A Genetic Simulated Annealing Algorithm for the Capacitated Arc Routing Problem

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Abstract—The Capacitated Arc Routing Problem (CARP) is a combinatorial optimization problem that is NP-hard. In this paper, we propose a Genetic Simulated Annealing Algorithm to solve the CARP. The algorithm uses simulated annealing to explore the solution space and genetic algorithms to maintain a population of potential solutions. In addition to the basic move operators, we also incorporate a new move operator called the Merge-Split operator introduced by Yi Mei et al. [1]. The algorithm is then tested on the various benchmark instances of the CARP.

I. Introduction

The Capacitated Arc Routing Problem (CARP), which is one form of the Arc Routing Problem (ARP), is a combinatorial optimization problem. It is first introduced by Golden et al. [2] in 1984. Unlike the Vehicle Routing Problem (VRP), the CARP serves customers on arcs instead of nodes. The problem can be briefly described as follows:

1) Given a graph $G = (V, E)$, and a set of arcs $A \subseteq E$. Each arc $a \in A$ has a demand q_a, and each edge $e \in E$ has a cost c_e.
2) A fleet of vehicles with a capacity Q is available.
3) Each route starts and ends at the depot, and the total demand of each route does not exceed the capacity of the vehicle.
4) The objective is to find a set of routes that minimizes the total cost of the routes.

It is evident that the CARP has many real-world applications, such as garbage collection, street sweeping, and snow removal. However, the CARP is NP-hard, which means that it is difficult to find an exact solution in a reasonable amount of time. Therefore, many heuristic and metaheuristic algorithms have been proposed to solve the CARP. Representative heuristic algorithms include Path-Scanning [2], Augmenting Path [3], Ulusoy's Route First-Cluster Second, etc. Representative metaheuristic algorithms include Memetic Algorithm (MA) [4], Genetic Algorithm (GA) [5], Variable Neighborhood Search (VNS) [6], etc.

In this paper, we propose a Genetic Simulated Annealing Algorithm (GSA) to solve the CARP. The algorithm uses simulated annealing with move operators to explore the solution space. A genetic algorithm is used to maintain a population of diverse solutions and to guide the search process. In addition to the basic move operators, we also incorporate a new move operator called the Merge-Split operator introduced by Yi Mei et al. [1].

The rest of the paper is organized as follows. Section II formally defines the CARP. Section III describes the proposed algorithm in detail. Section IV presents the experimental setup and results. Finally, Section V concludes the paper.

II. Preliminary

The Capacitated Arc Routing Problem (CARP) can be formally defined as follows. A solution s to the CARP is represented as a set of routes $s = \{R_1, R_2, \dots, R_m\}$. Each route R_i is represented as a sequence of tasks $R_i = \{\tau_{i1}, \tau_{i2}, \dots, \tau_{il_i}\}$, where $\tau_{ij} \in T$ is a task and l_i is the number of tasks in route R_i. Attributes of task are: beginning vertex of task τ_{ij} as $head(\tau_{ij})$, ending vertex of task τ_{ij} as $tail(\tau_{ij})$, demand of task τ_{ij} as $d(\tau_{ij})$, and cost of task τ_{ij} as $c(\tau_{ij})$. The depot is represented as 0. The cost of the shortest path between two vertices i and j is denoted as $pc(i, j)$.

The constraints of the CARP are as follows:

$$\sum_{k=1}^{m} l_k = |T| \tag{1}$$

$$\tau_{i_1 j_1} \neq \tau_{i_2 j_2}, \forall (i_1, j_1 \neq i_2, j_2) \tag{2}$$

$$\tau_{i_1 j_1} \neq inv(\tau_{i_2 j_2}), \forall (i_1, j_1 \neq i_2, j_2) \tag{3}$$

$$\sum_{i=1}^{l_k} d(\tau_{kj}) \leq Q, \forall k \tag{4}$$

$$\tau_{ij} \in T \tag{5}$$

To explain the constraints, Equation 1, 2, and 3 ensure that each task is assigned to exactly one route. Equation 4 ensures that the demand of each route does not exceed the capacity of the vehicle. Equation 5 defines the set of tasks.

The objective function of the CARP is to minimize the total cost of the routes, which is $TC(s)$ defined as follows:

$$TC(s) = \sum_{k=1}^{m} RC(R_k) \tag{6}$$

where $RC(R_k)$ is the cost of route R_k defined as follows:

$$RC(R_k) = \sum_{i=1}^{l_k} c(\tau_{ki}) + pc(0, head(\tau_{k1})) + pc(tail(\tau_{kl_k}), 0) + \sum_{i=2}^{l_k} pc(tail(\tau_{ki-1}), head(\tau_{ki})) \tag{7}$$

III. Methodology

A. General Workflow

The proposed Genetic Simulated Annealing Algorithm (GSA) is a hybrid algorithm that combines simulated annealing and genetic algorithms. The general workflow of the algorithm is as follows:

1) Initialize the population with heuristic solutions.
2) For each individual in the population, apply simulated annealing with move operators to explore the solution space.
3) Select the best individuals from parent population and offspring population.
4) If the stopping criterion is met, return the best individual, otherwise go to step 2.

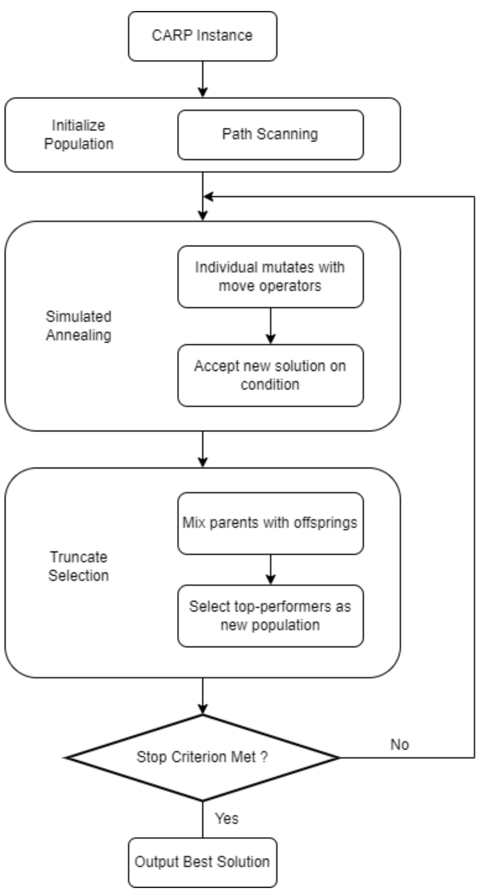


Fig. 1: General workflow of the proposed algorithm

B. Detailed Implementation

1) Initialization: For the initialization of the population, we use the Path-Scanning algorithm [2] to generate a set of feasible solutions. The Path-Scanning algorithm is a simple and effective heuristic algorithm for the CARP. It starts with an empty route and iteratively adds the nearest serviceable task to the route, until the capacity of the vehicle can no longer accommodate any remaining tasks. The algorithm then starts a new route and repeats the process until all tasks are assigned to routes. If multiple tasks are equidistant, the algorithm selects the task with one of the following strategies:

1) The task closest to the depot.
2) The task farthest from the depot.
3) The task with the smallest demand-cost ratio.
4) The task with the largest demand-cost ratio.
5) If less than half of the capacity is used, used rule 1. Otherwise, use rule 2.

Algorithm 1: Path-Scanning Algorithm

```
Input: A set of tasks T, a capacity Q
Output: A set of routes R
R ← ∅;
r ← {(depot, depot)};
d_cur ← 0;
while T ≠ ∅ do
    dist_min ← ∞;
    t_min ← ∅;
    t_prev ← r.last();
    foreach τ ∈ T do
        dist ← pc(tail(t_prev), head(τ));
        if dist < dist_min and d_cur + d(τ) ≤ Q then
            dist_min ← dist;
            t_min ← τ;
        end
    end
    if t_min = ∅ then
        R ← R ∪ {r};
        r ← {(depot, depot)};
        d_cur ← 0;
    else
        r ← r ∪ {t_min};
        d_cur ← d_cur + d(t_min);
        T ← T \ {t_min};
    end
end
R ← R ∪ {r};
return R
```

Note: $(depot, depot)$ is a dummy task that represents the depot as the beginning of the route.

The Path-Scanning algorithm guarantees that the generated solutions are feasible, but they may not be optimal. Hence, we apply simulated annealing to improve the quality of the solutions.

2) Simulated Annealing: Simulated annealing is a probabilistic optimization algorithm that is inspired by the annealing process in metallurgy. The algorithm starts with an initial solution and iteratively explores the solution space by accepting worse solutions with a certain probability. The probability of accepting a worse solution is determined by the temperature parameter, which decreases over time. The algorithm terminates when the temperature reaches a certain threshold. In the context of the CARP, the solution space is explored by applying move operators to the solution.

Algorithm 2: Simulated Annealing

Input: A solution s, a temperature T, a cooling rate α
Output: A new solution s, the best solution encountered s^*
$s^* \leftarrow s$;
while $T > 0$ do
    $s' \leftarrow \mathrm{ApplyMove}(s)$;
    $\Delta E \leftarrow TC(s') - TC(s)$;
    if $\Delta E < 0$ or $\mathrm{rand}(0,1) < \exp(-\Delta E/T)$ then
        $s \leftarrow s'$;
        if $TC(s) < TC(s^*)$ then
            $s^* \leftarrow s$;
        end
    end
    $T \leftarrow T \times \alpha$;
end
return s, s^*

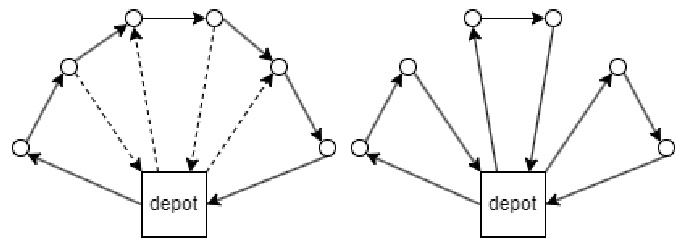
One modification we made to the basic simulated annealing algorithm is to keep track of the best solution encountered during the process. This is because the best solution may deteriorate during the exploration process, thus may not be the final solution. However, we want to keep the diversity of the population, so we also keep the final solution.

Another Implementation detail is the initial temperature. When running on large instances, the move operators may generate relatively large changes to the solution, which suggests a higher initial temperature. Hence, for instances with sum of demands greater than 3000, we set the initial temperature to 100, otherwise, we set it to 1.

3) Merge-Split Operator: In addition to the basic move operators, we also incorporate a new move operator called the Merge-Split operator introduced by Yi Mei et al. [1]. The purpose of intruding the Merge-Split operator is to improve the “step size” of the search process. Basic move operators such flip, swap, and reinsert only make small changes to the solution. Hence, the search process may get stuck in a local minimum. The Merge-Split operator regenerates the solution by merging and splitting routes, which can make larger changes to the solution and help escape local minima. The Merge-Split operator is defined as follows:

1) Merge: Select arbitrary number of routes $R_1, R_2, \ldots, R_n$. Then generate an unordered list of tasks $T_{merge} = \{t \in \bigcup_{i=1}^{n} R_i\}$. Apply the Path-Scanning algorithm to generate a new single route (we ignore the capacity constraint in this step). The generated route is considered as an ordered list of tasks.
2) Split: Apply Ulusoy’s splitting algorithm to split this ordered list of tasks into multiple routes. Replace the selected routes with the generated routes.

The Ulusoy’s splitting algorithm is a heuristic algorithm that splits a route into multiple routes. It starts with an empty route and iteratively adds tasks following the order of the list. When the capacity of the vehicle is reached, the algorithm starts a new route.



(a) Merge (b) Split

Fig. 2: Merge-Split operator

4) Genetic Algorithm: The genetic algorithm is used to maintain a population of diverse solutions and to guide the search process.

Algorithm 3: Genetic Algorithm

Input: A population P
Output: A new population P'
$P' \leftarrow P$;
$s_{best} \leftarrow \mathrm{BestSolution}(P)$;
foreach $s \in P'$ do
    $s, s^* \leftarrow \mathrm{SimulatedAnnealing}(s, T, \alpha)$;
    $s_{best} \leftarrow \mathrm{BestSolution}(s^*, s_{best})$;
end
$P' \leftarrow P \cup P' \cup \{s_{best}\}$;
$P'.sort()$;
$P' \leftarrow P'[0 : |P|]$;
return P'

We modify the genetic algorithm to select the best individuals from the parent population, offspring population, and the best solution encountered during the simulated annealing process. This is because the simulated annealing process may generate a better solution than the parent population and offspring population, as mentioned earlier.

IV. Experiments

A. Setup

We first experiment the proposed algorithm with different population sizes and cooling rates. Since smaller instances are easier to solve and show little difference in results, we use a relatively large *egl*-S1-A dataset for this experiment. The best set of parameters is selected for the subsequent experiments.

We then test the proposed Genetic Simulated Annealing Algorithm (GSA) on the benchmark instances of the *gdb*, *egl*, and *val* datasets. The *gdb* dataset contains the smallest instances, then the *val* dataset, and the *egl* dataset contains the largest instances. This allows us to evaluate the robustness of the algorithm on different scales of the problem.

The algorithm is implemented in Python and run on a computer with an Intel Core i9-12900H CPU and 16GB of RAM. The environment is Ubuntu 22.04.3 LTS with Python 3.9.7. For each instance, we conduct 5 independent runs with a runtime limit of 60 seconds. The random seed is set to 1, 2, 3, 4, and 5 for each run, respectively.

B. Results

The results of the first experiment are shown in Table I. The table shows the average cost of the solutions found by the algorithm.

Population Size	Cooling Rate	Avg Cost
5	0.999	5170.2
10	0.999	5223.6
20	0.999	5199.8
5	0.99	5226
10	0.99	5184.2
20	0.99	5203.4

TABLE I: Results of different population sizes and cooling rates on the *egl*-S1-A dataset

The results of the second experiment is shown in Table II, III, and IV. The table shows the cost of the best solution found by the algorithm and the average cost of the solutions. We also incorporate the theoretical lower bound and result of MEANS [1] for comparison. The columns labeled Gap_1 and Gap_2 show the percentage gap between the best solution found by the algorithm and the lower bound, and the result of MEANS, respectively.

C. Analysis

For the first experiment, we observe that a population size of 5 and a cooling rate of 0.999 yield the best results. This is because a smaller population size allows the algorithm to run the simulated annealing process more times within the same time limit. A higher cooling rate allows the algorithm to explore the solution space further. This is consistent with the intuition that a higher cooling rate allows the algorithm to escape local minima more easily.

Dataset	Lower Bound	MEANS		GSA		Gap_1	Gap_2
		Avg	Best	Avg	Best		
1	316	316	316	316	316	0	0
2	339	339	339	339	339	0	0
3	275	275	275	275	275	0	0
4	287	287	287	287	287	0	0
5	377	377	377	377	377	0	0
6	298	298	298	298	298	0	0
7	325	325	325	325	325	0	0
8	348	348.7	348	353.2	350	0.574713	0.574713
9	303	303	303	308	303	0	0
10	275	275	275	275	275	0	0
11	395	395	395	395	395	0	0
12	458	458	458	461.2	458	0	0
13	536	536	536	540.8	536	0	0
14	100	100	100	100	100	0	0
15	58	58	58	58	58	0	0
16	127	127	127	127	127	0	0
17	91	91	91	91	91	0	0
18	164	164	164	164	164	0	0
19	55	55	55	55	55	0	0
20	121	121	121	121	121	0	0
21	156	156	156	156	156	0	0
22	200	200	200	200	200	0	0
23	233	233	233	234.6	233	0	0
Avg	253.7826	253.813	253.7826	254.6435	253.8696	0.034264	0.034264
No. best	-	22	23	18	22	-	-

TABLE II: Results on the *gdb* dataset

Dataset	Lower Bound	MEANS		GSA		Gap_1	Gap_2
		Avg	Best	Avg	Best		
1A	173	173	173	174.4	173	0	0
1B	173	173	173	175.4	173	0	0
1C	245	245	245	246.4	245	0	0
2A	227	227	227	227	227	0	0
2B	259	259	259	261	260	0.3861	0.3861
2C	457	457.2	457	471.4	469	2.625821	2.625821
3A	81	81	81	81.2	81	0	0
3B	87	87	87	87.2	87	0	0
3C	138	138	138	138.2	138	0	0
4A	400	400	400	405.6	400	0	0
4B	412	412	412	416.8	412	0	0
4C	428	431.1	428	442	434	1.401869	1.401869
4D	526	532.9	530	545.8	540	2.661597	1.886792
5A	423	423	423	423	423	0	0
5B	446	446	446	446.6	446	0	0
5C	473	474	474	474	474	0.211416	0
5D	573	582.9	577	600.4	597	4.188482	3.466205
6A	223	223	223	223.4	223	0	0
6B	233	233	233	238.2	233	0	0
6C	317	317	317	318.6	317	0	0
7A	279	279	279	285.2	279	0	0
7B	283	283	283	284	283	0	0
7C	334	334	334	340.4	336	0.598802	0.598802
8A	386	386	386	387.2	386	0	0
8B	395	395	395	395	395	0	0
8C	518	525.9	521	538.6	537	3.667954	3.071017
9A	323	323	323	328.8	326	0.928793	0.928793
9B	326	326	326	328	326	0	0
9C	332	332	332	335.2	332	0	0
9D	385	391	391	399.4	393	2.077922	0.511509
10A	428	428	428	432.2	429	0.233645	0.233645
10B	436	436	436	440.4	439	0.688073	0.688073
10C	446	446	446	449.8	446	0	0
10D	525	533.6	531	539.8	538	2.47619	1.318267
Avg	343.8235	345.1059	344.5294	349.4294	346.9706	0.915312	0.708554
No. best	-	26	28	3	21	-	-

TABLE III: Results on the *val* dataset

For the second experiment, we observe that the proposed Genetic Simulated Annealing Algorithm (GSA) performs well on the *gdb* and *val* datasets. The algorithm finds the best solution for 22 out of 23 instances in the *gdb* dataset and 21 out of 28 instances in the *val* dataset. And the average gap between the best solution found by GSA and the lower bound is less than 0.1% for the *gdb* dataset and less than 1% for the *val* dataset. This indicates that the algorithm is effective in finding near-optimal solutions for small and medium-sized instances.

However, the algorithm struggles with the *egl* dataset, which contains larger instances. The average gap between the best solution found by GSA and the lower bound is around 5%. This suggests that the algorithm may require more sophisticated techniques or additional runtime to solve large instances effectively.

Dataset	Lower Bound	MEANS		GSA		Gap_1	Gap_2
		Avg	Best	Avg	Best		
E1-A	3548	3548	3548	3585	3561	0.366404	0.366404
E1-B	4498	4516.5	4498	4555	4543	1.000445	1.000445
E1-C	5566	5601.6	5595	5703.8	5687	2.173913	1.644325
E2-A	5018	5018	5018	5040.6	5027	0.179354	0.179354
E2-B	6305	6341.4	6317	6450.8	6430	1.982554	1.788824
E2-C	8243	8355.7	8335	8660.4	8615	4.51292	3.359328
E3-A	5898	5898.8	5898	6031.4	5996	1.66158	1.66158
E3-B	7704	7802.9	7775	7999.4	7946	3.141225	2.199357
E3-C	10163	10321.9	10292	10534	10457	2.892847	1.603187
E4-A	6408	6475.2	6456	6681.6	6658	3.901373	3.128872
E4-B	8884	9023	8998	9409.6	9305	4.738856	3.411869
E4-C	11427	11645.8	11561	12043.4	11859	3.78052	2.577632
S1-A	5018	5039.8	5018	5206.2	5152	2.670387	2.670387
S1-B	6384	6433.4	6388	6707	6689	4.777569	4.71196
S1-C	8493	8518.3	8518	8736.4	8640	1.730837	1.432261
S2-A	9824	9959.2	9895	10515	10397	5.832655	5.073269
S2-B	12968	13231.6	13147	13896.2	13789	6.330969	4.883243
S2-C	16353	16509.8	16430	17577.2	17408	6.451416	5.952526
S3-A	10143	10312.7	10257	10932.6	10862	7.088633	5.898411
S3-B	13616	13876.6	13749	14620.6	14463	6.220623	5.193105
S3-C	17100	17305.8	17207	18452.6	18353	7.327485	6.66008
S4-A	12143	12419.2	12341	13434	13123	8.070493	6.336602
S4-B	16093	16441.2	16337	17626	17468	8.544087	6.922936
S4-C	20375	20767.2	20538	22673	22531	10.5816	9.703963
Avg	9673.833	9806.817	9754.833	10294.66	10206.63	5.507555	4.631465
No. best	-	2	5	0	0	-	-

TABLE IV: Results on the *egl* dataset

V. Conclusion

The proposed Genetic Simulated Annealing Algorithm (GSA) is a hybrid algorithm that combines simulated annealing and genetic algorithms. The algorithm is effective in finding near-optimal solutions for small and medium-sized instances of the Capacitated Arc Routing Problem (CARP), showing a gap of less than 1% between the best solution found by GSA and the lower bound for the *gdb* and *val* datasets. However, the algorithm struggles with large instances, which suggests that more sophisticated techniques or additional runtime may be required to solve large instances effectively, with a gap of around 5% for the *egl* dataset. The algorithm can be further improved by incorporating more advanced move operators, exploring different population initialization strategies, and tuning the parameters based on the characteristics of the instances.

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