

1. The sample space of this algorithm is all possible permutations of the list
 l. The probabilities of this sample space only vary with the position of n
 because the orders of all the other elements are equally likely.

$$\text{Prob}[n \text{ is in } i^{\text{th}} \text{ position}] = \frac{1}{2^i}$$

$$2. \text{Prob}[l \text{ is not in } i_1 \dots i_m] = \begin{cases} \frac{1}{2} & l = n \\ (1 - \frac{1}{2n})^m & l \neq n \end{cases}$$

If n does not appear in i then that is the probability of not choosing n, m times. Similarly the probability of l not appearing in i is 1- the probability of choosing l, m times.

$$3. P_l = \frac{1}{2} + \frac{1}{2}(1 - \frac{1}{2n})\frac{1}{2}(1 - \frac{1}{2n})^2 + \dots + \frac{1}{2}(1 - \frac{1}{2n})^{m-1}$$

$$= \frac{1}{2} \sum_{i=1}^m \frac{1}{2}(1 - \frac{1}{2n})^{i-1} = n(1 - (1 - \frac{1}{2n})^m)$$

The sum comes from the chance of n being picked at each i over our element not being picked. So it is the probability that n was picked multiplied by the probability l was not picked before hand summed over each possible position of n.

$$4. E[B_l] = 1 \cdot \text{prob}[l \text{ is before } n] + 0 \cdot \text{prob}[n \text{ is before } l] = 1 - P_l = 1 - n(1 - (1 - \frac{1}{2n})^m)$$

$$5. E[T] = \sum_{l=0}^{n-1} E[B_l]$$

$$= n(1 - n(1 - (1 - \frac{1}{2n})^m)) = n - n^2 + n^2(1 - \frac{1}{2n})^m$$