Probability and Statistics

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Section 2.1

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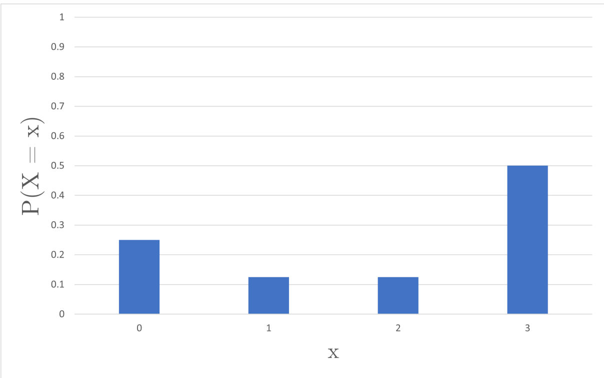
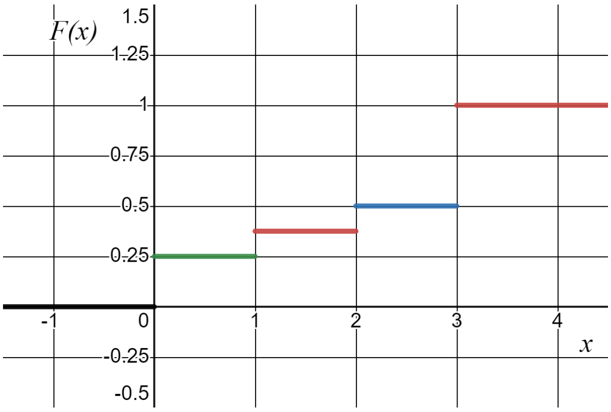
P46 Q1





P46 Q15

A 队在 5 局比赛中至少获胜 3 次的概率为:

$$
\begin{aligned}
P\{X \geqslant 3\} &= P\{X=3\}+P\{X=4\}+P\{X=5\} \\
&= C_2^2\ 0.4^3 \cdot 0.6^0 + C_3^2\ 0.4^3 \cdot 0.6^1 + C_4^2\ 0.4^3 \cdot 0.6^2 \\
&= 0.064 + 0.1152 + 0.13824 \\
&= 0.31744
\end{aligned}
$$

A 队在 7 局比赛中至少获胜 4 次的概率为:

$$
\begin{aligned}
P\{X \geqslant 4\} &= P\{X=4\}+P\{X=5\}+P\{X=6\}+P\{X=7\} \\
&= C_3^3\ 0.4^4 \cdot 0.6^0 + C_4^3\ 0.4^4 \cdot 0.6^1 + C_5^3\ 0.4^4 \cdot 0.6^2 + C_6^3\ 0.4^4 \cdot 0.6^3 \\
&= 0.0256 + 0.06144 + 0.09216 + 0.110592 \\
&= 0.289792
\end{aligned}
$$

综上，5 局 3 胜制对 A 队有利。

P47 Q31

a.

$$
\begin{aligned}
P\{X \geqslant 1\} &= 1 - P\{X = 0\} \\
&= 1 - \frac{(\lambda \cdot \frac{1}{6})^0}{0!} e^{-\lambda \cdot \frac{1}{6}} \\
&= 1 - e^{-\lambda \cdot \frac{1}{6}} \\
&= 1 - e^{-\frac{1}{3}} \\
&\approx 0.28347
\end{aligned}
$$

b.

$$
\begin{aligned}
P\{X = 0\} &= \frac{(2t)^0}{0!} e^{-2t} \\
&= e^{-2t} \\
&\leqslant 0.5
\end{aligned}
$$

故 $t \geqslant \frac{\ln 2}{2} \approx 0.34657$ 小时。

补充 1

$$
\begin{aligned}
\sum_{x=1}^{3} P\{X = x\} = \sum_{x=1}^{3} c(\frac{2}{3})^x \\
=& c \cdot (\frac{2}{3} + \frac{4}{9} + \frac{8}{27}) \\
=& c \cdot \frac{38}{27} \\
=& 1
\end{aligned}
$$

故 $c = \frac{27}{38}$。

补充 2

已知

$$
P\{X = k\} = \frac{\lambda^k}{k!} e^{-\lambda}
$$

使第 k 项除以第 $k-1$ 项，得

$$\frac{P\{X=k\}}{P\{X=k-1\}}=\frac{\lambda^k}{k!}e^{-\lambda}\cdot\frac{(k-1)!}{\lambda^{k-1}}e^{\lambda}=\frac{\lambda}{k}$$

易知当 $k\leqslant\lfloor\lambda\rfloor$ 时 $\frac{\lambda}{k}\geqslant 1$，故 $P\{X=k\}$ 在 $k=\lfloor\lambda\rfloor$ 时取得最大值。

补充 3

(1)

该随机变量服从超几何分布，故有：

$$\begin{aligned}P\{X=0\}=&\frac{C_{13}^3}{C_{15}^3}\\=&\frac{22}{35}\end{aligned}$$

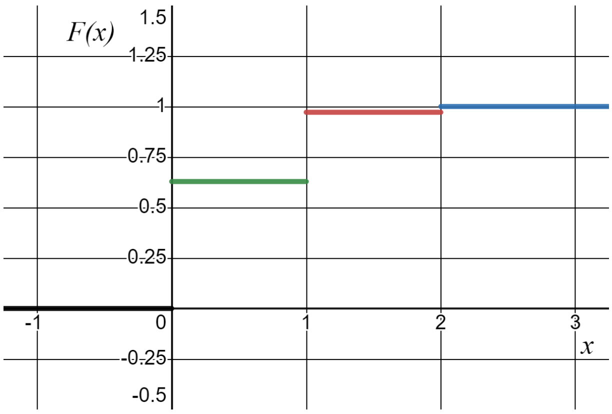
$$\begin{aligned}P\{X=1\}=&\frac{C_{13}^2\cdot C_2^1}{C_{15}^3}\\=&\frac{12}{35}\end{aligned}$$

$$\begin{aligned}P\{X=2\}=&\frac{C_{13}^1\cdot C_2^2}{C_{15}^3}\\=&\frac{1}{35}\end{aligned}$$

综上：

X	0	1	2
$P\{X=x\}$	$\frac{22}{35}$	$\frac{12}{35}$	$\frac{1}{35}$

(2)



(3)

$$
\begin{gathered}
P\{X \leqslant \frac{1}{2}\} = P\{X = 0\} = \frac{22}{35} \\
P\{1 < X \leqslant \frac{3}{2}\} = 0 \\
P\{1 \leqslant X \leqslant \frac{3}{2}\} = P\{X = 1\} = \frac{12}{35} \\
P\{1 < X < 2\} = 0
\end{gathered}
$$

补充 4

(1)

记在一年中死亡的人数为 X，该随机变量服从泊松分布。

若要使保险公司亏本，则有 $2500 \cdot 12 < 2000X$，即 $X > 15$。由泊松定理可知 $\lambda = np = 2500 \times 0.002 = 5$。则有：

$$
\begin{aligned}
P\{X > 15\} =& 1 - P\{X \leqslant 15\} \\
=& 1 - \sum_{x=0}^{15} \frac{5^x}{x!} e^{-5} \\
\approx& 1 - 0.99993 \\
=& 0.00007
\end{aligned}
$$

(2)

解 $2500 \cdot 12 - 2000X \geqslant 10000$, $2500 \cdot 12 - 2000X \geqslant 20000$, 分别得到 $X \leqslant 10$, $X \leqslant 5$。

$$
\begin{aligned}
P\{X \leqslant 10\} =& \sum_{x=0}^{10} \frac{5^x}{x!} e^{-5} \\
\approx& 0.98630
\end{aligned}
$$

$$
\begin{aligned}
P\{X \leqslant 5\} =& \sum_{x=0}^{5} \frac{5^x}{x!} e^{-5} \\
\approx& 0.61596
\end{aligned}
$$