# Theory Assignment 1 Answer

## 1.

### a)

| Integer part | Quotient | Remainder | Coefficient |
|---|---|---|---|
| $234/3 =$ | 78 | 0 | $a_0 = 0$ |
| $78/3 =$ | 26 | 0 | $a_1 = 0$ |
| $26/3 =$ | 8 | 2 | $a_2 = 2$ |
| $8/3 =$ | 2 | 2 | $a_3 = 2$ |
| $2/3 =$ | 0 | 2 | $a_4 = 2$ |

| Fraction part | Integer | Fraction | Coefficient |
|---|---|---|---|
| $0.5 \times 3 =$ | 1 | 0.5 | $a_{-1} = 1$ |
| $0.5 \times 3 =$ | 1 | 0.5 | $a_{-2} = 1$ |
| $0.5 \times 3 =$ | 1 | 0.5 | $a_{-3} = 1$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |

Therefore, $\boxed{(234.5)_{10} \approx (22200.11)_3}$.

### b)

| Integer part | Quotient | Remainder | Coefficient |
|---|---|---|---|
| $234/12 =$ | 19 | 6 | $a_0 = 6$ |
| $19/12 =$ | 1 | 7 | $a_1 = 7$ |
| $1/12 =$ | 0 | 1 | $a_2 = 1$ |

| Fraction part | Integer | Fraction | Coefficient |
|---|---|---|---|
| $0.5 \times 12 =$ | 6 | 0.0 | $a_{-1} = 6$ |

Therefore, $\boxed{(234.5)_{10} = (176.6)_{12}}$.

### c)

$$\begin{aligned} D = &= 4 \times 6^2 + 3 \times 6^1 + 5 \times 6^0 \\ &= 144 + 18 + 5 \\ &= 167 \end{aligned}$$

Therefore, $\boxed{(435)_6 = (167)_{10}}$.

### d)

| Radix r | Integer | | . | Fraction | |
|---|---|---|---|---|---|
| 2 | $\underbrace{0\ 1\ 0}$ | $\underbrace{1\ 1\ 0}$ | . | $\underbrace{0\ 1\ 0}$ | $\underbrace{1\ 0\ 0}$ |
| 8 | 2 | 6 | . | 2 | 4 |

Therefore, $\boxed{(10110.0101)_2 = (26.24)_8}$.

# 2.

## a)

Since all numbers are smaller than 7 and no carry is needed, the operation is correct in any number system that radix $\boxed{r \geq 7}$.

## b)

Assuming the operation is in base $r$, we first convert the operation into base 10:

$$\begin{aligned}
LHS &= (302)_r/(20)_r \\
&= (3r^2 + 0r^1 + 2r^0)_{10}/(2r^1 + 0r^0)_{10} \\
&= (\frac{3r^2+2}{2r})_{10} \\
RHS &= (12.1)_r \\
&= (1r^1 + 2r^0 + 1r^{-1})_{10} \\
&= (\frac{r^2+2r+1}{r})_{10}
\end{aligned}$$

Since $LHS = RHS$, we have:

$$\begin{aligned}
\frac{3r^2+2}{2r} &= \frac{r^2+2r+1}{r} \\
3r^2 + 2 &= 2r^2 + 4r + 2 \\
r^2 - 4r &= 0
\end{aligned}$$

Therefore, $r = 0$ or $r = 4$. Since $r \neq 0$, we have $\boxed{r = 4}$.

# 3.

## a)

$$\begin{aligned}
(a'+c)(a'+c')(a+b+c'd) &= (a'+cc')(a+b+c'd) \\
&= a'(a+b+c'd) \\
&= a'a + a'b + a'c'd \\
&= a'b + a'c'd \\
&= \boxed{a'(b+c'd)}
\end{aligned}$$

## b)

$$\begin{aligned}
abc'd + a'bd + abcd &= (abc' + a'b + abc)d \\
&= (a'b + ab(c+c'))d \\
&= (a'b + ab)d \\
&= (a'+a)bd \\
&= \boxed{bd}
\end{aligned}$$

# 4.

## a)

$$\begin{aligned}(a+c)(a'+b+c)(a'+b'+c) &= (a+c)(a'+c+bb') \\ &= (a+c)(a'+c) \\ &= c+aa' \\ &= \boxed{c}\end{aligned}$$

## b)

$$\begin{aligned}F(a,b,c) &= \sum(0,1,2,3,5) \\ &= a'b'c' + a'b'c + a'bc' + a'bc + ab'c \\ &= a'b'(c+c') + a'b(c+c') + ab'c \\ &= a'b' + a'b + ab'c \\ &= a'(b+b') + ab'c \\ &= a' + ab'c \\ &= a'(1+b'c) + ab'c \\ &= a' + a'b'c + ab'c \\ &= a' + (a+a')b'c \\ &= \boxed{a' + b'c}\end{aligned}$$

# 5.

## a)

$$\begin{aligned}F(a,b,c,d) &= bd' + acd' + ab'c + a'c' \\ &= (a+a')b(c+c')d' + a(b+b')cd' + ab'c(d+d') + a'(b+b')c'(d+d') \\ &= abcd' + a'bcd' + abc'd' + a'bc'd' + ab'cd' + ab'cd + a'bc'd + a'b'c'd + a'b'c'd' \\ &= \boxed{\sum(0,1,4,5,6,10,11,12,14)} \\ &= \boxed{\prod(2,3,7,8,9,13,15)}\end{aligned}$$

## b)

$$
\begin{aligned}
F(x,y,z) &= (x'+z)(y+x') \\
&= x'+yz \\
&= x'(y+y')(z+z') + (x+x')yz \\
&= x'yz + x'y'z + x'yz' + x'y'z' + xyz \\
&= \boxed{\sum(0,1,2,3,7)} \\
&= \boxed{\prod(4,5,6)}
\end{aligned}
$$

# 6.

## a)

$$
\begin{aligned}
F_1(A,B,C) &= \sum(2,3,7) \\
&= A'BC' + A'BC + ABC \\
&= A'B(C+C') + ABC \\
&= A'B + ABC \\
&= A'B(1+C) + ABC \\
&= A'B + A'BC + ABC \\
&= A'B + (A+A')BC \\
&= A'B + BC \\
&= \boxed{B(A'+C)}
\end{aligned}
$$

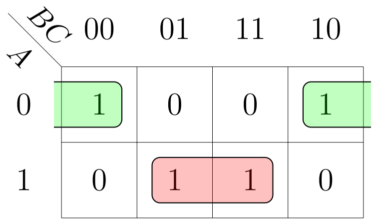
$$
\begin{aligned}
F_2(A,B,C) &= \sum(0,2,5,7) \\
&= A'B'C' + A'BC' + AB'C + ABC \\
&= A'(B+B')C' + A(B+B')C \\
&= A'C' + AC \\
&= \boxed{(A \oplus C)'}
\end{aligned}
$$

## b)

| A \ BC | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 |
| 1 | 0 | 0 | 1 | 0 |

Hence, the sum of product terms for $F_1$ is:

$$\boxed{F_1(A,B,C) = A'B + BC}$$

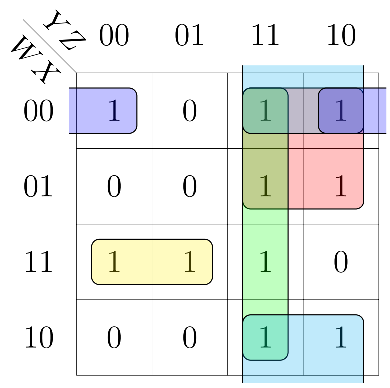



Hence, the sum of product terms for $F_2$ is:

$$\boxed{F_2(A,B,C) = AC + A'C'}$$

# 7.

## a)




Hence, the sum of product terms is:

$$\boxed{F(W,X,Y,Z) = W'Y + WXY' + W'X'Z' + X'Y + YZ}$$

## b)

| AB \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | 0 | 0 | 1 |
| 01 | 0 | 0 | 0 | 0 |
| 11 | 0 | 0 | 1 | 0 |
| 10 | 1 | 0 | 1 | 1 |

Hence, the sum of product terms is:

$$\boxed{F(A,B,C,D) = ACD + B'D'}$$

## 8.

We can use the following karnaugh map to find the truth table of both functions:

| AB \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 1 | 0 | 1 |
| 01 | 0 | 1 | 0 | 1 |
| 11 | 1 | 1 | 0 | 1 |
| 10 | 0 | 1 | 0 | 1 |

$$f = abd' + c'd + a'cd' + b'cd'$$

| AB \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | 0 | 0 | 1 |
| 01 | 1 | 1 | 1 | 0 |
| 11 | 1 | 0 | 1 | 0 |
| 10 | 1 | 0 | 1 | 1 |

$$g' = a'b'd + bcd' + ac'd$$

Since $F = fg$, we simply find the common minterms to be the minterms of $F$, and find the simplest sum of product terms using the following karnaugh map:

| AB \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 0 | 0 | 1 |
| 01 | 0 | 1 | 0 | 0 |
| 11 | 1 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 | 1 |

Therefore, the sum of product terms for $F$ is:

$$\boxed{F = abc'd' + a'bc'd + b'cd'}$$

# 9.

## a)

| AB \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | x | 1 | x | 1 |
| 01 | 1 | x | 1 | 0 |
| 11 | 0 | 0 | 0 | 0 |
| 10 | 1 | 1 | 1 | 0 |

The simplest sum of product terms is:

$$F(A,B,C,D) = A'B' + A'C' + A'D + B'C' + B'D$$

To implement $F$ using only NAND gates, we convert $F$ into a NAND form:

$$\begin{aligned} F(A,B,C,D) &= A'B' + A'C' + A'D + B'C' + B'D \\ &= ((A'B' + A'C' + A'D + B'C' + B'D)')' \\ &= \boxed{((A'B')'(A'C')'(A'D)'(B'C')'(B'D)')'} \end{aligned}$$

## b)

| AB \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | x | 1 | x | 1 |
| 01 | 1 | x | 1 | 0 |
| 11 | 0 | 0 | 0 | 0 |
| 10 | 1 | 1 | 1 | 0 |

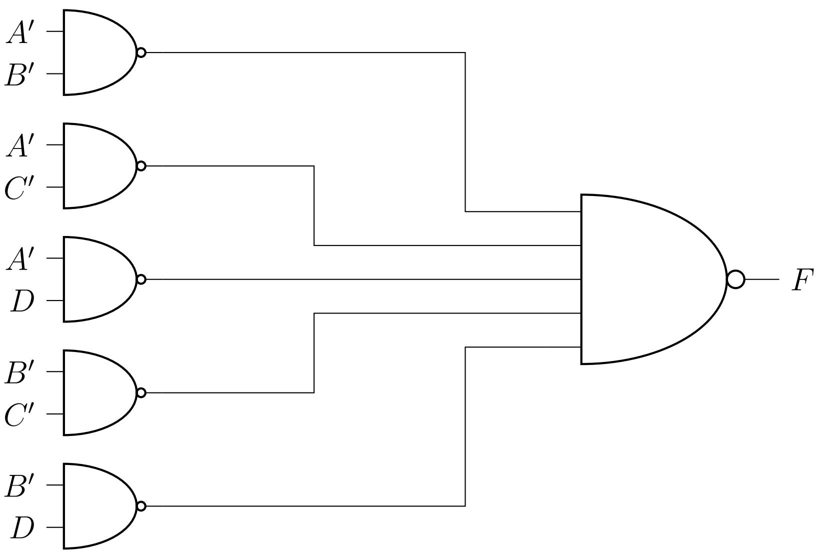
The simplest product of sum terms is:

$$
\begin{aligned}
F'(A,B,C,D) &= AB + ACD' + BCD' \\
F(A,B,C,D) &= (AB + ACD' + BCD')' \\
&= (AB)'(ACD')'(BCD')' \\
&= (A'+B')(A'+C'+D)(B'+C'+D)
\end{aligned}
$$

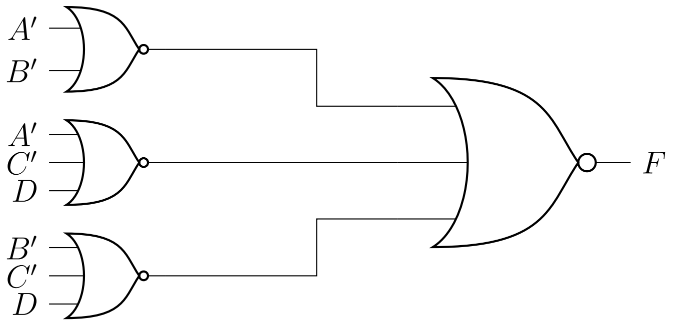
To implement $F$ using only NOR gates, we convert $F$ into a NOR form:

$$
\begin{aligned}
F(A,B,C,D) &= (A'+B')(A'+C'+D)(B'+C'+D) \\
&= (((A'+B')(A'+C'+D)(B'+C'+D))')' \\
&= \boxed{((A'+B')'+(A'+C'+D)'+(B'+C'+D)')'}
\end{aligned}
$$

## c)




NAND implementation




NOR implementation