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Section 3.2

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P76 Q3

这是一个多项分布。令玩家一，玩家二，玩家三赢得游戏的次数分别为 x_1，x_2，x_3，则每个人赢得游戏次数的联合频率函数为：

$$P\{X_1 = x_1, X_2 = x_2, X_3 = x_3\} = \begin{cases} \frac{10!}{3^{10} \cdot x_1! x_2! x_3!} & x_1 + x_2 + x_3 = 10 \\ 0 & \text{otherwise} \end{cases}$$

补充 1

(X, Y) 的频率函数可由下表表示：

X \ Y	1	3
0	0	$\frac{1}{8}$
1	$\frac{3}{8}$	0
2	$\frac{3}{8}$	0
3	0	$\frac{1}{8}$

补充 2

(X, Y) 的联合频率函数及边缘频率函数可由下表表示：

X \ Y	0	1	$P_X(x)$
-1	0	$\frac{1}{3}$	$\frac{1}{3}$
0	$\frac{1}{3}$	0	$\frac{1}{3}$
1	0	$\frac{1}{3}$	$\frac{1}{3}$
$P_Y(y)$	$\frac{1}{3}$	$\frac{2}{3}$	1

补充 3

二维随机变量 (X_1, X_2) 的频率函数可由下式表示：

$$P\{X_1 = x_1, X_2 = x_2\} = P\{X_1 = x_1 | X_2 = x_2\} \cdot P\{X_2 = x_2\}$$

故有：

$$\begin{aligned}
P\{X_1 = 0, X_2 = 0\} =& P\{X_1 = 0 | X_2 = 0\} \cdot P\{X_2 = 0\} \\
=& P\{Y \leqslant 1 | Y \leqslant 2\} \cdot P\{Y \leqslant 2\} \\
=& \frac{F(1)}{F(2)} \cdot F(2) \\
=& F(1) \\
=& 1 - e^{-1}
\end{aligned}$$

$$\begin{aligned}
P\{X_1 = 0, X_2 = 1\} =& P\{X_1 = 0 | X_2 = 1\} \cdot P\{X_2 = 1\} \\
=& P\{Y \leqslant 1 | Y > 2\} \cdot P\{Y > 2\} \\
=& \frac{0}{1 - F(2)} \cdot (1 - F(2)) \\
=& 0
\end{aligned}$$

$$\begin{aligned}
P\{X_1 = 1, X_2 = 0\} =& P\{X_1 = 1 | X_2 = 0\} \cdot P\{X_2 = 0\} \\
=& P\{Y > 1 | Y \leqslant 2\} \cdot P\{Y \leqslant 2\} \\
=& \frac{F(2) - F(1)}{F(2)} \cdot F(2) \\
=& F(2) - F(1) \\
=& 1 - e^{-2} - (1 - e^{-1}) \\
=& e^{-1} - e^{-2}
\end{aligned}$$

$$\begin{aligned}
P\{X_1 = 1, X_2 = 1\} =& P\{X_1 = 1 | X_2 = 1\} \cdot P\{X_2 = 1\} \\
=& P\{Y > 1 | Y > 2\} \cdot P\{Y > 2\} \\
=& \frac{1 - F(2)}{1 - F(2)} \cdot (1 - F(2)) \\
=& 1 - F(2) \\
=& e^{-2}
\end{aligned}$$

综上，(X_1, X_2) 的联合频率函数及边缘频率函数可由下表表示：

X_1 \ X_2	0	1	$P_{X_1}(x_1)$
0	$1-e^{-1}$	0	$1-e^{-1}$
1	$e^{-1}-e^{-2}$	e^{-2}	e^{-1}
$P_{X_2}(x_2)$	$1-e^{-2}$	e^{-2}	1