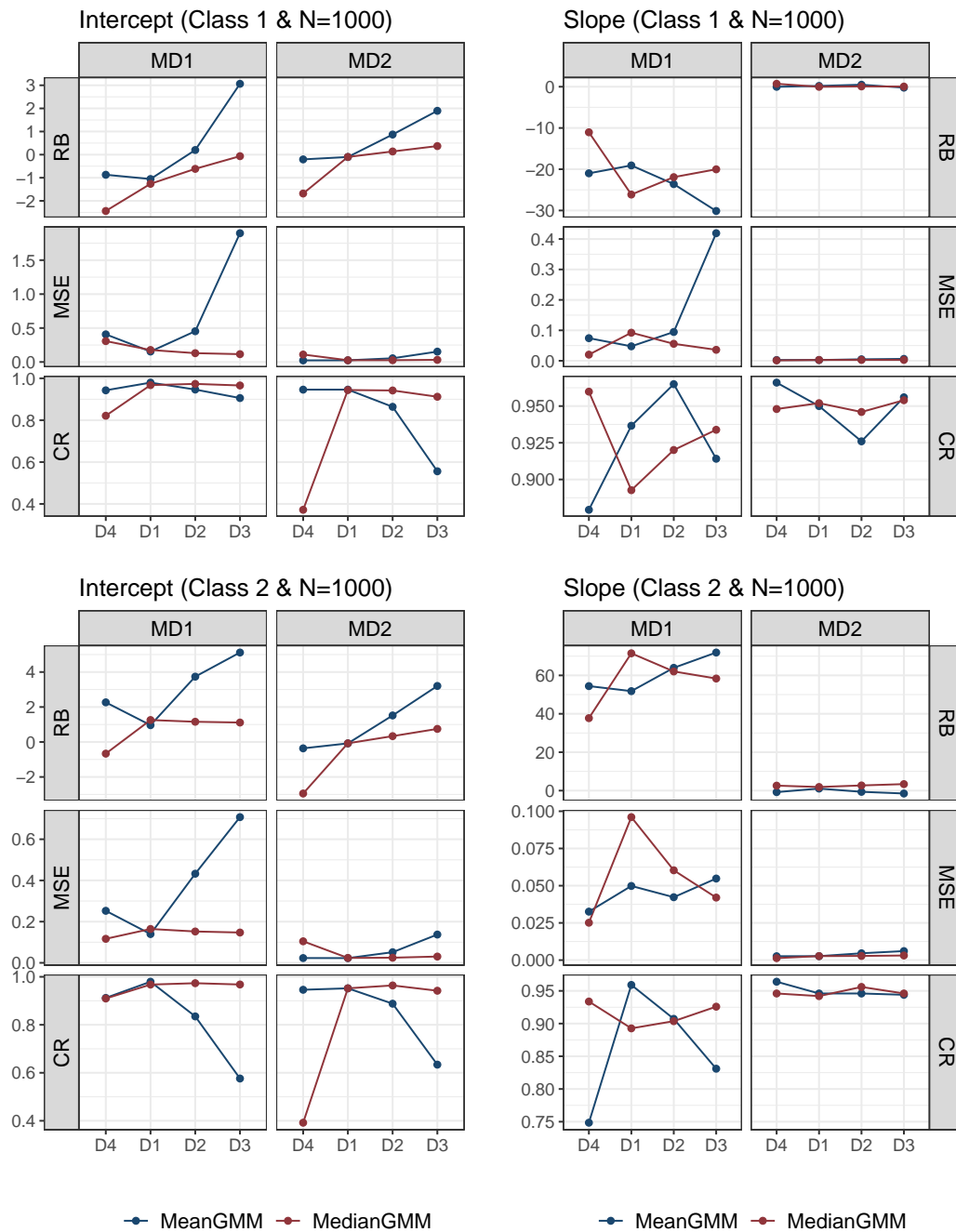


Figure 9. Estimation results for the intercept and slope parameters when $N=1000$ and mixing proportions were balanced. RB represents relative bias, and CR represents coverage rate.

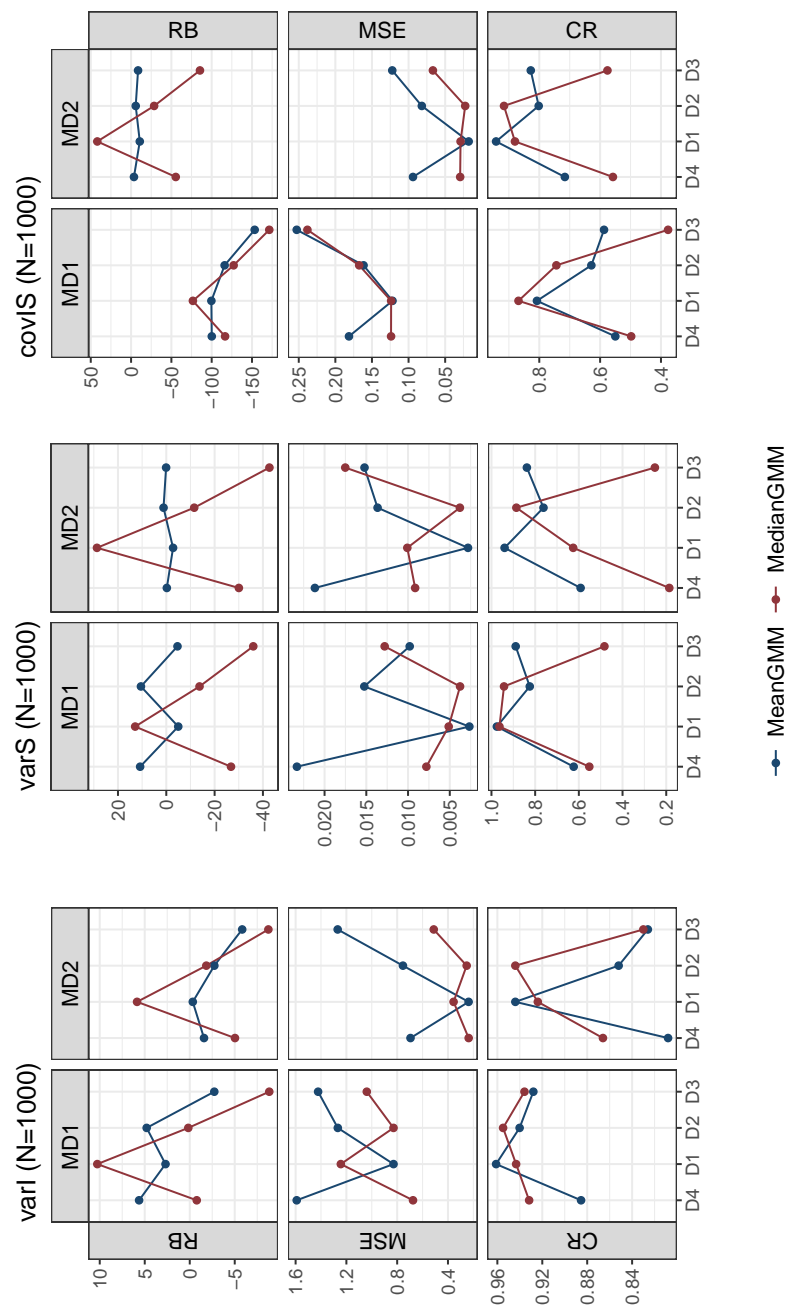


Figure 10. Estimation results for the parameters in Ψ when $N=1000$ and mixing proportions were balanced. RB represents relative bias, and CR represents coverage rate. varI shows results for intercept variance estimates, varS shows results for slope variance estimates, and covIS shows results for intercept-slope covariance estimates.