Data Structure and Algorithm Analysis(H)
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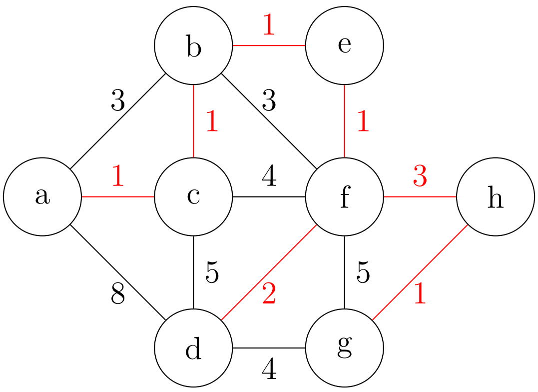
Work Sheet 15

Mengxuan Wu

Question 15.1

Prim's Algorithm

Suppose we begin with the vertex a.

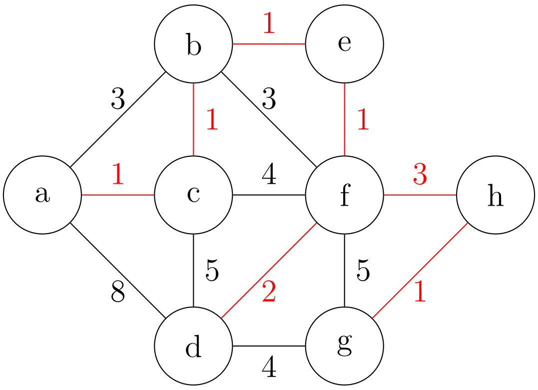


Edges with order considered:

Edge	Weight	Order
ac	1	1
cb	1	2
be	1	3
ef	1	4
fd	2	5
fh	3	6
hg	1	7

The total weight is 10.

Kruskal's Algorithm



Edges with order considered:

Edge	Weight	Order
ac	1	1
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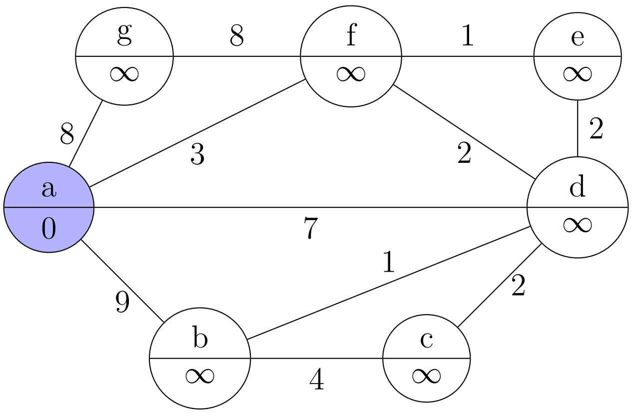
fh	3	7

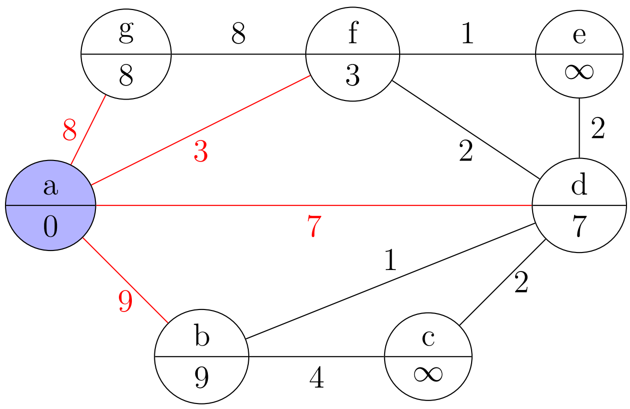
The total weight is 10.

Question 15.2

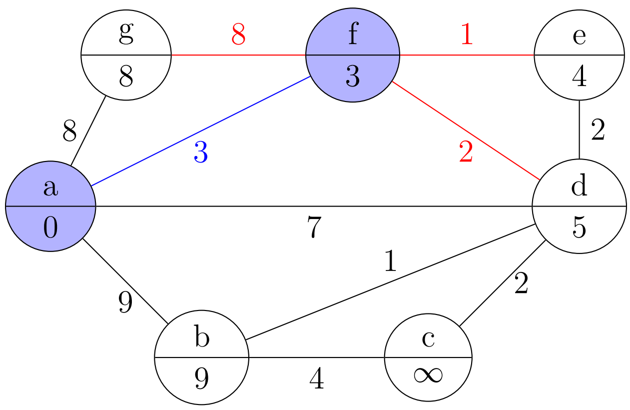
Step 1:



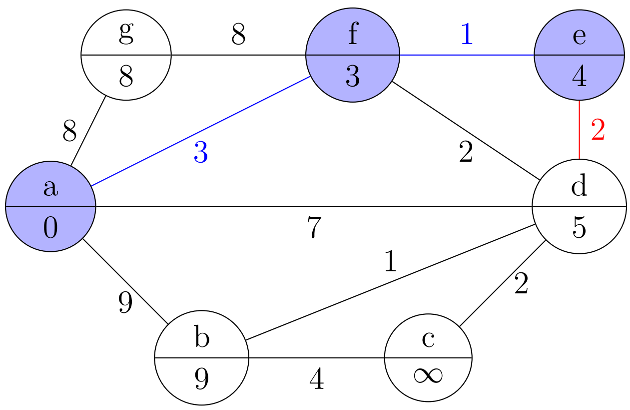
Step 2:



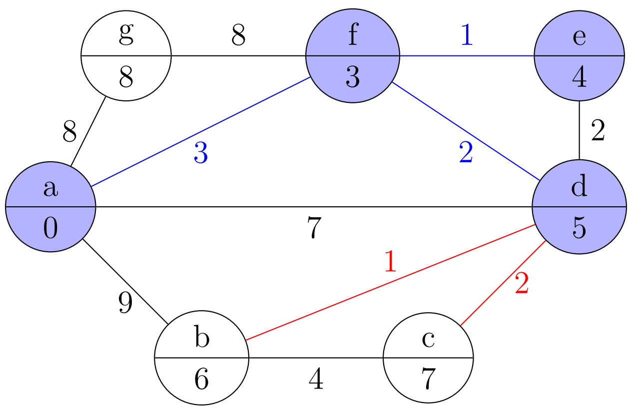
Step 3:



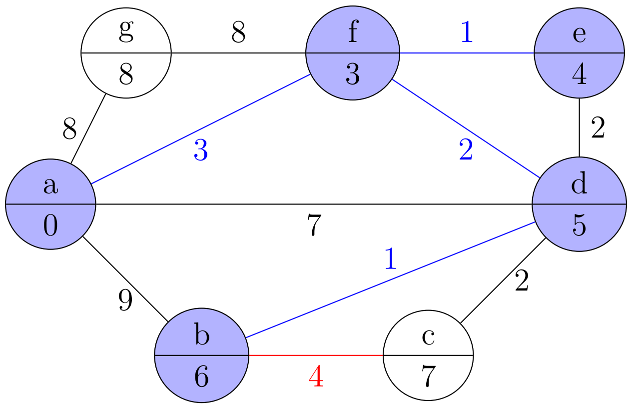
Step 4:



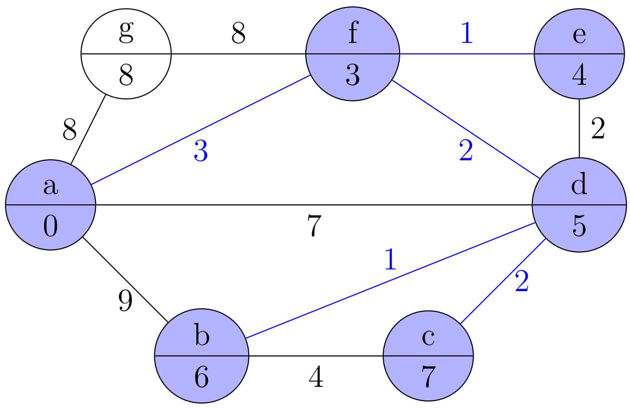
Step 5:



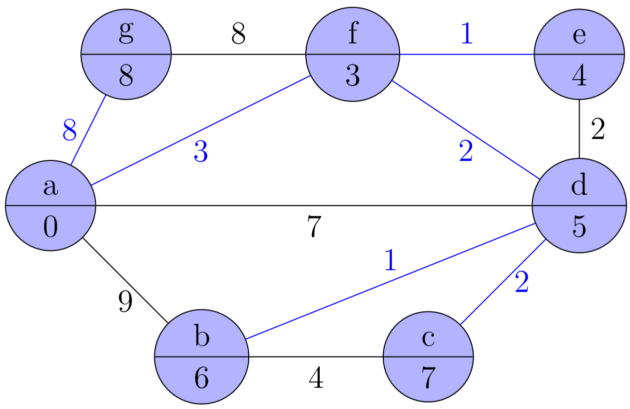
Step 6:



Step 7:

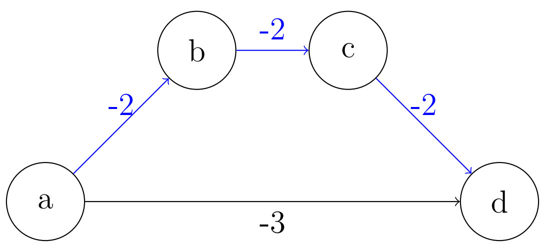


Step 8:



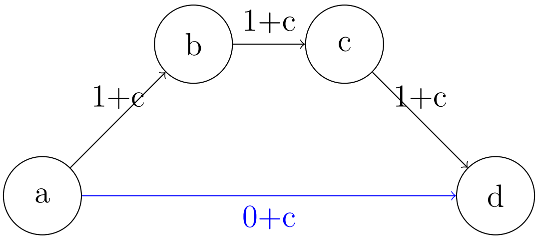
Question 15.3

Here is an example where the claim can be false.



This is a directed acyclic graph. The shortest path from a to d is $a \to b \to c \to d$ with weight -6. The lower path, $a \to d$, has weight -3.

However, if we add a constant weight to all edges to make them positive, the graph becomes:



Here c represents a non-negative constant. In this case, the shortest path from a to d is $a \to d$ with weight c. The upper path, $a \to b \to c \to d$, has weight $3 + 3c$.

The real problem with this claim is that: for two distinct paths p_1 and p_2 from s to t, the number of edges in p_1 and p_2 can be different. Let n_1 and n_2 be the number of edges in p_1 and p_2 respectively. If the sum of weights of edges in p_1 is W_1 before adding a constant weight c to all edges, then the sum of weights of edges in p_1 after adding c is $W_1 + n_1 c$. Similarly, the sum of weights of edges in p_2 after adding c is $W_2 + n_2 c$. In this case, the difference $W_1 - W_2$ will change after adding c, becomes $W_1 + n_1 c - W_2 - n_2 c = (W_1 - W_2) + (n_1 - n_2)c$. It is possible that $W_1 - W_2 < 0$ but $n_1 - n_2 > 0$, making $W_1 + n_1 c - W_2 - n_2 c > 0$. Then the shortest path changes from p_1 to p_2 after adding c.