Data Structure and Algorithm Analysis(H)
Southern University of Science and Technology
Mengxuan Wu
12212006

Work Sheet 12

Mengxuan Wu

Question 12.1

1.

No.

Consider $a_1 = [0, 3)$, $a_2 = [2, 4)$, $a_3 = [3, 6)$. To choose the activity of the least duration, we should choose a_2 and stop. But the optimal solution is to choose a_1 and a_3.

2.

No.

Consider $a_1 = [0, 2)$, $a_2 = [2, 4)$, $a_3 = [4, 6)$, $a_4 = [6, 8)$ (first 4 are optimal solution), $a_5 = [3, 5)$, $a_6 = a_7 = [1, 3)$, $a_8 = a_9 = [5, 7)$. Then a_5 is the greedy choice because it only overlaps twice, but choosing a_5 will lead to a solution of 3 activities, while the optimal solution is 4 activities.

3.

Yes.

Proof.

Let set S_k be the set of activities that finish before a_k starts, and A_k be the optimal solution of S_k. If a_m is the last-to-start activity in A_k, and a_n is the last-to-start activity in A_{k+1}, then if $a_m \neq a_n$, we can replace a_m with a_n and still get a compatible solution with same number of activities. Hence, a_n is in one of maximum-size subset of mutually compatible activities. □

4.

No.

Consider $a_1 = [0, 8)$, $a_2 = [1, 2)$, $a_3 = [2, 3)$. Then a_1 is the greedy choice, but the optimal solution is to choose a_2 and a_3.

Question 12.2

Without loss of generality, we assume that the items in knapsack are sorted by their value per unit weight in decreasing order. Then we can proof that in a fractional knapsack problem, the greedy choice is to choose the item with the largest value per unit weight.

Proof.

Let K_i be the knapsack after the ith item is added, and A_i be the optimal solution of K_i. Then if the $i+2$th item is added to K_i before all fraction of the $i+1$th item is added, then we can replace the $i+2$th item with the $i+1$th item and get a better solution. Hence, the greedy choice is to choose the item with the largest value per unit weight. □

Question 12.3

(a)

The greedy solution is to find the farthest point that Eddy can reach, and stop for supply there.

(b)

Proof.

Let S_k be the set of supply points that Eddy can reach after station k, and A_k be the optimal solution of starting from station k. If s_m is the farthest supply point Eddy can reach in he starts from station k, then if the first stop in A_k is not s_m, the subproblem (distances after the first stop) will be longer than the subproblem that chooses s_m, and will produce a worse or equal solution. Hence, s_m is in one of A_k. □