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Section 4.3

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P119 Q54

$$\begin{aligned}
Cov(U,V) =& Cov(Z+X, Z+Y) \\
=& Cov(Z,Z) + Cov(Z,Y) + Cov(X,Z) + Cov(X,Y) \\
=& Cov(Z,Z) \\
=& \sigma_Z^2
\end{aligned}$$

$$\begin{aligned}
\rho_{UV} =& \frac{Cov(U,V)}{\sqrt{Var(U)Var(V)}} \\
=& \frac{\sigma_Z^2}{\sqrt{(\sigma_X^2+\sigma_Z^2)(\sigma_Y^2+\sigma_Z^2)}}
\end{aligned}$$

P119 Q60

$$\begin{aligned}
Cov(X,Y) =& Cov(SY, Y) \\
=& E(SY^2) - E(SY)E(Y) \\
=& 0 - 0E(Y) \\
=& 0
\end{aligned}$$

$$
\begin{aligned}
F_X(x) =& P\{X \leqslant x\} \\
=& P\{SY \leqslant x\} \\
=& \frac{1}{2}P\{Y \leqslant x\} + \frac{1}{2}P\{Y \geqslant -x\} \\
=& \frac{1}{2}F_Y(x) + \frac{1}{2}[1 - F_Y(-x)] \\
=& \frac{1}{2}F_Y(x) - \frac{1}{2}F_Y(-x) + \frac{1}{2}
\end{aligned}
$$

可得 f_X 为

$$
\begin{aligned}
f_X(x) =& \frac{1}{2}f_Y(x) + \frac{1}{2}f_Y(-x) \\
=& \frac{1}{2}f_Y(x) + \frac{1}{2}f_Y(x) \\
=& f_Y(x)
\end{aligned}
$$

故 $f_X(1 \cdot y) = f_Y(y) \neq \frac{1}{2}f_Y(y) = f_S(1)f_Y(y)$，故 X 与 Y 不独立。

补充 1

(1)

$$
\begin{aligned}
E(X) =& \int_{-\infty}^{+\infty} x f_X(x) \mathrm{d}x \\
=& \int_{-\infty}^{+\infty} x \frac{1}{2} e^{-|x|} \mathrm{d}x \\
=& \frac{1}{2}\int_{-\infty}^{0} x e^{x} \mathrm{d}x + \frac{1}{2}\int_{0}^{+\infty} x e^{-x} \mathrm{d}x \\
=& \frac{1}{2}(x-1)e^{x}\Big|_{-\infty}^{0} + \frac{1}{2}(-x-1)e^{-x}\Big|_{0}^{+\infty} \\
=& -\frac{1}{2} + \frac{1}{2} \\
=& 0
\end{aligned}
$$

$$\begin{aligned}
D(X) =&E(X^2)-[E(X)]^2 \\
=&E(X^2) \\
=&\int_{-\infty}^{+\infty} x^2 f_X(x)\mathrm{d}x \\
=&\int_{-\infty}^{+\infty} x^2 \frac{1}{2}e^{-|x|}\mathrm{d}x \\
=&\frac{1}{2}\int_{-\infty}^{0} x^2 e^{x}\mathrm{d}x+\frac{1}{2}\int_{0}^{+\infty} x^2 e^{-x}\mathrm{d}x \\
=&\frac{1}{2}(x^2-2x+2)e^x\Big|_{-\infty}^{0}+\frac{1}{2}(-x^2-2x-2)e^{-x}\Big|_{0}^{+\infty} \\
=&1+1 \\
=&2
\end{aligned}$$

(2)

当 $x \geqslant 0$ 时，有

$$\begin{aligned}
f_{X\big||X|}(x) =&\frac{P\{X=x,|X|=x\}}{P\{|X|=x\}} \\
=&1 \\
\neq&\frac{1}{2}e^{-x}
\end{aligned}$$

因此 X 与 $|X|$ 不独立。

(3)

$$\begin{aligned}
Cov(X,|X|) =&E(X|X|)-E(X)E(|X|) \\
=&E(X|X|) \\
=&\int_{-\infty}^{+\infty} x|x|f_X(x)\mathrm{d}x \\
=&\int_{-\infty}^{0} -x^2\frac{1}{2}e^{x}\mathrm{d}x+\int_{0}^{+\infty} x^2\frac{1}{2}e^{-x}\mathrm{d}x \\
=&-1+1 \\
=&0
\end{aligned}$$

因此 X 与 $|X|$ 不相关。

补充 2

$$\begin{aligned}E(X) &= \int_0^2 \int_0^2 x f_{XY}(x,y)\mathrm{d}x\mathrm{d}y \\ &= \int_0^2 \int_0^2 x \frac{x+y}{8}\mathrm{d}x\mathrm{d}y \\ &= \frac{7}{6}\end{aligned}$$

$$\begin{aligned}E(Y) &= \int_0^2 \int_0^2 y f_{XY}(x,y)\mathrm{d}x\mathrm{d}y \\ &= \int_0^2 \int_0^2 y \frac{x+y}{8}\mathrm{d}x\mathrm{d}y \\ &= \frac{7}{6}\end{aligned}$$

$$\begin{aligned}Cov(X,Y) &= E(XY) - E(X)E(Y) \\ &= \int_0^2 \int_0^2 xy f_{XY}(x,y)\mathrm{d}x\mathrm{d}y - \frac{7}{6}\cdot\frac{7}{6} \\ &= \frac{4}{3} - \frac{7}{6}\cdot\frac{7}{6} \\ &= -\frac{1}{36}\end{aligned}$$

$$\begin{aligned}\rho_{XY} &= \frac{Cov(X,Y)}{\sqrt{Var(X)Var(Y)}} \\ &= \frac{-\frac{1}{36}}{\sqrt{(E(X^2)-[E(X)]^2)(E(Y^2)-[E(Y)]^2)}} \\ &= \frac{-\frac{1}{36}}{\sqrt{(\frac{5}{3}-\frac{7}{6}\cdot\frac{7}{6})^2}} \\ &= -\frac{1}{11}\end{aligned}$$

$$\begin{aligned}D(X+Y) &= D(X) + D(Y) + 2Cov(X,Y) \\ &= \frac{11}{36} + \frac{11}{36} + 2\times(-\frac{1}{36}) \\ &= \frac{5}{9}\end{aligned}$$

补充 3

$$
\begin{aligned}
Cov(Z,W) =& Cov(\alpha X+\beta Y, \alpha X-\beta Y)\\
=& \alpha^2 Cov(X,X)-\beta^2 Cov(Y,Y)\\
=& \alpha^2\sigma^2-\beta^2\sigma^2\\
=& \sigma^2(\alpha^2-\beta^2)
\end{aligned}
$$

$$
\begin{aligned}
\rho_{ZW} =& \frac{Cov(Z,W)}{\sqrt{Var(Z)Var(W)}}\\
=& \frac{\sigma^2(\alpha^2-\beta^2)}{\sqrt{(\alpha^2+\beta^2)\sigma^2(\alpha^2+\beta^2)\sigma^2}}\\
=& \frac{\alpha^2-\beta^2}{\alpha^2+\beta^2}
\end{aligned}
$$