

# Digital Logic(H)

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## Theory Assignment 1

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1.

a)

Integer part	Quotient	Remainder	Coefficient	Fraction part	Integer	Fraction	Coefficient
$234/3 =$	78	0	$a_0 = 0$	$0.5 \times 3 =$	1	0.5	$a_{-1} = 1$
$78/3 =$	26	0	$a_1 = 0$	$0.5 \times 3 =$	1	0.5	$a_{-2} = 1$
$26/3 =$	8	2	$a_2 = 2$	$0.5 \times 3 =$	1	0.5	$a_{-3} = 1$
$8/3 =$	2	2	$a_3 = 2$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$2/3 =$	0	2	$a_4 = 2$				

Therefore,  $\boxed{(234.5)_{10} \approx (22200.11)_3}$ .

b)

Integer part	Quotient	Remainder	Coefficient	Fraction part	Integer	Fraction	Coefficient
$234/12 =$	19	6	$a_0 = 6$	$0.5 \times 12 =$	6	0.0	$a_{-1} = 6$
$19/12 =$	1	7	$a_1 = 7$				
$1/12 =$	0	1	$a_2 = 1$				

Therefore,  $\boxed{(234.5)_{10} = (176.6)_{12}}$ .

c)

$$\begin{aligned} D &= 4 \times 6^2 + 3 \times 6^1 + 5 \times 6^0 \\ &= 144 + 18 + 5 \\ &= 167 \end{aligned}$$

Therefore,  $\boxed{(435)_6 = (167)_{10}}$ .

d)

Radix r	Integer			.	Fraction		
2	0	1	0	.	0	1	0
8	$\underbrace{\quad\quad}_2$		$\underbrace{\quad\quad}_6$	.	$\underbrace{\quad\quad}_2$	$\underbrace{\quad\quad}_4$	0

Therefore,  $\boxed{(10110.0101)_2 = (26.24)_8}$ .

**2.****a)**

Since all numbers are smaller than 7 and no carry is needed, the operation is correct in any number system that radix  $\boxed{r \geq 7}$ .

**b)**

Assuming the operation is in base  $r$ , we first convert the operation into base 10:

$$\begin{aligned}
 LHS &= (302)_r / (20)_r \\
 &= (3r^2 + 0r^1 + 2r^0)_{10} / (2r^1 + 0r^0)_{10} \\
 &= \left(\frac{3r^2 + 2}{2r}\right)_{10} \\
 RHS &= (12.1)_r \\
 &= (1r^1 + 2r^0 + 1r^{-1})_{10} \\
 &= \left(\frac{r^2 + 2r + 1}{r}\right)_{10}
 \end{aligned}$$

Since  $LHS = RHS$ , we have:

$$\begin{aligned}
 \frac{3r^2 + 2}{2r} &= \frac{r^2 + 2r + 1}{r} \\
 3r^2 + 2 &= 2r^2 + 4r + 2 \\
 r^2 - 4r &= 0
 \end{aligned}$$

Therefore,  $r = 0$  or  $r = 4$ . Since  $r \neq 0$ , we have  $\boxed{r = 4}$ .

**3.****a)**

$$\begin{aligned}
 (a' + c)(a' + c')(a + b + c'd) &= (a' + cc')(a + b + c'd) \\
 &= a'(a + b + c'd) \\
 &= a'a + a'b + a'c'd \\
 &= a'b + a'c'd \\
 &= \boxed{a'(b + c'd)}
 \end{aligned}$$

**b)**

$$\begin{aligned}
 abc'd + a'bd + abcd &= (abc' + a'b + abc)d \\
 &= (a'b + ab(c + c'))d \\
 &= (a'b + ab)d \\
 &= (a' + a)bd \\
 &= \boxed{bd}
 \end{aligned}$$

4.

a)

$$\begin{aligned}
(a+c)(a'+b+c)(a'+b'+c) &= (a+c)(a'+c+bb') \\
&= (a+c)(a'+c) \\
&= c+aa' \\
&= \boxed{c}
\end{aligned}$$

b)

$$\begin{aligned}
F(a,b,c) &= \sum(0,1,2,3,5) \\
&= a'b'c' + a'b'c + a'bc' + a'bc + ab'c \\
&= a'b'(c+c') + a'b(c+c') + ab'c \\
&= a'b' + a'b + ab'c \\
&= a'(b+b') + ab'c \\
&= a' + ab'c \\
&= a'(1+b'c) + ab'c \\
&= a' + a'b'c + ab'c \\
&= a' + (a+a')b'c \\
&= \boxed{a' + b'c}
\end{aligned}$$

5.

a)

$$\begin{aligned}
F(a,b,c,d) &= bd' + acd' + ab'c + a'c' \\
&= (a+a')b(c+c')d' + a(b+b')cd' + ab'c(d+d') + a'(b+b')c'(d+d') \\
&= abcd' + a'bcd' + abc'd' + a'bc'd' + ab'cd' + ab'cd + a'bc'd + a'b'c'd + a'b'c'd' \\
&= \boxed{\sum(0,1,4,5,6,10,11,12,14)} \\
&= \boxed{\prod(2,3,7,8,9,13,15)}
\end{aligned}$$

b)

$$\begin{aligned}
 F(x, y, z) &= (x' + z)(y + x') \\
 &= x' + yz \\
 &= x'(y + y')(z + z') + (x + x')yz \\
 &= x'yz + x'y'z + x'yz' + x'y'z' + xyz \\
 &= \sum(0, 1, 2, 3, 7) \\
 &= \prod(4, 5, 6)
 \end{aligned}$$

6.

a)

$$\begin{aligned}
 F_1(A, B, C) &= \sum(2, 3, 7) \\
 &= A'BC' + A'BC + ABC \\
 &= A'B(C + C') + ABC \\
 &= A'B + ABC \\
 &= A'B(1 + C) + ABC \\
 &= A'B + A'BC + ABC \\
 &= A'B + (A + A')BC \\
 &= A'B + BC \\
 &= \boxed{B(A' + C)}
 \end{aligned}$$

$$\begin{aligned}
 F_2(A, B, C) &= \sum(0, 2, 5, 7) \\
 &= A'B'C' + A'BC' + AB'C + ABC \\
 &= A'(B + B')C' + A(B + B')C \\
 &= A'C' + AC \\
 &= \boxed{(A \oplus C)'}
 \end{aligned}$$

b)

$\begin{smallmatrix} BC \\ A \end{smallmatrix}$	00	01	11	10
0	0	0	1	1
1	0	0	1	0

Hence, the sum of product terms for  $F_1$  is:

$$F_1(A, B, C) = A'B + BC$$

$\begin{smallmatrix} BC \\ A \end{smallmatrix}$	00	01	11	10
0	1	0	0	1
1	0	1	1	0

Hence, the sum of product terms for  $F_2$  is:

$$F_2(A, B, C) = AC + A'C'$$

7.

a)

$\begin{smallmatrix} YZ \\ WX \end{smallmatrix}$	00	01	11	10
00	1	0	1	1
01	0	0	1	1
11	1	1	1	0
10	0	0	1	1

Hence, the sum of product terms is:

$$F(W, X, Y, Z) = W'Y + WXY' + W'X'Z' + X'Y + YZ$$

b)

$\begin{array}{c} CD \\ AB \end{array}$	00	01	11	10
00	1	0	0	1
01	0	0	0	0
11	0	0	1	0
10	1	0	1	1

Hence, the sum of product terms is:

$$F(A, B, C, D) = ACD + B'D'$$

8.

We can use the following karnaugh map to find the truth table of both functions:

$\begin{array}{c} CD \\ AB \end{array}$	00	01	11	10
00	0	1	0	1
01	0	1	0	1
11	1	1	0	1
10	0	1	0	1

$\begin{array}{c} CD \\ AB \end{array}$	00	01	11	10
00	1	0	0	1
01	1	1	1	0
11	1	0	1	0
10	1	0	1	1

$$f = abd' + c'd + a'cd' + b'cd'$$

$$g' = a'b'd + bcd' + ac'd$$

Since  $F = fg$ , we simply find the common minterms to be the minterms of  $F$ , and find the simplest sum of product terms using the following karnaugh map:

$\begin{array}{c} CD \\ \backslash AB \end{array}$	00	01	11	10
00	0	0	0	1
01	0	1	0	0
11	1	0	0	0
10	0	0	0	1

Therefore, the sum of product terms for  $F$  is:

$$F = abc'd' + a'bc'd + b'cd'$$

9.

a)

$\begin{array}{c} CD \\ \backslash AB \end{array}$	00	01	11	10
00	x	1	x	1
01	1	x	1	0
11	0	0	0	0
10	1	1	1	0

The simplest sum of product terms is:

$$F(A, B, C, D) = A'B' + A'C' + A'D + B'C' + B'D$$

To implement  $F$  using only NAND gates, we convert  $F$  into a NAND form:

$$\begin{aligned}
 F(A, B, C, D) &= A'B' + A'C' + A'D + B'C' + B'D \\
 &= ((A'B' + A'C' + A'D + B'C' + B'D)')' \\
 &= \boxed{((A'B')'(A'C')'(A'D)'(B'C')'(B'D)')'}
 \end{aligned}$$

b)

$\begin{array}{c} C/D \\ \backslash AB \end{array}$	00	01	11	10
00	x	1	x	1
01	1	x	1	0
11	0	0	0	0
10	1	1	1	0

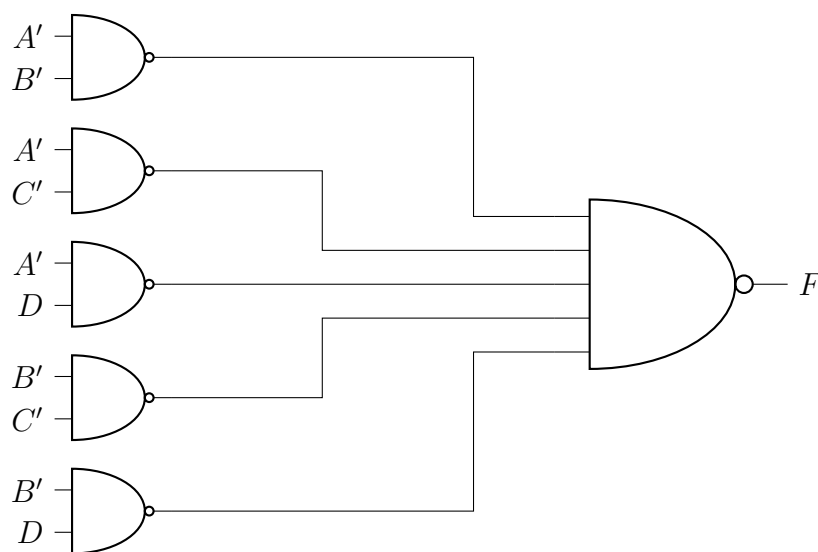
The simplest product of sum terms is:

$$\begin{aligned}
 F'(A, B, C, D) &= AB + ACD' + BCD' \\
 F(A, B, C, D) &= (AB + ACD' + BCD')' \\
 &= (AB)'(ACD')'(BCD')' \\
 &= (A' + B')(A' + C' + D)(B' + C' + D)
 \end{aligned}$$

To implement  $F$  using only NOR gates, we convert  $F$  into a NOR form:

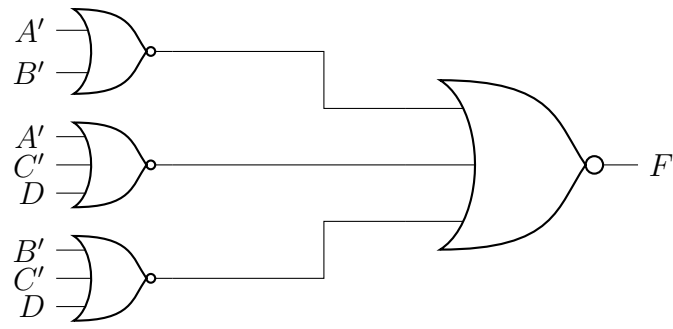
$$\begin{aligned}
 F(A, B, C, D) &= (A' + B')(A' + C' + D)(B' + C' + D) \\
 &= (((A' + B')(A' + C' + D)(B' + C' + D)))' \\
 &= \boxed{((A' + B')' + (A' + C' + D)' + (B' + C' + D)')}
 \end{aligned}$$

c)



NAND implementation





NOR implementation