Probability and Statistics
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Section 4.1

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P116 Q6

a.

$$
\begin{aligned}
E(x) &= \int_{-\infty}^{\infty} x f(x) \mathrm{d}x \\
&= \int_{0}^{1} x \cdot 2x \mathrm{d}x \\
&= \frac{2}{3}
\end{aligned}
$$

b.

Y 的分布函数为

$$
\begin{aligned}
F_Y(y) =& P\{Y \leqslant y\} \\
=& P\{X^2 \leqslant y\} \\
=& P\{X \leqslant \sqrt{y}\} \\
=& \int_{0}^{\sqrt{y}} 2x \mathrm{d}x \\
=& y \ (0 \leqslant y \leqslant 1)
\end{aligned}
$$

Y 的密度函数为

$$
f_Y(y) = \begin{cases} \frac{\mathrm{d}F_Y(y)}{\mathrm{d}y} = 1 & (0 \leqslant y \leqslant 1) \\ 0 & \text{otherwise} \end{cases}
$$

Y 的期望为

$$
\begin{aligned}
E(Y) &= \int_{0}^{1} y \cdot 1 \mathrm{d}y \\
&= \frac{1}{2}
\end{aligned}
$$

c.

直接计算 X^2 的期望为

$$E(X^2) = \int_0^1 x^2 \cdot 2x\mathrm{d}x$$
$$= \frac{1}{2}x^4\Big|_0^1$$
$$= \frac{1}{2}$$

d.

根据方差的定义有

$$Var(x) = \int_{-\infty}^{\infty} (x - E(x))^2 f(x)\mathrm{d}x$$
$$= \int_0^1 \left(x - \frac{2}{3}\right)^2 \cdot 2x\mathrm{d}x$$
$$= \frac{1}{18}$$

由定理 4.2.2 计算方差为

$$Var(x) = E(x^2) - [E(x)]^2$$
$$= \frac{1}{2} - \left(\frac{2}{3}\right)^2$$
$$= \frac{1}{18}$$

P117 Q15

从一种彩票中购买两张的期望为

$$E(X) = \sum_{x=0}^{C_n^2} c \cdot P(X = x)$$
$$= c \cdot \frac{n-1}{C_n^2}$$
$$= \frac{2c}{n}$$

从两种彩票中各购买一张的期望为

$$E(Y) = 2 \cdot \sum_{x=0}^{n} c \cdot P(Y = x)$$
$$= 2 \cdot c \cdot \frac{1}{n}$$
$$= \frac{2c}{n}$$

可知，两种彩票的期望相同。

P117 Q20

当 X 为泊松随机变量时，有

$$
\begin{aligned}
E\left(\frac{1}{1+X}\right) &= \sum_{x=0}^{\infty} \frac{1}{1+x} \cdot \frac{e^{-\lambda}\lambda^x}{x!} \\
&= \sum_{x=0}^{\infty} \frac{e^{-\lambda}\lambda^x}{(x+1)!} \\
&= \frac{e^{-\lambda}}{\lambda} \sum_{x=0}^{\infty} \frac{\lambda^{x+1}}{(x+1)!} \\
&= \frac{e^{-\lambda}}{\lambda} \sum_{x=1}^{\infty} \frac{\lambda^{x}}{x!} \\
&= \frac{e^{-\lambda}}{\lambda} \cdot (e^{\lambda} - 1)
\end{aligned}
$$

P117 Q21

由题可知 $X \sim U(0,1)$，则 $S = X^2$ 的期望为

$$
\begin{aligned}
E(S) &= \int_0^1 x^2 P(x) \mathrm{d}x \\
&= \int_0^1 x^2 \mathrm{d}x \\
&= \frac{1}{3}
\end{aligned}
$$

P117 Q31

由题可知 $X \sim U(1,2)$，则 $Y = \frac{1}{X}$ 的期望为

$$
\begin{aligned}
E(Y) &= \int_1^2 \frac{1}{x} \cdot \frac{1}{2-1} \mathrm{d}x \\
&= \ln 2
\end{aligned}
$$

易知 X 的期望为 $\frac{1+2}{2} = \frac{3}{2}$。明显有

$$
E\left(\frac{1}{X}\right) \neq \frac{1}{E(X)}
$$

补充 1

(1)

$$
\begin{aligned}
E(Y) &= \int_{-\infty}^{\infty} 2x \cdot P\{X = x\}\mathrm{d}x \\
&= \int_{0}^{\infty} 2x \cdot e^{-x}\mathrm{d}x \\
&= -2(x+1)e^{-x}\Big|_{0}^{\infty} \\
&= 2
\end{aligned}
$$

(2)

$$
\begin{aligned}
E(Y) &= \int_{-\infty}^{\infty} e^{-2x} \cdot P\{X = x\}\mathrm{d}x \\
&= \int_{0}^{\infty} e^{-2x} \cdot e^{-x}\mathrm{d}x \\
&= -\frac{1}{3}e^{-3x}\Big|_{0}^{\infty} \\
&= \frac{1}{3}
\end{aligned}
$$

补充 2

$$
\begin{aligned}
E(X) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x \cdot f(x,y)\mathrm{d}x\mathrm{d}y \\
&= \int_{0}^{1}\int_{y}^{1} x \cdot 12y^2\mathrm{d}x\mathrm{d}y \\
&= \int_{0}^{1} 12y^2 \int_{y}^{1} x\mathrm{d}x\mathrm{d}y \\
&= \int_{0}^{1} 12y^2 \cdot \frac{1}{2}(1-y^2)\mathrm{d}y \\
&= \int_{0}^{1} 6y^2 - 6y^4\mathrm{d}y \\
&= 2y^3 - \frac{6}{5}y^5\Big|_{0}^{1} \\
&= \frac{4}{5}
\end{aligned}
$$

$$\begin{aligned}
E(Y) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} y \cdot f(x,y)\mathrm{d}x\mathrm{d}y \\
&= \int_0^1\int_y^1 y \cdot 12y^2\mathrm{d}x\mathrm{d}y \\
&= \int_0^1 12y^3 \cdot (1-y)\mathrm{d}y \\
&= \int_0^1 12y^3 - 12y^4\mathrm{d}y \\
&= 3y^4 - \frac{12}{5}y^5\Big|_0^1 \\
&= \frac{3}{5}
\end{aligned}$$

$$\begin{aligned}
E(XY) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy \cdot f(x,y)\mathrm{d}x\mathrm{d}y \\
&= \int_0^1\int_y^1 xy \cdot 12y^2\mathrm{d}x\mathrm{d}y \\
&= \int_0^1 12y^3\int_y^1 x\mathrm{d}x\mathrm{d}y \\
&= \int_0^1 12y^3 \cdot \frac{1}{2}(1-y^2)\mathrm{d}y \\
&= \int_0^1 6y^3 - 6y^5\mathrm{d}y \\
&= \frac{3}{2}y^4 - y^6\Big|_0^1 \\
&= \frac{1}{2}
\end{aligned}$$

$$\begin{aligned}
E(X^2+Y^2) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x^2+y^2) \cdot f(x,y)\mathrm{d}x\mathrm{d}y \\
&= \int_0^1\int_y^1 (x^2+y^2) \cdot 12y^2\mathrm{d}x\mathrm{d}y \\
&= \int_0^1\int_y^1 12x^2y^2 + 12y^4\mathrm{d}x\mathrm{d}y \\
&= \int_0^1 4y^2 + 12y^4 - 16y^5\mathrm{d}y \\
&= \frac{4}{3}y^3 + \frac{12}{5}y^5 - \frac{8}{3}y^6\Big|_0^1 \\
&= \frac{16}{15}
\end{aligned}$$