L'algoritmo di Bellman-Ford (*single-source shortest-path algorithm*)

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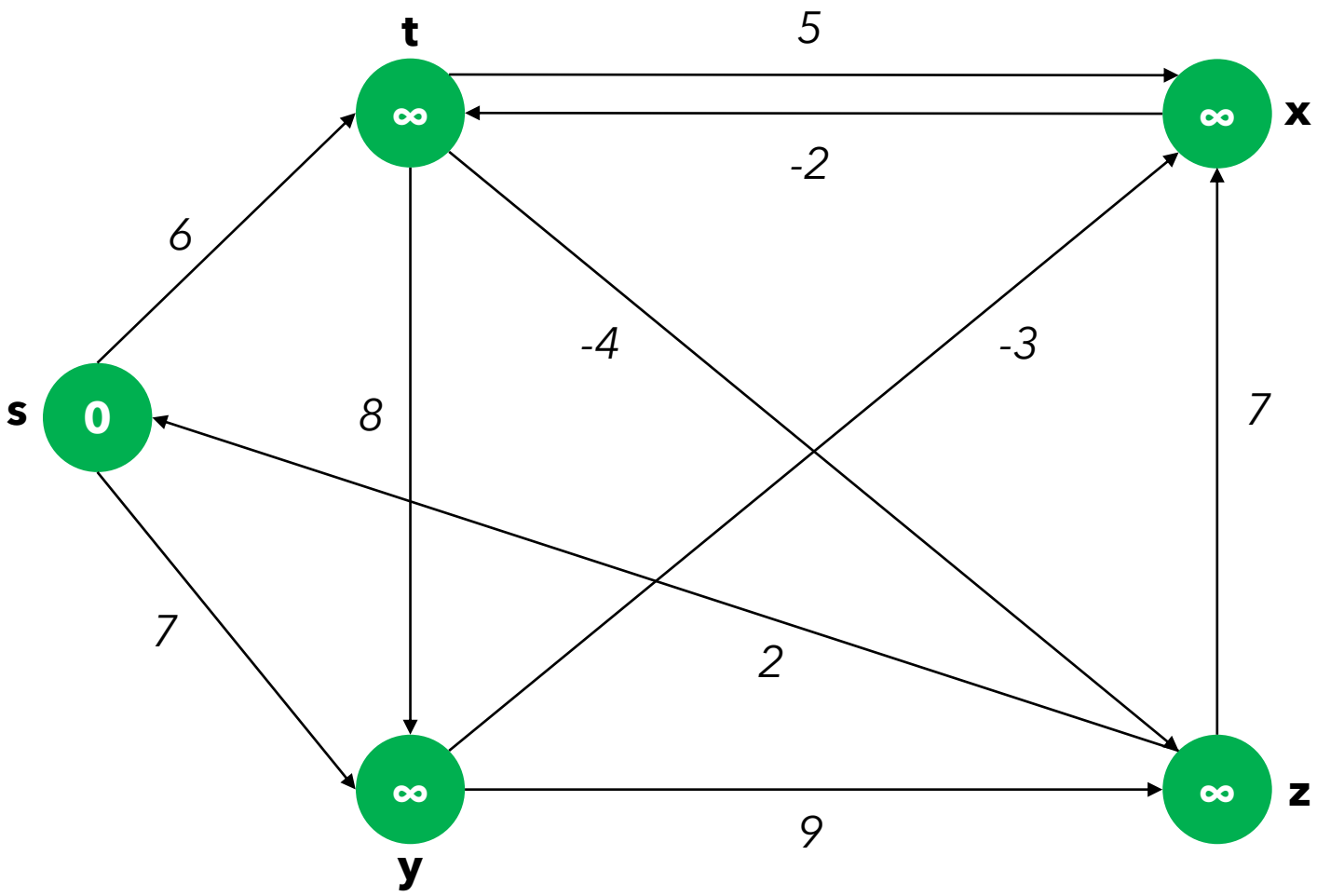
L'algoritmo di Bellman-Ford



vogliamo trovare i cammini di costo minimo da **s** verso tutti gli altri nodi

le chiavi dei nodi sono valorizzate con il costo dello shortest path da **s** noto all'iterazione corrente (inizialmente infinito)

a differenza dell'algoritmo di Dijkstra, sono ammessi costi negativi

L’algoritmo di Bellman-Ford

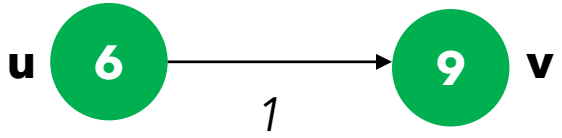
```
bellman_ford_part_1(G):
  repeat |G.V| - 1 times:
    for each edge (u, v) in G:
      if u.cost + cost(u, v) < v.cost:
        v.cost = u.cost + cost(u, v)
        v.predecessor = u
```

- `s`: sorgente da cui calcolare i cammini minimi verso tutti gli altri nodi
- `|G.V|`: numero di nodi del grafo `G`
- `u.cost`: costo minimo noto all’iterazione corrente del path `s -> u`
- `cost(u, v)`: costo dell’arco `(u, v)`

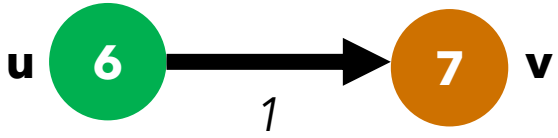
L’algoritmo di Bellman-Ford

```
bellman_ford_part_1(G):
  repeat |G.V| - 1 times:
    for each edge (u, v) in G:
      if u.cost + cost(u, v) < v.cost:
        v.cost = u.cost + cost(u, v)
        v.predecessor = u
```

le istruzioni interne al ciclo interno effettuano il **rilassamento**:



situazione pre-rilassamento



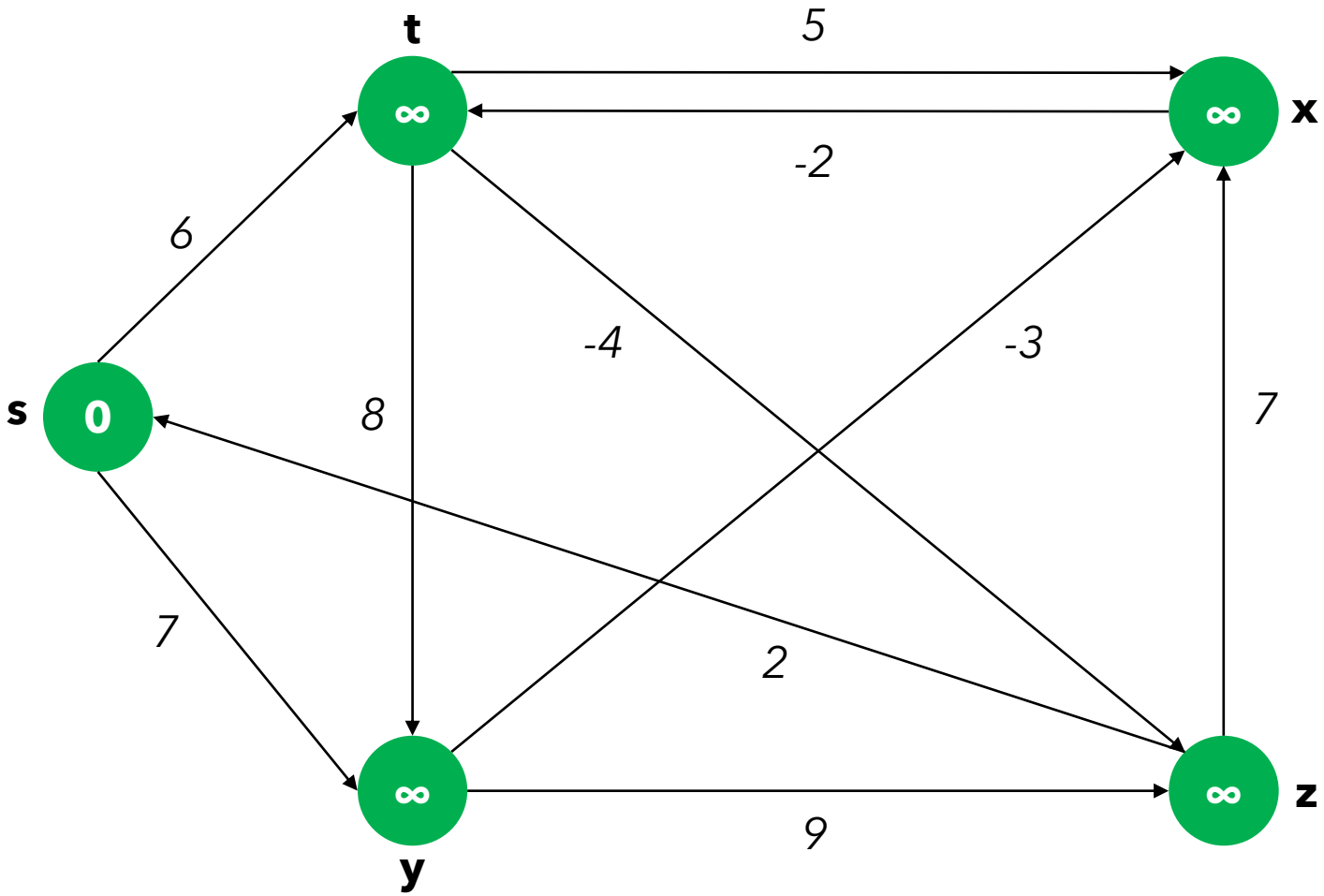
situazione post-rilassamento

L’algoritmo di Bellman-Ford

```
bellman_ford_part_1(G):
  repeat |G.V| - 1 times:
    for each edge (u, v) in G:
      if u.cost + cost(u, v) < v.cost:
        v.cost = u.cost + cost(u, v)
        v.predecessor = u
```

- il nostro grafo di esempio ha 5 nodi, quindi il ciclo esterno verrà eseguito 4 volte
- quindi si tratterà di eseguire 4 passate su tutti gli archi

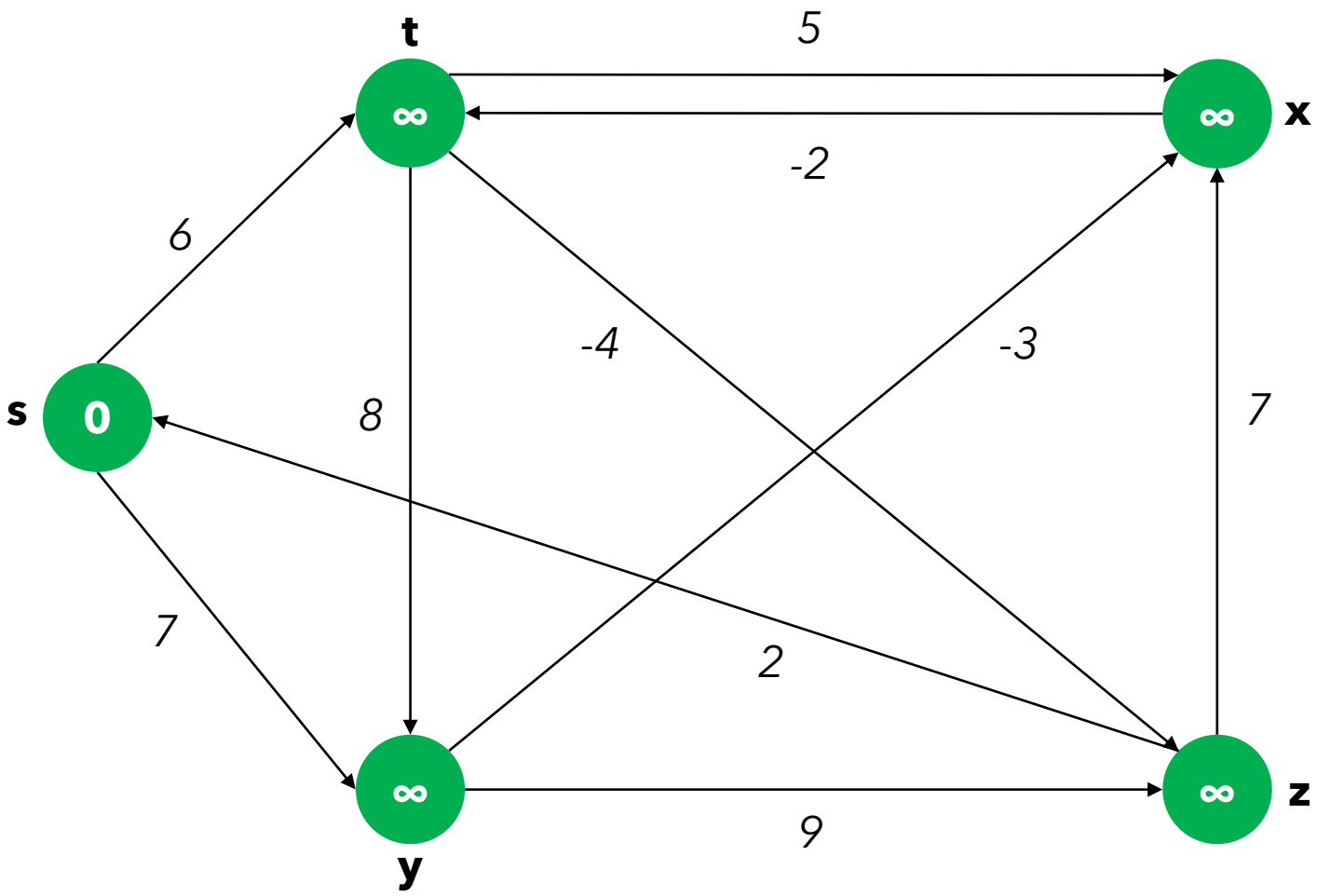
L'algoritmo di Bellman-Ford



iteriamo sugli archi del grafo ordinati (arbitrariamente) così:

```
t -> x
t -> y
t -> z
x -> t
y -> x
y -> z
z -> x
z -> s
s -> t
s -> y
```

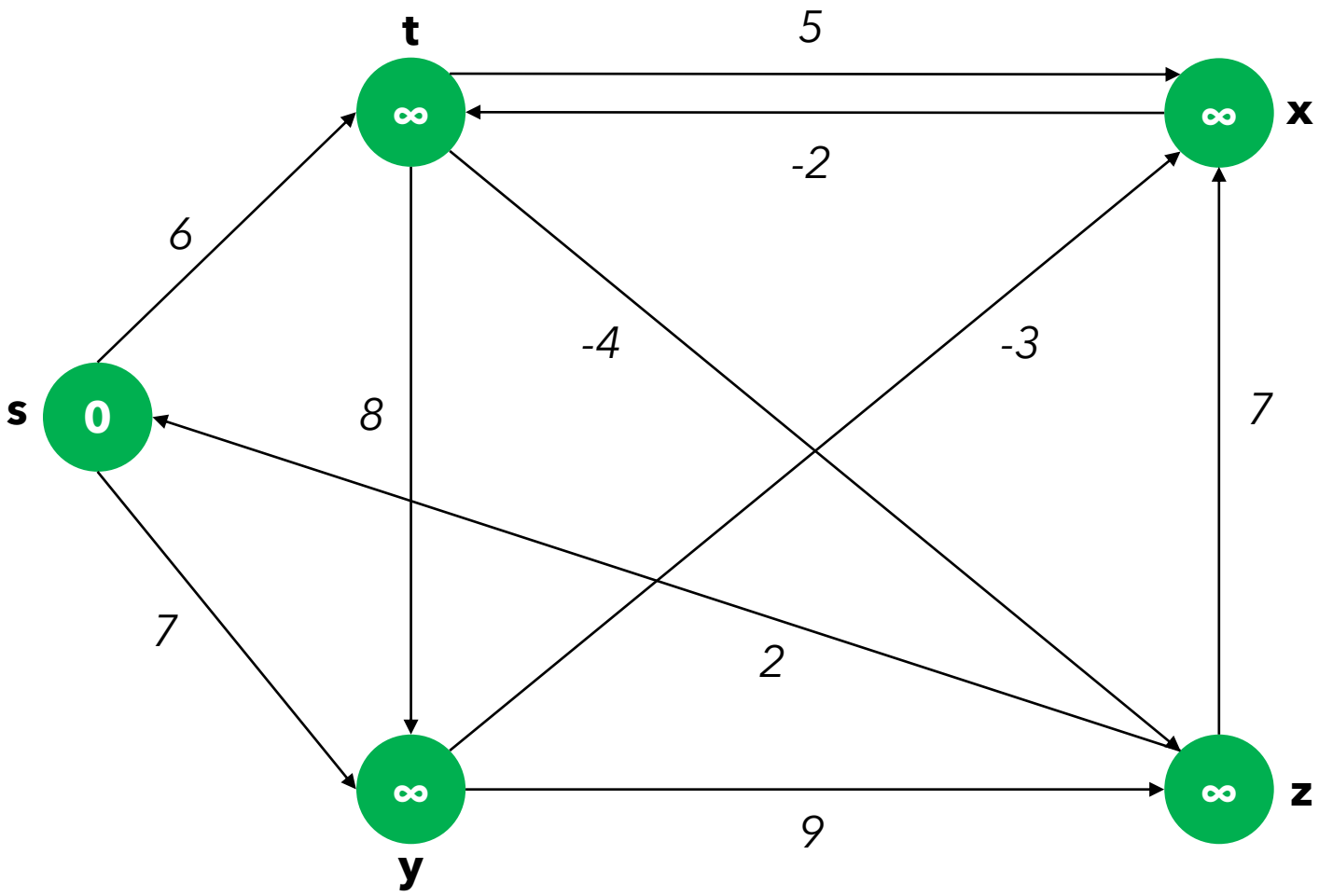
Bellman-Ford – 1st pass



t -> x
t -> y
t -> z
x -> t
y -> x
y -> z
z -> x
z -> s
s -> t
s -> y

rilassamento di t -> x (nessun effetto)

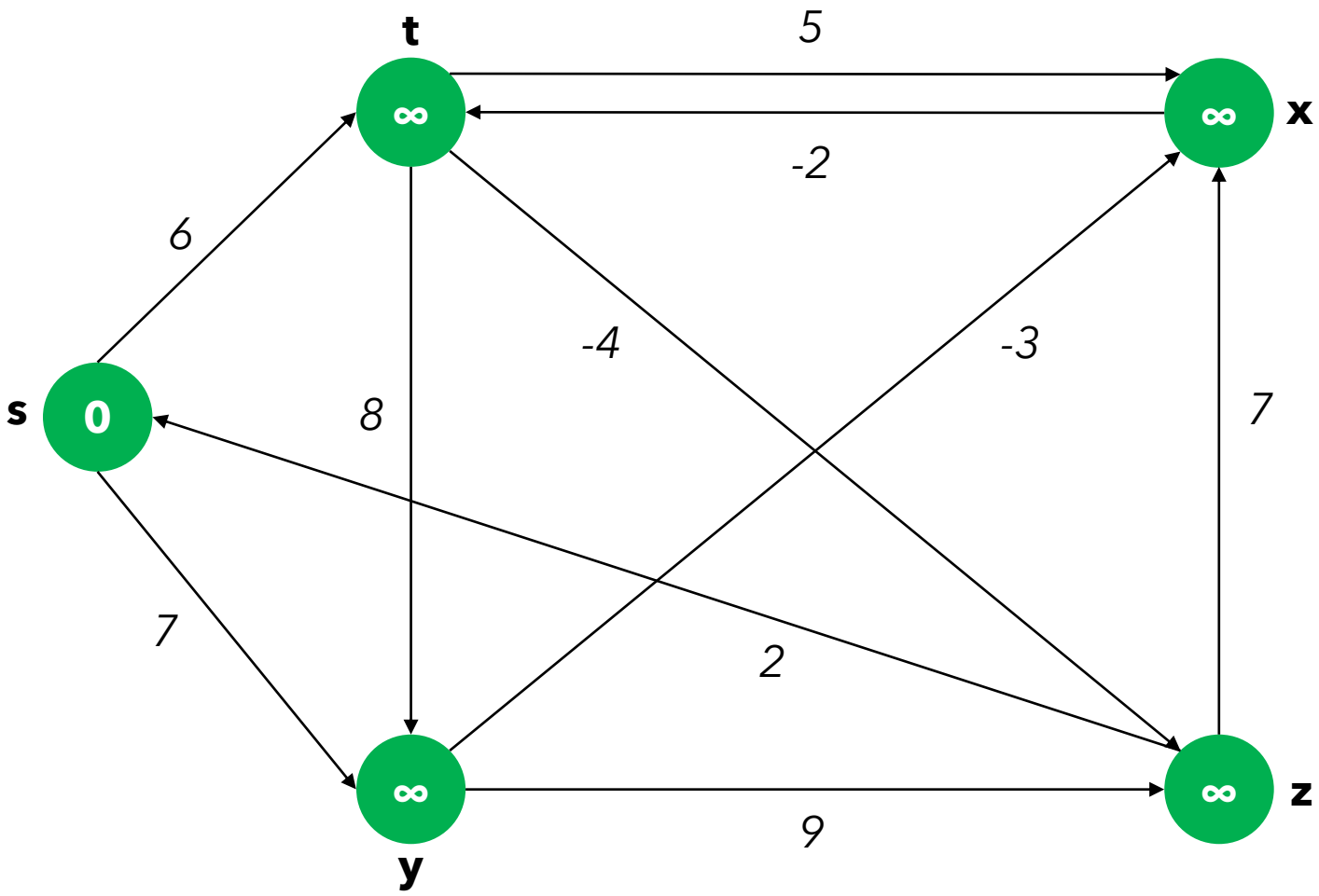
Bellman-Ford – 1st pass




```
t -> x
t -> y
t -> z
x -> t
y -> x
y -> z
z -> x
z -> s
s -> t
s -> y
```

rilassamento di `t -> y`
(nessun effetto)

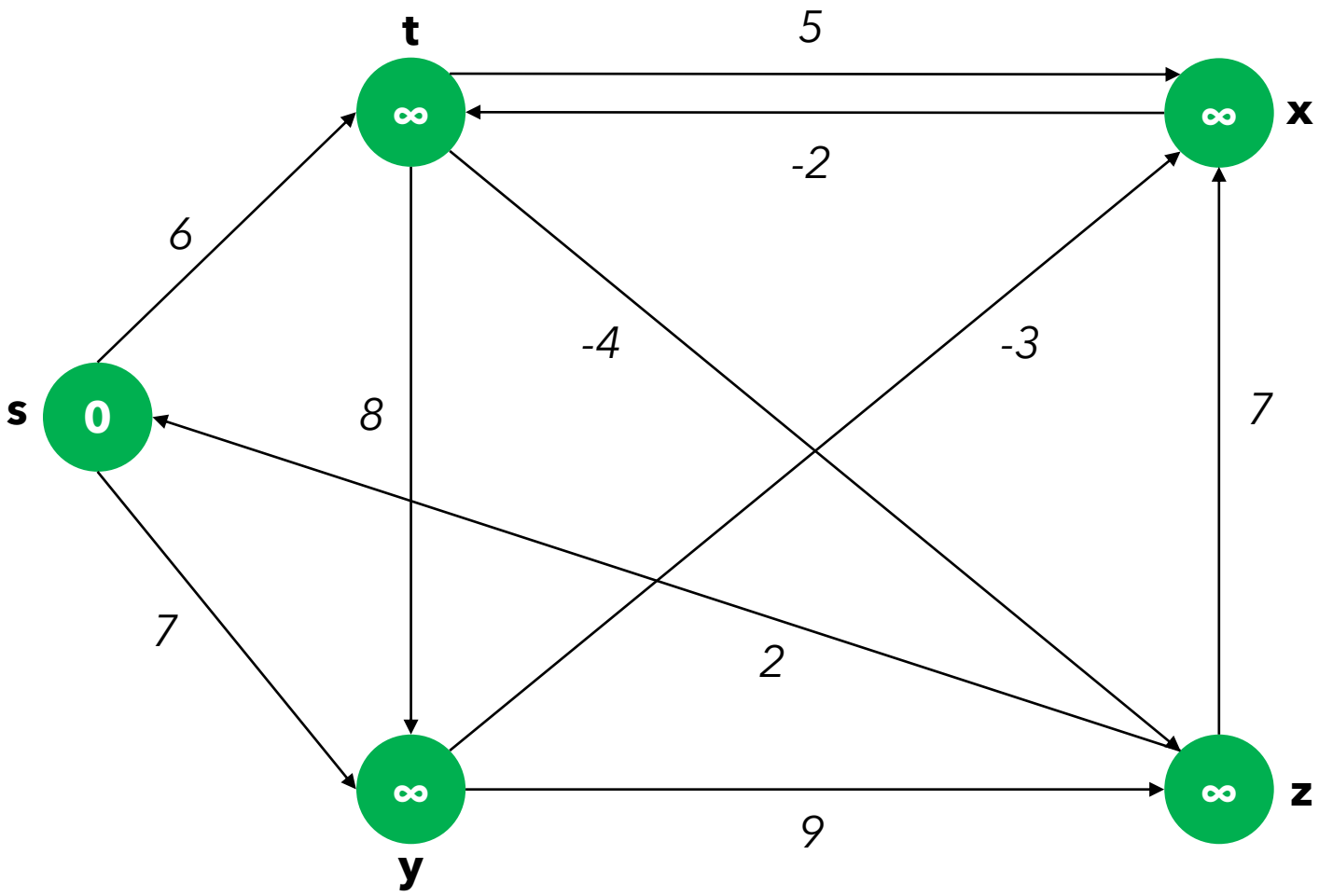
Bellman-Ford – 1st pass




```
t -> x
t -> y
t -> z
x -> t
y -> x
y -> z
z -> x
z -> s
s -> t
s -> y
```

rilassamento di `t -> z`
(nessun effetto)

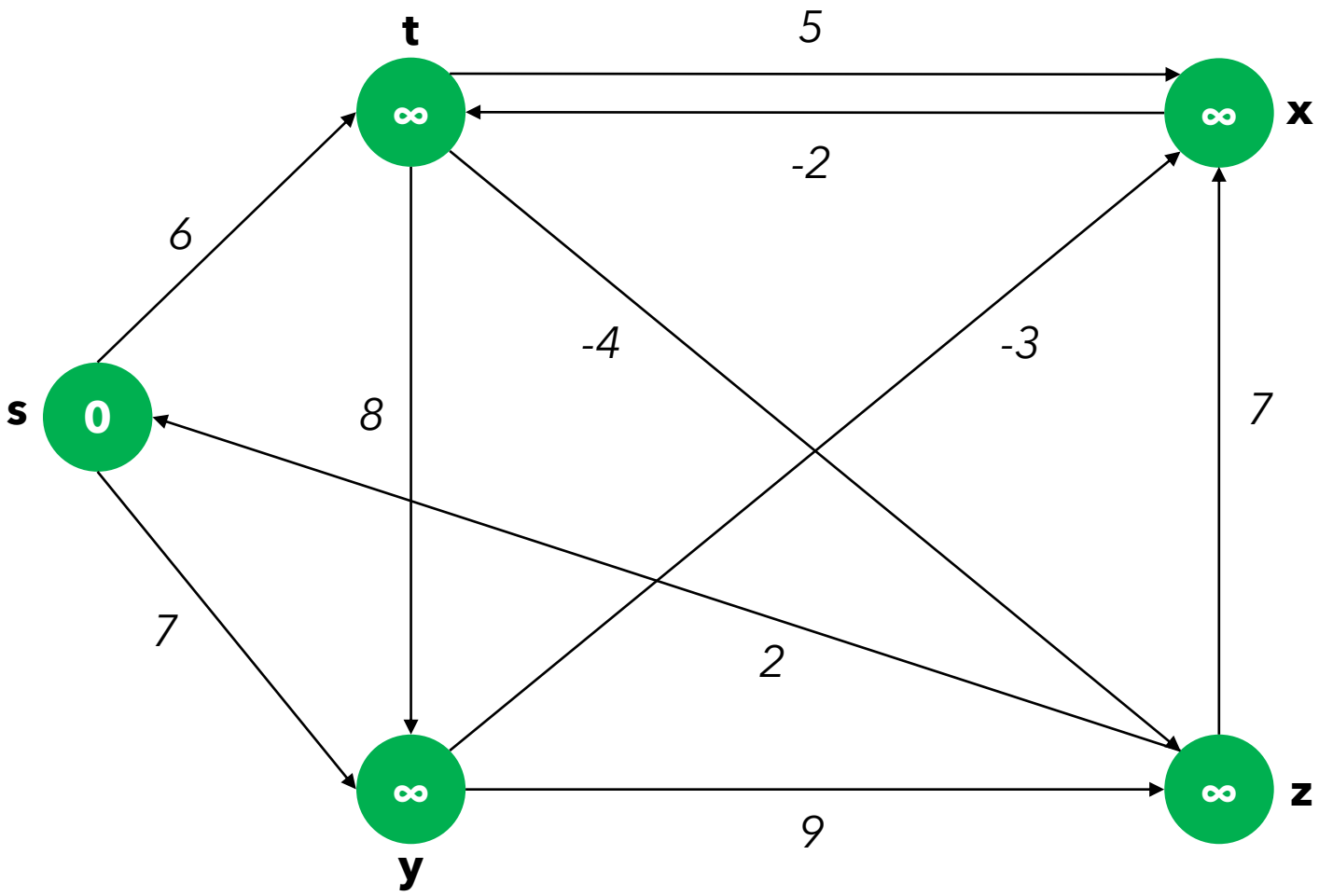
Bellman-Ford – 1st pass




```
t -> x
t -> y
t -> z
x -> t
y -> x
y -> z
z -> x
z -> s
s -> t
s -> y
```

rilassamento di `x -> t`
(nessun effetto)

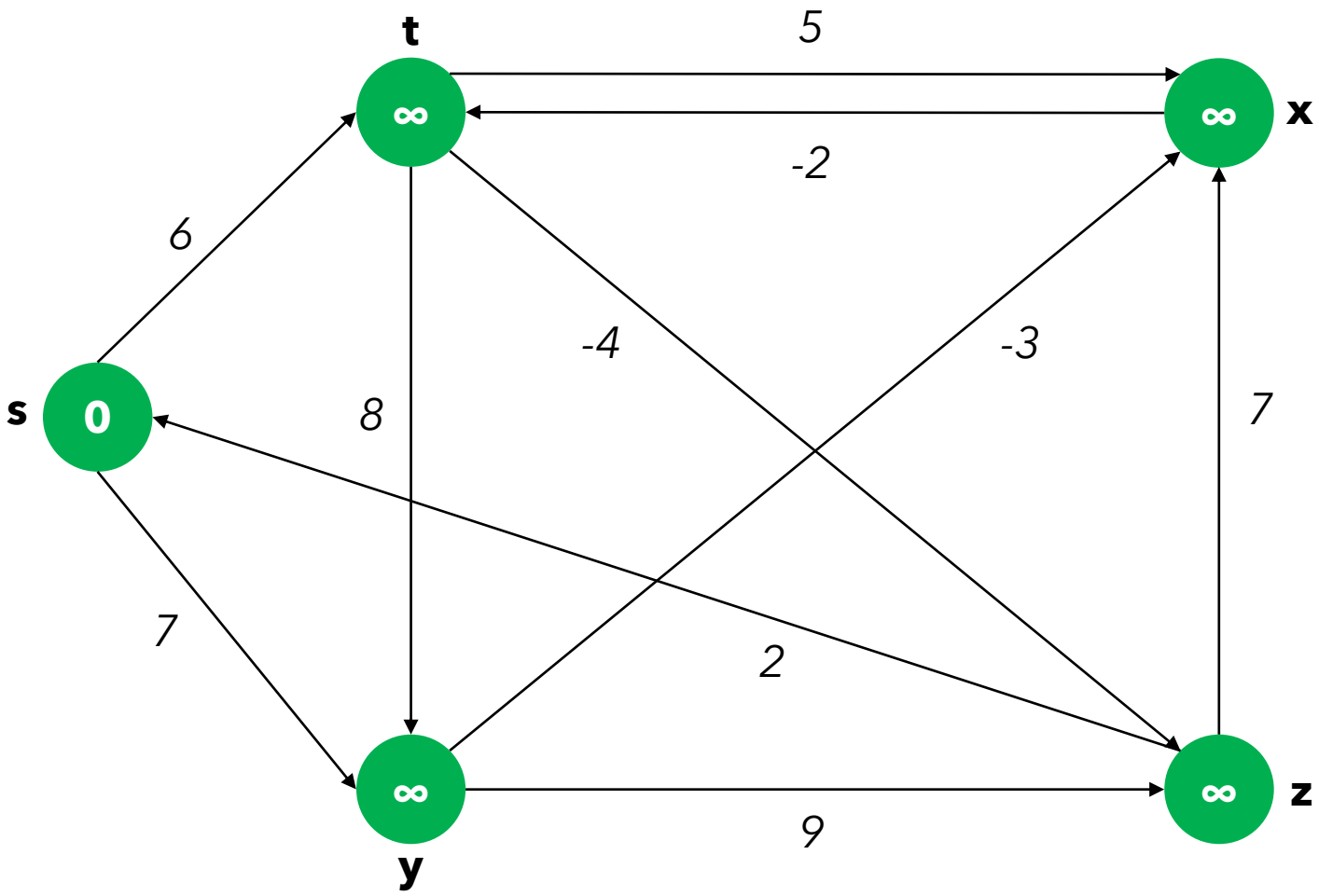
Bellman-Ford – 1st pass




```
t -> x
t -> y
t -> z
x -> t
y -> x
y -> z
z -> x
z -> s
s -> t
s -> y
```

rilassamento di y -> x (nessun effetto)

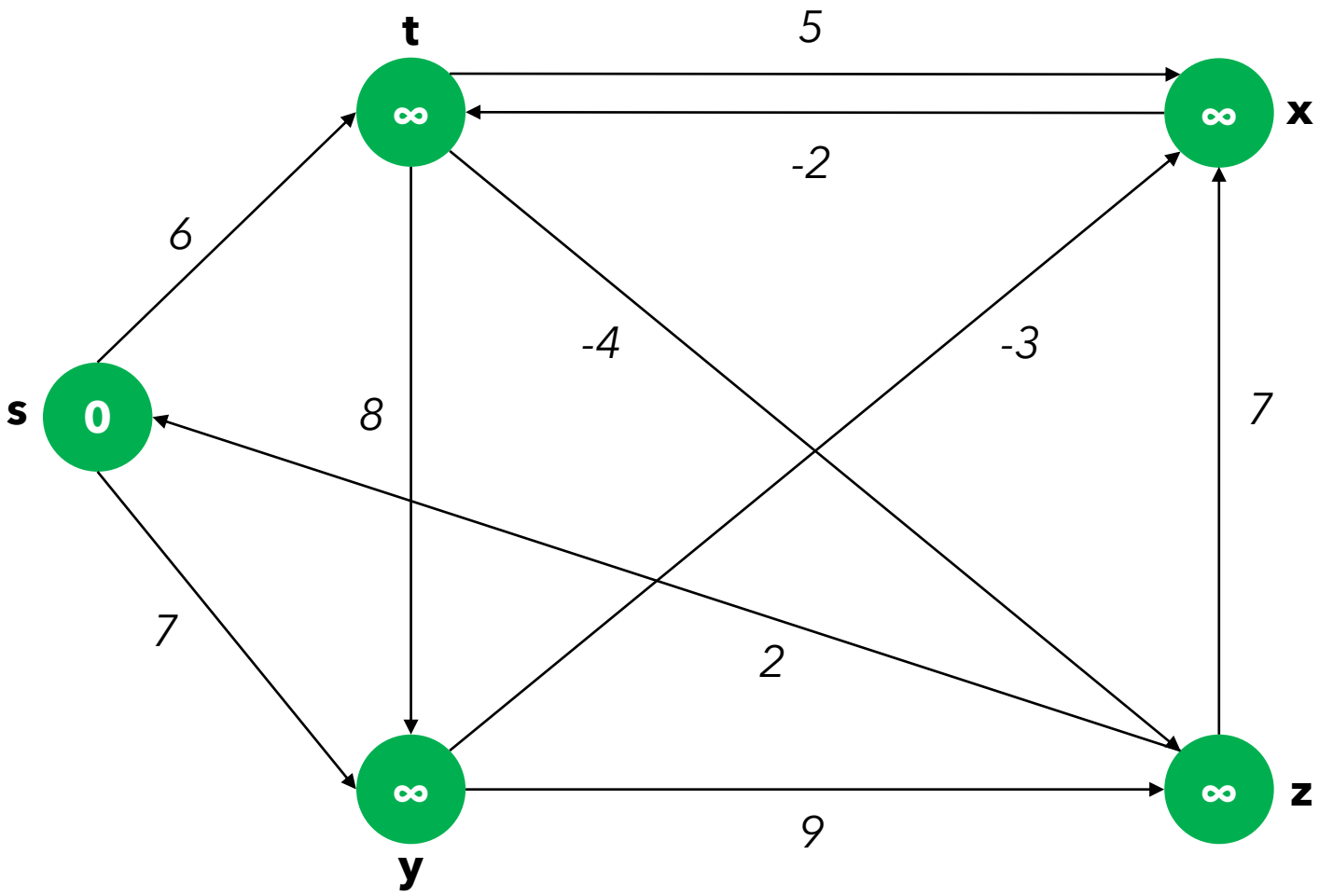
Bellman-Ford – 1st pass




```
t -> x
t -> y
t -> z
x -> t
y -> x
y -> z
z -> x
z -> s
s -> t
s -> y
```

rilassamento di y -> z (nessun effetto)

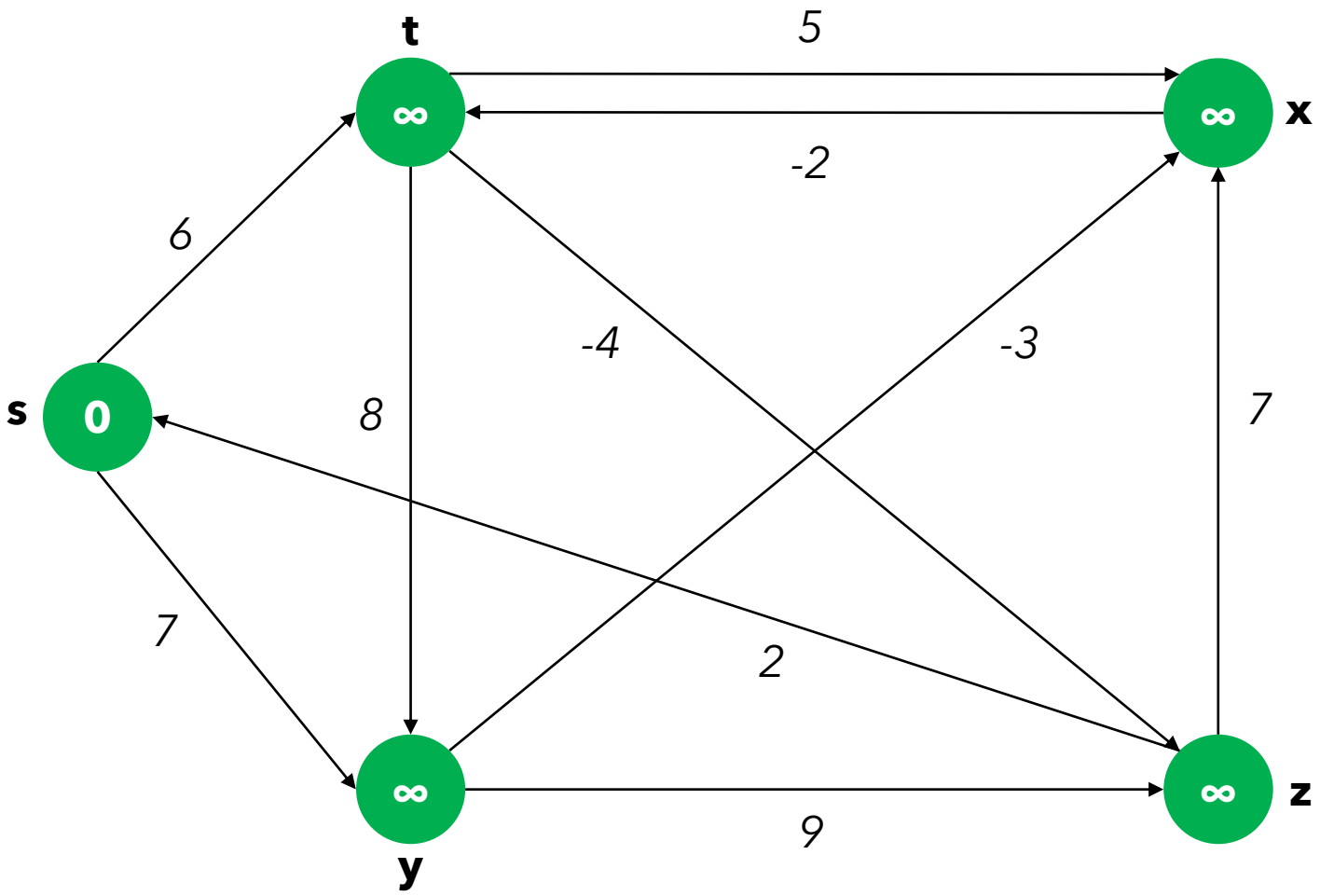
Bellman-Ford – 1st pass




```
t -> x
t -> y
t -> z
x -> t
y -> x
y -> z
z -> x
z -> s
s -> t
s -> y
```

rilassamento di z -> x (nessun effetto)

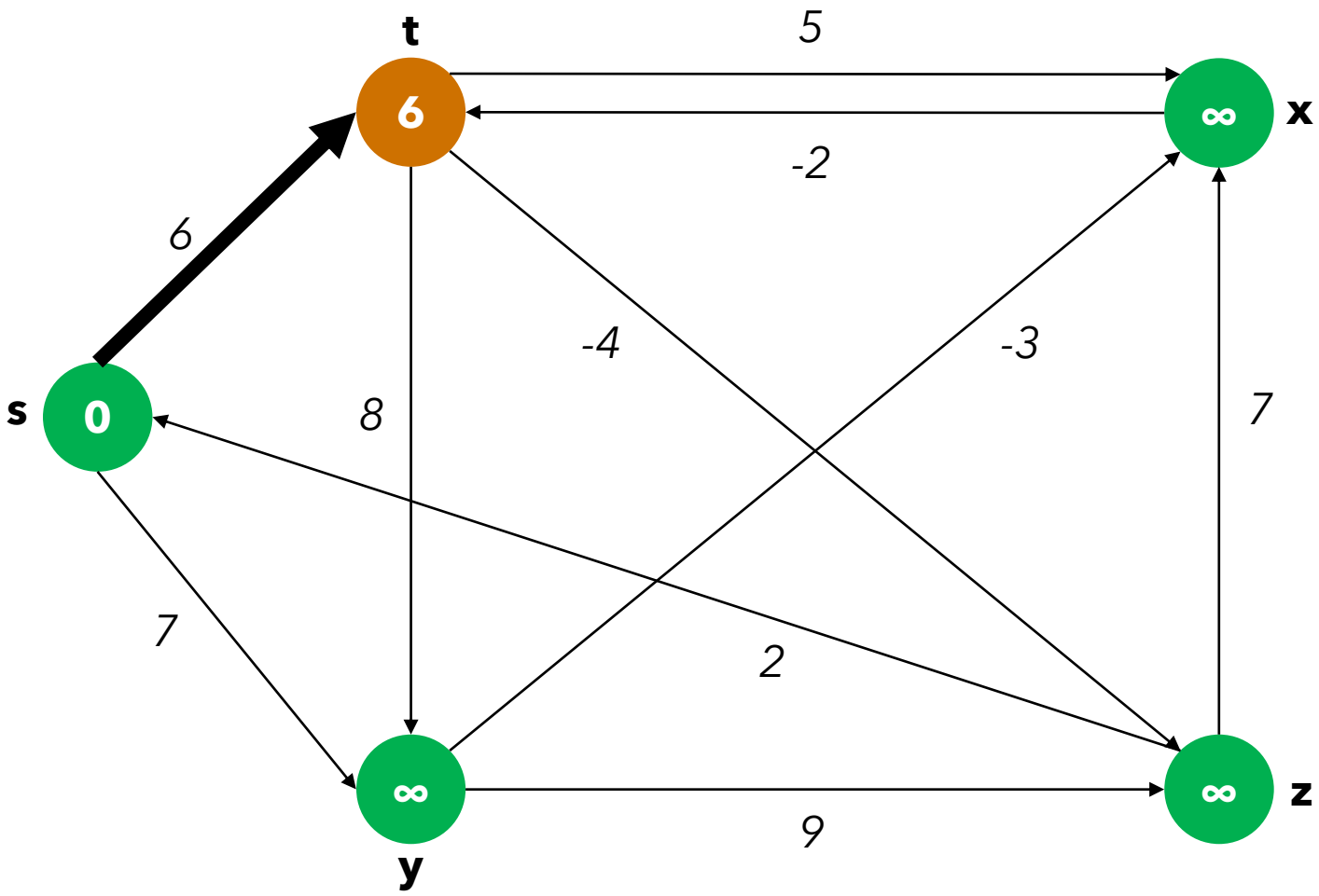
Bellman-Ford – 1st pass




```
t -> x
t -> y
t -> z
x -> t
y -> x
y -> z
z -> x
z -> s
s -> t
s -> y
```

rilassamento di z -> s (nessun effetto)

Bellman-Ford – 1st pass

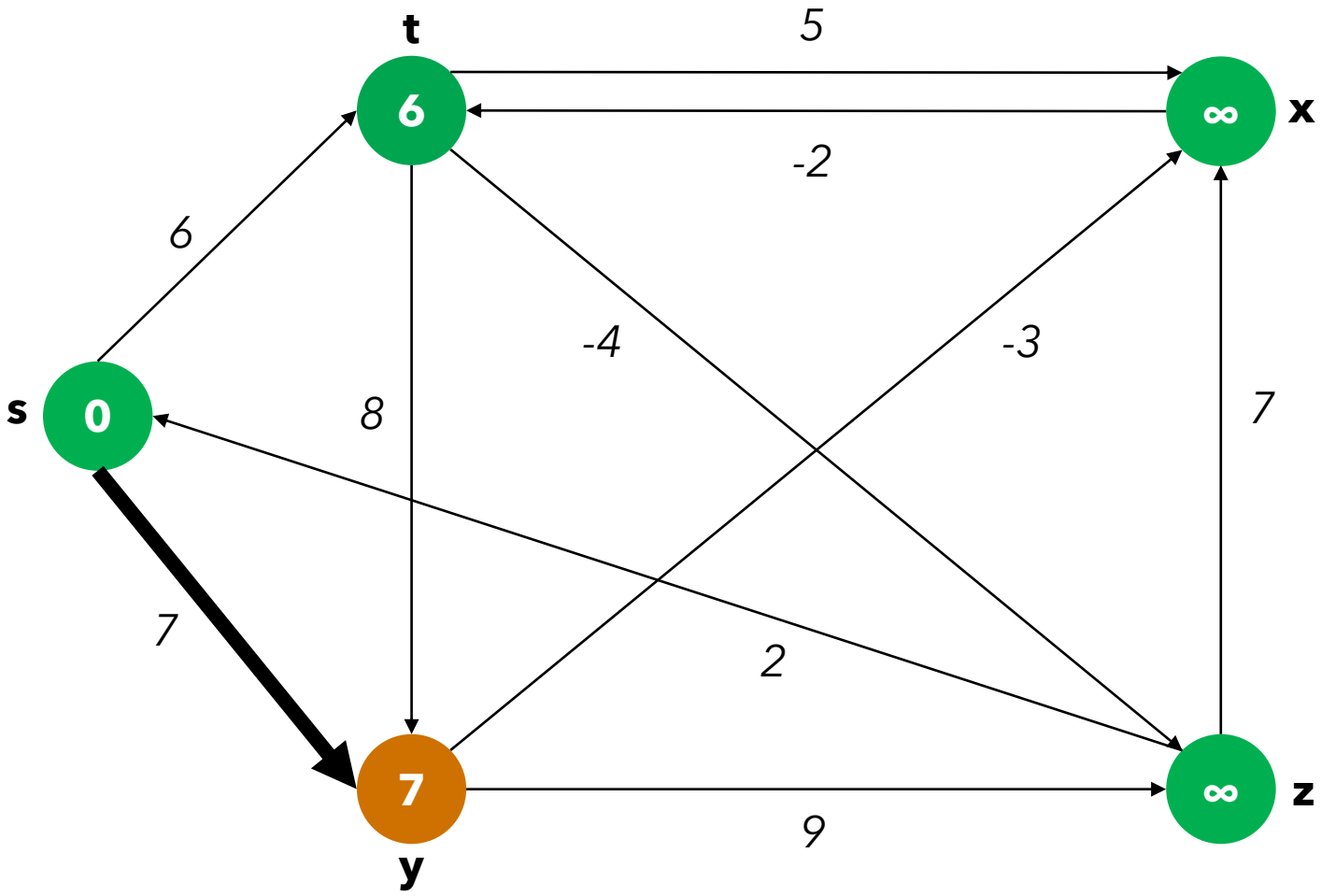



```
t -> x
t -> y
t -> z
x -> t
y -> x
y -> z
z -> x
z -> s
s -> t
s -> y
```

rilassamento di `s -> t`

`t.predecessor = s`

Bellman-Ford – 1st pass

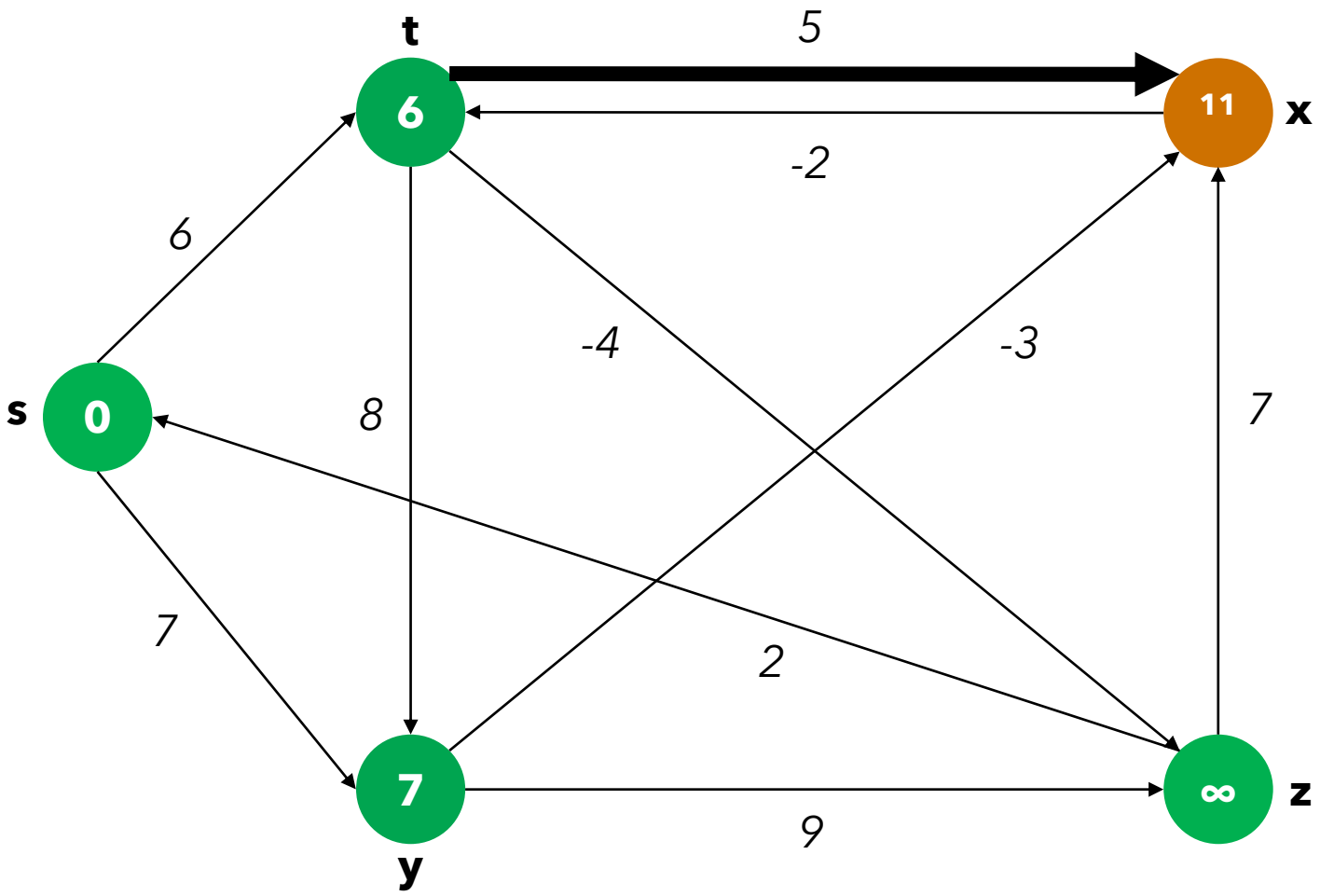



```
t -> x
t -> y
t -> z
x -> t
y -> x
y -> z
z -> x
z -> s
s -> t
s -> y
```

rilassamento di s -> y

y.predecessor = s

Bellman-Ford – 2nd pass

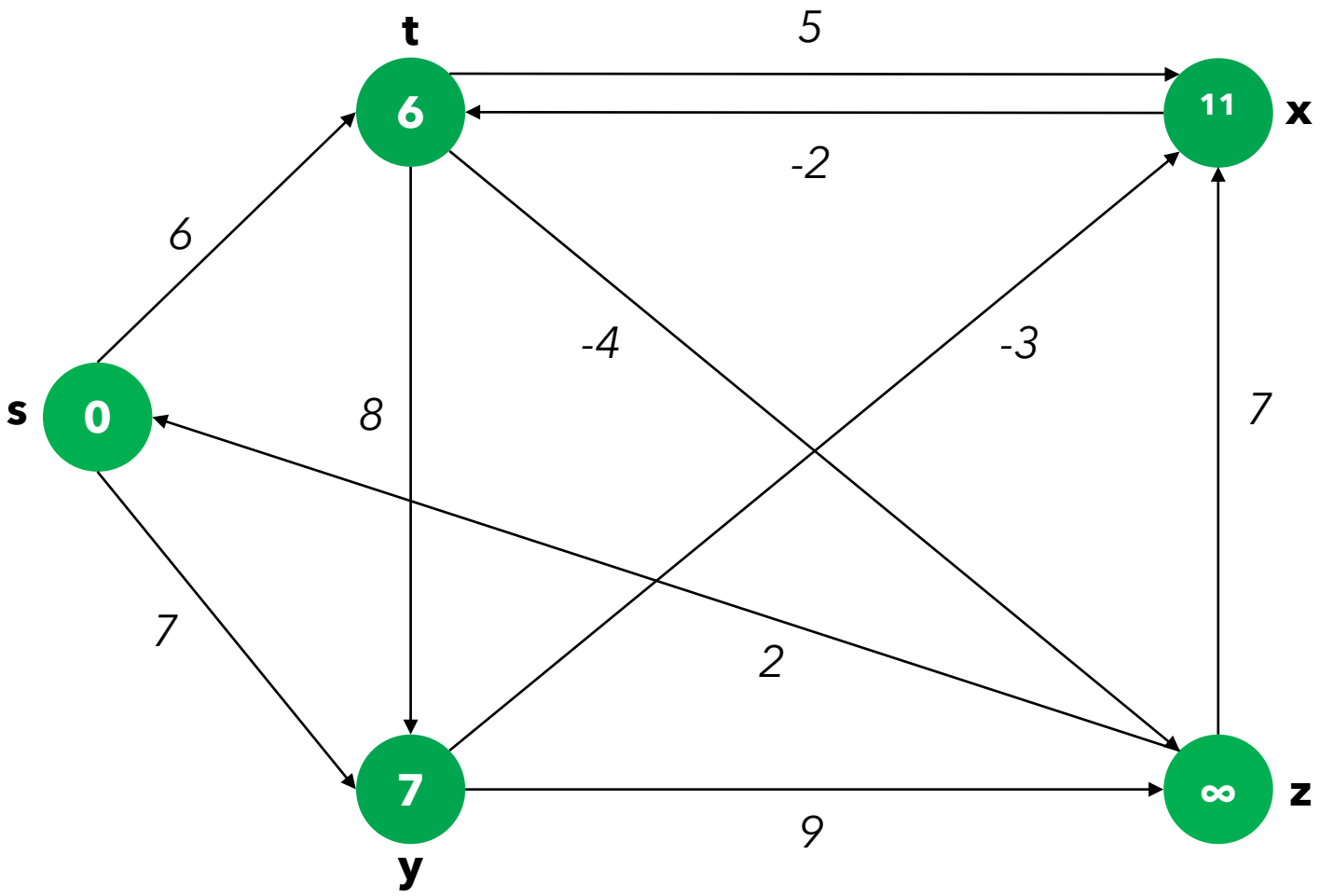



```
t -> x
t -> y
t -> z
x -> t
y -> x
y -> z
z -> x
z -> s
s -> t
s -> y
```

rilassamento di `t -> x`

`x.predecessor = t`

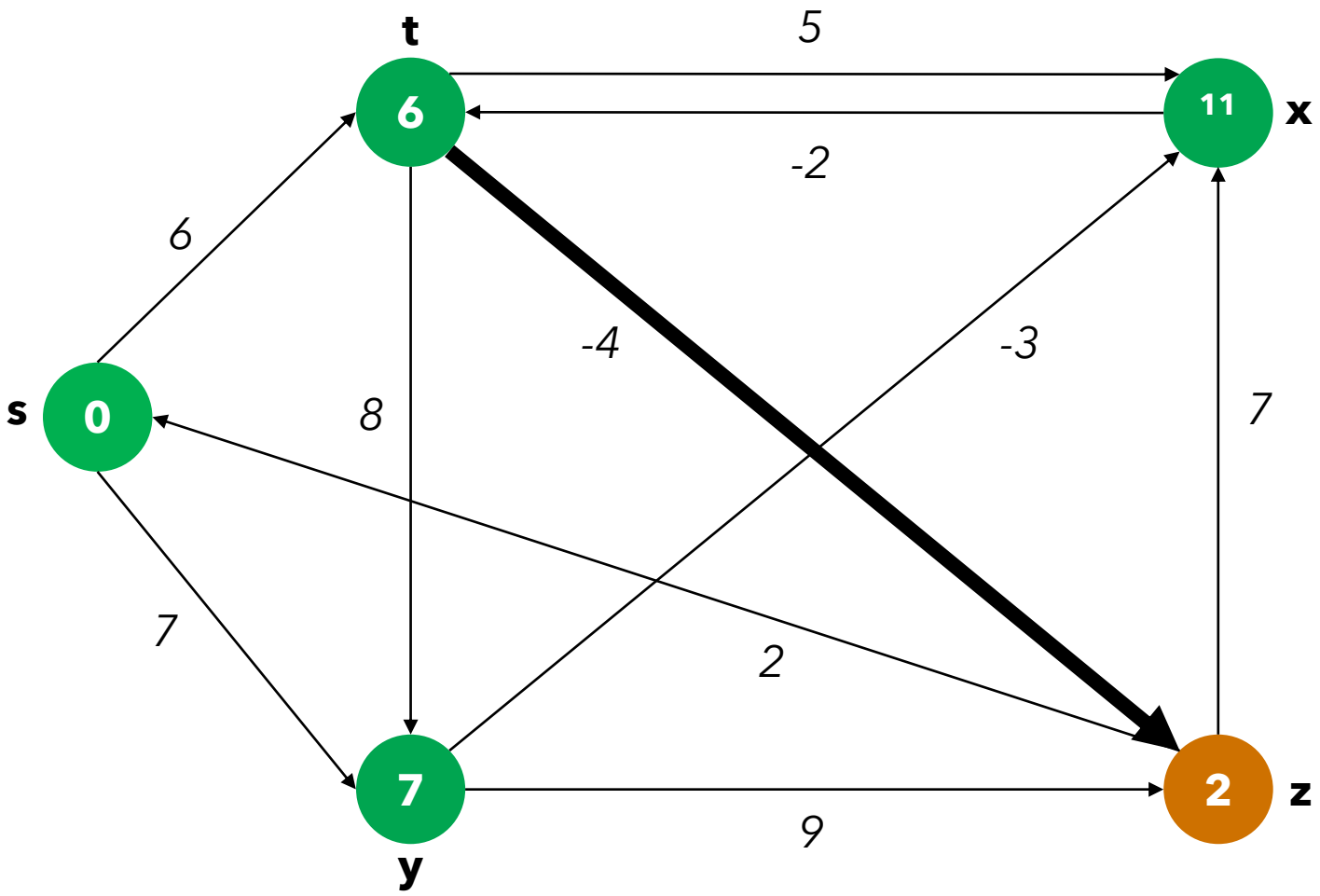
Bellman-Ford – 2nd pass




```
t -> x
t -> y
t -> z
x -> t
y -> x
y -> z
z -> x
z -> s
s -> t
s -> y
```

rilassamento di `t -> y`
(nessun effetto)

Bellman-Ford – 2nd pass

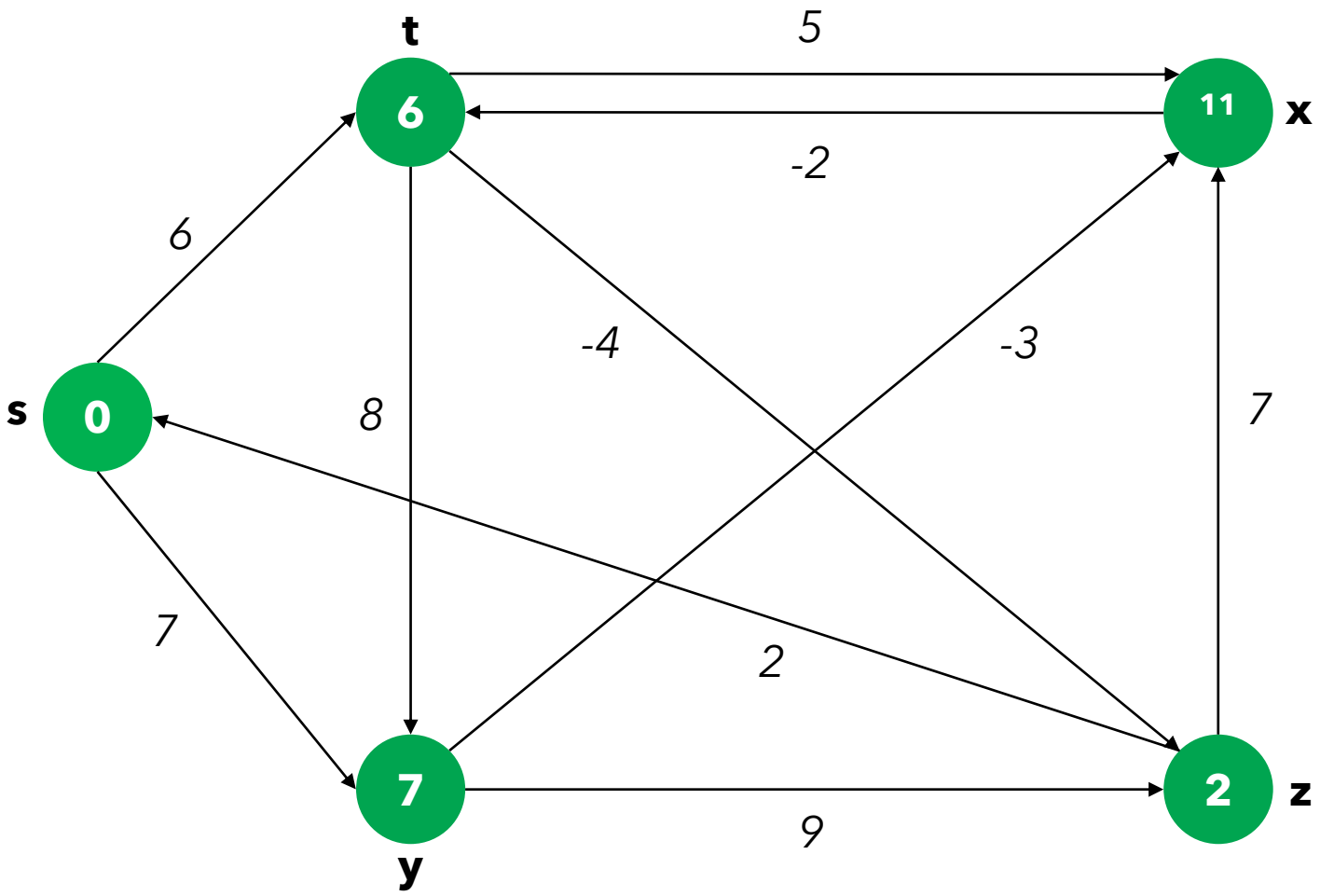



```
t -> x
t -> y
t -> z
x -> t
y -> x
y -> z
z -> x
z -> s
s -> t
s -> y
```

rilassamento di `t -> z`

`z.predecessor = t`

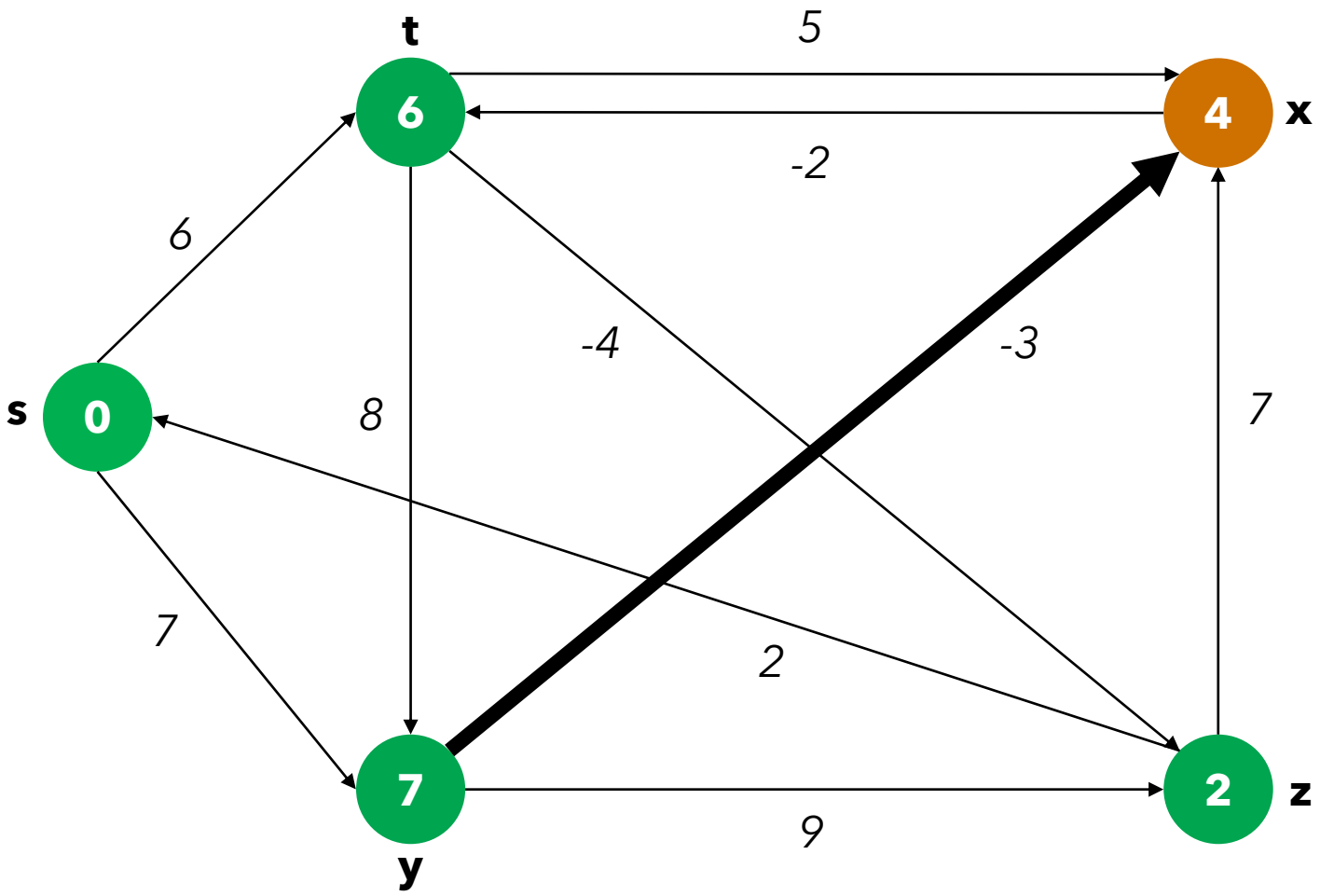
Bellman-Ford – 2nd pass




```
t -> x
t -> y
t -> z
x -> t
y -> x
y -> z
z -> x
z -> s
s -> t
s -> y
```

rilassamento di `x -> t`
(nessun effetto)

Bellman-Ford – 2nd pass

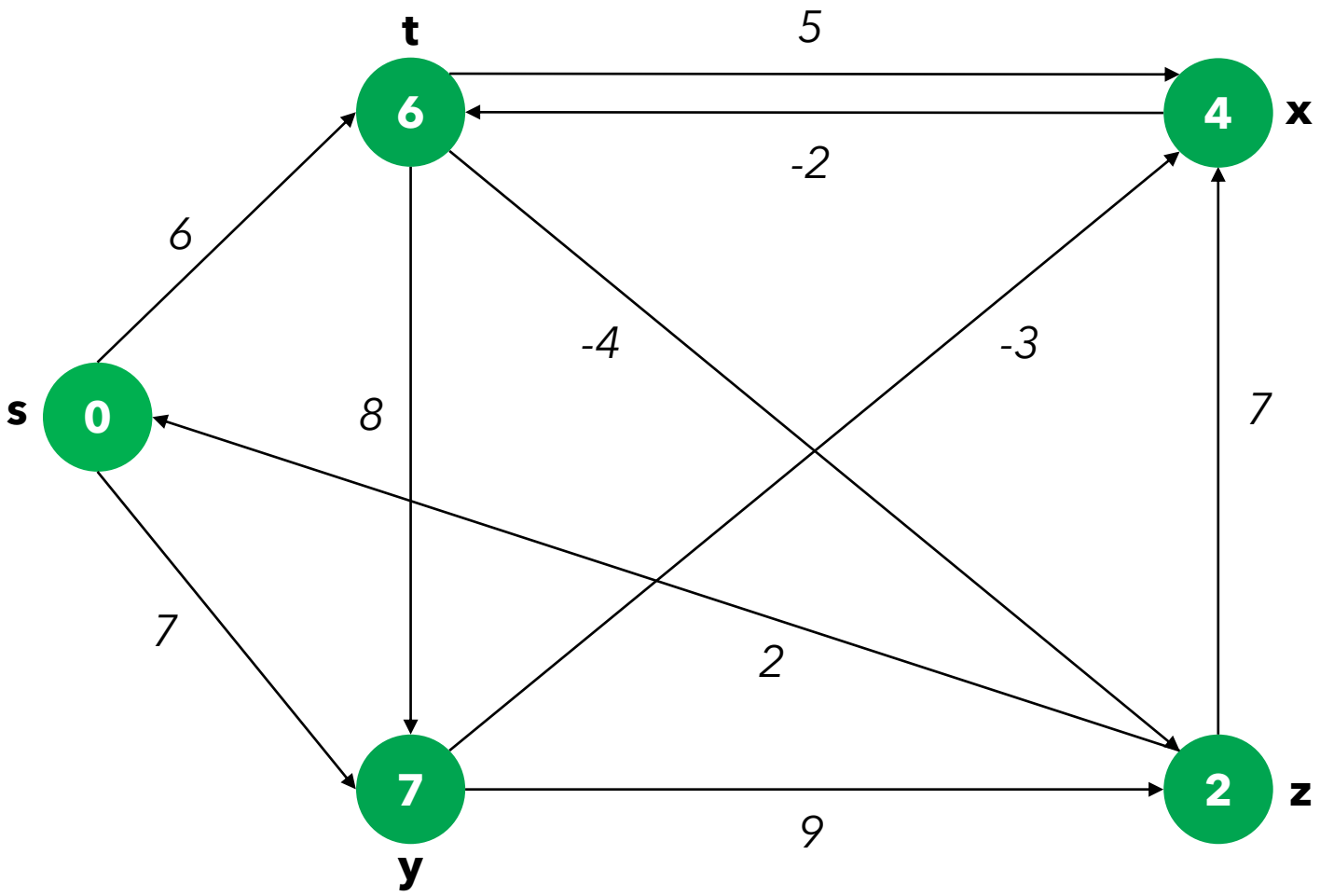



```
t -> x
t -> y
t -> z
x -> t
y -> x
y -> z
z -> x
z -> s
s -> t
s -> y
```

rilassamento di y -> x

x.predecessor = y

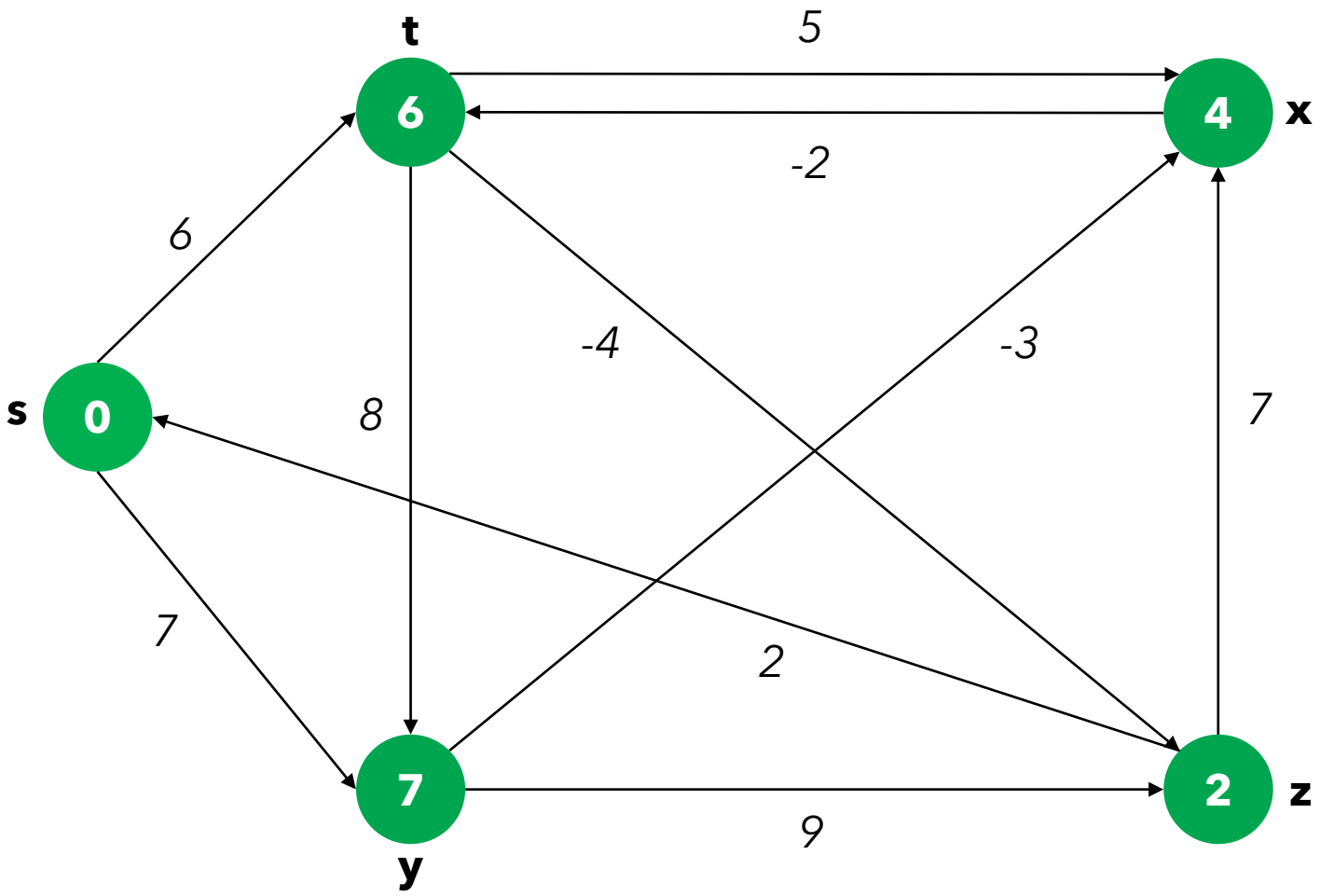
Bellman-Ford – 2nd pass




```
t -> x
t -> y
t -> z
x -> t
y -> x
y -> z
z -> x
z -> s
s -> t
s -> y
```

rilassamento di y -> z (nessun effetto)

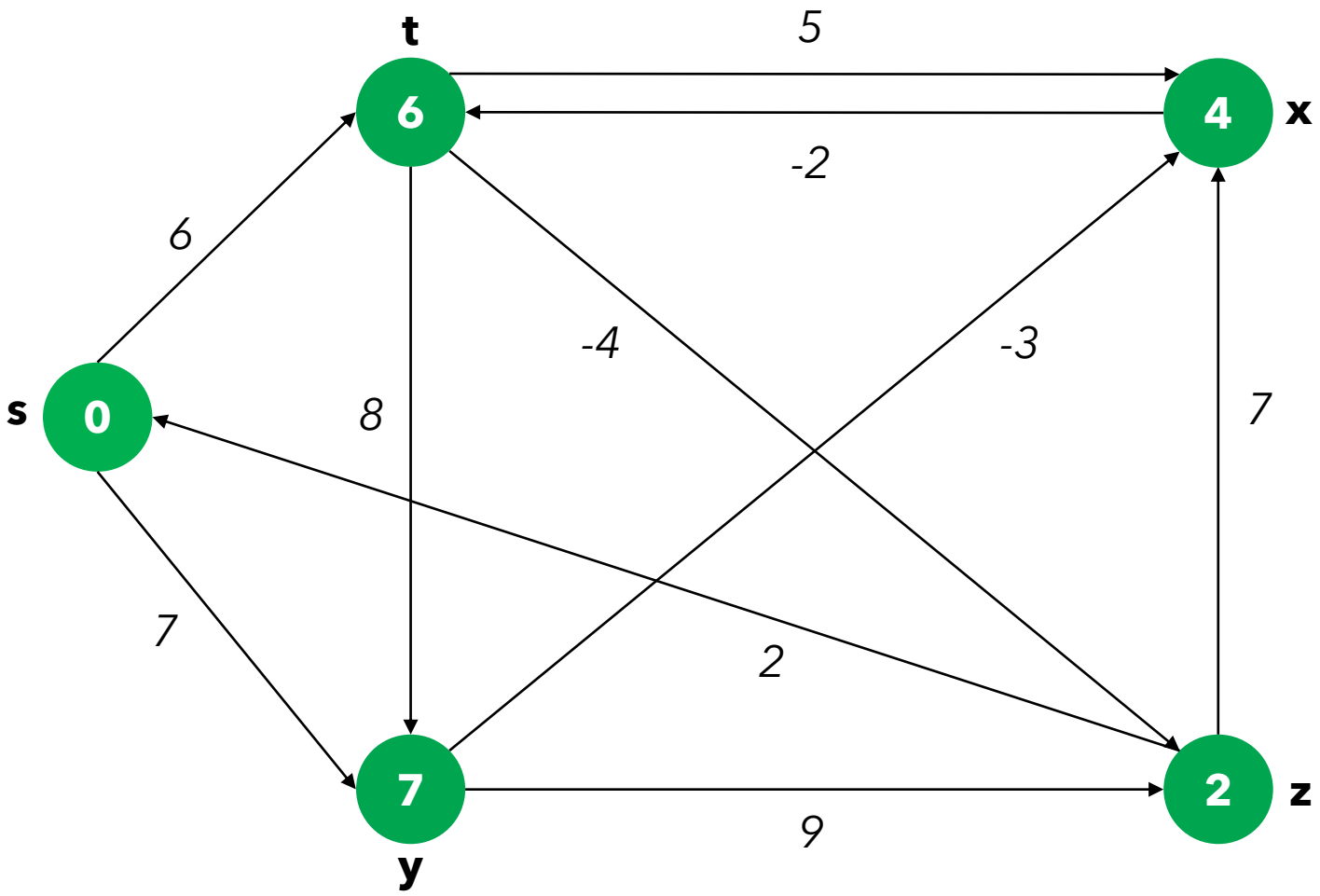
Bellman-Ford – 2nd pass




```
t -> x
t -> y
t -> z
x -> t
y -> x
y -> z
z -> x
z -> s
s -> t
s -> y
```

rilassamento di z -> x (nessun effetto)

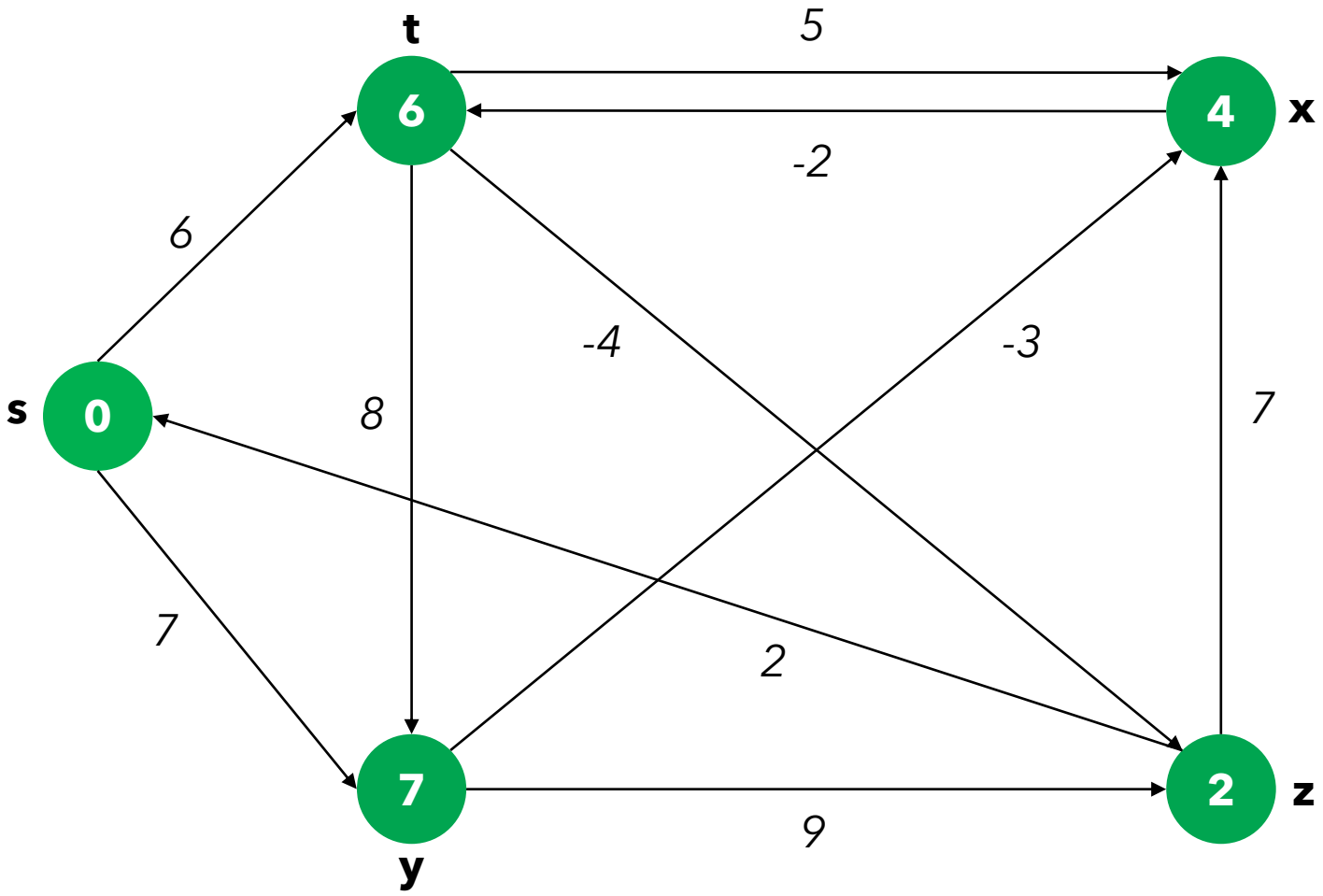
Bellman-Ford – 2nd pass




```
t -> x
t -> y
t -> z
x -> t
y -> x
y -> z
z -> x
z -> s
s -> t
s -> y
```

rilassamento di z -> s (nessun effetto)

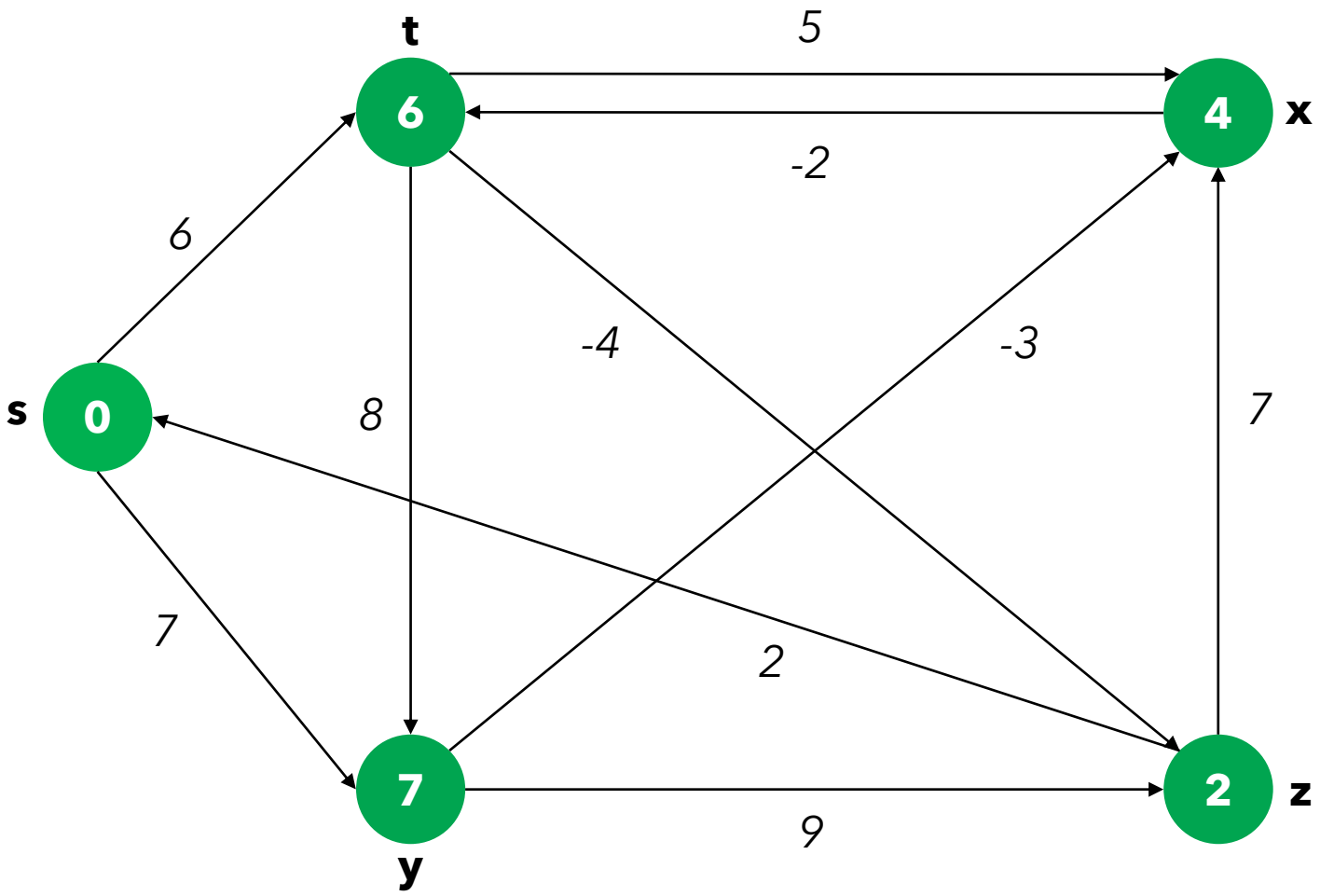
Bellman-Ford – 2nd pass




```
t -> x
t -> y
t -> z
x -> t
y -> x
y -> z
z -> x
z -> s
s -> t
s -> y
```

rilassamento di `s -> t`
(nessun effetto)

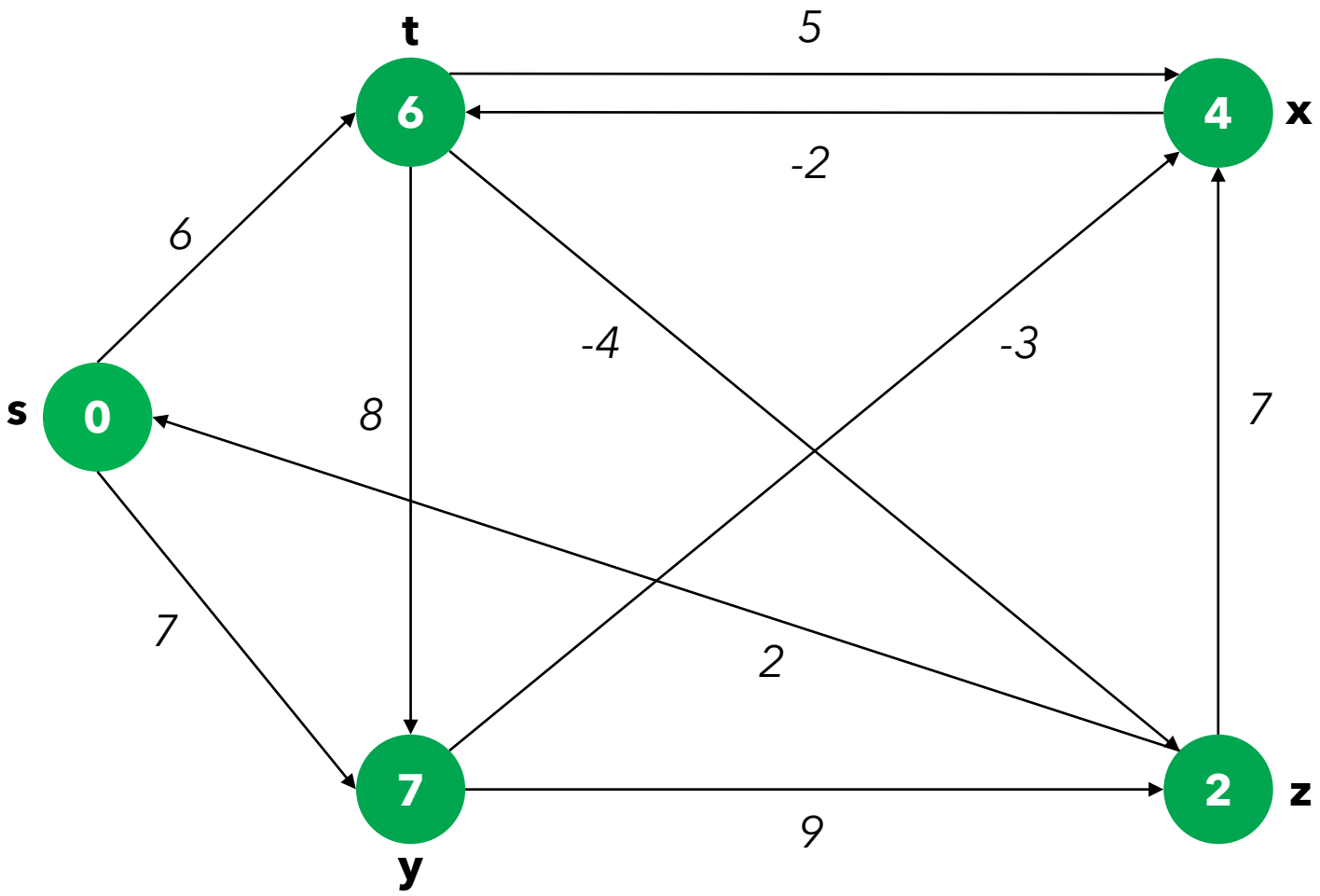
Bellman-Ford – 2nd pass




```
t -> x
t -> y
t -> z
x -> t
y -> x
y -> z
z -> x
z -> s
s -> t
s -> y
```

rilassamento di s -> y (nessun effetto)

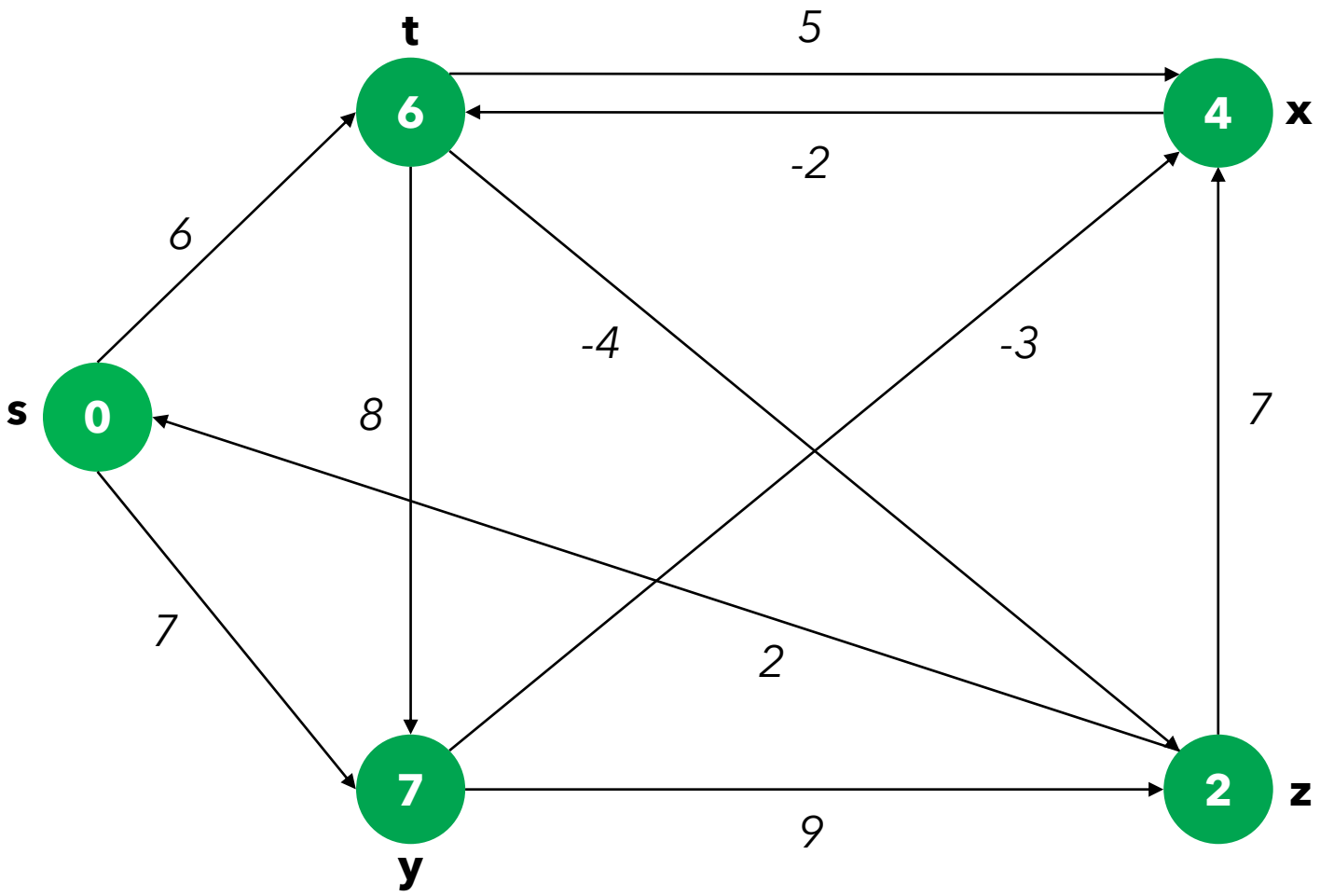
Bellman-Ford – 3rd pass



t -> x
t -> y
t -> z
x -> t
y -> x
y -> z
z -> x
z -> s
s -> t
s -> y

rilassamento di t -> x (nessun effetto)

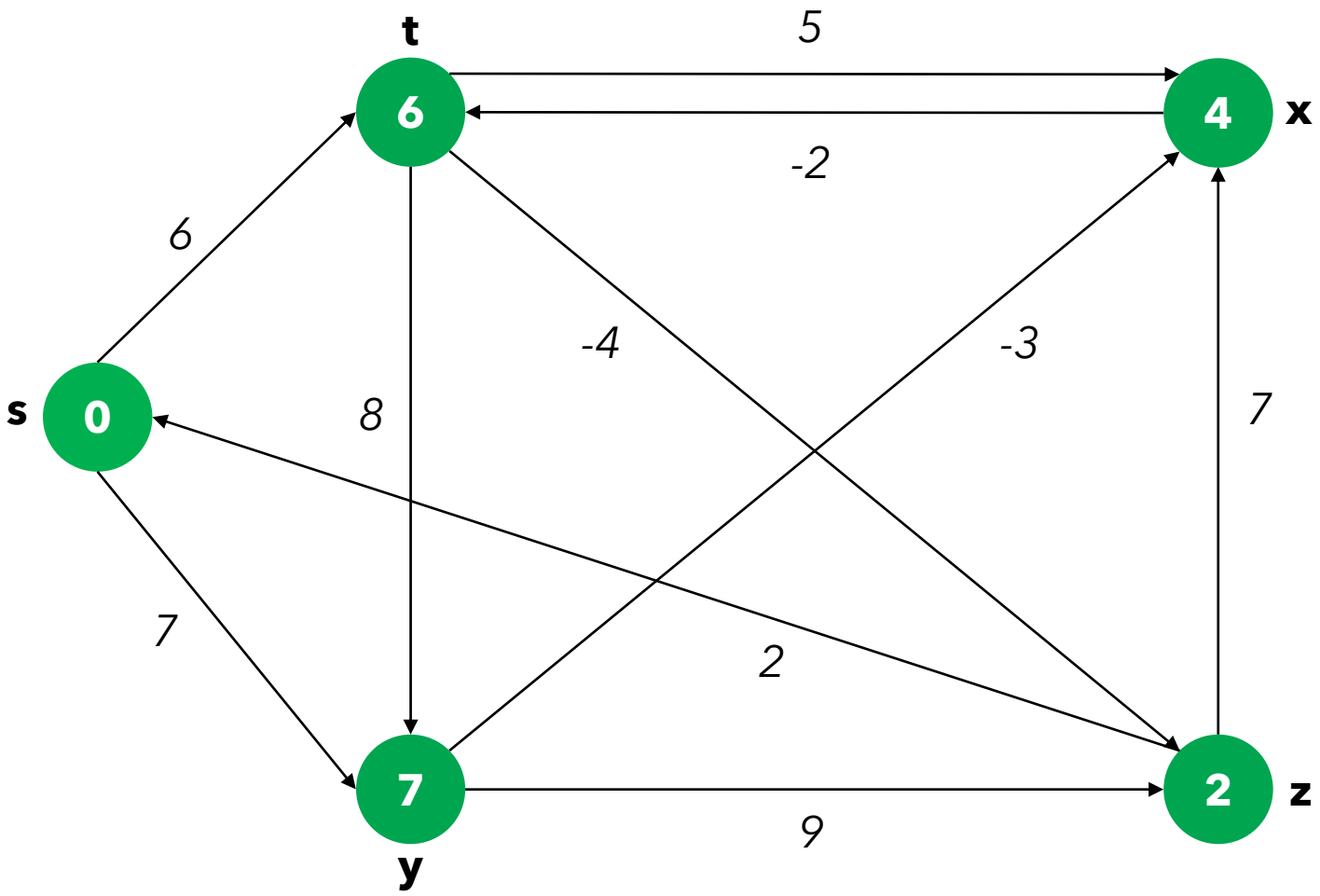
Bellman-Ford – 3rd pass




```
t -> x
t -> y
t -> z
x -> t
y -> x
y -> z
z -> x
z -> s
s -> t
s -> y
```

rilassamento di `t -> y`
(nessun effetto)

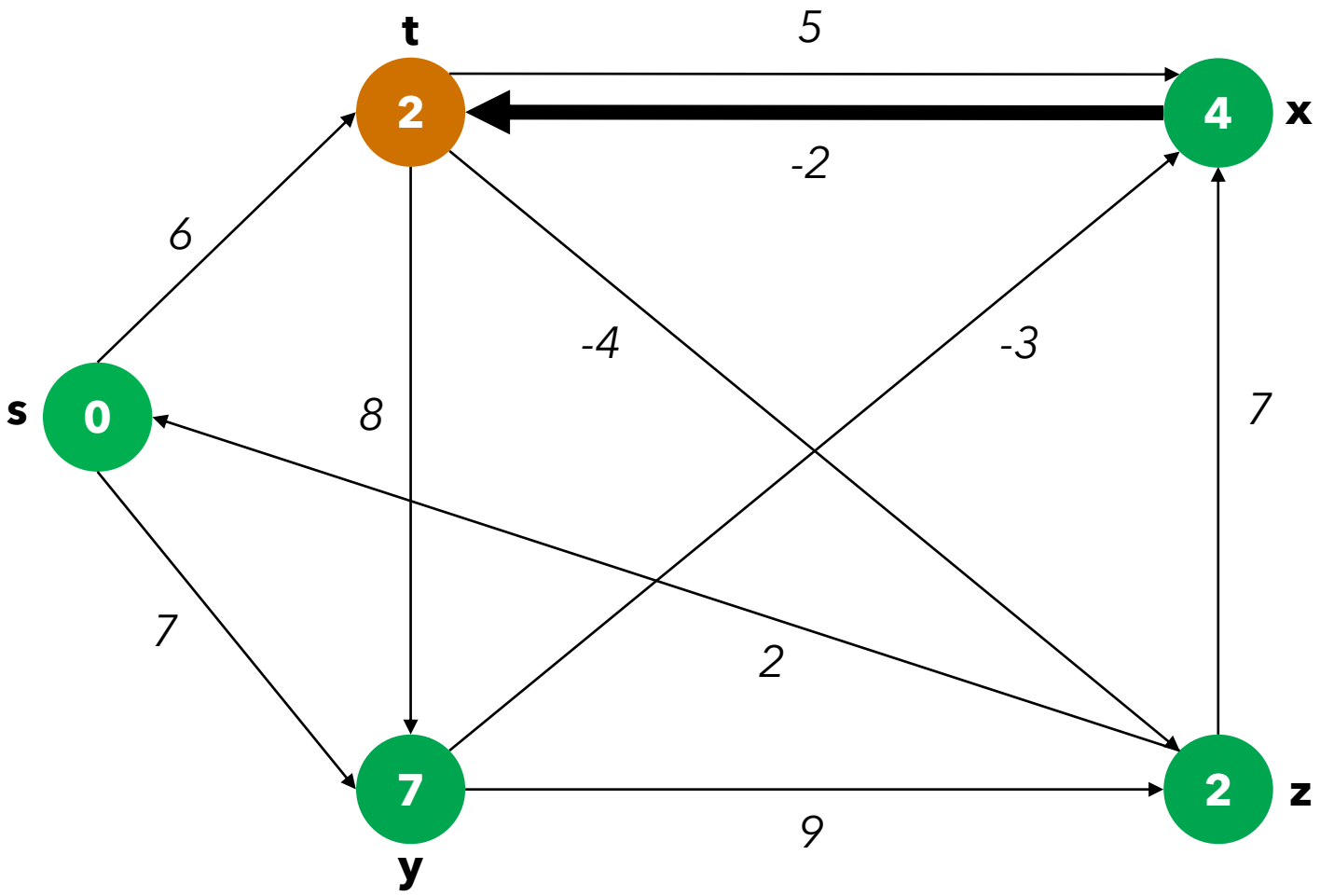
Bellman-Ford – 3rd pass




```
t -> x
t -> y
t -> z
x -> t
y -> x
y -> z
z -> x
z -> s
s -> t
s -> y
```

rilassamento di `t -> z`
(nessun effetto)

Bellman-Ford – 3rd pass

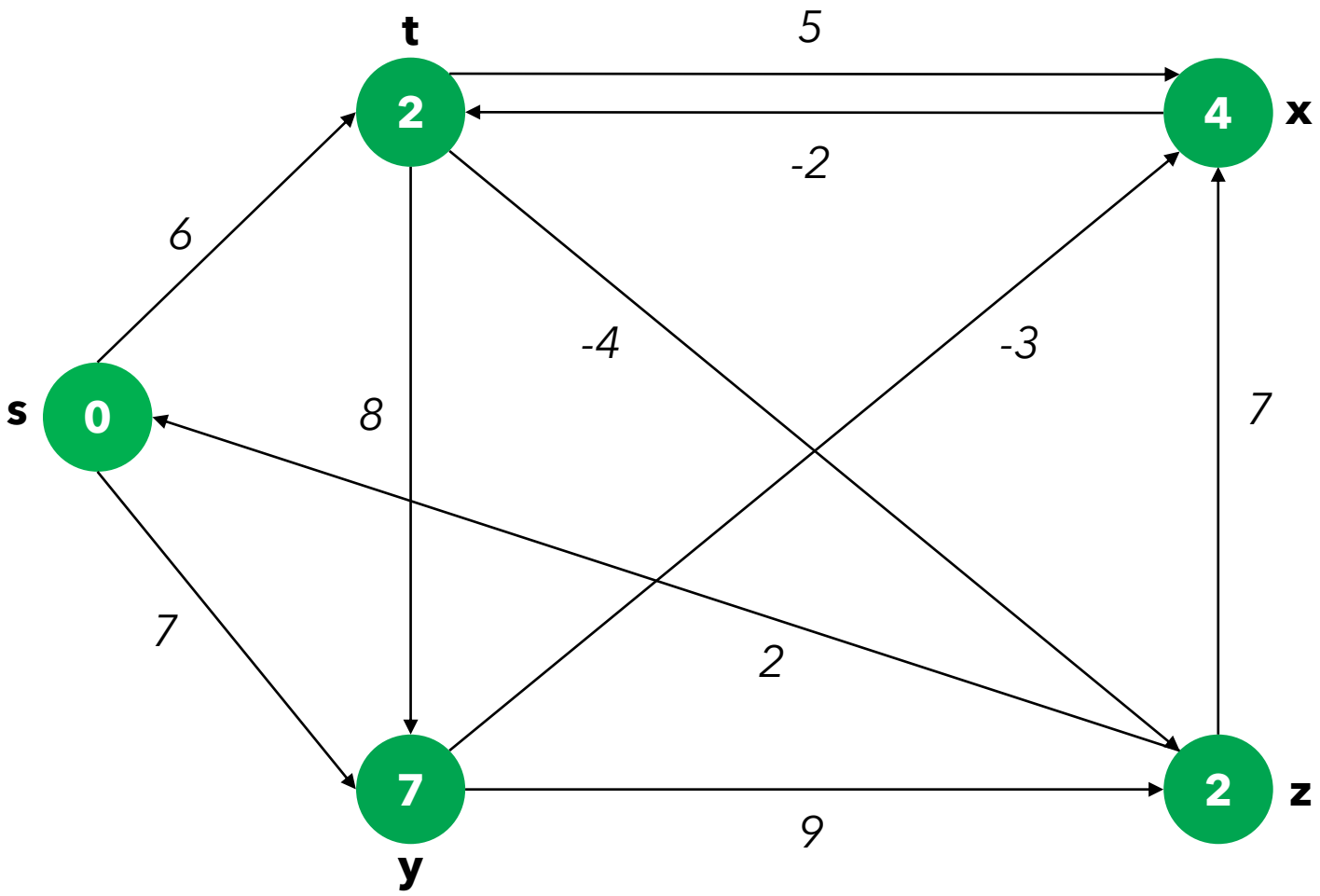



```
t -> x
t -> y
t -> z
x -> t
y -> x
y -> z
z -> x
z -> s
s -> t
s -> y
```

rilassamento di `t -> z`

`t.predecessor = x`

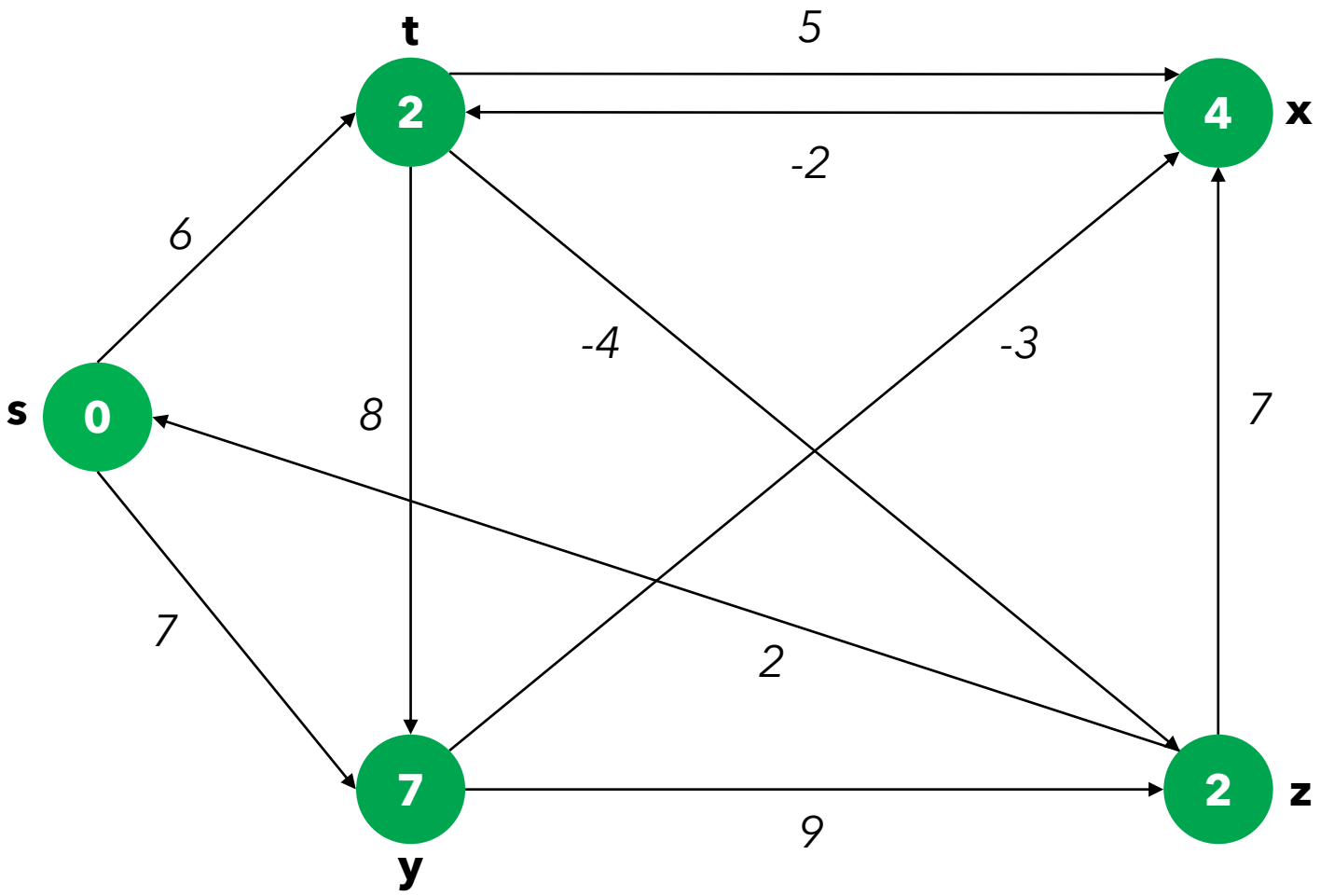
Bellman-Ford – 3rd pass




```
t -> x
t -> y
t -> z
x -> t
y -> x
y -> z
z -> x
z -> s
s -> t
s -> y
```

rilassamento di y -> x (nessun effetto)

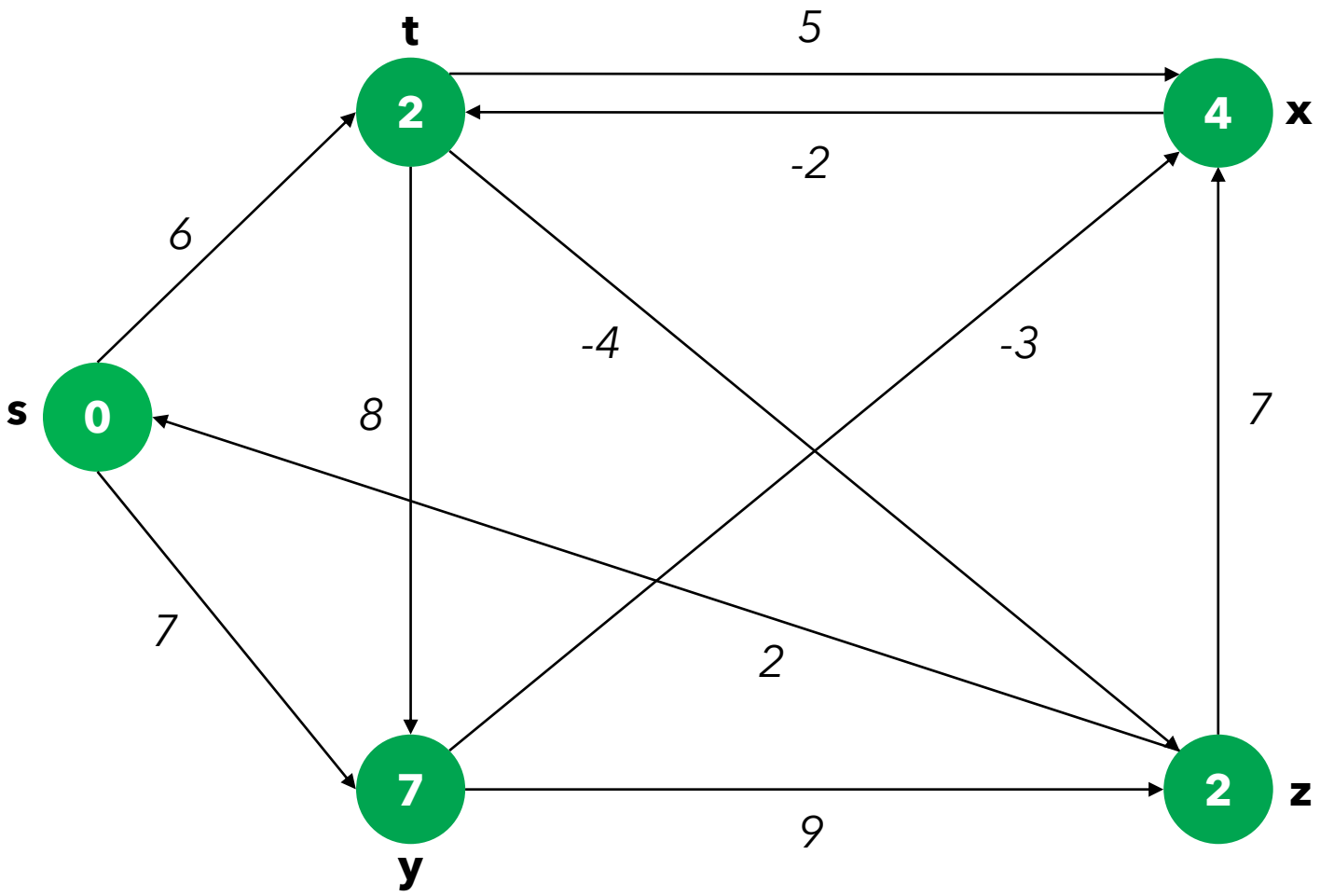
Bellman-Ford – 3rd pass




```
t -> x
t -> y
t -> z
x -> t
y -> x
y -> z
z -> x
z -> s
s -> t
s -> y
```

rilassamento di y -> z (nessun effetto)

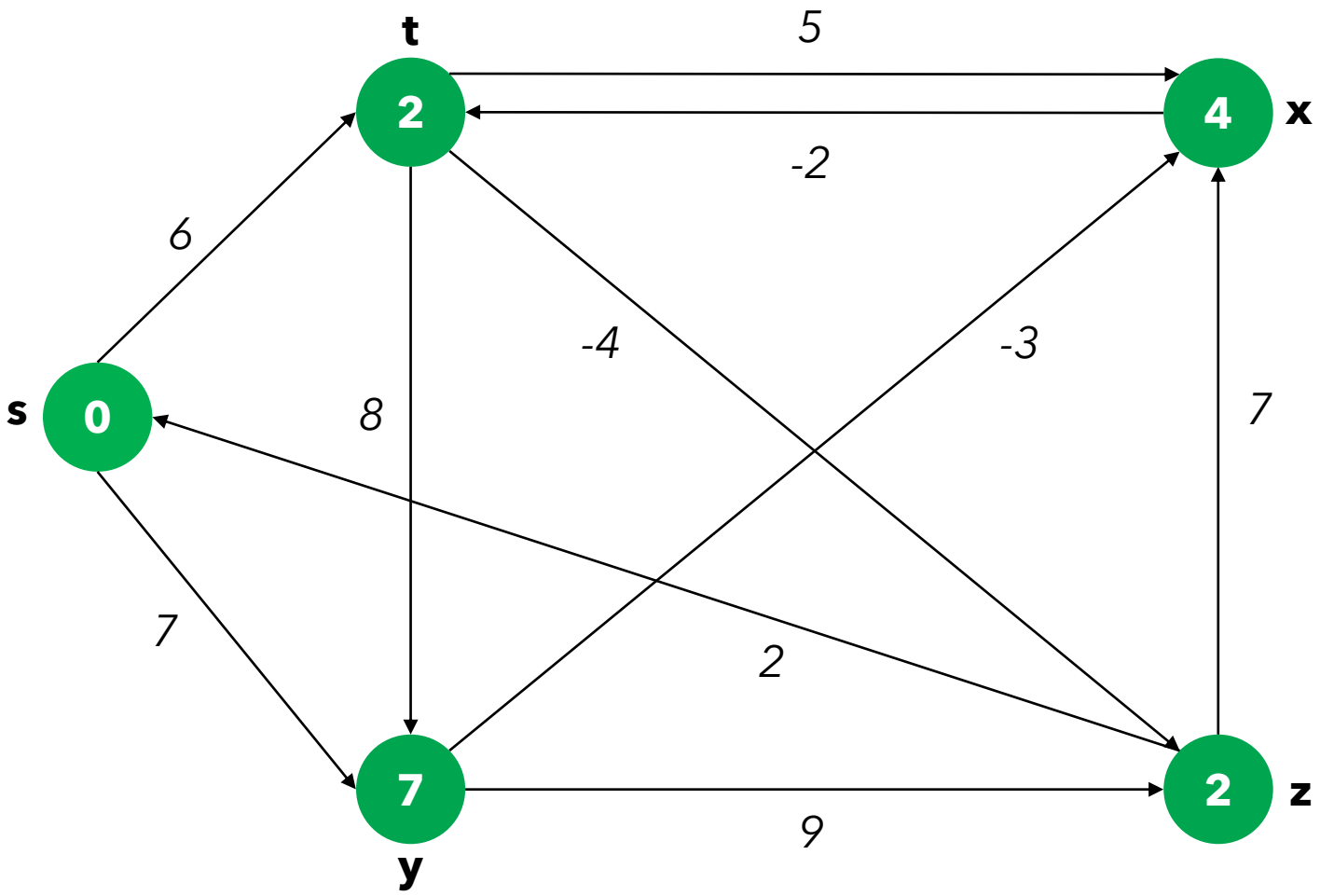
Bellman-Ford – 3rd pass




```
t -> x
t -> y
t -> z
x -> t
y -> x
y -> z
z -> x
z -> s
s -> t
s -> y
```

rilassamento di z -> x (nessun effetto)

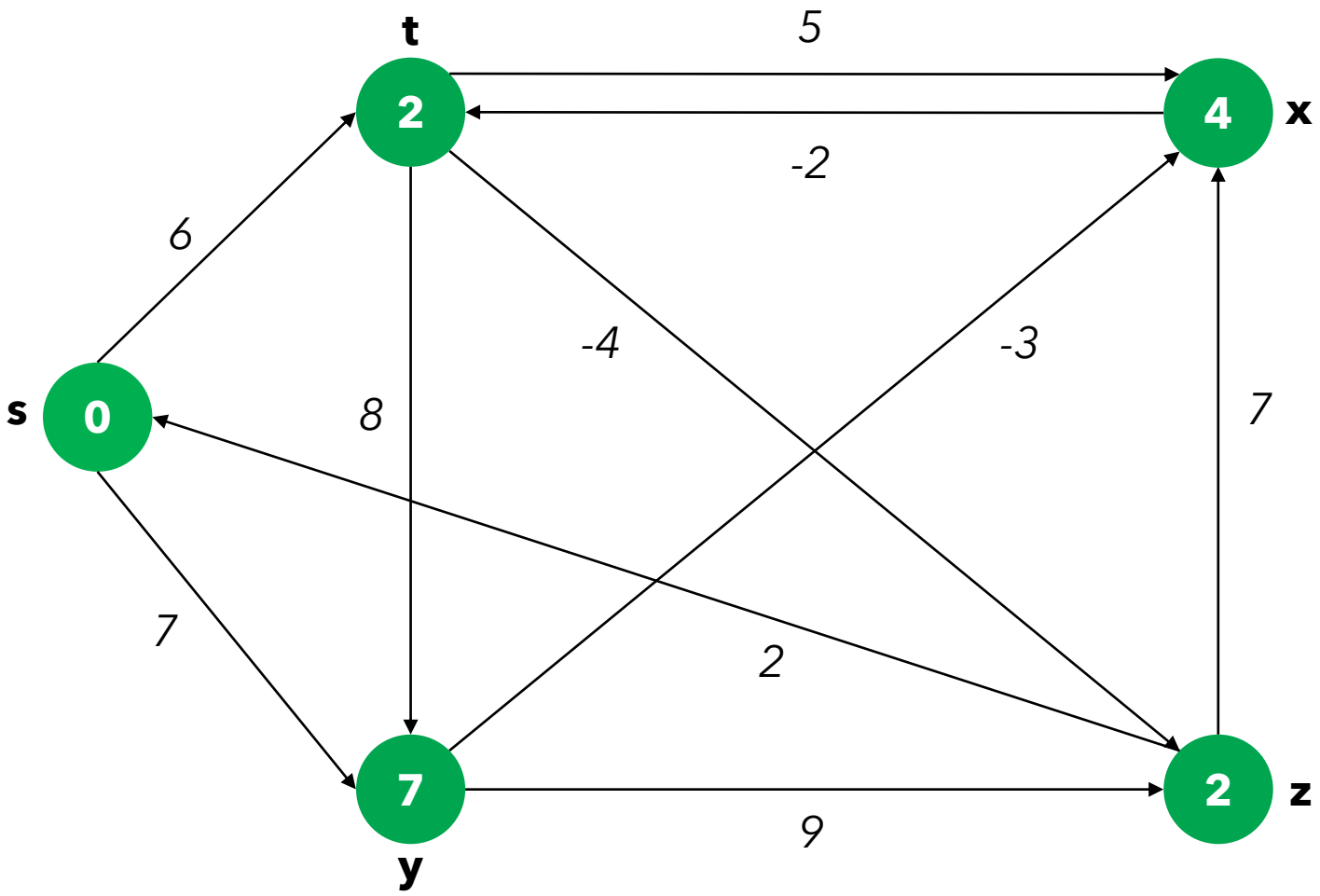
Bellman-Ford – 3rd pass




```
t -> x
t -> y
t -> z
x -> t
y -> x
y -> z
z -> x
z -> s
s -> t
s -> y
```

rilassamento di z -> s (nessun effetto)

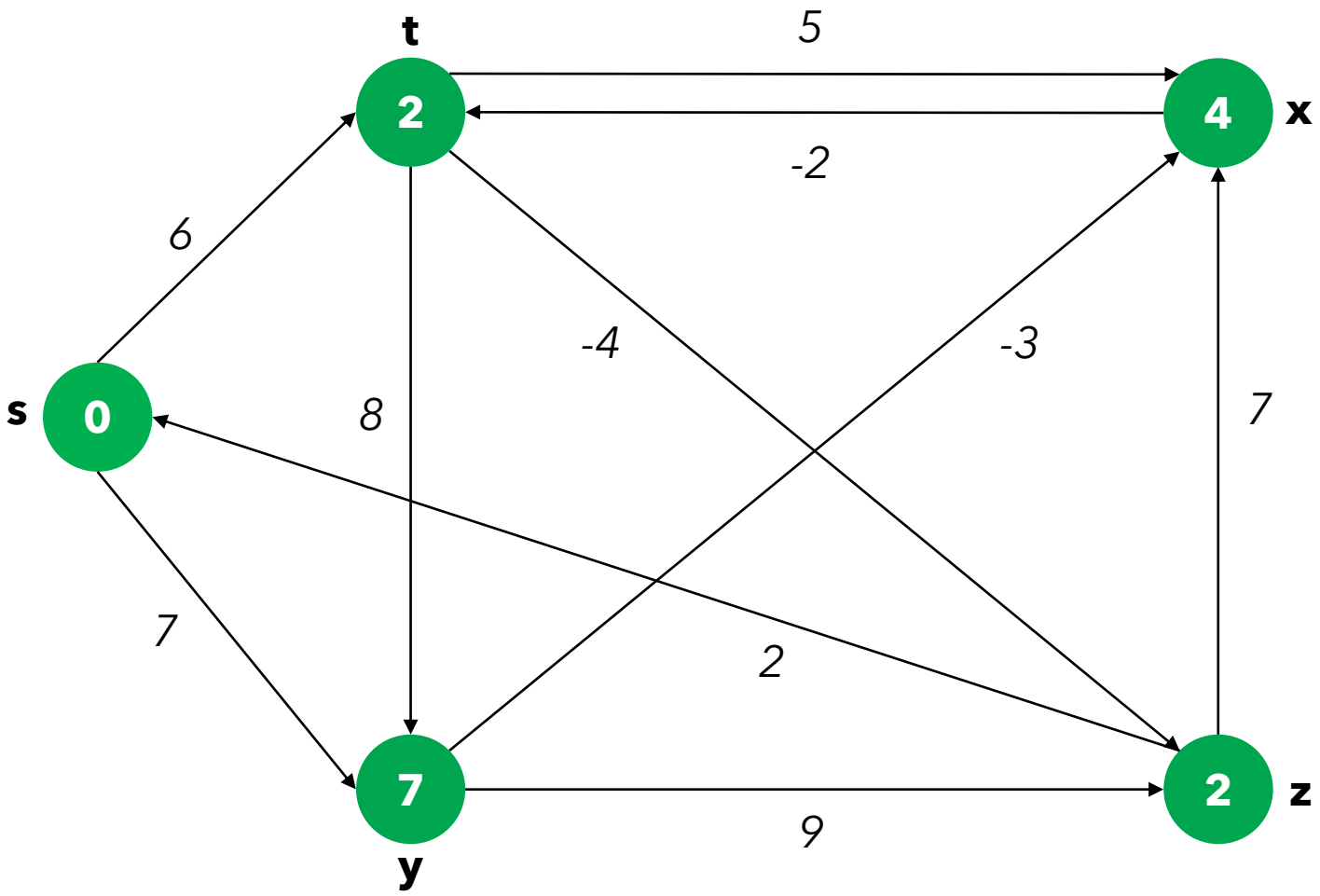
Bellman-Ford – 3rd pass




```
t -> x
t -> y
t -> z
x -> t
y -> x
y -> z
z -> x
z -> s
s -> t
s -> y
```

rilassamento di `s -> t`
(nessun effetto)

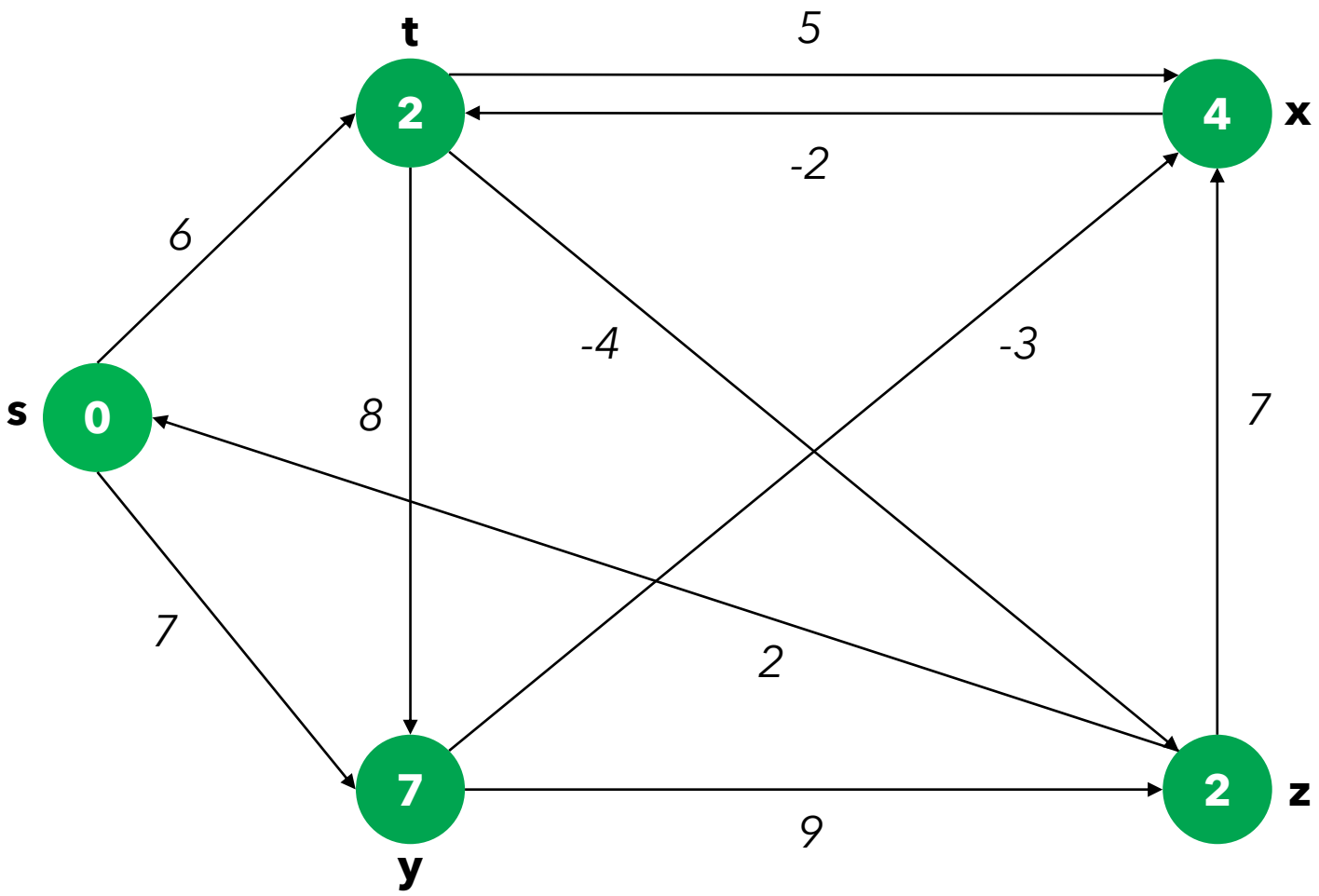
Bellman-Ford – 3rd pass




```
t -> x
t -> y
t -> z
x -> t
y -> x
y -> z
z -> x
z -> s
s -> t
s -> y
```

rilassamento di s -> y (nessun effetto)

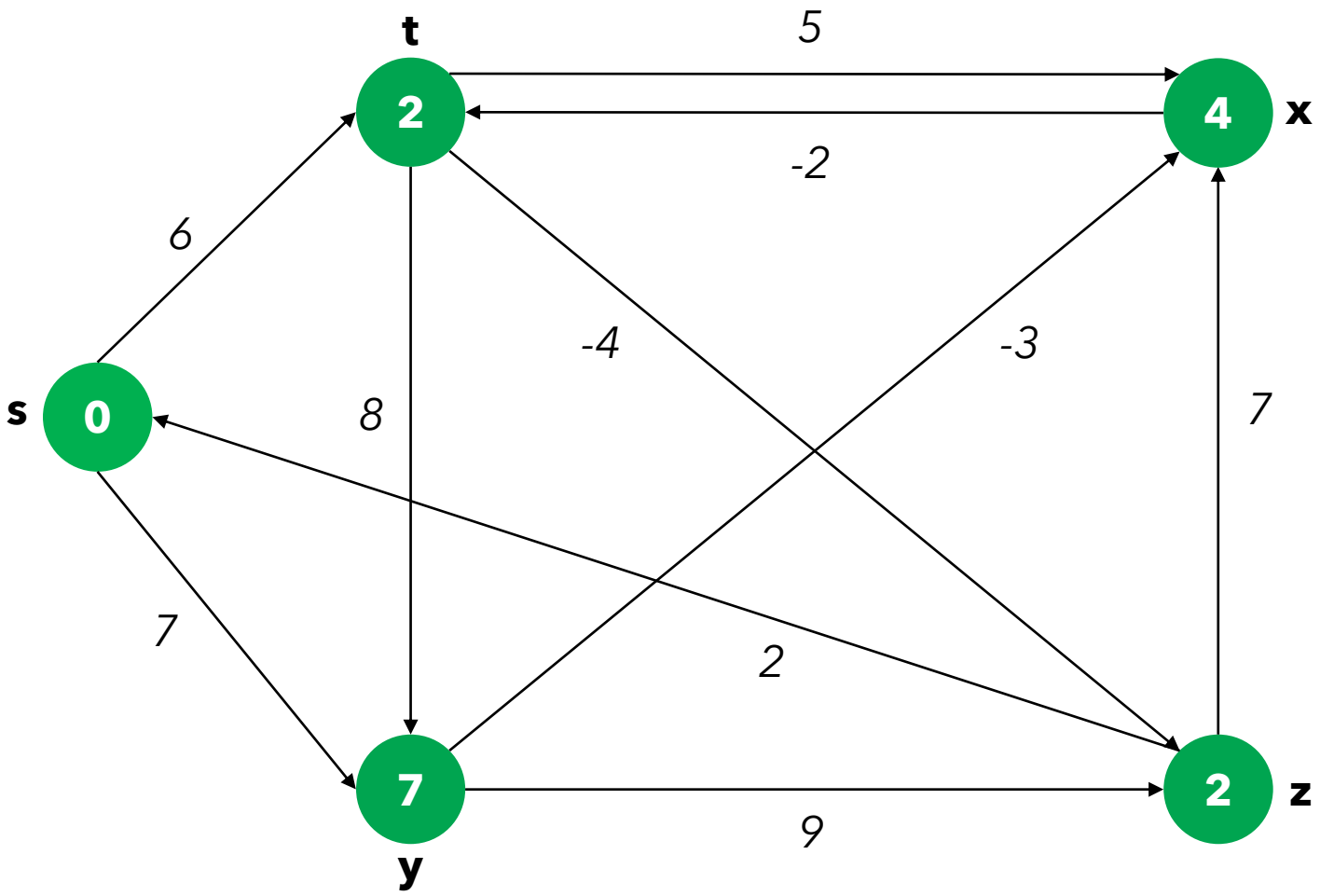
Bellman-Ford – 4th pass



t -> x
t -> y
t -> z
x -> t
y -> x
y -> z
z -> x
z -> s
s -> t
s -> y

rilassamento di t -> x (nessun effetto)

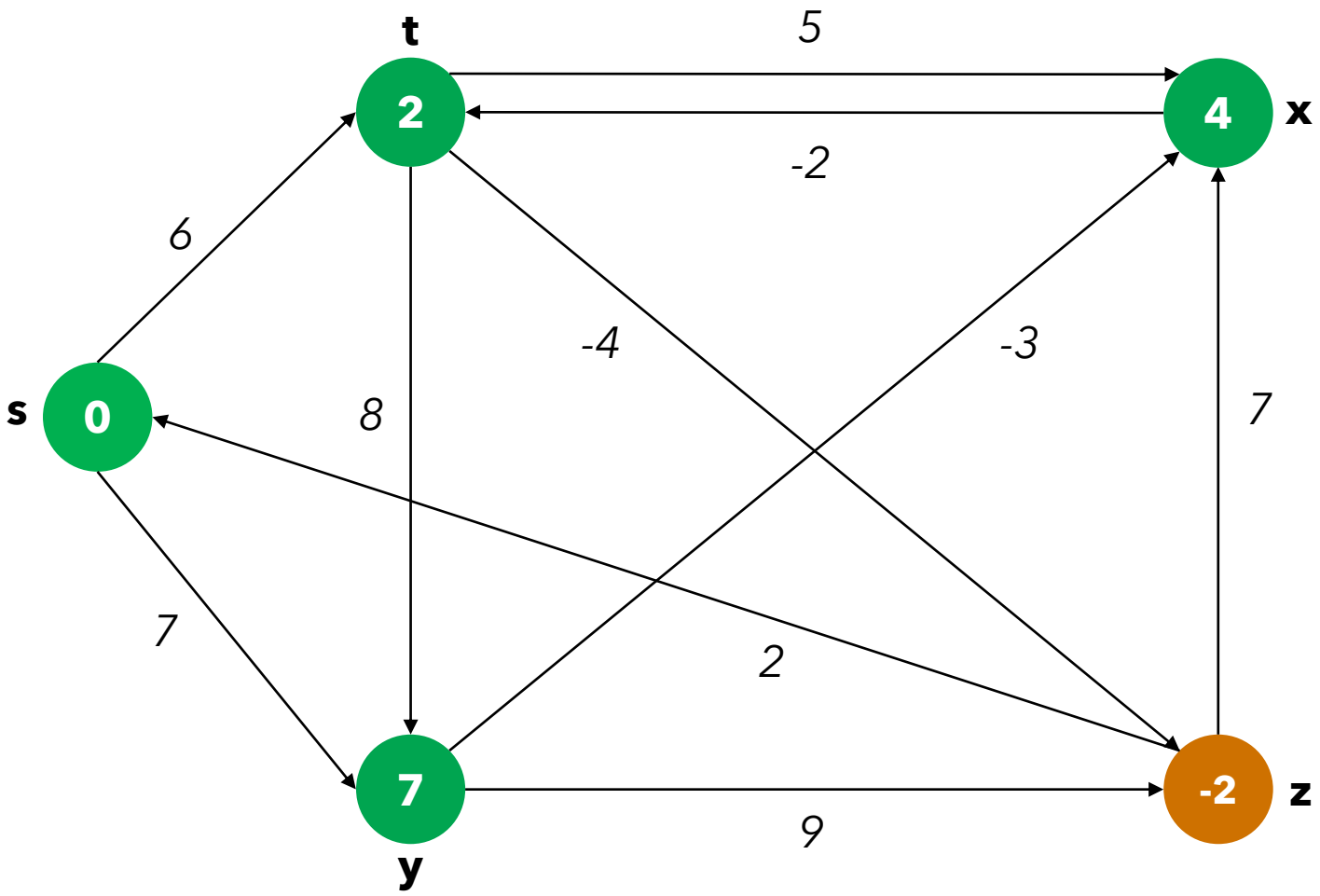
Bellman-Ford – 4th pass




```
t -> x
t -> y
t -> z
x -> t
y -> x
y -> z
z -> x
z -> s
s -> t
s -> y
```

rilassamento di `t -> y`
(nessun effetto)

Bellman-Ford – 4th pass

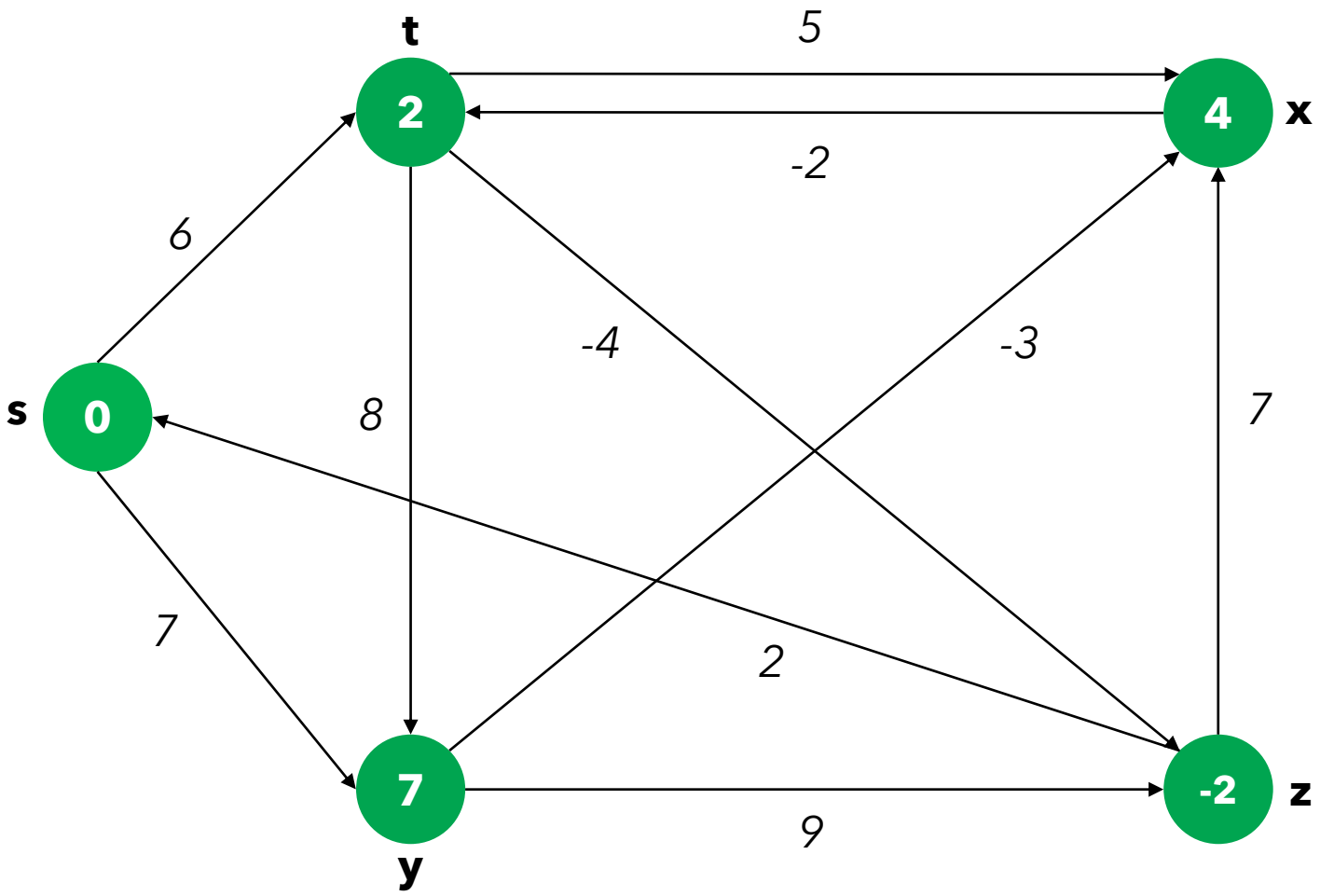



```
t -> x
t -> y
t -> z
x -> t
y -> x
y -> z
z -> x
z -> s
s -> t
s -> y
```

rilassamento di `t -> z`

`z.predecessor = t`

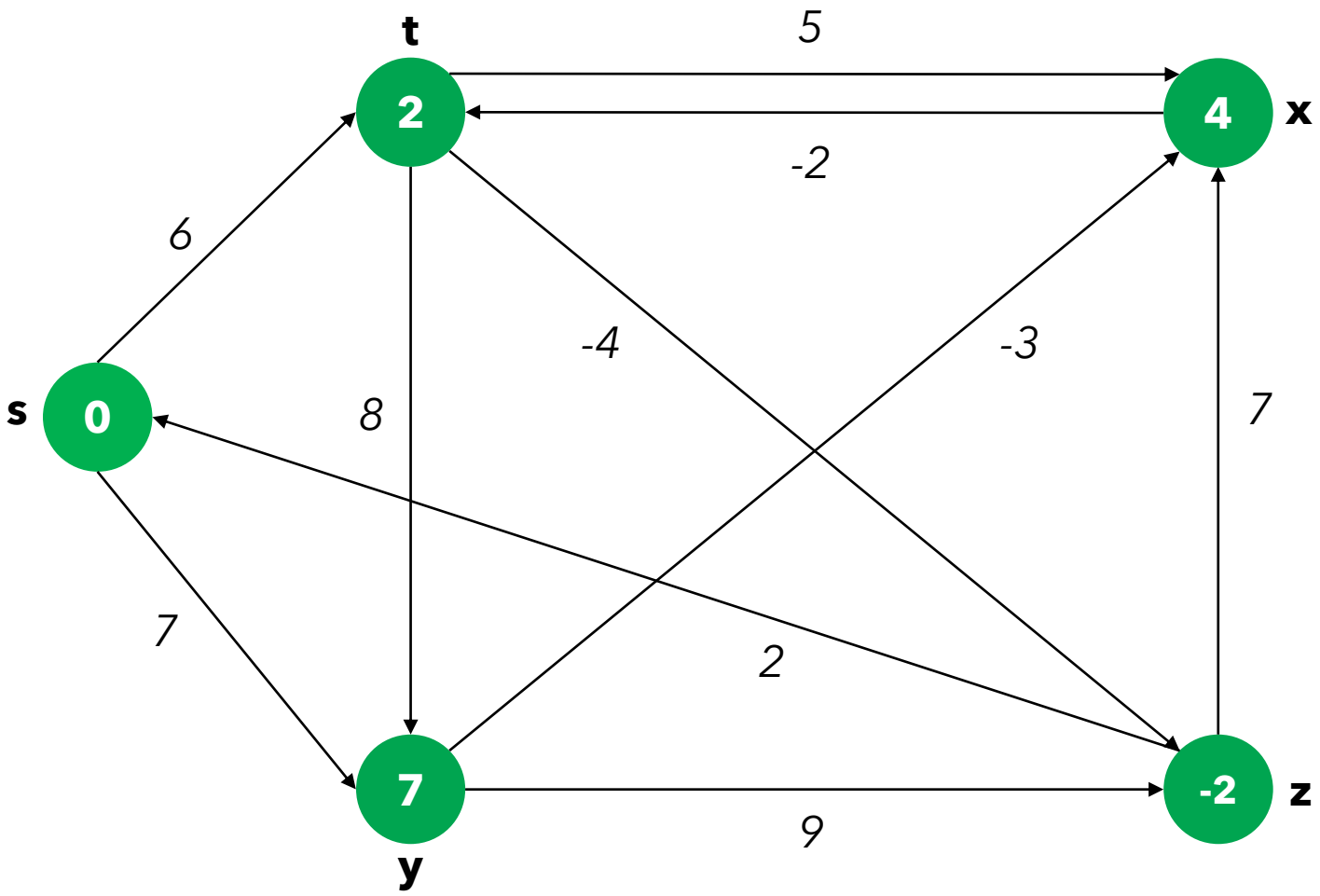
Bellman-Ford – 4th pass




```
t -> x
t -> y
t -> z
x -> t
y -> x
y -> z
z -> x
z -> s
s -> t
s -> y
```

rilassamento di `x -> t`
(nessun effetto)

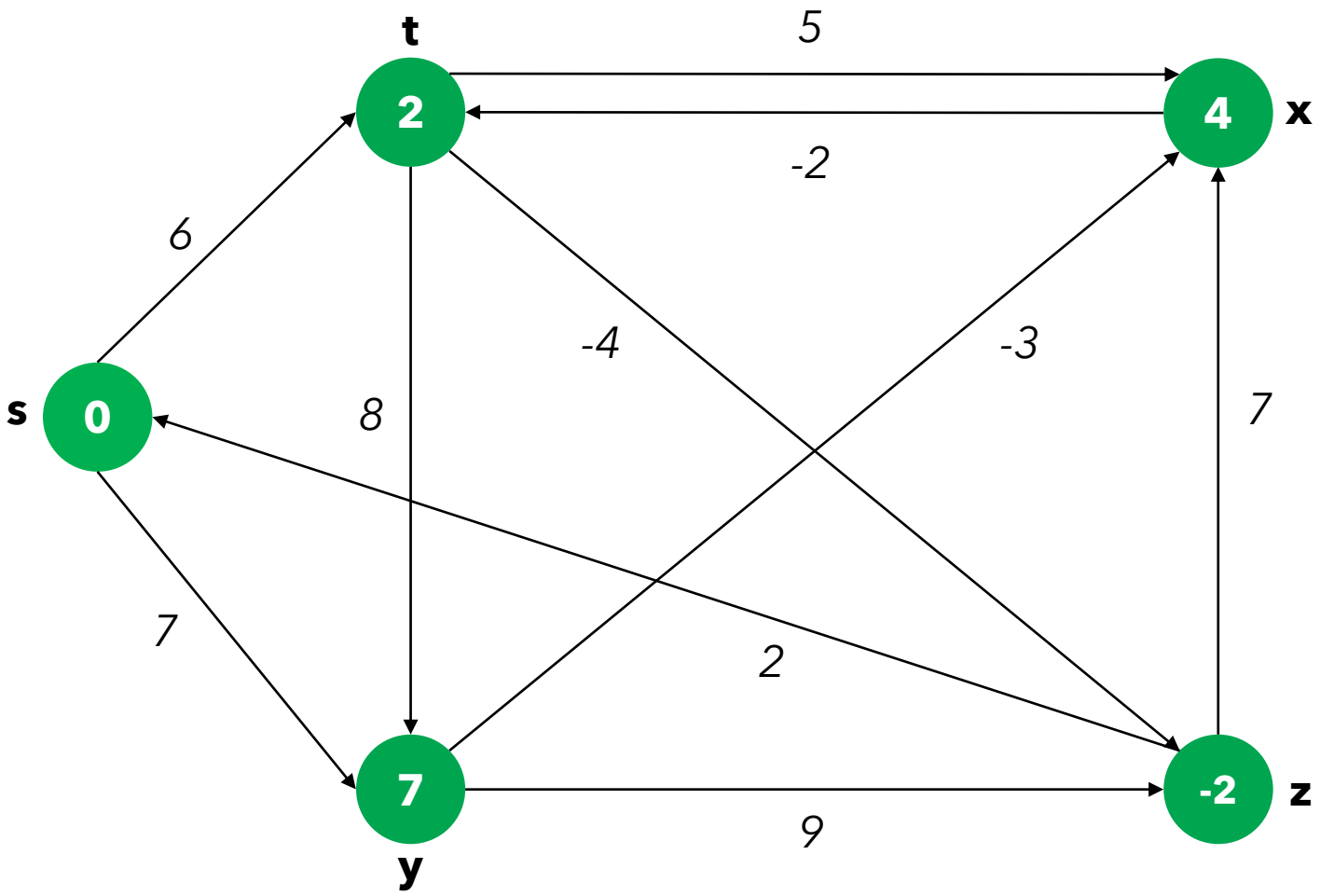
Bellman-Ford – 4th pass




```
t -> x
t -> y
t -> z
x -> t
y -> x
y -> z
z -> x
z -> s
s -> t
s -> y
```

rilassamento di y -> x (nessun effetto)

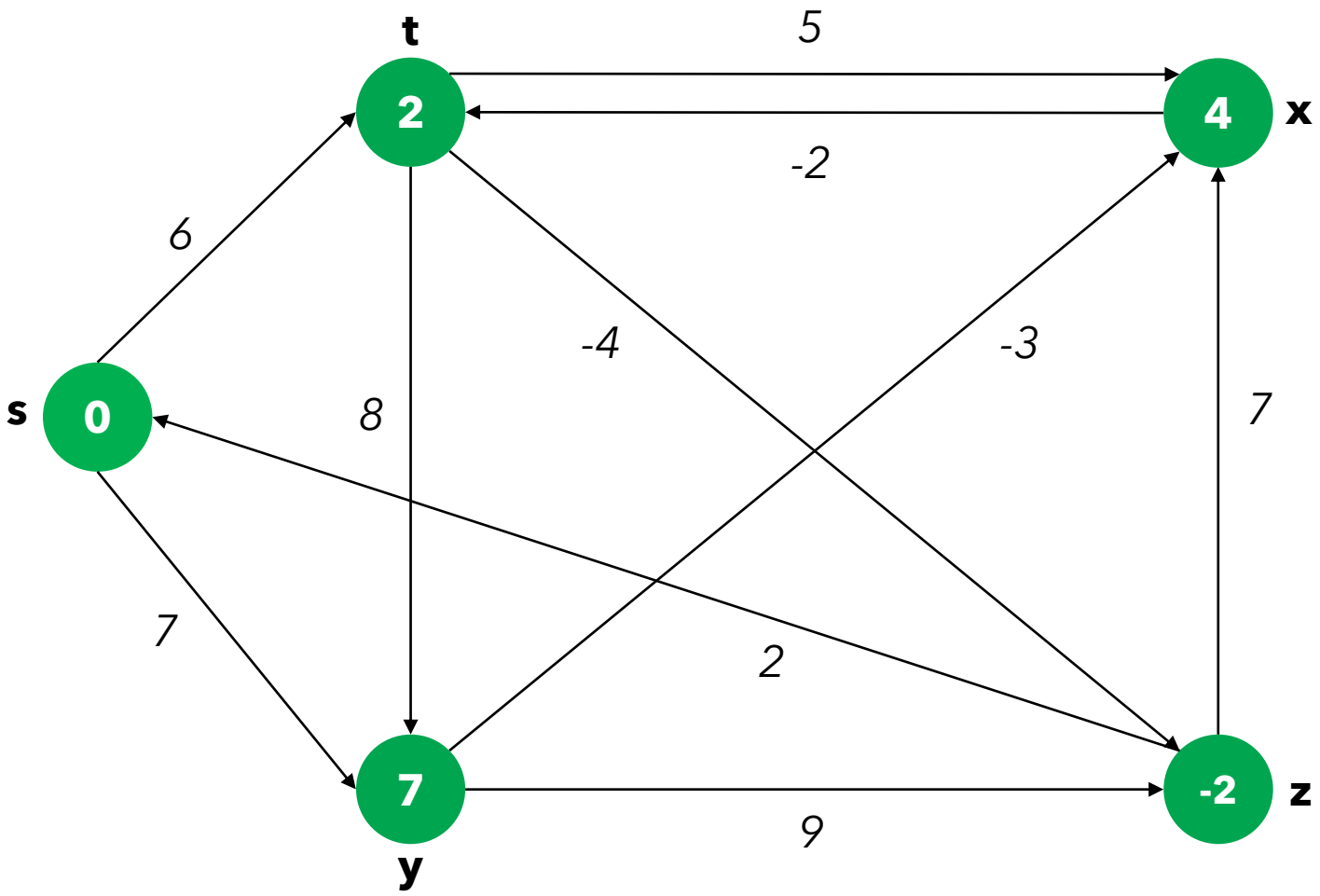
Bellman-Ford – 4th pass




```
t -> x
t -> y
t -> z
x -> t
y -> x
y -> z
z -> x
z -> s
s -> t
s -> y
```

rilassamento di y -> z (nessun effetto)

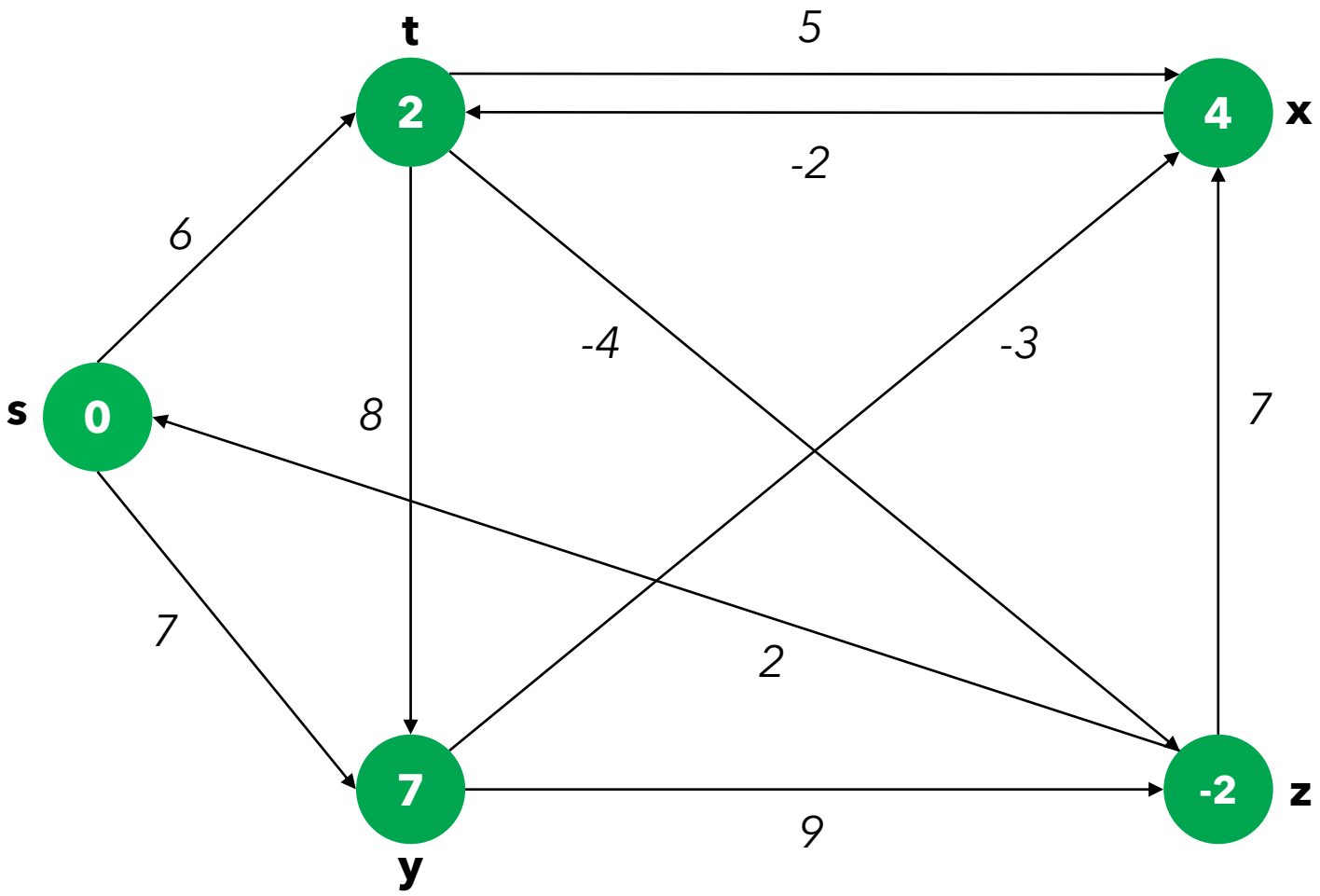
Bellman-Ford – 4th pass




```
t -> x
t -> y
t -> z
x -> t
y -> x
y -> z
z -> x
z -> s
s -> t
s -> y
```

rilassamento di z -> x (nessun effetto)

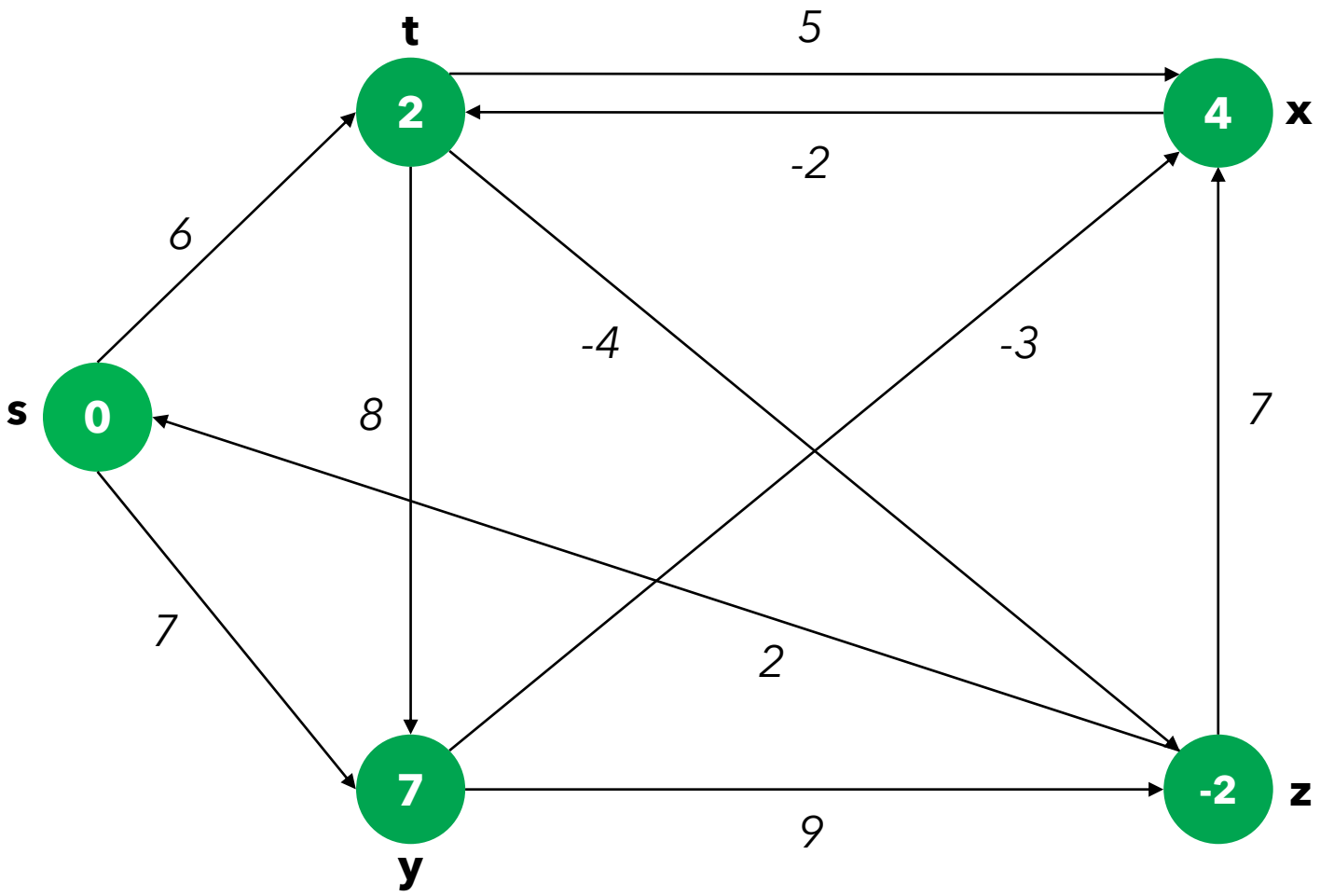
Bellman-Ford – 4th pass




```
t -> x
t -> y
t -> z
x -> t
y -> x
y -> z
z -> x
z -> s
s -> t
s -> y
```

rilassamento di z -> s (nessun effetto)

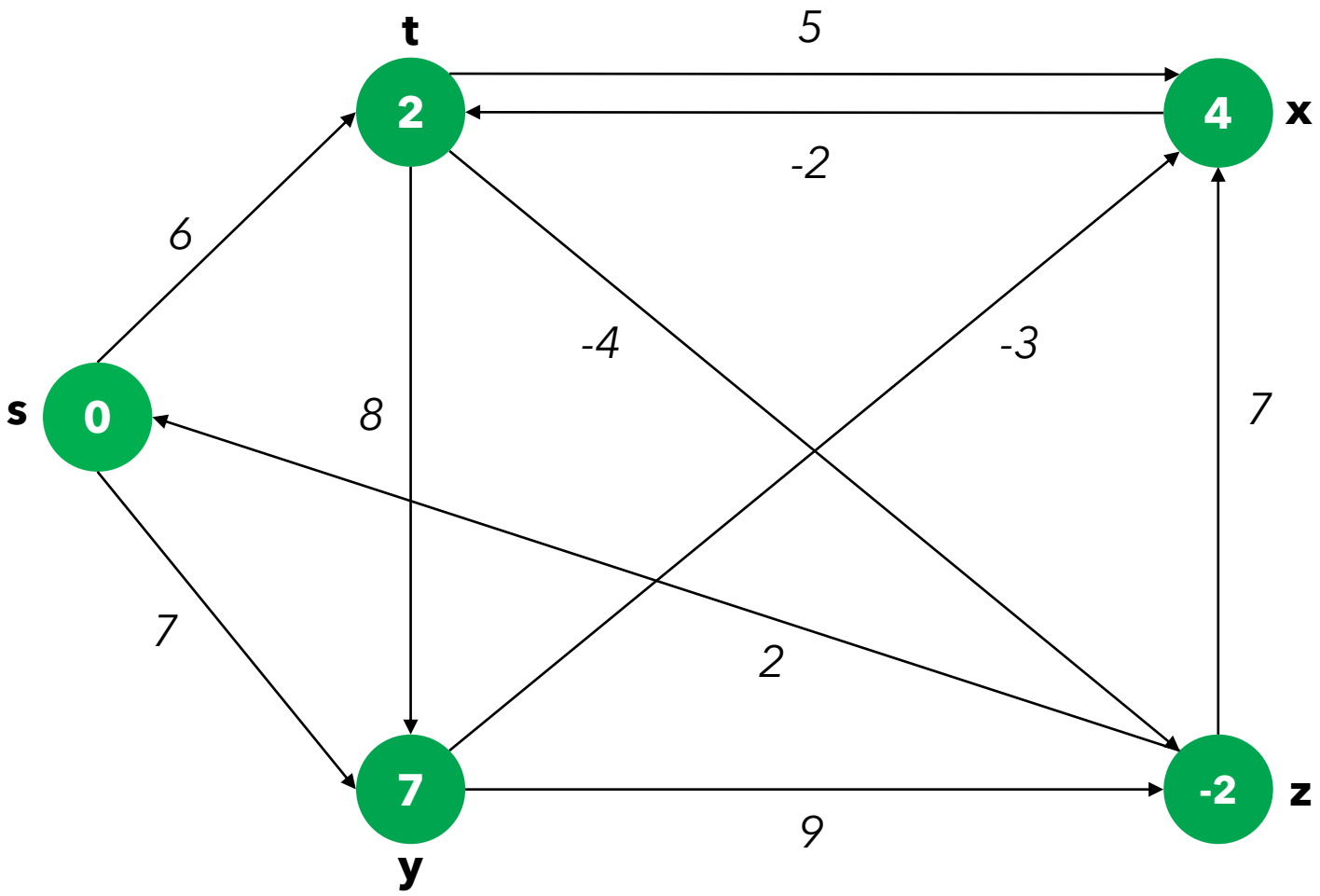
Bellman-Ford – 4th pass




```
t -> x
t -> y
t -> z
x -> t
y -> x
y -> z
z -> x
z -> s
s -> t
s -> y
```

rilassamento di `s -> t`
(nessun effetto)

Bellman-Ford – 4th pass




```
t -> x
t -> y
t -> z
x -> t
y -> x
y -> z
z -> x
z -> s
s -> t
s -> y
```

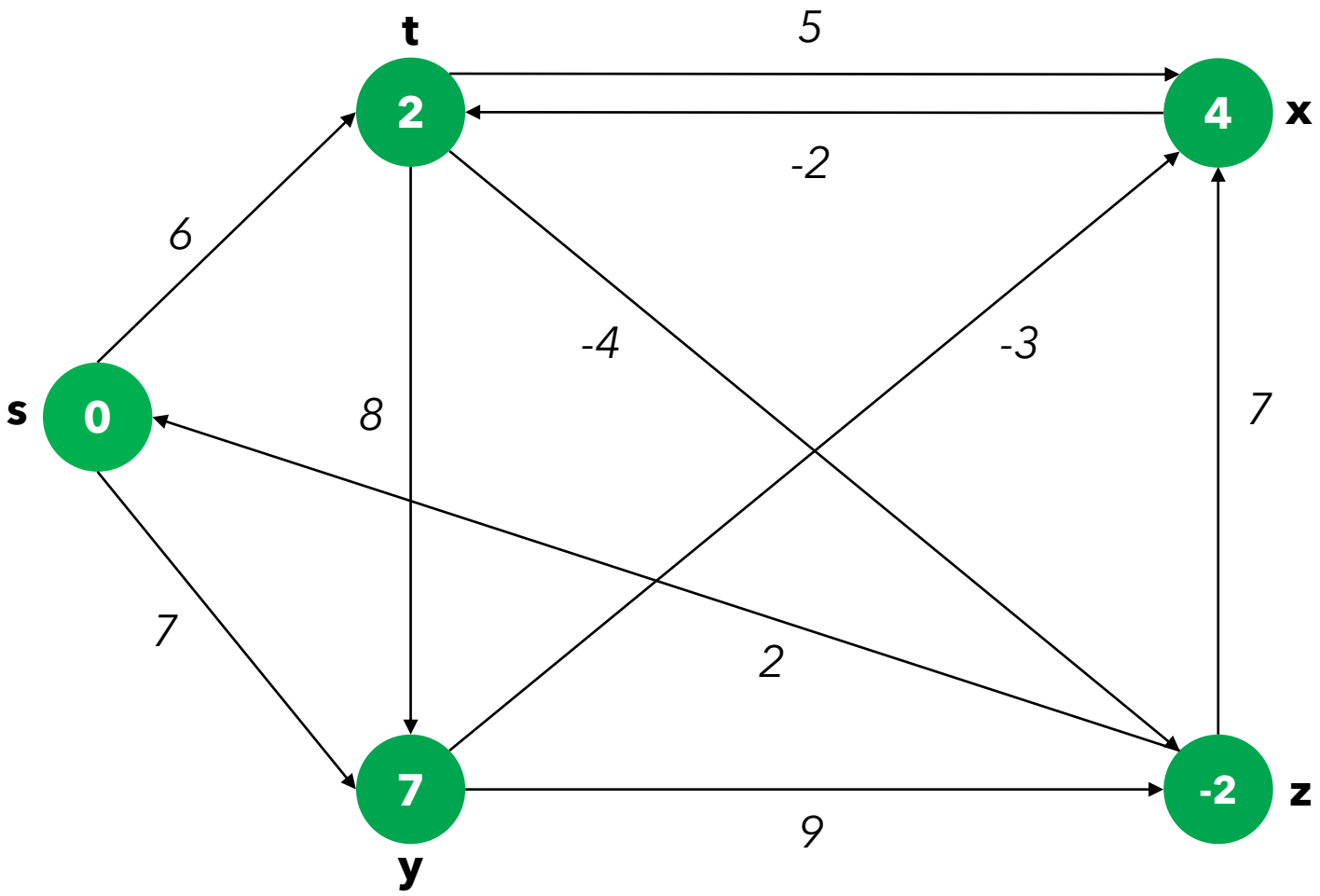
rilassamento di s -> y (nessun effetto)

Bellman-Ford – verifica cicli di costo negativo

```
bellman_ford_part_2(G):
  for each edge (u, v) in G:
    if v.cost > u.cost + cost(u, v):
      return TRUE
  return FALSE
```

- se c'è un ciclo di costo negativo, la procedura restituisce TRUE
- altrimenti, restituisce FALSE

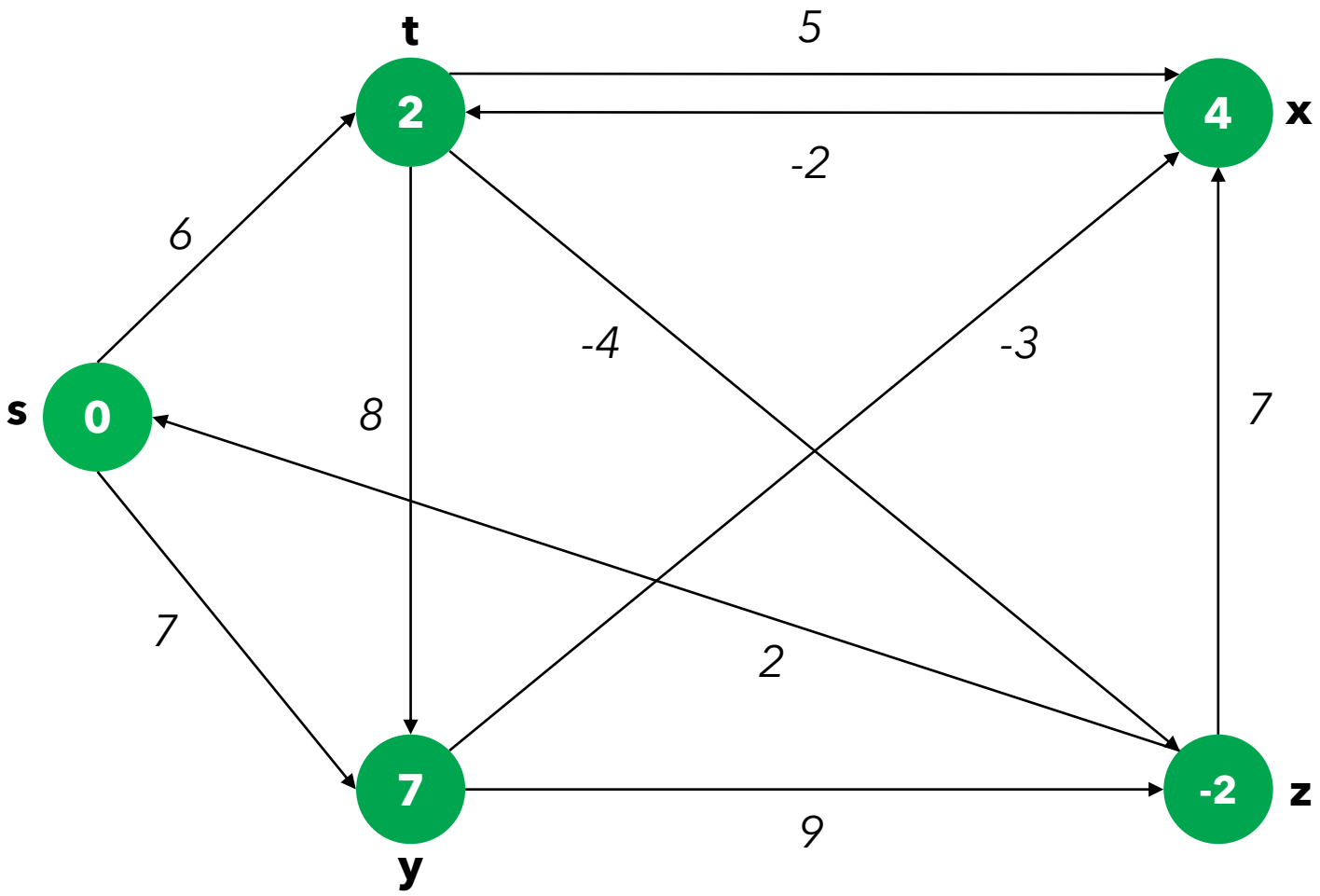
Bellman-Ford – verifica cicli di costo negativo



t -> x
t -> y
t -> z
x -> t
y -> x
y -> z
z -> x
z -> s
s -> t
s -> y

```
x.cost > t.cost + cost(t, x)
-> FALSE
```

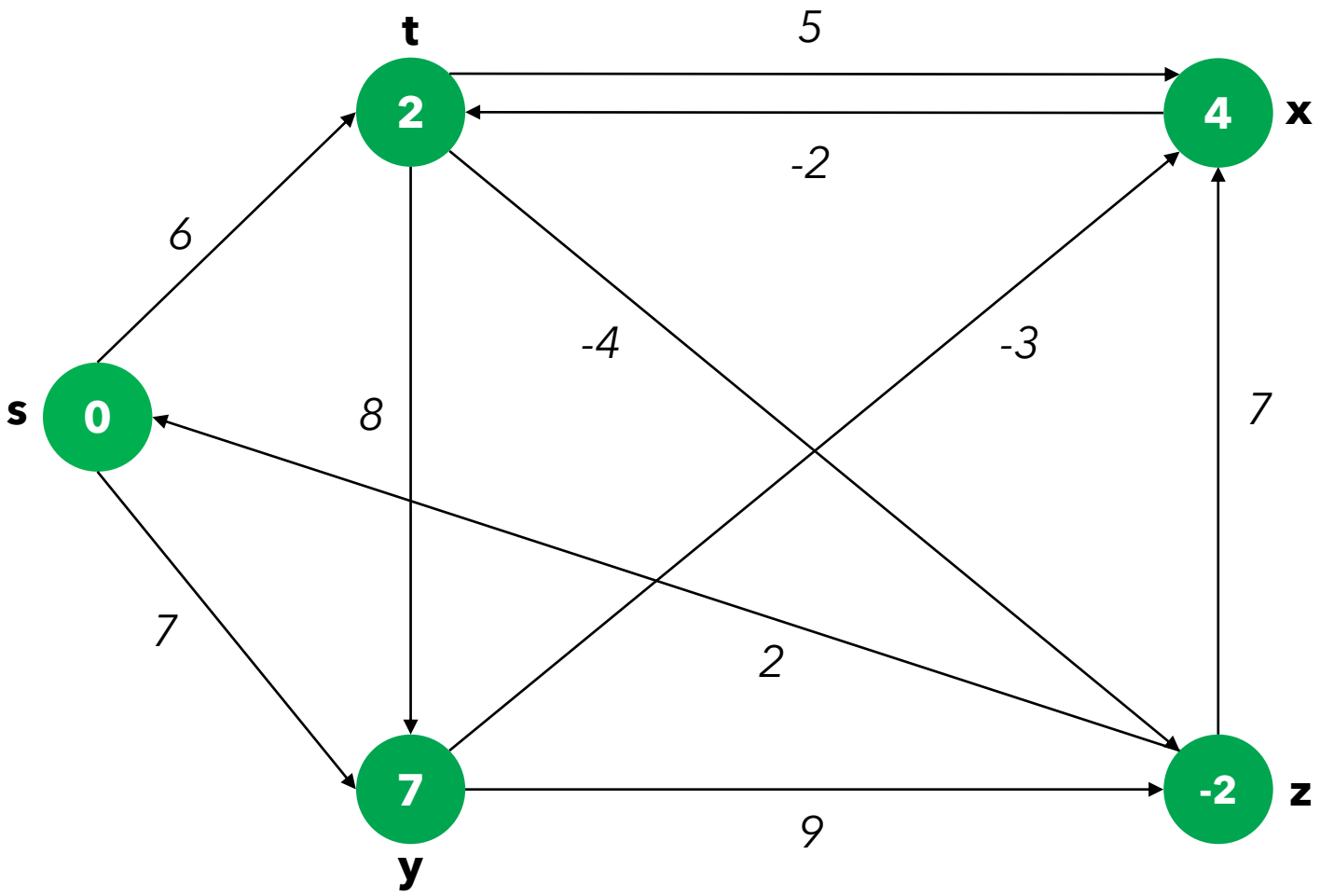
Bellman-Ford – verifica cicli di costo negativo




```
t -> x
t -> y
t -> z
x -> t
y -> x
y -> z
z -> x
z -> s
s -> t
s -> y
```

```
y.cost > t.cost + cost(t, y)
-> FALSE
```

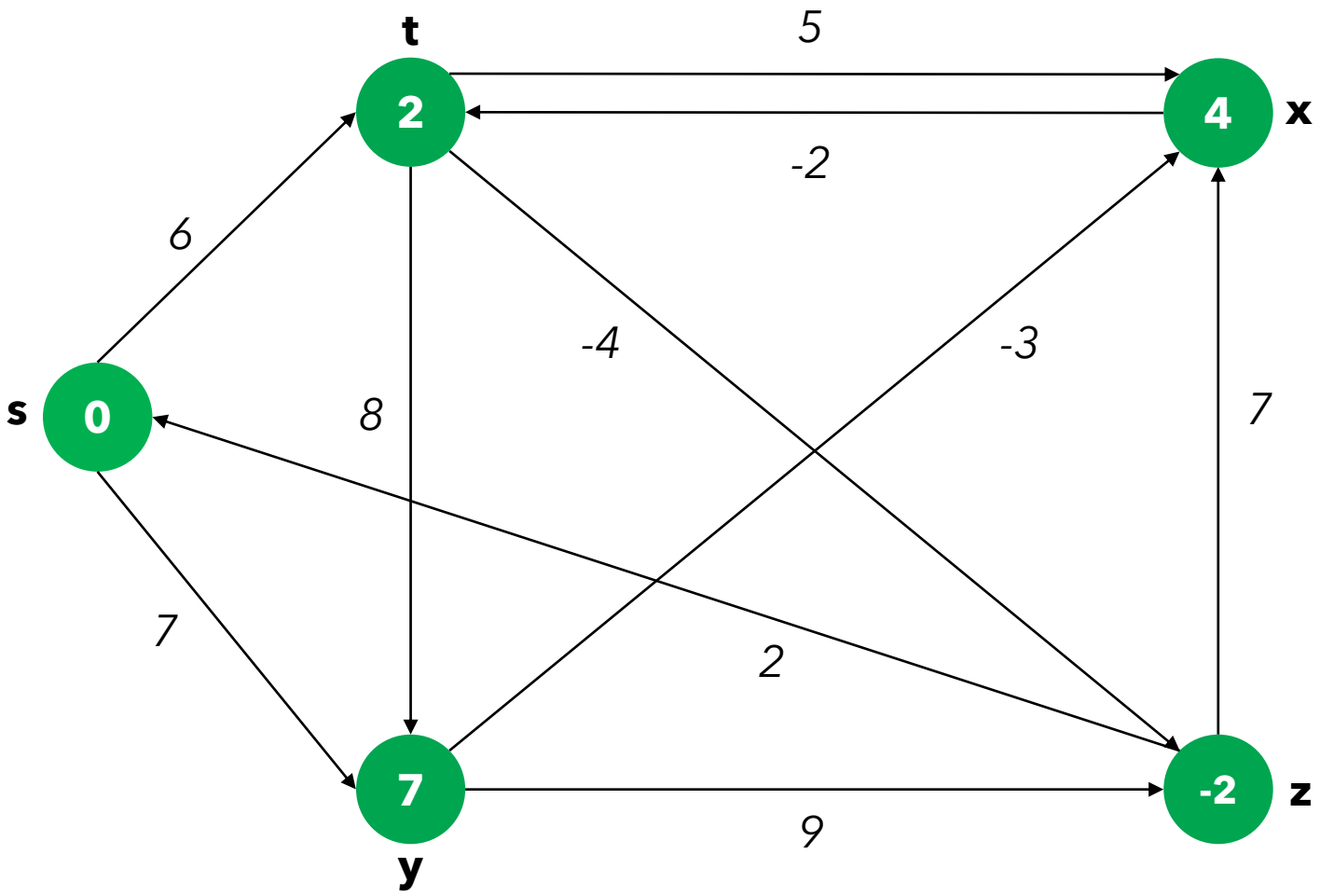
Bellman-Ford – verifica cicli di costo negativo




```
t -> x
t -> y
t -> z
x -> t
y -> x
y -> z
z -> x
z -> s
s -> t
s -> y
```

```
z.cost > t.cost + cost(t, z)
-> FALSE
```

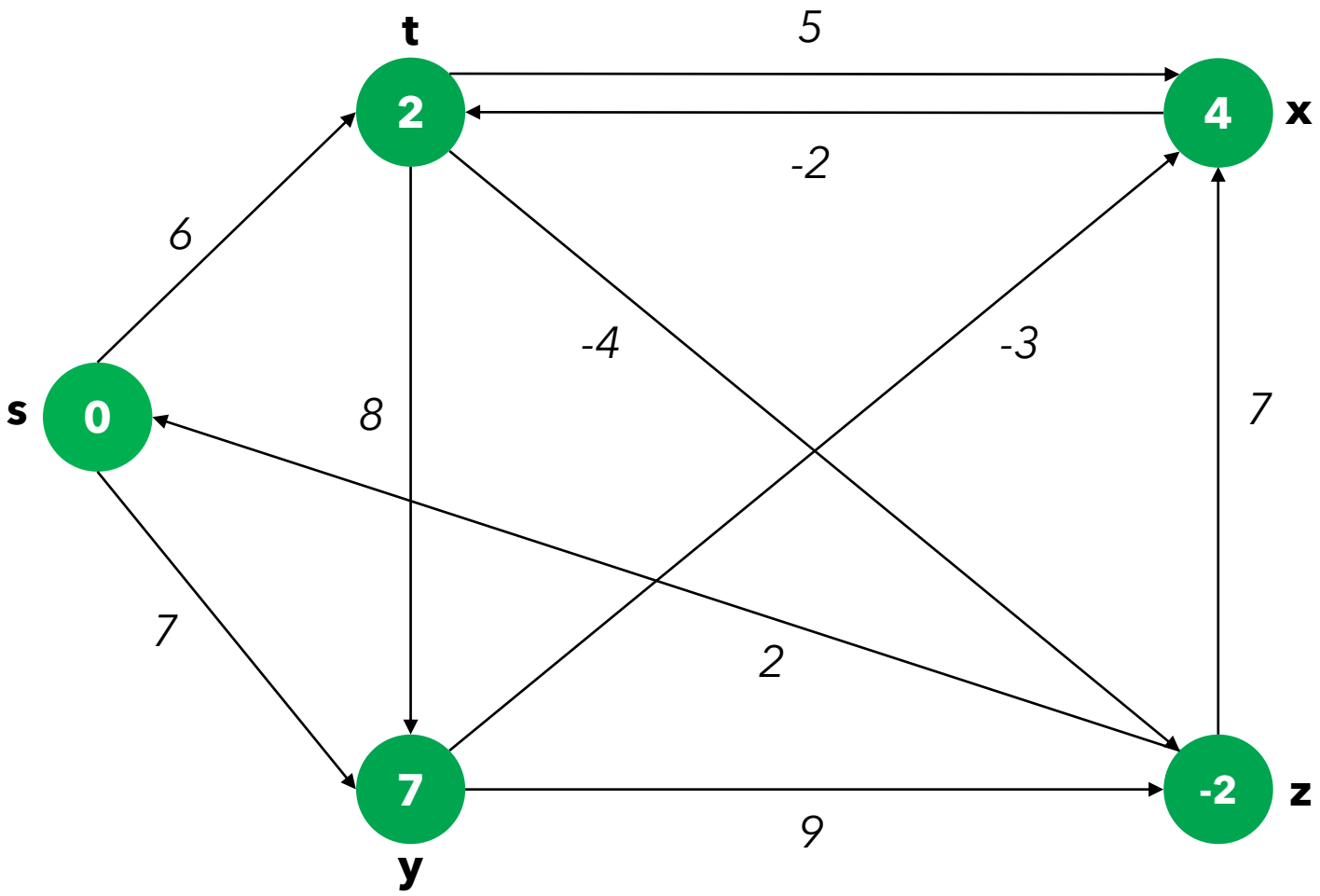
Bellman-Ford – verifica cicli di costo negativo




```
t -> x
t -> y
t -> z
x -> t
y -> x
y -> z
z -> x
z -> s
s -> t
s -> y
```

```
t.cost > x.cost + cost(x, t)
-> FALSE
```

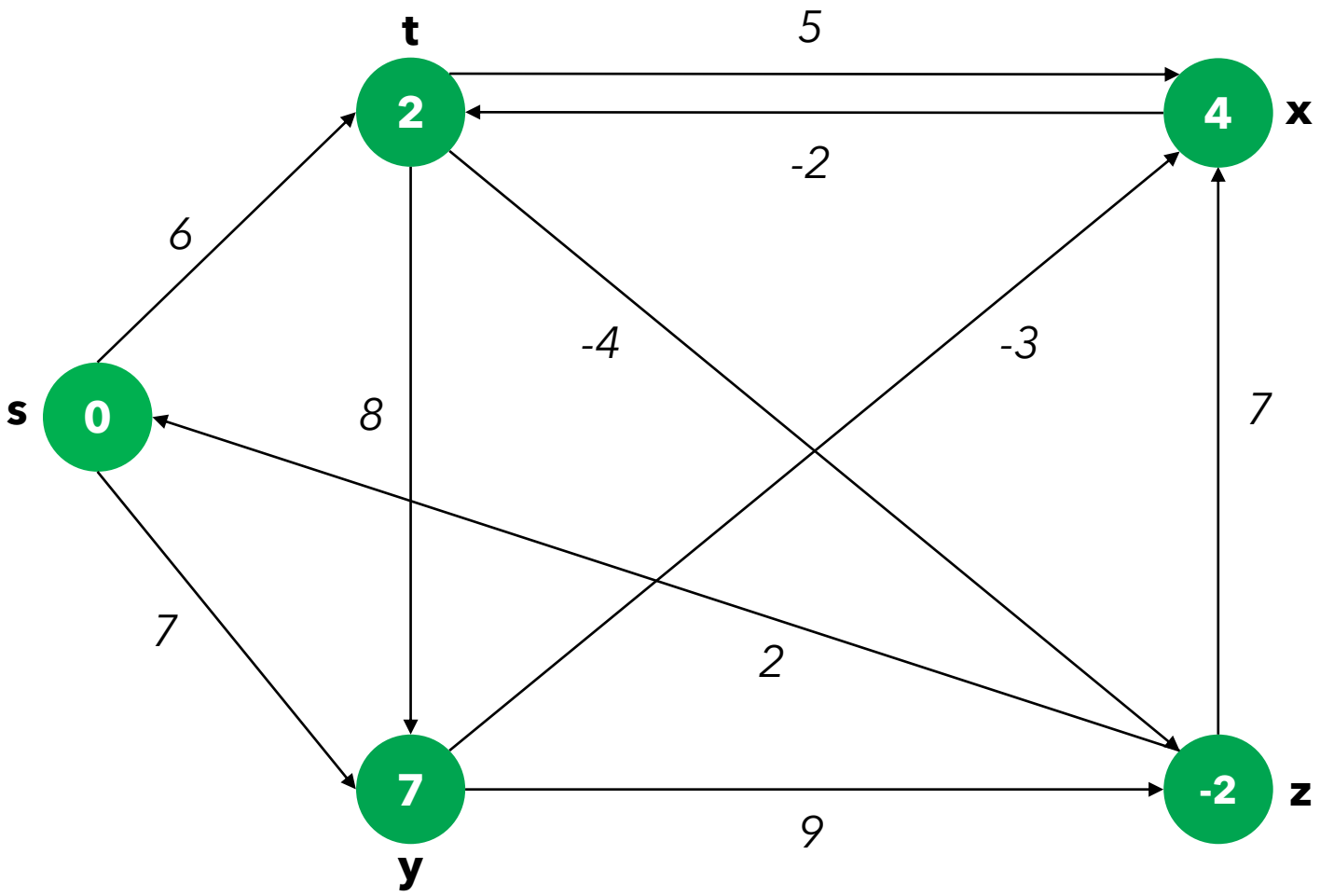
Bellman-Ford – verifica cicli di costo negativo




```
t -> x
t -> y
t -> z
x -> t
y -> x
y -> z
z -> x
z -> s
s -> t
s -> y
```

```
x.cost > y.cost + cost(y, x)
-> FALSE
```

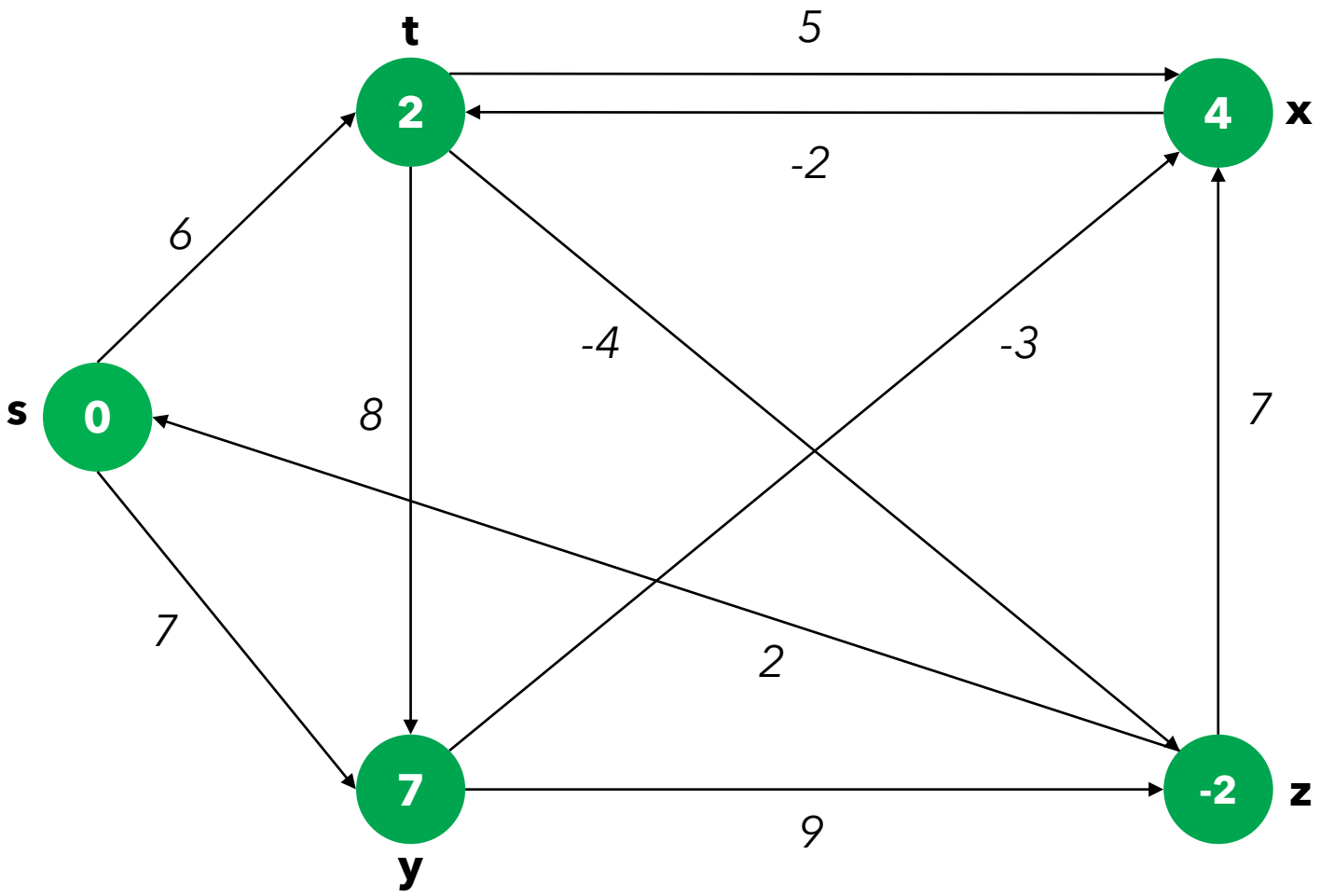
Bellman-Ford – verifica cicli di costo negativo




```
t -> x
t -> y
t -> z
x -> t
y -> x
y -> z
z -> x
z -> s
s -> t
s -> y
```

```
z.cost > y.cost + cost(y, z)
-> FALSE
```

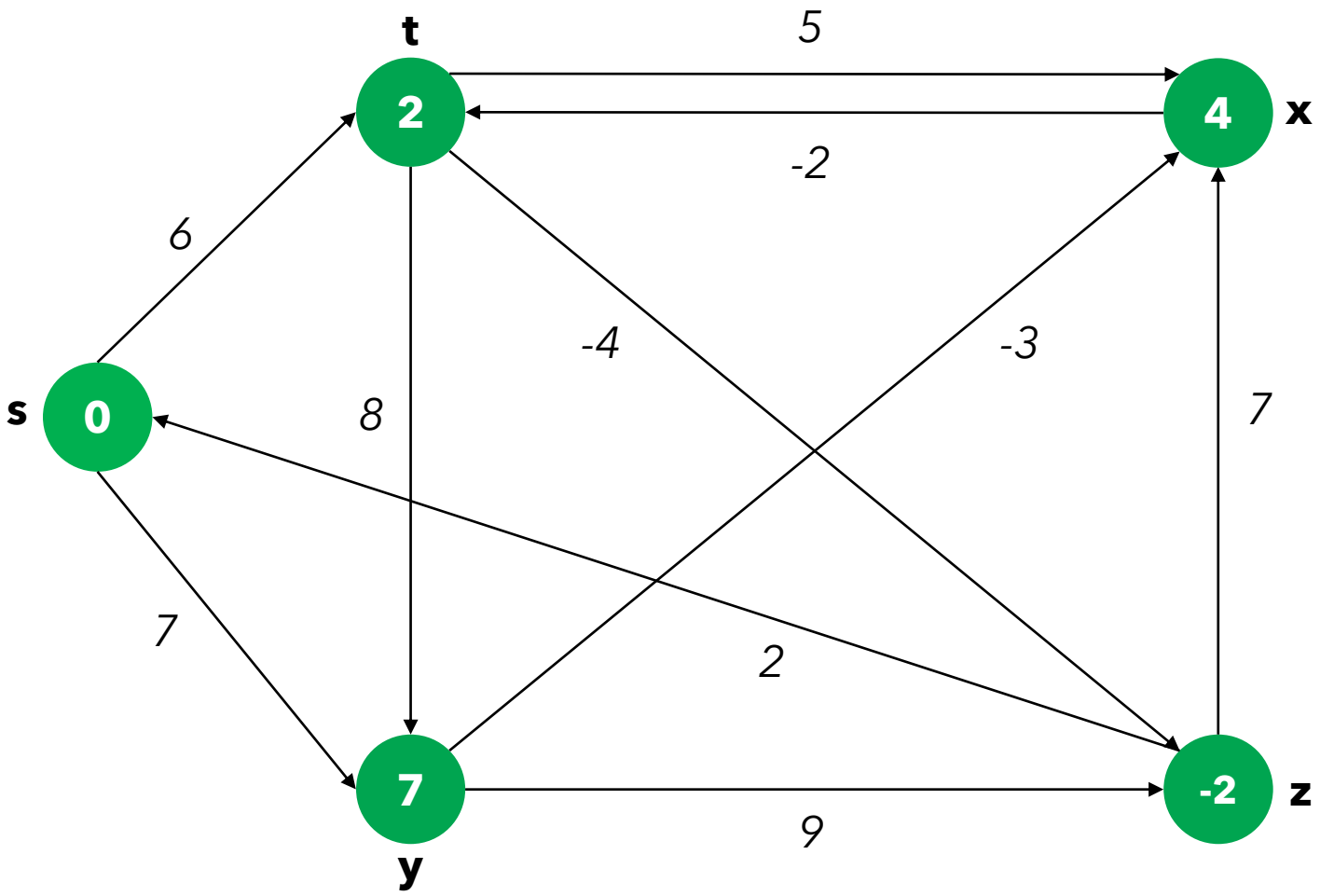
Bellman-Ford – verifica cicli di costo negativo




```
t -> x
t -> y
t -> z
x -> t
y -> x
y -> z
z -> x
z -> s
s -> t
s -> y
```

```
x.cost > z.cost + cost(z, x)
-> FALSE
```

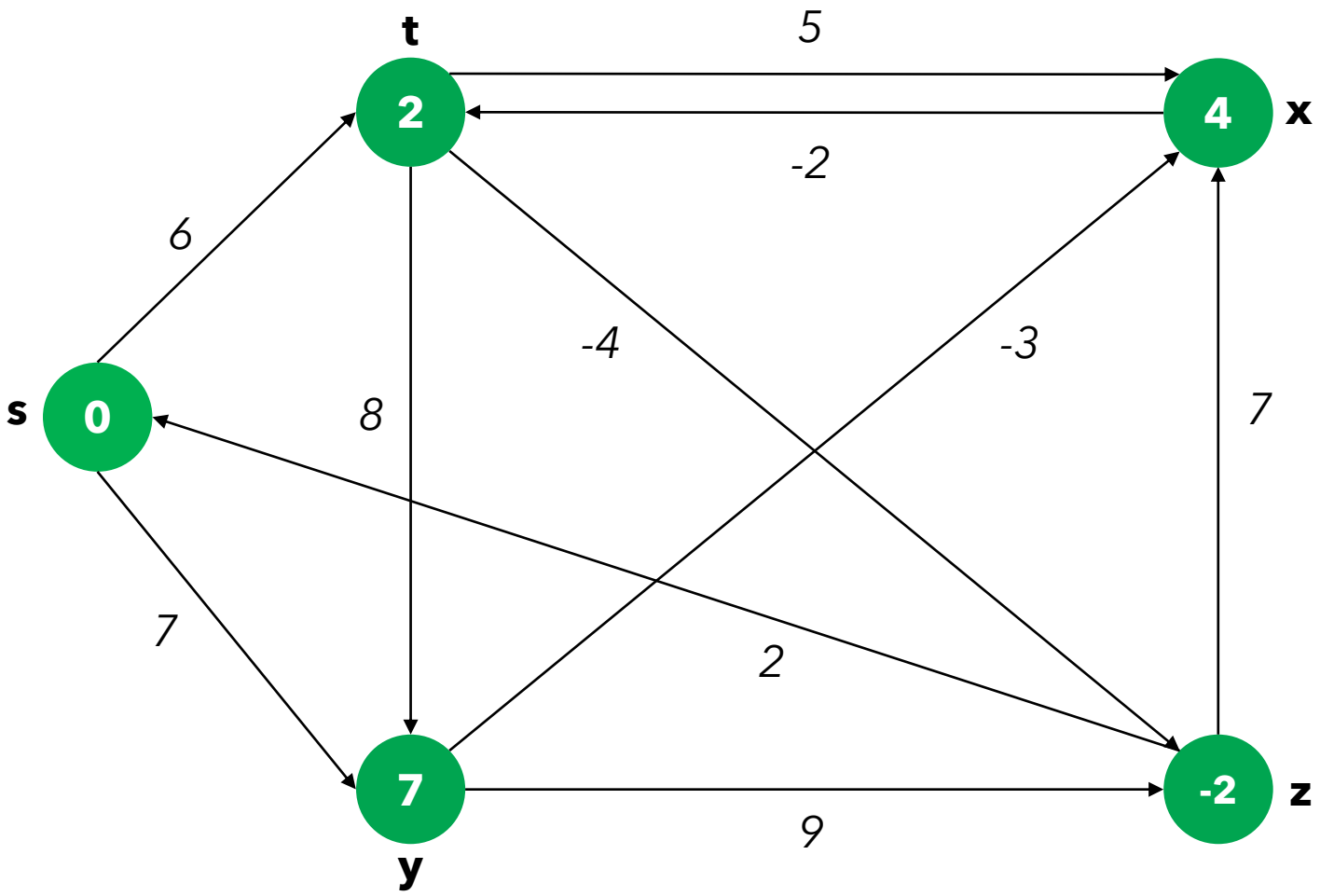
Bellman-Ford – verifica cicli di costo negativo




```
t -> x
t -> y
t -> z
x -> t
y -> x
y -> z
z -> x
z -> s
s -> t
s -> y
```

```
s.cost > z.cost + cost(z, s)
-> FALSE
```

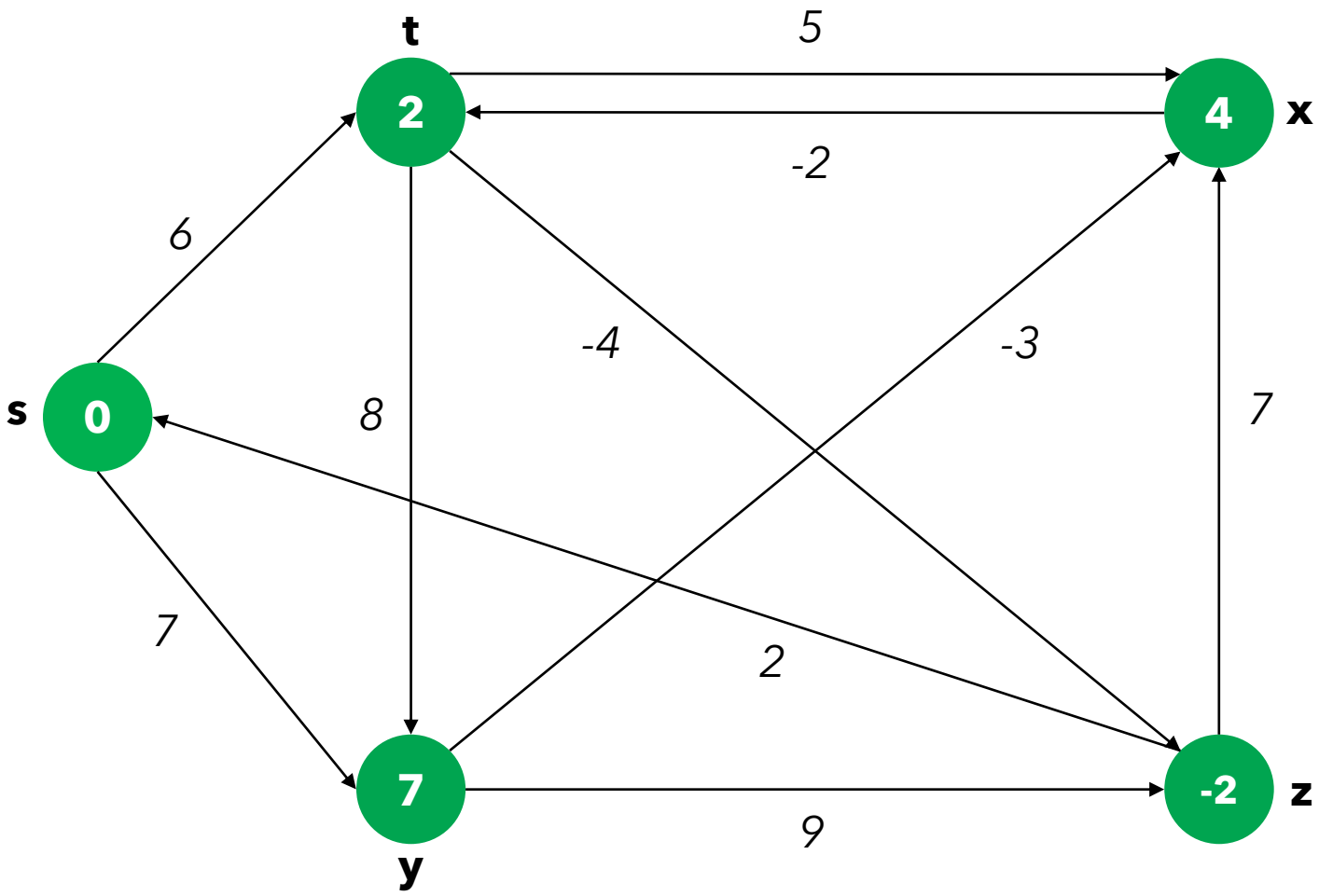
Bellman-Ford – verifica cicli di costo negativo




```
t -> x
t -> y
t -> z
x -> t
y -> x
y -> z
z -> x
z -> s
s -> t
s -> y
```

```
t.cost > s.cost + cost(s, t)
-> FALSE
```

Bellman-Ford – verifica cicli di costo negativo

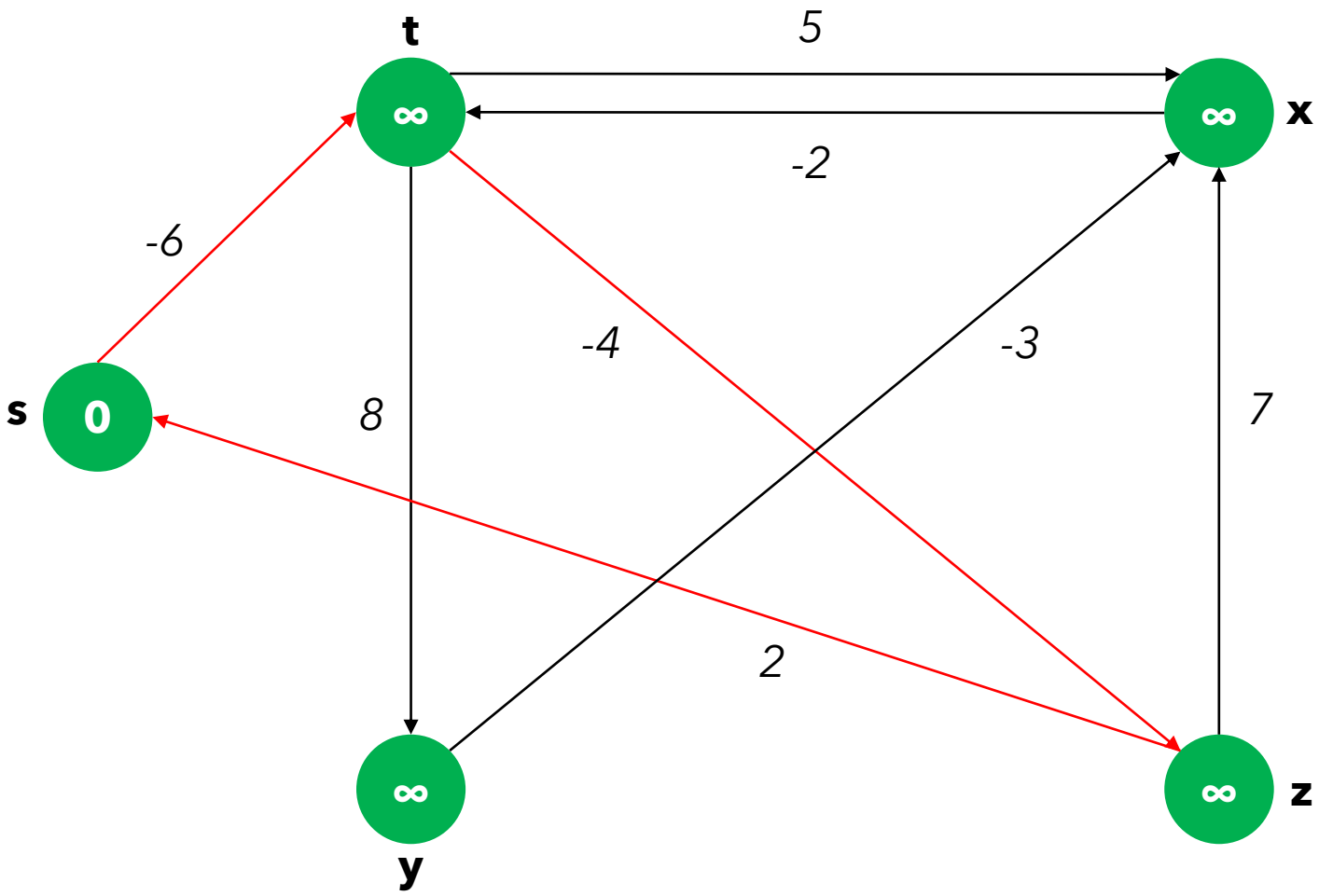



```
t -> x
t -> y
t -> z
x -> t
y -> x
y -> z
z -> x
z -> s
s -> t
s -> y
```

```
y.cost > s.cost + cost(s, y)
-> FALSE
```

no negative weight cycle

Bellman-Ford



eseguire le 2 sottoprocedure dell'algoritmo e verificare che il ciclo di costo negativo (indicato in rosso) venga rilevato correttamente