Speed Up Your Database Systems

CS307 Lab12, Spring 2019

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- How to search efficiently?

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  - Strategy: Divide and Conquer,
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But ...

Data are not stored in multiple orders

What is index?

Think about a Chinese Dictionary

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- Words are listed in alphabetical order

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- Words are listed in alphabetical order
- We still have
  - 音序检字法
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- How is this achieved?

What is index?

Concept

An *index* is a data structure which improves the **efficiency** of **retrieving** data with specific values from a database.

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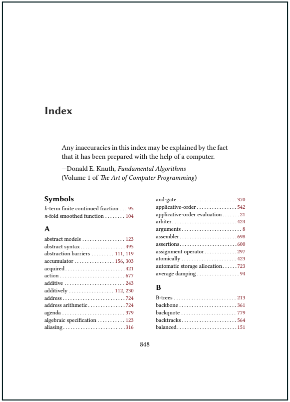
Remark

- Indexes in databases is similar to indexes in books
  - Data are stored in *pages* (blocks)
  - Indexes locate a row by (filename, block #, offset)



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848

Figure: Index page of *Structure and Interpretation of Computer Programs*, 2nd ed., by H. Abelson, G. J. Sussman and J. Sussman, licensed under CC BY-SA 4.0.

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An *index* is a data structure which improves the **efficiency** of **retrieving** data with specific values from a database.

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- Indexes in databases is similar to indexes in books
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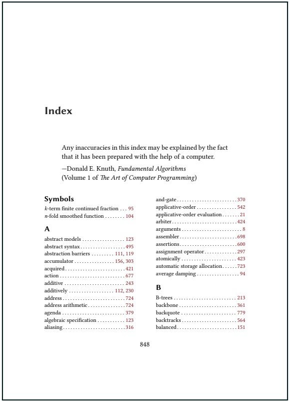
- Indexing is beyond the scope of Relational Theory



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  - `PRIMARY KEY` constraint
  - `UNIQUE` constraint

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  - `PRIMARY KEY` constraint
  - `UNIQUE` constraint
- Why does your DBMS create them automatically?

How to get an index?

Or, you can `CREATE INDEX`[1] on your own:

SQL Command Syntax

```
CREATE INDEX index_name
ON table_name (column_name [, ...]);
```

[1] https://www.postgresql.org/docs/current/sql-createindex.html

Index Taxonomy

- In terms of storage structure, is the index completely separated with the data records?
  - No $\implies$ **Integrated index**, e.g.
    - PK index in a MySQL InnoDB database
    - PK index in a SQL Server database
  - Yes $\implies$ **External index**, e.g.
    - Indexes in a PostgreSQL database
    - Indexes in a MySQL MyISAM database

[2] Someone may consider *primary index* as the alias of *integrated index*, and consider *secondary index* as the alias of *external index*. It depends on who you are talking with.

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    - Indexes in a MySQL MyISAM database
- Does the index specify the order in which records are stored in the data file?
  - Yes $\Longrightarrow$ **Clustered index** (a.k.a. primary index[2])
  - No $\Longrightarrow$ **Non-clustered index** (a.k.a. secondary index)

---

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- Does every search key in the data file correspond to an index entry?
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- Does every search key in the data file correspond to an index entry?
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- Does the search key contain more than one attribute?
  - Yes $\Longrightarrow$ **Multi-key index** (Multi-column index)
  - No $\Longrightarrow$ **Single-key index** (Single-column index)

Index implementation

Data Structures for Indexes

- Ordered
  - *B*-tree and its variants (e.g. B^+-tree)
  - Generalizations of *B*-tree, e.g. *R*-tree
  - ...

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- Unordered
  - Hash tables
  - Bitmap
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Question

What is the common limitation of unordered indexes?

Index implementation: B-tree definition[3]

Definition

A *B-tree of order m* satisfies that

1. For every node, # of children $=$ # of keys $+ 1$

[3] Here we mainly follow D. E. Knuth's definition [2]. Note that the *B*-tree defined in *CLRS* [3] is slightly different. Read `https://stackoverflow.com/a/45826413` for further comparison.

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4. **(Always balanced)** All leaves appear on the same level

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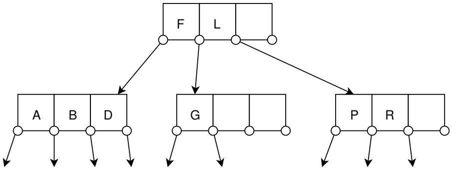
Index implementation: *B*-tree definition

Terminology

- $\lceil m/2 \rceil$ is the called the *minimum branching factor* (a.k.a. *minimum degree*) of the tree
- A B-tree of order m is usually called a *$\lceil m/2 \rceil$-m tree*, like 2-3 tree, 2-4 tree, 3-5 tree, 3-6 tree, ...



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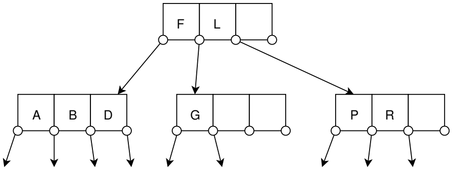
In practice, the order *m* is much larger (typically on the order of 100)



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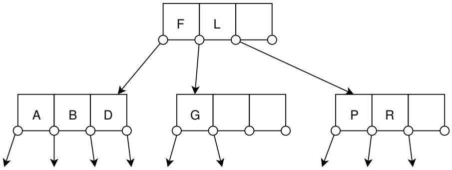

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Question

Note that a *B*-tree usually waste some space. Why do we prefer *B*-tree to balanced BST (e.g. AVL tree, red-black tree, etc.)?

Index implementation: Why *B*-trees?

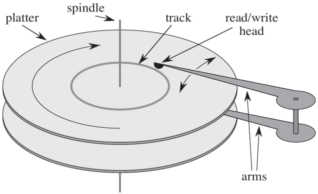

Figure: A typical disk drive (Retrieved from *CLRS* 3rd ed., pp. 485. [3])

Memory Hierarchy [4]

Technology	Access time
DRAM	50-70 ns
Magnetic disk	5-20 ms

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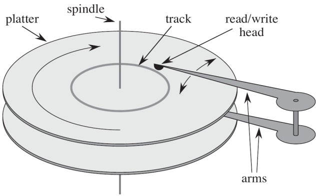

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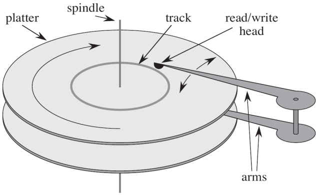

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- Height of a B-tree

$$h \leq 1 + \log_{\lceil m/2 \rceil} \left(\frac{n+1}{2} \right)$$

(What's the height of a B-tree of order 200 with two million search keys?)

Index implementation: *B*-tree operations

Tree operations:

- Search
- Insert
- Delete
- Update: Delete + Insert

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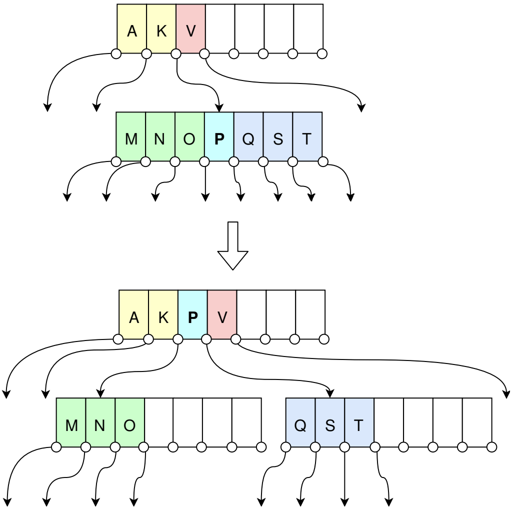
What is special in *B*-tree?

- Split
- Merge

Index implementation: Split a node in a *B*-tree



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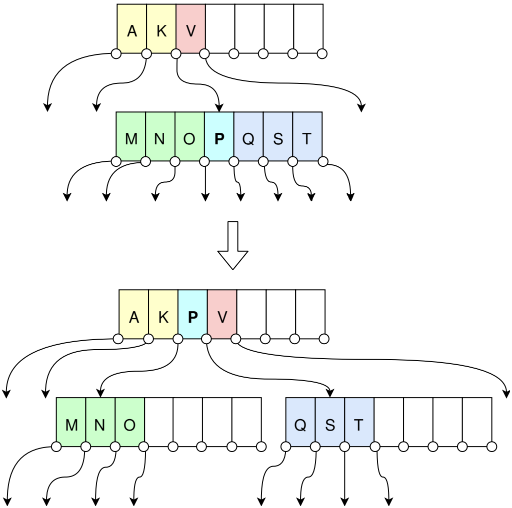

Observation

In this example, the height of the *B*-tree doesn't change after splitting.

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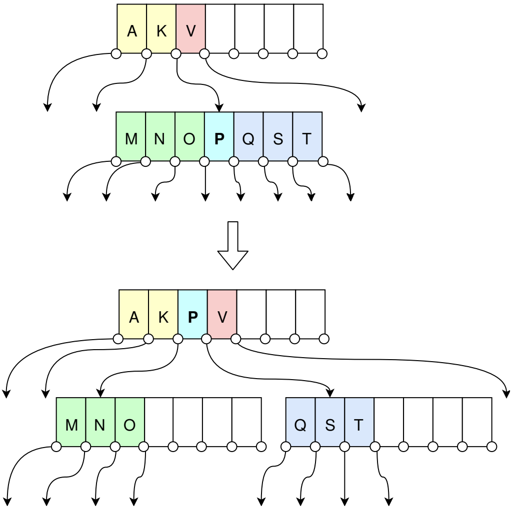


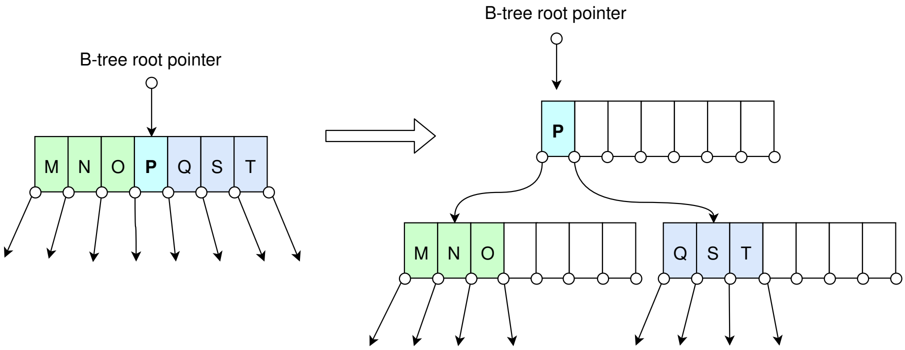
Observation

In this example, the height of the *B*-tree doesn't change after splitting.

Question

What if the parent node is also full?

Index implementation: Split the root node of a *B*-tree



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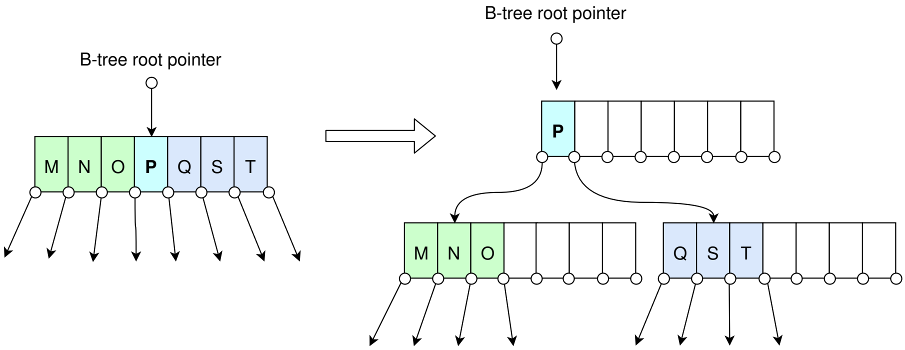


Remark

Note that the height of the *B*-tree is increased by one. This is the only way that a *B*-tree increases its height.

Index implementation: Delete a key in a *B*-tree

More complicated than insertion. May involve

- Rotation
- Merging

Suggestions:

- Watch the video [9].
- Read the related chapters [2, 3].

Index implementation: B^+-tree vs. B-tree

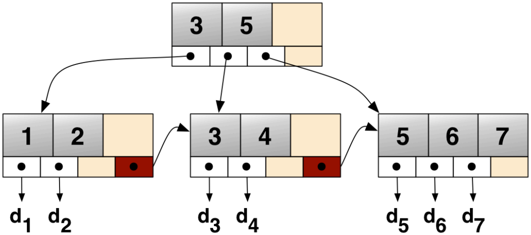


Figure: An example of B^+-tree (published by Grundprinzip under CC BY 3.0)

B-tree	B^+-tree
Data stored in every node	Data stored only in leaves
No sequential links	Leaves linked sequentially

Index implementation: Benefits of B^+-trees

Data stored only in leaves $\implies$ Larger m and smaller height

Index implementation: Benefits of B^+-trees

Data stored only in leaves	$\Longrightarrow$	Larger m and smaller height
Leaves linked sequentially	$\Longrightarrow$	Better support for range query

Negative impacts of indexes: Storage overhead

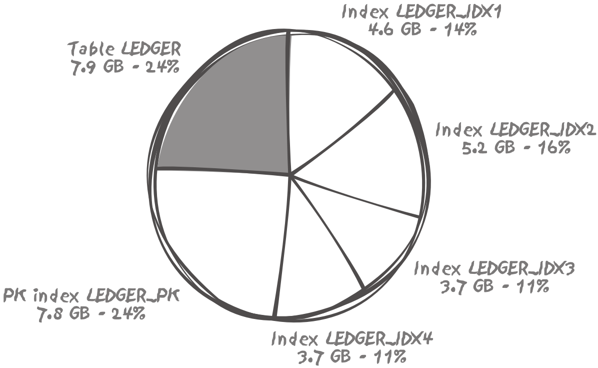


Figure: A real-life case: Data versus Index out of a 33 GB total (Retrieved from *The Art of SQL*, ch. 3, pp. 57. [5])

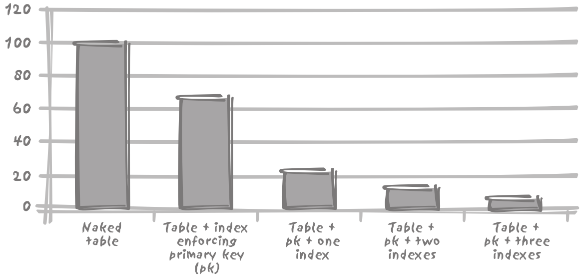
Reminder

The more indexes created, the more storage space consumed.

Negative impacts of indexes: Processing overhead

Reminder

Indexing may speed up data retrieval, but will bring overhead to `INSERT`, `UPDATE`, and `DELETE` operations.



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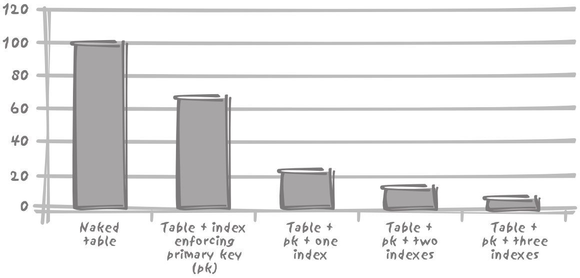
Indexes and **Backup**:

- Logical backup
- Physical backup



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  - `=`
  - `<`, `<=`, `>`, `>=`, `BETWEEN`
  - `IN`
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  - `LIKE` (prefix matching)

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- Longest prefix matching of multi-key indexes
  - When specifying a composite primary key or creating a multi-key index, the order of columns matters.
  - If required, you may consider creating indexes for (col1, col2, col3), (col2), and (col3) respectively.

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SELECT attr1, attr2
FROM table
WHERE function(column) = search_key;
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  - Or, utilize *indexes on expressions* [4]

```
CREATE INDEX idx_name ON table1(function(col1));
```

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- Low selectivity

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Reminder

- Full scan $\neq$ Bad scheme
- Index retrieval $\neq$ Good scheme

Contents

How `JOIN` works

Some widely used `JOIN` algorithms:

- Nested-loop join
- Hash join
- Sort-merge join

How JOIN works: Nested loop join

```
for each row in t1 match C1(t1)
    for each row in t2 match P(t1, t2)
        if C2(t2)
           add t1|t2 to the result
```

Improvement: Block nested-loop join

How `JOIN` works: Hash join

1. Create a hash table for the smaller table `t1` in the memory
2. Scan the larger table `t2`. For each record `r`,
    1. compute the hash value of `r.join_attribute`
    2. map to corresponding rows in `t1` using the hash table

How `JOIN` works: Sort-merge join (a.k.a. merge join)

1. Sort tables `t1` and `t2` respectively according to the join attributes
2. Perform an interleaved scan of `t1` and `t2`. When encountering a matched value, join the related rows together.

How `JOIN` works: Sort-merge join (a.k.a. merge join)

1. Sort tables `t1` and `t2` respectively according to the join attributes
2. Perform an interleaved scan of `t1` and `t2`. When encountering a matched value, join the related rows together.

Observation

When there are indexes on the join attributes, step 1, the most expensive operation, can be skipped because `t1` and `t2` are already sorted in this scenario.

Query optimizer

- Equivalence rules in Relational Algebra
  Example:

$$\sigma_{\theta_1 \wedge \theta_2}(R_1 \bowtie_\theta R_2) = (\sigma_{\theta_1}(R_1)) \bowtie_\theta (\sigma_{\theta_2}(R_2))$$

Query optimizer

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- Cost estimation

Query optimizer

- Equivalence rules in Relational Algebra
  Example:

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- Cost estimation
- Limitation

EXPLAIN the execution plan

- So, has the DBMS used the index?
- Which algorithm did it use on earth?

```
EXPLAIN SELECT *
FROM movies m INNER JOIN countries c
    ON m.country = c.country_code;
```

```
Hash Join (cost=6.16..201.16 rows=9538 width=49)
  Hash Cond: (m.country = c.country_code)
  ->  Seq Scan on movies m (cost=0.00..169.38 rows=9538 width=31)
  ->  Hash  (cost=3.85..3.85 rows=185 width=18)
        ->  Seq Scan on countries c (cost=0.00..3.85 rows=185 width=18)
```

EXPLAIN the execution plan

Some tools like *pgAdmin* can visualize the execution plan for you

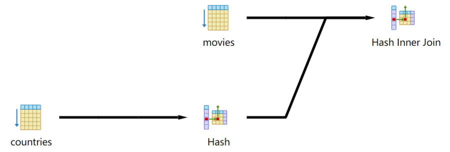

Figure: Execution plan visualized by pgAdmin 4

Reminder

- Full scan $\neq$ Bad scheme; Index retrieval $\neq$ Good scheme.
- When data stored are different, which implies different statistics, DBMS may have different execution plan for the same query.

Contents

Parameterized SQL

Parsing SQL costs time $\longrightarrow$ DBMS cache parsed SQL in memory

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Essentially the same statement, but caching doesn't help :(

```
String sql = "INSERT␣INTO␣students(name)␣VALUES␣("
        + stuName + ")";
```

Parameterized SQL

Parsing SQL costs time → DBMS cache parsed SQL in memory

Essentially the same statement, but caching doesn't help :(

```
String sql = "INSERT␣INTO␣students(name)␣VALUES␣("
        + stuName + ")";
```

Solution: variable binding

```
String sql = "INSERT␣INTO␣students(name)␣VALUES␣(?)";
PreparedStatement insert = con.prepareStatement(sql);
// ...
insert.setString(1, student.getName());
insert.executeUpdate();
con.commit();
```

Parameterized SQL

Why SQL parameterization is so important? Avoid SQL injection!

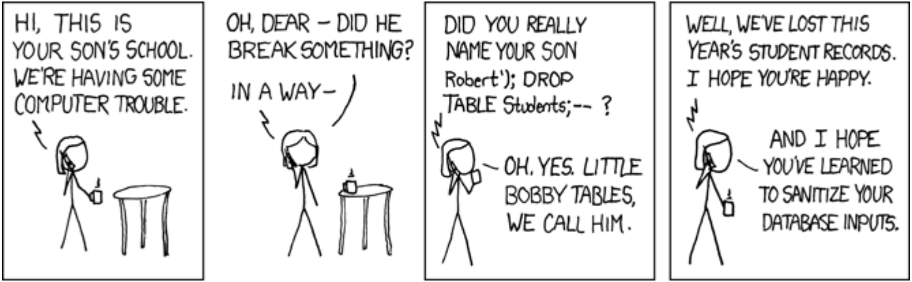

Figure: Exploits of a mom (Retrieved from `https://xkcd.com/327/`)

字符拼接一时爽，删库脱库火葬场!

Caching fetched data

Some DBMS can even cache query result. (Or you can consider other solutions like Memcached.)

Useful if

1. High-read
2. Low-write

Contents

Performance consideration beyond a single query

- Overhead outside the DBMS
  - TCP/IP protocol stack overhead
  - Context switch
- Multiple small operations vs. Batch operation
- Hardware consideration

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